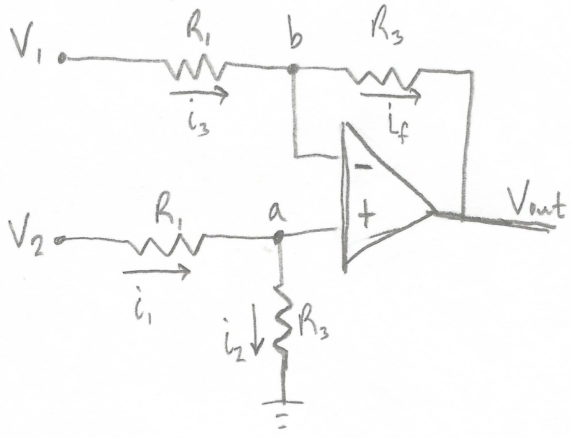
Problem 1 -



- subscripts on i don't match component subscripts due to repeated #'s on R_1 & R_3.

components -

① $i_3 = \dfrac{V_1 - V_b}{R_1}$

② $i_1 = \dfrac{V_2 - V_a}{R_1}$

③ $i_2 = \dfrac{V_a - 0}{R_3}$

④ $i_f = \dfrac{V_b - V_{out}}{R_3}$

nodes (assuming no current flows into op-amps on inputs)

⑤ $i_3 = i_f$

⑥ $i_1 = i_2$

other op-amp assumptions -

$V_a = V_b \Rightarrow$

solving:

use ⑤ with ① & ④

$$\frac{V_1 - V_b}{R_1} = \frac{V_b - V_{out}}{R_3} \Rightarrow$$

$$V_1 R_3 - V_b R_3 = R_1 V_b - R_1 V_{out} \Rightarrow$$

⑦ $$V_b = \frac{V_1 R_3 + V_{out} R_1}{R_1 + R_3}$$

use ② & ③ with ⑥ $\Rightarrow$

$$\frac{V_2 - V_a}{R_1} = \frac{V_a}{R_3} \Rightarrow$$

$$V_a = \frac{V_2 R_3}{R_1 + R_3} \quad ⑧$$

sub ⑧ into ⑦ since $V_a = V_b \Rightarrow$

$$\frac{V_2 R_3}{\cancel{R_1 + R_3}} = \frac{V_1 R_3 + V_{out} R_1}{\cancel{R_1 + R_3}} \Rightarrow$$

$$\boxed{V_{out} = \frac{R_3}{R_1}(V_2 - V_1)}$$

Problem 6.41

From 6.5.5 and 6.5.6 :

$$\frac{di_a}{dt} = \frac{1}{L_a}\left[-R_a i_a - K_b \Omega + v_a(t)\right]$$

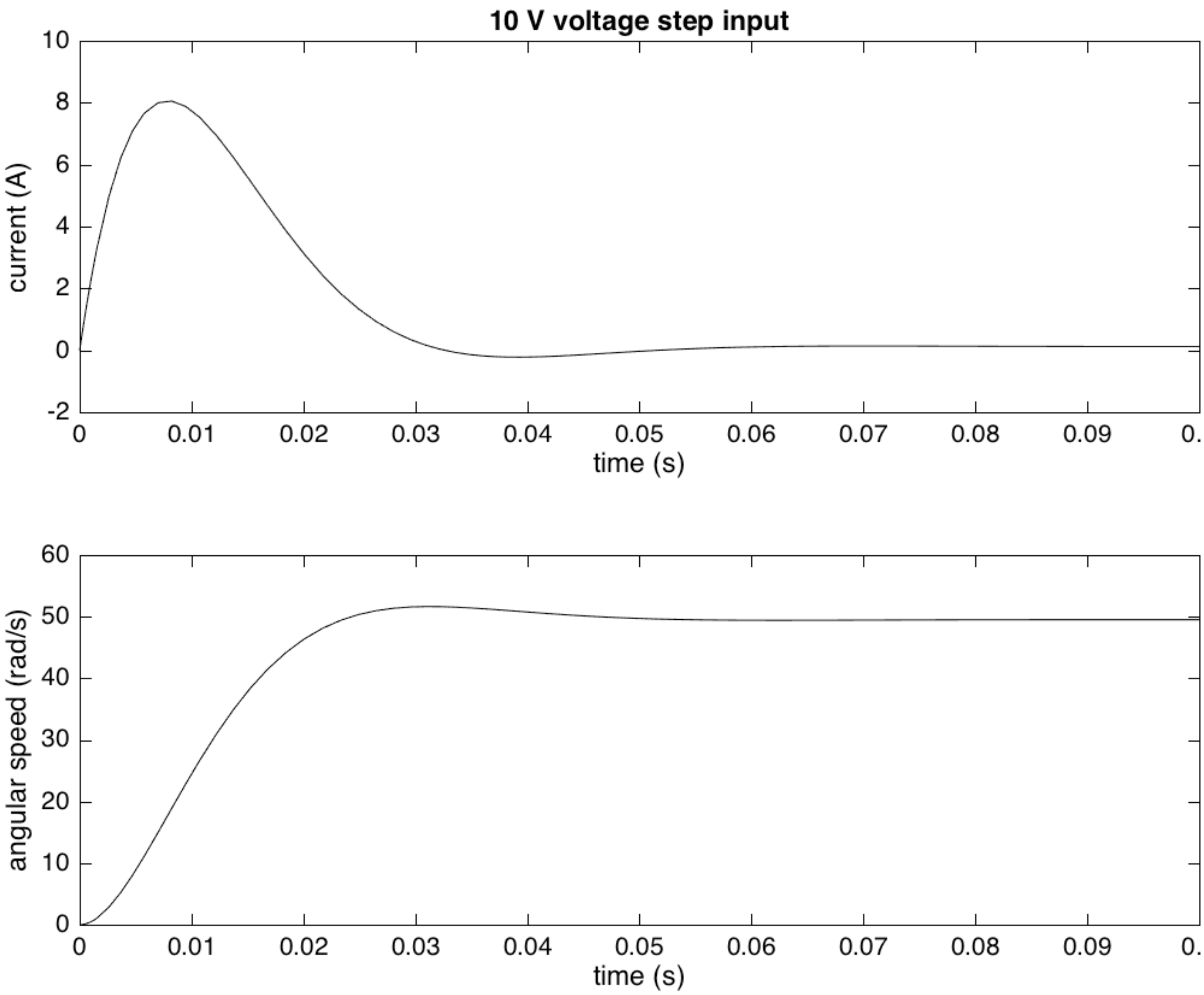
$$\frac{d\Omega}{dt} = \frac{1}{J}\left[K_t i_a - c\Omega - T_L(t)\right]$$

(a) Plot response with $v_a(t) = 10$ V , $T_L(t) = 0$ N-m

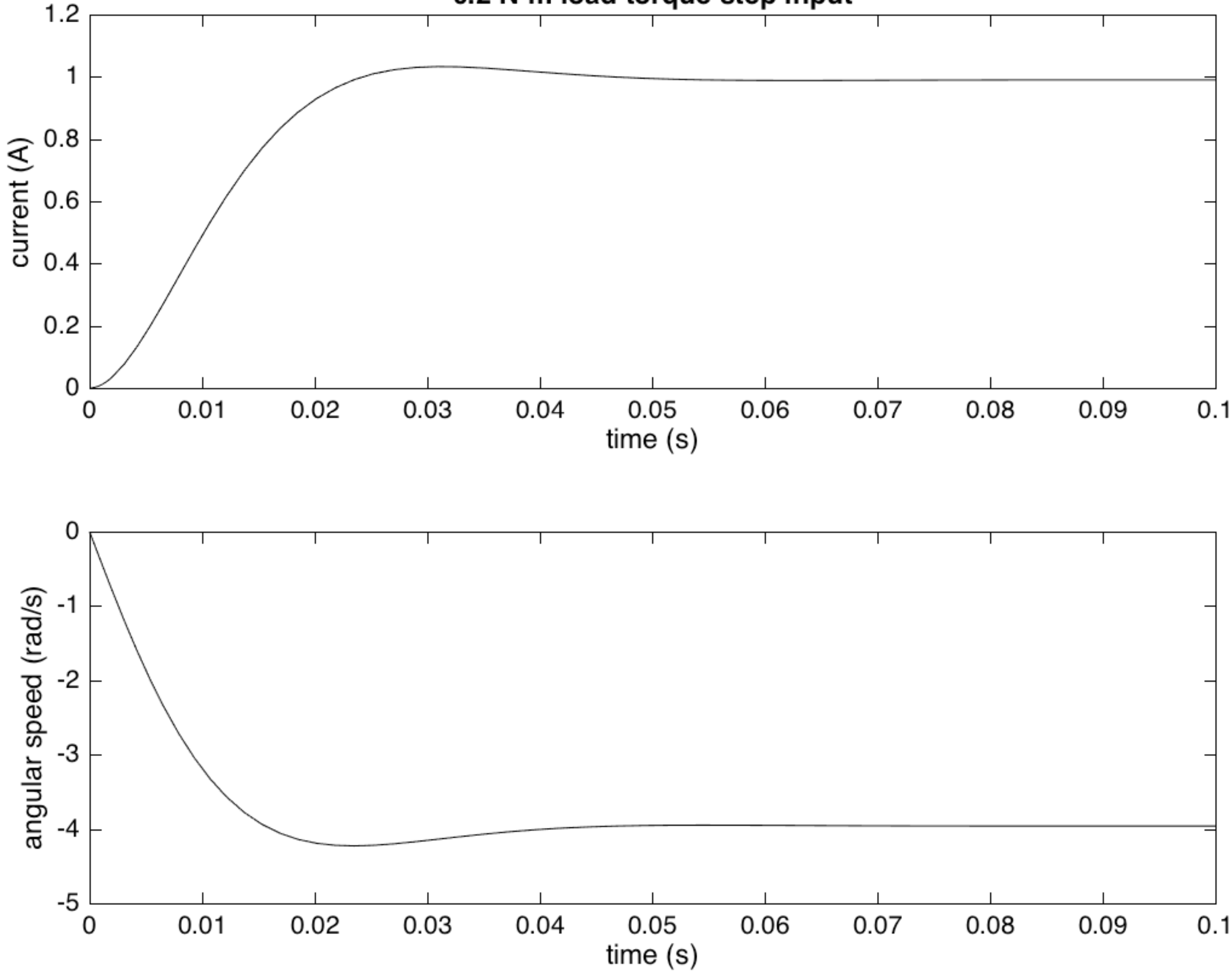
(b) Plot response with $v_a(t) = 0$ V , $T_L(t) = 0.2$ N-m

```matlab
% Plots the load speed and armature current for a
% motor in response to a voltage step or load torque
% input

% Simulate response
x0 = [0 0];
tspan = [0 0.1];
[t,x] = ode45('p6_41_eom', tspan, x0);
ia = x(:,1);
om = x(:,2);

% Plots
figure(1); clf;
subplot(211);
plot(t,ia);
xlabel('time (s)');
ylabel('current (A)');
% title('10 V voltage step input');
title('0.2 N-m load torque step input');
subplot(212);
plot(t,om);
xlabel('time (s)');
ylabel('angular speed (rad/s)');
```

```matlab
function xdot = p6_41_eom(t,x)

% Motor parameters
Kt = 0.2;       % Torque constant (N*m/A)
Kb = Kt;        % Back EMF constant (N*m/A)
c = 5e-4;       % Damping coefficient (N*m*s/rad)
Ra = 0.8;       % Armature resistance (ohms)
La = 0.004;     % Armature inductance (H)
J =0.0005;      % Motor inertia (kg*m^2)

ia = x(1);
om = x(2);

% va = 10;
va = 0;
% TL = 0;
TL = 0.2;

iadot = 1/La*(-Ra*ia - Kb*om + va);
omLdot = 1/J*(Kt*ia - c*om - TL);

xdot = [iadot;omLdot];
```


10 V voltage step input
current (A)
10
8
6
4
2
0
-2
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
time (s)
angular speed (rad/s)
60
50
40
30
20
10
0
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
time (s)

0.2 N-m load torque step input



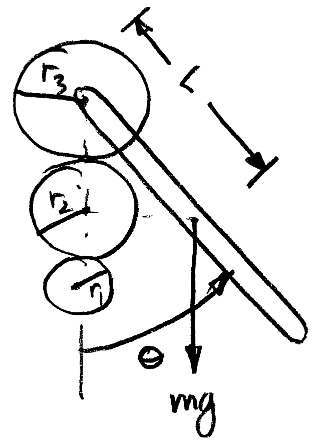
Motor/Arm Model

$c = 0$

The radii of the gears are r_1, r_2, r_3

The equivalent inertia seen by the motor is I_e.

Find a model for the arm output θ



The load torque felt by the motor is due to the weight of the arm. The inertias of the gears and arm are already lumped into the equivalent inertia, and the damping is zero.

Gravitational torque on gear 3:

$$T_{L_3} = mgL\sin\theta$$

Gear ratio:

$$\omega_1 = \frac{r_2}{r_1}\omega_2$$

$$\omega_2 = \frac{r_4}{r_3}\omega_3$$

$$\omega_1 = \frac{r_2 r_4}{r_1 r_3}\omega_3 \Rightarrow \theta_1 = \frac{r_2 r_4}{r_1 r_3}\theta \qquad \left(\omega_3 = \frac{d\theta}{dt}\right)$$

$$T_1 = \frac{r_1 r_3}{r_2 r_4} T_3$$

The gravitational torque on the motor is:

$$T_{L_1} = \frac{r_1 r_3}{r_2 r_4} T_{L_3} = \frac{r_1 r_3}{r_2 r_4} mgL\sin\theta$$

The motor model is:

$$\frac{di_a}{dt} = \frac{1}{L_a}\left(v_a - R_a i_a - K_b \omega_1\right)$$

$$\frac{d\omega_1}{dt} = \frac{1}{I_e}\left(K_T i_a - c\,\cancelto{0\ (c=0)}{\omega_1} - T_{L_1}\right)$$

But we don't want the model in terms of ω_1, so use $\omega_1 = \frac{r_2 r_4}{r_1 r_3}\omega_3$

$$\boxed{\begin{aligned} \frac{di_a}{dt} &= \frac{1}{L_a}\left(v_a - R_a i_a - K_b \frac{r_2 r_4}{r_1 r_3}\omega_3\right) \\ \frac{d\omega_3}{dt} &= \frac{r_1 r_3}{r_2 r_4}\frac{1}{I_e}\left(K_T i_a - \frac{r_1 r_3}{r_2 r_4} mgL\sin\theta\right) \end{aligned}}$$

We need one more state equation to relate θ to ω_3:

$$\boxed{\frac{d\theta}{dt} = \omega_3}$$