# MNIST 手写字识别程序入门

# - 一、部分基础

## ▼ 1.1 线性回归基础

### 1.1.1 最小二乘法曲线拟合问题

## 1.1.2 模型假设

理想的线性模型

$$y=0.99x+9.31; x\in [1:1:10]$$

假设的线性模型

$$f(x; a, b) = ax + b$$

## 1.1.3 最小二乘法确定参数

$$f(x; a, b) = ax + b$$

均方误差函数

$$S = \sum_{i=1}^{n} (y_i - (ax_i + b))^2$$

对误差求极限, 找误差函数的最小值点

$$egin{aligned} rac{\partial S}{\partial a} &= -2(\sum_{i=1}^n x_i y_i - b \sum_{i=1} n x_i - a \sum_{i=1}^n x_i^2) \ rac{\partial S}{\partial b} &= -2(\sum_{i=1}^n y_i - n b - a \sum_{i=1}^n x_i) \end{aligned}$$

令偏导数为零,求得凸函数的极小值点处的a,b 值

$$a = rac{n\sum x_iy_i - \sum x_i\sum y_i}{n\sum x_i^2 - (\sum x_i)^2} \ b = rac{\sum x_i^2\sum y_i - \sum x_i\sum x_iy_i}{n\sum x_i^2 - (\sum x_i)^2}$$

下面用两种代码的实现方式实现上述曲线拟合过程

• 用户自定义函数代码实现

• 调用numpy的最小二乘法的线性拟合问题Istsq函数实现

```
# 用户自定义函数代码实现
import numpy as np
import matplotlib.pyplot as plt
def calcAB(x,y):
    n = len(x)
    sumX, sumY, sumXY, sumXX = 0, 0, 0, 0
    for i in range(0,n):
         sumX
              += x[i]
         sumY
              += y[i]
         sumXX += x[i]*x[i]
        sumXY += x[i]*y[i]
    a = (n*sumXY - sumX*sumY)/(n*sumXX - sumX*sumX)
    b = (sumXX*sumY - sumX*sumXY)/(n*sumXX-sumX*sumX)
    return a,b,
\# xi = [1,2,3,4,5,6,7,8,9,10]
\# yi = [10,11.5,12,13,14.5,15.5,16.8,17.3,18,18.7]
xi = [1,2,3,4,5,6,7,8,9,10]
\# yi = [1 for i in range(10)]
yi = [0] * 10
print(yi)
for num in xi:
  yi[num -1] = num*1.201030944 + 9.8678999766
a,b=calcAB(xi,yi)
print("y = 1.201030944*x + 9.8678999766", '\n', "f(x;a,b) = (a)%3.5fx + (b)%3.5f
x = np.linspace(0,10)
y = a * x + b
plt.plot(x,y,'red', label='fitting curve')
plt.scatter(xi,yi, label='primitive curve')
plt.legend(loc='right')
plt.title('last square method fitting curve')
plt.show()
     [0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
     y = 1.201030944*x + 9.8678999766
      f(x;a,b) = (a)1.20103x + (b)9.86790
                   last square method fitting curve
      22
      20
      18
                                            fitting curve
      16
                                            primitive curve
      14
      12
      10
                            4
                                                     10
# 最小二乘法的线性拟合问题lstsq函数实现
import numpy as np
import matplotlib.pyplot as plt
x = [1,2,3,4,5,6,7,8,9,10]
y = [10, 11.5, 12, 13, 14.5, 15.5, 16.8, 17.3, 18, 18.7]
y = [0] * 10
for num in x:
  y[num -1] = num*1.201030944 + 9.8678999766
```

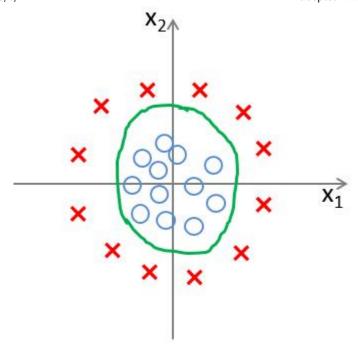
```
A = np.vstack([x,np.ones(len(x))]).T
a,b = np.linalg.lstsq(A,y, rcond=-1)[0]
print("1.理想曲线:","y = 1.201030944*x + 9.8678999766")
print('2.拟合曲线:', "f(x;a,b) = %10.5fx + %10.5f" %(a,b))
x = np.array(x)
y = np.array(y)
plt.plot(x,y,'o',label='idea curve',markersize=10)
plt.plot(x,a*x+b,'r',label='fitting curve')
plt.legend(loc='upper left')
plt.title(' the least-squares solution to a linear matrix equation.')
plt.show()
      1.理想曲线: y = 1.201030944*x + 9.8678999766
      2.拟合曲线: f(x;a,b) =
                                          1.20103x +
                                                               9.86790
            the least-squares solution to a linear matrix equation.
        22
                 idea curve
                 fitting curve
        20
       18
       16
       14
       12
                               4
                                           6
                                                       8
                                                                  10
```

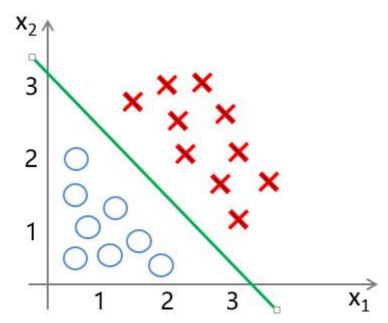
```
# 矩阵的转置
```

import numpy as np

```
x = np.array([[1, 2, 3, 4, 5], [1, 2, 3, 5, 4]])
print(x,x.T, x.transpose())
    [[1 2 3 4 5]
      [1 2 3 5 4]] [[1 1]
      [2 2]
      [3 3]
      [4 5]
      [5 4]] [[1 1]
      [2 2]
      [3 3]
      [4 5]
      [5 4]]
```

# ▼ 1.2 逻辑回归问题





## 1.2.1 线性可分逻辑回归模型

• 本文只针对--->线性可分且特征数为n种的情况下,目标找到n维的超平面

$$z(x^{(i)}) = heta_0 + heta_1 x_1^{(i)} + heta_2 x_2^{(i)} {+} \ldots {+} heta_n x_n^{(i)}$$

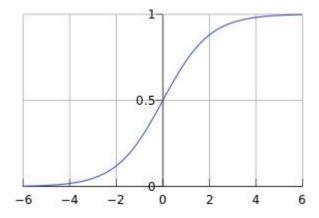
## 1.2.2 线性输出转逻辑输出

逻辑回归作为分类算法,和前面的线性回归模型的最大不同之处在于它的输出是 **0/1** 。那么如何将输出值转换成0/1呢?在此介绍一下sigmoid 函数。

• sigmoid 函数定义

$$sigmoidfunction; g(z) = rac{1}{1 + e^{-z}}$$

• 函数图像如下



函数的意义如下公式所示

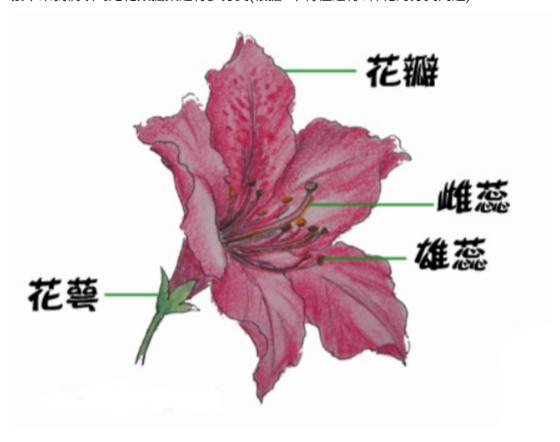
$$y = \left\{ egin{array}{ll} 1, & ext{if } g(z) \geq 0.5 \ 0, & ext{otherwise} \end{array} 
ight.$$

其中 y 表示分类结果。sigmoid 函数实际表达的是将样本分为"1"类的概率,这将在本文的最后一部分进行详细解释。

# ▼ 1.3 多分类问题

## 1.3.1 Iris莺尾花数据分类(3种结果分类)

接下来我们以莺尾花数据集进行多分类(根据4个特征进行3种花的分类问题)



# 1.3.2 莺尾花数据集介绍

下面截取iris数据集原始iris.csv文件中的部分内容

**C**→

```
5.1,3.5,1.4,0.2,Iris-setosa
4.9,3.0,1.4,0.2,Iris-setosa
4.7,3.2,1.3,0.2,Iris-setosa
4.6,3.1,1.5,0.2,Iris-setosa
5.0,3.6,1.4,0.2,Iris-setosa
5.4,3.9,1.7,0.4,Iris-setosa
4.6,3.4,1.4,0.3,Iris-setosa
5.0,3.4,1.5,0.2,Iris-setosa
4.4,2.9,1.4,0.2,Iris-setosa
4.9,3.1,1.5,0.1,Iris-setosa
5.4,3.7,1.5,0.2,Iris-setosa
4.8,3.4,1.6,0.2,Iris-setosa
```

Iris 数据集中对原始iris.csv文件中的数据解释如下

|   | sepal_len | sepal_width | petal_len | petal_width | class       |
|---|-----------|-------------|-----------|-------------|-------------|
| 0 | 4.9       | 3.0         | 1.4       | 0.2         | Iris-Setosa |
| 1 | 4.7       | 3.2         | 1.3       | 0.2         | Iris-Setosa |
| 2 | 4.6       | 3.1         | 1.5       | 0.2         | Iris-Setosa |
| 3 | 5.0       | 3.6         | 1.4       | 0.2         | Iris-Setosa |
|   |           |             |           |             |             |

每个数据都是以都好分隔,换行符结束。

下面将用代码的方式,了解iris我们能获取到的数据有哪些。

```
import numpy as np
from sklearn import datasets

# import some data to play with
iris = datasets.load_iris()

print(iris.target)
print(type(iris.target),len(iris.target),type(iris))

print(len(iris.data), iris.data.shape, iris.data[range(1,150,1),3])
print("\n")
print(iris.feature_names,iris.target_names,iris.filename)
```

### ▼ 1.3.3 对Iris数据可视化操作

| ₽ |   | sepal_len | sepal_width | petal_len | petal_width | class       |
|---|---|-----------|-------------|-----------|-------------|-------------|
|   | 0 | 4.9       | 3.0         | 1.4       | 0.2         | Iris-setosa |
|   | 1 | 4.7       | 3.2         | 1.3       | 0.2         | Iris-setosa |
|   | 2 | 4.6       | 3.1         | 1.5       | 0.2         | Iris-setosa |
|   | 3 | 5.0       | 3.6         | 1.4       | 0.2         | Iris-setosa |
|   | 4 | 5.4       | 3.9         | 1.7       | 0.4         | Iris-setosa |

最后类别一列,感觉前面的'Iris-'有点多余 即把class这一列的数据按'-'进行切分 取切分后的第二个数据,为了好看一点点

111

df['class'] = df['class'].apply(lambda x: x.split('-')[1])

#### # 查看数据信息 df.describe()

| _ |       |            |             |            |             |
|---|-------|------------|-------------|------------|-------------|
| ₽ |       | sepal_len  | sepal_width | petal_len  | petal_width |
|   | count | 149.000000 | 149.000000  | 149.000000 | 149.000000  |
|   | mean  | 5.848322   | 3.051007    | 3.774497   | 1.205369    |
|   | std   | 0.828594   | 0.433499    | 1.759651   | 0.761292    |
|   | min   | 4.300000   | 2.000000    | 1.000000   | 0.100000    |
|   | 25%   | 5.100000   | 2.800000    | 1.600000   | 0.300000    |
|   | 50%   | 5.800000   | 3.000000    | 4.400000   | 1.300000    |
|   | 75%   | 6.400000   | 3.300000    | 5.100000   | 1.800000    |
|   | max   | 7.900000   | 4.400000    | 6.900000   | 2.500000    |

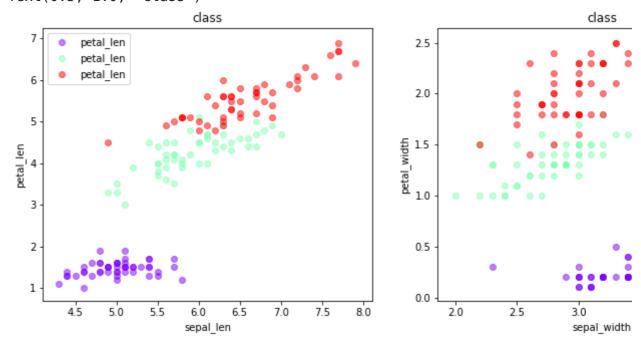
使用describe()可以很方便的查看数据的大致信息,可以看到数据是没有缺失值的,总共有145条,每一列的最大值、最小值、平均值都可以查看。

### ▼ 1.3.4 Iris 数据可视化

为了比较直观的查看数据的分布,用matplotlib进行了简单的可视化展示,查看数据的分布,画个图。

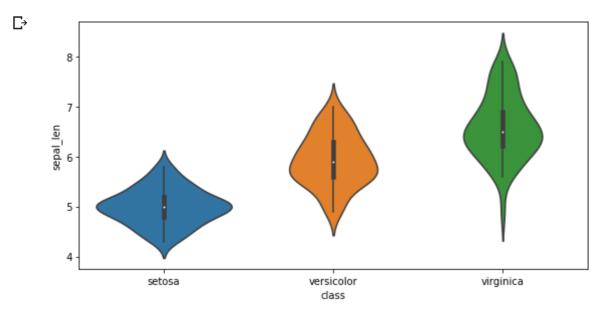
```
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.cm as cm
%matplotlib inline
def scatter_plot_by_category(feat, x, y):
     alpha = 0.5
     gs = df.groupby(feat)
     cs = cm.rainbow(np.linspace(0, 1, len(gs)))
    for g, c in zip(gs, cs):
    plt.scatter(g[1][x], g[1][y], color=c, alpha=alpha)
plt.figure(figsize=(20,5))
plt.subplot(131)
scatter_plot_by_category('class', 'sepal_len', 'petal_len')
plt.xlabel('sepal_len')
plt.ylabel('petal_len')
plt.legend(loc='upper left')
plt.title('class')
# 为了节省篇幅,省了第二、三个图的代码
plt.subplot(132)
scatter_plot_by_category('class', 'sepal_width', 'petal_width')
plt.xlabel('sepal_width')
plt.ylabel('petal_width')
plt.title('class')
plt.subplot(133)
scatter plot by category('class', 'sepal len', 'sepal width')
plt.xlabel('sepal_len')
plt.ylabel('sepal_width')
plt.title('class')
```

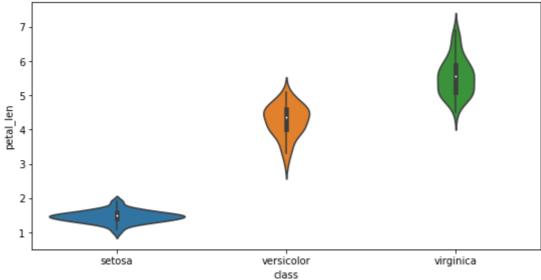
### r→ Text(0.5, 1.0, 'class')



import seaborn as sb

```
plt.figure(figsize=(20, 10))
for column_index, column in enumerate(df.columns):
    if column == 'class':
        continue
    plt.subplot(2, 2, column_index + 1)
    sb.violinplot(x='class', y=column, data=df)
```





## ▼ 1.3.5 Iris 模型训练

以下提供两种模型训练方法,一种是线性回归的方法,另一种是决策树的方法进行模型的训练

#### 线性回归方法进行Iris模型训练

```
区分花(多分类):softmax回归
from sklearn import datasets
import
import numpy as np
import pandas as pd
import tensorflow as tf
# 数据预处理
```

sepal\_width

petal\_width

```
# import some data to play with
iris = datasets.load iris()
data=pd.read_csv('./data/iris.data',names=['e_cd','e_kd','b_cd','b_kd','cat'])
# 独热编号
data['c1']=np.array(data['cat']=='Iris-setosa').astype(np.float32)
data['c2']=np.array(data['cat']=='Iris-versicolor').astype(np.float32)
data['c3']=np.array(data['cat']=='Iris-virginica').astype(np.float32)
del data['cat']
# 合并行
target=np.stack([data.c1.values,data.c2.values,data.c3.values]).T
shuju=np.stack([data.e cd.values,data.e kd.values,data.b cd.values,data.b kd.val
print(target.shape,shuju.shape)
# 定义网络
x=tf.placeholder("float", shape=[None, 4])
y=tf.placeholder("float", shape=[None, 3])
weight=tf.Variable(tf.truncated normal([4,3]))
bias=tf.Variable(tf.truncated normal([3]))
combine input=tf.matmul(x,weight)+bias
pred=tf.nn.softmax(combine input)
# 损失函数
loss=tf.reduce mean(tf.nn.softmax cross entropy with logits(labels=y,logits=comb
correct pred=tf.equal(tf.argmax(pred,1),tf.argmax(y,1))
accuracy=tf.reduce mean(tf.cast(correct pred,tf.float32))
# 梯度下降
train step=tf.train.AdadeltaOptimizer(0.05).minimize(loss)
sess=tf.Session()
sess.run(tf.global variables initializer())
for i in range(10000):
    index=np.random.permutation(len(target))
    shuju=shuju[index]
    target=target[index]
    sess.run(train_step,feed_dict={x:shuju,y:target})
    if i%1000==0:
        print(sess.run((loss,accuracy),feed dict={x:shuju,y:target}))
```

#### ▼ 使用决策树进行模型的计算

因为鸢尾花数据集很简单,特征已经全部提取好了,而且也很纯,所以就直接放到机器学习算法里面训练了。这里使用的是决策树分类算法。

```
import numpy as np import matplotlib.pyplot as plt from mpl_toolkits.mplot3d import Axes3D data = np.random.randint(0, 255, size=[40, 40, 40]) x, y, z = data[0], data[1], data[2] ax = plt.subplot(111, projection='3d') # 创建一个三维的绘图工程 # 将数据点分成三部分画,在颜色上有区分度 ax.scatter(x[:10], y[:10], z[:10], c='y') # 绘制数据点 ax.scatter(x[:10:20], y[:10:20], z[:10:20], c='r') ax.scatter(x[:30:40], y[:30:40], z[:30:40], c='g') ax.set_zlabel('Z') # 坐标轴 ax.set_ylabel('Y') ax.set_xlabel('X') plt.show()
```

C→

```
250
200
150 2
100
50
0
0
150 7
```

```
# 首先对数据进行切分,即分出数据集和测试集
from sklearn.model selection import train test split
all classes = df['class'].values
(X train,
 X<sup>-</sup>test,
X train,
 Y test) = train test split(all inputs, all classes, train size=0.8, random stat
# 使用决策树算法进行训练
from sklearn.tree import DecisionTreeClassifier
# 定义一个决策树对象
decision tree classifier = DecisionTreeClassifier()
model = decision tree classifier.fit(training inputs, training classes)
# 所得模型的准确性
print(decision tree classifier.score(testing inputs, testing classes))
# 使用训练的模型进行预测,为了偷懒,
# 直接把测试集里面的数据拿出来了三条
print(X_test[0:3])
print(Y_test[0:3])
model.predict(X test[0:3])
```

/usr/local/lib/python3.6/dist-packages/sklearn/model\_selection/\_split.py:2
FutureWarning)

NameError: name 'training\_inputs' is not defined

SEARCH STACK OVERFLOW

```
print( doc )
```

```
# Code source: Gaël Varoquaux
# Modified for documentation by Jaques Grobler
# License: BSD 3 clause
import matplotlib.pyplot as plt
from mpl toolkits.mplot3d import Axes3D
from sklearn import datasets
from sklearn.decomposition import PCA
# import some data to play with
iris = datasets.load iris()
X = iris.data[:, :2] # we only take the first two features.
y = iris.target
x_{min}, x_{max} = X[:, 0].min() - .5, X[:, 0].max() + .5
y = min, y = x = x[:, 1].min() - .5, x[:, 1].max() + .5
plt.figure(2, figsize=(8, 6))
plt.clf()
# Plot the training points
plt.xlabel('Sepal length')
plt.ylabel('Sepal width')
plt.xlim(x min, x max)
plt.ylim(y_min, y_max)
plt.xticks(())
plt.yticks(())
# To getter a better understanding of interaction of the dimensions
# plot the first three PCA dimensions
fig = plt.figure(1, figsize=(8, 6))
ax = Axes3D(fig, elev=-150, azim=110)
X reduced = PCA(n components=3).fit transform(iris.data)
ax.set title("First three PCA directions")
ax.set_xlabel("1st eigenvector")
ax.w_xaxis.set_ticklabels([])
ax.set_ylabel("2nd eigenvector")
ax.w_yaxis.set_ticklabels([])
ax.set_zlabel("3rd eigenvector")
ax.w zaxis.set ticklabels([])
plt.show()
```

# - 二、MNIST手写字识别

# - tensorflow 入门

```
import tensorflow as tf
import numpy as np
# 使用 NumPy 生成假数据(phony data), 总共 100 个点.
x_data = np.float32(np.random.rand(2, 100)) # 随机输入
y data = np.dot([0.100, 0.200], x data) + 0.300
# 构造一个线性模型
b = tf.Variable(tf.zeros([1]))
W = tf.Variable(tf.random uniform([1, 2], -1.0, 1.0))
y = tf.matmul(W, x data) + b
# 最小化方差
loss = tf.reduce_mean(tf.square(y - y_data))
optimizer = tf.train.GradientDescentOptimizer(0.5)
train = optimizer.minimize(loss)
# 初始化变量
init = tf.initialize_all_variables()
# 启动图 (graph)
sess = tf.Session()
sess.run(init)
# 拟合平面
for step in range(201):
    sess.run(train)
    if step % 20 == 0:
        print(step, sess.run(W), sess.run(b))
# 得到最佳拟合结果 W: [[0.100 0.200]], b: [0.300]
```

# → 初始化变量

将输入输入x\_data 用随机输入的方式输入2行100列的二维数据。

y\_data 则是通过线性模型进行矩阵的运算,具体的运算公式如下

$$y = w.*x + b; ==> y_data = [0.100, 0.200].*x_data + 0.300$$

v\_data 的输出结果就是训练后要对比的结果,一般就是训练数据参考的label

## 注意

最终的到的y\_data 结果是一个一维100列的数据,注意这个b=0.300代表的是参与到式子 里是所有元素都为0.300的一个一行100列的数组。

```
import tensorflow as tf
import numpy as np

# 使用 NumPy 生成假数据(phony data), 总共 100 个点.
x_data = np.float32(np.random.rand(2, 100)) # 随机输入
y_data = np.dot([0.100, 0.200], x_data) + 0.300

import tensorflow as tf
x = tf.constant([[1., 1.], [2., 2.]])
y = tf.reduce_mean(x) # 1.5
with tf.Session as sess:
    print(sess.run(tf.reduce_mean(x)))
```

```
AttributeError Traceback (most recent call last <ipython-input-8-aaac89929b03> in <module>()
    2 x = tf.constant([[1., 1.], [2., 2.]])
    3 y = tf.reduce_mean(x) # 1.5
----> 4 with tf.Session as sess:
    5 print(sess.run(tf.reduce_mean(x)))

AttributeError: __enter__

SEARCH STACK OVERFLOW
```

## → 构建一个线性模型

y = w. \*x + b

• 简单的逻辑回归模型

输入x是2行100列的数组;

# 构造一个线性模型

import numpy as np

print(type(x data))

```
b = tf.Variable(tf.zeros([1]), name="biases") # 定义初值为0的一维向量
W = tf.Variable(tf.random_uniform([1, 2], -1.0, 1.0), name="weights") # 均匀分布
y = tf.matmul(W, x_data) + b #

双击(或按回车键)即可修改
```

### # 最小化方差

```
loss = tf.reduce_mean(tf.square(y - y_data))
optimizer = tf.train.GradientDescentOptimizer(0.5)
train = optimizer.minimize(loss)
```

x data = np.random.rand(100).astype(np.float32)