2.1) a) nonlinear due to $y\ddot{y}$ term

b) nonlinear due to $\sin y$ term

c) nonlinear due to $\sqrt{y}$ term

2.6) a) $x(t) = 10 + t^2$

$X(s) = \mathcal{L}\{x(t)\}$

$X(s) = \frac{10}{s} + \frac{2}{s^3}$ (lines 3 & 5 of Table 2.2.1)

b) $x(t) = 6te^{-5t} + e^{-3t}$

$= 6 \frac{1}{1!} t^1 e^{-5t} + e^{-3t}$

$x(t) = \frac{6}{(s+5)^2} + \frac{1}{s+3}$ (lines 6 and 7 of Table 2.2.1)

2.14) d) $\dot{x} + 7x = 4t \qquad x(0) = 5$

Take Laplace x-form:

$$sX(s) - x(0) + 7X(s) = \frac{4}{s^2}$$

$$(s+7)X(s) = \frac{4}{s^2} + 5$$

$$X(s) = \frac{5s^2+4}{s^2(s+7)} = \frac{5}{s+7} + \frac{4}{s^2(s+7)}$$

$$= 5\left(\frac{1}{s+7}\right) + \frac{4}{49}\left(\frac{7^2}{s^2(s+7)}\right)$$

$$= 5e^{-7t} + \frac{4}{49}\left(7t - 1 + e^{-7t}\right)$$

$$= \frac{245}{49}e^{-7t} + \frac{4}{7}t - \frac{4}{49} + \frac{4}{49}e^{-7t}$$

$$x(t) = \frac{4}{7}t - \frac{4}{49} + \frac{249}{49}e^{-7t}$$

2.15) a) $\ddot{x} + 10\dot{x} + 21x = 0 \qquad x(0) = 4, \ \dot{x}(0) = -3$

Take Laplace x-form:

$$s^2X(s) - sx(0) - \dot{x}(0) + 10[sX(s) - x(0)] + 21X(s) = 0$$

$$(s^2 + 10s + 21)X(s) = 4s - 3 + 40$$

$$X(s) = \frac{4s+37}{s^2+10s+21} = 4\frac{(s + 37/4)}{(s+3)(s+7)}$$

From line 14 with $p = \frac{37}{4}$, $a = 3$, $b = 7$:

$$x(t) = 4 \cdot \frac{1}{4}\left[\frac{25}{4}e^{-3t} - \frac{9}{4}e^{-7t}\right]$$

$$x(t) = \frac{25}{4}e^{-3t} - \frac{9}{4}e^{-7t}$$

Section 2.4

a) $\dot{x} + 2x = f(t)$

$$(s+2)X = \frac{1}{s} \implies X = \frac{1}{s(s+2)} = \frac{1}{2}\left(\frac{2}{s(s+2)}\right)$$

$$x(t) = \frac{1}{2} - \frac{1}{2}e^{-2t}$$

b) $\ddot{x} + 5\dot{x} + 6x = f(t)$

$$(s^2+5s+6)X = \frac{1}{s} \implies X = \frac{1}{s(s+2)(s+3)}$$

$$= \frac{1}{(s+0)(s+2)(s+3)}$$

a b c

$$x(t) = \frac{e^{0t}}{2\cdot 3} + \frac{e^{-2t}}{1\cdot(-2)} + \frac{e^{-3t}}{-3\cdot(-1)}$$

$$x(t) = \frac{1}{6} - \frac{1}{2}e^{-2t} + \frac{1}{3}e^{-3t}$$

c) $\ddot{x} + 4\dot{x} + 4 = f(t)$

$$(s^2+4s+4)X = \frac{1}{s} \implies (s+2)^2 X = \frac{1}{s}$$

$$X = \frac{1}{s(s+2)^2}$$

$$X = \frac{1}{4}\,\frac{2^2}{s(s+2)^2}$$

$$x(t) = \frac{1}{4}\left[1-(2t+1)e^{-2t}\right]$$

$$x(t) = \frac{1}{4} - \left(\frac{1}{2}t + \frac{1}{4}\right)e^{-2t}$$

d) $\ddot{x} + 4x = f(t)$

$$(s^2+4)X = \frac{1}{s} \implies X = \frac{1}{s(s^2+4)} = \frac{1}{4}\,\frac{2^2}{s(s^2+2^2)}$$

$$x(t) = \frac{1}{4}(1-\cos 2t)$$

Section 2.4 cont.

e) $\ddot{x} + 2\dot{x} + 4x = f(t)$

$$(s^2 + 2s + 4)X = \frac{1}{s} \implies X = \frac{1}{s(s^2+2s+4)}$$

$$= \frac{1}{s\left[(s+\underset{a}{1})^2 + (\underset{b}{\sqrt{3}})^2\right]}$$

$$x(t) = \frac{1}{4}\left[1 - \left(\frac{1}{\sqrt{3}}\sin\sqrt{3}t + \cos\sqrt{3}t\right)e^{-t}\right]$$

2.23) a) $\ddot{x} + 8\dot{x} + 15x = 30 \qquad x(0) = 10 \qquad \dot{x}(0) = 4$

$$s^2X - s\,x(0) - \dot{x}(0) + 8sX - 8x(0) + 15X = \frac{30}{s}$$

$$(s^2 + 8s + 15)X = \frac{30}{s} + 10s + 84$$

$$s(s+3)(s+5)X = 10s^2 + 84s + 30$$

$$X = \frac{10s}{(s+3)(s+5)} + \frac{84}{(s+3)(s+5)} + \frac{30}{s(s+3)(s+5)}$$

$$= 10\frac{s+0}{(s+3)(s+5)} + 84\frac{1}{(s+3)(s+5)} + 30\frac{1}{(s+0)(s+3)(s+5)}$$

$$x(t) = \underbrace{10\cdot\tfrac{1}{2}\left[(0-3)e^{-3t} - (0-5)e^{-5t}\right] + 84\left[\tfrac{1}{2}\left(e^{-3t} - e^{-5t}\right)\right]}_{\text{free}}$$

$$+ 30\left[\frac{1}{3\cdot 5} + e^{-3t}\cdot\frac{1}{2\cdot(-3)} + e^{-5t}\cdot\frac{1}{(-5)(-2)}\right] \leftarrow \text{forced}$$

$$= (-15 + 42 - 5)e^{-3t} + (25 - 42 + 3)e^{-5t} + 2$$

forced

$$x(t) = 22e^{-3t} - 14e^{-5t} + 2$$

Forced: $x(t) = 2 - 5e^{-3t} + 3e^{-5t}$

Free: $x(t) = 25e^{-3t} - 17e^{-5t}$ ← free response is total response minus forced response

Steady: $x(t) = 2$

Transient: $x(t) = 22e^{-3t} - 14e^{-5t}$

b) $\ddot{x} + 10\dot{x} + 25x = 75 \qquad x(0) = 10, \quad \dot{x}(0) = 4$

$$s^2X - s\,x(0) - \overset{4}{\dot{x}(0)} + 10sX - \overset{100}{10x(0)} + 25X = \frac{75}{s}$$

$$(s^2 + 10s + 25)X = \frac{75}{s} + 10s + 104$$

$$X = \frac{75}{s(s+5)^2} + \frac{10s}{(s+5)^2} + \frac{104}{(s+5)^2}$$

$$= 3\frac{5^2}{s(s+5)^2} + \frac{10s+50}{(s+5)^2} + \frac{104-50}{(s+5)^2}$$

$$= 3\frac{5^2}{s(s+5)^2} + 10\frac{1}{(s+5)} + 54\frac{1}{(s+5)^2}$$

(cont.)

(2.23 b cont.)

$$x(t) = \overbrace{3\left[1-(5t+1)e^{-5t}\right]}^{\text{forced}} + \overbrace{10e^{-5t} + 54\left[te^{-5t}\right]}^{\text{free}}$$

$$x(t) = \underbrace{3}_{\text{steady state}} + \underbrace{7e^{-5t} + 39te^{-5t}}_{\text{transient}} \sim \text{total}$$

c) $\ddot{x} + 25x = 100 \qquad x(0) = 10 \qquad \dot{x}(0) = 4$

$$s^2X - s\,x(0) - \dot{x}(0) + 25X = \frac{100}{s}$$

$$(s^2+25)X = \frac{100}{s} + 10s + 4$$

$$X = 4\frac{5^2}{s(s^2+5^2)} + 10\frac{s}{s^2+5^2} + \frac{4}{5}\frac{5}{s^2+5^2}$$

$$x(t) = \underbrace{4(1-\cos 5t)}_{\text{forced}} + \underbrace{10\cos 5t + \frac{4}{5}\sin 5t}_{\text{free}}$$

$$x(t) = \underbrace{4 + 6\cos 5t + \frac{4}{5}\sin 5t}_{\text{steady}} \sim \text{total response}$$

There is no transient response

d) $\ddot{x} + 8\dot{x} + 65x = 130 \qquad x(0) = 10 \qquad \dot{x}(0) = 4$

$$s^2X - s\,x(0) - \dot{x}(0) + 8sX - 8x(0) + 65X = \frac{130}{s}$$

$$(s^2+8s+65)X = \frac{130}{s} + 10s + 84$$

$$\left[(s+4)^2+7^2\right]X = \frac{130}{s} + 10s + 84$$

$$X = \frac{130}{s\left[(s+4)^2+7^2\right]} + \frac{10s}{\left[(s+4)^2+7^2\right]} + \frac{84}{\left[(s+4)^2+7^2\right]}$$

$$X = \;'' \; + 10\frac{(s+4)}{\left[(s+4)^2+7^2\right]} + \frac{44}{7}\frac{7}{\left[(s+4)^2+7^2\right]}$$

$$x(t) = \underbrace{\frac{130}{65}\left[1-\left(\frac{4}{7}\sin 7t + \cos 7t\right)e^{-4t}\right]}_{\text{forced}} + \underbrace{10e^{-4t}\cos 7t + \frac{44}{7}e^{-4t}\sin 7t}_{\text{free}}$$

(cont.)

(2.23 d) cont)

$$x(t) = \overbrace{2 - \left(\frac{8}{7}\sin 7t + \cos 7t\right)e^{-4t}}^{\text{forced}} + \overbrace{10e^{-4t}\cos 7t + \frac{44}{7}e^{-4t}\sin 7t}^{\text{free}}$$

$$x(t) = \underbrace{2}_{\text{steady}} + \overbrace{\frac{36}{7}e^{-4t}\sin 7t + 8e^{-4t}\cos 7t}^{\text{transient}}$$

Section 2.5

a) $$X = \frac{10s}{s^2+8s+15} + \frac{84}{s^2+8s+15} + \frac{30}{s(s^2+8s+15)}$$

$$x_{ss} = \lim_{s\to 0} s\,X(s)$$

$$= \lim_{s\to 0} \cancel{s}\,\frac{10s^2+84s+30}{\cancel{s}(s^2+8s+15)} = \frac{30}{5}$$

$$\underline{\underline{x_{ss} = 2}}$$

b) $$X = \frac{10s^2+104s+75}{s(s+5)^2}$$

$$x_{ss} = \lim_{s\to 0} \cancel{s}\,\frac{10s^2+104s+75}{\cancel{s}(s+5)^2} = \frac{75}{25}$$

$$\underline{\underline{x_{ss} = 3}}$$

c) $$X = \frac{10s^2+4s+100}{s(s^2+25)}$$

Roots of X are $\underline{s_{1,2} = \pm 5j}$

Because roots do not have negative real part, FVT cannot be used.

d) $$X = \frac{10s^2+84s+130}{s(s^2+8s+65)}$$

$$x_{ss} = \lim_{s\to 0} s\,\frac{10s^2+84s+130}{s(s^2+8s+65)} = \frac{130}{65}$$

$$x_{ss} = \underline{\underline{2}}$$

2.32) a) $5\dot{x} + 7x = 15 f(t)$

$(5s + 7)X = 15F$

$\frac{X}{F} = \frac{15}{5s+7}$ char. root: $s = -7/5$

b) $3\ddot{x} + 30\dot{x} + 63x = 5f(t)$

$(3s^2 + 30s + 63)X = 5F$

$\frac{X}{F} = \frac{5/3}{s^2 + 10s + 21}$ char roots: $s = -3$, $s = -7$

c) $\ddot{x} + 10\dot{x} + 21x = 4f(t)$

$(s^2 + 10s + 21)X = 4F$

$\frac{X}{F} = \frac{4}{s^2 + 10s + 21}$ char. roots: $s = -3$, $s = -7$

d) $\ddot{x} + 14\dot{x} + 49x = 7f(t)$

$(s^2 + 14x + 49)X = 7F$

$\frac{X}{F} = \frac{7}{s^2 + 14x + 49}$ char. roots: $s = -7$, $s = -7$

e) $\ddot{x} + 14\dot{x} + 58x = 6\dot{f}(t) + 4f(t)$

$(s^2 + 14s + 58)X = (6s + 4)F$

$\frac{X}{F} = \frac{6s + 4}{s^2 + 14s + 58}$ char. roots: $s = -7 \pm 3j$

f) $5\dot{x} + 7x = 4\dot{f}(t) + 15f(t)$

$(5s + 7)X = (4s + 15)F$

$\frac{X}{F} = \frac{4s + 15}{5s + 7}$ char root: $s = -\frac{7}{5}$

2.34) $\dot{x} = -2x + 5y \qquad \dot{y} = f(t) - 6y - 4x$

$$(s+2)X - 5Y = 0$$

$$4X + (s+6)Y = F$$

$$\underbrace{\begin{bmatrix} (s+2) & -5 \\ 4 & (s+6) \end{bmatrix}}_{A} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} F$$

$$\det A = (s+2)(s+6) + 20 = s^2 + 8s + 32$$

$$\frac{X}{F} = \frac{\det \begin{bmatrix} 0 & -5 \\ 1 & s+6 \end{bmatrix}}{\det A}$$

$$\boxed{\frac{X}{F} = \frac{5}{s^2 + 8s + 32}}$$

$$\frac{Y}{F} = \frac{\det \begin{bmatrix} s+2 & 0 \\ 4 & 1 \end{bmatrix}}{\det A}$$

$$\boxed{\frac{Y}{F} = \frac{s+2}{s^2 + 8s + 32}}$$

2.36) a) $\dot{x} = -4x + 2y + f(t)$ $\quad \dot{y} = -9y - 5x + g(t)$

$$(s+4)X - 2Y = F$$
$$5X + (s+9)Y = G$$

$$\underbrace{\begin{bmatrix} s+4 & -2 \\ 5 & s+9 \end{bmatrix}}_{A} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} F + \begin{bmatrix} 0 \\ 1 \end{bmatrix} G$$

$$\det A = (s+4)(s+9) + 10 = s^2 + 13s + 46$$

To find X/F, let $G = 0$:

$$\frac{X}{F} = \frac{\det \begin{bmatrix} 1 & -2 \\ 0 & s+9 \end{bmatrix}}{\det A}$$

$$\boxed{\frac{X}{F} = \frac{s+9}{s^2+13s+46}}$$

to find X/G, let $F = 0$:

$$\frac{X}{G} = \frac{\det \begin{bmatrix} 0 & -2 \\ 1 & s+9 \end{bmatrix}}{\det A}$$

$$\boxed{\frac{X}{G} = \frac{2}{s^2+13s+46}}$$