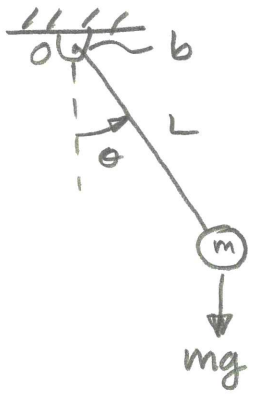
Problem 1 Pendulum:



EOM:

$$I_o\ddot{\theta} + b\dot{\theta} + mgL\sin\theta = 0$$

$$I_o = mL^2$$

$$\boxed{mL^2\ddot{\theta} + b\dot{\theta} + mgL\sin\theta = 0}$$

For small θ, $\sin\theta \approx \theta$

$$mL^2\ddot{\theta} + b\dot{\theta} + mgL\theta = 0$$

or

$$\boxed{\ddot{\theta} + \frac{b}{mL^2}\dot{\theta} + \frac{g}{L}\theta = 0}$$

$$\boxed{\omega_n = \sqrt{\frac{g}{L}}}$$

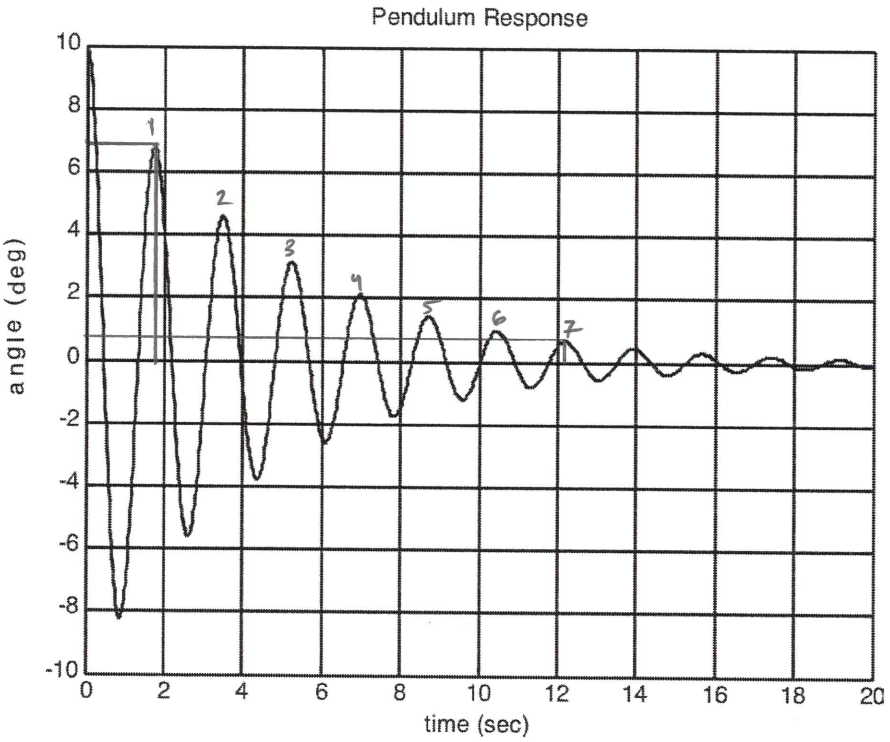
$$2\zeta\omega_n = \frac{b}{mL^2} \Rightarrow \zeta = \frac{b}{2mL^2\omega_n} = \frac{b}{2mL^2\sqrt{\frac{g}{L}}}$$

$$\boxed{\zeta = \frac{b}{2m\sqrt{gL^3}}}$$

$$\sigma = \zeta\omega_n = \frac{b}{2m\sqrt{gL^3}}\sqrt{\frac{g}{L}}$$

$$\boxed{\sigma = \frac{b}{2mL^2}}$$

Problem 2



Use the log decrement method.

$t_1 = 1.7\,s \qquad x_1 = 7$

$t_7 = 12.2\,s \qquad x_7 = 0.6$

Find the oscillation period:

$$T = \frac{t_7 - t_1}{7-1} = \frac{12.2 - 1.7}{6} = 1.75\ s$$

Find the damped natural frequency:

$$\omega_d = \frac{2\pi}{T} = 3.59\ rad/s$$

Find the damping ratio:

$$\zeta = \frac{\frac{1}{7-1}\ln\left(\frac{7}{0.6}\right)}{\sqrt{4\pi^2 + \left(\frac{1}{7-1}\ln\left(\frac{7}{0.6}\right)\right)^2}} = 0.065$$

Find the natural frequency:

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = 3.6 \text{ rad/s}$$

Compare with theoretical results to find model parameters:

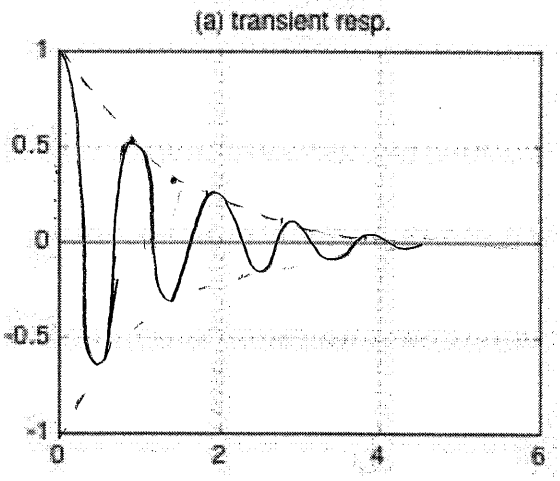
$$\omega_n = 3.6 = \sqrt{\frac{g}{L}} \Rightarrow \boxed{L = 0.76 \text{ m}}$$

$$2\zeta\omega_n = \frac{b}{mL^2} \Rightarrow m = \frac{b}{2\zeta\omega_n L^2} = \frac{1}{2(0.065)(3.6)(0.76)^2}$$

$$m = 3.7 \text{ kg}$$

$$W = mg = \boxed{36.3 \text{ N}}$$

Problem 3



$\tau = \frac{1}{\zeta \omega_n}$

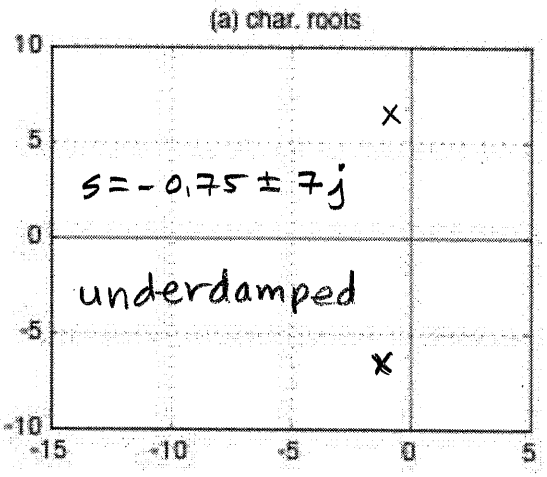
$= \frac{1}{0.75}$

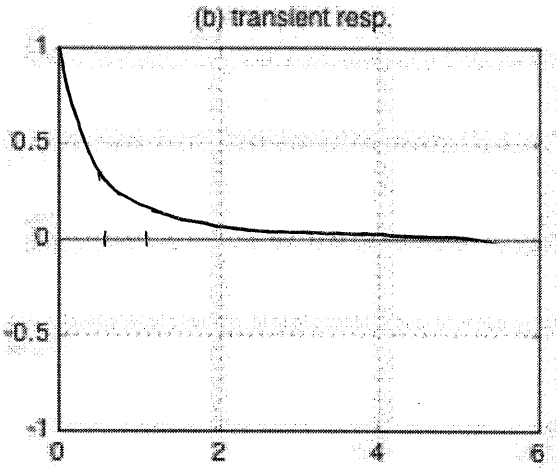
$\tau = \frac{4}{3}$

$\omega_d = 7$ rad/s

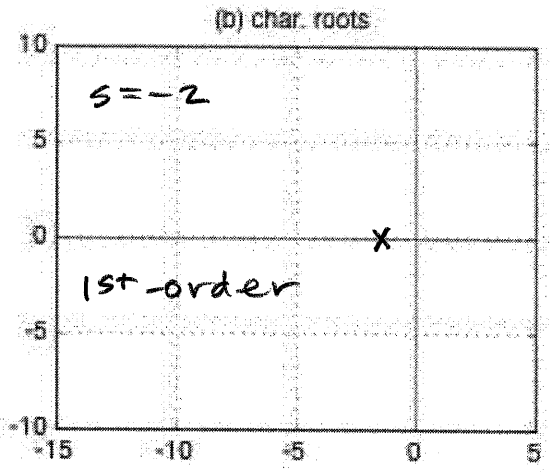
$f_d = 1.11$ Hz

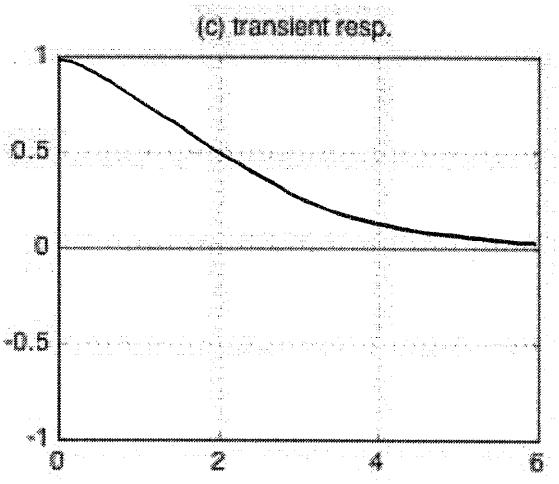
$T = 0.9$s

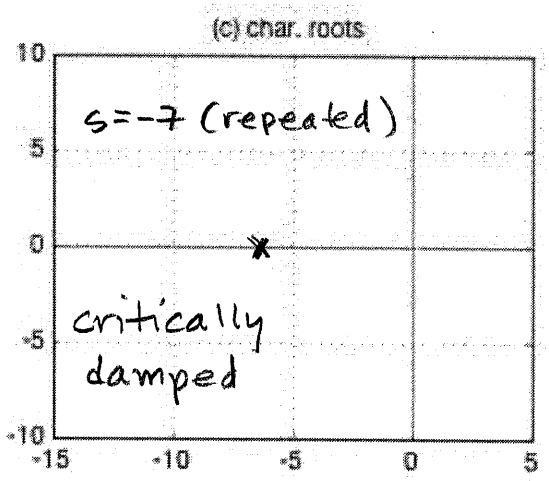




$\tau = \frac{1}{2}$

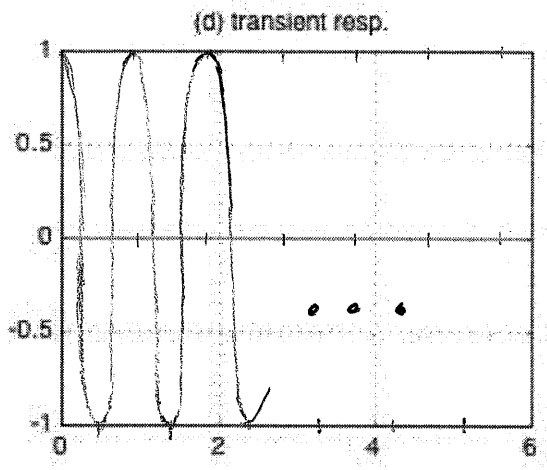


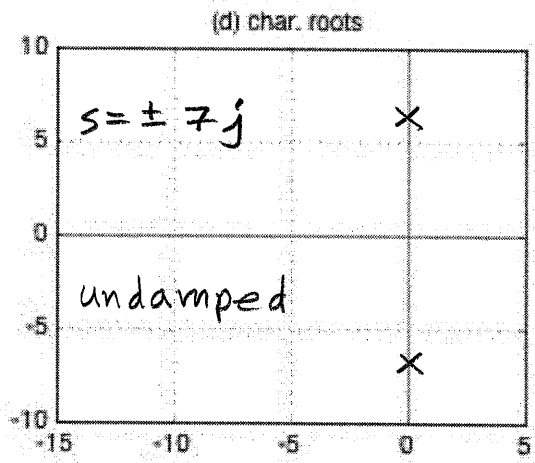




$\omega_d = 7$ rad/s

$T = 0.9$ s





$\tau_1 = \frac{1}{49} = 0.02$

$\tau_2 = \frac{1}{1} = 1$ s