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Higher order diff.

$\frac{d^3y}{dx^3} = y''' \Rightarrow D^3$

$\frac{d^2y}{dx^2} = y'' \Rightarrow D^2$ C.F P.I

અલગ-અલગ y_c y_p

$\frac{dy}{dx} = y' \Rightarrow D$ $y_c = c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x} + \ldots$

સરખા $y_c = (c_1 + c_2 x + c_3 x^2 + \ldots) e^{m_1 x}$

G.S $= y_c + y_p$ complex $y_c = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$

(1) Solve the eq^n $(D^2 + 7D + 12)y = e^{-3t}$

$y'' + 7y' + 12y = e^{-3t}$

$\frac{d^2y}{dx^2} + 7\frac{dy}{dx} + 12y = e^{-3t}$

→ Auxilary eq^n :- $(D^2 + 7D + 12) = 0$

$\therefore D^2 + 4D + 3D + 12 = 0$

$D(D+4) + 3(D+4) = 0$

$\therefore (D+4)(D+3) = 0$

$\therefore D + 4 = 0$ or $D + 3 = 0$

$\therefore D = -4$ $D = -3$

$\therefore y_c = c_1 e^{m_1 x} + c_2 e^{m_2 x}$

$= c_1 e^{-4x} + c_2 e^{-3x}$

* $(D^2 - 4D + 4)y = e^{-5t}$

Auxi ⇒ $D^2 - 4D + 4 = 0$

$D^2 - 2D - 2D + 4 = 0$

$\therefore D(D-2) - 2(D-2) = 0$

$\therefore (D-2)(D-2) = 0$

$\therefore D = 2, 2$

$y_c = (c_1 + c_2 x) e^{mx}$

$= (c_1 + c_2 x) e^{2x}$

(*) $(D^2+9)y = \sin 4x$

$\frac{d^2y}{dx^2} + 9y = $ ______

$y'' + 9y = $ ______

→ Auxi eqⁿ :- $(D^2+9) = 0$

$\therefore D^2 = -9$

$\therefore D^2 = 9i^2$

$\therefore D = \pm 3i$

$\therefore \alpha \pm \beta i$

$\alpha = 0, \quad \beta = 3$

$y_c = e^{\alpha x}[c_1 \cos\beta x + c_2 \sin\beta x]$

$= 1[c_1 \cos 3x + c_2 \sin 3x]$

(*) $(D^2 - 5D + 9)y = \cos 3x$

$\therefore D^2 - 5D + 9 = 0$

$\therefore AD^2 + BD + C = 0$

$\therefore A = 1, \ B = -5, \ C = 9$

$\alpha, \beta = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$= \frac{5 \pm \sqrt{25 - 4(1)(9)}}{2(1)}$

$= \frac{5 \pm \sqrt{25-36}}{2}$

$= \frac{5 \pm \sqrt{-11}}{2}$

$= \frac{5 \pm \sqrt{11 i^2}}{2}$

$= \frac{5 \pm \sqrt{11}\, i}{2}$

$= \frac{5}{2} \pm \frac{\sqrt{11}}{2} i$

$\alpha \pm \beta i$

$\alpha = \frac{5}{2} \qquad \beta = \frac{\sqrt{11}}{2}$

$y_c = e^{\alpha x}[c_1 \cos\beta x + c_2 \sin\beta x]$

$= e^{5/2 x}[c_1 \cos\frac{\sqrt{11}}{2}x + c_2 \sin\frac{\sqrt{11}}{2}$

* $(D^3 + D^2 - D - 1)y = \cos 2x$

Ans: $D^3 + D^2 - D - 1 = 0$

	D^3	D^2	D	const.
$D = 1$	1	1	-1	-1
	0	1	2	1
	1	2	1	0
	D^2	D	con	

$D = 1 \qquad (D^2 + 2D + 1) = 0$

$D = 1 \qquad D^2 + D + D + 1 = 0$

$D(D+1) + 1(D+1) = 0$

$(D+1)(D+1) = 0$

$D = 1 \quad D = -1, \quad D = -1$

$D = -1, -1, 1$

$y_c = (c_1 + c_2 x)e^{-x} + c_3 e^{x}$

method : 1

જમણી બાજુ e^{ax} વાળુ પદ આપેલ હોય ત્યારે.

* Solve the eq^n $(D^2 + 7D + 12)y = e^{3x}$

→ Auxilary eq^n :-

$D^2 + 7D + 12 = 0$

$\therefore D^2 + 4D + 3D + 12 = 0$

$\therefore D(D+4) + 3(D+4) = 0$

$\therefore \boxed{(D+3)(D+4) = 0}$

$\therefore D = -3, -4$

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$$y_c = c_1 e^{m_1 x} + c_2 e^{m_2 x}$$
$$= c_1 e^{-3x} + c_2 e^{-4x}$$

$$y_p = \frac{1}{f(D)} \cdot e^{3x}$$

$$= \frac{1}{(D+3)(D+4)} \cdot e^{3x}$$

$$= \frac{1}{6 \times 7} e^{3x}$$

$$y_p = \frac{1}{42} e^{3x}$$

$$G.S = y_c + y_p$$

$$= c_1 e^{-3x} + c_2 e^{-4x} + \frac{1}{42} e^{3x}$$

(7) solve $(D^2 + 5D + 6)y = e^x$

$\therefore D^2 + 5D + 6 = 0$

$\therefore D^2 + 3D + 2D + 6 = 0$

$\therefore D(D+3) + 2(D+3) = 0$

$\therefore (D+3)(D+2) = 0$

$\therefore D+3 = 0$ & $D+2 = 0$

$\therefore D = -3 \qquad D = -2$

$$y_c = c_1 e^{m_1 x} + c_2 e^{m_2 x}$$
$$= c_1 e^{-3x} + c_2 e^{-2x}$$

$$y_p = \frac{1}{f(D)} e^x$$

$$= \frac{1}{(D+3)(D+2)} e^x$$

$$= \frac{1}{4 \times 3} e^x$$

$$= \frac{1}{12} e^x$$

$$y = y_p + y_c$$
$$= \frac{1}{12} e^x + c_1 e^{-3x} + c_2 e^{-2x}$$

Ⓠ Solve, $(D^2 + 2D + 1)y = e^{-x}$

$\rightarrow$ $D^2 + 2D + 1 = 0$

$\therefore D^2 + D + D + 1 = 0$

$\therefore D(D+1) + 1(D+1) = 0$

$\therefore (D+1)(D+1) = 0$

$\therefore D = -1, -1$

$$y_c = (c_1 + c_2 x) e^{mx}$$
$$= (c_1 + c_2 x) e^{-x}$$

$$y_p = \frac{1}{f(D)} e^{-x}$$

$$= \frac{1}{(D+1)^2} \cdot e^{-x}$$

$$\frac{x^r}{r!}$$

$$= \frac{x^2}{2!} \cdot e^{-x}$$

$$y_p = \frac{x^2}{2} e^{-x}$$

* Solve, $(D^3 - 5D^2 + 8D - 4)y = e^{2x}$

→ Auxi eqⁿ: $D^3 - 5D^2 + 8D - 4 = 0$

	D^3	D^2	D	cons.
$D=1$	1	-5	8	-4
	0	1	-4	4
	1	-4	4	0
	D^2	D	cons.	

$(D-1)[D^2 - 4D + 4] = 0$

$\therefore (D-1)(D-2)^2 = 0$

$D = 1, 2, 2$

$$y_c = (c_1 + c_2 x)e^{2x} + c_3 e^x$$

$$y_p = \frac{1}{(D-1)(D-2)^2} \cdot e^{2x}$$

$$= \frac{1}{(1)(D-2)^2} \cdot e^{2x}$$

$$= \frac{x^2}{2!} \cdot e^{2x}$$

$$y_p = \frac{1}{2} x^2 \cdot e^{2x}$$

(x) $(D^2+7D+12)y = \cosh 2x$

$(D^2+7D+12)y = \left(\frac{e^{2x}+e^{-2x}}{2}\right)$

$\therefore (D^2+7D+12)y = \frac{1}{2}e^{2x} + \frac{1}{2}e^{-2x}$

$\therefore y_p = \frac{1}{(D+3)(D+4)} \frac{1}{2}e^{2x} + \frac{1}{(D+3)(D+4)} \frac{1}{2}e^{-2x}$

$= \frac{1}{5\times 6} \frac{1}{2} e^{2x} + \frac{1}{(1)(2)} \frac{1}{2} e^{-2x}$

$= \frac{1}{60} e^{2x} + \frac{1}{4} e^{-2x}$

$y_c = -3, -4$

$y_c = c_1 e^{-3x} + c_2 e^{-4x}$

$\cosh x = \frac{e^{x}+e^{-x}}{2}$

$\sinh x = \frac{e^{x}-e^{-x}}{2}$

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Method : 2

(જ્યારે $\sin ax$ & $\cos ax$ આપેલ હોય ત્યારે)

* Solve; $(D^2+3D+2)y = \sin 2x$

→ Auxilary eqⁿ:

$$D^2+3D+2=0$$
$$D^2+2D+D+2=0$$
$$D(D+2)+1(D+2)=0$$
$$(D+2)(D+1)=0$$
$$D=-1,\ -2$$

$$y_c = c_1 e^{m_1 x} + c_2 e^{m_2 x}$$
$$= c_1 e^{-x} + c_2 e^{-2x}$$

$$y_p = \frac{1}{f(D)} \cdot \sin 2x \qquad (D^2=-a^2)$$

$$= \frac{1}{D^2+3D+2} \cdot \sin 2x$$

$$= \frac{1}{-4+3D+2} \sin 2x$$

$$= \frac{1}{3D-2} \sin 2x$$

$$= \left(\frac{1}{3D-2} \times \frac{3D+2}{3D+2}\right) \cdot \sin 2x$$

$$= \frac{(3D+2)\sin 2x}{9D^2-4}$$

$$= \frac{(3D+2)\ \sin 2x}{9(-4)-4}$$

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$$= -\frac{(3D+2)}{40} \sin 2x$$

$$= -\frac{1}{40}\left[3D \sin 2x + 2\sin 2x\right]$$

$$= \frac{-1}{40}\left[3\cos 2x\,(2) + 2\sin 2x\right]$$

$$y_p = \frac{-1}{40}\left[6\cos 2x + 2\sin 2x\right]$$

$$y = y_c + y_p$$

* Solve $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - \frac{dy}{dx} - y = \cos 2x$

→ A.E $(D^3 + D^2 - D - 1) = 0$

$D^2(D+1) - 1(D+1) = 0$

$\therefore (D+1)(D^2-1) = 0$

$\therefore (D+1)(D+1)(D-1) = 0$

$\therefore D = -1, -1, 1$

$$y_c = (C_1 + C_2 x)e^{-x} + C_3 e^{x}$$

$$y_p = \frac{1}{f(D)} \cdot \cos 2x$$

$$= \frac{1}{(D^3+D^2-D-1)} \cdot \cos 2x$$

$$= \frac{1}{-4D-4-D-1} \cdot \cos 2x$$

$$= \frac{1}{-5D-5} \cdot \cos 2x$$

$$= \frac{1}{-5(D+1)} \cdot \cos 2x$$

$$= -\frac{1}{5} \cdot \frac{1}{D+1} \cdot \cos 2x$$

$$= \frac{-1}{5} \cdot \frac{1}{D+1} \times \frac{D-1}{D-1} \cdot \cos 2x$$

$$= \frac{-1}{5} \cdot \frac{(D-1)}{D^2-1} \cdot \cos 2x$$

$$= \frac{1}{25} (D-1) \cdot \cos 2x$$

$$= \frac{1}{25} \cdot [D \cdot \cos 2x - \cos 2x]$$

$$y_p = \frac{1}{25} [-\sin 2x \cdot 2 - \cos 2x]$$

$$y = y_c + y_p$$

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$(D^2+9)y = \cos 2x + \sin 2x$

A.E

$D^2+9=0$

$D^2=-9$

$D = \pm 3i$

$\alpha \pm \beta e^{i}$ $\qquad \alpha=0 \qquad \beta=3$

$y_c = e^{\alpha x}[c_1 \cos\beta x + c_2 \sin\beta x]$

$= [c_1 \cos 3x + c_2 \sin 3x]$

$y_p = \frac{1}{f(D)} \cdot (\cos 2x + \sin 2x)$

$= \frac{1}{f(D)} \cdot \cos 2x + \frac{1}{f(D)} \sin 2x$

$= \frac{1}{D^2+9} \cos 2x + \frac{1}{D^2+9} \sin 2x$

$= \frac{1}{-4+9} \cos 2x + \frac{1}{-4+9} \sin 2x$

$= \frac{1}{5} \cos 2x + \frac{1}{5} \sin 2x$

$y_p = \frac{1}{5}(\cos 2x + \sin 2x)$

$y = y_c + y_p$

$*$ $(D^2+7D+12)y = \sin x . \sin 4x$

$= \sin 4x \ \sin x$

$= \left(-\frac{1}{2}\right) -2\sin 4x \sin x$

$= -\frac{1}{2}\left[\cos 5x - \cos 3x\right]$

$\Downarrow$

<u>As earlier.</u>

Method : 3 (જ્યારે સમીકરણ x^m ઘાતવાળું પદ આપેલ હોય ત્યારે

$*$ Solve, $\boxed{\frac{d^2y}{dx^2} + 4y = 4x^2}$

$(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 \ldots$

$(1-x)^{-1} = 1 + x + x^2 + x^3 + x^4 \ldots$

$\Rightarrow$ $y = y_c + y_p$

for y_c = Auxilary eq^n :-

$D^2+4=0$

$D^2=-4$

$D^2 = 4i^2$

$\therefore D = \pm 2i$

$\alpha \pm \beta i$

$\alpha = 0 \quad \beta = 2$

$y_c = e^{\alpha x}\left[c_1 \cos \beta x + c_2 \sin \beta x\right]$

$= \left[c_1 \cos 2x + c_2 \sin 2x\right]$

$y_p = \frac{1}{f(D)} . 4x^2$

$= \frac{1}{D^2+4} . 4x^2$

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$$= \frac{1}{4\left[\frac{D^2}{4}+1\right]} \cdot 4x^2$$

$$= \left(1+\frac{D^2}{4}\right)^{-1} \cdot x^2$$

$$= \left(1-\frac{D^2}{4}+\left(\frac{D^2}{4}\right)^2-\left(\frac{D^2}{4}\right)^3+ \cdots\right) \cdot x^2$$

$$= \left(1-\frac{D^2}{4}\right) \cdot x^2.$$

$$= x^2 - \frac{1}{4}x^2 D^2.$$

$$= x^2 - \frac{1}{\cancel{4}_2} \cdot \cancel{2}$$

$$= x^2 - \frac{1}{2}.$$

* $y''' + 5y'' + 6y' = x^3,$

A.E

$D^3 + 5D^2 + 6D = 0$

$D[D^2 + 5D + 6] = 0$

$D = 0,\quad D^2 + 3D + 2D + 6 = 0$

$(D+3)(D+2) = 0$

$D = 0, -2, -3$

$y_c = c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x}$

$= c_1 + c_2 e^{-2x} + c_3 e^{-3x}$

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$$y_p = \frac{1}{f(D)} \cdot x^3$$

$$= \frac{1}{D^3+5D^2+6D} \cdot x^3$$

$$= \frac{1}{6D\left[\frac{D^3+5D^2}{6D}+1\right]} \cdot x^3$$

$$= \frac{1}{6D}\left[1+\frac{D^2+5D}{6}\right]^{-1} \cdot x^3$$

$$= \frac{1}{6D}\left[1-\left(\frac{D^2+5D}{6}\right)+\left(\frac{D^2+5D}{6}\right)^2-\left(\frac{D^2+5D}{6}\right)^3+\cdots\right] \cdot x^3$$

$$= \frac{1}{6D}\left[1-\frac{D^2}{6}-\frac{5D}{6}+\frac{1}{36}\left(D^4+10D^3+25D^2\right)\right] \cdot x^3$$

$$= \frac{1}{6D}\left[x^3-\frac{1}{6}D^2x^3-\frac{5}{6}Dx^3+\frac{10D^3}{36} \cdot x^3+\frac{25}{36}D^2 \cdot x^3\right]$$

$$= \frac{1}{6D}\left[x^3-\frac{1}{\cancel{6}} \cdot \cancel{6}x-\frac{5}{6} \cdot \frac{\cancel{3}x^2}{2}+\frac{\cancel{10}^{5}}{\cancel{36}_{\cancel{6}\,3}} \cdot \cancel{6}+\frac{25}{\cancel{36}_{6}} \cdot \cancel{6}x\right]$$

$$= \frac{1}{6D}\left[x^3-x-\frac{5}{2}x^2+\frac{5}{3}+\frac{25}{6}x\right]$$

$$= \frac{x^4}{24}-\frac{x^2}{12}-\frac{5x^3}{36}+\frac{5x}{18}+\frac{25x^2}{72}$$

Method 4: (જમણી બાજુ e^{ax} સામેલ હોય ત્યારે) આગળ લાવી બાકીના પદોમાં $D \to D+a$ મૂકવું

$\star$ Solve,

$$(D^2-4D+3)y = e^x \cos 2x$$

$$y = y_c + y_p$$

$\rightarrow$ Auxilary eqn:-

$$(D^2-4D+3)=0$$

$$(D^2-3D+D+3)=0$$

$$(D-3)(D-1)=0$$

$$\therefore D=1,3$$

$$y_c = c_1 e^{m_1 x} + c_2 e^{m_2 x}$$

$$= c_1 e^x + c_2 e^{3x}$$

$$y_p = \frac{1}{f(D)} \cdot e^x \cdot \cos 2x$$

$$= \frac{1}{(D-3)(D-1)} e^x \cdot \cos 2x$$

$$= e^x \cdot \frac{1}{(D+1-3)(D+1-1)} \cos 2x$$

$$= e^x \cdot \frac{1}{(D-2)D} \cos 2x$$

$$= e^x \cdot \frac{1}{D^2-2D} \cdot \cos 2x$$

$$= e^x \cdot \frac{1}{-4-2D} \cdot \cos 2x$$

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$$= e^x \cdot \frac{1}{-4-2D} \times \frac{-4+2D}{-4+2D} \cdot \cos 2x$$

$$= \frac{e^x \cdot (-4+2D)}{16-4D^2} \cdot \cos 2x$$

$$= \frac{e^x \cdot (-4+2D) \cos 2x}{32}$$

$$= \frac{e^x}{32} \left[-4\cos 2x + 2D\cos 2x\right]$$

$$= \frac{e^x}{32} \left[-4\cos 2x + 2(-\sin 2x) \cdot 2\right]$$

$$y_p = \frac{e^x}{32} \left[-4\cos 2x - 4\sin 2x\right]$$

* Solve, $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2e^x \cos\left(\frac{x}{2}\right)$

Auxi eq^n :-

$(D^2 - 3D + 2) = 0$

$(D-2)(D-1) = 0$

$D = 2, 1$

$y_c = c_1 e^{m_1 x} + c_2 e^{m_2 x}$

$= c_1 e^x + c_2 e^{2x}$

$$y_p = \frac{1}{f(D)} \cdot 2e^x \cdot \cos\left(\frac{x}{2}\right)$$

$$= \frac{1}{(D+1-2)(D+1-1)} \, 2e^{x} . \cos x/2$$

$$= 2e^{x} . \frac{1}{(D-1).D} \cos x/2$$

$$= 2e^{x} . \frac{1}{D^2 - D} \cos x/2$$

$$= 2e^{x} . \frac{1}{-1/4 - D} . \cos x/2$$

$$= 8e^{x} . \frac{1}{-1-4D} \cos x/2$$

$$= 8e^{x} . \frac{1}{-1-4D} \times \frac{-1+4D}{-1+4D} \cos x/2$$

$$= 8.e^{x} . \frac{(-1+4D)}{(1-16D^2)} . \cos x/2$$

$$= \frac{8}{5} e^{x} . (-1+4D) \cos x/2$$

$$= \frac{8}{5} . e^{x} [- \cos x/2 + 4^{2}(-\sin x/2) . 1/2]$$

$$y_p = \frac{8}{5} e^{x} [- \cos x/2 - 2 \sin x/2]$$

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* Method of Undetermined Co-efficient

* Using the Method of undetermined coefficient Solve the diff. eqⁿ $y'' + 4y = 8x^2$

⟹

$$y = y_c + y_p$$

→ Auxilary eqⁿ :-

$$(D^2+4) = 0$$
$$D^2 = -4 = 4i^2$$
$$D = \pm 2i$$
$$\alpha \pm \beta i$$
$$\alpha = 0 , \beta = 2$$

$$y_c = e^{\alpha x}[c_1 \cos\beta x + c_2 \sin\beta x]$$
$$= c_1 \cos 2x + c_2 \sin 2x$$

Let $y_p = A_1x^2 + A_2x + A_3$

$$y_p' = A_1 . 2x + A_2$$
$$y_p'' = 2A_1$$

$$y'' + 4y = 8x^2$$
$$(2A_1) + 4(A_1x^2 + A_2x + A_3) = 8x^2$$
$$2A_1 + 4A_1x^2 + 4A_2x + 4A_3 = 8x^2$$
$$4A_1x^2 + 4A_2x + (2A_1 + 4A_3) = 8x^2$$

$4A_1 = 8$ | $4A_2 = 0$ | $2A_1 + 4A_3 = 0$

$A_1 = 2$ | $A_2 = 0$ | $4 + 4A_3 = 0$

$4A_3 = -4$

$A_3 = -1$

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$$\boxed{y_p = 2x^2 - 1}$$

* ~~$(D^2 - 4D + 3) = \sin 3x$~~

$(D^2 - 2D + 3)y = x^3 + \sin x$

→

$D^2 - 2D + 3 = 0$

$\therefore AD^2 + BD + C = 0$

$A = 1, \quad B = -2, \quad C = 3$

$$\alpha, \beta = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{2 \pm \sqrt{4 - 12}}{2(1)}$$

$$= \frac{2 \pm \sqrt{-8}}{2}$$

$$= \frac{2 \pm \sqrt{8}i}{2}$$

$= 1 \pm \sqrt{2}i$

$\alpha \pm \beta i$

$\alpha = 1, \quad \beta = \sqrt{2}$

$y_c = e^{\alpha x} [c_1 \cos \beta x + c_2 \sin \beta x]$

$= e^{x} [c_1 \cos \sqrt{2}x + c_2 \sin \sqrt{2}x]$

$y_p = A_1 x^3 + A_2 x^2 + A_3 x + A_4 + A_5 \sin x + A_6 \cos x$

$y' = 3A_1 x^2 + 2A_2 x + A_3 + A_5 \cos x - A_6 \sin x$

$$y'' = 6A_1x + 2A_2 - A_5\sin x - A_6\cos x$$

$$\therefore 6A_1x + 2A_2 - A_5\sin x - A_6\cos x - 2[3A_1x^2 + 2A_2x + A_3 + A_5\cos x - A_6\sin x] + 3[A_1x^3 + A_2x^2 + A_3x + A_4 + A_5\sin x + A_6\cos x] = x^3 + \sin x$$

$$\therefore 6A_1x + 2A_2 - A_5\sin x - A_6\cos x - 6A_1x^2 - 4A_2x - 2A_3 - 2A_5\cos x + 2A_6\sin x + 3A_1x^3 + 3A_2x^2 + 3A_3x + 3A_4 + 3A_5\sin x + 3A_6\cos x = x^3 + \sin x$$

$\therefore 3A_1 = 1$

$\boxed{A_1 = 1/3}$

$\therefore -6A_1 + 3A_2 = 0$

$\therefore -6\left(\frac{1}{3}\right) + 3A_2 = 0$

$\therefore -2 + 3A_2 = 0$

$\therefore \boxed{A_2 = 2/3}$

$\therefore 6A_1 - 4A_2 + 3A_3 = 0$

$\therefore 6\left(\frac{1}{3}\right) - 4\left(\frac{2}{3}\right) + 3A_3 = 0$

$\therefore 2 - \frac{8}{3} + 3A_3 = 0$

$\therefore -\frac{2}{3} + 3A_3 = 0$

$\therefore 3A_3 = 2/3$

$\therefore \boxed{A_3 = 2/9}$

$\therefore 2A_2 - 2A_3 + 3A_4 = 0$

$\therefore 2\left(\frac{2}{3}\right) - 2\left(\frac{2}{9}\right) + 3A_4 = 0$

$\therefore \frac{4}{3} - \frac{4}{9} + 3A_4 = 0$

$\therefore 3A_4 = \frac{4}{9} - \frac{4}{3}$

$\therefore 3A_4 = \frac{4-12}{9}$

$\therefore \boxed{A_4 = -\frac{8}{27}}$

$\therefore -A_5 + 2A_6 + 3A_5 = 1$

$\therefore \boxed{2A_5 + 2A_6 = 1}$

$-2A_5 + 3A_6 - A_6 = 0$

$\boxed{-2A_5 + 2A_6 = 0}$

$$\begin{aligned} \cancel{2A_5} + 2A_6 &= 1 \\ -\cancel{2A_5} + 2A_6 &= 0 \\ \hline 4A_6 &= 1 \end{aligned}$$

$\boxed{A_6 = 1/4}$

$\rightarrow 2A_5 + \cancel{2}\left(\frac{1}{\cancel{4}_{2}}\right) = 1$

$2A_5 = 1 - 1/2$

$2A_5 = 1/2$

$\boxed{A_5 = 1/4}$

Solve, $(D^2 - 5D + 6)y = x.\underset{(V)}{\cos 2x}$

(જમણી બાજુ x અને તેની સાથે બીજુ કોઈ પદ ગુણાકાર માં આવેલ હોય ત્યારે):-

$P.I = \frac{1}{f(D)}.x.\underset{V}{\phi}$

$= x.\frac{1}{f(D)}.V - \frac{f'(D)}{[f(D)]^2}.V$

A.E

$D^2 - 5D + 6 = 0$

$\therefore (D-2)(D-3) = 0$

$\therefore D = 2, 3$

$y_c = c_1 e^{m_1 x} + c_2 e^{m}$

$= c_1 e^{2x} + c_2 e^{3.}$

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$$y_p = x \cdot \frac{1}{D^2 - 5D + 6} \cdot \cos 2x - \frac{2D - 5}{[D^2 - 5D + 6]^2} \cdot \cos 2x$$

$$= x \cdot \frac{1}{2 - 5D} \cos 2x - \frac{2D - 5}{(2 - 5D)^2} \cos 2x$$

$$= x \cdot \frac{1}{2 - 5D} \times \frac{2 + 5D}{2 + 5D} \cos 2x - \frac{(2D - 5)}{(4 - 20D + 25D^2)} \cos 2x$$

$$= x \cdot \frac{[2 + 5D]}{4 - 25D^2} \cdot \cos 2x - \frac{(2D - 5)}{(-96 - 20D)} \cos 2x$$

$$= \frac{1}{104} \cdot x \, [2 \cos 2x + 5(-\sin 2x) 2] - \frac{(2D - 5)}{-4(24 + 5D)(24 - 5D)} \times (24 - 5D) \cdot \cos 2x$$

$$= \frac{x}{104} [2 \cos 2x - 10 \sin 2x] + \frac{1}{4} \frac{(2D - 5)(24 - 5D)}{(576 - 25D^2)} \cdot \cos 2x$$

$$= \frac{x}{104} [2 \cos 2x - 10 \sin 2x] + \frac{1}{2704} [48D - 10D^2 - 120 + 25D] \cdot \cos 2x$$

$$= \frac{x}{104} [2 \cos 2x - 10 \sin 2x] + \frac{1}{2704} [73D - 10D^2 - 120] \cos 2x$$

$$= \frac{x}{104} [2 \cos 2x - 10 \sin 2x] + \frac{1}{2704} [-73 \sin 2x \cdot 2 - 10(-4 \cos 2x) - 120 \cos 2x]$$

$$= \frac{x}{104} [2 \cos 2x - 10 \sin 2x] + \frac{1}{2704} [-146 \sin 2x + 40 \cos 2x - 120 \cos 2x]$$

Variation of Parameter

* Using method of Variation of Parameter solve the diff equation $\frac{d^2y}{dx^2} + 4y = \tan 2x$

→ Auxilary eqⁿ :-

$$D^2 + 4 = 0$$
$$\therefore D^2 = -4$$
$$\therefore D^2 = 4i^2$$
$$\Rightarrow D = \pm 2i$$
$$\alpha \pm \beta i$$
$$\alpha = 0, \quad \beta = 2$$

$$y_c = e^{\alpha x}[c_1 \cos\beta x + c_2 \sin\beta x]$$
$$= c_1 \cos 2x + c_2 \sin 2x$$
$$= c_1 y_1 + c_2 y_2$$

$$y_1 = \cos 2x \qquad y_2 = \sin 2x$$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} \cos 2x & \sin 2x \\ -\sin 2x \cdot 2 & \cos 2x \cdot 2 \end{vmatrix}$$

$$= 2\cos^2 2x + 2\sin^2 2x$$
$$= 2(\cos^2 2x + \sin^2 2x)$$
$$= 2(1)$$
$$= 2$$

$$y_p = -y_1 \int \frac{y_2}{W} R(x)\,dx + y_2 \int \frac{y_1}{W} R(x)\,dx$$

$$= -\cos 2x \int \frac{\sin 2x}{2} \cdot \tan 2x\,dx + \sin 2x \int \frac{\cos 2x}{2} \cdot \tan 2x\,dx$$

$$= -\frac{\cos 2x}{2} \int \frac{\sin 2x \cdot \sin 2x}{\cos 2x}\,dx + \frac{\sin 2x}{2} \int \sin 2x\,dx$$

$$= -\frac{\cos 2x}{2} \int \frac{1-\cos^2 2x}{\cos 2x}\,dx + \frac{\sin 2x}{2}\left(\frac{-\cos 2x}{2}\right)$$

$$= -\frac{\cos 2x}{2} \int (\sec 2x - \cos 2x)\,dx + \frac{1}{4}\sin 2x \cdot \cos 2x$$

$$= -\frac{\cos 2x}{2}\left[\frac{\log|\sec 2x + \tan 2x|}{2} - \frac{\sin 2x}{2}\right] - \frac{1}{4}\sin 2x \cos 2x$$

(7) Solve, the diff eq^n $\frac{d^3y}{dx^3} + \frac{dy}{dx} = \operatorname{cosec} x$ by Using Method of Variation of Parameter.

→ Auxilary eq^n:

$D^3 + D = 0$

$D(D^2+1) = 0$

$D = 0, \quad D^2 + 1 = 0$

$D^2 = -1$

$D = \pm i$

$y_c = c_1 e^{0x} + e^{0x}[c_2 \cos x + c_3 \sin x]$

$= c_1 + c_2 \cos x + c_3 \sin x$

$c_1 y_1 + c_2 y_2 + c_3 y_3$

$y_1 = 1 \qquad y_2 = \cos x \qquad y_3 = \sin x$

$$W = \begin{vmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{vmatrix}$$

$$= \begin{vmatrix} 1 & \cos x & \sin x \\ 0 & -\sin x & \cos x \\ 0 & -\cos x & -\sin x \end{vmatrix}$$

$$= 1 \begin{vmatrix} -\sin x & \cos x \\ -\cos x & -\sin x \end{vmatrix}$$

$$= \sin^2 x + \cos^2 x$$

$$\boxed{W = 1}$$

$$W_1 = \begin{vmatrix} 0 & \cos x & \sin x \\ 0 & -\sin x & \cos x \\ 1 & -\cos x & -\sin x \end{vmatrix} = 1$$

$$W_2 = \begin{vmatrix} 1 & 0 & \sin x \\ 0 & 0 & \cos x \\ 0 & 1 & -\sin x \end{vmatrix} = -\cos x$$

$$W_3 = \begin{vmatrix} 1 & \cos x & 0 \\ 0 & -\sin x & 0 \\ 0 & -\cos x & 1 \end{vmatrix} = -\sin x$$

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$$y_p = y_1 \int \frac{w_1}{w} R(x)\,dx + y_2 \int \frac{w_2}{w} R(x)\,dx$$
$$+ y_3 \int \frac{w_3}{w} R(x)\,dx$$

$$= \int \operatorname{cosec} x\,dx + \cos x \int -\cos x \cdot \operatorname{cosec} x\,dx + \sin x \int -\sin x \cdot \operatorname{cosec} x\,dx$$

$$= \log|\operatorname{cosec} x - \cot x| + \cos x \cdot \int -\frac{\cos x}{\sin x}\,dx + \sin x \int -\frac{\sin x}{\sin x}\,dx$$

$$= \log|\operatorname{cosec} x - \cot x| - \cos x \cdot \log|\sin x| - \sin x \cdot x$$