Math 308: Midterm 1 Solutions

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Instructions:

- Your Name: ______________________________ ID Number: _______________
- You may have one piece of paper on which you put notes, plus scratch paper.
- You may use a calculator, though it will likely be of no use, since there is a table of reduced row echelon forms at the end of the exam.
- In every question, assume that you are working over the real numbers (no complex numbers, no numbers modulo p).
- Be careful. If you can think of any possible way to double check your work do so. Try to get a perfect score!
- All questions are true or false and worth 4 points (for a total of 100).

Questions (circle T or F):

1. **T** There is a system of linear equations that has exactly one solution.
2. **F** There is a system of linear equations that has exactly two solutions.
3. **T** There is a system of linear equations that has infinitely many solutions.
4. **T** The vector (π, e) is a linear combination of the vectors $(1, 2)$ and $(3, 4)$ in $\mathbb{R}^2$.
5. **T** The vector $(9, 8, 7)$ is a linear combination of the vectors $(1, 2, 3)$ and $(4, 5, 6)$.
6. **F** The vectors $(9, 10, 11, 12)$, $(1, 2, 3, 4)$ and $(5, 6, 7, 8)$ in $\mathbb{R}^4$ are linearly independent.
7. **F** We have $\mathrm{span}((10, 20), (20, 10)) = \mathrm{span}((1, 2))$.
8. **F** We have $\mathrm{span}((1, 2, 3, 4)) = \mathrm{span}((4, 3, 2, 1))$.
9. **T** We have $\mathrm{span}((1, 2), (3, 4)) = \mathrm{span}((0, 1), (1, 0))$.
10. **F** Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$. Then $\{x : Ax = 0\} = \mathrm{span}((1, 2, 1))$.
11. **F** Any two vectors in $\mathbb{R}^4$ are linearly independent.

12. **T** Any four vectors in $\mathbb{R}^3$ are linearly dependent.

13. **F** There are exactly three subspaces of $\mathbb{R}^2$.

14. **T** If A is a square $n \times n$ matrix, then $A + A^T$ is necessarily symmetric.

15. **T** If A and B are 2×2 matrices, then $(A + B)(B + A)^T$ is necessarily symmetric.

16. **T** If A is symmetric, then A^T is necessarily also symmetric.

17. **F** Suppose A is an $n \times r$ matrix (i.e., has n rows and r columns), that B is an $r \times m$ matrix, and that C is an $m \times s$ matrix. Then ABC is defined and is an $s \times n$ matrix.

18. **T** Suppose A, B, C are all 3×3 matrices. Then $AB + (B + C) = C + (B + AB)$?

19. **F** If $A + I = \begin{pmatrix} 2 & 2 \\ 2 & 5 \end{pmatrix}$, then there is exactly one x that such that $Ax = \begin{pmatrix} 2013 \\ 2014 \end{pmatrix}$.

20. **F** The empty set is a subspace of $\mathbb{R}^2$.

21. **T** The set of points on the line defined by $y = 2x$ is a subspace of $\mathbb{R}^2$.

22. **F** The set of points on the line defined by $y = 2x + 1$ is a subspace of $\mathbb{R}^2$.

23. **F** The set of solutions to the equation $x^2 + y^2 = 1$ (a circle) is a subspace of $\mathbb{R}^2$.

24. **T** The set of pairs $\{(x, y) : x \geq 0, y \geq 0\}$, where both x and y are nonnegative (i.e., the first quadrant), is a subset of $\mathbb{R}^2$.

25. **T** If S_1, S_2, S_3 are subspaces of $\mathbb{R}^4$, then the intersection $S_1 \cap S_2 \cap S_3$ is also a subspace of $\mathbb{R}^4$.

Some Echelon Forms:

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 10 & 20 \\ 20 & 10 \\ 1 & 2 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 9 & 8 & 7 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$