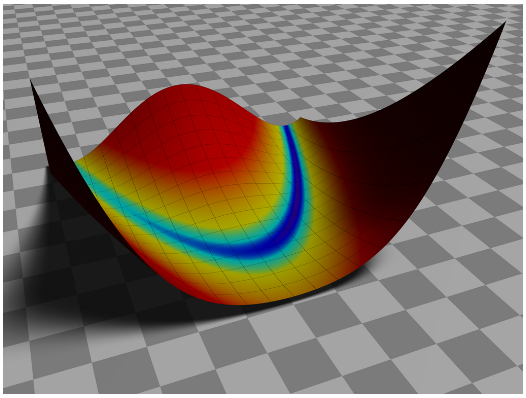
# Approximate Reliability Methods

Lecture 30



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## Outline

Worst-Case Tolerances

Transmitted Variance

These methods allow us to *approximately* satisfy these constraints:

$$\text{Prob}[c(x) \leq 0] \geq R$$

but in a much simpler way.

## Worst-Case Tolerances

Consider a constraint of the following form:

$$c(x, p) \leq 0$$

where $x$ and $p$ are random variabiles.

Assume: probability distributions for $x$ and $p$ are unknown, but we do know their tolerances:

$$\begin{aligned} x_1 &= 5.0 \pm 0.4 \\ x_2 &= 2.1 \pm 0.1 \\ p_1 &= 5.3 \pm 0.3 \\ &\vdots \end{aligned}$$

Let's assume the worst-case where every extreme combines simultaneously (and assume a first-order approximation).

$$\Delta c = \sum_{i=1}^{n} \left| \frac{\partial c}{\partial x_i} \Delta x_i \right| + \sum_{j=1}^{m} \left| \frac{\partial c}{\partial p_j} \Delta p_j \right|$$

We can now form a new tighter constraint (i.e., more reliable):

$$c(x) + \Delta c \leq 0$$

## Procedure:

1. Compute deterministic optimum (what you've already been doing).
2. Estimate worst-case $\Delta c$ at the deterministic optimum.
3. Adjust constraint to $c(x) + \Delta c \leq 0$ and reoptimize (start from the solution you just found).

Limitations:

- Worst-case assumes simultaneous extremes and so tends to be overly conservative.
- Assumes reliable optimum is near deterministic optimum.

## Transmitted Variance

Assume: probability distributions for $x$ and $p$ are known and are Gaussian

$$
\begin{aligned}
x_1 &= \mathcal{N}(5.0, 0.13) \\
x_2 &= \mathcal{N}(2.1, 0.033) \\
p_1 &= \mathcal{N}(5.3, 0.1) \\
&\vdots
\end{aligned}
$$

$$\sigma_c^2 = \sum_{i=1}^{n} \left( \frac{\partial c}{\partial x_i} \sigma_{x_i} \right)^2 + \sum_{j=1}^{m} \left( \frac{\partial c}{\partial p_j} \sigma_{p_j} \right)^2$$

Form new constraint as:

$$c(x) + k\sigma_c \le 0$$

where $k$ is user chosen.

$k = 2$ implies reliability of 97.72%. Why not 95%?

What if you have two constraints both with $k = 2$. What is the overall reliability?

# Same procedure:

1. Compute deterministic optimum (what you've already been doing).
2. Estimate transmitted variance $\sigma_c$.
3. Adjust constraint to $c(x) + k\sigma_c \leq 0$ and reoptimize.

Limitations:

- Assumes reliable optimum is near deterministic optimum.
- Assumes output distribution is normally distributed.
- Assumes constraints are uncorrelated.

# Work an example in small groups

$$
\begin{aligned}
\text{min.} \quad & 4x_1^2 + 2x_2^2 + x_3^2 \\
\text{s.t.} \quad & 6x_1 + 2x_2 + 4x_3 \geq 12 \\
& x_1 - 4x_2 + 7x_3 \leq 10
\end{aligned}
$$

The deterministic optimum is:

$$x^* = [0.69, 0.59, 1.67]$$

$$f^* = 5.105$$

- Find $\Delta c$ assuming worst-case with tolerances $\Delta x_1 = \Delta x_2 = \Delta x_3 = 0.3$
- Find $\sigma c$ assuming normal distributions with $\sigma_{x_1} = \sigma_{x_2} = \sigma_{x_3} = 0.1$
- If you used $k = 1$ for each constraint. What would be the overall reliability?