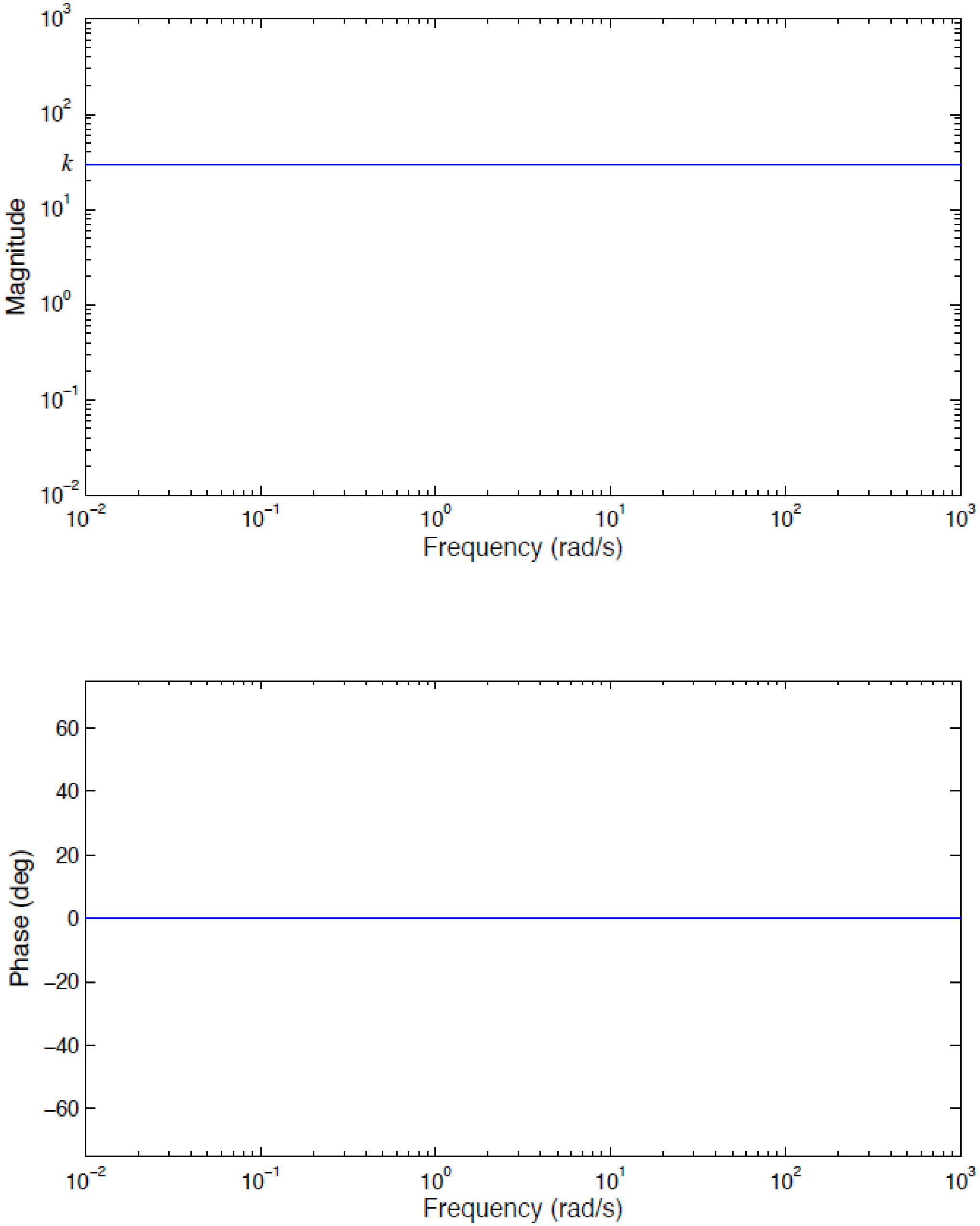
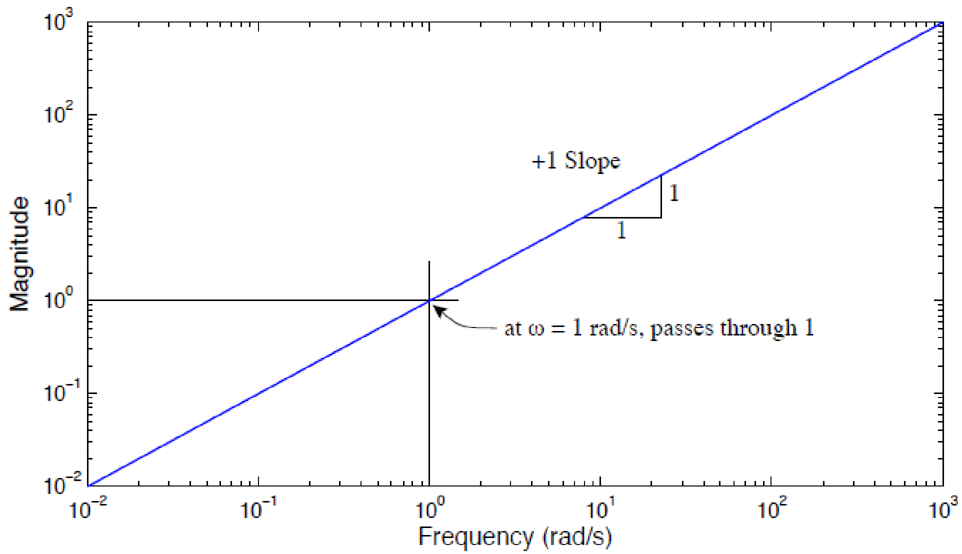
Types of Terms for Sketching Rules

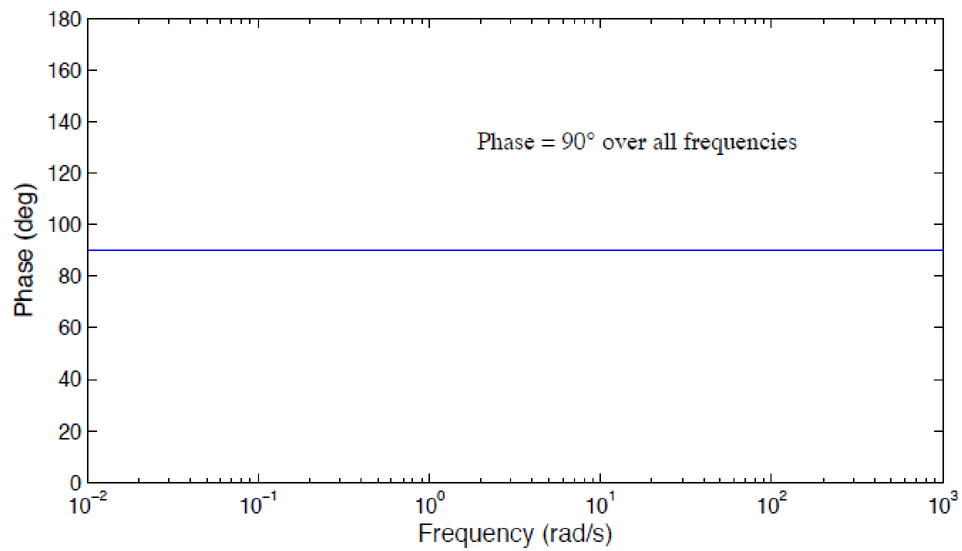
1. k (constant)
2. $(j\omega)^n$
3. $(j\omega)^{-n}$
4. $j\omega\tau + 1$
5. $\dfrac{1}{j\omega\tau + 1}$
6. $\left(\dfrac{j\omega}{\omega_n}\right)^2 + 2\zeta\left(\dfrac{j\omega}{\omega_n}\right) + 1$
7. $\dfrac{1}{\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_n}\right) + 1}$



$$k$$

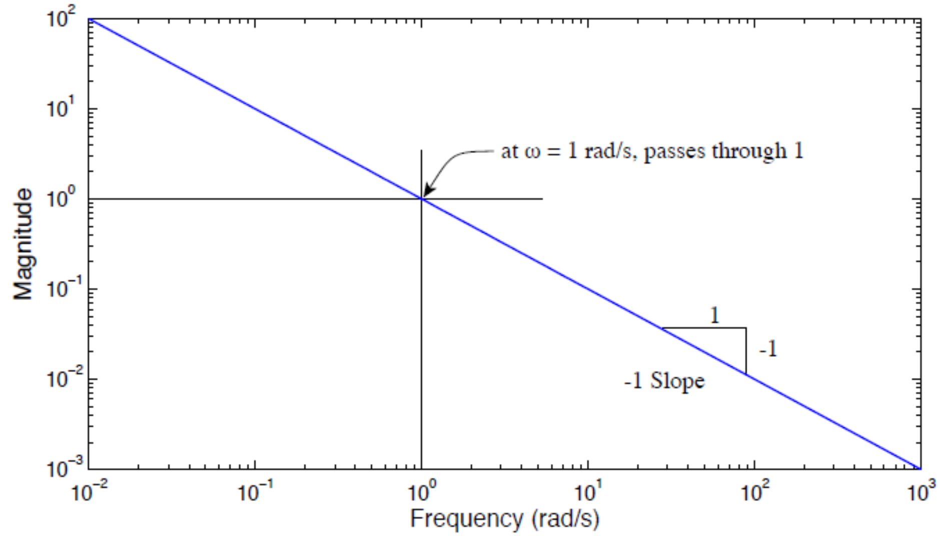
- magnitude plot is a horizontal line having magnitude k for all ω
- phase plot is $\phi = 0$ for all ω



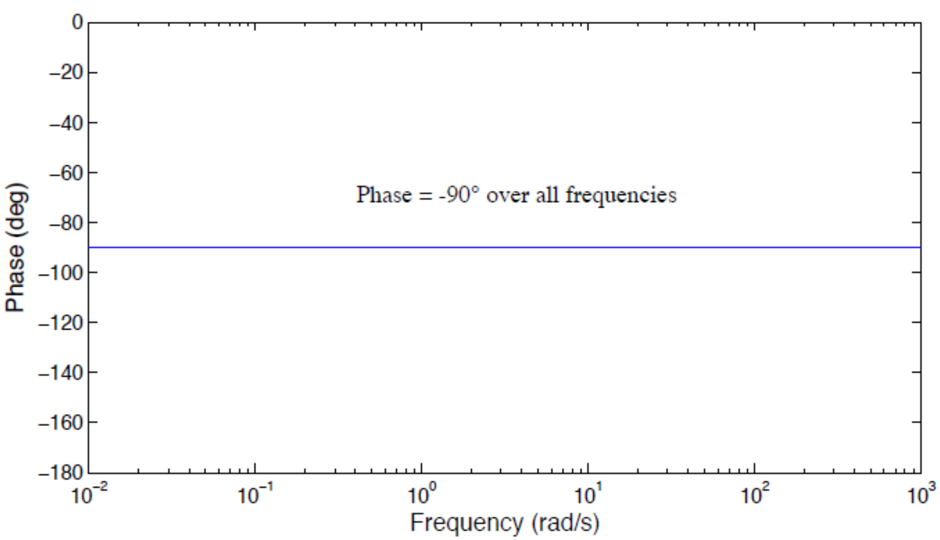


$$(j\omega)^n$$

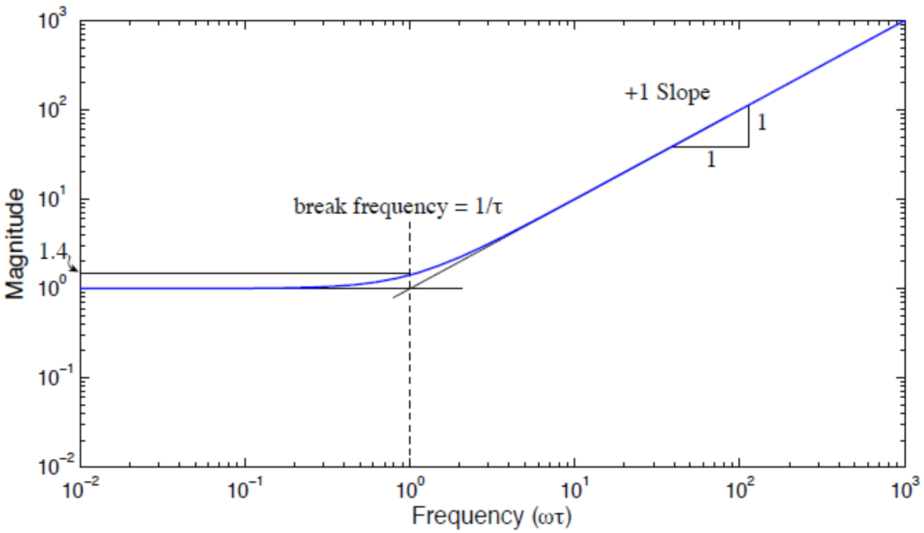
- magnitude plot is a straight line with slope n passing through 1 at $\omega = 1$
- phase plot is $\phi = n \times 90^\circ$ for all ω



$$(j\omega)^{-n}$$



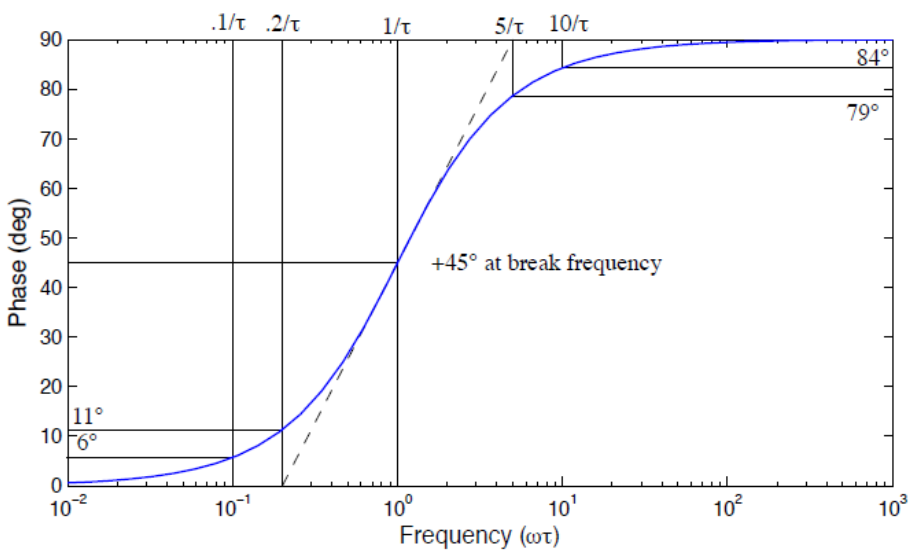
- magnitude plot is a straight line with slope $-n$ passing through 1 at $\omega = 1$ rad/sec
- phase plot is $\phi = -n \times 90^\circ$ for all ω



$$j\omega\tau + 1$$

$$\text{For } \omega \ll \frac{1}{\tau}, \quad |j\omega\tau + 1| \approx 1$$

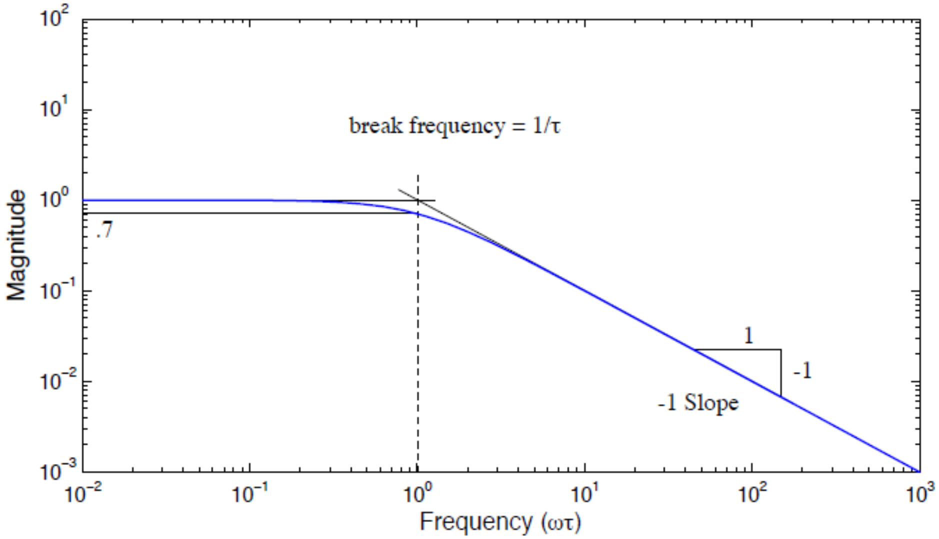
$$\text{For } \omega \gg \frac{1}{\tau}, \quad |j\omega\tau + 1| \approx \tau|j\omega| \leftarrow +1 \text{ slope}$$



$$\text{For } \omega \ll \frac{1}{\tau}, \quad \angle(j\omega\tau + 1) \approx \angle 1 = 0^\circ$$

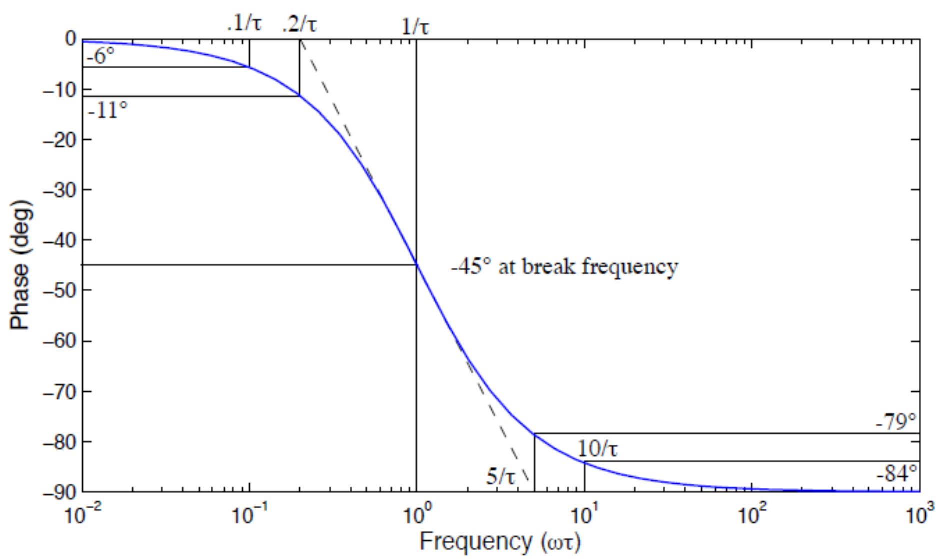
$$\text{For } \omega \gg \frac{1}{\tau}, \quad \angle(j\omega\tau + 1) \approx \angle(j\omega\tau) = 90^\circ$$

$$\frac{1}{j\omega\tau+1}$$



$$\text{For } \omega \ll \frac{1}{\tau}, \quad \left|\frac{1}{j\omega\tau+1}\right| \approx 1$$

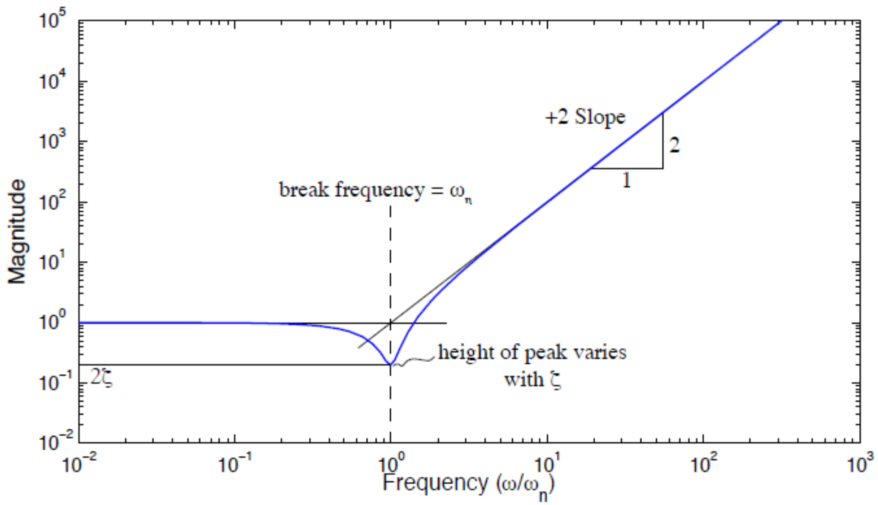
$$\text{For } \omega \gg \frac{1}{\tau}, \quad \left|\frac{1}{j\omega\tau+1}\right| \approx \frac{1}{\tau|j\omega|} \leftarrow -1 \text{ slope}$$



$$\text{For } \omega \ll \frac{1}{\tau}, \quad \angle\left(\frac{1}{j\omega\tau+1}\right) \approx \angle 1 = 0^\circ$$

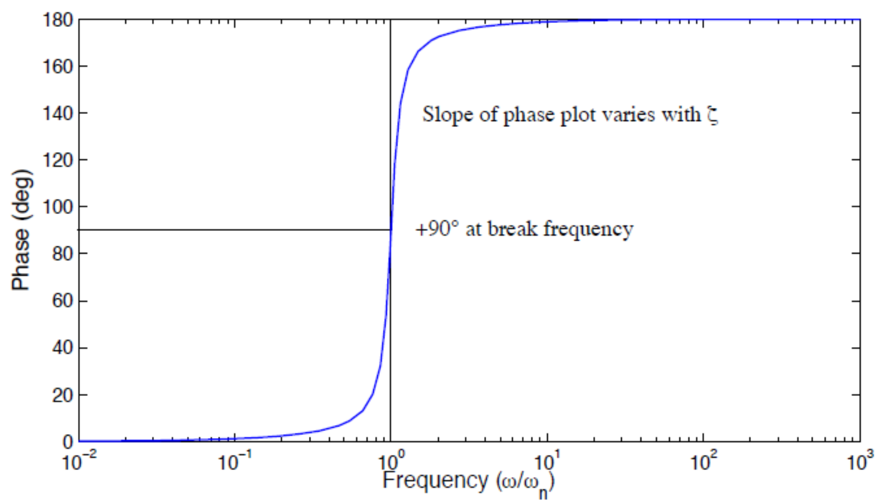
$$\text{For } \omega \gg \frac{1}{\tau}, \quad \angle\left(\frac{1}{j\omega\tau+1}\right) \approx \angle\frac{1}{j\omega\tau} = -90^\circ$$

$$\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_n}\right) + 1$$



For $\omega \ll \omega_n$, $\left|\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_n}\right) + 1\right| \approx 1$

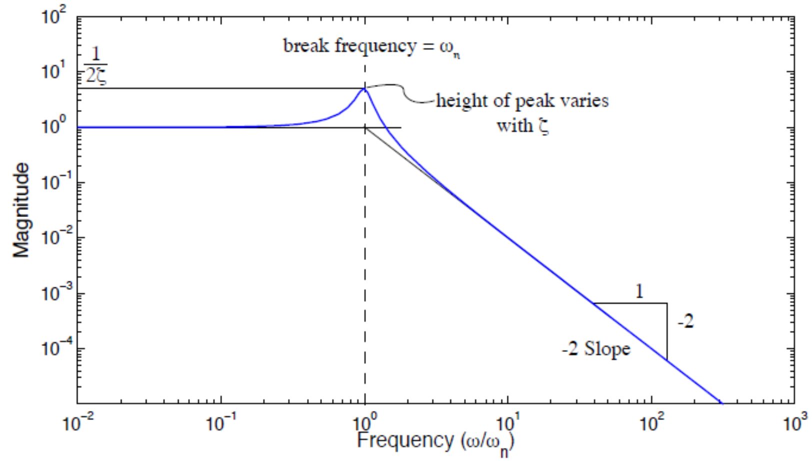
For $\omega \gg \omega_n$, $\left|\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_n}\right) + 1\right| \approx \left|\left(\frac{j\omega}{\omega_n}\right)^2\right| \leftarrow +2$ slope



For $\omega \ll \omega_n$, $\angle\left[\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_n}\right) + 1\right] \approx \angle 1 = 0^\circ$

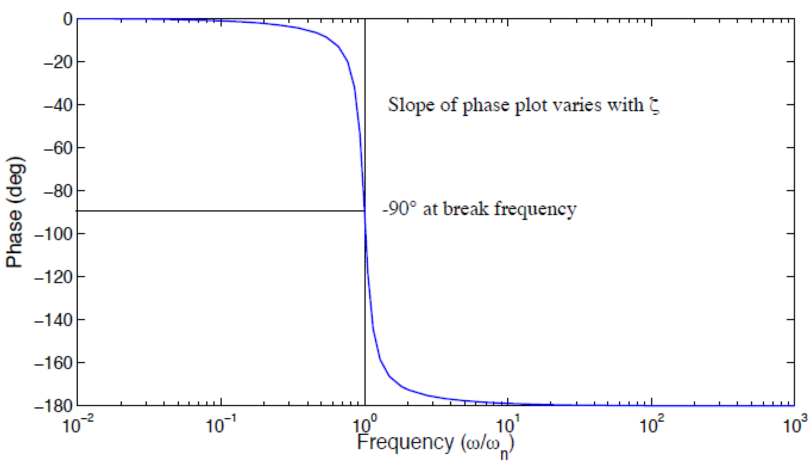
For $\omega \gg \omega_n$, $\angle\left[\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_n}\right) + 1\right] \approx \angle\left(\frac{j\omega}{\omega_n}\right)^2 = 180^\circ$

$$\frac{1}{\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_n}\right) + 1}$$



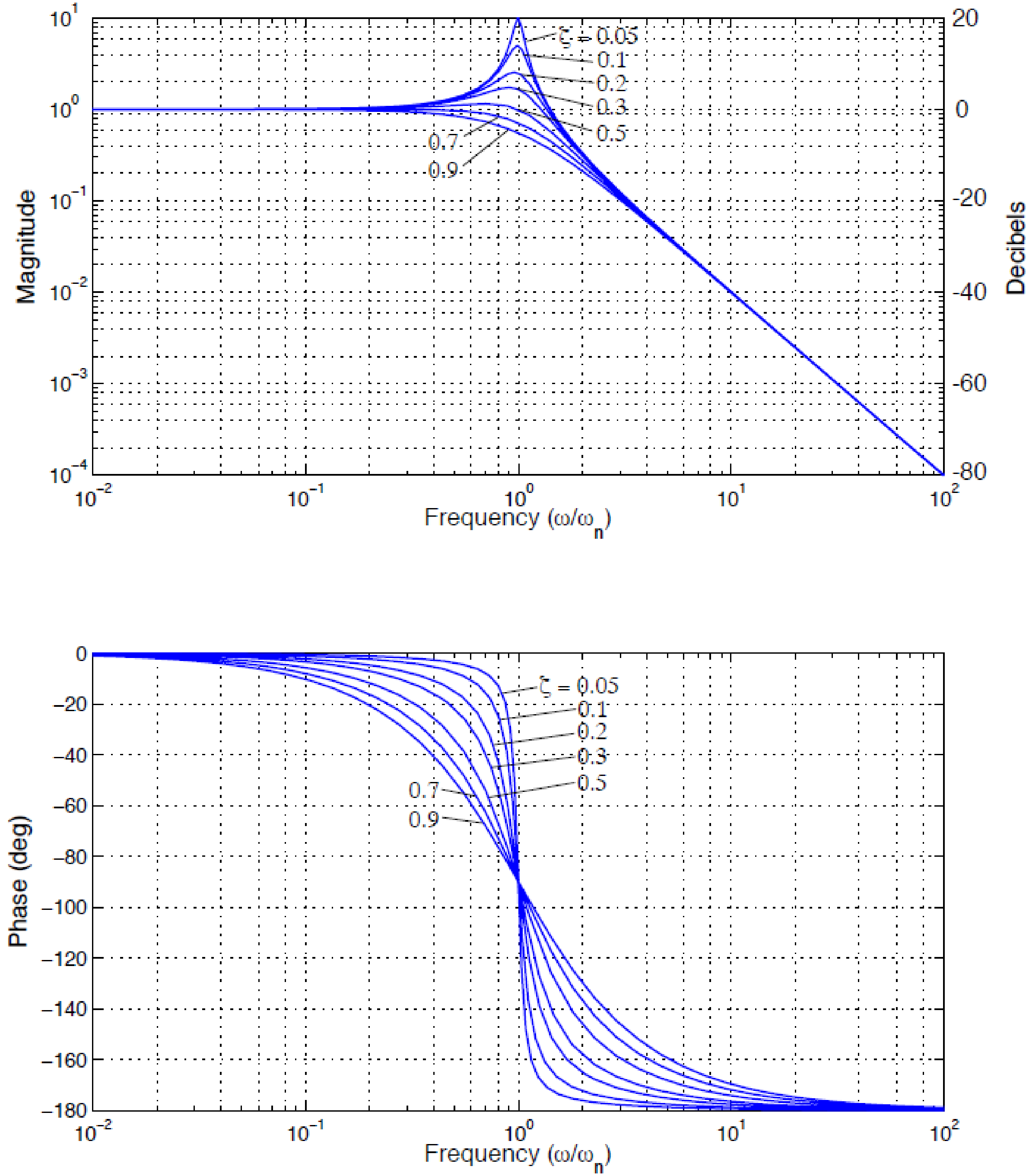
For $\omega \ll \omega_n$, $$\left|\frac{1}{\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_n}\right) + 1}\right| \approx 1$$

For $\omega \gg \omega_n$, $$\left|\frac{1}{\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_n}\right) + 1}\right| \approx \left|\left(\frac{j\omega}{\omega_n}\right)^{-2}\right| \leftarrow -2 \text{ slope}$$

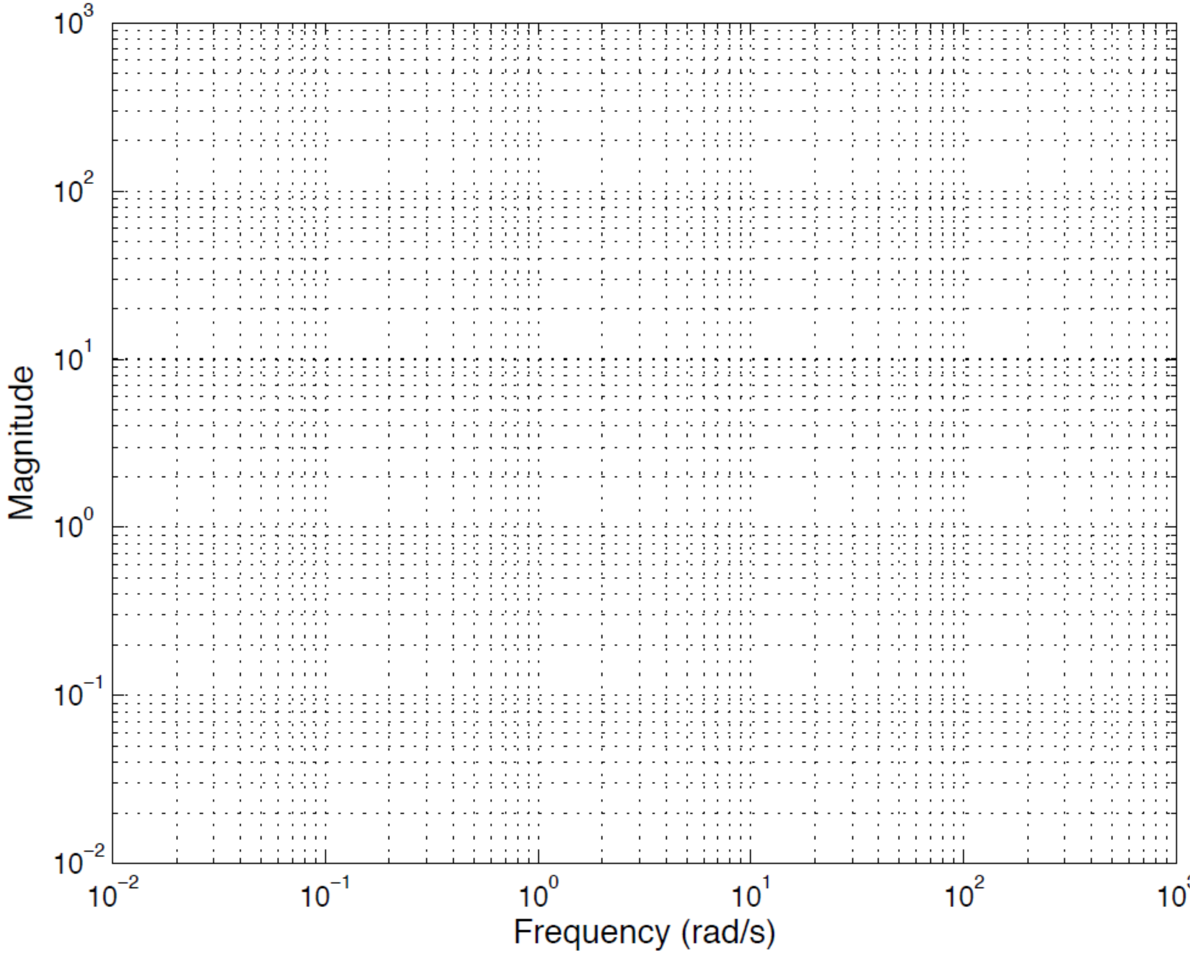


For $\omega \ll \omega_n$, $$\angle\left[\frac{1}{\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_n}\right) + 1}\right] \approx \angle 1 = 0^\circ$$

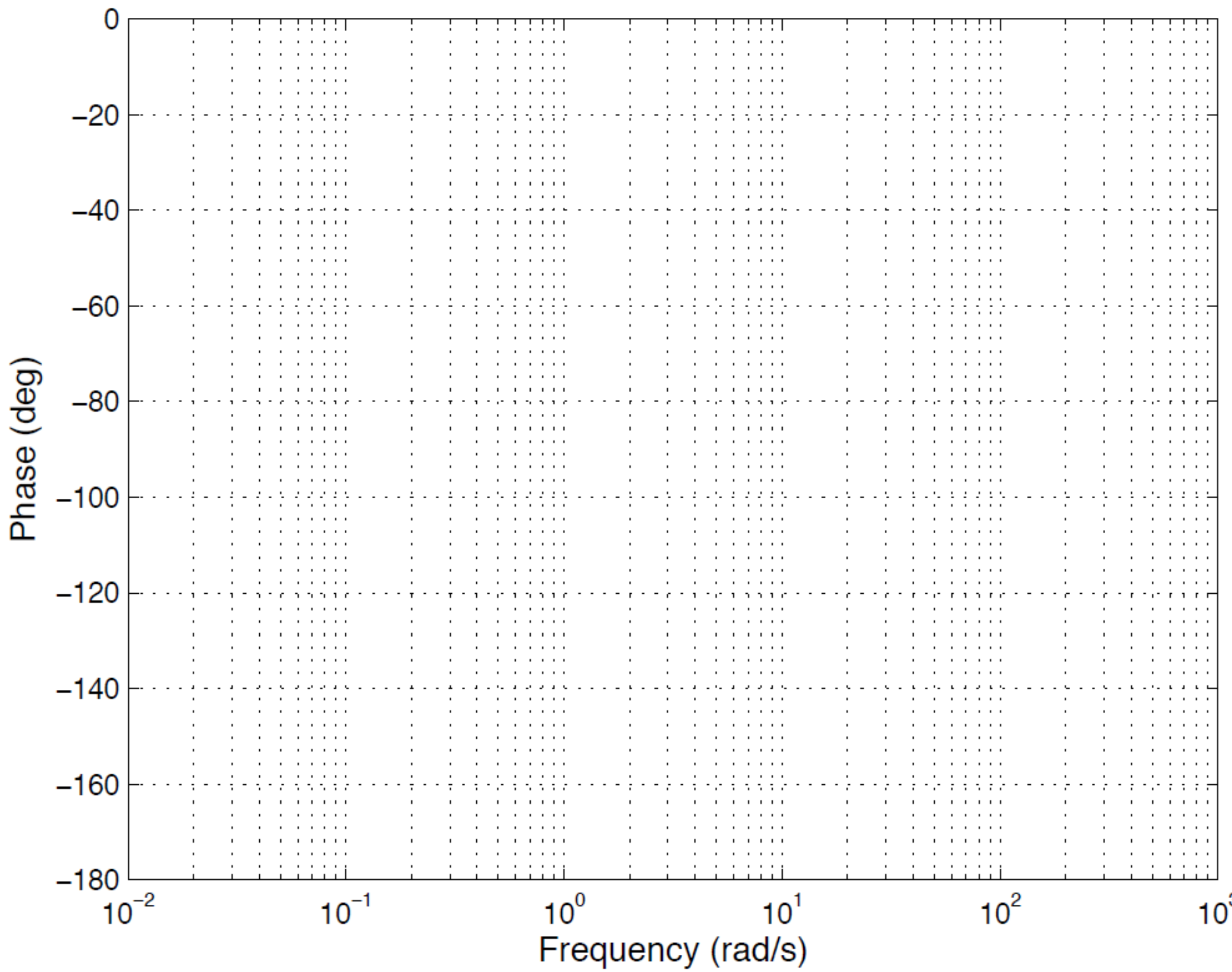
For $\omega \gg \omega_n$, $$\angle\left[\frac{1}{\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_n}\right) + 1}\right] \approx \angle\left(\frac{j\omega}{\omega_n}\right)^{-2} = -180^\circ$$


ζ = 0.05
0.1
0.2
0.3
0.5
0.7
0.9
Magnitude
Decibels
Frequency (ω/ω_n)
Phase (deg)

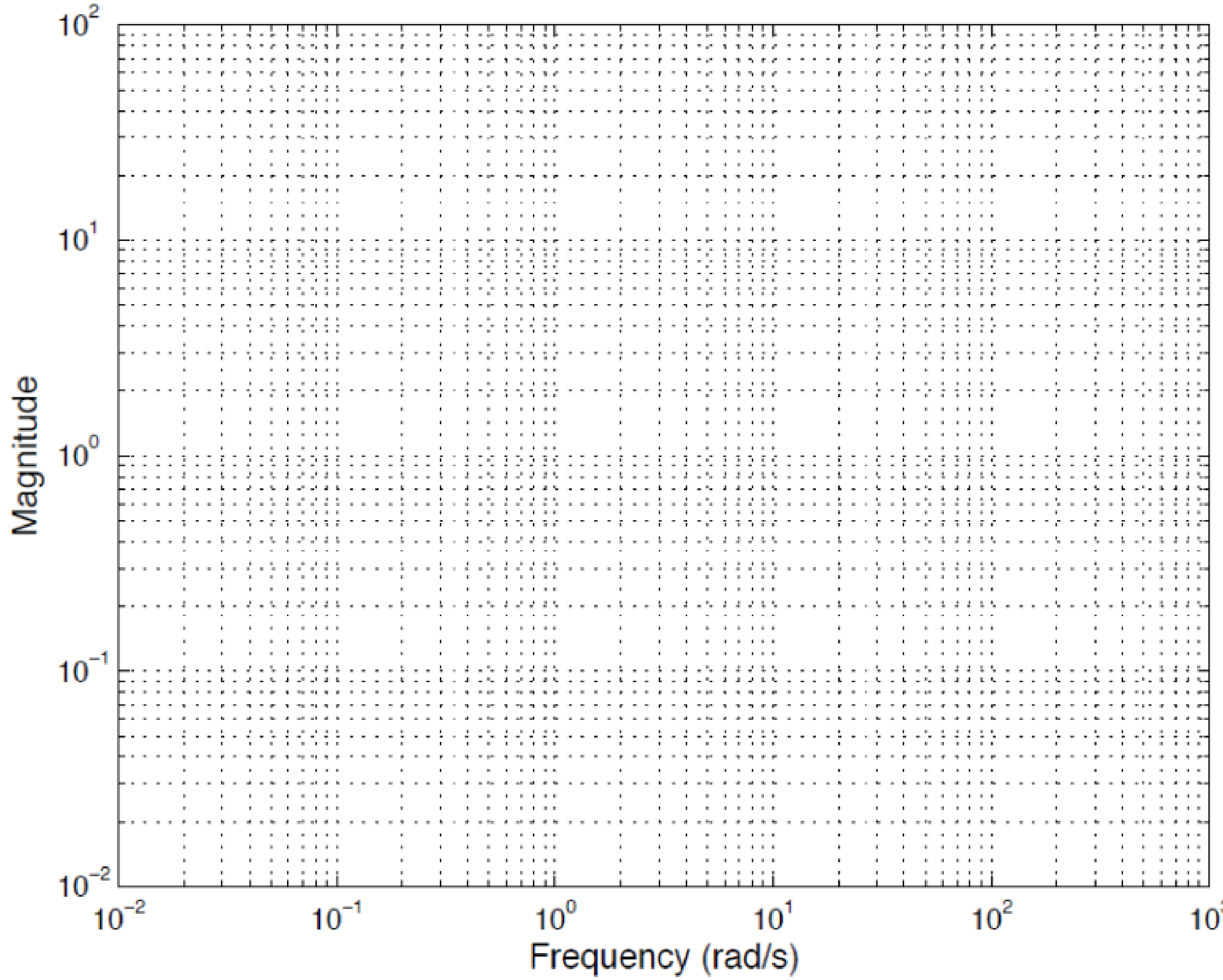
Sketching Examples



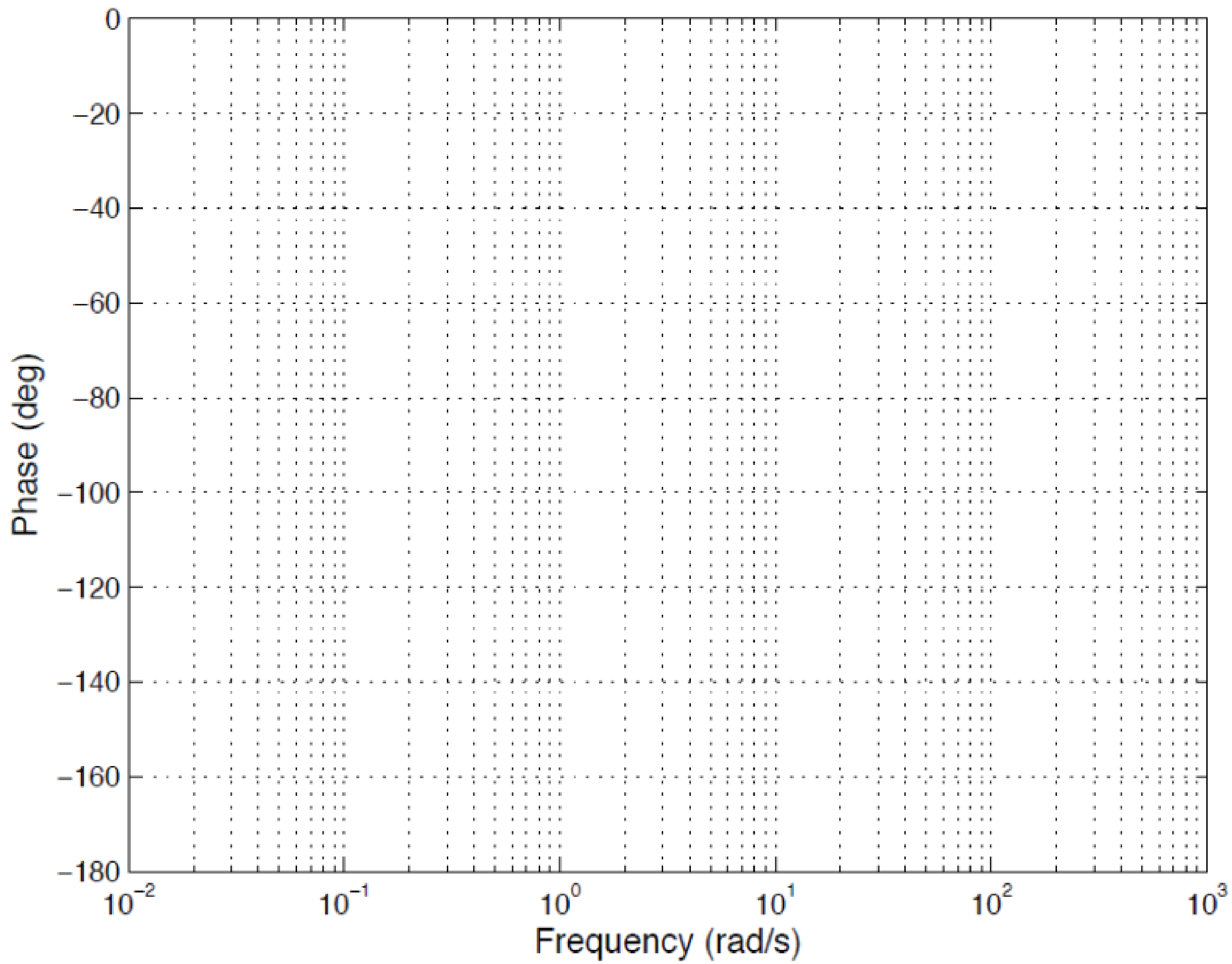
$$G(s) = \frac{2000(s+0.5)}{s(s+10)(s+50)}$$



$$G(s) = \frac{2000(s+0.5)}{s(s+10)(s+50)}$$



$$G(s) = \frac{20,000s + 20,000}{s^3 + 40s^2 + 10,000s}$$



$$G(s) = \frac{20{,}000s + 20{,}000}{s^3 + 40s^2 + 10{,}000s}$$