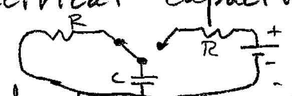
First - Order Systems

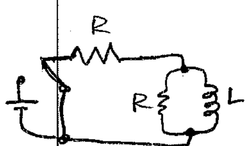
- A system that can store energy in only one form and location is called a "first-order" dynamic system.
  - The mathematical equation describing its motion can be written in terms of a single variable and its 1st derivative only.

Examples:

1. A single mass moving against friction (motor rotor, viscocity test)
2. A single electrical capacitance with resistors



3. A single inductance with resistors



4. A single mechanical spring with friction (torque motor)
5. A single thermal capacitance with thermal resistance (363 thermocouple)

- Masses, springs, inductors, capacitors store energy
- Resistors, friction dissipate energy

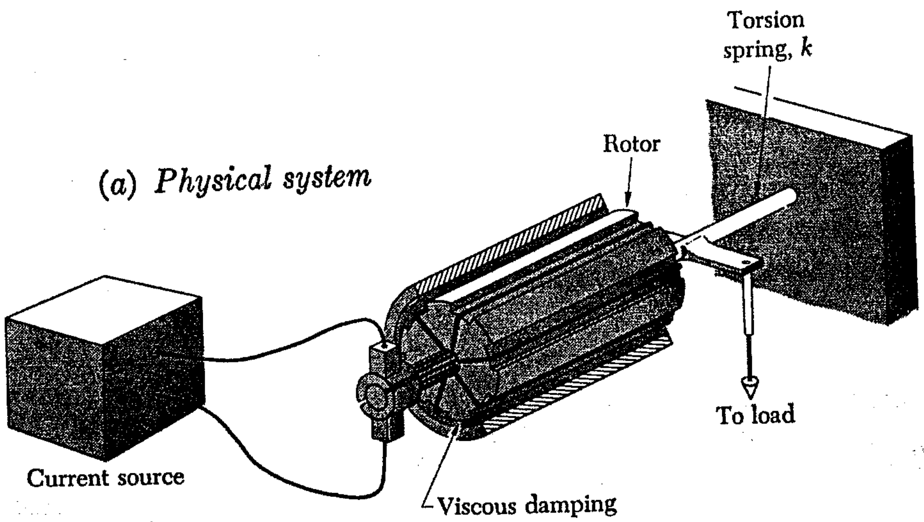
In each case, the system returns naturally to a state of static equilibrium

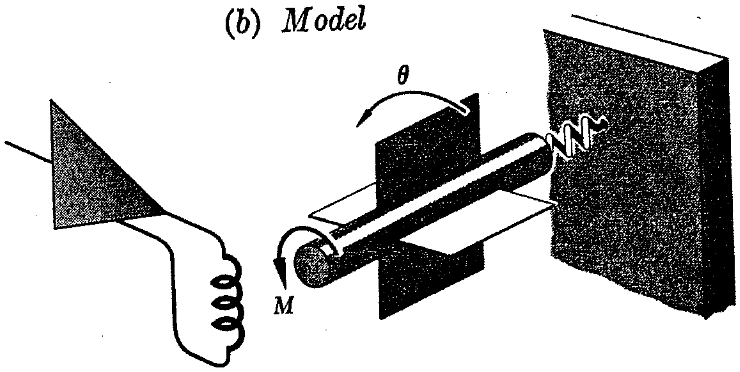
- "motion" will stop
- state will remain constant

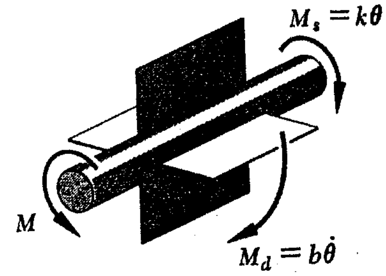
Example : Torque motor

$$b\dot{\theta} + k\theta = M(t)$$

this assumes that rotor inertia is negligible.







$$\Sigma M^* = 0$$

$$M_s + M_d - M = 0$$

$$b\dot{\theta} + k\theta = M(t)$$

This assumes inertia is negligible

Natural Motion (Unforced Motion)

- Motor torque M is absent

$$\boxed{b\dot{\theta} + k\theta = 0}$$

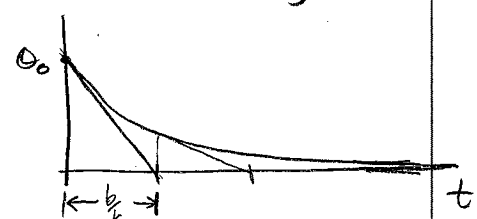
Motion of System by Physical Reasoning:

- 2 forces acting at any instant
  - spring torque $k\theta$ — opposing displacement
  - damping torque $b\dot{\theta}$ — opposing velocity
- These torques are equal and opposite at each instant:

$$b\dot{\theta} = -k\theta$$

- Spring attempt to return rotor to neutral position (where spring force is zero)
- Damping torque opposes this motion, such that the velocity at any instant is proportional to the twist of the spring. As the rotor approaches the neutral position, its velocity $\rightarrow 0$, so that the rotor comes to rest w/o overshooting the neutral position.

$$\dot{\theta} = -\frac{k}{b}\theta$$



- Rate of return toward 0 is always proportional to displacement from neutral

$$\dot{\theta}(0) = -\frac{k}{b}\theta_0 = -\frac{\theta_0}{b/k}$$

$$\tau = \text{time constant} = \frac{b}{k}$$

- Can sketch performance graphically based upon this fact:

Mathematical Solution — Laplace Transform

$$b\dot{\theta} + k\theta = 0$$

Take Laplace transform:

$$b\left[s\Theta(s) - \theta(0)\right] + k\,\Theta(s) = 0$$

where $\Theta(s) = \mathcal{L}\{\theta(t)\}$

$$(bs + k)\,\Theta(s) = b\,\theta(0)$$

Note: For $\theta(0) = 0$ (zero initial conditions), we have

$$(bs+k)\Theta(s) = 0$$

$$\Rightarrow \boxed{bs + k = 0} \leftarrow \text{Characteristic Equation}$$

$$\boxed{s = -\frac{k}{b}} \leftarrow \text{Root of Char. Eqn. a.k.a. eigenvalue}$$

Determines completely the dynamic behavior of the natural motions

$$\Theta(s) = \frac{1}{s + k/b}\,\theta(0)$$

From Laplace transform tables,

$$\theta(t) = \mathcal{L}^{-1}\{\Theta(s)\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s + k/b}\,\theta(0)\right\}$$

$$\boxed{\theta(t) = \theta_0 e^{-\frac{k}{b}t}} \qquad \text{where } \theta(0) = \theta_0$$

Note agreement with physical reasoning:

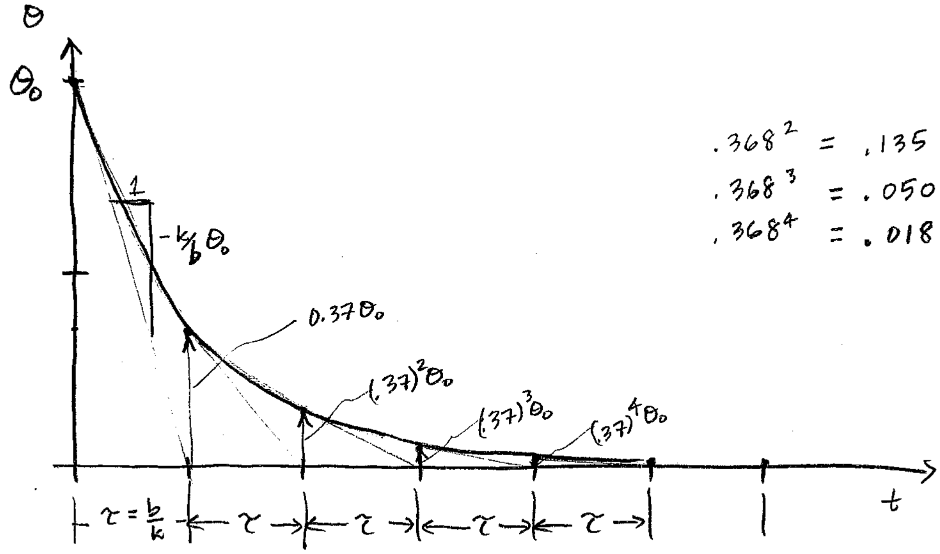
$$\theta(0) = \theta_0 \quad , \quad \dot{\theta}(0) = -\frac{k}{b}\theta_0$$

Plotting time response:

Consider "decay" of θ at the end of one time constant, $t = \tau = \frac{b}{k}$:

$$\theta(\tau) = \theta_0 e^{-\left(\frac{k}{b}\cdot\frac{b}{k}\right)} = \theta_0 e^{-1} = \frac{\theta_0}{e}$$

$$\boxed{\theta(\tau) \approx 0.368\,\theta_0}$$



$.368^2 = .135$
$.368^3 = .050$
$.368^4 = .018$

- τ depends only on the physical parameters of the system (in this case b, k).
- It is independent of the initial position or any torques applied to the system
- It characterizes the free motion of the system — the natural response.

<u>Stability</u>

The response of a 1^{st}-order system is stable if its characteristic root is less than 0.

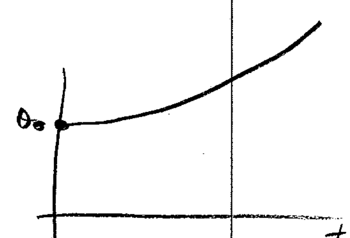
$$s = -\frac{k}{b} < 0.$$

motion decays

- There is a possibility that the solution to the CE could be positive. What will this mean?

$$\theta = \Theta\, e^{st} \qquad \text{for } s > 0$$

implies a growing exponential.



When the <u>natural</u> motion grows w/o bound, we say that the system is <u>unstable</u>.

- When a root of a system's CE is a positive number, the system will be unstable.

Forced Motion

Response to a step input:

$$b\dot{\theta} + k\theta = M_o u(t) \qquad u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

$$\theta(0) = 0$$

Physical Reasoning:

- Long after step is applied, the rotor will be stationary at a new position where spring torque is equal and opposite to M_o.

- From physical reasoning, mathematical solution must contain a constant (to represent final displacement) and an exponential term (to represent the transient).

Take Laplace transform of EOM:

$$b\left[s\Theta(s) - \cancel{\theta(0)}^{\,0}\right] + k\Theta(s) = \frac{M_o}{s}$$

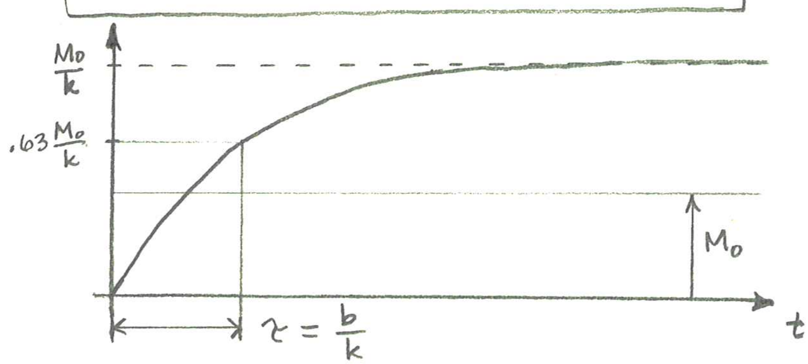
$$(bs+k)\Theta(s) = \frac{M_o}{s}$$

$$\Theta(s) = \frac{M_o/b}{s(s+k/b)} \qquad \left(= \frac{M_o/b}{(s-0)(s-(-k/b))} \right)$$

From the Laplace transform tables:

$$\theta(t) = \frac{M_o}{b} \cdot \frac{1}{0-(-k/b)} \cdot \left(e^{0t} - e^{-(k/b)t} \right)$$

$$\boxed{\theta(t) = \frac{M_o}{k}\left[1 - e^{-(k/b)t}\right]}$$



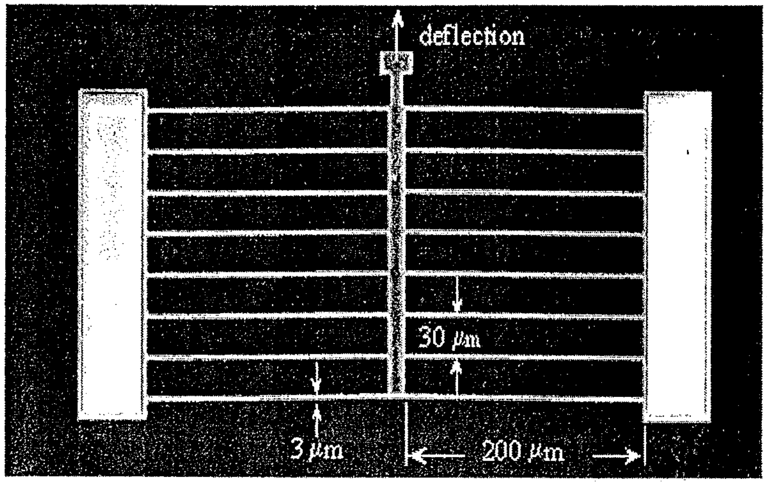


Figure 1.1 - Thermomechanical In-plane Microactuator or (TIM)

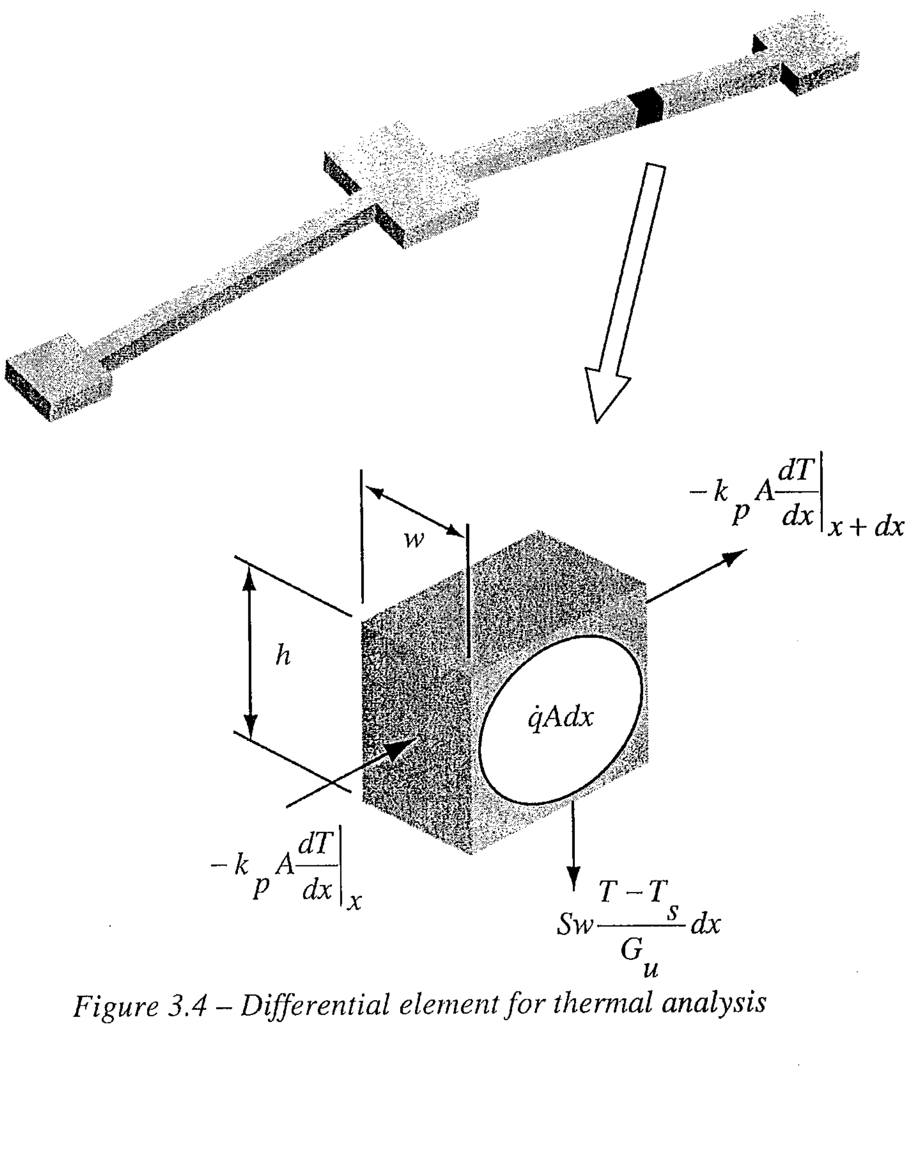


Figure 3.4 – Differential element for thermal analysis

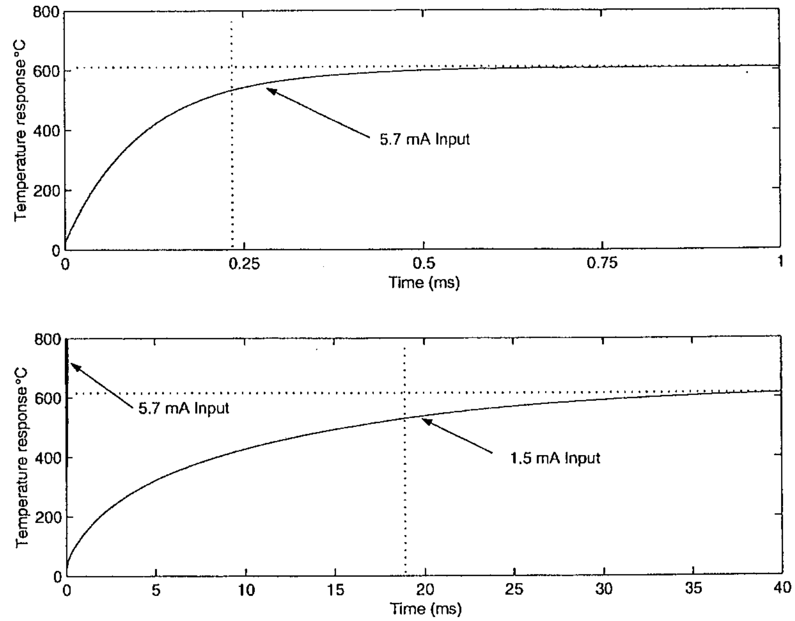


Figure 4.8 - (Top) Temperature response of legs in air, and (Bottom) response of legs in a high vacuum.

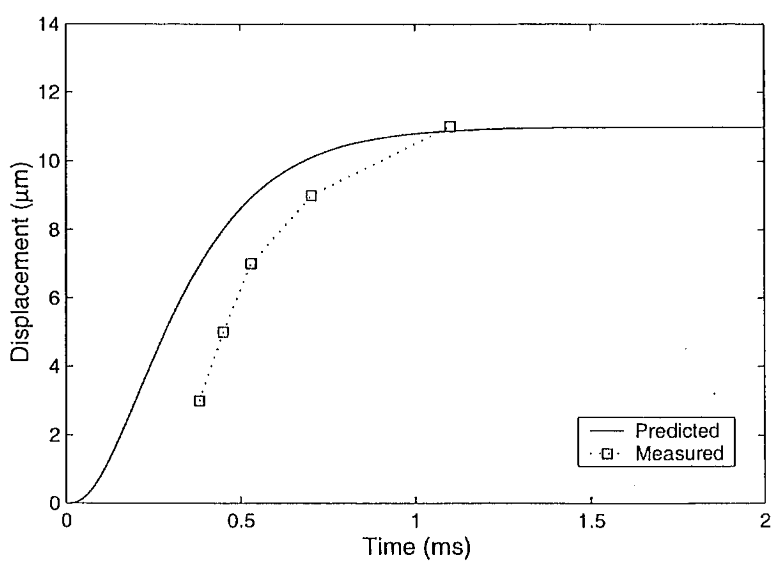
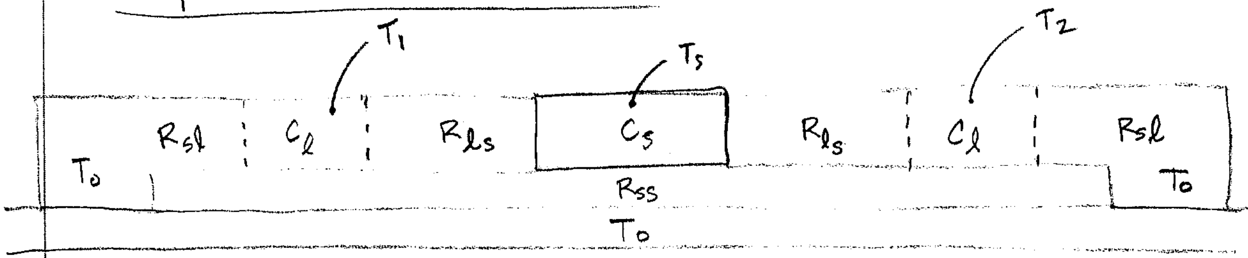


Figure 4.9 – Transient response predictions from model compared to measured data

Simple 3rd-order TIM model



$$C_s \dot{T}_s = \frac{T_1 - T_s}{R_{ls}} + \frac{T_2 - T_s}{R_{ls}} + \frac{T_o - T_s}{R_{ss}}$$

$$C_l \dot{T}_1 = q + \frac{T_o - T_1}{R_{sl}} + \frac{T_s - T_1}{R_{ls}}$$

$$C_l \dot{T}_2 = q + \frac{T_o - T_2}{R_{sl}} + \frac{T_s - T_2}{R_{ls}}$$

$$\dot{T}_s = \frac{1}{C_s}\left[\frac{1}{R_{ls}} T_1 - \frac{1}{R_{ls}} T_s + \frac{1}{R_{ls}} T_2 - \frac{1}{R_{ls}} T_s + \frac{1}{R_{ss}} T_o - \frac{1}{R_{ss}} T_s \right]$$

$$\dot{T}_s = \frac{1}{C_s}\left[-\left(\frac{2}{R_{ls}} + \frac{1}{R_{ss}}\right) T_s + \frac{1}{R_{ls}} T_1 + \frac{1}{R_{ls}} T_2 + \frac{1}{R_{ss}} T_o \right]$$

$$\dot{T}_1 = \frac{1}{C_l}\left[-\left(\frac{1}{R_{sl}} + \frac{1}{R_{ls}}\right) T_1 + \frac{1}{R_{ls}} T_s + \frac{1}{R_{sl}} T_o + q \right]$$

$$\dot{T}_2 = \frac{1}{C_l}\Big[-\left(\frac{1}{R_{sl}} + \frac{1}{R_{ls}}\right) T_2 + \frac{1}{R_{ls}} T_s + \frac{1}{R_{sl}} T_o + q$$