## Problem 8.3

a) $16\dot{x} + 14x = 0, \quad x(0) = 6$

$\frac{16}{14}\dot{x} + x = 0 \Rightarrow \boxed{\tau = \frac{16}{14} = \frac{8}{7} = 1.14}$

$\boxed{x(t) = 6e^{-\frac{7}{8}t}}$

b) $12\dot{x} + 5x = 15, \quad x(0) = 3$

$\frac{12}{5}\dot{x} + x = 3 \Rightarrow \boxed{\tau = \frac{12}{5} = 2.4}$

$\boxed{x(t) = 3e^{-\frac{5}{12}t}}$

c) $13\dot{x} + 6x = 0, \quad x(0) = -2$

$\frac{13}{6}\dot{x} + x = 0 \Rightarrow \boxed{\tau = \frac{13}{6} = 2.17}$

$\boxed{x(t) = -2e^{-\frac{6}{13}t}}$

d) $7\dot{x} - 5x = 0, \quad x(0) = 9$

The system is unstable (pole at $s = +\frac{5}{7}$)

so $\tau$ is undefined.

$\boxed{x(t) = 9e^{\frac{5}{7}t}}$

# BLOCK ON AN INCLINED PLANE

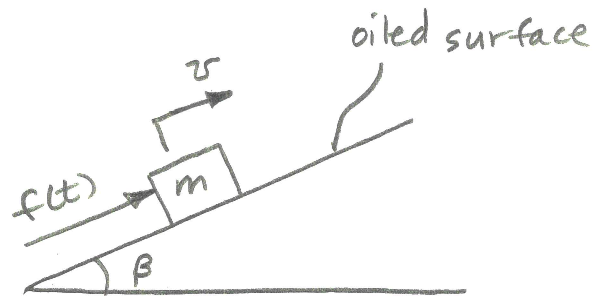
EOM:

$$m\dot{v} + bv = f(t) - mg\sin\beta$$

$f(t) = A\,u_s(t)$ (a step)

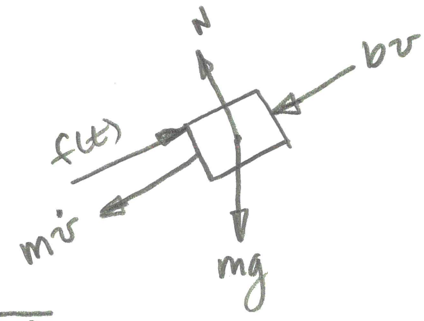
and

$v(0) = 0$




Laplace:

$$(ms+b)V(s) = \frac{A}{s} - \frac{mg\sin\beta}{s}$$




$$V(s) = \frac{A - mg\sin\beta}{s(ms+b)}$$

$$V(s) = \frac{A - mg\sin\beta}{m}\,\frac{m}{b}\,\frac{b/m}{s(s+b/m)}$$

$$V(s) = \frac{A - mg\sin\beta}{b}\,\frac{b/m}{s(s+b/m)}$$

but $\tau = \frac{m}{b}$ from the original equation

$$V(s) = \frac{A - mg\sin\beta}{b}\,\frac{1/\tau}{s(s+1/\tau)}$$

$m = 80$ kg

$b = 12$ N·s/m

$A = 400$ N

$\beta = 20°$

$$V(s) = 10.97\,\frac{0.15}{s(s+0.15)}$$

a) $v_{ss} = \lim_{s \to 0} s\,V(s)$

$$= \lim_{s \to 0} \cancel{s} \frac{10.97\,(0.15)}{\cancel{s}\,(s+0.15)}$$

$$\boxed{v_{ss} = 10.97 \text{ m/s}}$$

b) $\tau = \dfrac{m}{b} = \dfrac{80}{12} = \boxed{6.67 \text{ s}}$

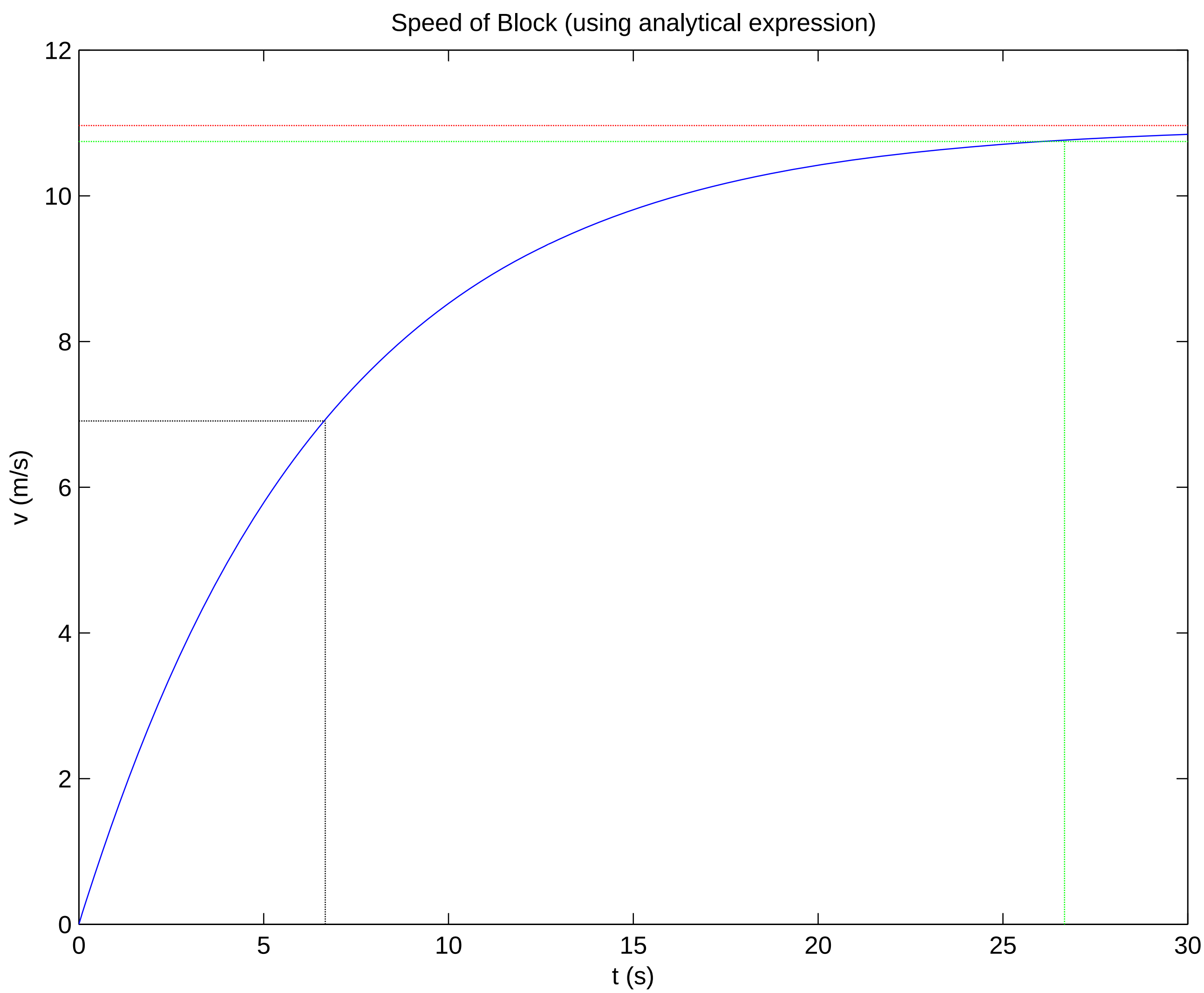
c) $t_s = 4\tau = 4(6.67) = \boxed{26.7 \text{ s}}$

d) $v(t) = \mathcal{L}^{-1}\left\{10.97 \dfrac{0.15}{s(s+0.15)}\right\}$

$$\boxed{v(t) = 10.97\left(1 - e^{0.15t}\right)}$$

```
% Block sliding on an oiled inclined plane with a step input

clear;
clc;

% Model parameters
m = 80;         % kg
b = 12;         % N*s/m
beta = 20;      % deg
g = 9.81;       % m/s^2

% Step input
A = 400;        % N

% Time constant
tau = m/b;

% Time vector
t = 0:0.1:30;


%--------------------------------------------------
% Method 1: 'step' command with transfer function

% % Transfer function V(s)/F(s) = G(s), where F(s) includes
% % applied force and gravity force
% num = 1;
% den = [m b];
% G = tf(num,den);
%
% % Find unit step response
% [v,t] = step(G,t);
%
% % Convert to non-unit step response due to the applied force
% % and gravity
% v = (A-m*g*sind(beta))*v;
%
% % Plot result
% figure;
% plot(t,v);
% xlabel('t (s)');
```

```
% ylabel('v (m/s)');
% title('Speed of Block (using step command)');


%-------------------------------------------------
% Method 2: Analytical expression

clear v;

% Calculate speed using analytical expression
v = ((A-m*g*sind(beta))/b)*(1-exp(-t/tau));

% Plot result
figure;
plot(t,v);
xlabel('t (s)');
ylabel('v (m/s)');
title('Speed of Block (using analytical expression)');

% Add lines of interest to plot
v_ss = (A-m*g*sind(beta))/b
v_tau = 0.63*v_ss
hold on;
plot([0 tau],[v_tau v_tau],'k:');       % v(tau)
plot([tau tau],[0 v_tau],'k:');         % t = tau
plot(t,v_ss*ones(size(t)),'r:');        % v_ss
plot(t,0.98*v_ss*ones(size(t)),'g:');   % 0.98*v_ss
plot([4*tau 4*tau],[0 0.98*v_ss],'g:');
```



Speed of Block (using analytical expression)
v (m/s)
t (s)
0
2
4
6
8
10
12
0
5
10
15
20
25
30