ME 335 HW 11

1. i) $G(s) = \dfrac{s}{(s+a)(s+b)} \rightarrow G(j\omega) = \dfrac{j\omega}{(j\omega+a)(j\omega+b)}$

a) set $j\omega = 0 \Rightarrow$

$$G(j\omega)\Big|_{\omega=0} = \frac{0}{ab} = 0$$

b) "s" term order = 1

slope of low-frequency mag. plot = 1

c) $m = 1$, $p = 2$, $m-p = -1 \rightarrow$ high-frequency slope is -1

d) $n = 1$, so phase at low frequencies = $90°$

e) $m-p=-1$, so phase at high frequencies = $-1 \times 90° = -90°$

f) break frequencies at $\omega = a$, and $\omega = b$, $\quad G(s) = \dfrac{s}{ab\left(\frac{s}{a}+1\right)\left(\frac{s}{b}+1\right)}$

ii) $G(s) = \dfrac{500(s+10)}{s(s^2+200s+10{,}000)}$

a) at $s = j\omega = 0$, $G = \dfrac{500(10)}{0(10{,}000)} \Rightarrow \infty$

b) $n = -1$, low-freq. magnitude slope $= -1$

c) $m = 1$, $p = 3$, $m-p = -2$, high freq. magnitude slope $= -2$

d) low-frequency phase $= -90°$

e) high-frequency phase $= -180°$

f) $G(s) = \dfrac{5{,}000\left(\frac{s}{10}+1\right)}{10{,}000\, s\left(\frac{s^2}{10{,}000} + \frac{1}{50}s + 1\right)}$

$\tau_1 = \frac{1}{10}$

$\omega_n = \sqrt{10{,}000} = 100$ (2 identical roots)

critical frequencies at 10 and 100 rad/s

iii) $G(s) = \dfrac{7{,}500\, s(s + 1{,}000)}{(s^2 + 100s + 625)(s^2 + 180s + 22{,}500)}$

a) $G(s)\Big|_{s=0} = \dfrac{7{,}500(0)(1{,}000)}{(625)(22{,}500)} = 0$

b) $n=1$, slope $= 1$

c) $m=2$, $p=4$, $m-p=-2 \Rightarrow$ high-frequency slope

d) $n \times 90^\circ = 90^\circ$

e) $m-p=-2$, $-2 \times 90^\circ = -180^\circ$

f) $G(s) = \dfrac{7.5(s)\left(\frac{s}{1{,}000} + 1\right)}{(22{,}500)(s+93.3)(s+6.7)\left(\frac{s^2}{22{,}500} + \frac{180s}{22{,}500} + 1\right)} = \dfrac{7.5(s)\left(\frac{s}{1{,}000} + 1\right)}{625(22{,}500)\left(\frac{s}{93.3}+1\right)\left(\frac{s}{6.7}+1\right)\left(\frac{s^2}{22{,}500} + \frac{180s}{22{,}500} + 1\right)}$

critical freq:

$\frac{1}{\tau_1} = 1{,}000$

$\omega_n = 150$

$\frac{1}{\tau_2} = 93.3$

$\frac{1}{\tau_3} = 6.70$

iv) $G(s) = \dfrac{K\left(\frac{s^2}{f^2} + \frac{gs}{f} + 1\right)}{(as+1)(bs+1)(cs+1)^2}$, assume $g<2$

a) $G(0) = \frac{K}{1} = K$

b) $n=0 \rightarrow$ slope $=0$

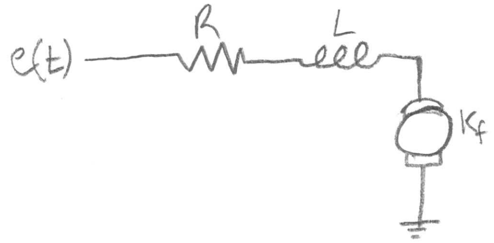
c) $m=2$, $p=4$, $m-p=-2=$ high-frequency slope

d) $n \times 90^\circ = 0^\circ$ phase at low frequency

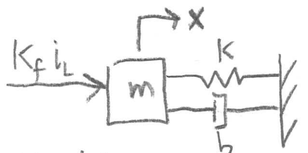
e) $-2 \times 90^\circ = -180^\circ$ phase at high frequency

f) break frequencies are at f, $\frac{1}{a}$, $\frac{1}{b}$, and $\frac{1}{c}$. Note that $\frac{1}{c}$ term appears twice.

2.

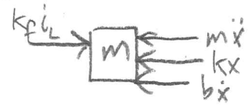


① $e(t) = Ri_L + L\dfrac{di_L}{dt} + K_f\dot{x}$



$\Rightarrow$

fbd:



② $m\ddot{x} + b\dot{x} + kx = K_f i_L$

3. a) xfer function for ①

$E(s) = RI(s) + LsI(s) + K_f sX(s)$ ③

xfer function for ②

$ms^2X(s) + bsX(s) + kX(s) = K_f I(s)$ ④

solve ③ for $I(s)$

$(R+Ls)I(s) = E(s) - K_f sX(s) \Rightarrow$

$I(s) = \dfrac{E(s) - K_f sX(s)}{R+Ls}$, sub into ④

$ms^2X(s) + bsX(s) + kX(s) = \dfrac{K_f E(s) - K_f^2 sX(s)}{R+Ls} \Rightarrow$

$\left[(R+Ls)(ms^2+bs+k) + K_f^2 s\right]X(s) = K_f E(s)$

$$\boxed{\begin{aligned}\frac{X(s)}{E(s)} &= \frac{K_f}{(R+Ls)(ms^2+bs+k)+K_f^2 s}\\ &= \frac{K_f}{mLs^3 + (mR+bL)s^2 + (bR+kL+K_f^2)s + kR}\end{aligned}}$$

b) $\dfrac{X(s)}{E(s)} = \dfrac{K_f/mL}{s^3 + \left(\frac{R}{L} + \frac{b}{m}\right)s^2 + \left(\frac{bR}{mL} + \frac{k}{m} + \frac{K_f^2}{mL}\right)s + \frac{kR}{mL}}$

$K_f/mL = \dfrac{0.75}{.05(.005)} = 3{,}000$

$\dfrac{R}{L} + \dfrac{b}{m} = \dfrac{8}{0.005} + \dfrac{.01}{.05} = 1600.2$

$\dfrac{bR}{mL} + \dfrac{k}{m} + \dfrac{K_f^2}{mL} = 4{,}570$

$\dfrac{kR}{mL} = 3{,}200{,}000$

$\dfrac{X}{E} = \dfrac{3{,}000}{s^3 + 1600.2s^2 + 4570s + 3{,}200{,}000}$ ✓

c)

i) at low frequencies ($s=0$)

$$\frac{X}{E} = \frac{3{,}000}{3{,}200{,}000} = 0.0009375$$

ii) no "s" terms, so slope is 0

iii) $m=0$, $p=3$ $\quad m-p=-3$, high-frequency slope is -3

iv) phase at low frequencies is 0 ($n=0$)

v) phase at high frequencies is $(m-p)\times 90 = -270°$

d) if $\omega = 100$ rad/s

$$G(j\omega) = \frac{3{,}000}{(j100)^3 + 1600.2(j100)^2 + 4{,}570(j100) + 3{,}200{,}000}$$

$$= \frac{3{,}000}{-12{,}802{,}000 + j(-543{,}000)}$$

$$|G| = \frac{3{,}000}{\sqrt{(12{,}802{,}000)^2 + (543{,}000)^2}} = \boxed{2.34 \times 10^{-4}}$$

$$\angle G = \angle N - \angle D = 0 - \tan^{-1}\left(\frac{-543{,}000}{-12{,}802{,}000}\right) = \boxed{-182.4° \text{ or } 177.6°}$$

e) $$G(s) = \frac{3{,}000}{(s+1599)(s^2 + 1.607s + 2{,}002)}$$ ← factored using MATLAB "roots"

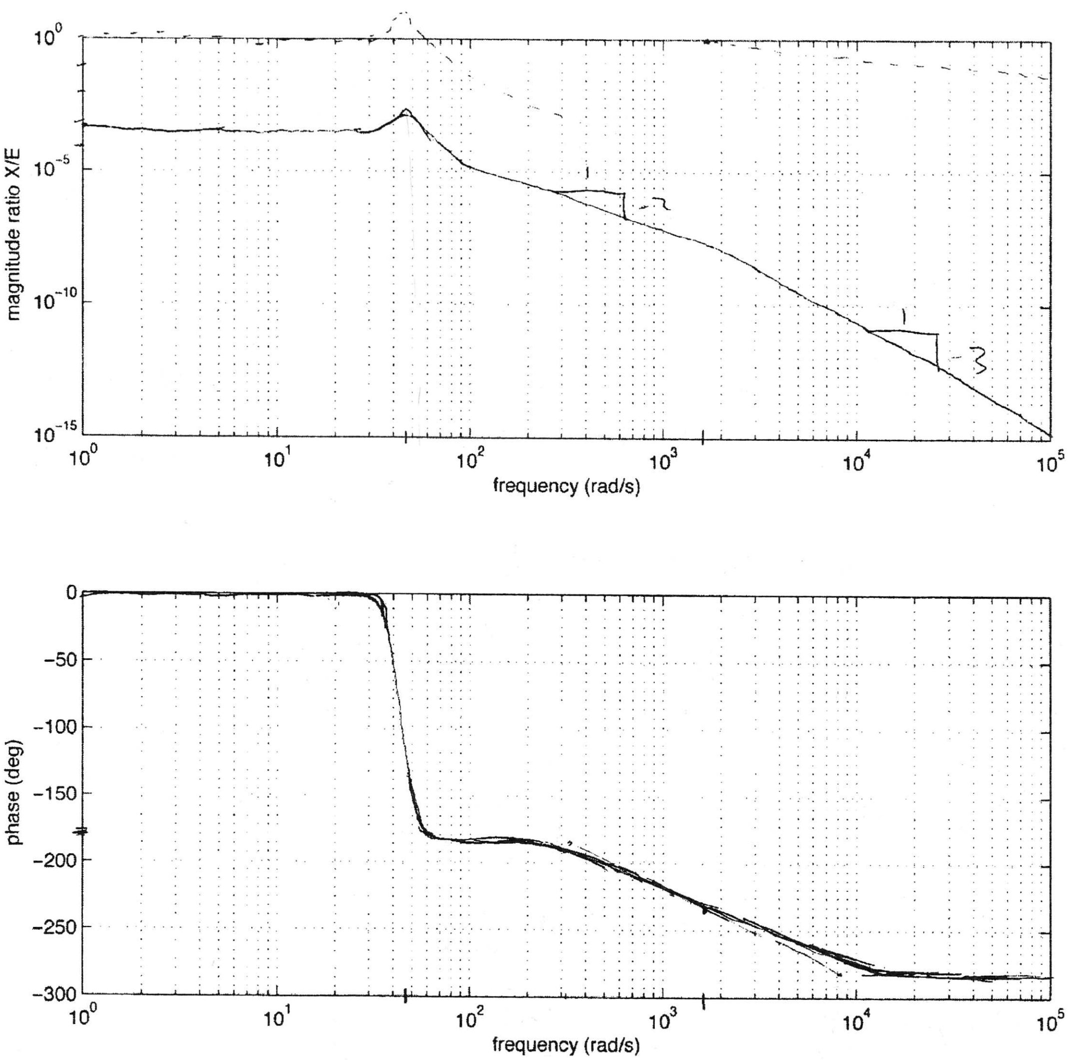
so

$$G(s) = \frac{3000/[(1599)(2002)]}{\left(\frac{s}{1599}+1\right)\left(\frac{s^2}{2002} + \frac{1.607s}{2002} + 1\right)}$$

$$\boxed{\tau = \frac{1}{1599} \qquad \omega_n = \sqrt{2{,}002} = 44.7 \text{ rad/s}}$$

$$2\zeta = \frac{1.607}{2{,}002} \Rightarrow \zeta = .018$$

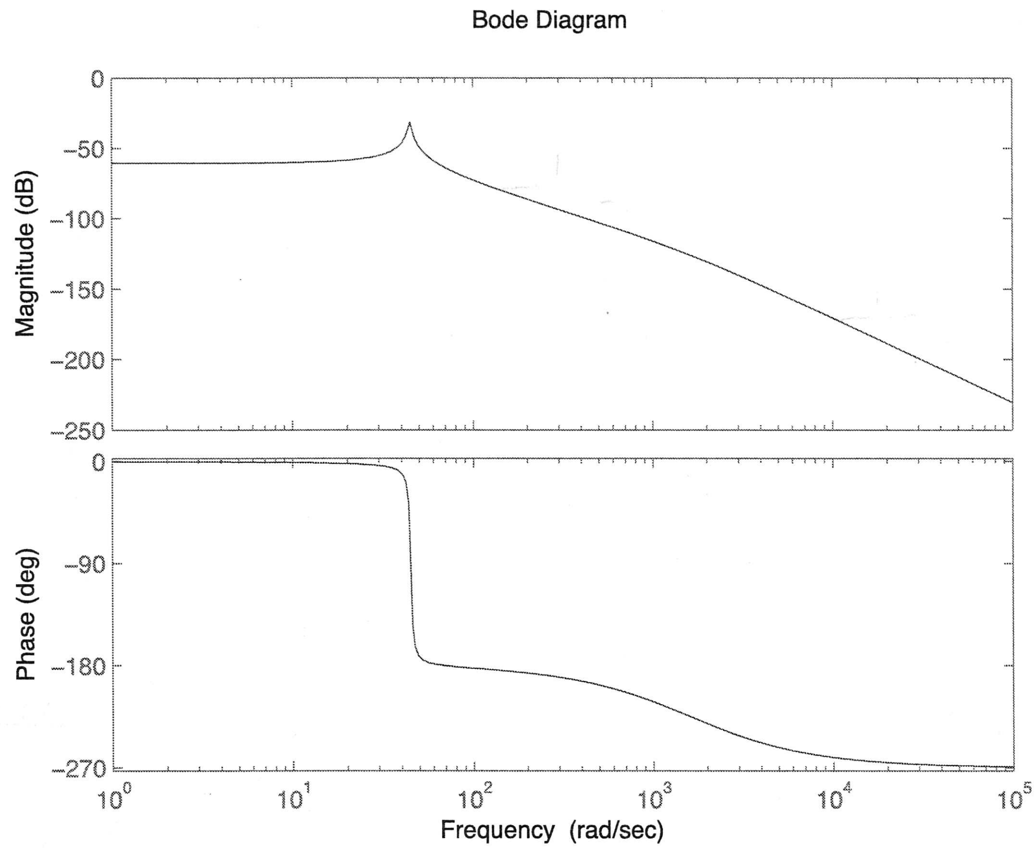
$$\boxed{\frac{1}{2\zeta} = 27.8}$$



$.2(1599) = 320$

$5(1599) = 8000$

Part f): the Bode diagram:



and the Matlab code:

```
%prob2f.m

num = 3000;
den = [1 1600.2 4570 3200000];
sys = tf(num,den);

bode(sys)
```

5 pts for
MATLAB plots