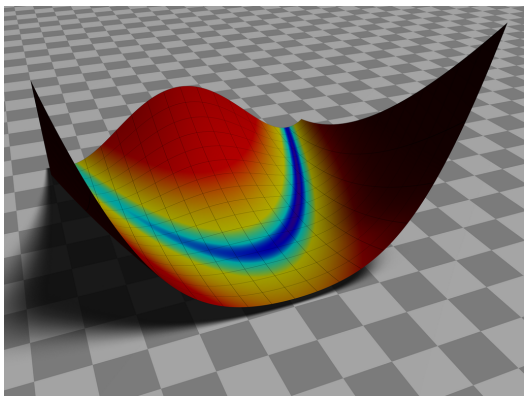
Trust Region Methods

Lecture 7



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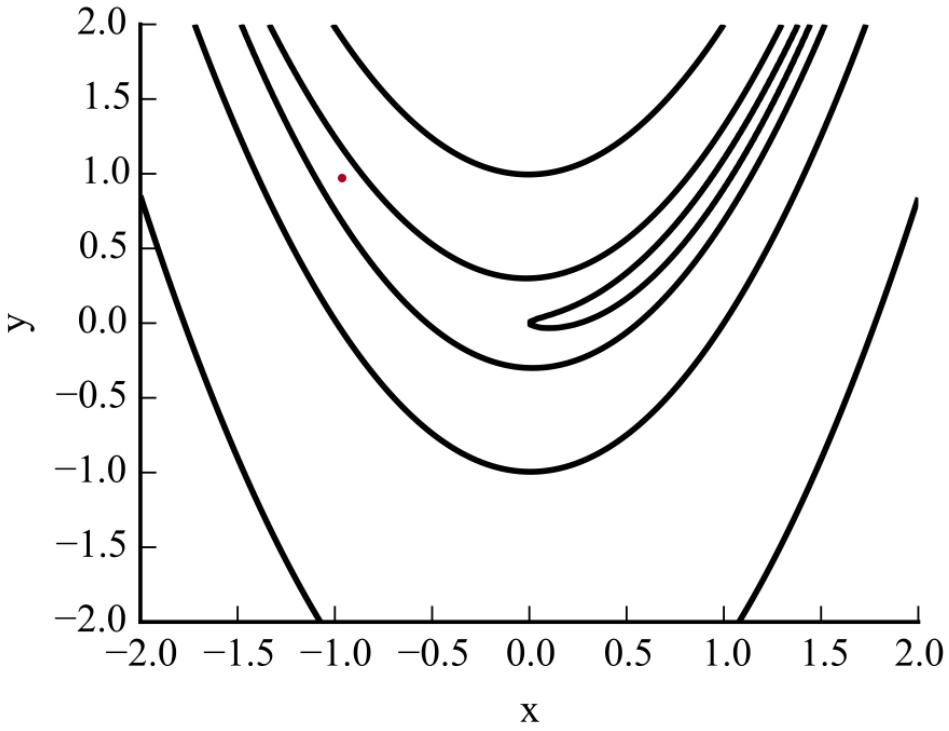
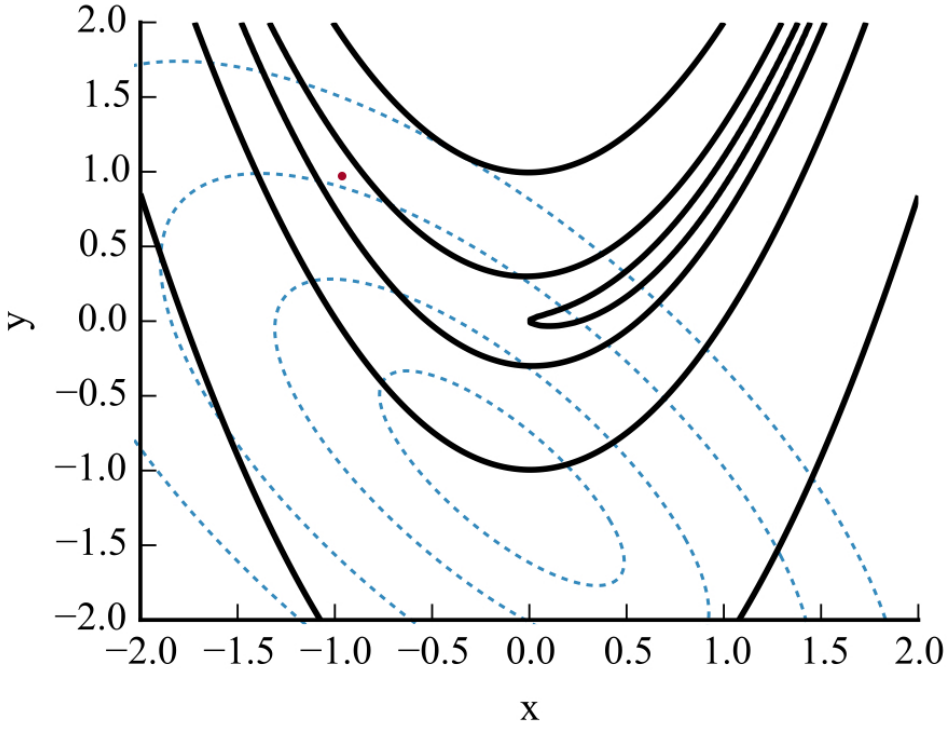
Outline

Trust Region Methods

Current methods: pick search direction, then pick step along that direction.

Trust region: pick a "step" first, then choose a direction. Also, when "backtracking", the direction can change.

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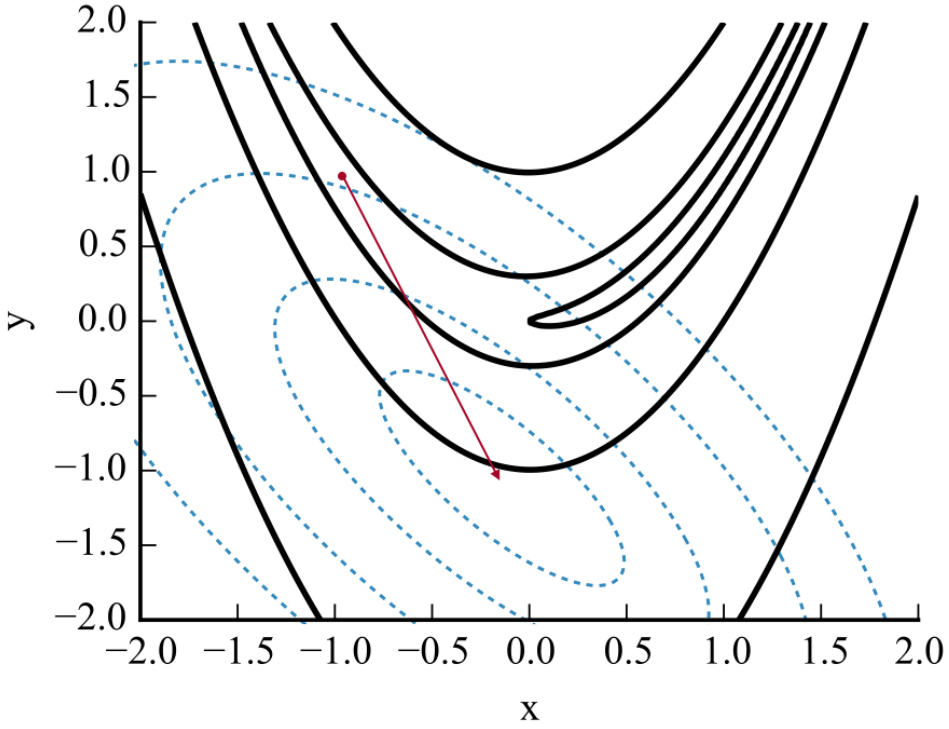
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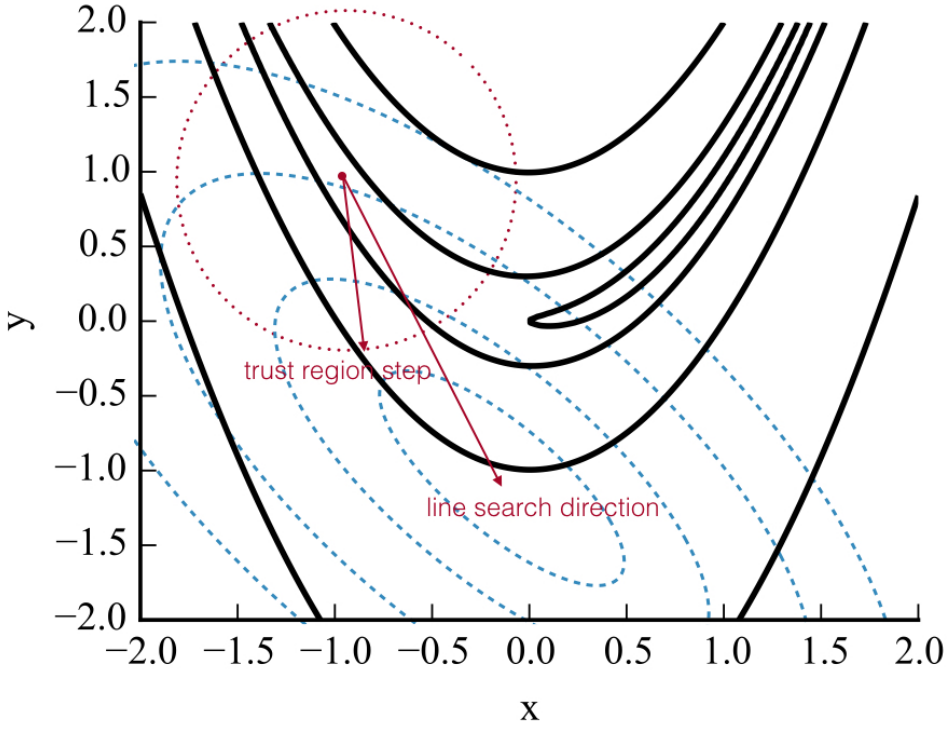
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y
trust region step
line search direction
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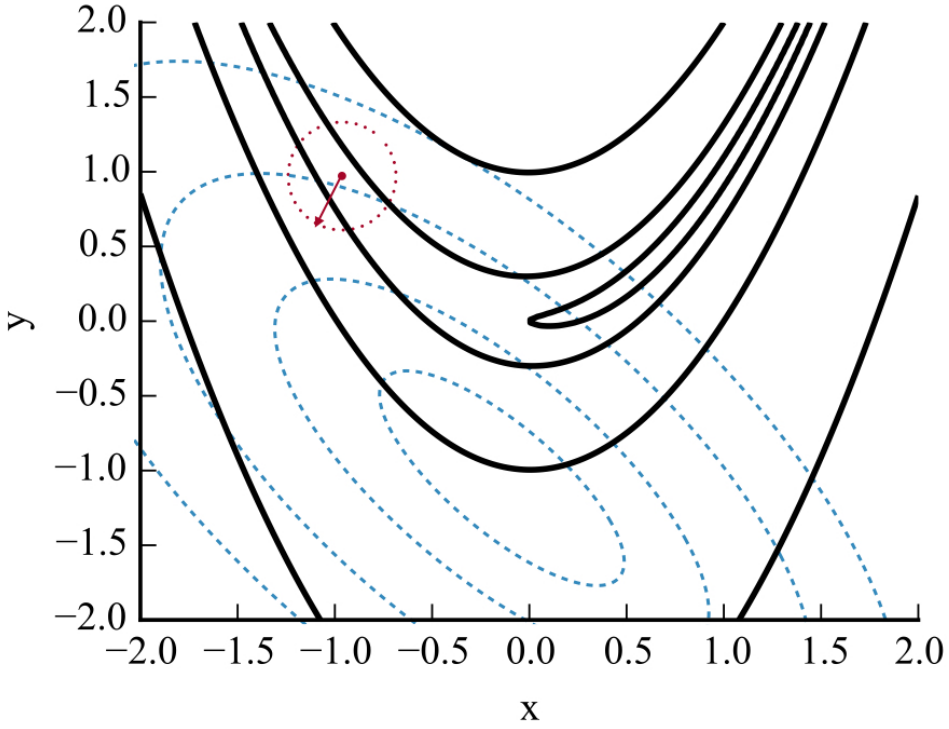
x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x

Solve:

$$\begin{aligned} \text{minimize} \quad & m_k(p) \\ \text{with repsect to} \quad & ||p|| \leq \Delta_k \end{aligned}$$

Usually we use a local quadratic model (with an approximation to the Hessian), and the Euclidean norm.

$$\begin{aligned} \text{minimize} \quad & f_k + g_k^T s + \frac{1}{2} s^T H_k s \\ \text{with respect to} \quad & ||s||_2 \leq \Delta_k \end{aligned}$$

If H_k is positive definite and $||H_k^{-1} g_k|| \leq \Delta_k$, solution is easy

Otherwise, solution is hard so we resort to an appproixmate solution because that is all that is really needed anyway. (We won't get into details here).

Resize Trust Region Radius

Estimate accuracy of trust-region model with:

$$\frac{\text{actual decrease}}{\text{predicted decrease}}$$

$$\rho_k = \frac{f(x_k) - f(x_k + s_k)}{m(0) - m(s_k)}$$

What is the sign for the predicted decrease?

$$\rho_k = \frac{f(x_k) - f(x_k + s_k)}{m(0) - m(s_k)}$$

Predicted decrease is always nonnegative. Thus,

- if ρ_k is negative $=>$ bad (means function increased)
- if ρ_k is close to 1 $=>$ good (model agreement is close)
- if ρ_k larger than 1 this is also good, more than expected decrease
- if ρ_k is positive but not close to 1, OK. just keep the trust region.

Example Algorithm:

Start with initial guess x_0, and inital trust region size Δ_0
Repeat:

Estimate Hessian
(Approximately) solve for optimal step location
Assess accuracy of trust region model

$$\rho_k = \frac{f(x_k) - f(x_k + s_k)}{m(0) - m(s_k)}$$

Resize and move trust region

Resize trust region:

if $\rho_k < 0.25$

$\Delta_{k+1} = \Delta_k/4$ // Model is poor; shrink

elseif $\rho_k > 0.75$ and $\|s_k\| = \Delta_k$

$\Delta_{k+1} = \min(2\Delta_k, \hat{\Delta})$ // Model is good and new point on edge; expand

else

$\Delta_{k+1} = \Delta_k$ // Model is reasonable; keep size

end if

Move trust region:

if $\rho_k \leq \eta \in [0, 0.25)$

$x_{k+1} = x_k$ // Keep trust region centered about the same point

else

$x_{k+1} = x_k + s_k$ // Move center of trust region to new point

end if