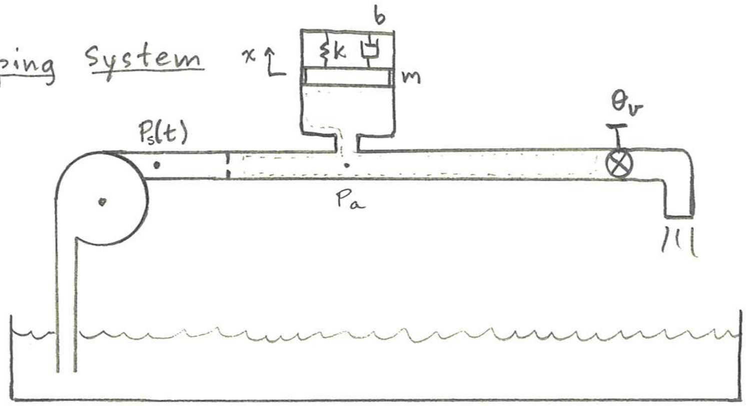
<u>Pumping System</u>



- distinct pressures : $P_s(t)$, P_a ← gauge (relative to P_{atm})

- continuity equation for c.v. around a :

$$Q_{in} - Q_{out} = \dot{V} + \frac{V_0}{\beta} \dot{P}_a$$

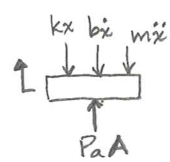
Assume $\frac{V_0}{\beta}$ large enough for compressibility effects to be significant.

- physical relations

$$Q_{in} = k_1 \sqrt{P_s(t) - P_a}$$

$$Q_{out} = k_2 \theta_v \sqrt{P_a}$$

$$\dot{V} = A v \qquad F_f = P_a A$$



- Newton's 2nd Law :

$$m\dot{v} + bv + kx = P_a A$$

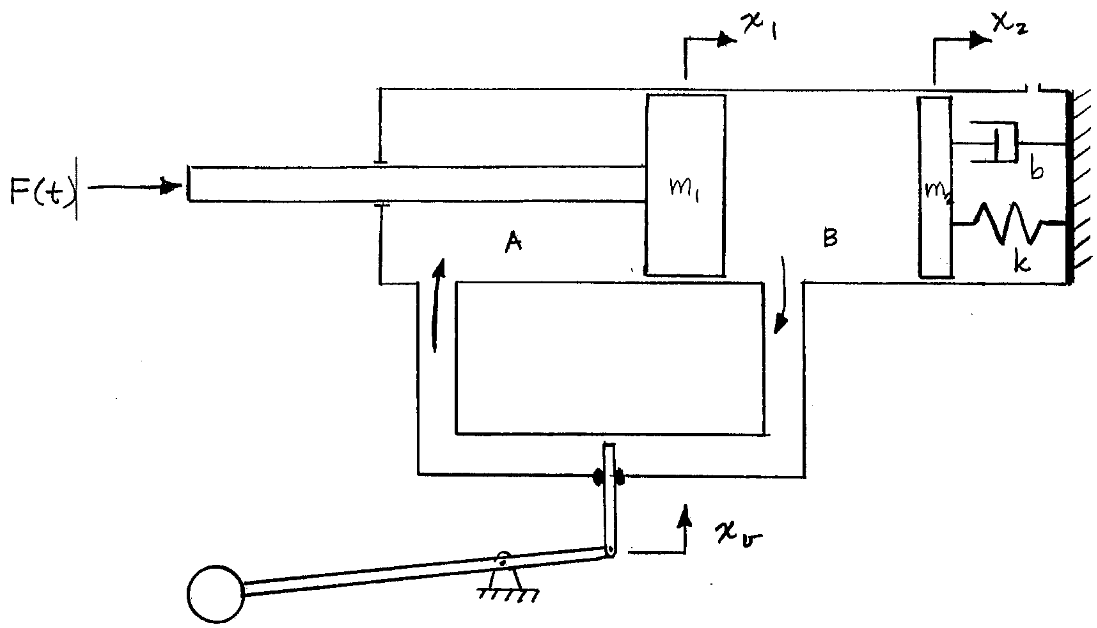
$$\Rightarrow \boxed{\begin{aligned} \dot{v} &= -\frac{b}{m} v - \frac{k}{m} x + \frac{A}{m} P_a \\ \dot{x} &= v \end{aligned}}$$

- Combine relations

$$k_1 \sqrt{P_s(t) - P_a} - k_2 \theta_v(t) \sqrt{P_a} = Av + \frac{V_0}{\beta} \dot{P}_a$$

$$\boxed{\dot{P}_a = \frac{\beta}{V_0} \left[k_1 \sqrt{P_s(t) - P_a} - k_2 \theta_v(t) \sqrt{P_a} - Av \right]}$$

Example : Fluid Brake



- Pressure nodes: P_a , P_b

- Continuity Equs:

$$Q_{in,a} - \cancel{Q_{out,a}}^{\,0} = \dot{V}_a + \frac{V_{o,a}}{\beta}\dot{P}_a$$

$$\cancel{Q_{in,b}}^{\,0} - Q_{out,b} = \dot{V}_b + \frac{V_{o,b}}{\beta}\dot{P}_b$$

- Define $\dot{V}$'s

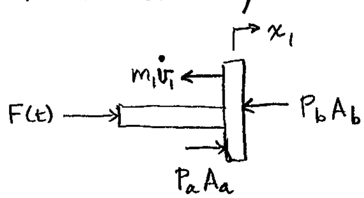
$$\dot{V}_a = A_a v_1$$

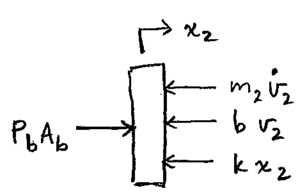
$$\dot{V}_b = A_b(v_2 - v_1)$$

- Flow relations:

$$Q_{in,a} = Q_{out,b} = k x_v \sqrt{|P_b - P_a|}\,\text{sign}(P_b - P_a)$$

- Mechanical System:





$$m_1 \dot{v}_1 + P_b A_b - P_a A_a - F(t) = 0$$

$$\boxed{\begin{aligned} \dot{v}_1 &= \frac{A_a}{m_1} P_a - \frac{A_b}{m_1} P_b + \frac{F(t)}{m_1} \\ \dot{x}_1 &= v_1 \end{aligned}}$$

$$m_2 \dot{v}_2 + b v_2 + k x_2 - P_b A_b = 0$$

$$\boxed{\begin{aligned} \dot{v}_2 &= -\frac{b}{m_2} v_2 - \frac{k}{m_2} x_2 + \frac{A_b}{m_2} P_b \\ \dot{x}_2 &= v_2 \end{aligned}}$$

- Combine relations

$$\dot{P}_a = \frac{\beta}{V_{o,a}} \left[Q_{in,a} - \dot{V}_a \right]$$

$$\boxed{\dot{P}_a = \frac{\beta}{V_{o,a}} \left[k x_v \sqrt{|P_b - P_a|}\, sign(P_b - P_a) - A_a v_1 \right]}$$

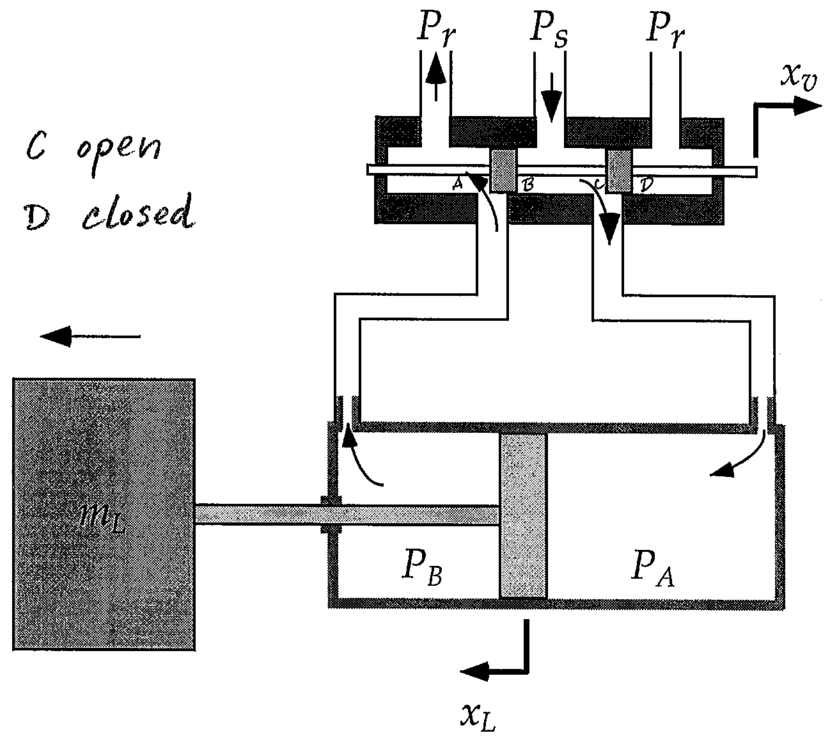
$$\dot{P}_b = \frac{\beta}{V_{o,b}} \left[-Q_{out,b} - \dot{V}_b \right]$$

$$\boxed{\dot{P}_b = \frac{\beta}{V_{o,b}} \left[-k x_v \sqrt{|P_b - P_a|}\, sign(P_b - P_a) - A_b (v_2 - v_1) \right]}$$

4-way servovalve basics

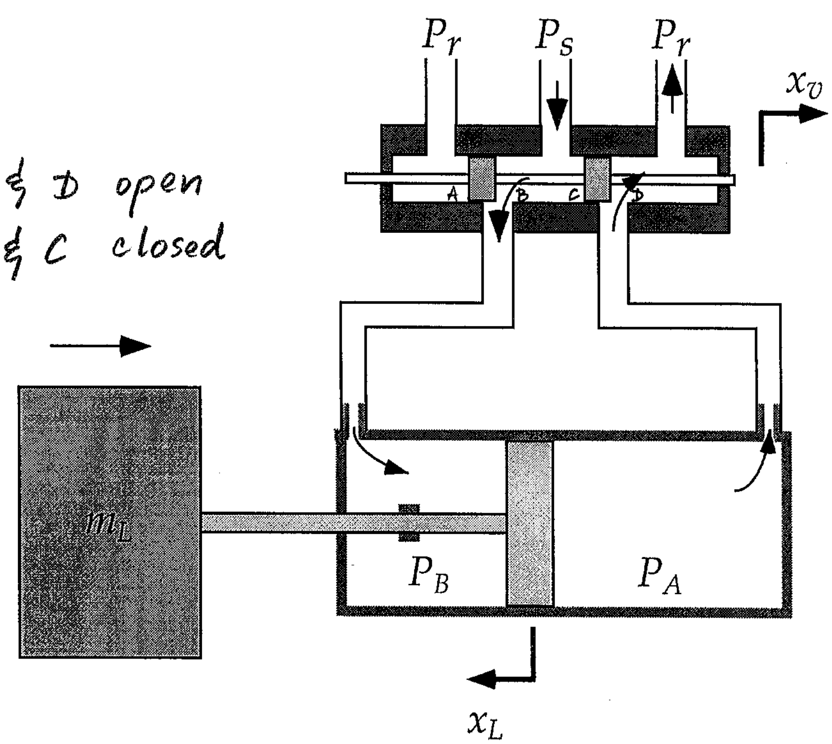
$x_v > 0$

orifices A & C open
B & D closed

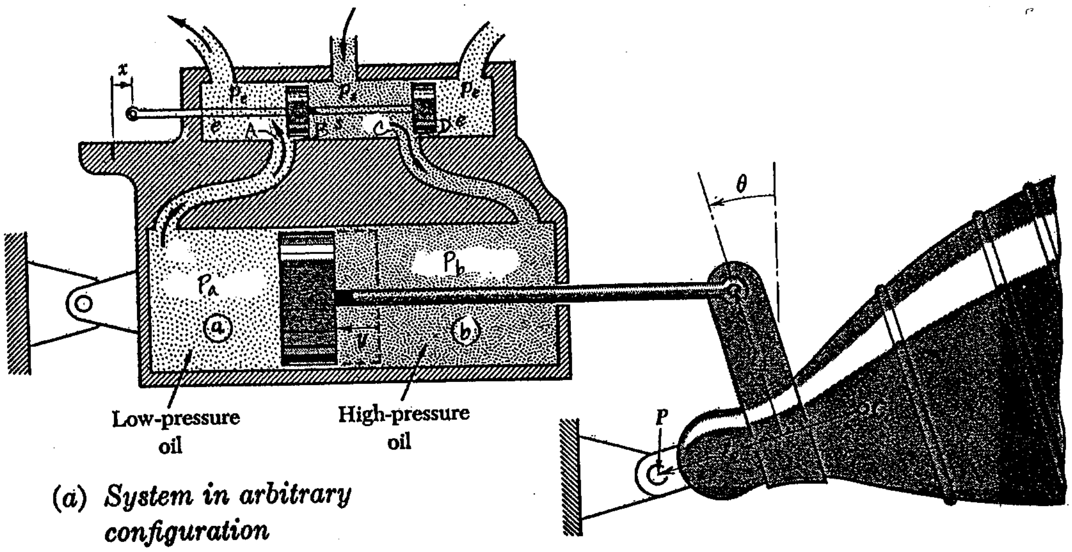


$x_v < 0$

orifices B & D open
A & C closed



A Complex Fluid-Power Example:



(a) *System in arbitrary configuration*

Include in model:

1) Orifice resistance
2) Fluid compressibility
3) Leakage across piston
4) Seal friction
5) Piston mass
6) Engine inertia
7) Pivot Friction

Consider case where $x > 0$ $\Rightarrow$ ports A & C open

- Distinct pressures : P_s , P_e , P_a , P_b

  (P_s, P_e: known inputs)

- Continuity Equations :

$$Q_{in,a} - Q_{out,a} = \dot{V}_a + \frac{V_{0,a}}{\beta}\dot{P}_a$$

$$Q_{in,b} - Q_{out,b} = \dot{V}_b + \frac{V_{0,b}}{\beta}\dot{P}_b$$

- $\dot{V}_a = -A_a v_y$ $\qquad$ $\dot{V}$ terms

$$\dot{V}_b = A_b v_y$$

- Flows

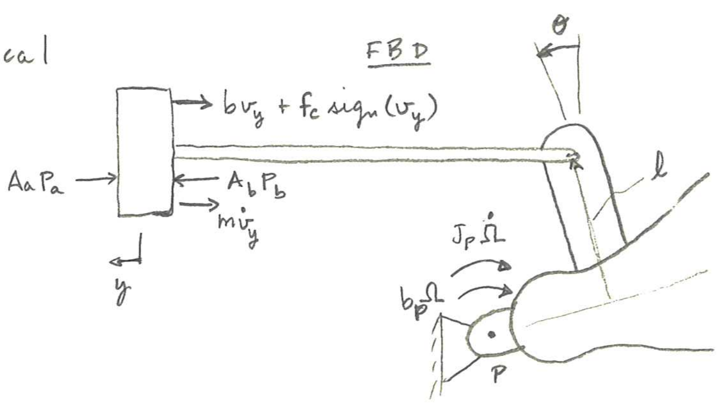
$$Q_{in,a} = k_\ell \sqrt{|P_b - P_a|}\ \text{sign}(P_b - P_a)$$

$$Q_{out,a} = k_v x \sqrt{|P_a - P_e|}\ \text{sign}(P_a - P_e)$$

$$Q_{in,b} = k_v x \sqrt{|P_s - P_b|}\ \text{sign}(P_s - P_b)$$

$$Q_{out,b} = k_\ell \sqrt{|P_b - P_a|}\ \text{sign}(P_b - P_a)$$

- Mechanical



* Assume small angles !

$\Rightarrow$ $y = \ell\theta$

$v_y = \ell\Omega$

$$\Sigma M_P^* = 0 \qquad (m\dot{v}_y + bv_y + f_c\,\text{sign}(v_y) + A_a P_a - A_b P_b)\ell + J_p\dot{\Omega} + b_p\Omega = 0$$

$$(m\ell\dot{\Omega} + b\ell\Omega + f_c\,\text{sign}(\Omega) + A_a P_a - A_b P_b)\ell + J_p\dot{\Omega} + b_p\Omega = 0$$

$$(J_p + m\ell^2)\dot{\Omega} + (b_p + b\ell^2)\Omega + f_c\ell\,\text{sign}(\Omega) + A_a\ell P_a - A_b\ell P_b = 0$$

$$\boxed{\Rightarrow \quad \begin{aligned} \dot{\Omega} &= \frac{1}{J_p + m l^2}\left[-(b_p + b l^2)\Omega - f_c l \, \text{sign}(\Omega) + A_b l P_b - A_a l P_a\right] \\ \dot{\theta} &= \Omega \end{aligned}}$$

- Combine Relations

$$\dot{P}_a = \frac{\beta}{V_{o,a}}\left[Q_{in,a} - Q_{out,a} - \dot{V}_a\right]$$

$$\boxed{\dot{P}_a = \frac{\beta}{V_{o,a}}\left[k_l \sqrt{|P_b - P_a|}\, \text{sign}(P_b - P_a) - k_v x \sqrt{|P_a - P_e|}\, \text{sign}(P_a - P_e) + A_a l \Omega\right]}$$

$$\dot{P}_b = \frac{\beta}{V_{o,b}}\left[Q_{in,b} - Q_{out,b} - \dot{V}_b\right]$$

$$\boxed{\dot{P}_b = \frac{\beta}{V_{o,b}}\left[k_v x \sqrt{|P_s - P_b|}\, \text{sign}(P_s - P_b) - k_l \sqrt{|P_b - P_a|}\, \text{sign}(P_b - P_a) - A_b l \Omega\right]}$$

* What happens if we neglect fluid compliance (even if the fluid is truly incompressible)?

- State equations for P_a and P_b (ODEs) become coupled nonlinear algebraic expressions in P_a and P_b ($\dot{P}_a = \dot{P}_b = 0$). The equations do not have a closed form solution for P_a and P_b.

- Better to include compliance – even though it may be very small.