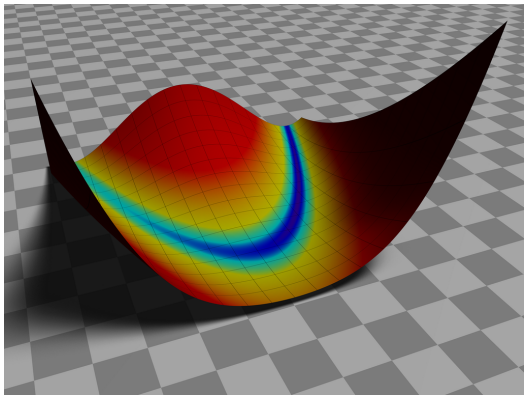
Integer and Discrete Programming

Lecture 27



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Outline

Integer Programming

Excel Example

Integer Programming

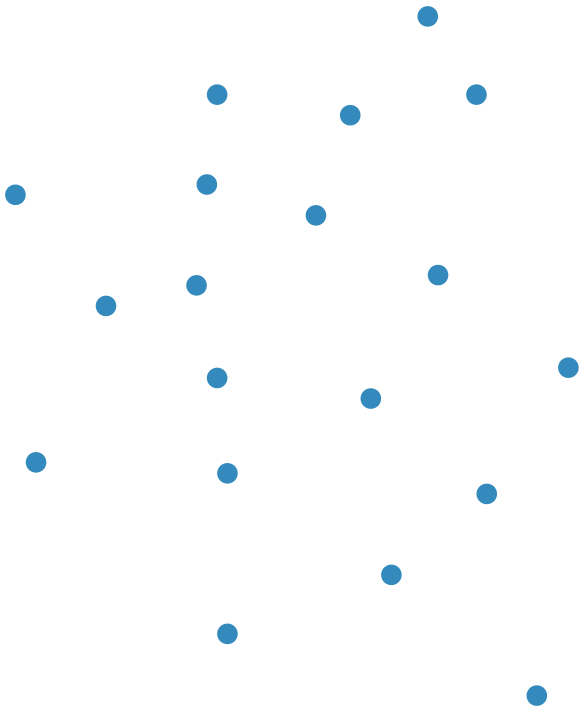
- binary: 0 or 1
- integer: -1, 0, 1, 2, . . .
- discrete: titanium, steel, aluminum, . . .

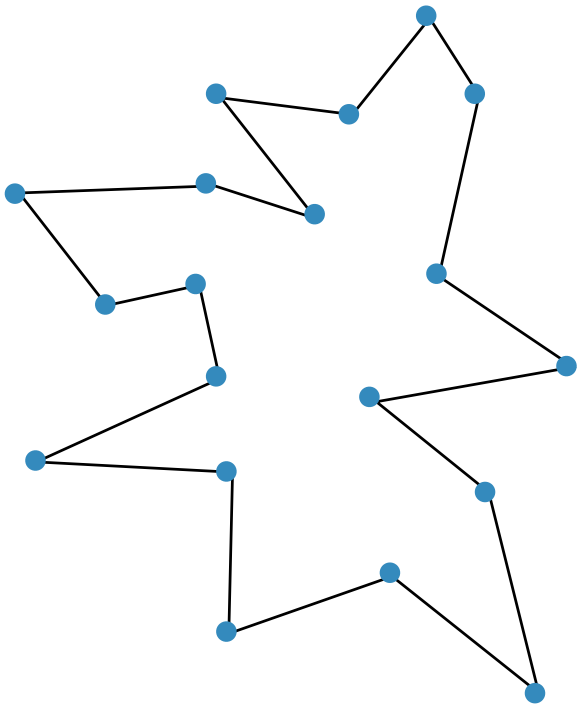
For our purposes today we will call of these integer problems.

Often used for scheduling, resource allocation, etc.

Integer programming is NP-hard

Traveling salesman problem: given a list of cities, what is the shortest possible route that visits each city exactly ones and returns to the starting city.





Number of combinations: $(n-1)!$

Imagine using all 50 state capitals in the U.S. That is

$$49! = 6.08 \times 10^{62}$$

Impossible to use a brute force approach.

Can we avoid using integer/discrete variables?

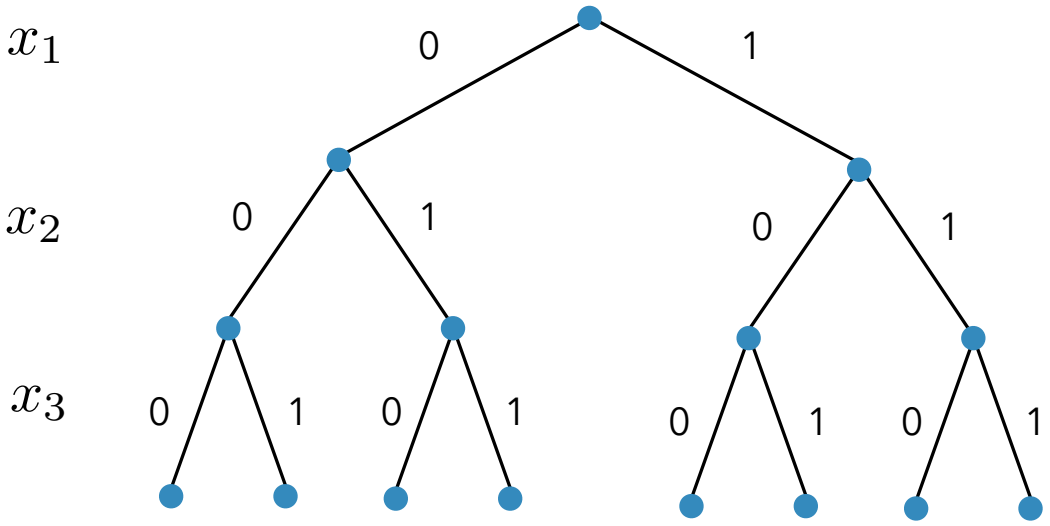
- If number of combinations for the integer variables is small, then we can perform several independent optimizations.
- Sometimes we can justifiable optimize with a continuous representation, then round to integer values afterwards (usually only justifiable if magnitude of variables is large).

Usually the only problems we can handle effectively are *linear* integer programming problems.

$$\begin{aligned} \text{minimize} \quad & c^T x \\ \text{subject to} \quad & Ax \leq b \\ & x_i \text{ is integer for some or all i} \end{aligned}$$

Called mixed-integer if some integer some not.

Branch and Bound Algorithm



Basic Idea

First solve the problem as a continous LP and look at the solution:

$$x^* = [3.4, 2.4, 5.3, 6.3]$$

Let's say we require the first three to be integers.

Pick one of the integer variables (e.g., x_1) and solve two new LP problems:

1. Original LP but with constaint that $x_1 \geq 4$
2. Original LP but with constaint that $x_1 \leq 3$

Four possible outcomes:

1. Found a solution, and all variables that are required to be integer are integer (save this as best solution we've found so far).
2. No feasible solution (backtrack).
3. The solution cannot be improved beyond our best solution so far (backtrack).
4. Solution can be improved, but some of our integer variables are still fractions (move deeper in the tree).

Implementations:

- intlinprog (Matlab)
- CVX
- Gurobi (Python, Matlab, C, etc.)
- CPLEX
- PuLP (Python)
- Solver (Excel)

If not linear, the optimization is difficult to solve with any reliability. However, we have already noted that some evolutionary algorithms, like a genetic algorithm, can be used with integer nonlinear programming problems.

Matlab Traveling Salesman Example

$$\binom{n}{x} = \binom{200}{2} = 19,900$$

Excel Example