Mechanical Translation and Rotation

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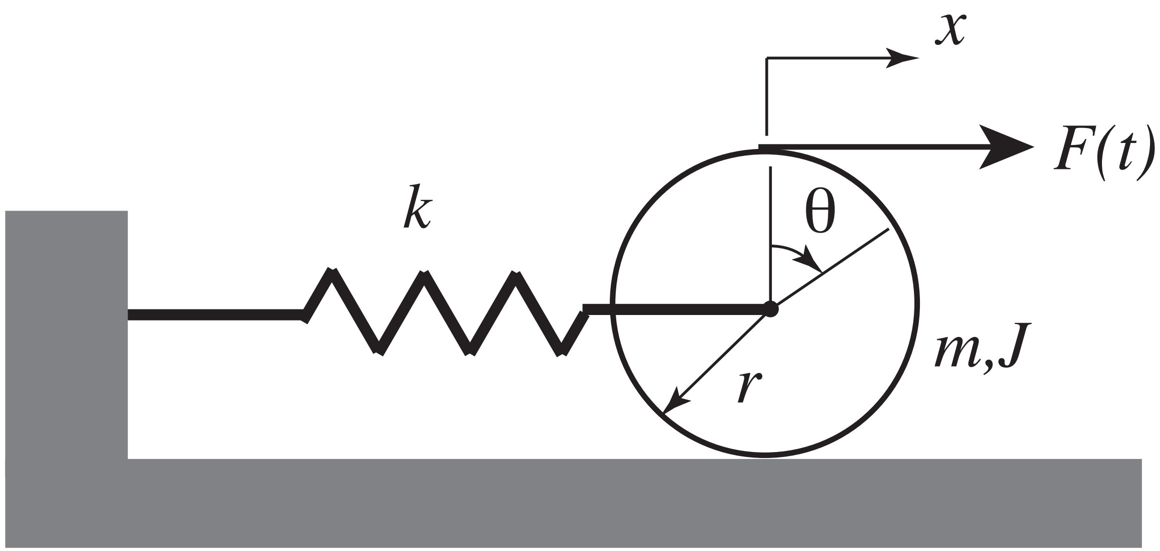
Deriving equations of motion for systems mixing translation and rotation

Five steps:

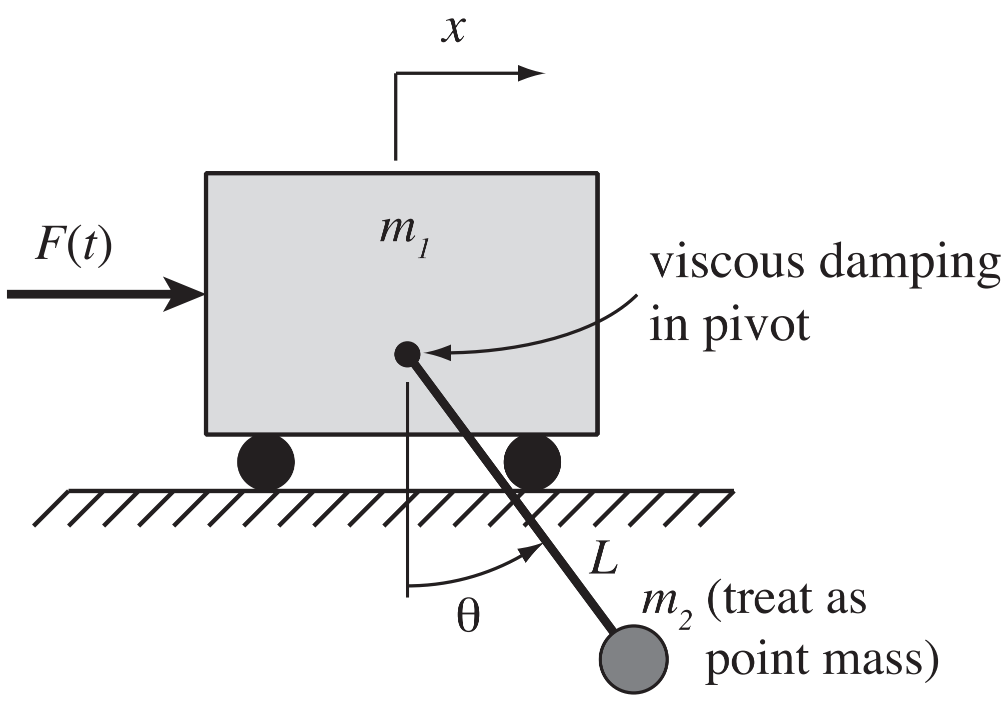
1. Define system geometry
2. Apply force and moment balance relations
3. Define kinematic relationships
4. Define constitutive relations for system elements
5. Combine relations to get EOM

Example – Rolling wheel



$$\frac{3}{2}m\ddot{x} + kx = 2F(t)$$

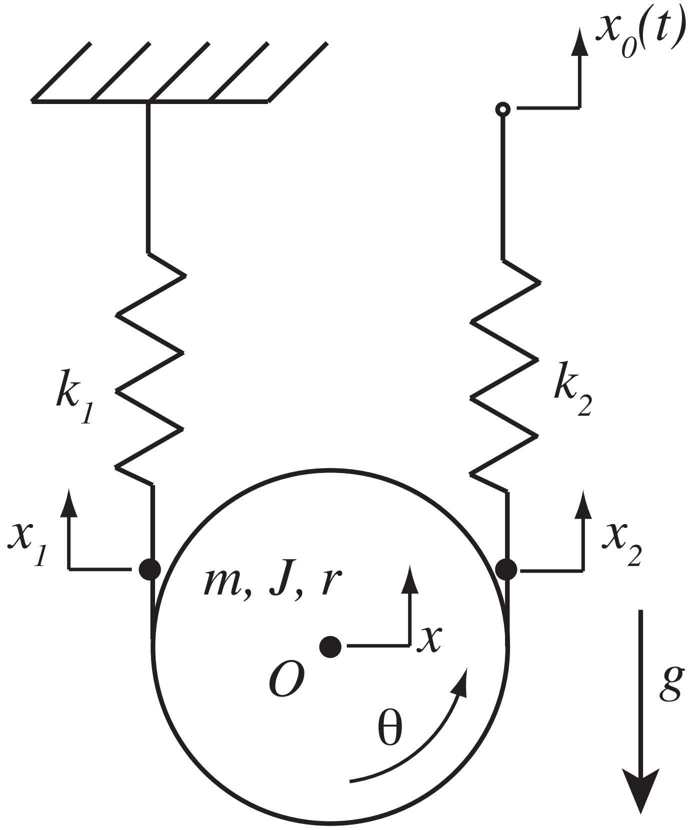
Example – Gantry crane



$$(m_2 L\cos\theta)\ddot{x} + m_2 L^2\ddot{\theta} + b\dot{\theta} + m_2 g L\sin\theta = 0$$

$$(m_1 + m_2)\ddot{x} + (m_2 L\cos\theta)\ddot{\theta} - (m_2 L\sin\theta)\dot{\theta}^2 = F(t)$$

Example – Thingamajig



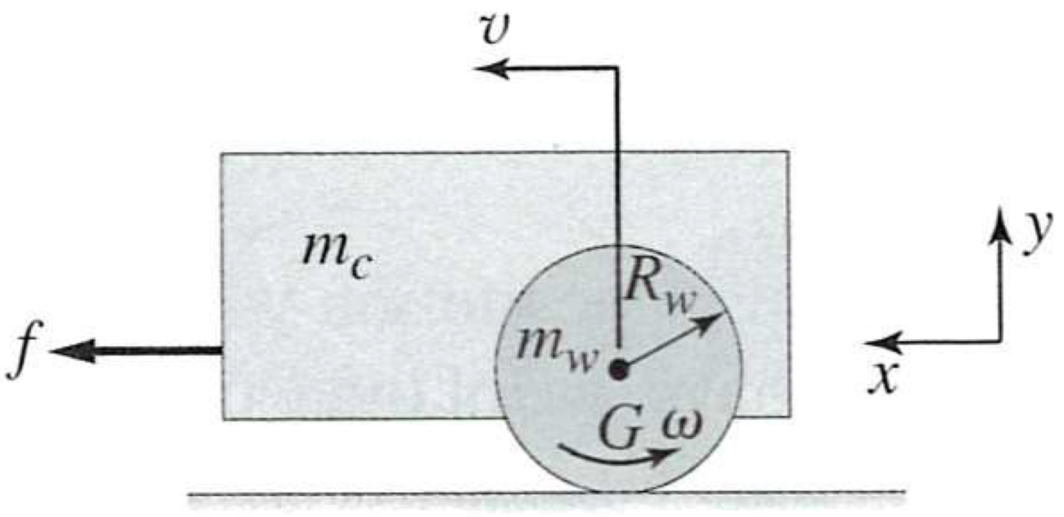
$$m\ddot{x} + k_1(x - r\theta) + k_2[x + r\theta - x_0(t)] = 0$$

$$J\ddot{\theta} + k_1 r(r\theta - x) + k_2 r[x + r\theta - x_0(t)] = 0$$

Equivalent mass/inertia

- Useful when you have a system of translating and/or rotating parts whose motions are *directly coupled*
  - Geared, wheeled, belt driven, etc.
  - Motions are related by kinematics (algebra)
- Equivalent mass or equivalent inertia?
  - View as mass if external force is applied
  - View as inertia if external torque is applied
- Simplified analysis involves kinetic energy
- Approach:
  - Write kinetic energy of system in terms of component velocities
  - Express kinetic energy in terms of single component velocity – equivalent mass/inertia will be evident
  - Lump mass/inertia elements into single element
  - Draw FBD and develop EOM

Example – Rolling cart (3.3.7)



$$m_e = m_c + 2m_w + \frac{2J_w}{R_w^2}$$

$$\begin{aligned}\dot{v} &= \frac{f}{m_e}\\ &= \frac{400\ \mathrm{N}}{130\ \mathrm{kg}}\\ \dot{v} &= 3.08\ \mathrm{m/s^2}\end{aligned}$$

What’s wrong with this model?