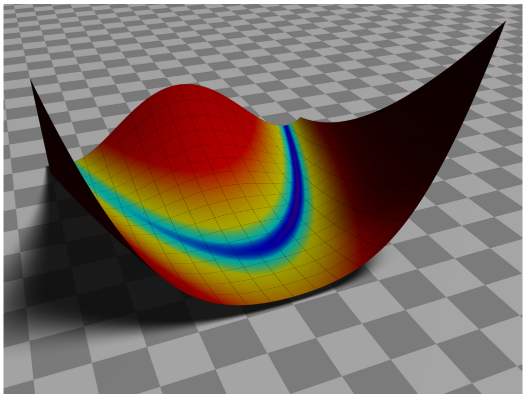
Optimization Under Uncertainty II

Lecture 29



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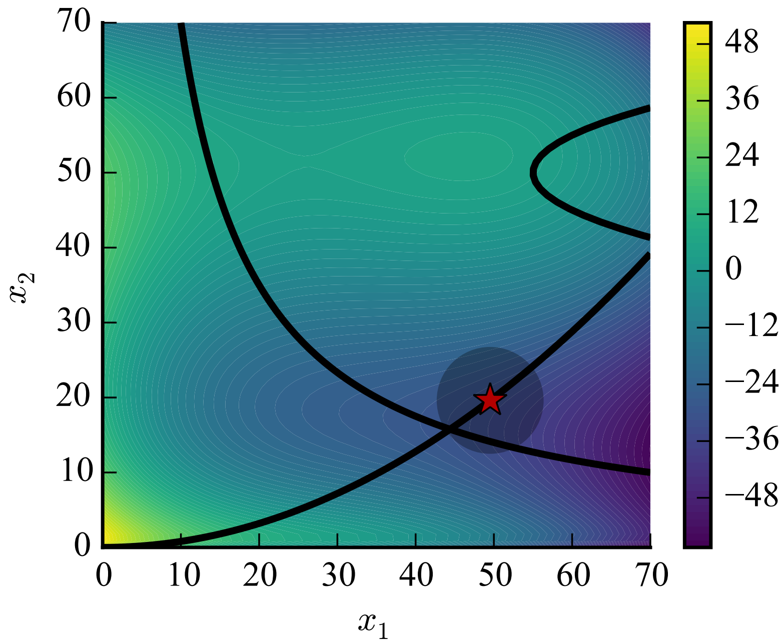
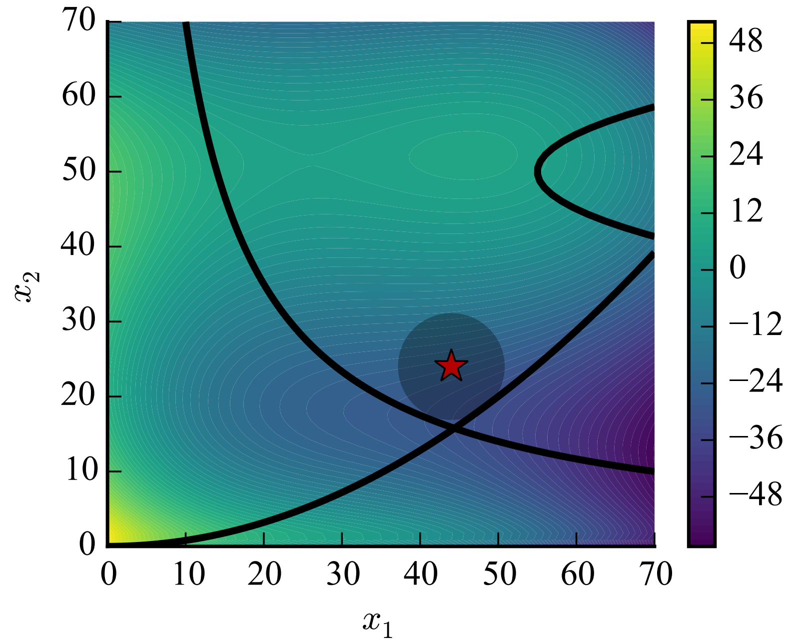
Outline

Reliability

Forward Propagation

Reliability





If standards exist for your industry, safety factors can be useful.

If not, or if better estimates are needed, statistical approaches are effective.

Reliability should always be considered when performing optimization.

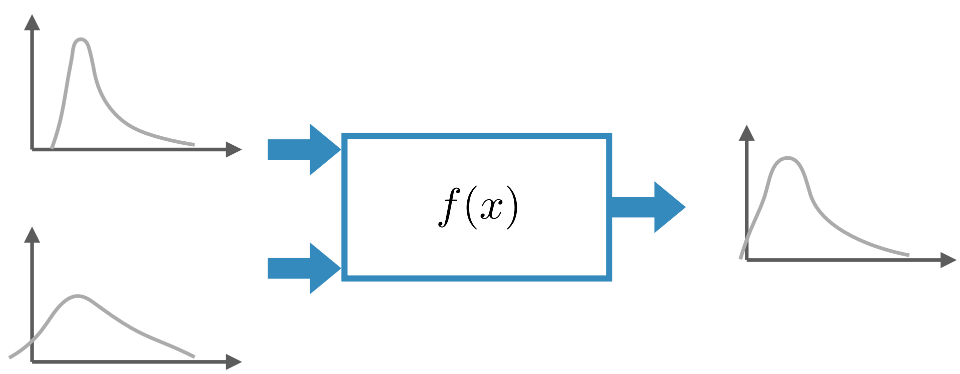
$$\text{Prob}[c(x) \leq 0] \geq R$$

What is R? You choose what is acceptable.

1-Sided Normal Distribution

standard deviations	1-sided reliability
1	84.1%
2	97.7%
3	99.865%

Forward Propagation



Get out your laptops.

Consider the following objective and constraint

$$\begin{aligned} f(x) &= x_1^2 + 2x_2^2 + 3x_3^2 \\ c(x) &= x_1 + x_2 + x_3 \le 3.5 \end{aligned}$$

At the point:

$$x = [1, 1, 1]$$

the standard deviation in x is (normally distributed)

$$\sigma_x = [0.0, 0.06, 0.2]$$

Compute the following:

- Output statistics for f (mean, standard deviation, histogram)
- Reliability of c

Helpful functions:

- randn, np.random.randn
- nnz, np.count_nonzero

Is 99.9% good? Maybe.

If 99.9% were good enough:

- 350 babies will be given to the wrong parents each day.
- 22,000 checks will be deducted from the wrong bank accounts in the next hour.
- 100 flights per day would be unsafe.

Monte Carlo Simulation

1. Sample N points from input probability distribution (x_i).
2. Evaluate output $f_i = f(x_i)$.
3. Compute statistics on f_i.

We can also easily evaluate

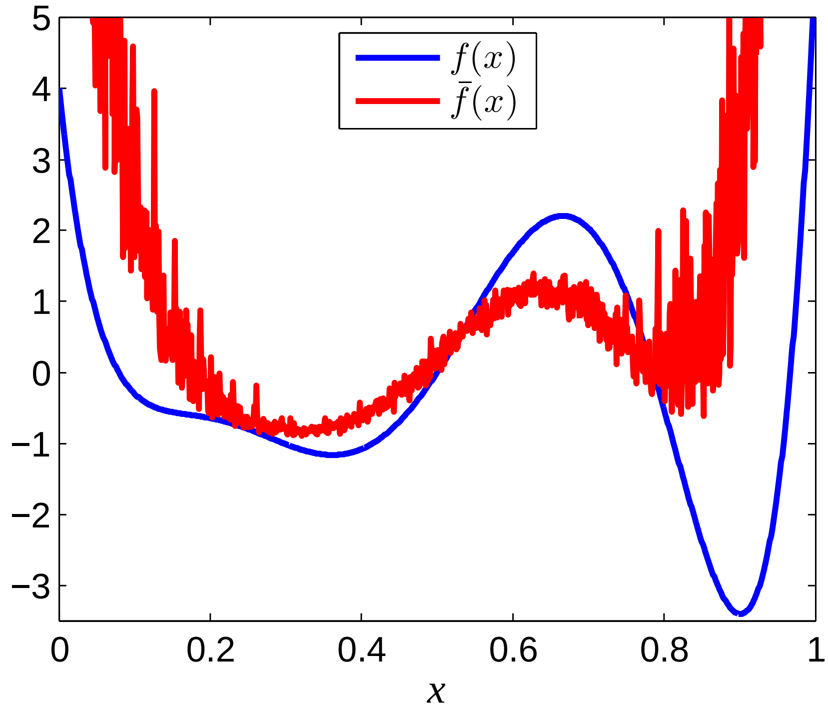
$$\text{Prob}[c(x) \leq 0]$$

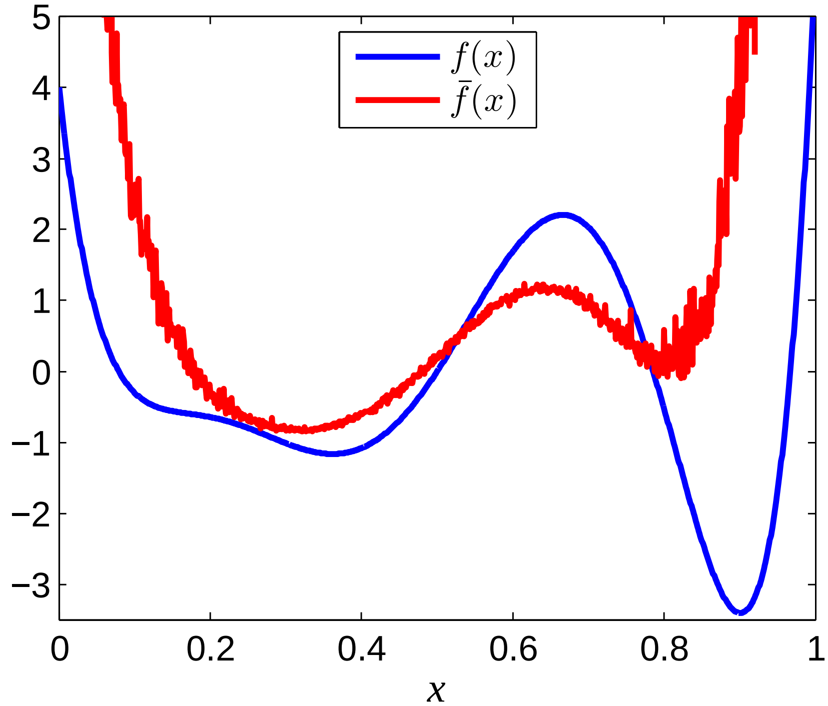
Advantages:

- Code is a black box.
- Easy to parallelize.
- Convergence rate is independent of the dimension of the stochastic input space.

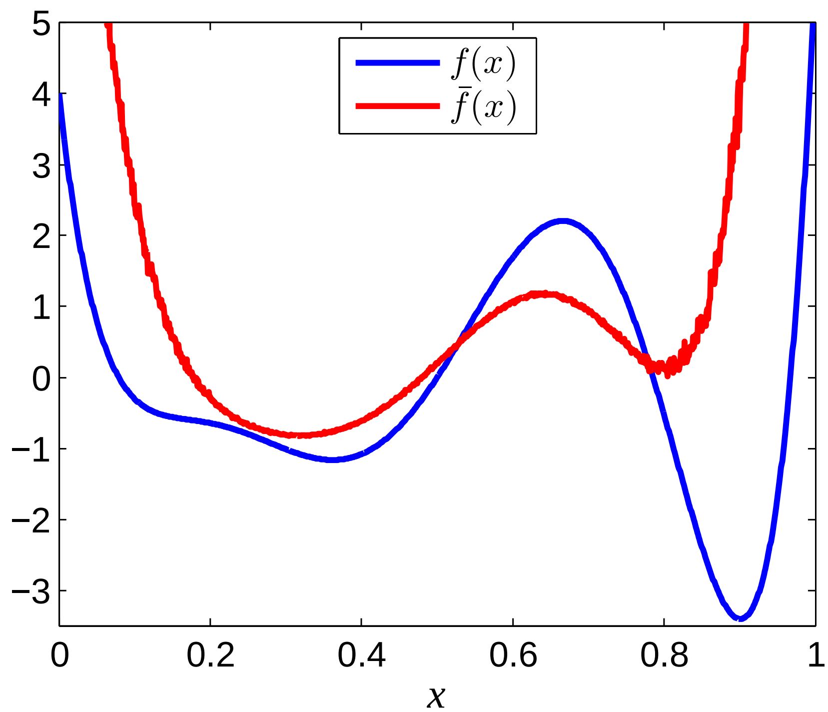
Disadvantage:

Convergence rate is slow: requires *a lot* of samples.





$f(x)$
$\bar{f}(x)$
5
4
3
2
1
0
−1
−2
−3
0
0.2
0.4
0.6
0.8
1
x



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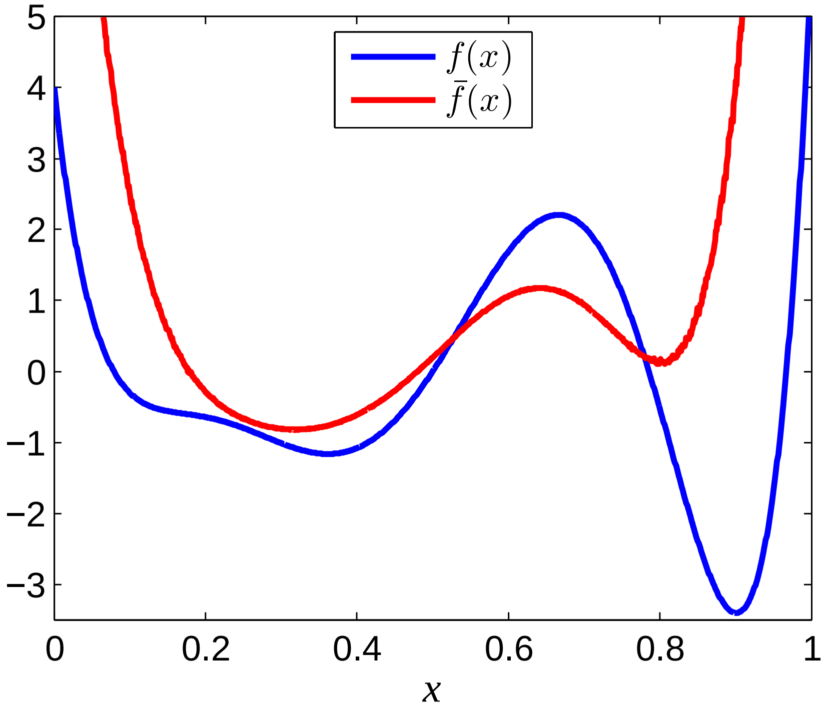
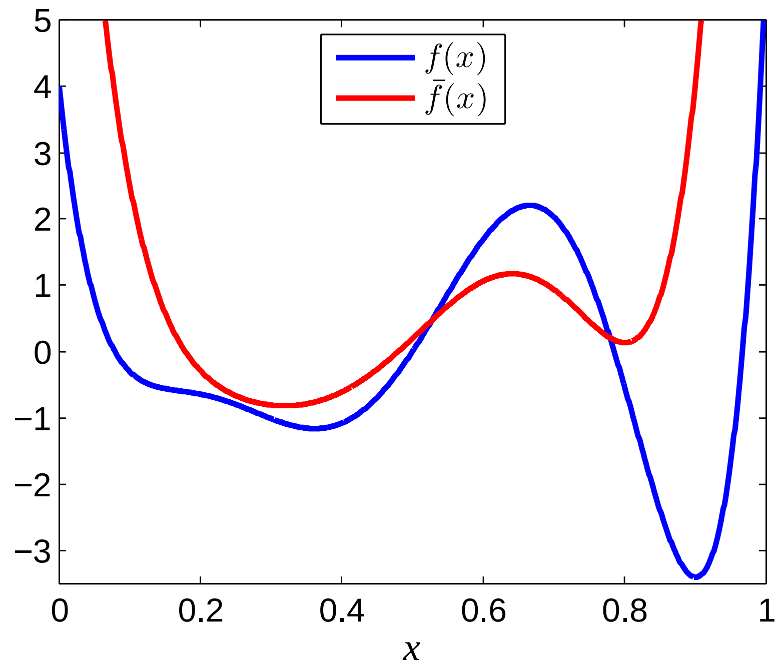


Stochastic Collocation

A more efficient integration technique

$$\int_a^b f(x)dx = \sum_{i=1}^{\infty} w_i f(x_i) \approx \sum_{i=1}^{N} w_i f(x_i)$$

the bassis functions f are carefully chosen.



Requires 6 function calls instead of 100,000 with Monte Carlo

Advantages:

- Black box
- Very efficient
- Does not work for all distribution types (but other methods like Polynomical Chaos can be used)

Disadvantage: Does not scale well with large numbers of stochastic input variables.

Other methods exist:

- Polynomial Chaos Expansion
- Spectral methods (e.g., Stochastic Galerkin)