Systems Similarity

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Conservation laws

- Inflow – Outflow = Rate of Creation (or Storage)
- How does this general statement of the conservation laws apply to
  - Mechanical systems? $$\sum F = \frac{d}{dt}(mv)$$
  - Electrical systems? $$\sum i = C\frac{dV}{dt}$$
  - Fluid systems? $$\sum \dot{m} = \frac{d}{dt}(\rho V) = \dot{\rho}V + \rho\dot{V}$$
  - Thermo-fluid systems? $$\underbrace{\sum \dot{Q}_h}_{\text{heat transfer}} - \overbrace{\dot{W}}^{\text{work}} + \underbrace{\dot{m}\left(h + \frac{v^2}{2g} + zg\right)}_{\text{mass flow across boundaries}} = \frac{d}{dt}\underbrace{\left(mu + \frac{mv^2}{2} + mzg\right)_{cv}}_{\text{energy stored}}$$

Power variables

domain	effort variable	flow variable
mechanical translation	force - F	velocity - v
mechanical rotation	torque - T	angular velocity - Ω
electrical	voltage - V	current - i
fluid	pressure - P	volumetric flow rate - Q

$$\text{power} = \text{effort} \times \text{flow}$$

Energy variables

domain	momentum variable	displacement variable
mechanical translation	linear momentum - p	linear displacement - x
mechanical rotation	angular momentum - H	angular displacement - θ
electrical	flux linkage - λ	charge - q
fluid	pressure momentum - p_P	volume - V

For a spring: $$PE = \int \text{power}\ dt = \int Fv\ dt = \int kx\ dx = \frac{1}{2}kx^2$$

For a mass: $$KE = \int \text{power}\ dt = \int vF\ dt = \int v\ dp = \int mv\ dv = \frac{1}{2}mv^2$$

Fundamental relationships

$$\text{flow} = \frac{d}{dt}(\text{displacement})$$

$$\text{effort} = \frac{d}{dt}(\text{momentum})$$

Dissipative elements (Resistance)

$$\text{effort} = \text{resistance} \times \text{flow}$$

domain	constitutive relation	physical description
mechanical translation	$F = bv$	friction, damping
mechanical rotation	$\tau = b\Omega$	friction, damping
electrical	$V = Ri$	electrical resistance
fluid	$P = RQ$	fluid drag, resistance

Energy storage elements (Capacitance)

$$\text{effort} = \frac{1}{\text{capacitance}} \times \text{displacement}$$

$$\text{capacitance} \times \frac{d}{dt}\text{effort} = \text{flow}$$

domain	constitutive relation	alternat relation	physical description
mechanical translation	$F = kx$	$\frac{1}{k}\frac{dF}{dt} = v$	linear spring
mechanical rotation	$\tau = k\theta$	$\frac{1}{k}\frac{d\tau}{dt} = \Omega$	torsional spring
electrical	$V = \frac{1}{C}q$	$C\frac{dV}{dt} = i$	capacitor
fluid	$\Delta P = \frac{1}{C}\Delta V$	$C\frac{dP}{dt} = Q$	compliance

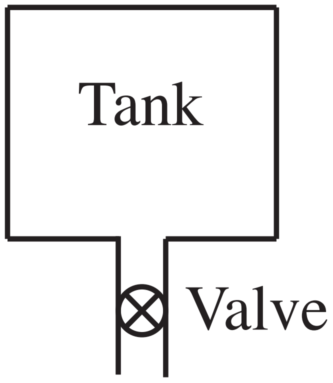
Energy storage elements (Inertia)

$$\text{flow} = \frac{1}{\text{inertia}} \times \text{momentum}$$

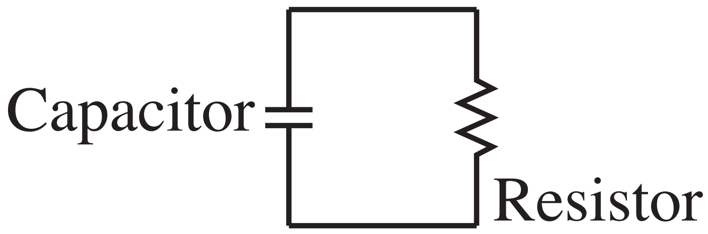
$$\text{inertia} \times \frac{d}{dt}\text{flow} = \text{effort}$$

domain	constitutive relation	alternate relation	physical description
mechanical translation	$v = \frac{1}{m}p$	$m\frac{dv}{dt} = F$	mass
mechanical rotation	$\Omega = \frac{1}{J}H$	$J\frac{d\Omega}{dt} = \tau$	mass moment of inertia
electrical	$i = \frac{1}{L}\lambda$	$L\frac{di}{dt} = V$	inductance
fluid	$Q = \frac{1}{I}p_P\mathrm{V}$	$I\frac{dQ}{dt} = P$	fluid inertia

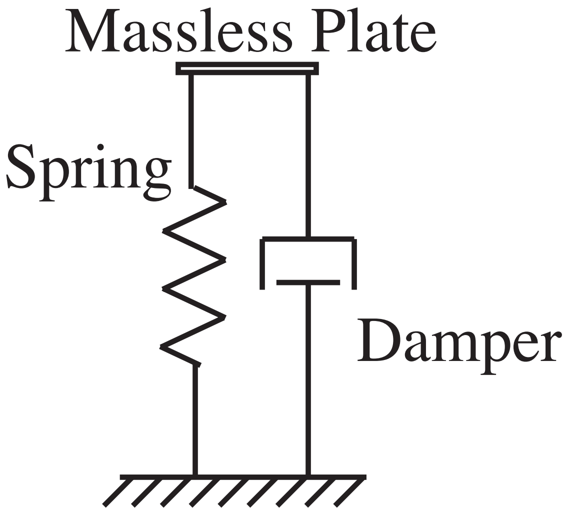
Similar systems (Capacitance-effort)



Toilet tank



Flash bulb

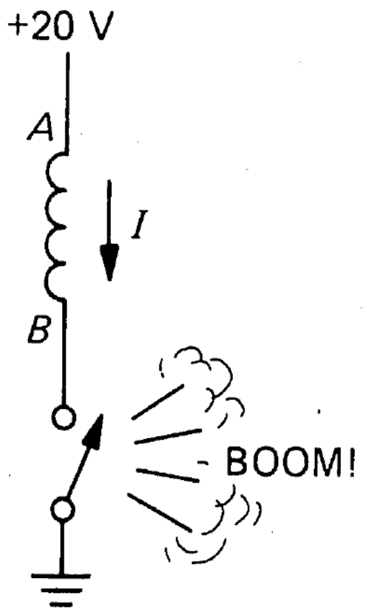


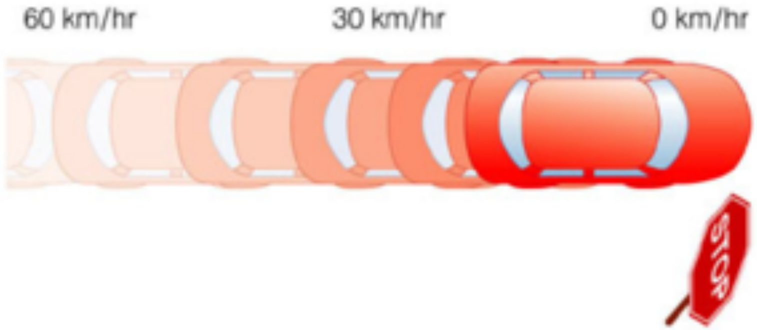
Door-closing mechanism

Similar systems (Inertia-flow)

Long pipe
with valve

DC motor with
transistor switch





Car quickly
decelerating

Similar systems (Inertia-flow)

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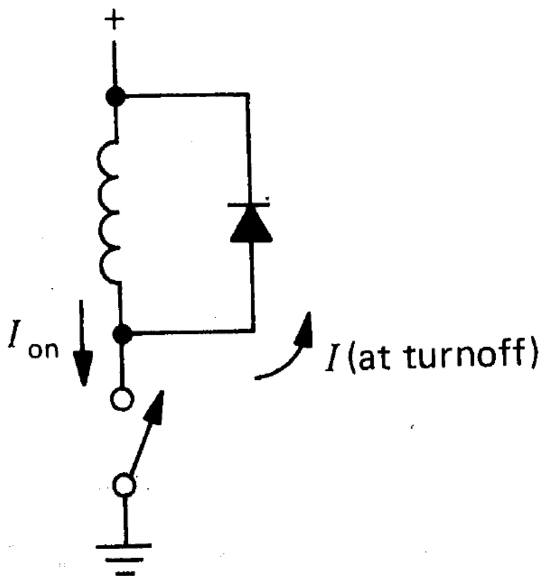

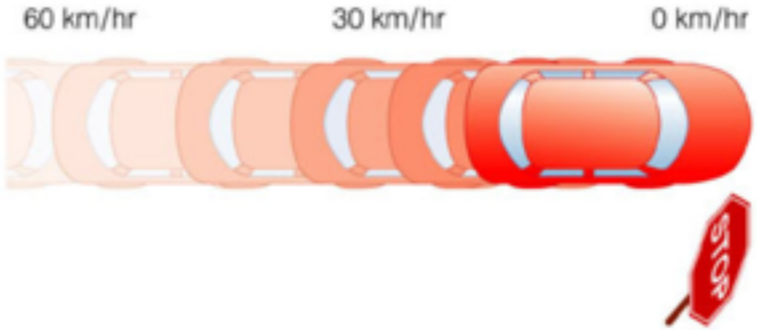

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