Math 308: Midterm 2

by William Stein

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Instructions:

- Your Name: ____
  ID Number: ____________________
- You may have one piece of paper with notes, plus scratch paper.
- You may use a calculator (but shouldn't need one).
- Be careful. Triple check your answer to every question. Make sure you understand exactly what is being asked.
- All questions are true or false and worth 5 points (for a total of 100 points).

Questions (circle T or F):

1. **T** **F** The function $f : \mathbb{R}^2 \to \mathbb{R}^2$ given by $f(x, y) = (-2y, 3x + 1)$ is a linear transformation.

2. **T** **F** Suppose that a matrix A has characteristic polynomial $f(x) = x \cdot (x^2 - 15x - 18)$. Then A has exactly 2 distinct real eigenvalues.

3. **T** **F** If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is any linear transformation, then the set of vectors v such that $Tv = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ is a subspace of $\mathbb{R}^2$.

4. **T** **F** There is a basis for $\mathbb{R}^2$ consisting of eigenvectors for $A = \begin{pmatrix} 1 & 2013 \\ 0 & 1 \end{pmatrix}$.

5. **T** **F** The characteristic polynomial of $A = \begin{pmatrix} -1 & 0 & 2 \\ -1 & -3 & -1 \\ -3 & -1 & 1 \end{pmatrix}$ is
$$f(X) = (X - 3) \cdot (X^2 + 4).$$

6. **T** **F** Suppose A and B are $n \times n$ matrices with $|A| = 5$ and $|B| = 7$. Then
$$|A^2 B A^{-1} B^{-1} A^{-1}| = 1.$$

7. **T** **F** If A is a symmetric matrix, then $|A| = 1$.

8. **T** **F** Suppose A is an $n \times n$ matrix with $n \geq 1$. If the equation $Ax = 0$ has a unique solution, then $|A| = 0$.

9. **T** **F** The function $T : \mathbb{R}^2 \to \mathbb{R}^5$ given by $f(x, y) = (x, x + y, x - y, 0, y)$ is a linear transformation.

10. **T** **F** We have $|A| = 112$, where $A = \begin{pmatrix} 1 & -4 & 2 & 1 & -2 \\ 0 & -7 & 2 & 2 & 1 \\ 0 & 0 & -2 & 7 & 1 \\ 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$.

11. **T** **F** We have $|A^T| = -112$, where A is from the previous problem.

12. **T** **F** Let T be the linear transformation of $\mathbb{R}^2$ that rotates a vector by 90 degrees clockwise around the origin. Then the matrix corresponding to T is $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

13. **T** **F** The matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ has no real eigenvalues.

14. **T** **F** If S and T are both linear transformstions $\mathbb{R}^3 \to \mathbb{R}^3$, then

$$f(v) = 2S(v) - 3T(v)$$

is necessarily also a linear transformation.

15. **T** **F** If A is an $n \times n$ matrix of integers, then the determinant of A must necessarily also be an integer.

16. **T** **F** If A is any $n \times n$ matrix and B is the reduced row echelon form of A, then $|A| = |B|$.

17. **T** **F** If A is any 2×2 matrix and k any scalar, then $|kA| = k^4 \cdot |A|$.

18. **T** **F** If v is an eigenvector with eigenvalue λ of a matrix A, then

$$A^{2013}v = \lambda^{2013}v.$$

19. **T** **F** There are infinitely many linear transformations $T : \mathbb{R}^2 \to \mathbb{R}^2$.

20. **T** **F** Suppose A is any invertible 2×2 matrix with the two distinct eigenvalues -1 and $+1$. Then we must have $A^2 = I_2$, where I_2 is the 2×2 identity matrix.