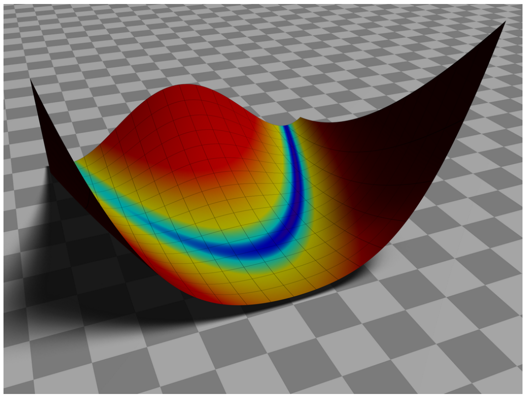
Line Search

Lecture 2



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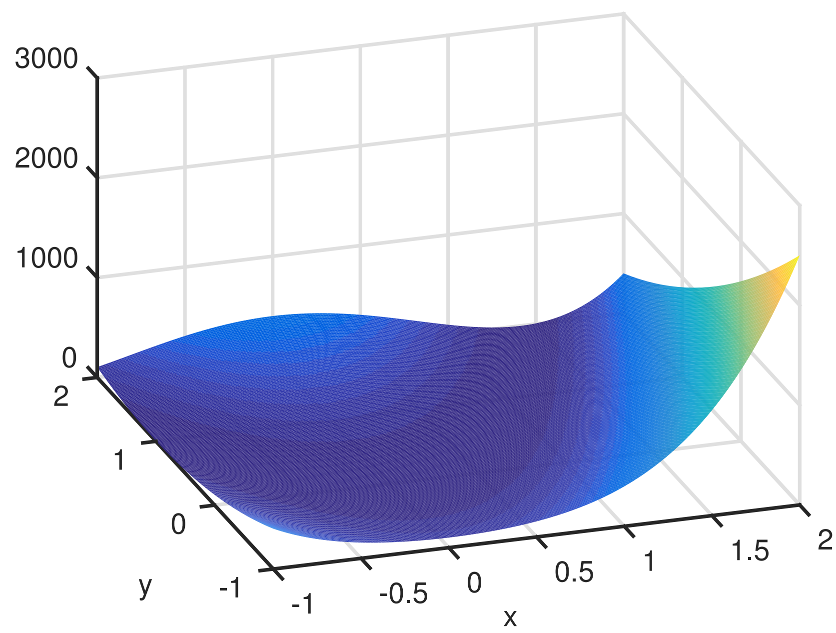
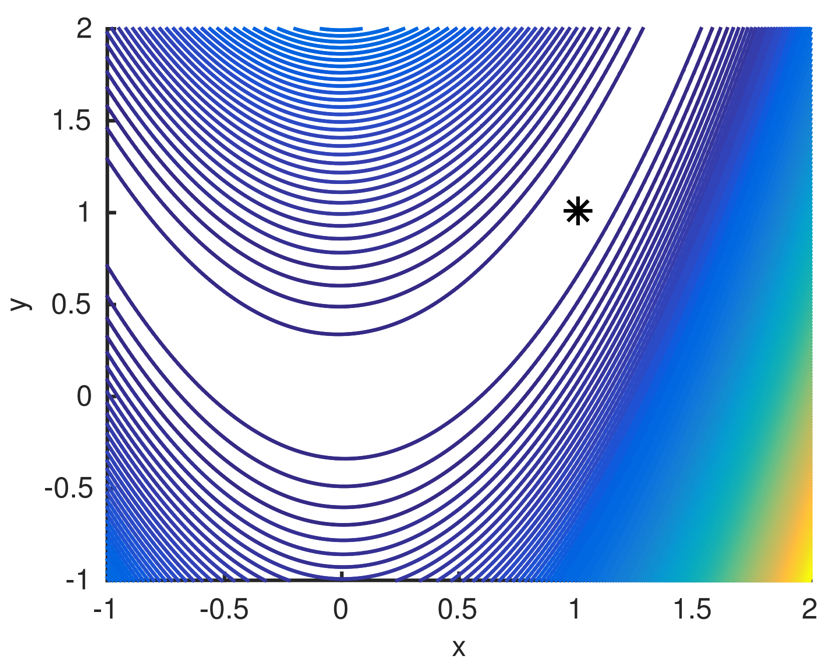
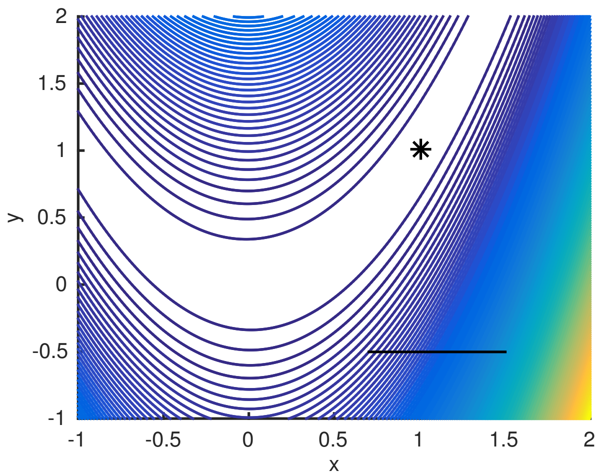
Outline

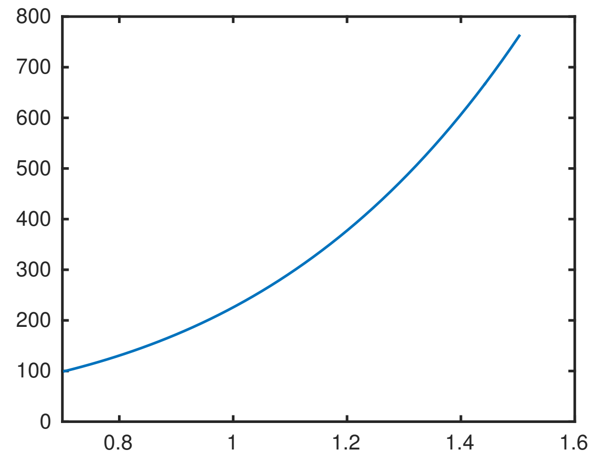
- Line Search Motivation
- 1D Optimization
- Bisection
- Newton's Method
- Brent's Method

Line Search Motivation









We will first study 1D optimization to help us with line searches, but remember a line search is not the same as 1D optimization.

1D Optimization

How do we know when we have reached a local minimum?

$$f'(x^*) = 0$$
$$f''(x^*) \geq 0$$

Root finding methods can help us find a minimum (although root finding alone is not sufficient).

Bisection

Bisection is a simple (recursive) bracketing method. At each iteration we evaluate at the midpoint of the previous bracket.

Bracketing methods are robust but slow.

Example: Refrigeration Tank

Minimize the cost of a cylindrical refrigeration tank with a volume of 50 m^3.

- Circular ends cost \$10 per m^2
- Cylindrical walls cost \$6 per m^2
- Refrigerator costs \$80 per m^2 over its life

Let d be the tank diameter, and L the height.

$$\begin{aligned} f &= 10 \cdot 2 \left(\frac{\pi d^2}{4} \right) + 6(\pi d L) + 80 \left(\frac{2\pi d^2}{4} + \pi d L \right) \\ &= 45\pi d^2 + 86\pi d L \end{aligned}$$

However, L is a function of d because our volume is constrained. We could add a constraint to the problem

$$\frac{\pi d^2}{4} L = V$$

but it is easier to express V as a function of d and make the problem unconstrained.

$$L = \frac{4V}{\pi d^2} = \frac{200}{\pi d^2}$$

Tip: Sometimes equality constraints can be explicitly computed. This reduces the dimensionality of the problem, and removes a constraint.

$$\begin{aligned} \text{minimize} \quad & 45\pi d^2 + \frac{17200}{d} \\ \text{with respect to} \quad & d \\ \text{subject to} \quad & d \geq 0 \end{aligned}$$

See refrig_line Matlab example under Resources

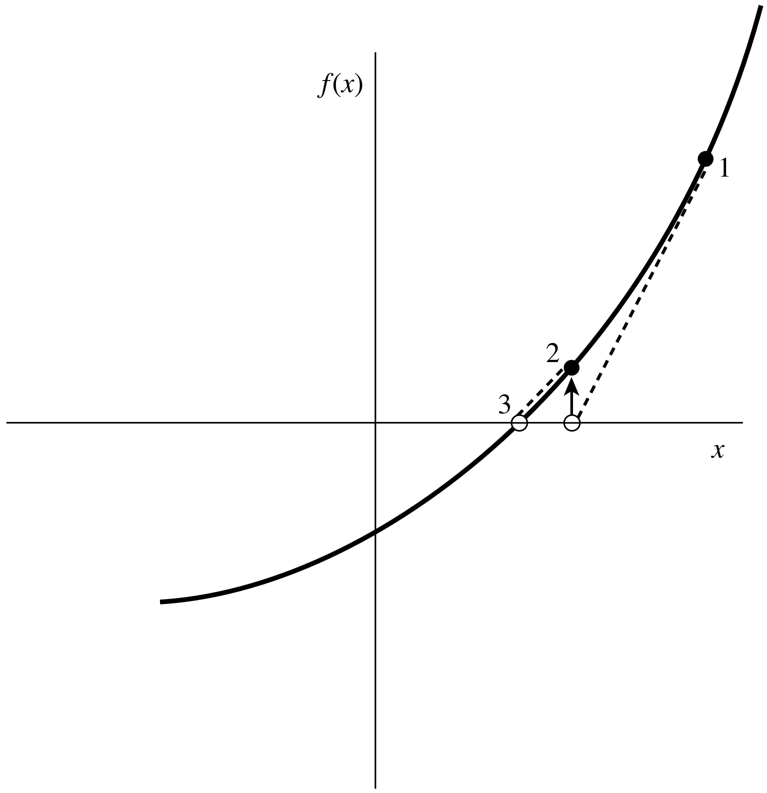
Newton's Method

We can do better by using gradient information (derivative for root finding, meaning second derivative for optimization)

$$f(x_{k+1}) = f(x_k) + (x_{k+1} - x_k) f'(x_k) + \mathcal{O}((x_{k+1} - x_k)^2)$$

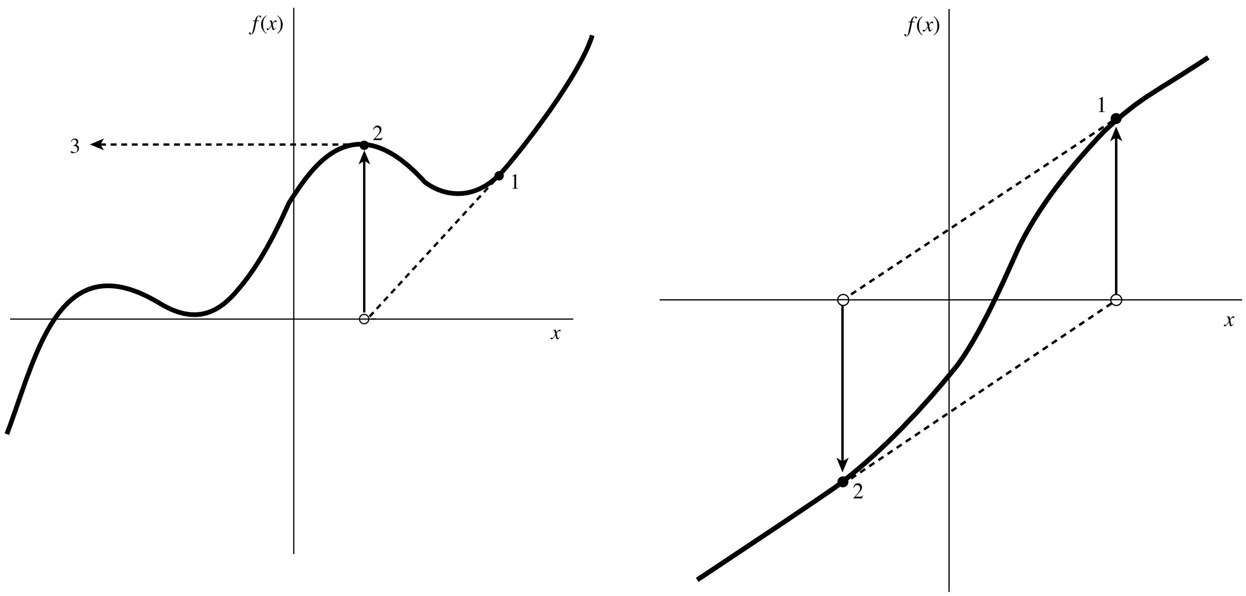
Let's pick the step that would predict landing at the root for the next iteration (i.e., $f(x_{k+1}) = 0$)

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}.$$



Remember, that we are interested in minimization, not root finding. Or in other words, we need the root of the first derivative:

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$



Brent’s Method

- Bracketing methods are robust, but slow (linear convergence)
- Interpolatory methods are fast (quadratic), but not guaranteed to converge
- Brent’s method (and other variations) combine the advantages of both.

Brent’s method is generally the best method for 1D root finding. There are different, but related, versions for root finding and minimization.