# Math 308: Midterm 1

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## Instructions:

- Your Name: _____ ID Number: _____

- You may have one piece of paper on which you put notes, plus scratch paper.

- You may use a calculator, though it will likely be of no use, since there is a table of reduced row echelon forms at the end of the exam.

- In every question, assume that you are working over the real numbers (no complex numbers, no numbers modulo $p$).

- Be careful. If you can think of any possible way to double check your work do so. Try to get a perfect score!

- All questions are true or false and worth 4 points (for a total of 100).

## Questions (circle T or F):

1.   **T**   **F**   There is a system of linear equations that has exactly one solution.

2.   **T**   **F**   There is a system of linear equations that has exactly two solutions.

3.   **T**   **F**   There is a system of linear equations that has infinitely many solutions.

4.   **T**   **F**   The vector $(\pi, e)$ is a linear combination of the vectors $(1, 2)$ and $(3, 4)$ in $\mathbb{R}^2$.

5.   **T**   **F**   The vector $(9, 8, 7)$ is a linear combination of the vectors $(1, 2, 3)$ and $(4, 5, 6)$.

6.   **T**   **F**   The vectors $(9, 10, 11, 12)$, $(1, 2, 3, 4)$ and $(5, 6, 7, 8)$ in $\mathbb{R}^4$ are linearly independent.

7.   **T**   **F**   We have $\text{span}((10, 20), (20, 10)) = \text{span}((1, 2))$.

8.   **T**   **F**   We have $\text{span}((1, 2, 3, 4)) = \text{span}((4, 3, 2, 1))$.

9.   **T**   **F**   We have $\text{span}((1, 2), (3, 4)) = \text{span}((0, 1), (1, 0))$.

10.   **T**   **F**   Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$. Then $\{x : Ax = 0\} = \text{span}((1, 2, 1))$.

11.   **T**   **F**   Any two vectors in $\mathbb{R}^4$ are linearly independent.

12.  **T**  **F**  Any four vectors in $\mathbb{R}^3$ are linearly dependent.

13.  **T**  **F**  There are exactly three subspaces of $\mathbb{R}^2$.

14.  **T**  **F**  If $A$ is a square $n \times n$ matrix, is $A + A^T$ necessarily symmetric?

15.  **T**  **F**  If $A$ and $B$ are $2 \times 2$ matrices, is $(A + B)(B + A)^T$ necessarily symmetric?

16.  **T**  **F**  If $A$ is symmetric, is $A^T$ necessarily also symmetric?

17.  **T**  **F**  Suppose $A$ is an $n \times r$ matrix (i.e., has $n$ rows and $r$ columns), that $B$ is an $r \times m$ matrix, and that $C$ is an $m \times s$ matrix. Then $ABC$ is defined and is an $s \times n$ matrix.

18.  **T**  **F**  Suppose $A, B, C$ are all $3 \times 3$ matrices. Then $AB + (B + C) = C + (B + AB)$?

19.  **T**  **F**  If $A + I = \begin{pmatrix} 2 & 2 \\ 2 & 5 \end{pmatrix}$, then there is exactly one $x$ that such that $Ax = \begin{pmatrix} 2013 \\ 2014 \end{pmatrix}$.

20.  **T**  **F**  The empty set is a subspace of $\mathbb{R}^2$.

21.  **T**  **F**  The set of points on the line defined by $y = 2x$ is a subspace of $\mathbb{R}^2$.

22.  **T**  **F**  The set of points on the line defined by $y = 2x + 1$ is a subspace of $\mathbb{R}^2$.

23.  **T**  **F**  The set of solutions to the equation $x^2 + y^2 = 1$ (a circle) is a subspace of $\mathbb{R}^2$.

24.  **T**  **F**  The set of pairs $\{(x, y) : x \geq 0, y \geq 0\}$, where both $x$ and $y$ are nonnegative (i.e., the first quadrant), is a subset of $\mathbb{R}^2$.

25.  **T**  **F**  If $S_1, S_2, S_3$ are subspaces of $\mathbb{R}^4$, then the intersection $S_1 \cap S_2 \cap S_3$ is also a subspace of $\mathbb{R}^4$.

**Some Echelon Forms:**

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 10 & 20 \\ 20 & 10 \\ 1 & 2 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 9 & 8 & 7 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$