ME EN 335
Homework #1

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1. One of the best ways to learn about dynamic systems is to begin to notice real-life dynamic systems. Find examples of at least four dynamic systems: one mechanical, one electrical, one fluid, and one mixed-domain system (e.g., electromechanical, fluid-mechanical, etc.). Provide a simple diagram that describes each system's function and list the important variables that would appear in a model of these systems. You do not need to create a mathematical model.

2. Complete Problem 4.4 from the text.

3. Complete Problem 4.8, part a, from the text. Use $E = 3\text{x}10^7$ psi for steel.

4. Complete Problem 4.9 from the text.

5. Complete Problem 4.11 from the text. Include both positive and negative displacements.

6. Complete Problem 4.17 from the text. The figure for this problem is at the top of page 234 and the figure label is oddly placed.

7. Complete Problem 4.28 from the text.

8. Complete Problem 4.52 from the text.

9. Complete Problem 4.55 from the text. You do not need to derive the transfer function

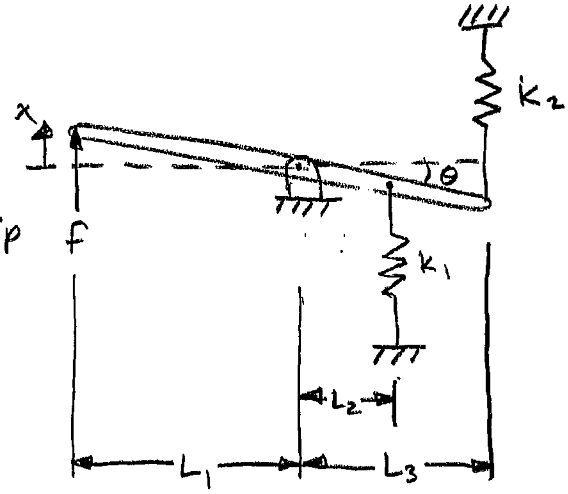
10. Complete Problem 4.70 from the text.

Problem 1

Consider this as one problem with four separate parts (4 pts each). For each of the subsystems (mechanical, electrical, fluid, mixed-domain), give yourself full credit if you made a good effort to provide a diagram and list of important variables.

Problem 4.4

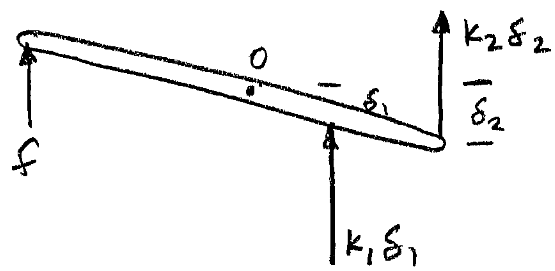
Want equivalent spring constant k_e, which is the relationship between x and f:



$f = k_e x$

From the FBD:

$[\Sigma M_o = 0]$



$f L_1 = k_1 \delta_1 L_2 + k_2 \delta_2 L_3$

But $\delta_1 = L_2 \theta$

$\delta_2 = L_3 \theta$

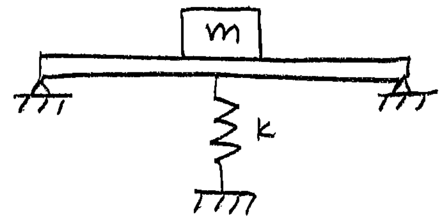
And $x = L_1 \theta \Rightarrow \theta = \frac{x}{L_1}$

$f L_1 = k_1 L_2^2 \theta + k_2 L_3^2 \theta = \frac{k_1 L_2^2 + k_2 L_3^2}{L_1} x$

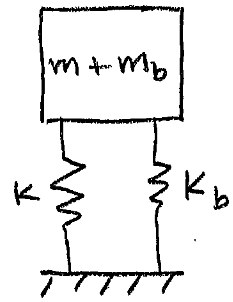
$f = \frac{k_1 L_2^2 + k_2 L_3^2}{L_1^2} x = k_e x$

$$\boxed{k_e = \frac{k_1 L_2^2 + k_2 L_3^2}{L_1^2}}$$

Problem 4.8.a



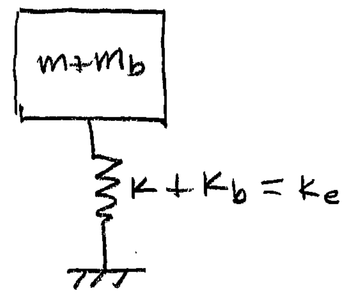
The system can be modeled as two parallel springs supporting a mass:



k_b = stiffness of beam

m_b = some portion of the mass of beam

This can be reduced to a mass with a single spring:



The stiffness of the beam is

$$k_b = \frac{48EI}{L^3} = \frac{4Ewh^3}{L^3}$$

$$k_b = \frac{4(3\times10^7)(12)(1)^3}{(36)^3} = 30864 \text{ lb/in} = 3.70\times10^5 \text{ lb/ft}$$

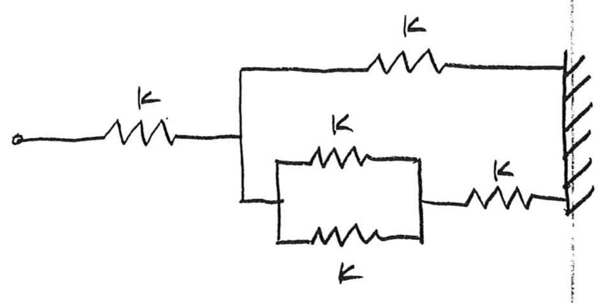
Since the springs are in parallel, the equivalent stiffness is $k_e = k + k_b$

We want $k_e = 2k_b$, so $2k_b = k + k_b$

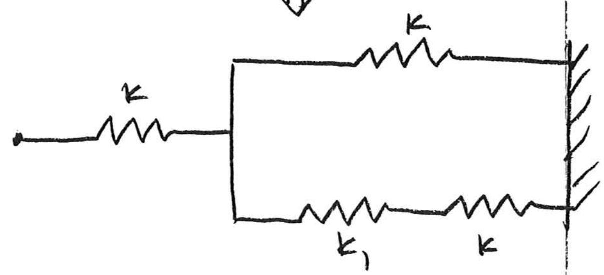
$\Rightarrow k = k_b$

$$\boxed{k = 3.7\times10^5 \text{ lb/ft} = 30864 \text{ lb/in}}$$

Problem 4.9

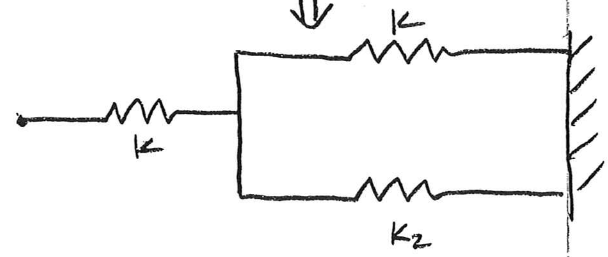


$$k_1 = k \| k = 2k$$



$$k_2 = \frac{k k_1}{k + k_1} = \frac{2k^2}{k + 2k}$$

$$k_2 = \frac{2k^2}{3k} = \frac{2k}{3}$$



$$k_3 = k + k_2 = \frac{5k}{3}$$

$$k_e = \frac{k k_3}{k + k_3}$$

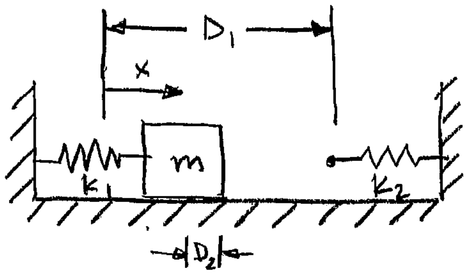
$$k_e = \frac{\frac{5k^2}{3}}{k + \frac{5k}{3}} = \frac{\frac{5k^2}{3}}{\frac{8k}{3}}$$

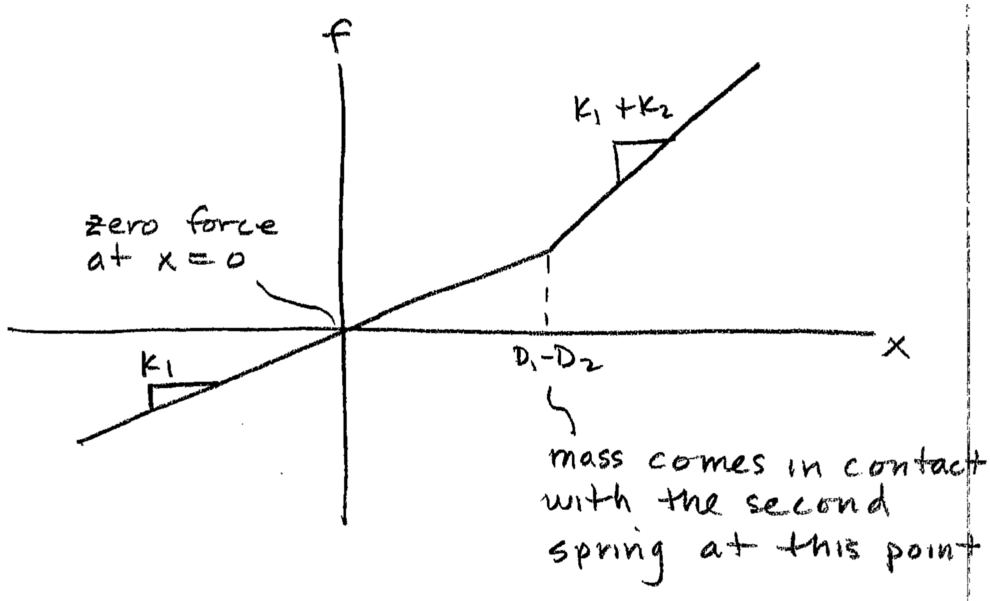
$$\boxed{k_e = \frac{5k}{8}}$$

Problem 4.11

K_1 unstretched when

$x = 0$





Problem 4.17

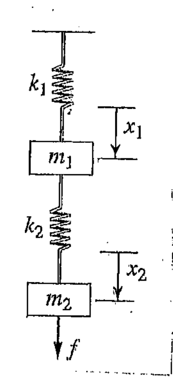
The deflection of each system is measured from static equilibrium.
Assume $x_2 > x_1$ in each case.

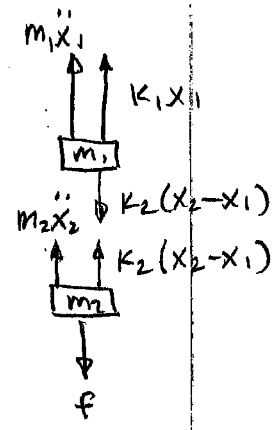
a) Mass 1:

$$k_2(x_2-x_1) - k_1 x_1 - m_1\ddot{x}_1 = 0$$

or

$$\boxed{m_1\ddot{x}_1 + k_2(x_1-x_2) + k_1 x_1 = 0}$$





Mass 2:

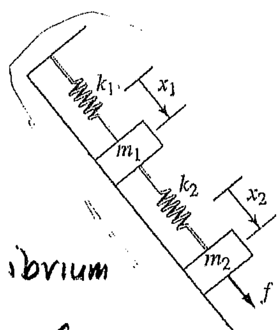
$$f - k_2(x_2-x_1) - m_2\ddot{x}_2 = 0$$

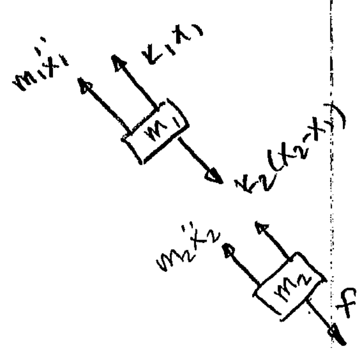
or

$$\boxed{m_2\ddot{x}_2 + k_2(x_2-x_1) = f}$$

Note that we didn't include gravitational forces because we measured displacements from static equilibrium.

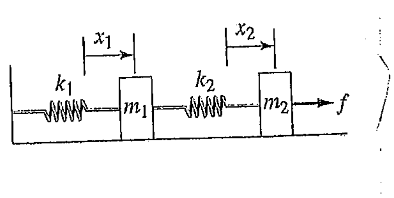
b) Note that the FBDs are identical to part a) because we are measuring from static equilibrium





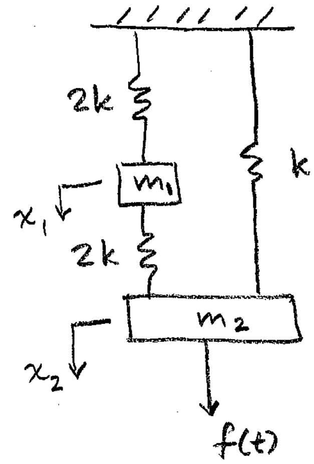
The EOMs are therefore identical.

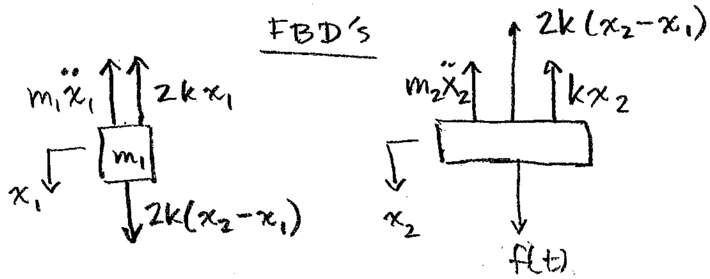
c) Same here.



Problem 4.28

Assume x_1 and x_2 are measured relative to static equilibrium





Mass 1 :

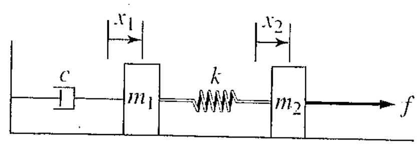
$$m_1\ddot{x}_1 + 2kx_1 - 2k(x_2-x_1) = 0$$

$$\boxed{m_1\ddot{x}_1 + 2kx_1 + 2k(x_1-x_2) = 0}$$

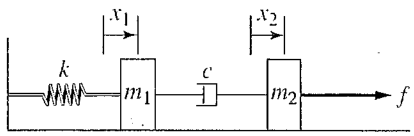
Mass 2 :

$$\boxed{m_2\ddot{x}_2 + kx_2 + 2k(x_2-x_1) = f(t)}$$

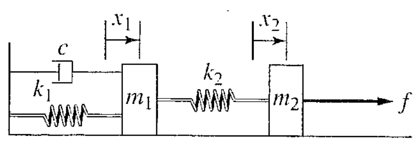
Problem 4.52



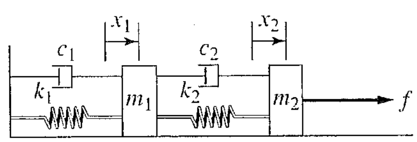
(a)



(b)

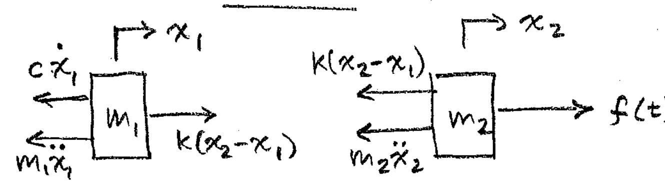


(c)



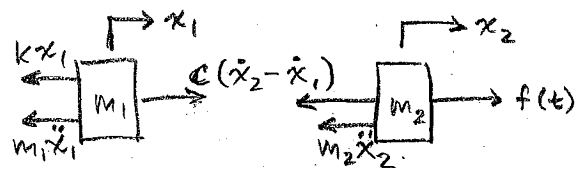
(d)

(a) FBD's



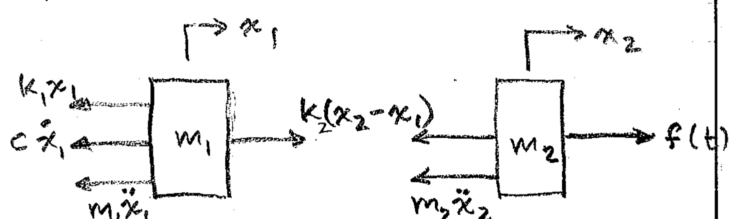
$$\boxed{\begin{aligned} m_1\ddot{x}_1 + c\dot{x}_1 + k(x_1-x_2) &= 0 \\ m_2\ddot{x}_2 + k(x_2-x_1) &= f(t) \end{aligned}}$$

(b) FBD's



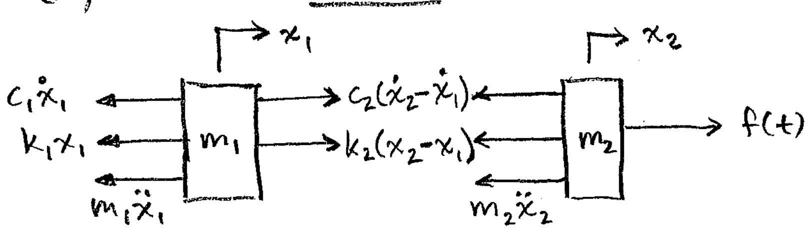
$$\boxed{\begin{aligned} m_1\ddot{x}_1 + c(\dot{x}_1-\dot{x}_2) + kx_1 &= 0 \\ m_2\ddot{x}_2 + c(\dot{x}_2-\dot{x}_1) &= f(t) \end{aligned}}$$

(c) FBD's



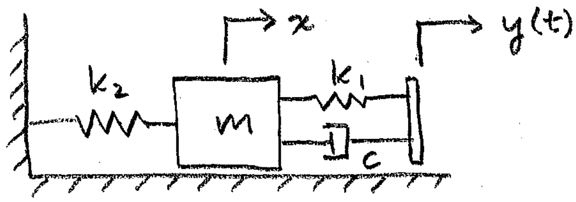
$$\boxed{\begin{aligned} m_1\ddot{x}_1 + c\dot{x}_1 + k_1x_1 + k_2(x_1-x_2) &= 0 \\ m_2\ddot{x}_2 + k_2(x_2-x_1) &= f(t) \end{aligned}}$$

(d) FBD's



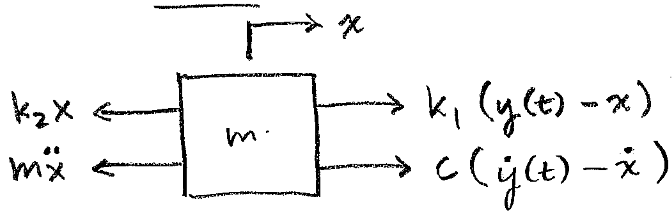
$$\boxed{\begin{aligned} m_1\ddot{x}_1 + c_1\dot{x}_1 + c_2(\dot{x}_1-\dot{x}_2) + k_1x_1 + k_2(x_1-x_2) &= 0 \\ m_2\ddot{x}_2 + c_2(\dot{x}_2-\dot{x}_1) + k_2(x_2-x_1) &= f(t) \end{aligned}}$$

Problem 4.55



Assume equilibrium position corresponds to $x = y = 0$.

FBD

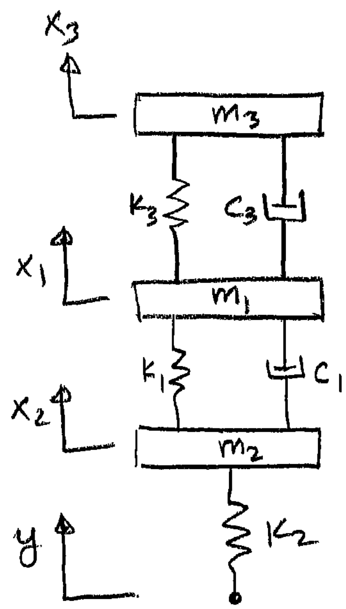


$$m\ddot{x} + k_2 x - k_1(y(t) - x) - c(\dot{y}(t) - \dot{x}) = 0$$

$$\boxed{m\ddot{x} + c\dot{x} + (k_1 + k_2)x = c\dot{y}(t) + k_1 y(t)}$$

Problem 4.70

Assume $y > x_2 > x_1 > x_3$



Mass 3:

$$k_3(x_1-x_3)+c_3(\dot{x}_1-\dot{x}_3)-m_3\ddot{x}_3=0$$

$$\boxed{m_3\ddot{x}_3+c_3(\dot{x}_3-\dot{x}_1)+k_3(x_3-x_1)=0}$$

Mass 1:

$$k_1(x_2-x_1)+c_1(\dot{x}_2-\dot{x}_1)-k_3(x_1-x_3)$$
$$-c_3(\dot{x}_1-\dot{x}_3)-m_1\ddot{x}_1=0$$

$$\boxed{\begin{aligned}m_1\ddot{x}_1&+c_1(\dot{x}_1-\dot{x}_2)+c_3(\dot{x}_1-\dot{x}_3)\\&+k_1(x_1-x_2)+k_3(x_1-x_3)=0\end{aligned}}$$

Mass 2:

$$k_2(y-x_2)-k_1(x_2-x_1)-c_1(\dot{x}_2-\dot{x}_1)$$
$$-m_2\ddot{x}_2=0$$

$$\boxed{\begin{aligned}m_2\ddot{x}_2+c_1(\dot{x}_2-\dot{x}_1)+k_2x_2+k_1(x_2-x_1)\\=k_2y\end{aligned}}$$