Formulae And Results

✡ Trigonometric Identities And Formulae :

(1) $\cos^2\theta + \sin^2\theta = 1$

(2) $\sec^2\theta - \tan^2\theta = 1$

(3) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

(4) $\sin 2\theta = 2\sin\theta\cos\theta$

$= \dfrac{2\tan\theta}{1+\tan^2\theta}$

(5) $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 2\cos^2\theta - 1 \longrightarrow 1 + \cos 2\theta = 2\cos^2\theta$

$= 1 - 2\sin^2\theta \longrightarrow 1 - \cos 2\theta = 2\sin^2\theta$

$= \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

(6) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

(7) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

(8) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

(9) $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

(10) $\cos(-\theta) = \cos\theta$, $\sin(-\theta) = -\sin\theta$

$\tan(-\theta) = -\tan\theta$, $\cot(-\theta) = -\cot\theta$

$\sec(-\theta) = \sec\theta$, $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

(11) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

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(12). $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

(13) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

(14) $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

(15) $\tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

(16) $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

(17) $2\sin\alpha\cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$

(18) $2\cos\alpha\sin\beta = \sin(\alpha+\beta) - \sin(\alpha-\beta)$

(19) $2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$

(20) $2\sin\alpha\sin\beta = -\cos(\alpha+\beta) + \cos(\alpha-\beta)$

(21) $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

(22) $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

(23) $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

(24) $\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

(25) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

(26) $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

(27) $\sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2}$

(28) $\sin^{-1}(-x) = -\sin^{-1}x$

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(29) $\cos^{-1}(-x) = \pi - \cos^{-1}x$

(30) $\tan^{-1}(-x) = -\tan^{-1}x$

✡ Trigonometric Table :

θ°	0	30	45	60	90	180	270	360
θ^R	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	∞	0

✡ Differential Formulae :

(1) $\frac{d}{dx}(x^n) = nx^{n-1}$

(2) $\frac{d}{dx}(\log x) = \frac{1}{x}$

(3) $\frac{d}{dx}(e^x) = e^x$

(4) $\frac{d}{dx}(a^x) = a^x \log a$

(5) $\frac{d}{dx}(\sin x) = \cos x$

(6) $\frac{d}{dx}(\cos x) = -\sin x$

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(7) $\frac{d}{dx}(\tan x) = \sec^2 x$

(8) $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

(9) $\frac{d}{dx}(\sec x) = \sec x \tan x$

(10) $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

(11) $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

(12) $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$

(13) $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

(14) $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$

(15) $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$

(16) $\frac{d}{dx}(\operatorname{cosec}^{-1} x) = -\frac{1}{|x|\sqrt{x^2-1}}$

(17) $\frac{d}{dx}(\sinh x) = \cosh x$, $\frac{d}{dx}(\cosh x) = \sinh x$

(18) $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

(19) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$, $\frac{d}{dx}\left(\frac{1}{v}\right) = -\frac{1}{v^2}\frac{dv}{dx}$

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☆ Integral Formulae :

(1) $\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad , \quad n \neq -1$

(2) $\int \sin x \, dx = -\cos x + c$

(3) $\int \cos x \, dx = \sin x + c$

(4) $\int \tan x \, dx = \log|\sec x| + c$

(5) $\int \cot x \, dx = \log|\sin x| + c$

(6) $\int \sec x \, dx = \log|\sec x + \tan x| + c$

(7) $\int \operatorname{cosec} x \, dx = \log|\operatorname{cosec} x - \cot x| + c$

(8) $\int \sec^2 x \, dx = \tan x + c$

(9) $\int \operatorname{cosec}^2 x \, dx = -\cot x + c$

(10) $\int \sec x \tan x \, dx = \sec x + c$

(11) $\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + c$

(12) $\int e^x dx = e^x + c$

(13) $\int a^x dx = \frac{a^x}{\log a} + c$

(14) $\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c$

(15) $\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + c$

(16) $\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + c$

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(17) $\int \frac{1}{\sqrt{x^2 \pm a^2}}\,dx = \log\left|x + \sqrt{x^2 \pm a^2}\right| + c$

(18) $\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$

(19) $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2+a^2}\right| + c$

(20) $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2-a^2}\right| + c$

(21) $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$

(22) $\int f(ax+b)\,dx = \frac{1}{a}F(ax+b) + c$ where $\frac{d}{dx}(F(x)) = f(x)$

(23) $\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + c$

(24) $\int \frac{f'(x)}{\sqrt{f(x)}}\,dx = 2\sqrt{f(x)} + c$

(25) $\int [f(x)]^n f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + c$

(26) $\int uv\,dx = u\int v\,dx - \int\left[\frac{du}{dx} \times \int v\,dx\right]dx$ Rule : LIATE

(27) $\int uv\,dx = uv_1 - u'v_2 + u''v_3 - u'''v_4 + \cdots\cdots$

(28) $\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx) + c$

(29) $\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx) + c$