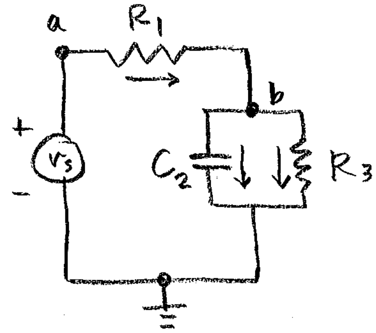
Problem 6.8



state: e_2

input: $v_s(t)$

Node voltages:

$$e_a = v_s(t)$$

$$e_b = e_2$$

KCL (b): $i_1 = i_2 + i_3$

Component relations:

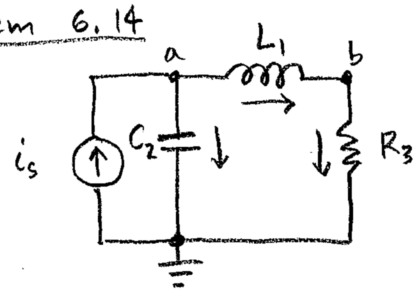
$$i_1 = \frac{e_a - e_b}{R_1} \qquad \dot{e}_2 = \frac{1}{C_2} i_2 \qquad i_3 = \frac{e_b}{R_3}$$

State equations:

$$\dot{e}_2 = \frac{1}{C_2} i_2 = \frac{1}{C_2}(i_1 - i_3) = \frac{1}{C_2}\left[\frac{1}{R_1}(v_s(t) - e_2) - \frac{e_2}{R_3}\right]$$

$$\dot{e}_2 = \frac{1}{C_2}\left[\frac{1}{R_1} v_s(t) - \left(\frac{1}{R_1} + \frac{1}{R_3}\right) e_2\right]$$

Problem 6.14



States : i_1 , e_2

Input: $i_s(t)$

Node voltages:

$e_a = e_2$

KCL (a) $i_s = i_1 + i_2$

(b) $i_1 = i_3$

Component relations :

$$\frac{di_1}{dt} = \frac{1}{L_1}(e_a - e_b) \qquad \dot{e}_2 = \frac{1}{C_2} i_2$$

$$i_3 = \frac{1}{R_3} e_b$$

State equations :

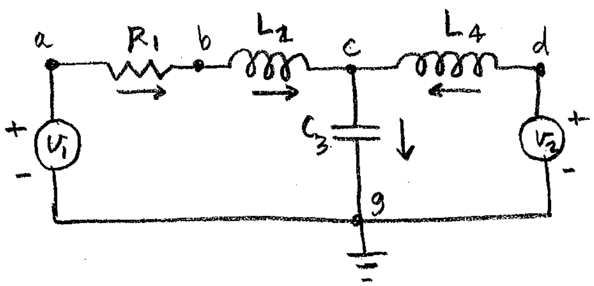
$$\frac{di_1}{dt} = \frac{1}{L_1}(e_2 - R_3 i_3)$$

$$\frac{di_1}{dt} = \frac{1}{L_1}(e_2 - R_3 i_1)$$

$$\dot{e}_2 = \frac{1}{C_2} i_2$$

$$\dot{e}_2 = \frac{1}{C_2}\left[i_s(t) - i_1\right]$$

Problem 6.16



States: $\dot{i}_2, e_3, \dot{i}_4$

Inputs: $v_1(t)$, $v_2(t)$

Node voltages:

$e_a = v_1(t)$, $e_d = v_2(t)$

$e_c = e_3$

Component relations:

$i_1 = \frac{1}{R_1}(e_a - e_b)$ $\qquad \frac{di_2}{dt} = \frac{1}{L_2}(e_b - e_c)$ $\qquad \dot{e}_3 = \frac{1}{C_3} i_3$

$\frac{di_4}{dt} = \frac{1}{L_4}(e_d - e_c)$

KCL ⓑ $i_1 = i_2$ $\qquad$ ⓒ $i_2 + i_4 = i_3$

State equations:

$$\frac{di_2}{dt} = \frac{1}{L_2}(e_b - e_3) = \frac{1}{L_2}\left[e_a - R_1 i_1 - e_3\right]$$

$$\frac{di_2}{dt} = \frac{1}{L_2}\left[v_1(t) - R_1 i_2 - e_3\right]$$

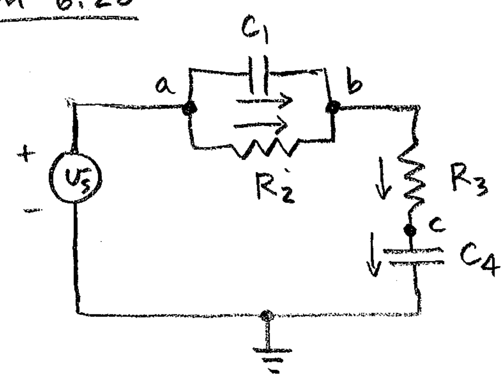
$$\dot{e}_3 = \frac{1}{C_3} i_3$$

$$\dot{e}_3 = \frac{1}{C_3}(i_2 + i_4)$$

$$\frac{di_4}{dt} = \frac{1}{L_4}(e_d - e_c)$$

$$\frac{di_4}{dt} = \frac{1}{L_4}\left[v_2(t) - e_3\right]$$

Problem 6.26



States : e_1, e_4

Input : $v_s(t)$

Node voltages:

$e_a = v_s(t) \qquad e_c = e_4$

$e_1 = e_a - e_b$

$e_b = v_s(t) - e_1$

Component Relations:

$$\dot{e}_1 = \frac{1}{C_1} i_1 \qquad i_2 = \frac{1}{R_2} e_1 \qquad i_3 = \frac{1}{R_3}(e_b - e_c)$$

$$= \frac{1}{R_3}\left[v_s(t) - e_1 - e_4\right]$$

$$\dot{e}_4 = \frac{1}{C_4} i_4$$

KCL: (b) $i_1 + i_2 = i_3$

(c) $i_3 = i_4$

State equations:

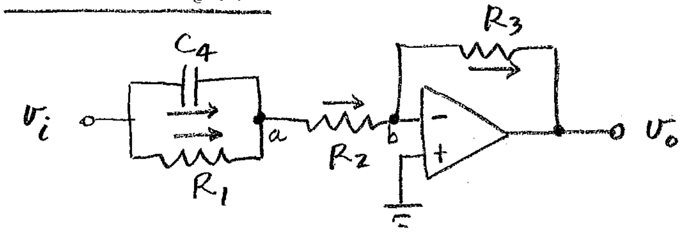
$$\dot{e}_1 = \frac{1}{C_1}(i_3 - i_2)$$

$$\dot{e}_1 = \frac{1}{C_1}\left[\frac{1}{R_3}\left(v_s(t) - e_1 - e_4\right) - \frac{1}{R_2} e_1\right]$$

$$\dot{e}_4 = \frac{1}{C_4} i_3$$

$$\dot{e}_4 = \frac{1}{C_4}\left[\frac{1}{R_3}\left(v_s(t) - e_1 - e_4\right)\right]$$

Problem 6.30



State : e_4

Input : $v_i(t)$

Node voltages : $e_4 = v_i - e_a \Rightarrow e_a = v_i(t) - e_4$

$$e_b = e^+ = 0$$

KCL: ⓐ $i_1 + i_4 = i_2$

ⓑ $i_2 = i_3$

Component Relations:

$$i_1 = \frac{1}{R_1}(v_i(t) - e_a) \qquad i_2 = \frac{1}{R_2}(e_a - e_b)$$

$$i_3 = \frac{1}{R_3}(e_b - v_o) \qquad \dot{e}_4 = \frac{1}{C_4} i_4$$

State equation:

$$i_1 = \frac{1}{R_1} e_4$$

$$i_2 = \frac{1}{R_2} e_a$$

$$\dot{e}_4 = \frac{1}{C_4} i_4 = \frac{1}{C_4}(i_2 - i_1)$$

$$= \frac{1}{C_4}\left[\frac{1}{R_2} e_a - \frac{1}{R_1} e_4\right]$$

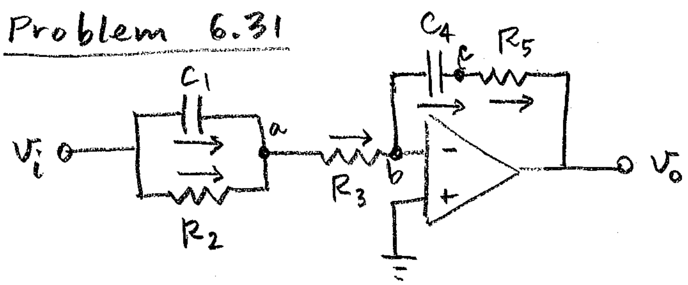
$$= \frac{1}{C_4}\left[\frac{1}{R_2}(v_1(t) - e_4) - \frac{1}{R_1} e_4\right]$$

$$\dot{e}_4 = \frac{1}{C_4}\left[\frac{1}{R_1} v_1(t) - \left(\frac{1}{R_1} + \frac{1}{R_2}\right) e_4\right]$$

$$v_o = \cancel{e_b}^{\,0} - R_3 i_3 = -\frac{R_3}{R_2}(v_1(t) - e_4)$$

$$v_o = -\frac{R_3}{R_2}\left[v_1(t) - e_4\right]$$

Problem 6.31



States: e_1, e_4
Input: $v_i(t)$

Node voltages: $e_a = v_i(t) - e_1$ $\quad e_b = 0$ $\quad e_4 = e_b - e_c$

$e_c = -e_4$

KCL: ⓐ $i_1 + i_2 = i_3$

ⓑ $i_3 = i_4$ $\quad$ ⓒ $i_4 = i_5$

Component Relations:

$$\dot{e}_1 = \frac{1}{C_1} i_1 \qquad i_2 = \frac{1}{R_2} e_1 \qquad i_3 = \frac{1}{R_3} e_a = \frac{1}{R_3}(v_i(t) - e_1)$$

$$\dot{e}_4 = \frac{1}{C_4} i_4 \qquad i_5 = \frac{1}{R_5}(e_c - v_o)$$

State equations:

$$\dot{e}_1 = \frac{1}{C_1} i_1 = \frac{1}{C_1}(i_3 - i_2)$$

$$= \frac{1}{C_1}\left[\frac{1}{R_3}(v_i(t) - e_1) - \frac{1}{R_2} e_1\right]$$

$$\dot{e}_1 = \frac{1}{C_1}\left[\frac{1}{R_3} v_i(t) - \left(\frac{1}{R_2} + \frac{1}{R_3}\right) e_1\right]$$

$$\dot{e}_4 = \frac{1}{C_4} i_4 = \frac{1}{C_4} i_3$$

$$\dot{e}_4 = \frac{1}{C_4}\left[\frac{1}{R_3}(v_i(t) - e_1)\right]$$

$$v_o = e_c - R_5 i_5 = -e_4 - R_5 i_3$$

$$v_o = -e_4 - \frac{R_5}{R_3}\left[v_i(t) - e_1\right]$$

Problem 7

(a)

States : e_2 , e_5

Input : $e_i(t)$

Node voltages :

$$e_A = e_2 \qquad e_B = 0 \qquad e_5 = \cancel{e_B}^{\,0} - e_0$$

KCL : Ⓐ $i_1 = i_2 + i_3 + i_4$

Ⓑ $i_3 = i_5$

Component Relations :

$$i_1 = \frac{1}{R_1}(e_i - e_2) \qquad \dot{e}_2 = \frac{1}{C_2} i_2 \qquad i_3 = \frac{1}{R_3} e_2$$

$$i_4 = \frac{1}{R_4}(e_A - e_0) \qquad \dot{e}_5 = \frac{1}{C_5} i_5$$

$$= \frac{1}{R_4}(e_2 + e_5)$$

State equations:

$$\dot{e}_2 = \frac{1}{C_2} i_2 = \frac{1}{C_2}\left[i_1 - i_3 - i_4 \right]$$

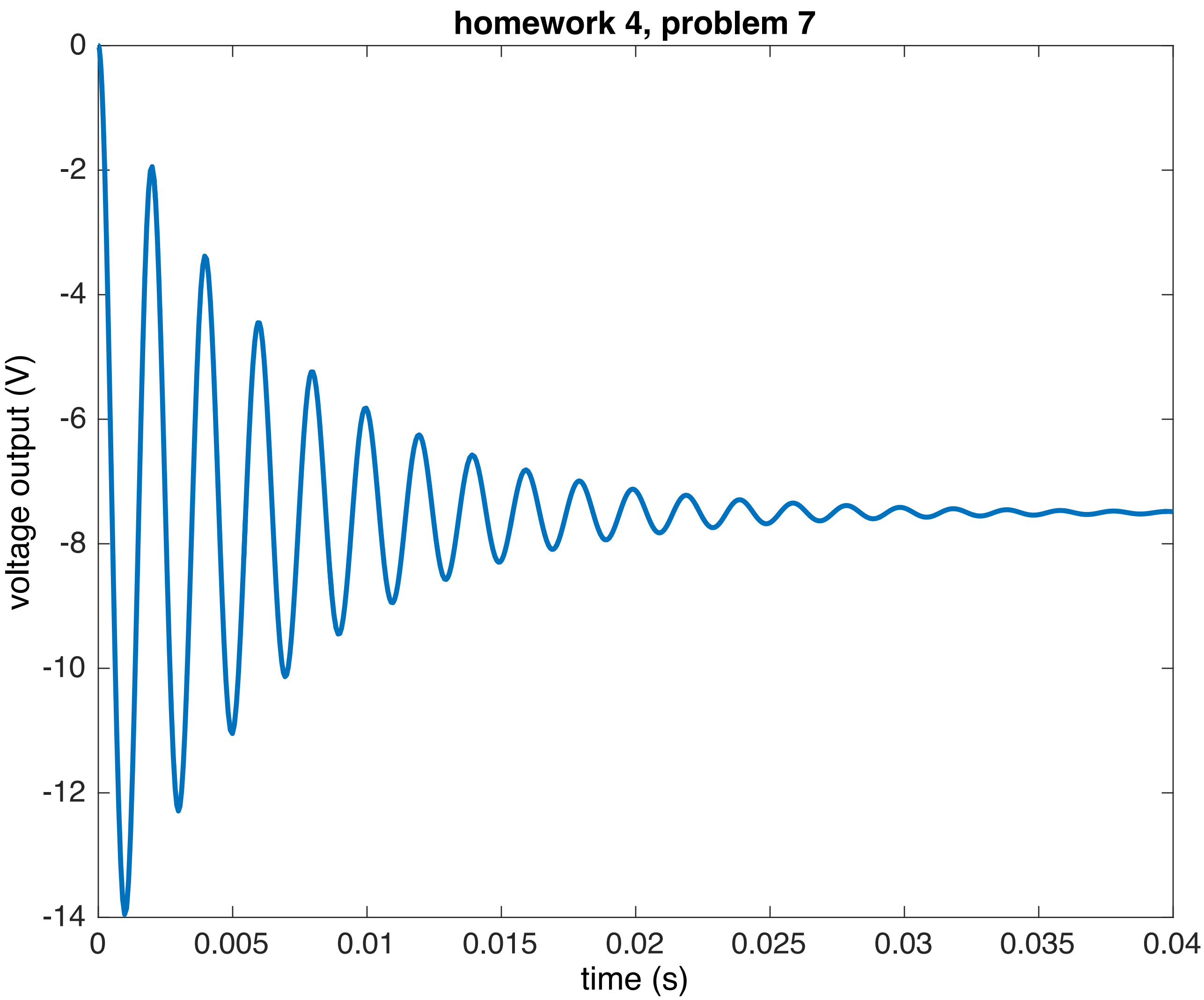
$$= \frac{1}{C_2}\left[\frac{1}{R_1}(e_i(t) - e_2) - \frac{1}{R_3} e_2 - \frac{1}{R_4}(e_2 + e_5) \right]$$

$$\dot{e}_2 = \frac{1}{C_2}\left[\frac{1}{R_1} e_i(t) - \left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4} \right) e_2 - \frac{1}{R_4} e_5 \right]$$

$$\dot{e}_5 = \frac{1}{C_5} i_5 = \frac{1}{C_5} i_3$$

$$\dot{e}_5 = \frac{1}{C_5 R_3} e_2$$

$$e_0 = -e_5$$


homework 4, problem 7
voltage output (V)
time (s)
0
-2
-4
-6
-8
-10
-12
-14
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04

```matlab
% This uses ode45 to simulate the step response of the op-amp circuit

tspan = [0 0.04];
x0 = [0; 0];

[t,x] = ode45(@opamp_eom,tspan,x0);

e2 = x(:,1);
e5 = x(:,2);

eo = -e5;    % output voltage

figure(1); clf;
plot(t,eo);
xlabel('time (s)');
ylabel('voltage output (V)');
title('homework 4, problem 7');
```

```matlab
function [xdot] = opamp_eom(t,x)
% opamp_eom.m  Function that implements state equations for op amp circuit
% Calculates responst to 7.5 V step input

% State variables: e2 and e5
e2 = x(1);
e5 = x(2);

% Model parameters
R1 = 10e3;
R3 = R1;
R4 = R1;
C2 = 1e-6;
C5 = 1e-9;

% Magnitude of step
A = 7.5;

e2dot = 1/C2*(1/R1*A - (1/R1+1/R3+1/R4)*e2 -1/R4*e5);
e5dot = 1/(C5*R3)*e2;

xdot = [e2dot; e5dot];

end
```