The Bees' Needs

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Abstract

The mysterious decline in the bee population of North America has left the agricultural community at a loss to explain what is happening and what might be the future consequences. Scientists agree that bees today face heightened obstacles like pesticides, parasites, and invasive species, but have yet to agree upon the exact effects of these obstacles on the bees, and the eventual effects on the human population. We use ODE modeling techniques to construct several versatile models that demonstrate the damaging effects these obstacles have on a hive. We discovered that pesticides are of a greater concern than parasites, though both can cause a hive to suffer a net loss of tens of thousands of bees each year.

1 Background and Motivation

One of every three bites of our food is dependent on bees [1]. As such, agricultural farmers and other food suppliers are very invested in the future of these creatures. Beekeepers in the US lost 40% of their colonies 2018-19, leading many to wonder what factors were negatively affecting bee populations [2]. Understanding the effects of parasites, pesticides, and invasive species on bees gives us the first step towards helping our striped friends and, in the long run, helping ourselves. In the last few years, scientists have run experiments exposing bees to various pesticides in labs [3] or introducing mites into a colony to try out a new type of cure [4], but we felt we could expand upon and connect their work though a model.

While many of the coefficients we use are based in previous research, we have tried to design our models in a way that would be easy to modify as more knowledge becomes available in the future.

2 Modeling

2.1 Early Models

Initially, we were unsure of how to create a single, united model that accounted for invasive species, parasites, and pesticides. As a result, our first attempt produced a function for the seasonal population fluctuation of a hive (Fig. 1), and 3 separate models (Fig. 2-4) for mites, pesticides, and invasive species.

Population Fluctuation

The population of a single hive can fluctuate by as much as 60 percent in a year, as the queen stops laying eggs in winter and more bees die due to the cold. We decided we could model this as the modified sine function $P(t) = \frac{1}{4}\sin(\frac{2\pi}{365}(t + 275)) + .7)$ This gives a function with highest population occurring at the start of July, and lowest population occurring at the end of December.

We assume a population loss of 50 percent in the winter, and a peak population of 30000 bees.

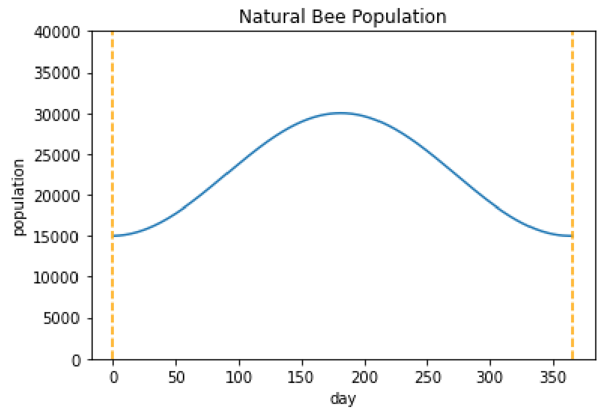

Figure 1: This graph shows the change in a hive's population over the course of a year using the assumptions above.

Mites

We chose to model the spread of mites with an SIR model. The spread rate of the mites and the recovery rate of the bees can be adjusted as necessary. The problem with the SIR model is that it assumes a constant population within the hive. This is not realistic. As discussed above, the population of a beehive ranges greatly throughout the year. The other major problem with this model is that it assumes immunity after a bee has recovered from mites. Mites are not germs and bees are still susceptible after recovery. This is addressed in later iterations.

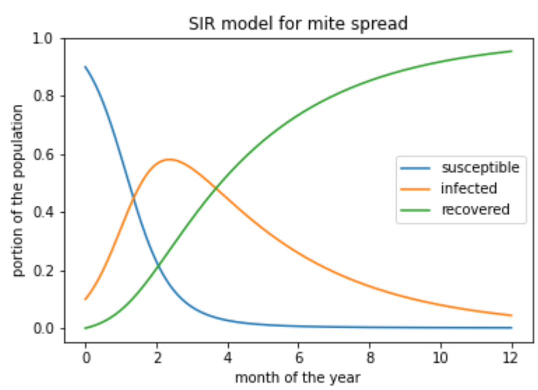

Figure 2: This graph shows the spread of mites within a hive over the period of a year. This graph assumes an infection rate of 1.3 and a recovery rate of .75.

Pesticides — NFD Model

We modeled the effect of pesticides using a modified SIR model. Because pesticides are transmitted to a hive via forager worker bees, we focus our model on the stages of a worker bee's life:

$$N' = -cN - qNF$$

$$F' = cN - dF - pF - qF^2$$

$$D' = dF + pF + qF^2 + qNF$$

Variables are defined as follows:

N: non-forager worker bees
F: forager bees
D: dead bees
c: proportion of non-foragers becoming foragers
d: natural death rate of foragers
p: amount of acute exposure to pesticides
q: amount of sublethal exposure to pesticides

Foraging only takes place in the summer, so this model does not include winter months. We assume the average lifespan of a worker bee is 40 days, with the last 10 days spent foraging. We assume no natural death for non-forager bees. Acute exposure occurs when a forager is quickly exposed to a large amount of pesticides, and dies before returning to the hive. Sublethal exposure doesn't kill immediately, and includes effects extending to non-forager bees (poisoned food, weakened immune systems, etc.). Our model ignores the queen bee and drones (male bees), as they only make up about 2 percent of the population. For this initial model, we assume a birth rate of 0. Pesticide toxicity levels correspond to values of p and q.

Coefficients: $c = .033, d = .005, p = [0, .01, .03, .05], q = [0, .001, .003, .005]$

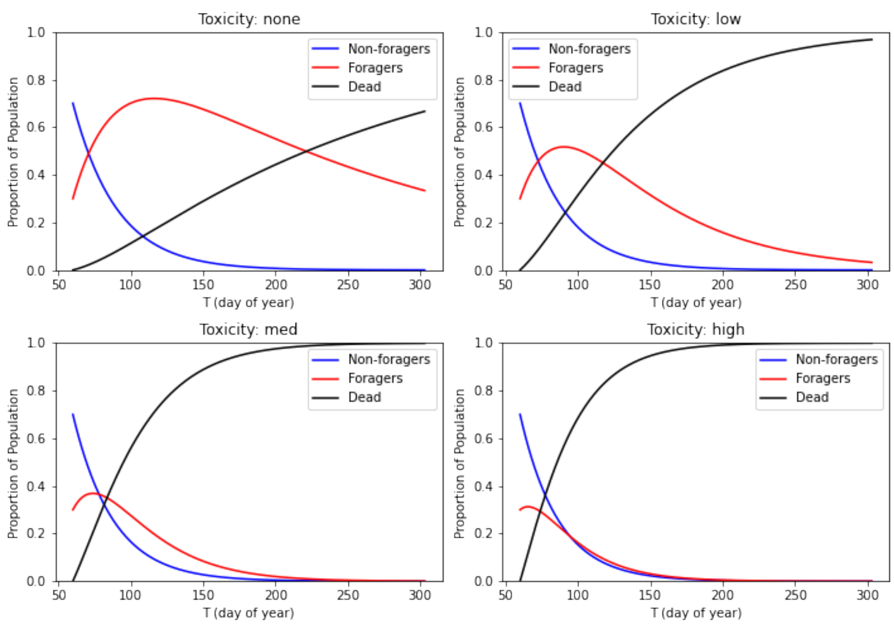

Figure 3: Change in a hive's population during the summer for hives exposed to pesticides of varying levels of toxicity.

Invasive Species

After doing some research on murder hornet attacks, we realized that they were too sudden and too drastic to be well formatted for an SIR model. We attempted to model it using the population equation from before, but ended up with something entirely unexciting (See Figure 4).

From this initial model, we came to understand that invasive species was not formatted in a way that would work well with the two other parts of our project, so we made the decision to leave it be (for now).

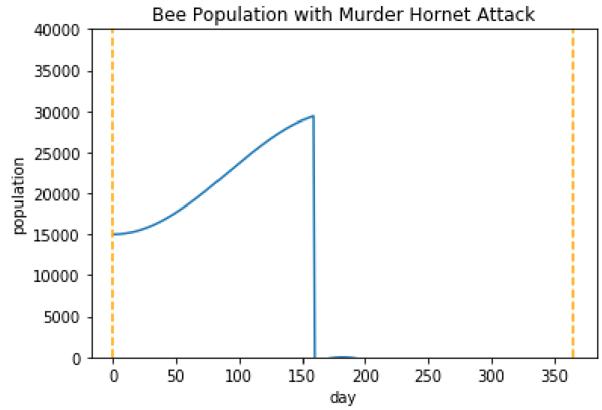

Figure 4: This graph shows the change in a hive's population over the course of a year for a hive that suffered a murder hornet attack. It was not particularly interesting.

2.2 Later Iterations

Once we identified some of our initial issues, we decided to focus on combining the elements of our models, and figuring out how to include a nonzero birth rate.

A Nonzero Birth Rate

The issue with the population fluctuation function that we came up with earlier was that it was already accounting for part of the natural death rate, but in a way that we could not understand or control well. Also, the pesticide model was designed with its own better, more interpretable death rate. So we choose instead to use the following function that accounted for only the fluctuating birth rate with parameter t day of the year (integer mod 365)

$$B(t) = \begin{cases} 0 & \text{if } t < 60 \text{ or } t > 304 \\ 250\sin(\frac{2\pi t}{214} + 90) + 1250) & \text{else} \end{cases}$$

This function makes the following assumptions: the queen bee lays no eggs before March or after October, and the queen bee lays most eggs at the start of July with a peak of 1500 eggs per day, but she has to work up to that from 1000 eggs a day beginning in March.

Combined Model Version 1

We experienced a lot of failure in our early attempts to combine the two SIR models due to the number of variables.

When combining the two models, we decided it would be best to do a 5-stage modification of SIR. The five stages are: Healthy non-foragers (H_n), Healthy foragers (H_f), Infected non-foragers (I_n), Infected foragers (I_f), and Dead (D). When we use the word "infected", we are talking about being infected with parasites. For the sake of labeling, a "healthy" bee could still be exposed to pesticides on a regular basis. Theoretically, a bee will go through at least three stages, but could go through all five stages. We define the following constants:

c, d, p, q: defined as in the NFD Model.

y: parasite infection rate

w: parasite recovery rate

These gave the following equations:

$$H_n' = -cH_n - y(I_n + I_f)H_n - qH_n(H_f + I_f) + wI_n$$
$$H_f' = cH_n - y(I_n + I_f)H_f - dH_f - pH_f - qH_f(H_f + I_f) + wI_f$$
$$I_n' = y(I_n + I_f)H_n - cI_n - wI_n - (1 - w)I_n - qI_n(H_f + I_f)$$
$$I_f' = y(I_n + I_f)H_f + cI_n - wI_f - (1 - w)I_f - pI_f - qI_f(H_f + I_f) - dI_f$$
$$D' = qH_n(H_f + I_f) + dH_f + pH_f + qH_f(H_f + I_f) + (1-w)I_n + qI_n(H_f + I_f) + (1-w)I_f + pI_f + qI_f(S_f + I_f)$$

This unsuccessful model served as a stepping stone to the next, more successful iteration of the combined model. One weakness of this model is the sheer number of parameters, especially for the dead equation D'. Furthermore, this iteration of our model helped us notice that the dead equation wasn't helpful in interpreting our results, because the quantity of dead bees would simply go off to infinity especially once a birth rate was added. Finally, we noticed some mathematical errors with this iteration of our model. The infected equation I_n' contains the term $-wI_n - (1-w)I_n$. This was meant to model the fact that some infected bees would recover, and the bees who didn't recover would die. However, although bees must *eventually* recover or die, these equations erroneously models a situation where all infected bees recover or die at each time step, zeroing out the quantities in I_n. The same problem exists in the I_f' equation. These problems are addressed in the next iteration.

Combined Model Version 2

With this model, we improved upon our previous attempt at a combined model. The biggest difference is that the equation for the dead bees D' has been removed. This allows for more interpretable results. We added a constant birth rate b, and a death by parasite parameter z. After some research, we learned that acute exposure doesn't contribute to nearly as many deaths as sublethal exposure, so the acute exposure coefficient p was absorbed into the sublethal exposure coefficient q to create a single coefficient representing exposure to parasites.

$$H_n' = -cH_n - y(I_n + I_f)H_n - q(H_f + I_f)H_n + wI_n + b$$
$$H_f' = cH_n - y(I_n + I_f)H_f - q(H_f + I_f)H_f + wI_f - dH_f$$
$$I_n' = -cI_n + y(I_n + I_f)H_n - q(H_f + I_f)I_n - zI_n - wI_n$$
$$I_f' = cI_n + y(I_n + I_f)H_f - q(H_f + I_f)I_f - zI_f - wI_f - dI_f$$

Some coefficients were adjusted to be more realistic in this iteration. We let $b = 1500$ because we know queen bees lay around 2000 eggs per day. Letting $c = 0.05$ reflects the fact that non-forager bees become foragers after about 30 days. Letting $d = 0.2$ reflects the fact that foragers die after about 10 days. [5]

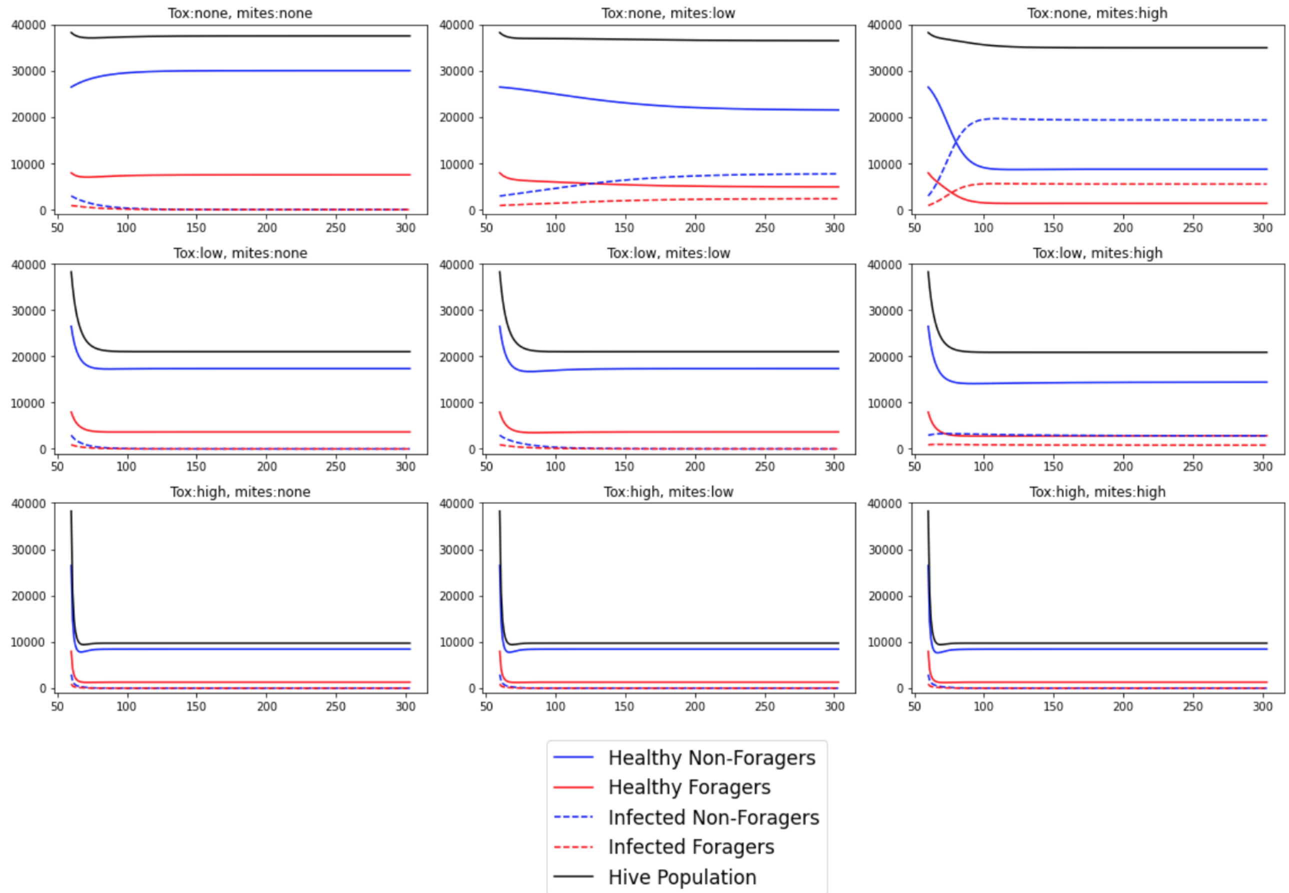

Figure 5: Combined effects for varying levels of pesticides and parasites.

Although there are some interesting interactions between the stages of this model, in Figure 5 we can see that for many combinations of parameters, the model quickly reaches an equilibrium and becomes rather uninteresting. This is due to the fact that the model doesn't currently include a term to represent the direct relationship between the population of foragers and non-foragers — less foragers means less food, lower birth rates, and an increasing number of starving bees. We spent some time trying to work out this "starvation" term, but were unsuccessful in incorporating it into an interpretable model. For the remainder of our efforts, we decided to move on from this combined model and focus on finalizing results with the two separate models.

2.3 Final Model(s)

Once we realized that we were not going to have a net population change of zero (that would invalidate everything we are trying to show here) we were able to move into the final stages of our project. As the population of our hive is constantly changing with varying birth and death rates, the SIR model standard technique of measuring against the percentage of the population was not going to work for us. That, and the fact that we really shouldn't be graphing death, because it was going to just keep increasing and wouldn't really add value or clarity to our model.

Mites with Nonzero Birth Rate

The previous mite model assumed the population of the hive was static and that once a bee recovered from having mites the bee was immune to mites from then onward. Both of these assumptions are false and are fixed in this iteration. This iteration of the model is a modified SI model defined as follows:

H: Healthy Bees

I: Infected Bees

N: Population of hive

γ: Infection rate

ω: Recovery rate

which gave the following equations:

$$I' = -\gamma IH + \omega I$$

$$H' = \gamma IH - \omega I$$

$$N' = \begin{cases} \text{(1-I)}\cos\theta N & \text{if } \cos\theta > 0 \\ \text{(1+I)}\cos\theta N & \text{if } \cos\theta < 0 \end{cases}$$

$$\text{where } \theta = \frac{t * pi}{180} + 90$$

This model returns a percentage of the hive population that is infected, the percentage of the population that is healthy and the number of bees at each time step. The definitions of H' and I' are classic SI model definitions. However the N' model is interesting. The cosine term models the population change based on time of year as bee hives reproduce differently depending on the season. The $1 \pm I$ term brings in the effect of the mites on the population. The mites dampen the growth rate when growth rate is positive and amplify the growth rate when growth rate is negative. This models how the mites add a burden to the hive by increasing the rate at which bees die off. The effects of different levels of initial infection on the population of a hive can be seen in Figure 6.

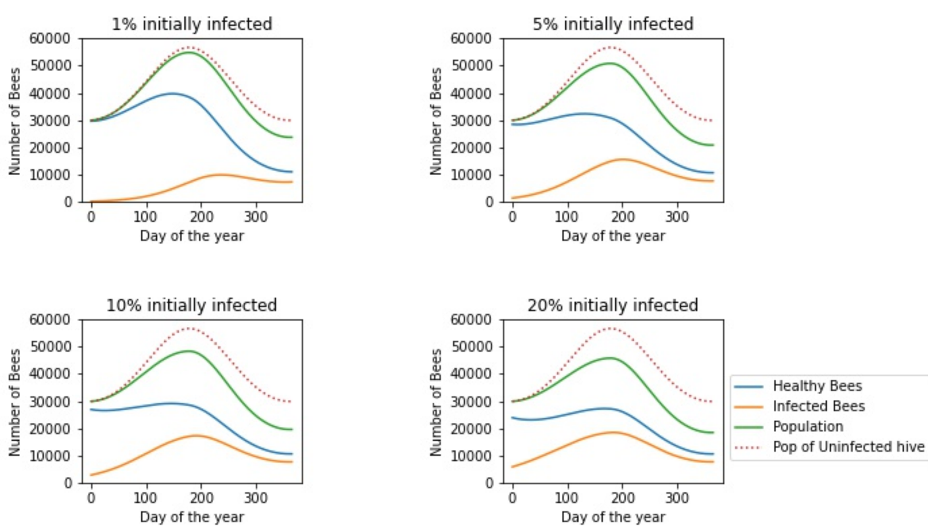

Figure 6: These graphs show the effect of differing levels of initial mite infection over the course of a year. These are modeled with a $\gamma = 1.3$ and $\omega = .75$

Pesticides with Nonzero Birth Rate

In this model, we wanted to take the necessary steps to make our model more realistic by adding a nonzero fluctuating birth rate and a fluctuating death rate. The way a hive operates in the winter is completely different than how it operates in the summer. In the summer, bees live 40 days on average, and the queen lays between 1000 and 1500 eggs per day. In the winter, bees live for 4-6 months, and the queen essentially stops laying eggs. To model this, we made piecewise functions for both birth and death rate. In the warmer months (March through October) the death rate for bees is 0.2 and the birth rate is modeled by the nonzero birth rate function from section 2.2. In the colder months (November through February) the death rate is 0.03 and the birth rate is 0.

Obviously, piecewise functions are an imperfect way of modeling nature. However, this was a large improvement upon the last model. The vertical lines on the graph mark the seasonal change in hive behavior (see Figure 7).

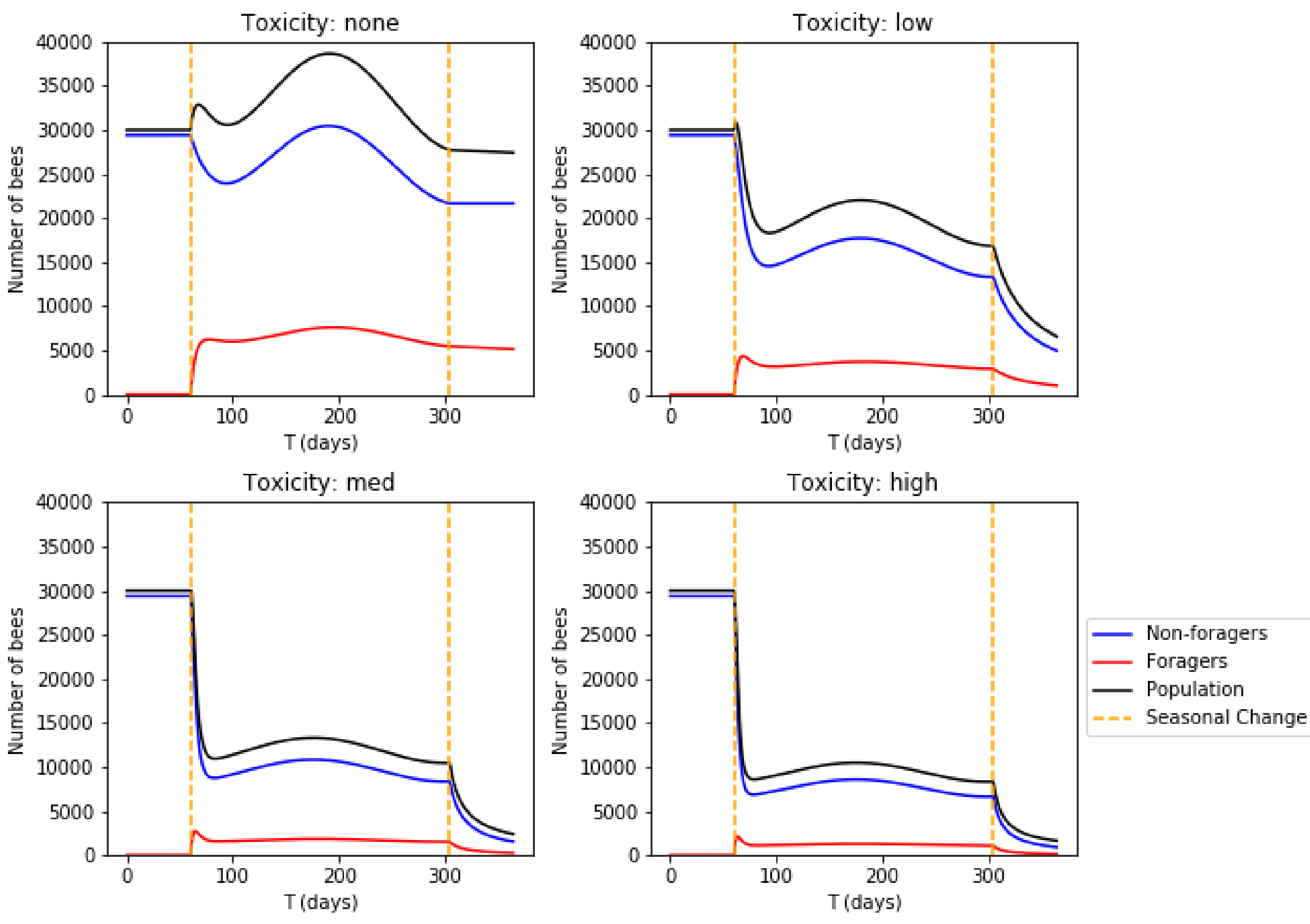

Figure 7: These graphs show the change in a hive's population over the course of a year for hives exposed to different levels of pesticides assuming a nonzero birth rate. Note that no bees forage in the winter.

3 Analysis and Results

The most apparent weakness in our methods is the way coefficients were selected and used. We had to come up with coefficients for life spans, birth rates, death rates, infection rates, toxicity levels, and so on. Much of the research we did lacked actual numbers to describe these rates of change, but could describe the effect that they had. For example, when finding the birth rate for bees, research told us that queen bees didn't really lay many eggs in the winter, but that they could lay up to 1500 eggs per day in the summer. Intuition told us that the queen would not lay 0 eggs on February 28th and then wake up on March 1st and lay 1500 eggs. Thus we tried to model a more gradual process using trig functions. For the toxicity levels of pesticides, we struggled to find concrete numbers to work with, but we had a vague description of the magnitude of the damage. So for those, some guesswork was involved. Knowing this weakness, our results are heavily dependent upon the assumptions we made, so any conclusions drawn from these models here will have to be seen in that light. That said, we do believe that we built models that are efficient and easy to work with, so that if we were to find more accurate numbers for any of the above coefficients, it would be a simple substitution.

This setup suggests that our model could perform well quantitatively if we had access to more detailed research, but as it stands, it just performs well qualitatively.

Based on the coefficients we have chosen, the combined model suggests that pesticides pose more of a threat than mites, as the mite-infected bees die off quickly in the presence of any pesticides. We also see that a hive with a severe mite infestation (20% initial infestation) only has a net loss of about 10,000 bees whereas a hive that is consistently exposed to low toxicity pesticides suffers a net loss of over 20,000 bees.

4 Conclusion

We are proud of the models we have produced. They are effective in that they are adaptable to new information. As we are more able to quantify the relationship between pesticides, parasites, and the death rate of bees, these models will continually improve.

We had initially hoped to use our models on a macro scale to model the current trajectory for the bee population in North America, but in the process of modeling the effects at a hive level, we realized there was so much to learn about a bee hive that we could not do a macro-level projection without sacrificing much accuracy and information that comes from the lower-level projections. Again, this was an issue of just not having enough time.

This project taught us how detail-oriented and research-heavy mathematical modeling can be. We began our research and at step one (a hive-level reconstruction) and we found so much work to be done that we didn't make it to step two, or even to the end of step one. Were we to continue this project in the future, we would want to do more research into the effects of pesticides, construct a combined model with a non-zero birth rate, and expand our model to project the future of bees in North America.

Also, we all want to be beekeepers now.

References

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