first order.

* Ordinary Diffⁿ eqⁿs.

Eliminating arbitrary Constant

$y = Ae^{2x} + Be^{-2x}$

where A & B are arbitrary const.

$\frac{dy}{dx} = 2Ae^{2x} + (-2)Be^{-2x}$

$\frac{d^2y}{dx^2} = 4Ae^{2x} + 4Be^{-2x}$

$= 4(Ae^{2x} + Be^{-2x})$

$\frac{d^2y}{dx^2} = 4y$

$\boxed{\frac{d^2y}{dx^2} - 4y = 0}$

which is required D.E

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Q7) y

Ex) $y = e^x(A\cos x + B\sin x) \rightarrow ①$

$\Rightarrow \frac{dy}{dx} = e^x(-A\sin x + B\cos x)$
$+ \underbrace{e^x(A\cos x + B\sin x)}_{y} \rightarrow ②$

$\therefore \frac{dy}{dx} - y = e^x(-A\sin x + B\cos x) \rightarrow ③$

$\frac{d^2y}{dx^2} = e^x(-A\cos x - B\sin x) + \frac{dy}{dx}$

$\frac{d^2y}{dx^2} = \frac{dy}{dx} - e^x(A\cos x + B\sin x).$

$\frac{d^2y}{dx^2} = \frac{dy}{dx} - y$

$$\boxed{\frac{d^2y}{dx^2} - \frac{dy}{dx} + y = 0}$$

Variable Seperable

$f(x)\,dx = g(y)\,dy$

apply Integration both sides

$\int f(x)\,dx = \int g(y)\,dy + C$

Ex: solve: $9yy' + 4x = 0$

$9y\frac{dy}{dx} + 4x = 0$

$9y\,dy + 4x\,dx = 0$

apply Integration

$\frac{9y^2}{2} + \frac{4x^2}{2} = C$

$9y^2 + 4x^2 = 2C$

$9y^2 + 4x^2 = C'$

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Q1 $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

$= e^{x} \cdot e^{-y} + x^2 e^{-y}$

$\frac{dy}{dx} = e^{-y}(e^{x} + x^2)$

$\therefore e^{y} dy = (e^{x} + x^2) dx$

apply Integration both sides,

$\therefore e^{y} = e^{x} + \frac{x^3}{3} + C$

GTU.

Q1 $3e^{x} \tan y \, dx + (1+e^{x}) \sec^2 y \, dy = 0$

$3e^{x} \tan y \, dx = -(1+e^{x}) \sec^2 y \, dy$

$\frac{3e^{x}}{(1+e^{x})} dx = \frac{\sec^2 y}{\tan y} dy$

$\frac{3e^{x}}{(1+e^{x})} dx = \cancel{\frac{1}{\sin y \cdot \cos y} dy}$

$3 \log(1+e^{x}) = -\log(\tan y) + \log 1$

$\log(1-e^{x})^3 = \log \frac{1}{\tan y} + \log c$

$(1-e^{x})^3 = \cancel{\tan y + c} \; \frac{c}{\tan y}$ $\quad (1-e^{x})^3 \tan y = c$

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Ex $xy' + y = 0 \quad y(2) = -2$

$x\frac{dy}{dx} = -y$

$\frac{dy}{y} = -\frac{dx}{x}$

$\log y = -\log x + \log c$

$y = \frac{c}{x}$

$xy = c \rightarrow (1)$

$(-2)(2) = c$

$\boxed{-4 = c}$

$\boxed{xy = -4}$

Reducible Variable Separable

$\frac{dy}{dx} = \cos x \cdot \cos y - \sin x \sin y$

$\frac{dy}{dx} = \cos(x+y)$

$$\boxed{x+y = v}$$

$$1+\frac{dy}{dx} = \frac{dv}{dx}$$

$$\boxed{\frac{dy}{dx} = \frac{dv}{dx} - 1}$$

$$\frac{dv}{dx} - 1 = \cos v$$

$$\boxed{\frac{dv}{dx} = 1+\cos v}$$

$$\frac{dv}{1+\cos v} = dx$$

$$\frac{1-\cos v}{1-\cos^2 v}\, dv = dx$$

$$\frac{1-\cos v}{\sin^2 v}\, dv = dx$$

$$[\operatorname{cosec}^2 v - \cot v \operatorname{cosec} v]\, dv = dx$$

apply Integration both side

$$-\cot v + \operatorname{cosec} v = x$$

$$-\cot(x+y) + \operatorname{cosec}(x+y) = x + c$$

H.W $\dfrac{dy}{dx} = (4x+y+1)^2$

Assume $\boxed{4x+y+1 = v}$

~~Exact eq.~~

Homogenous D.E

ex. solve:

$(x^2y - 2xy^2)\,dx - (x^3 - 3x^2y)\,dy = 0$

It is homogenous D.E

$(x^2y - 2xy^2)\,dx = (x^3 - 3x^2y)\,dy$

$\therefore \dfrac{dy}{dx} = \dfrac{x^2y - 2xy^2}{x^3 - 3x^2y}$

$\therefore \dfrac{dy}{dx} = \dfrac{x^3\left(\frac{y}{x} - 2\left(\frac{y}{x}\right)^2\right)}{x^3\left(1 - 3\frac{y}{x}\right)}$

$y = vx$ $\boxed{\frac{y}{x} = v}$

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$$\boxed{\frac{dy}{dx} = v + x\frac{dv}{dx}}$$

$$v + x\frac{dv}{dx} = \left(\frac{v - 2v^2}{1 - 3v}\right)$$

$$x\frac{dv}{dx} = \frac{v - 2v^2}{1 - 3v} - v$$

$$= \frac{\cancel{v} - 2v^2 - \cancel{v} + 3v^2}{1 - 3v}$$

$$x\frac{dv}{dx} = \frac{v^2}{1 - 3v}$$

$$\frac{1 - 3v}{v^2}\,dv = \frac{dx}{x}$$

$$\left(\frac{1}{v^2} - \frac{3}{v}\right)dv = \frac{1}{x}\,dx$$

separating the variable

$$-\frac{1}{v} - 3\ln v = \ln x + C$$

$$-\frac{x}{y} - 3\ln\left(\frac{y}{x}\right) = \ln x + C$$

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$(x^4+y^4)dx-(xy^3)dy=0.$

$$\frac{(x^4+y^4)}{(xy^3)}=\frac{dy}{dx}$$

$$\frac{x^4\left(1+\left(\frac{y}{x}\right)^4\right)}{x^4\left(\left(\frac{y}{x}\right)^3\right)}=\frac{dy}{dx}$$

$$\frac{y}{x}=v$$

$$\frac{dy}{dx}=v+x\frac{dv}{dx}.$$

$$v+x\frac{dv}{dx}=\frac{1+v^4}{v^3}$$

$$x\frac{dy}{dx}=\frac{1+\cancel{v^4}-\cancel{v^4}}{v^3}$$

$$\frac{dx}{x}=\frac{v^3}{1}dv$$

$$\ln x=+\frac{v^4}{4}+c$$

$$\log x=\frac{1}{4}\left(\frac{y}{x}\right)+c$$

ex: ~~$(x^2y - 2xy^2)\,dx$.~~

~~$2x^3y\,dx - (x^3+y^3)\,dy = 0$~~

$$\cancel{\frac{x^3y}{x^3+y^3} = \frac{dy}{dx}}$$

$$\cancel{x^3}\,\cancel{(\;y}$$

$$\frac{x^2y}{(x^3+y^3)} = \frac{dy}{dx}$$

$$\frac{\cancel{x^3}\left(\frac{y}{x}\right)}{\cancel{x^3}\left(1+\left(\frac{y}{x}\right)^3\right)} = \frac{dy}{dx}$$

$$\frac{y}{x} = v$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}.$$

$$v + x\frac{dv}{dx} = \frac{v}{1+v^3}.$$

$$x\frac{dv}{dx} = \frac{\cancel{v} - \cancel{v} - v^4}{1+v^3}$$

$$\frac{(1+v^3)}{v^4}\,dv = \frac{dx}{x}.$$

$$\left(\frac{1}{v^4} + \frac{1}{v}\right)dv = \frac{dx}{x} = -\frac{1}{3v^3} + \log v = \log x + c$$

$$= -\frac{x^3}{3y^3} + \log\left(\frac{y}{x}\right) = \log x + c$$

exet - non exact.

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ex $(x^3 + 3xy^2)dx + (3x^2y + y^3)dy = 0$

$M = x^3 + 3xy^2$ $N = 3x^2y + y^3$

$\frac{\partial M}{\partial y} = 6xy$ $\frac{\partial N}{\partial x} = 6xy$

exact.

$\int_{y\ constant} M\,dx + \int \left(\text{term of N only const y}\right) dy = 0$

$\int_{y\ con} (x^3 + 3xy^2)dx + \int y^3\, dy = 0$

$\frac{x^4}{4} + \frac{3}{2}x^2y^2 + \frac{y^4}{4} + C$.

$y e^x dx + (2y + e^x)dy = 0$

$M = ye^x$. $N = 2y + e^x$.

$\frac{\partial M}{\partial y} = e^x$ $\frac{\partial N}{\partial x} = e^x$.

$\int_{y\ const} M\,dx + \int \left(\text{term of N cont only y}\right) dy = 0$

$\int_{y\ con} ye^x dx + \int 2y\, dy = 0$

$ye^x + y^2 + c$

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(9) $[(x+1)e^{x} - e^{y}]dx - xe^{y}dy = 0 \cdot y(1) = 0.$

$m = (x+1)e^{x} - e^{y}. \qquad N = -xe^{y}.$

$\frac{\partial m}{\partial y} = -e^{y}. \qquad \frac{\partial N}{\partial x} = -e^{y}.$

$\int m\,dx + \int \left(\begin{matrix}\text{term of } N\\ \text{contain only } y\end{matrix}\right) dy = 0.$

$\int ((x+1)e^{x} - e^{y})dx + \int 0\,dy = c.$

$\int (x+1)e^{x}dx - e^{y}x = c$

$(x+1)e^{x} - e^{x} - e^{y}x = c$

$e^{x}(x) - xe^{y} = c$

$x(e^{x} - e^{y}) = c$

$(e^{1} - 1) = c$

$c = e - 1$

$x(e^{x} - e^{y}) = e - 1$

Non-exact diff^n eq^n

Case:1) If $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = f(x)$.

$$I.F = e^{\int f(x)\,dx}$$

ex:1) $(x^2+y^2+3)\,dx - 2xy\,dy = 0$

$\Rightarrow$ $M = (x^2+y^2+3)$ | $N = -2xy$

$\frac{\partial M}{\partial y} = 2y$ | $\frac{\partial N}{\partial x} = -2y$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \Rightarrow$ Non exact

$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}$

$= 2y + 2y = 4y$

$\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{1}{-2xy}\cdot(4y)$

$= \boxed{-2/x = f(x)}$

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$$I.F = e^{\int f(x)\,dx}$$

$$= e^{-\int \frac{2}{x} dx}$$

$$= e^{-2\ln x} = e^{+\ln x^{-2}}$$

$$\boxed{I.F = \frac{1}{x^2}}$$

$$\left(1 + \frac{y^2}{x^2} + \frac{3}{x^2}\right)dx - \frac{2y}{x}\,dy = 0$$

which is exact.

$$M^* = 1 + \frac{y^2}{x^2} + \frac{3}{x^2} \qquad N^* = \frac{-2y}{x}$$

The soln is

$$\int_{y\ \text{const}} M^*\,dx + \int \left(\begin{array}{c}\text{terms of } N^* \\ \text{Containing only } y\end{array}\right) dy = C$$

$$\int_{y\ \text{const}} \left(1 + \frac{y^2}{x^2} + \frac{3}{x^2}\right) dx + \int 0\, dy = C$$

$$x + y^2\left(-\frac{1}{x}\right) + 3\left(-\frac{1}{x}\right) = C$$

$$\boxed{x^2 - y^2 - 3 = Cx} \quad ; C = \text{const}$$

Case 2: $\frac{1}{M}\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right) = f(y)$

$I.F = e^{\int f(y)\,dy}$.

Ex: $(2xy^4e^y + 2xy^3 + y)dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0$

$\Rightarrow M = 2xy^4e^y + 2xy^3 + y$

$N = x^2y^4e^y - x^2y^2 - 3x$

$\frac{\partial M}{\partial y} = 2xy^4e^y + 8xe^y y^3 + 6xy^2 + 1$

$\frac{\partial N}{\partial x} = 2xy^4e^y - 2xy^2 - 3$

$\boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}} \Rightarrow$ given D.E is non exact

$\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}$

$= (2xy^4e^y - 2xy^2 - 3) - (2xy^4e^y + 8xe^y y^3 + 6xy^2 + 1)$

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$$= \cancel{2x y^4 e^y} - 2xy^2 - 3 - \cancel{2x y^4 e^y}$$

$$- 8x e^y y^3 - 6xy^2 - 1$$

$$= -8x e^y y^3 - 8xy^2 - 4$$

$$\frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)$$

$$= \frac{-4}{y} \frac{\cancel{(2xy^3 e^y + 2xy^2 + 1)}}{\cancel{(2xy^3 e^y + 2xy^2 + 1)}}$$

$$= -4/y = f(y)$$

$$I.F = e^{-\int 4/y \, dy}$$

$$= e^{-4 \log y} = e^{\log y^{-4}}$$

$$\boxed{I.F = \frac{1}{y^4}}$$

$$\left(2x e^y + \frac{2x}{y} + y^{-3}\right) dx$$

$$+ \left(x^2 e^y - \frac{x^2}{y^2} - 3x/y^4\right) dy$$

which is exact.

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$$\underset{y\,const}{\int} M^{*}dx + \int \begin{pmatrix} \text{terms of } N^{*} \\ \text{containing only } y \end{pmatrix} dy = c$$

$$\underset{y\,const}{\int} \left(2xe^{y} + \frac{2x}{y} + y^{-3}\right)dx + \int 0\,dy = c$$

$$= \frac{\cancel{2}x^2}{\cancel{2}}e^{y} + \frac{\cancel{2}}{y}\frac{x^2}{\cancel{2}} + y^{-3}x = C$$

$$= x^2e^{y} + \frac{x^2}{y} + \frac{x}{y^3} = C$$

Q. $xe^{x}(dx - dy) + e^{x}dx + ye^{y}dy = 0$

$xe^{x}dx - xe^{x}dy + e^{x}dx + ye^{y}dy = 0$

$(xe^{x} + e^{x})dx + (ye^{y} - xe^{x})dy = 0$

$M = xe^{x} + e^{x}$ $\qquad$ $N = ye^{y} - xe^{x}$

$\frac{\partial M}{\partial y} = 0$ $\qquad$ $\frac{\partial N}{\partial x} = -(xe^{x} + e^{x})$

$= -xe^{x} - e^{x}$

$$= \frac{-(xe^{x} + e^{x})}{xe^{x} + e^{x}}$$

$\boxed{= -1}$

I. F. $= e^{\int -1\,dy}$

$= e^{-y}$

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$[x e^{x} \cdot e^{-y} + e^{x} e^{-y}) \, dx + (y - x e^{x} e^{-y}) dy$

$\int\limits_{y\,con} m^{*} dx + \int \text{terms of } N^{+} \text{ contai only } y \; dy = c$

$\int (x e^{x} e^{-y} + e^{x} e^{-y}) dx + \int y \, dy = c$

$\int e^{-y} e^{x} (x + 1) \, dx + \frac{y^2}{2} = c$

$(x+1) e^{-y} e^{x} - e^{-y} e^{x} + \frac{y^2}{2} = c$

$e^{-y}(x e^{x} + \cancel{e^{x}} - \cancel{e^{x}}) + \frac{y^2}{2} = c$

$\boxed{x e^{x-y} + \frac{y^2}{2} = c}$

eg

$(x y^2 - e^{\frac{1}{x^3}}) dx - x^2 y \, dy = 0.$

$M = x y^2 - e^{\frac{1}{x^3}} \qquad N = -x^2 y.$

$\frac{\partial M}{\partial y} = 2xy \qquad \frac{\partial N}{\partial x} = -2xy$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ → non exact

$$\frac{1}{-x^4 y}(4xy)$$

$$\boxed{-\frac{4}{x}}$$

$$I.F = \frac{1}{x^4}$$

$$\left(\frac{y^2}{x^3} - \frac{e^{\frac{1}{x^3}}}{x^4}\right)dx - \frac{y^2}{x^3}dy$$

$$\int_{y\,\text{const}} M \, dx + \int \text{term of } N \text{ not containing } x \; dy = C$$

$$\int \frac{y^2}{x^3}dx - \int \frac{e^{x^{-3}}}{x^4}dx = C$$

$$= \frac{y^2}{-2x^2} + \frac{1}{3}\int e^{\frac{1}{x^3}} \cdot \frac{-3}{x^4}\,dx = C$$

$$-\frac{y^2}{2x^2} + \frac{1}{3}e^{\frac{1}{x^3}} = C$$

$\frac{1}{x^3}$

Case-3 $Mdx + Ndy = 0$ is a Homogenous equation.

$$I.F = \frac{1}{Mx + Ny} \quad , \quad Mx + Ny \neq 0$$

Ex:1 Ans

$$x^2 y\,dx - (x^3 + xy^2)\,dy = 0$$

3 3 3

It is homogenous D.E

$M = x^2 y$ | $N = -x^3 - xy^2$

$\frac{\partial M}{\partial y} = x^2$ | $\frac{\partial N}{\partial x} = -3x^2 - y^2$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \Rightarrow$ Nonexact

$Mx + Ny$

$x^3 y + (-x^3 y - xy^3)$

$\cancel{x^3 y} - \cancel{x^3 y} - xy^3$

$-xy^3 \neq 0$

$= \frac{1}{1-x-xy^3}$

$$\boxed{I.F = \frac{1}{-xy^3}}$$

$$\frac{-1}{xy^3} x^2 y \, dx + \frac{1}{xy^3} (x^3 + xy^2) dy = 0$$

$$-\frac{x}{y^2} dx + \left(\frac{x^2}{y^3} + \frac{1}{y}\right) dy = 0$$

which is exact

The solⁿ is

$$\int_{y\,const} M^* dx + \int \left(\text{terms of } N \text{ containing only } y\right) dy = c$$

$$\int_{y\,const} \left(\frac{-x}{y^2}\right) dx + \int \frac{1}{y} dy = c$$

$$\frac{-x^2}{2y^2} + \log y = c$$

case: 5 $f_1(x,y)\,y\,dx + f_2(x,y)\,x\,dy$

$$I.F = \frac{1}{Mx - Ny}, \quad Mx - Ny \neq 0$$

Q: $(x^2y^2+2)\,y\,dx + (2-x^2y^2)\,x\,dy = 0$

The eqn is form of

$y\,f_1(x,y)\,dx + f_2(x,y)\,x\,dy = 0$

$M = (x^2y^3 + 2y)$ | $N = 2x - x^3y^2$

$\frac{\partial M}{\partial y} = 3x^2y^2 + 2$ | $\frac{\partial N}{\partial x} = 2 - 3x^2y^2$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \Rightarrow$ Nonexact

$Mx - Ny$

$= x^3y^3 + \cancel{2xy} - \cancel{2xy} + x^3y^3$

$= 2x^3y^3 \neq 0$

$$I.F = \frac{1}{Mx + Ny}$$

$$= \frac{1}{2x^3y^3}$$

$$\left(\frac{x^{\cancel{3}}\cancel{y^3}}{2x^3\cancel{y^3}} + \frac{\cancel{x}y}{2x^3y^3}\right)dx + \left(\frac{\cancel{2}x}{\cancel{2}x^3y^3} - \frac{x^{\cancel{3}}y^2}{2x^3y^3}\right)dy$$

$$\left(\frac{1}{2x} - \frac{1}{x^3y^2}\right)dx + \left(\frac{1}{x^2y^3} - \frac{1}{2y}\right)dy = 0$$

which is exact

The soln is

$$\int_{y\ \text{const}} M^* dx + \int \left(\text{terms of } N^* \text{ containing } y\right) dy = c$$

$$\int_{y\ \text{const}} \left(\frac{1}{2x} - \frac{1}{x^3y^2}\right)dx + \int \left(-\frac{1}{2y}\right)dy = c$$

$$\frac{1}{2}\log x + \frac{x^{-2}}{2y^2} - \frac{1}{2}\log y = c$$

where c is an arbitrary const.