

Math 308: Last Homework Assignment

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Instructions:

- Due on midnight, Wednesday, June 5 via this page: <http://goo.gl/rd8zi>
- The final exam will be similar to this, but longer.

Questions (circle T or F):

1. **T F** A system of linear equations over \mathbf{R} has either no solutions or infinitely many solutions.
2. **T F** The span of the rows of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ has dimension 3.
3. **T F** There are 4 linearly independent vectors in \mathbf{R}^5 .
4. **T F** The upper half plane, i.e., the set of points $(x, y) \in \mathbf{R}^2$ with $y \geq 0$, is a subspace of \mathbf{R}^2 .
5. **T F** If A and B are any two square matrices (of the same size), then $\det(AB) = \det(BA)$ and $\det(A + B) = \det(B + A)$.
6. **T F** Let \mathbf{F}_3 be the finite field with 3 elements and let V be a 2-dimensional vector space over \mathbf{F}_3 . Then there are 81 linear transformations $V \rightarrow V$.
7. **T F** If A is any $n \times n$ matrix and B is the reduced row echelon form of A , then $\det(A) = \det(B^t)$, where B^t is the transpose of B .
8. **T F** Suppose A is a 3×3 matrix with eigenvalues 1, 2, 3. Then A must be diagonalizable.
9. **T F** Suppose A is a 3×3 matrix with eigenvalues 1 and -1 . Then $A^2 = I_3$.
10. **T F** Let \mathcal{P} be the vector space over \mathbf{R} of polynomials of degree at most 3 with real coefficients, and let $T : \mathcal{P} \rightarrow \mathcal{P}$ be the linear transformation $T(ax^2 + bx + c) = a + b + c$. Then there is a basis \mathcal{B} for \mathcal{P} such that $[T]_{\mathcal{P}, \mathcal{P}}$ is diagonal.
11. **T F** Let \mathcal{P} be the vector space over \mathbf{R} of polynomials of degree at most 3 with real coefficients, and let $T : \mathcal{P} \rightarrow \mathbf{R}^1$ be the linear transformation $T(ax^2 + bx + c) = a + b + c$. Then the kernel of T (i.e., the set of v with $T(v) = 0$) has dimension 1.
12. **T F** Suppose V and W are vector spaces over \mathbf{R} of dimensions 2 and 3, respectively. Then there is a linear transformation $T : V \rightarrow W$ such that $\ker(T) = 0$.
13. **T F** Suppose V and W are vector spaces over \mathbf{R} of dimensions 3 and 2, respectively. Then there is a linear transformation $T : V \rightarrow W$ such that $\ker(T) = 0$.
14. **T F** Every 2×2 matrix A is diagonalizable (over the complex numbers \mathbf{C}).

15. **T F** The matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ is diagonalizable over \mathbf{C} .
16. **T F** Let V be the vector space of all differentiable functions $\mathbf{R} \rightarrow \mathbf{R}$. The function that send f to its derivative is a linear transformation $V \rightarrow V$.
17. **T F** The dimension of the kernel of the derivative transformation (defined in the previous problem) is 1.
18. **T F** Let V be the set of all integrable functions $\mathbf{R} \rightarrow \mathbf{R}$, i.e., functions that have some antiderivative. Then V is a vector space.
19. **T F** Let V be the set of all integrable functions $\mathbf{R} \rightarrow \mathbf{R}$. The function that f to “the antiderivative of f that sends 0 to 0” is a linear transformation of V .
20. **T F** Let V be the set of all integrable functions $\mathbf{R} \rightarrow \mathbf{R}$. The function that f to “the antiderivative of f that sends 0 to 1” is a linear transformation of V .
21. **T F** The determinant of $A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{pmatrix}$ is 0.
22. **T F** If B and C are basis for a (finite dimensional) vector space V , then there must be some linear transformation $T : V \rightarrow V$ such that $[T]_{B,C}$ is the identity matrix.
23. **T F** Let V be the set of functions $\mathbf{R} \rightarrow \mathbf{R}$. Then the function $V \rightarrow \mathbf{R}^2$ that sends f to $(f(1), f(\pi))$ is a linear transformation.
24. **T F** Let V be the vector space over \mathbf{R} that is the span of $\sin(x)$, $\cos(x)$, and $\cos(3x)$. Let $T : V \rightarrow \mathbf{R}^1$ be the linear transformation that sends $f \in V$ to $f(0)$. Then the kernel of T has dimension 1.
25. **T F** Let V be the vector space over \mathbf{R} that is the span of $\sin(x)$, $\cos(x)$, and $\cos(3x)$. Let $T : V \rightarrow \mathbf{R}^1$ be the linear transformation that sends $f \in V$ to $f(0)$. Then T is surjective, i.e., for every $a \in \mathbf{R}^1$ there is $v \in V$ such that $T(v) = a$.
26. **T F** Suppose A is a matrix with characteristic polynomial $x(x-1)(x-2)(x-3)$. Then there is a basis \mathcal{B} of \mathbf{R}^4 such that $[A]_{\mathcal{B},\mathcal{B}} = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$.
27. **T F** Suppose A has characteristic polynomial $x^7 + x^3 - 3$. Then $\det(A) \neq 0$.