Problem 5.13

given: $2\ddot{x} + 5\dot{x} + 4x = 4y(t)$

find: state variable form

states:

$$\begin{bmatrix} \dot{x} \\ x \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} \ddot{x} \\ \\ \dot{x} \end{bmatrix} = \begin{bmatrix} \frac{d\dot{x}}{dt} \\ \\ \frac{dx}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{5}{2}\dot{x} - 2x + 2y(t) \\ \\ \dot{x} \end{bmatrix}$$

problem 5.15

given:
$$m_1 \ddot{x}_1 + k_1(x_1 - x_2) = f(t) \quad (1)$$
$$m_2 \ddot{x}_2 - k_1(x_1 - x_2) + k_2 x_2 = 0 \quad (2)$$

find: state variable form

solve (1) & (2) for highest order derivatives

$$\ddot{x}_1 = \frac{-k_1}{m_1}(x_1 - x_2) + \frac{f(t)}{m_1}$$
$$\ddot{x}_2 = \frac{k_1}{m_2}(x_1 - x_2) - \frac{k_2}{m_2} x_2$$

states?

$$\begin{bmatrix} \dot{x}_1 \\ x_1 \\ \dot{x}_2 \\ x_1 \end{bmatrix}$$

state variable form:

$$\begin{bmatrix} \ddot{x}_1 \\ \dot{x}_1 \\ \ddot{x}_2 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{d\dot{x}_1}{dt} \\ \frac{dx_1}{dt} \\ \frac{d\dot{x}_2}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{k_1}{m_1} x_1 + \frac{k_1}{m_1} x_2 + \frac{f(t)}{m_1} \\ \dot{x}_1 \\ \frac{k_1}{m_2} x_1 - \frac{(k_1 + k_2)}{m_2} x_2 \\ \dot{x}_2 \end{bmatrix}$$

Problem 5.36

$$5\ddot{y} = 5g - (900y + 1700y^3) \qquad \dot{y}(0) = 0$$

Two IC's on position: (a) $y(0) = 0.06$

(b) $y(0) = 0.1$

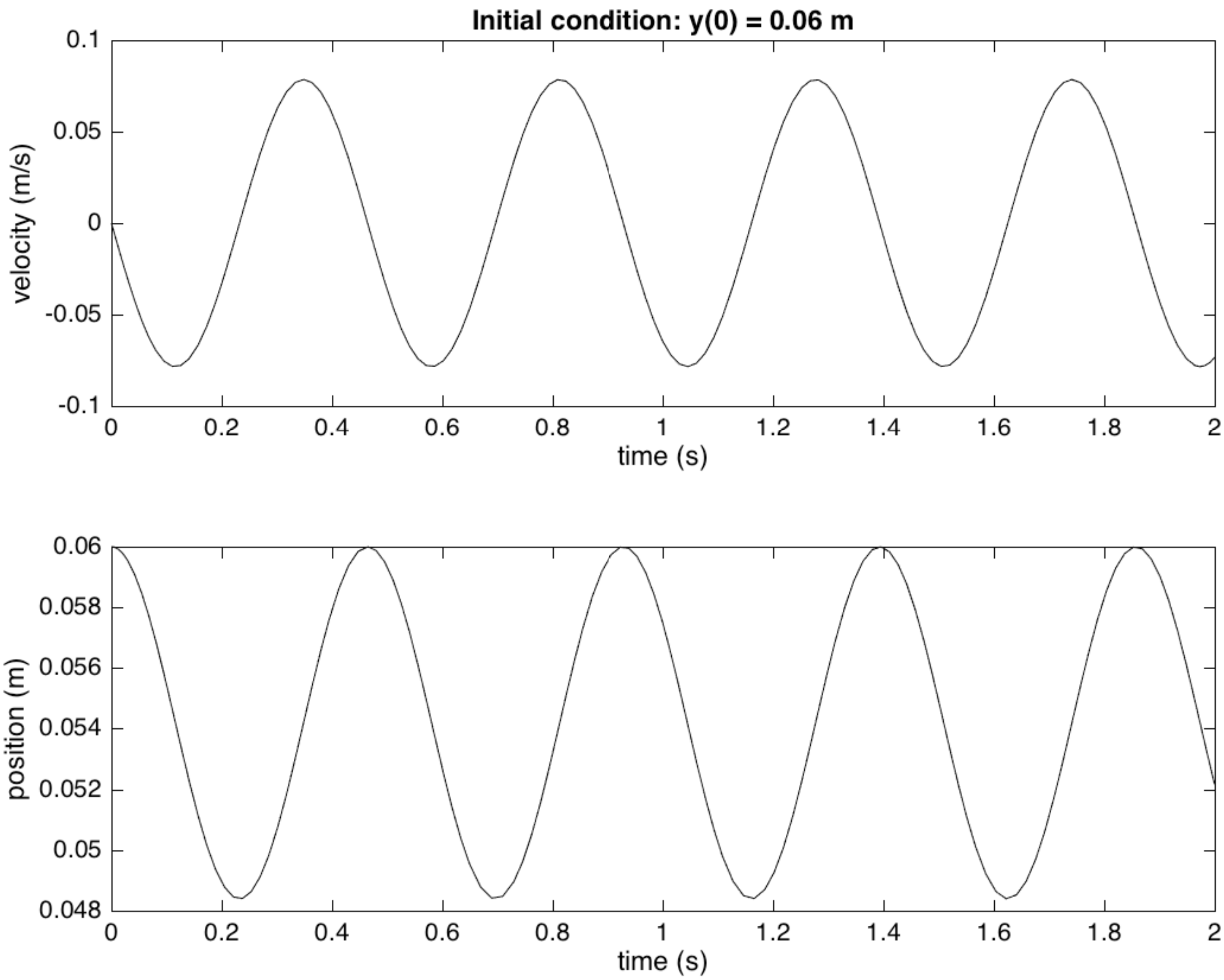
$$\ddot{y} = g - (180y + 340y^3)$$
$$\dot{y} = \dot{y}$$

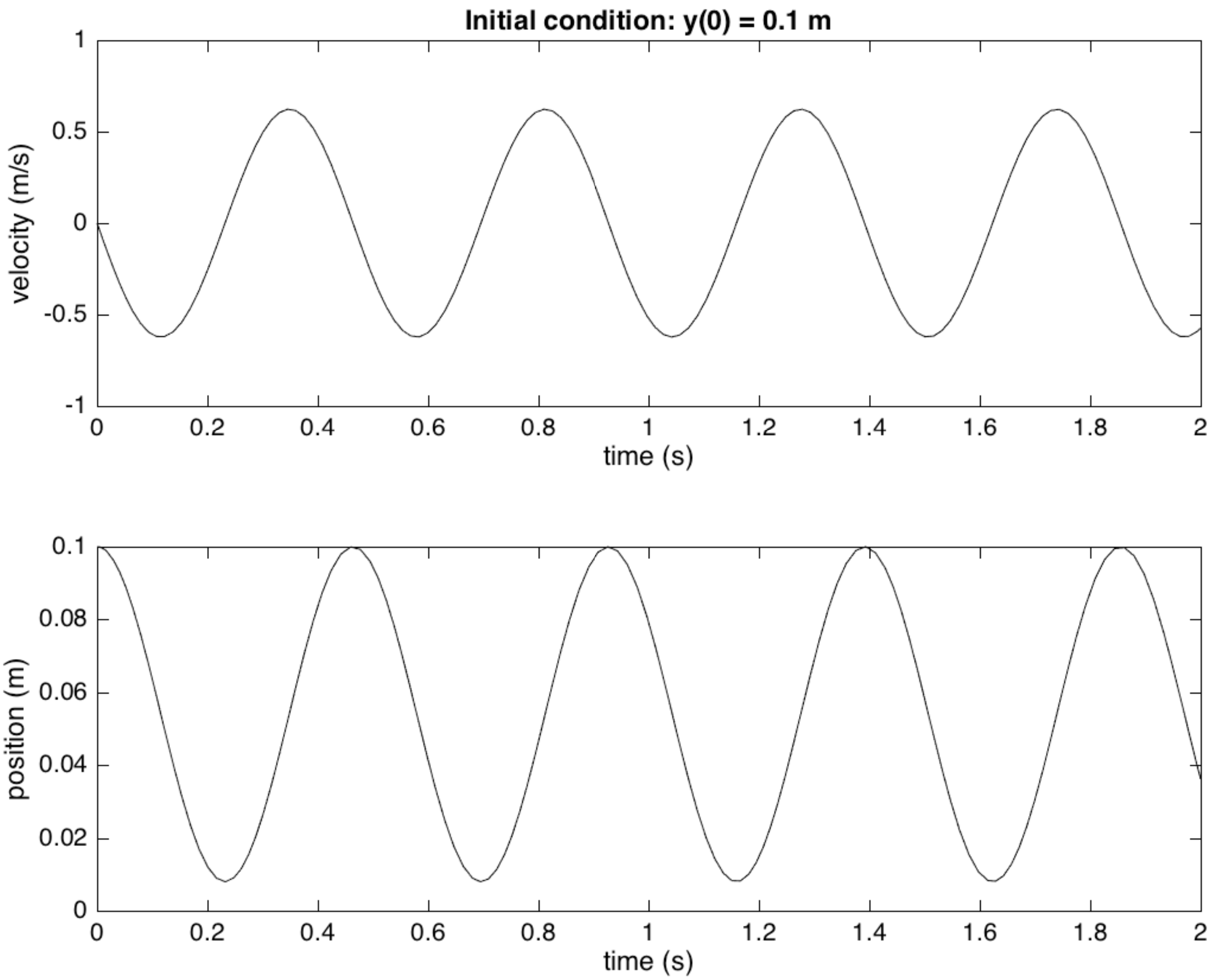
Let $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \dot{y} \\ y \end{bmatrix}$

$$\dot{x}_1 = g - (180x_2 + 340x_2^3)$$
$$\dot{x}_2 = x_1$$

To Matlab...

```
%Problem 5.36

clear all

%defining initial conditions
x0 = [0; 0.06];

%define time span of interest
tspan = [0 2];

%calling ode45 with the function that defines EOM
[t, x] = ode45(@p5_36_eom,tspan,x0);

figure(1); clf;
subplot(211);
plot(t, x(:,1))
xlabel('time (s)');
ylabel('velocity (m/s)');
subplot(212);
plot(t, x(:,2))
xlabel('time (s)');
ylabel('position (m)');




function dx = p5_36_eom(t,x)

%sizing xdot and initializing it to zero
dx = zeros(2,1);

g = 9.81;

% writing eom in state variable form
dx(1) = g - (180*x(2) + 340*x(2)^3);
dx(2) = x(1);
end
```


Initial condition: y(0) = 0.06 m
velocity (m/s)
0.1
0.05
0
-0.05
-0.1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
time (s)
position (m)
0.06
0.058
0.056
0.054
0.052
0.05
0.048
time (s)



Initial condition: y(0) = 0.1 m
1
0.5
0
-0.5
-1
velocity (m/s)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
time (s)
0.1
0.08
0.06
0.04
0.02
0
position (m)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
time (s)

Problem 5.39

$$L\ddot{\theta} + g\sin\theta = a(t)\cos\theta$$

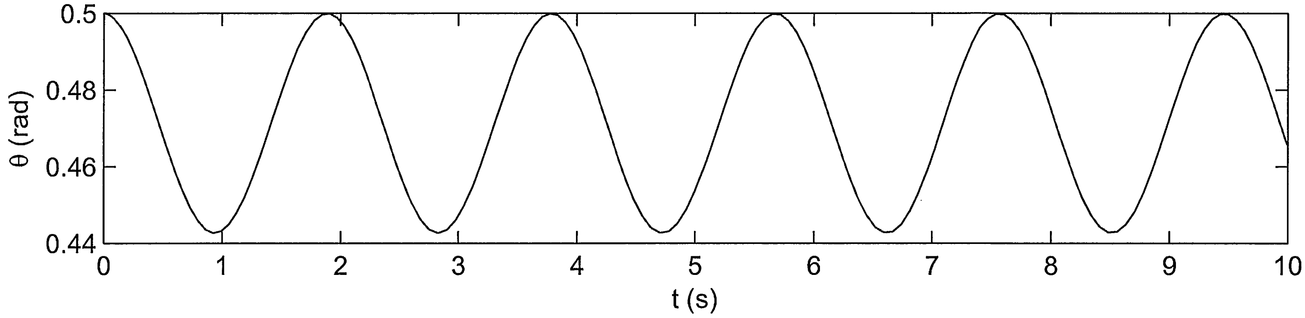
$a(t)$ is the input

State variable form:

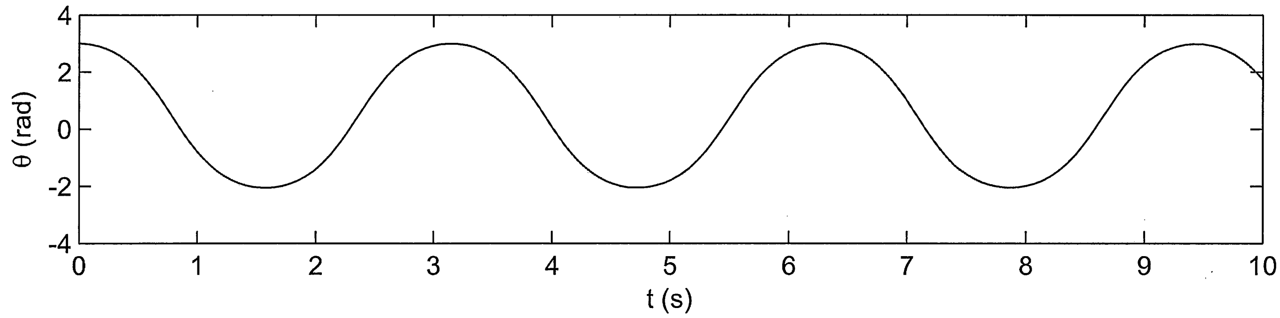
$$\left.\begin{aligned} x_1 &= \theta \\ x_2 &= \dot{\theta} = \dot{x}_1 \end{aligned}\right\} \quad \begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{g}{L}\sin x_1 + \frac{1}{L}a(t)\cos x_1 \end{aligned}$$

See the attached m-files and plots

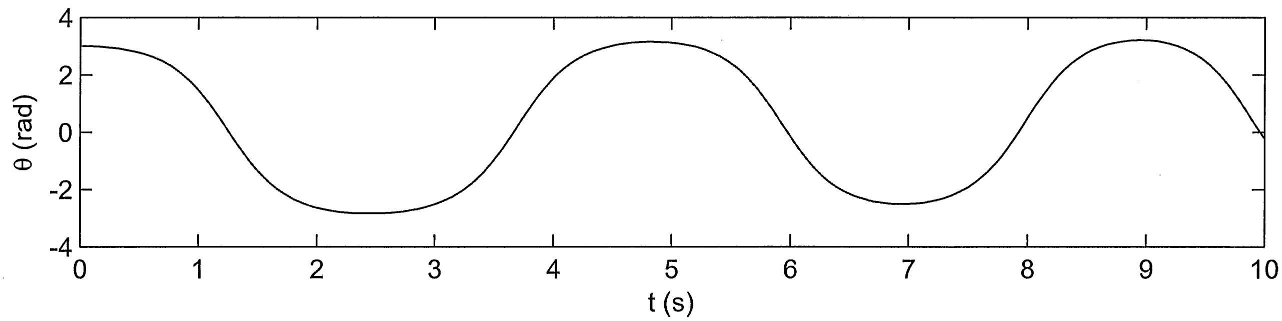
Palm 5.39.a



Palm 5.39.b



Palm 5.39.c



```
% Palm 5.39

clear;
clc;

% Parameters
global g L;
g = 9.81;
L = 1;
tspan = [0 10];


% Part a
x0 = [0.5 0];
[t,x] = ode45('palm5_39_ab', tspan, x0);
theta = x(:,1);
figure(1);
subplot(3,1,1);
plot(t,theta);
xlabel('t (s)');
ylabel('\theta (rad)');
title('Palm 5.39.a');


% Part b
x0 = [3 0];
[t,x] = ode45('palm5_39_ab', tspan, x0);
theta = x(:,1);
figure(1);
subplot(3,1,2);
plot(t,theta);
xlabel('t (s)');
ylabel('\theta (rad)');
title('Palm 5.39.b');
```

```
% Part c
x0 = [3 0];
[t,x] = ode45('palm5_39_c', tspan, x0);
theta = x(:,1);
figure(1);
subplot(3,1,3);
plot(t,theta);
xlabel('t (s)');
ylabel('\theta (rad)');
title('Palm 5.39.c');
```

```
function xdot = palm5_29_ab(t,x)

global g L;

a = 5;
xdot = [x(2); -g/L*sin(x(1)) + 1/L*a*cos(x(1))];
```

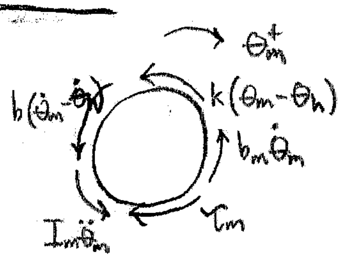
```
function xdot = palm5_29_c(t,x)

global g L;

a = 0.5*t;
xdot = [x(2); -g/L*sin(x(1)) + 1/L*a*cos(x(1))];
```

problem 5 of HW 3 -

given: - disk drive figure in homework & known parameters I_m, I_h, k, b, b_m
- states are $\dot{\theta}_m, \dot{\theta}_h, \theta_m, \theta_h$
- input is $\tau_m(t)$

find:
- EOM in state variable form
- find the order of system & if can be expressed in fewer variables

motor fbd -

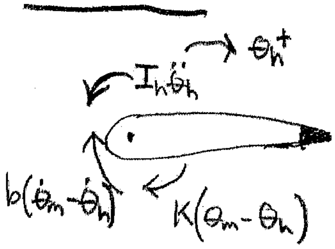


$\Sigma M^* = 0$:

$$-I_m\ddot{\theta}_m - b(\dot{\theta}_m-\dot{\theta}_h) - k(\theta_m-\theta_h) - b_m\dot{\theta}_m + \tau_m(t) = 0 \Rightarrow$$

$$\boxed{I_m\ddot{\theta}_m + b(\dot{\theta}_m-\dot{\theta}_h) + b_m\dot{\theta}_m + k(\theta_m-\theta_h) = \tau_m(t)}$$

head fbd -



$\Sigma M^* = 0$

$$-I_h\ddot{\theta}_h + b(\dot{\theta}_m-\dot{\theta}_h) + k(\theta_m-\theta_h) = 0 \Rightarrow$$

$$\boxed{I_h\ddot{\theta}_h + b(\dot{\theta}_h-\dot{\theta}_m) + k(\theta_h-\theta_m) = 0}$$

state variable form:

$$\begin{bmatrix} \dfrac{d\dot{\theta}_m}{dt} \\ \dfrac{d\theta_m}{dt} \\ \dfrac{d\dot{\theta}_h}{dt} \\ \dfrac{d\theta_h}{dt} \end{bmatrix} = \begin{bmatrix} \dfrac{-(b+b_m)}{I_m}\dot{\theta}_m - \dfrac{k}{I_m}\theta_m + \dfrac{b}{I_m}\dot{\theta}_h + \dfrac{k}{I_m}\theta_h + \dfrac{\tau_m}{I_m} \\ \dot{\theta}_m \\ \dfrac{b}{I_h}\dot{\theta}_m + \dfrac{k}{I_h}\theta_m - \dfrac{b}{I_h}\dot{\theta}_h - \dfrac{k}{I_h}\theta_h \\ \dot{\theta}_h \end{bmatrix}$$

system is fourth order with 4 state variables. Could use just 3 state variables since there are only 3 energy storage elements $\Rightarrow$

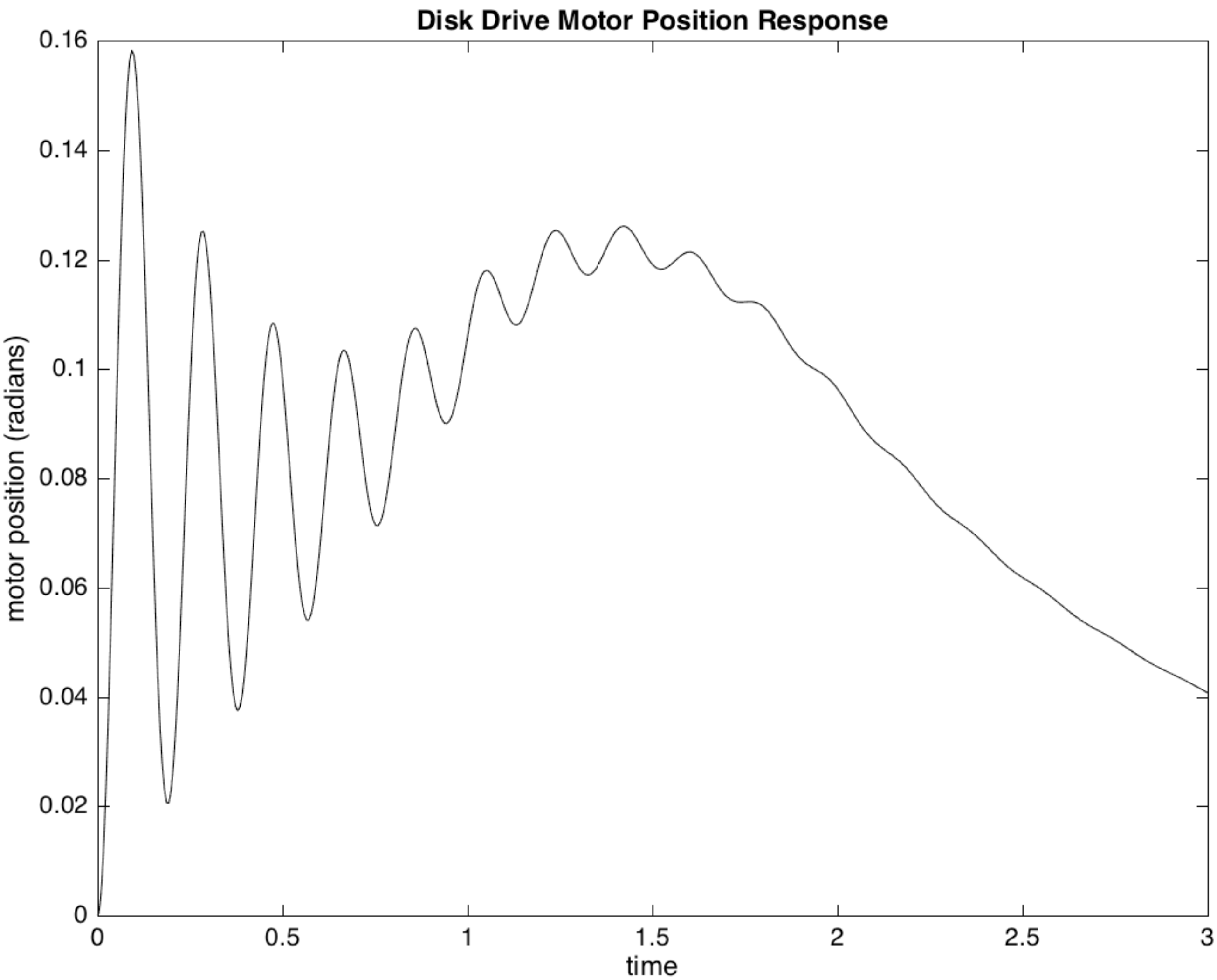
Example: (not required for homework)

states: $\begin{bmatrix} \dot{\theta}_m \\ \Delta\theta = \theta_m - \theta_h \\ \dot{\theta}_h \end{bmatrix} \Rightarrow$

$$\begin{bmatrix} \dfrac{d\dot{\theta}_m}{dt} \\ \dfrac{d\Delta\theta}{dt} \\ \dfrac{d\dot{\theta}_h}{dt} \end{bmatrix} = \begin{bmatrix} -\dfrac{(b+b_m)}{I_m}\dot{\theta}_m - \dfrac{k}{I_m}\Delta\theta + \dfrac{b}{I_m}\dot{\theta}_h + \dfrac{\tau_m}{I_m} \\ \dot{\theta}_m - \dot{\theta}_h \\ \dfrac{b}{I_h}\dot{\theta}_m + \dfrac{k}{I_h}\Delta\theta - \dfrac{b}{I_h}\dot{\theta}_h \end{bmatrix}$$

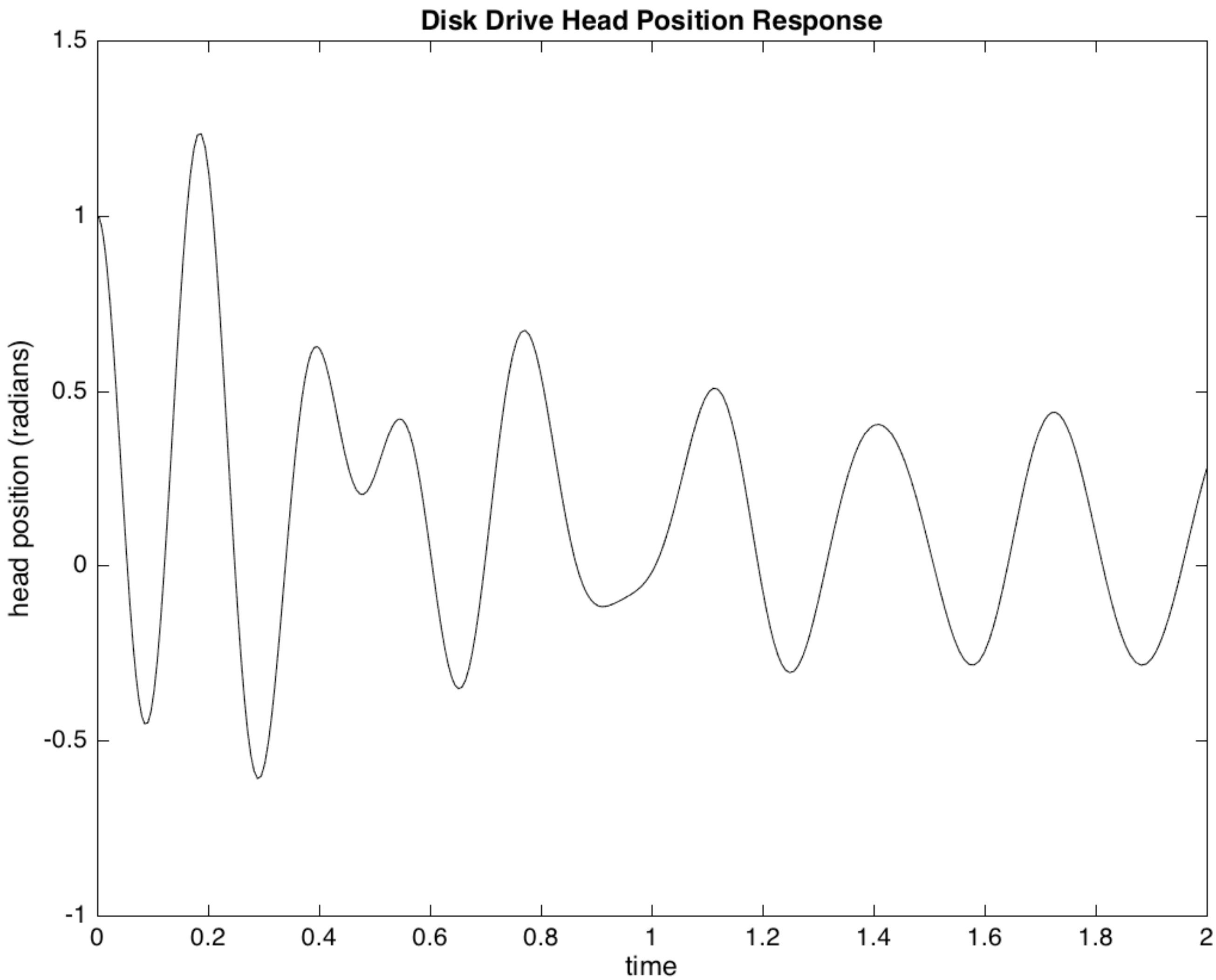
```
%Problem 6
%disk drive problem -- motor position

clear all

%defining initial conditions
x0 = [0; 0; 0; 1];

%define time range of interest
t_range = [0 3];

%calling ode45 with the function that defines our EOM in state variable form
[t, x] = ode45(@eom_disk_drive,t_range,x0);

%plotting only the motor position
plot(t, x(:,2))
xlabel('time');
ylabel('motor position (radians)');
title('Disk Drive Motor Position Response');
```

```
function xdot = eom_disk_drive(t,x)

%sizing xdot and initializing it to zero
xdot = zeros(4,1);

%define parameters, could do this just once and pass them in, but this is easier
Im = 0.1;   % kg-m^2
Ih = 0.01;  % kg-m^2
k = 10;     % N-m/rad
bm = 0.1;   % N-m-s/rad
b = 0.04;   % N-m-s/rad

%defining input
if t<1,
    tau_m = 0.02;      %problem 6
elseif t<2,
    tau_m = -0.02;
else
    tau_m = 0.0;
end

%tau_m = 10*cos(20*t); %problem 7

% writing eom in state variable form
xdot(1) = -(b+bm)/Im*x(1) - k/Im*x(2) + b/Im*x(3) + k/Im*x(4) + tau_m/Im;
xdot(2) = x(1);
xdot(3) = b/Ih*x(1) + k/Ih*x(2) - b/Ih*x(3) - k/Ih*x(4);
xdot(4) = x(3);
end
```


Disk Drive Motor Position Response
motor position (radians)
time
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0
0.5
1
1.5
2
2.5
3

```matlab
%Problem 7
%disk drive problem -- head position

clear all

%defining initial conditions
x0 = [0; 0; 0; 1];

%define time range of interest
t_range = [0 2];

%calling ode45 with the function that defines our EOM in state variable form
[t, x] = ode45(@eom_disk_drive,t_range,x0);

%plotting only the head position
plot(t, x(:,4))
xlabel('time');
ylabel('head position (radians)');
title('Disk Drive Head Position Response');
```

```matlab
function xdot = eom_disk_drive(t,x)

%sizing xdot and initializing it to zero
xdot = zeros(4,1);

%define parameters, could do this just once and pass them in, but this is easier
Im = 0.1;   % kg-m^2
Ih = 0.01;  % kg-m^2
k = 10;     % N-m/rad
bm = 0.1;   % N-m-s/rad
b = 0.04;   % N-m-s/rad

%defining input
%if t<1,
%    tau_m = 0.02;      %problem 6
%elseif t<2,
%    tau_m = -0.02;
%else
%    tau_m = 0.0;
%end

tau_m = 10*cos(20*t);   %problem 7

% writing eom in state variable form
xdot(1) = -(b+bm)/Im*x(1) - k/Im*x(2) + b/Im*x(3) + k/Im*x(4) + tau_m/Im;
xdot(2) = x(1);
xdot(3) = b/Ih*x(1) + k/Ih*x(2) - b/Ih*x(3) - k/Ih*x(4);
xdot(4) = x(3);
end
```


Disk Drive Head Position Response
head position (radians)
time
1.5
1
0.5
0
-0.5
-1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2