Taylor's Series

$$f(x) = f(a) + \frac{(x-a)}{1!}f'(a) + \frac{(x-a)^2}{2!}f''(a)$$

$$+ \frac{(x-a)^3}{3!}f'''(a) + \frac{(x-a)^4}{4!}f^{iv}(a) + \cdots$$

Ex: $f(x) = \sqrt{x}$ $\boxed{a=4}$

$\Rightarrow$ $f(x) = x^{1/2} = \sqrt{x}$

$f'(x) = \frac{1}{2}x^{-1/2}$

$f''(x) = -\frac{1}{4}x^{-3/2}$

$f'''(x) = \frac{3}{8}x^{-5/2}$

$f^{iv}(x) = -\frac{15}{16}x^{-7/2}$

& so on...

$f(4) = (2^2)^{1/2}$
$= 2$

$f'(4) = \frac{1}{2}(2^2)^{-1/2}$
$= 1/4$

$f''(4) = -\frac{1}{4}(2^2)^{-3/2}$
$= -1/32$

$f'''(4) = \frac{3}{8}(2^2)^{-5/2}$
$= \frac{3}{256}$

$f^{iv}(4) = -\frac{15}{16}(2^2)^{-7/2}$

$= -\frac{15}{16 \times 2^7} = -\frac{15}{2048}$

& so on

By Taylor's series.

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$$f(x) = f(4) + \frac{(x-4)}{1!} f'(4) + \frac{(x-4)^2}{2!} f''(4) +$$

$$\frac{(x-4)^3}{3!} f'''(4) + \frac{(x-4)^4}{4!} f^{iv}(4) + \text{---}$$

$$\sqrt{x} = 2 + (x-4)\cdot\left(\frac{1}{4}\right) + \frac{(x-4)^2}{2}\left(\frac{-1}{32}\right) +$$

$$\frac{(x-4)^3}{3!}\frac{3}{256} + \frac{(x-4)^4}{4!}\left(-\frac{15}{2048}\right) \text{---}$$

$4 \times 3 \times 2 \times 1$

$$\sqrt{x} = 2 + \frac{(x-4)}{4} - \frac{(x-4)^2}{64} + \frac{(x-4)^3}{512}$$

$$- \frac{5(x-4)^4}{16384} + \text{---}$$

Q: Express $f(x) = 2x^3 + 3x^2 - 8x + 7$ in terms of $(x-2)$.

$\Rightarrow f(x) = 2x^3 + 3x^2 - 8x + 7$

$f'(x) = 6x^2 + 6x - 8$

$f''(x) = 12x + 6$

$f'''(x) = 12$

$f^{iv}(x) = 0$

$f(2) = 16 + 12 - 16 + 7$

$= \underline{19}$

$f'(2) = 24 + 12 - 8$

$= \underline{28}$

$f''(2) = 24 + 6 = 30$

$f'''(2) = 12$

$f^{iv}(2) = 0$

By Taylor's series,

$$f(x) = f(2) + \frac{(x-2)}{1!} f'(2) + \frac{(x-2)^2}{2!} f''(2)$$

$$+ \frac{(x-2)^3}{3!} f'''(2) + \frac{(x-2)^4}{4!} f^{iv}(2) + \ldots$$

$$= 19 + \frac{(x-2)}{1}(28) + \frac{(x-2)^2}{\cancel{2}}(\cancel{30})^{15} +$$

$$\frac{(x-2)^3}{\cancel{3}\times\cancel{2}\times 1}(\cancel{12})^{2} + 0$$

$$= 19 + 28(x-2) + 15(x-2)^2 + 2(x-2)^3$$

(ii) $f(x) = \tan x \quad (x - \pi/4)$.

~~powers of~~

a $\tan 46°$, $\tan 44°$

$f(x) = \tan x$

$\Rightarrow$

$\tan \pi/4 = 1$

$\sec \pi/4 = \sqrt{2}$

$f'(x) = \sec^2 x$

$f''(x) = 2 \sec x (\sec x \tan x)$

$f''(x) = 2\sec^2 x \tan x$

2 1

$f'''(x) = 2\sec^2 x(\sec^2 x) +$
$\tan x(4\sec x \cdot \sec x \tan x)$,

$f'''(x) = 2\sec^4 x + 4\sec^2 x \tan^2 x$

$f^{iv}(x) =$ & so on - -

$f(\pi/4) = \tan \pi/4 = 1$

$f'(\pi/4) = \sec^2 \pi/4 = 2$

$f''(\pi/4) = 2(\sqrt{2})^2 \cdot 1 = 4$

$f'''(\pi/4) = 2(\sqrt{2})^4 + 4(\sqrt{2})^2(1)^2$

$= 8 + 8 = 16$

& so on - -

By taylor's series

$$f(x) = f(\pi/4) + \frac{(x-\pi/4)}{1!} f'(\pi/4) + \frac{(x-\pi/4)^2}{2!} f''(\pi/4) + \frac{(x-\pi/4)^3}{3!} f'''(\pi/4) + (x-\pi/4) - -$$

$$\tan x = 1 + (x-\pi/4)2 + \frac{(x-\pi/4)^2}{\cancel{2}} \cancel{4}^2 + \frac{(x-\pi/4)^3}{3 \times \cancel{2}} \cancel{16}^8 + - -$$

$$\tan x = 1 + 2(x - \pi/4) + 2(x - \pi/4)^2 + \frac{8}{3}(x - \pi/4)^3 + \text{———} \rightarrow (4)$$

$\Rightarrow$ $\boxed{x = 46°}$

$$x = \frac{46\pi}{180} = 0.8024$$

$$(x - \pi/4) = (0.8024 - 0.785)$$

$$= 0.0174$$

$$\tan 46' = 1 + 2(0.0174) + 2(0.0174)^2 + \frac{8}{3}(0.0174)^3 + \text{———}$$

$$= 1 + 0.0348 + 0.0006055 + 0.00001404$$

$$\boxed{\tan 46' = 1.0354}$$

$\Rightarrow$ $x = 44$

$$= \frac{44\pi}{180} = 0.7675$$

$$(x - \pi/4) = (~~0.8025~~$$

$$= (0.7675 - 0.785)$$

$$= -0.0175$$

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$\tanh h' = 1 + 2(-0.0174) + 2(-0.0174)^2 + \frac{4}{3}(-0.0174)^3 + \text{---}$

$= 1 - 0.0348 + 0.0006055 - 0.00001404$

$\tanh h' = 0.9657$

(18) : Expand $f(x) = \log x$ in powers of $(x-1)$ by 4 $\log 1$.

$f(x) = \log x$

$f'(x) = \frac{1}{x}$

$f''(x) = \frac{-1}{x^2} = -1x^{-2}$

$f'''(x) = \frac{2}{x^3} = 2x^{-3}$

$f^{IV}(x) = -6x^{-4} = \frac{-6}{x^4}$

$f^{V}(x) = \frac{24}{x^5}$ & so on

$f(1) = \log 1 = 0$

$f'(1) = 1$

$f''(1) = -1$

$f'''(1) = 2$

$f^{IV}(1) = -6$

$f^{V}(1) = 24$

& so on

By taylor series

$$f(x) = f(1) + \frac{(x-1)}{1!}f'(1) + \frac{(x-1)^2}{2!}f''(1) + \frac{(x-1)^3}{3!}f'''(1) + \cdots$$

$$\log x = 0 + \frac{(x-1)}{1!}(1) + \frac{(x-1)^2}{2!}(-1) + \frac{(x-1)^3}{3!}\,2 + \frac{(x-1)^4}{4!}(-6) + \frac{(x-1)^5}{5!}(24) + \cdots$$

$3\times2\times1$

$4\times3\times2\times1$

$$\log x = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \frac{(x-1)^5}{5} + \cdots \longrightarrow (1)$$

$\boxed{x = 1.1}$ Put in eqn (1)

$$\log 1.1 = (1.1-1) - \frac{(1.1-1)^2}{2} + \frac{(1.1-1)^3}{3} - \frac{(1.1-1)^4}{4} + \cdots$$

$$\log 1.1 = 0.1 - 0.005 + 0.000333 - 0.000025 + \cdots$$

$$\log 1.1 = 0.09530.$$

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Q. $f(x) = \sin x$ in powers of $(x - \frac{\pi}{2})$
& $\sin 91°$ $x = \pi/2$

$f(x) = \sin x$	$f\left(\frac{\pi}{2}\right) = 1$
$f'(x) = \cos x$	$f'\left(\frac{\pi}{2}\right) = \cos\frac{\pi}{2} = 0$
$f''(x) = -\sin x$	$f''\left(\frac{\pi}{2}\right) = -1$
$f'''(x) = -\cos x$	$f'''\left(\frac{\pi}{2}\right) = 0$
$f^{iv}(x) = \sin x$	$f^{iv}\left(\frac{\pi}{2}\right) = 1$
$f^{v}(x) = \cos x$	$f^{v}\left(\frac{\pi}{2}\right) = 0$
$f^{vi}(x) = -\sin x$	$f^{vi}\left(\frac{\pi}{2}\right) = -1$

$$f(x) = f(a) + \frac{(x-a)}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} f'''(a) + \frac{(x-a)^4}{4!} f^{iv}(a) + \dots$$

$$\sin x = 1 + \frac{(x-\pi/2)}{1!}(0) + \frac{(x-\pi/2)^2}{2}(-1) + 0 + \frac{(x-a)^4}{24}(1) + 0 + \frac{(x-a)^5}{120}(0) + \frac{(x-a)^6}{720}(-1)$$

$$= 1 - \frac{(x-\pi/2)^2}{2} + \frac{(x-a)^4}{24} - \frac{(x-a)^6}{720} + \dots$$

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$x = 91^\circ$

$x = 91 \times \frac{\pi}{180} = 1.587$

$\left(x - \frac{\pi}{2}\right) = \left(1.587 - 1.57\right) = 0.017$

$\sin 91^\circ = 1 - 0.000289 + 0.000000084$

$= \cancel{0.9998} = \underline{0.9998}$