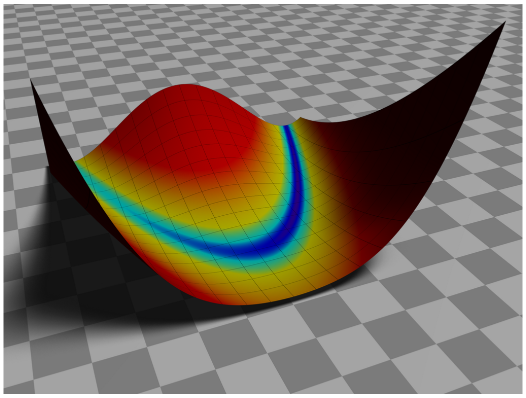
Optimization Under Uncertainty I

Lecture 28



ME EN 575
Andrew Ning
aning@byu.edu

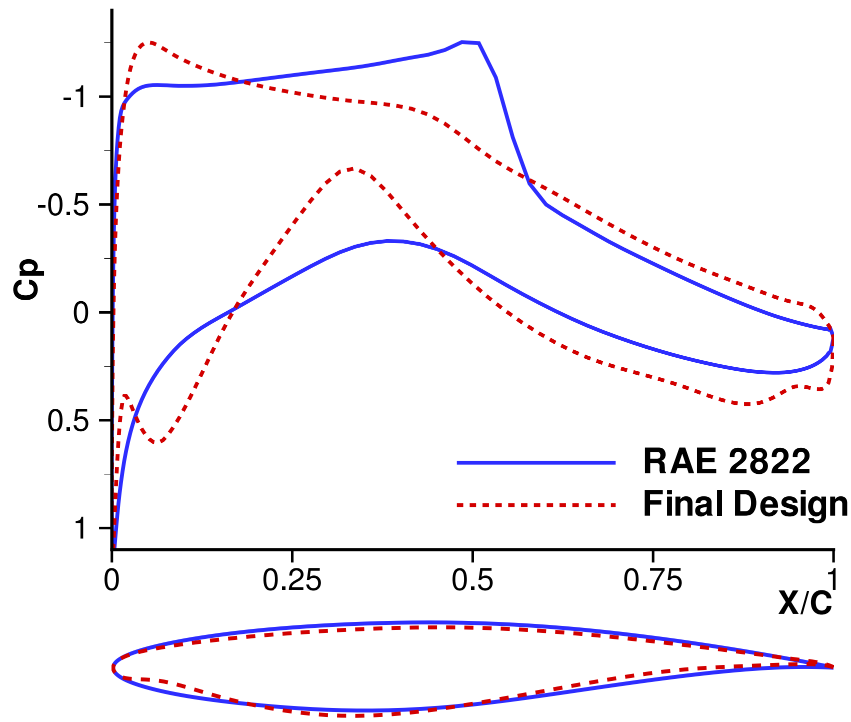
Outline

Introduction

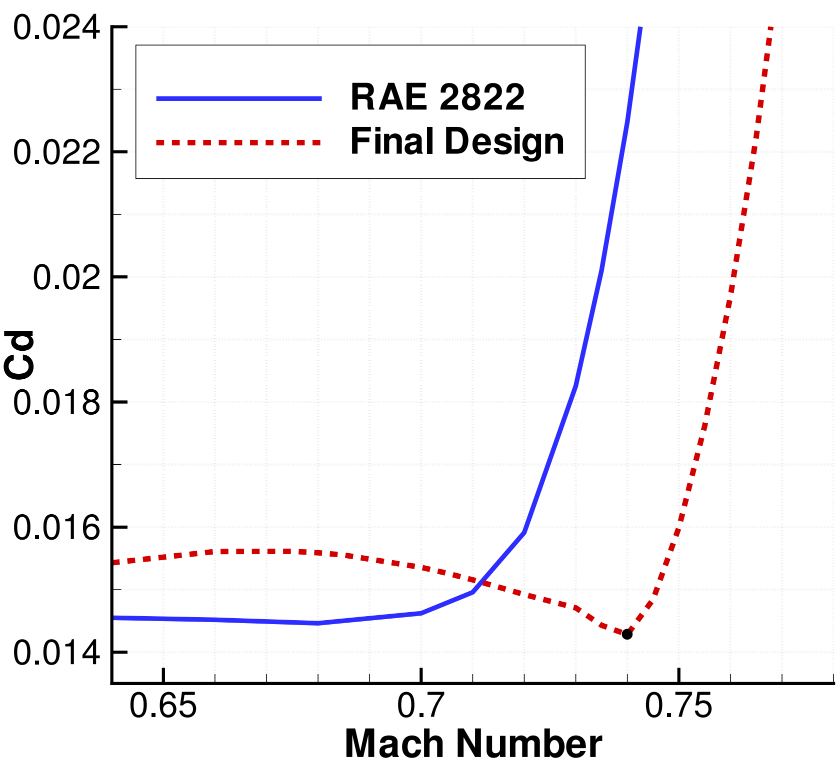
Robust Design

Introduction

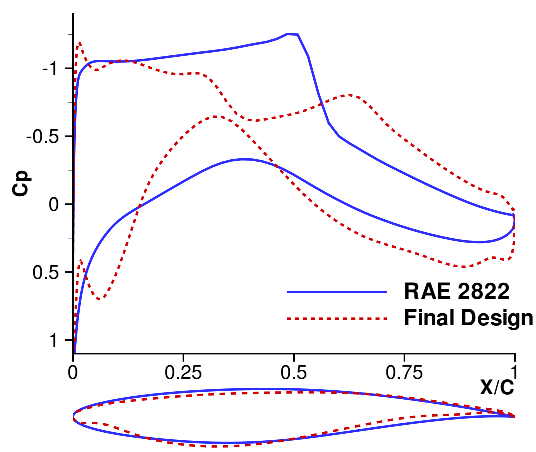
What are some sources of variability in engineering design?

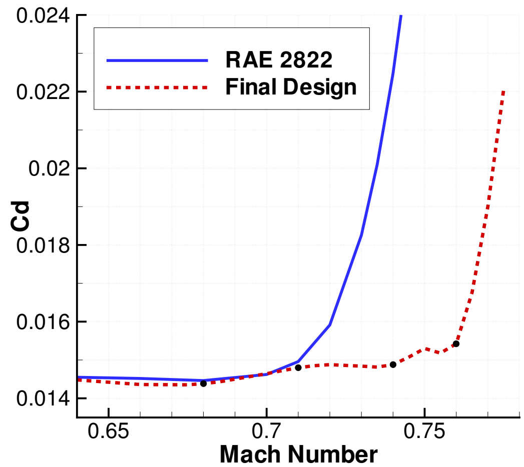


Cp
-1
-0.5
0
0.5
1
0
0.25
0.5
0.75
1
X/C
RAE 2822
Final Design



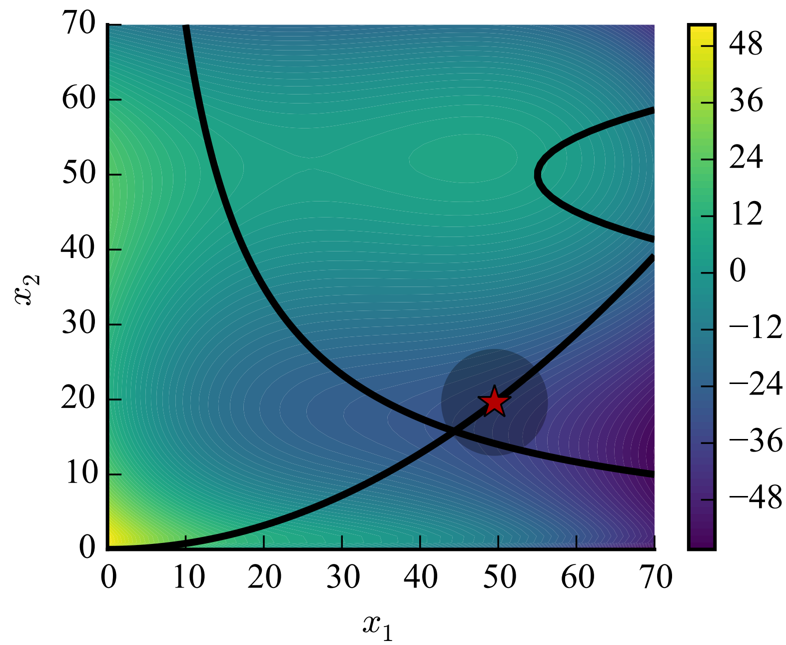
0.024
0.022
0.02
0.018
0.016
0.014
Cd
0.65
0.7
0.75
Mach Number
RAE 2822
Final Design





The multipoint optimization leads to a design that is much more robust.

What happens if you need to be at work right at 8AM and you devise a plan that will get you there right at 8AM?



Only using deterministic values leads to engineering designs that are *not* reliable.

- Robust: performance is less sensitive to inherent variability (objective function)
- Reliable: less prone to failure under inherent variability (constraints)

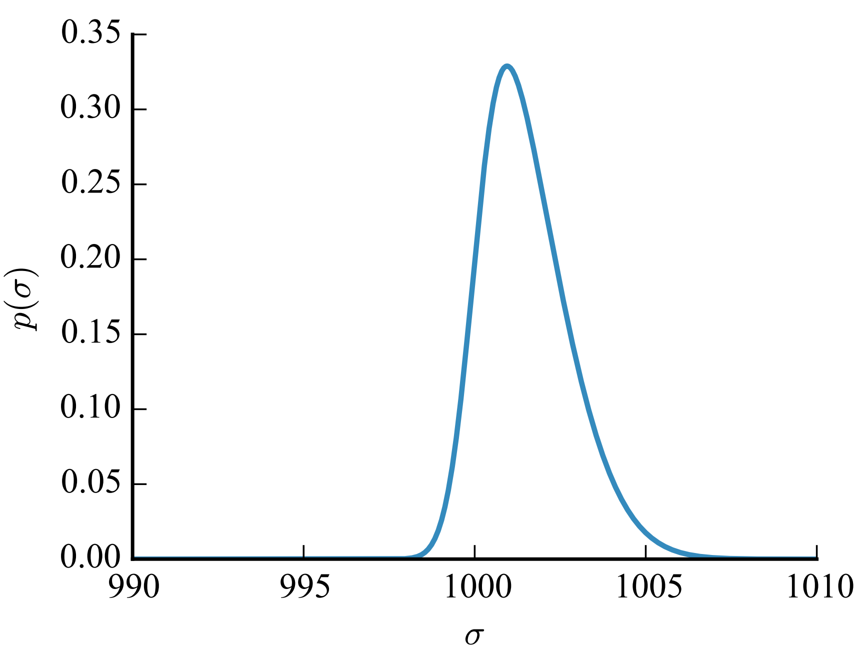
Mean:

$$\mu_x = \frac{1}{N}\sum_{i=1}^{N} x_i$$

Variance:

$$\sigma_x^2 = \frac{1}{N-1}\left(\sum_{i=1}^{N} x_i^2 - N\mu_x^2\right)$$

Probability distribution function (PDF):



$$\mathsf{Prob}[x = a] = p(a)$$

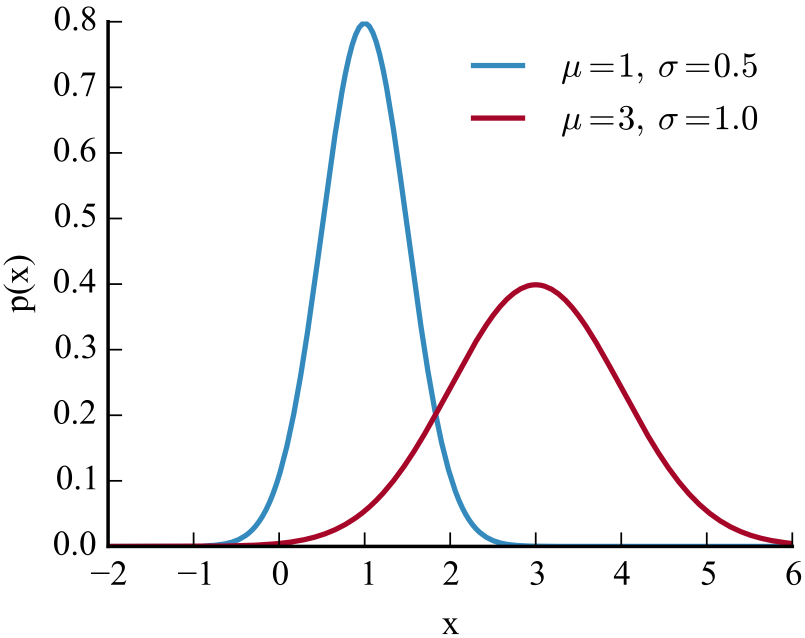
$$\mathsf{Prob}[a \leq x \leq b] = \int_a^b p(x)dx$$

$$\int_{-\infty}^{\infty} p(x)dx = 1$$

$$\mu_x = E[x] = \int_{-\infty}^{\infty} xp(x)dx$$

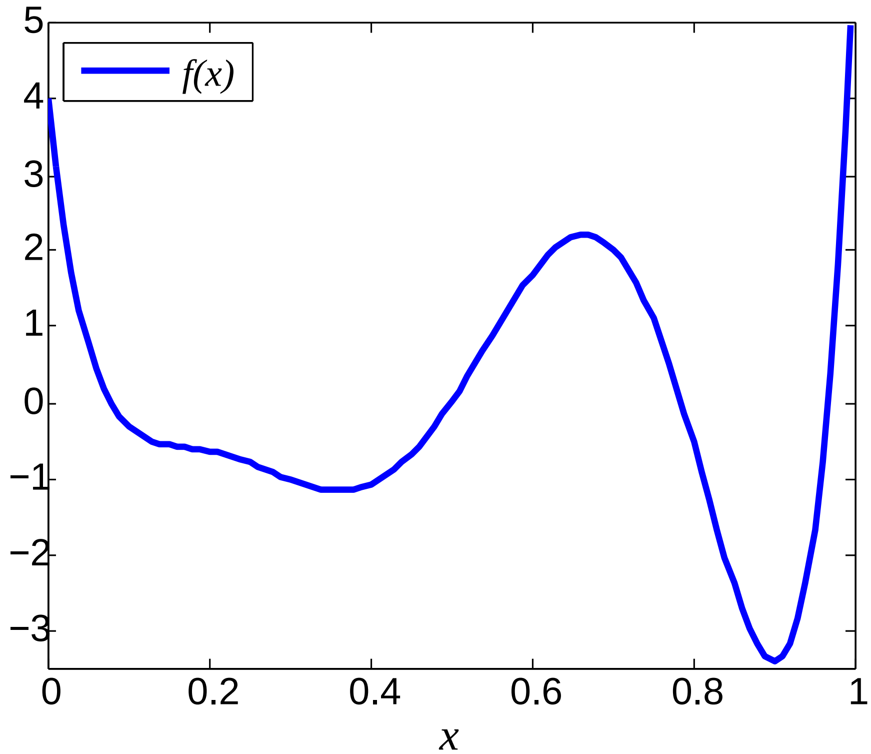
One of the most common distributions is the Gaussian or Normal distribution

$$p(x; \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \exp \frac{-(x-\mu)^2}{2\sigma^2}$$

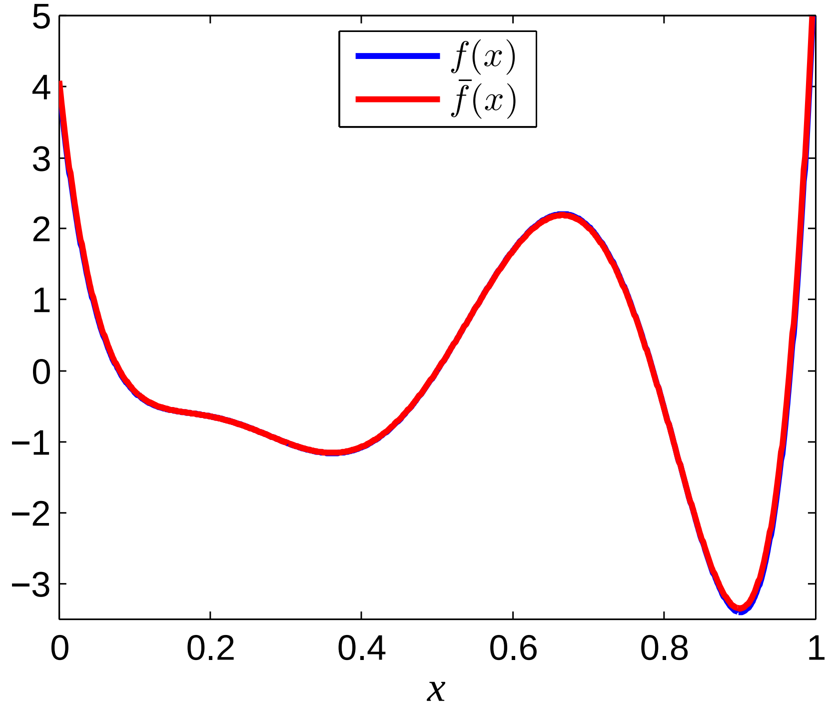


Robust Design

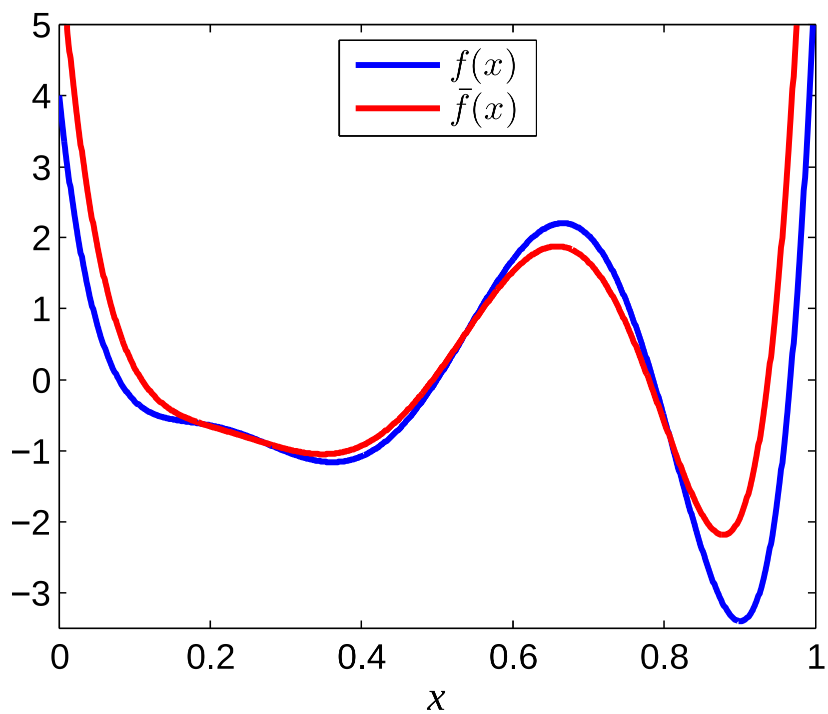
Where is the optimal point?



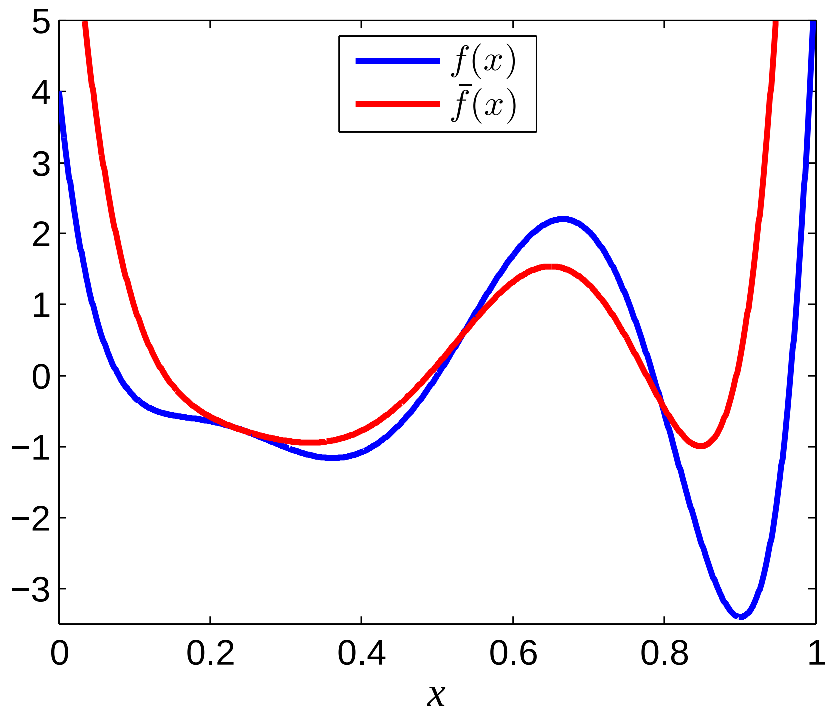
What if there is some variability in x?



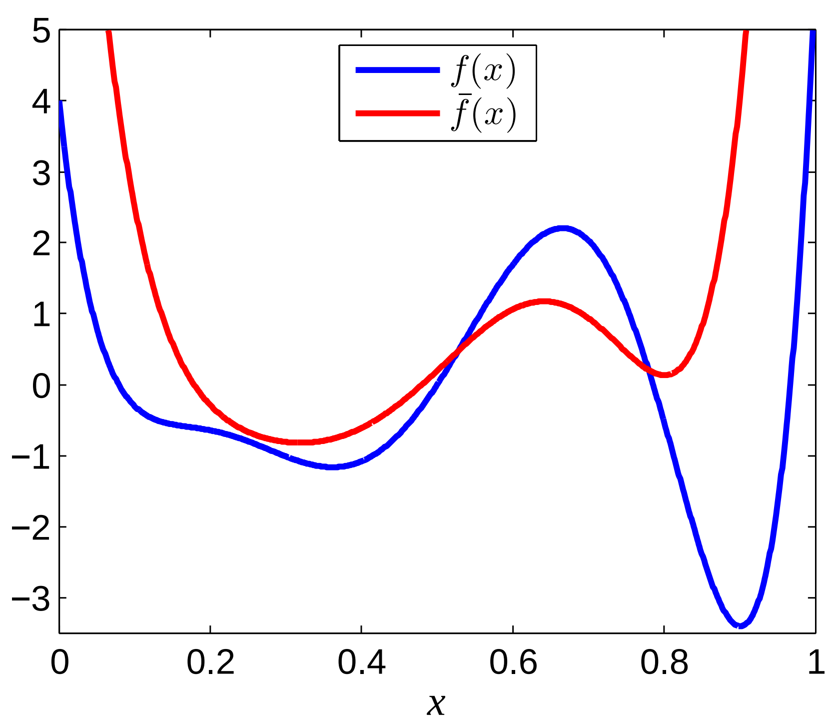
$\sigma = 0.01$



$\sigma = 0.05$



$$\sigma = 0.075$$

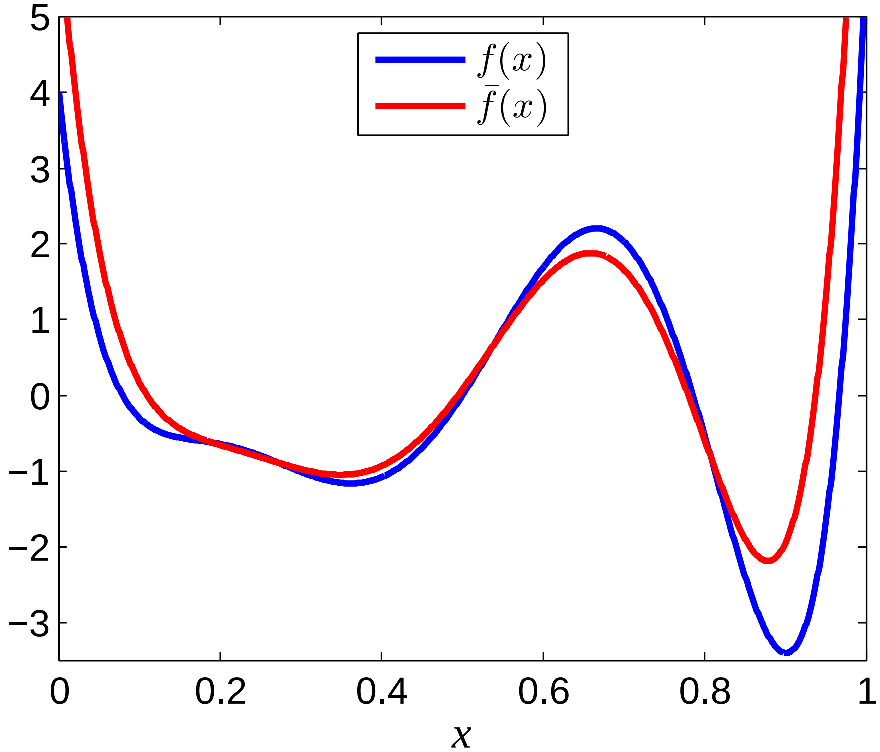


$$\sigma = 0.1$$

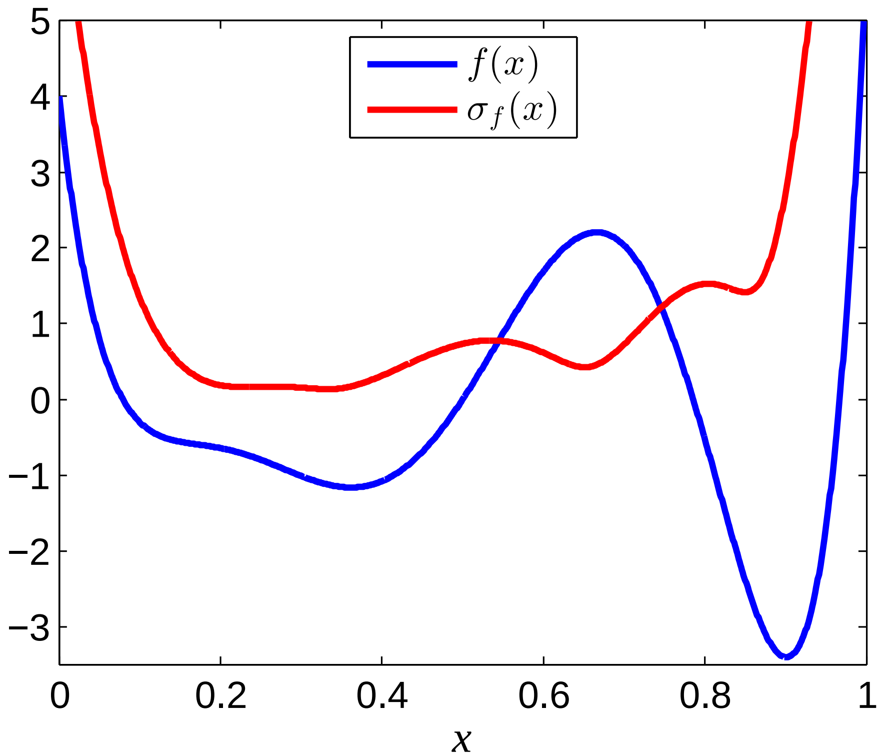
See notebook example.

Instead of optimizing deterministically what else can we do?

Minimize the mean of the function μ_f (with $\sigma_x = 0.05$)

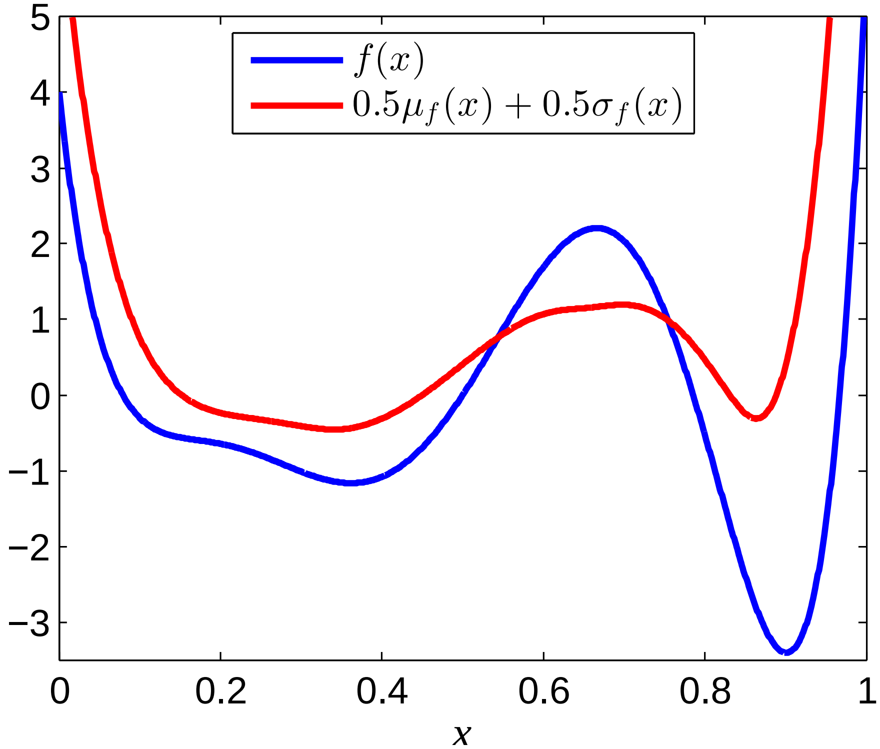


Minimize the standard deviation σ_f (with $\sigma_x = 0.05$) (with $\sigma = 0.05$)



Perform a multiobjective optimization.

Example below uses a weighted sum, but as we discussed there are better ways to do multiobjective optimization. (σ_x = 0.05)



Minimize a reliability metric:

$$\text{Prob}[f(x) > f_{crit}]$$