1.1 INTRODUCTION

Elasticity is a branch of physics which deals with the elastic property of materials. Elasticity is the property by virtue of which bodies regain original shape and size after removal of the deforming force.

when a deforming force acts on a body, there is a change in its length or shape or volume. Then, the body is said to strained. In this chapter, elastic properties of materials and their applications for different uses are discussed.

1.2 LOAD

A force which is applied to the structure is called a load.

Load causes stress, deformation and displacement in the structures.

1.2.1 CLASSIFICATION OF LOADS-BASED ON TIME

Static load: The load which are relatively constant for an extended time is called static load.

Dynamic load: The load which varies with respect to time is called dynamic load.

Normal load: The load which acts perpendicular to cross sectional area is called normal load.

Shear load: The load which tends to produce a sliding failure of a body along a plane parallel to the direction of load or force is called shear load.

Concentrated load: The load which is concentrated at a point is called concentrated load.

Distributed load: The load which is distributed over the length or area of the structure is called distributed load.

1.3 STRESS

Restoring force per unit area is called stress.

$$\sigma = F/A$$

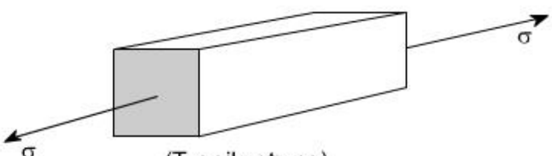
There are three types Longitudinal stress, volume stress and tangential (shearing) stress.

1.3.1 LONGITUDINAL STRESS

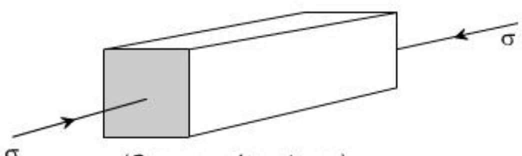
Longitudinal stress can be compressive or tensile.

In compressive length will decrease and in tensile length will increase.

$$\sigma_l = F/A$$



(Tensile stress)

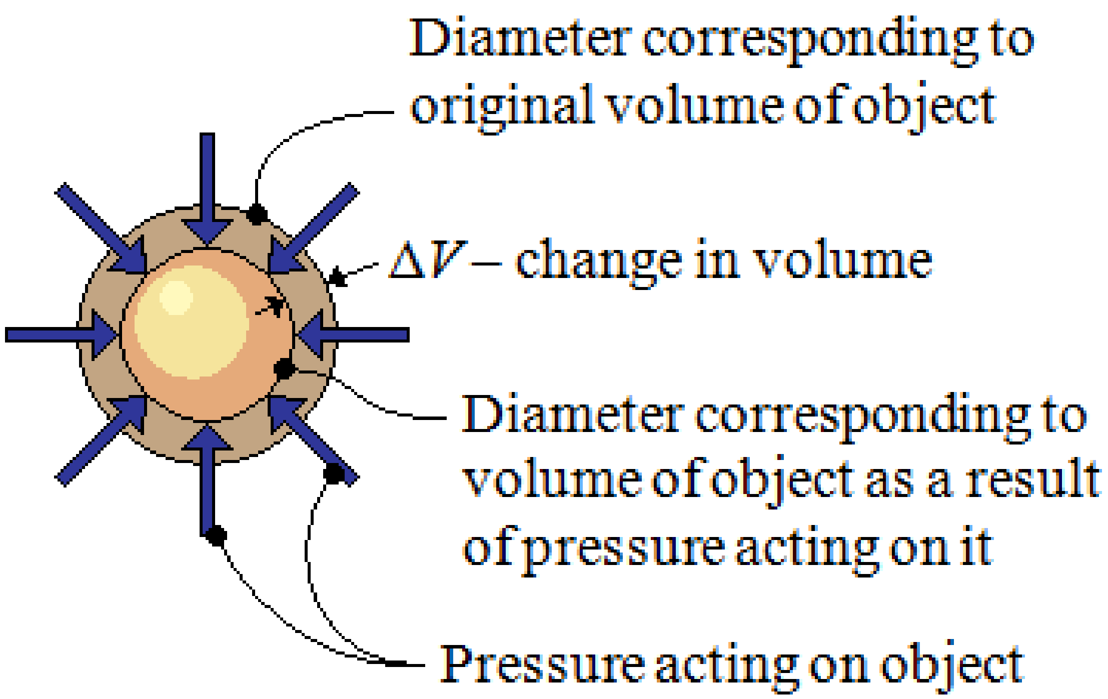


(Compressive stress)

1.3.2 Volume (Bulk) stress

Volume stress is nothing but pressure because to change the volume force must be applied normal to the surface.

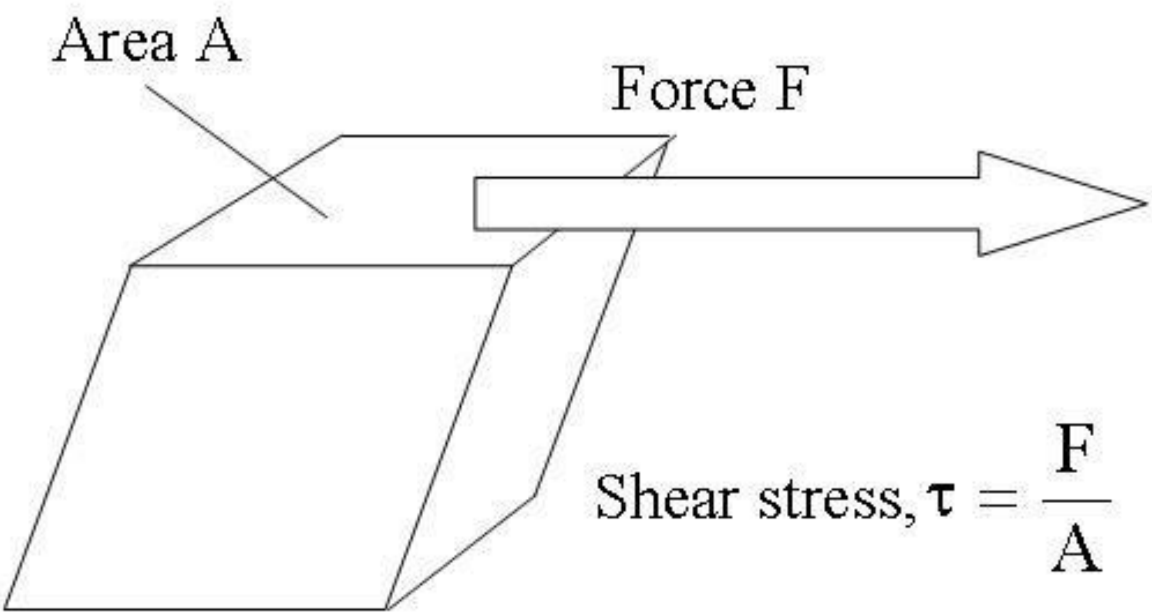
$$\sigma_V = \frac{F}{A} = P$$



1.3.3 Tangential stress

In tangential stress shape will change.

$$\sigma_T = F/A$$



1.4 Strain

Strain is ratio of change in dimension to original dimension.

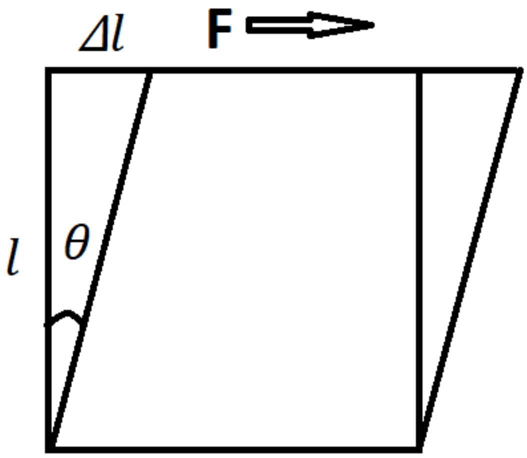
There are three types of strains.

Longitudinal strain, volume (bulk) strain and tangential (shearing) strain.

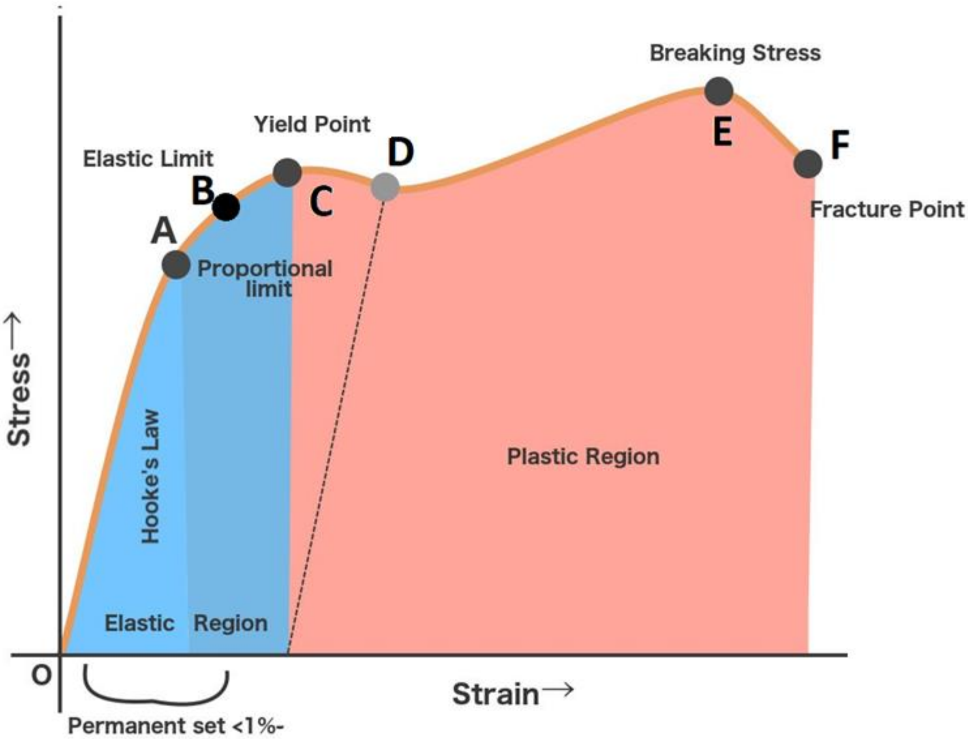
$$\text{longitudinal strain, } \epsilon_l = \Delta l / l$$

$$\text{volume strain, } \epsilon_V = \Delta V / V$$

$$\text{tangential strain, } \epsilon_T = \theta = \Delta l / l$$



1.5 Stress – strain curve



Portion OA: This portion is absolutely straight, where the stress is proportional to strain and the material obeys Hooke's law (σ=Yϵ). The value of stress at point A is called proportional limit.

Portion AB: In this portion, Hook's law is not obeyed, although the material may still be elastic. The point B indicates the elastic limit.

Portion BC: In this portion, the metal shows a strain even without increase in stress and the strain is not fully return when load is removed.

Portion CD: Yielding start in this portion and there is a drop of stress at the point D directly after yielding begins at C. The point D is termed as lower yield point and C is called upper yield point.

Portion DE: After yielding has taken place at D, further straining takes place at this portion by increasing the stress and the stress–strain curve continues to rise up to the point E. Strain in this portion is about 100 times that of portion O-A. At the point E, the bar begins to form a local neck. The point E is termed as ultimate tensile stress point.

Portion EF: In this portion, the load is falling off from the maximum and fracture at F takes place. The point F is termed as fracture or breaking point and the identical stress is called breaking stress.

1.5.1. Ductility

It is the property due to which material can be drawn into wires.

Higher the strain more is the ductility.

Ductility increases with temperature.

Gold is the most ductile material.

1.5.2. BRITTLENESS

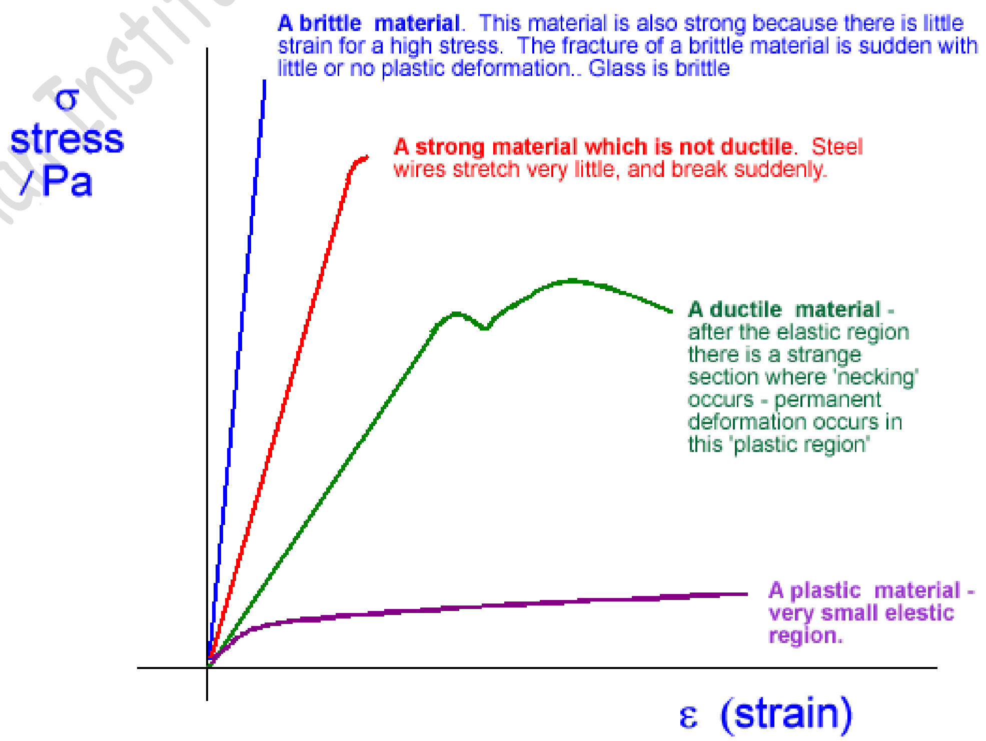
It is the property due to which material do not deform under load but suddenly breaks.

It is opposite to ductility.

In this material no or negligible strain.

1.5.3. PLASTICITY

It is the property due to which there is permanent deformation without breaking.



1.6 Hook's law

Stress is directly proportional to strain for small stress applied.

Ratio of stress to strain is constant and is known as coefficient of elasticity or modulus of elasticity.

$$\text{stress} \propto \text{strain}$$

$$\text{stress} = \text{E} \times \text{strain}$$

$$\text{E} = \frac{\text{stress}}{\text{strain}}$$

1.7 Types of elastic moduli

1.7.1. Young's modulus

$$\text{Y} = \frac{\text{longitudinal stress}}{\text{longitudinal strain}}$$

If load applied to a wire of length l and cross-section area $\text{A} = \pi\text{r}2$, is mg and extension in wire due to this is Δl. Then

$$\text{Y} = \frac{\text{F}/\text{A}}{\Delta\text{l}/\text{l}}$$

$$\text{Y} = \frac{\text{mgl}}{\pi\text{r}2\Delta\text{l}}$$

Unit of Young's modulus is N/m^2 or Pascal.

1.7.2. Bulk modulus

$$\text{B} = \frac{\text{volume stress}}{\text{volume strain}}$$

If load applied normal to substance, then only change in volume will be there. Thus volume stress is nothing but pressure. Due to normal force if change in volume is ΔV then,

$$\text{B} = \frac{\text{P}}{\Delta\text{V}/\text{V}}$$

$$\text{B} = \frac{\text{PV}}{\Delta\text{V}}$$

Unit of Bulk modulus is N/m^2 or Pascal.

1.7.3. Modulus of rigidity

$$\eta = \frac{\text{shearing stress}}{\text{shearing strain}}$$

If load applied tangential to surface of the substance shape of the substance will change.

$$\eta = \frac{\text{F}/\text{A}}{\Delta\text{l}/\text{l}}$$

$$\eta = \frac{\text{Fl}}{\text{A}\Delta\text{l}}$$

Unit of Modulus of rigidity is N/m^2 or Pascal.

1.8 Poisson's ratio

When stress is applied in one direction there will be change in other two directions.

If wire is stretched along its length, there is increase in length and decrease in diameter (cross sectional area).

The ratio of lateral strain to linear strain is called Poisson's ratio.

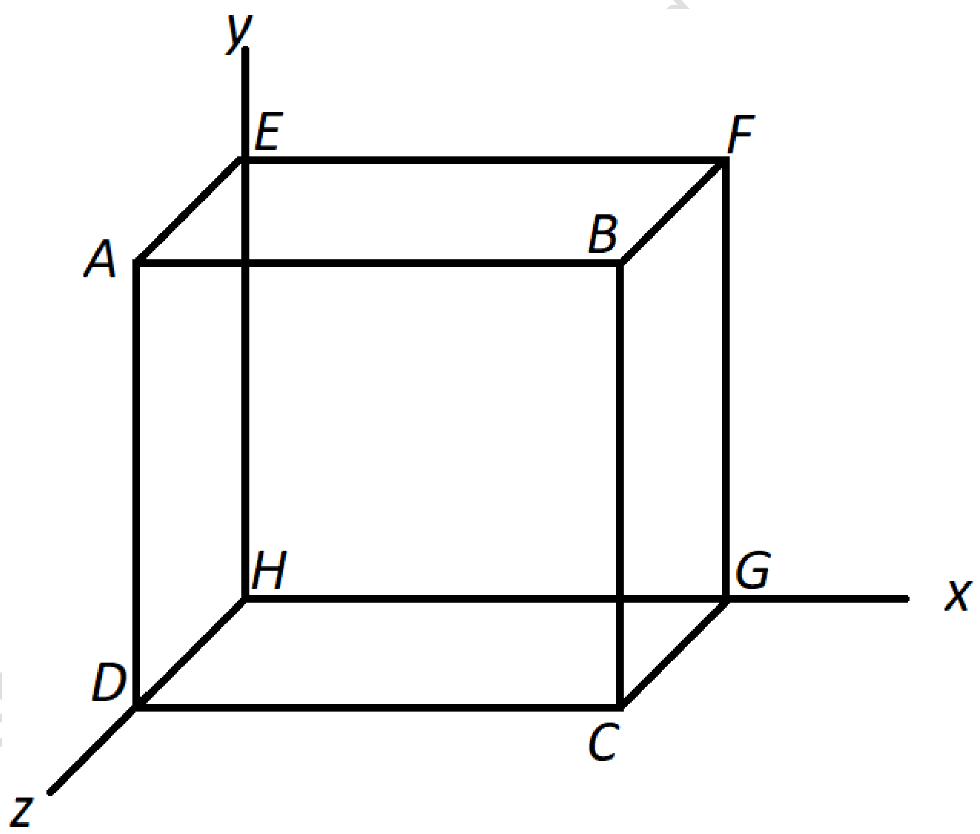
$$\mu = \frac{\Delta d/d}{\Delta l/l}$$

Value is less than 0.5

1.9 Relation between Y and B

Consider cube of length L.

Equal stress is applied normal to all surfaces.



Longitudinal strain is,

$$\epsilon = \frac{\Delta l}{l}$$

$$\text{Also from, } Y = \frac{\sigma}{\epsilon} \quad \Rightarrow \quad \epsilon = \frac{\sigma}{Y}$$

Comparing both

$$\frac{\Delta l}{l} = \frac{\sigma}{Y} \qquad --------1$$

Net strain along x direction is,

$$\epsilon_x = \frac{\Delta l}{l} - \frac{\Delta b}{b} - \frac{\Delta h}{h}$$

$$\epsilon_x = \frac{\Delta l}{l} - \mu\frac{\Delta l}{l} - \mu\frac{\Delta l}{l} \qquad \left(\because\ \mu = \frac{\frac{\Delta d}{d}}{\frac{\Delta l}{l}}\right)$$

$$\epsilon x = \frac{\Delta l}{l}(1-2\mu)$$

Using equation 1

$$\epsilon_x = \frac{\sigma}{Y}(1-2\mu) = \epsilon y = \epsilon z \quad -------- \ 2$$

Volume is $V = lbh$

Volume strain is

$$\frac{\Delta V}{V} = \frac{\Delta l}{l} + \frac{\Delta b}{b} + \frac{\Delta h}{h}$$

$$\therefore\ \epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z$$

Using equation 2 Volume strain is

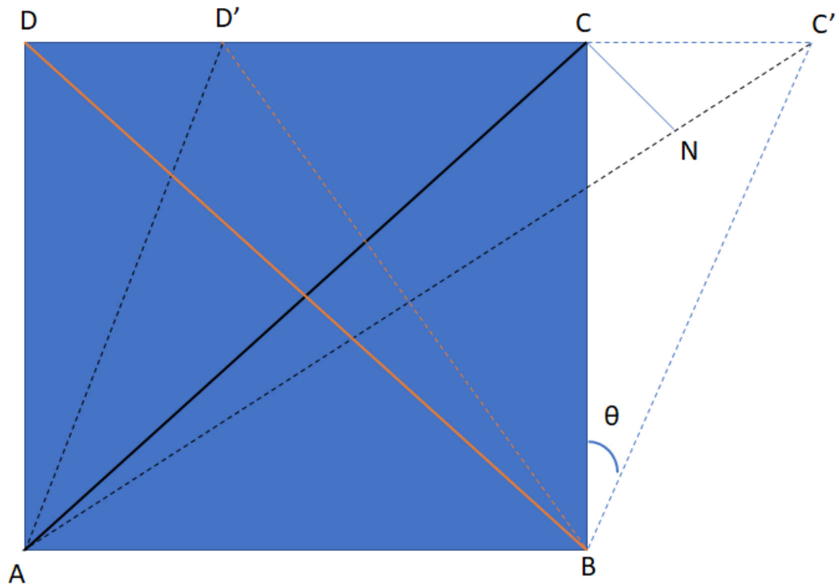
$$\epsilon_v = \frac{3\sigma}{Y}(1-2\mu) \ ----- 3$$

$$B = \frac{\sigma}{\epsilon_v} \qquad \Rightarrow \qquad \epsilon_v = \frac{\sigma}{B} \ ----4$$

Using equation 4 in 3

$$\frac{\sigma}{B} = \frac{3\sigma}{Y}(1-2\mu) \quad \Rightarrow \quad Y = 3B(1-2\mu)$$

1.10 Relation between Y and η



Longitudinal strain is,

$$\epsilon = \frac{\Delta l}{l}$$

$$\text{Also from, } Y = \frac{\sigma t}{\epsilon} \quad \Rightarrow \quad \epsilon = \frac{\sigma t}{Y}$$

Comparing both

$$\frac{\Delta l}{l} = \frac{\sigma t}{Y} \;------1$$

Net strain in AC

$$\epsilon_{AC} = \frac{AC' - AC}{AC} - \frac{BD' - BD}{BD} = \frac{\Delta l}{l} - \left(-\frac{\Delta b}{b}\right)$$

$$\epsilon_{AC} = \frac{\Delta l}{l} + \mu\frac{\Delta l}{l} = \frac{\Delta l}{l}(1+\mu)$$

Using equation 1

$$\epsilon_{AC} = \frac{\sigma t}{Y}(1+\mu) -----2$$

Net strain in AC

$$\epsilon_{AC} = \frac{AC' - AC}{AC} - \frac{BD' - BD}{BD} = \frac{\Delta l}{l} - \left(-\frac{\Delta b}{b}\right)$$

$$\epsilon_{AC} = \frac{\Delta l}{l} + \mu\frac{\Delta l}{l} = \frac{\Delta l}{l}(1+\mu)$$

Using equation 1

$$\epsilon_{AC} = \frac{\sigma t}{Y}(1+\mu) -----2$$

Strain

$$\epsilon = \frac{AC' - AC}{AC}$$

$$\epsilon = \frac{AC' - AN}{AC} = \frac{NC'}{AC} = \frac{CC' \cos 45}{BC\sqrt{2}}$$

$$\epsilon = \frac{CC'}{2BC} = \frac{\tan\theta}{2} = \frac{\theta}{2}$$

$$\epsilon = \frac{\theta}{2} \;----3$$

Using equation 2 in 3

$$\frac{\sigma t}{Y}(1+\mu) = \frac{\theta}{2}$$

$$\frac{\sigma t}{Y}(1+\mu) = \frac{\sigma t}{2\eta} \;----- \quad \left(\because \eta = \frac{\sigma t}{\theta}\right)$$

$$Y = 2\eta(1+\mu)$$

1.11 Relation between Y, B and η

We know that

$$Y = 2\eta(1+\mu) \text{ and } Y = 3B(1-2\mu)$$

$$\frac{Y}{2\eta} = (1+\mu) \text{ and } \frac{Y}{3B} = (1-2\mu)$$

$$\mu = \left(\frac{Y}{2\eta} - 1\right) \text{ and } 2\mu = \left(1 - \frac{Y}{3B}\right)$$

Comparing above equations

$$2\left(\frac{Y}{2\eta} - 1\right) = \left(1 - \frac{Y}{3B}\right)$$

$$\frac{Y}{\eta} - 2 = 1 - \frac{Y}{3B}$$

$$\frac{Y}{\eta} + \frac{Y}{3B} = 1 + 2$$

$$Y\left(\frac{1}{\eta} + \frac{1}{3B}\right) = 3$$

$$Y\left(\frac{3B+\eta}{3B\eta}\right) = 3$$

$$Y = 3\left(\frac{3B\eta}{3B+\eta}\right)$$

$$Y = \frac{9B\eta}{3B+\eta}$$

1.12 Factors affecting elasticity

1.12.1 Effect of stress

Due to application of large constant stress or repeated number of cycles of stresses on a body a gradual decrease in elasticity of a body results.

1.12.2 Effect of temperature

A change in temperature also affects the elastic properties of a material—rise in temperature usually decreases its elasticity and vice-versa. Exceptions are "invar steel" whose elasticity remains practically unaffected by any changes in temperature.

Thus, lead becomes quite elastic and rings like steel when struck by a wooden mallet, and cooled in liquid air.

Carbon filament, which is highly elastic at the ordinary temperature, becomes plastic when heated by the current passed through it, as a result, it can be easily distorted by a magnet brought near it.Generally, elasticity decreases with increase in temperature.

1.12.3 Effect of hammering and rolling

Operations like hammering and rolling, etc. help break up the crystal grains into smaller unit results in an increase or extension of their elastic properties.

1.12.4 Effect of annealing (heating or cooling)

Operations like annealing (i.e., heating and then cooling gradually) tends to produce a uniform pattern of orientation of the constituent crystals, by orienting them all in one particular direction and thus forming larger crystal grains, resulting in a decrease in their elastic properties or an increase in softness or plasticity of the material. Effect of impurities

Effect of impurities

It is well known that sometimes suitable impurities are deliberately added to metals to help bind their crystal grains better, without affecting their orientation. For example, carbon and potassium are added in minute quantities to molten iron and gold respectively for this purpose. Such impurities naturally affect the elastic properties of the metal to which they are added, enhancing or impairing them. In either case, the elastic properties are considerably strengthened.

1.13 Working stress

Working stress is defined as the stress that is developed in the body during working condition.

While considering working stress, other stresses are also considered in addition to its loading stress.

$$\text{Working stress} = \frac{\text{actual applied load}}{\text{cross section area under loading}}$$

1.14 Factor of safety

It is defined as the ratio of maximum stress to the working stress.

It is always greater than 1.

For ductile material, it is ratio of yield stress to working stress.

For brittle material, it is ratio of ultimate stress to working stress.

Some factors are also considered for factor of safety.

Type of materials

Behaviour of material under different loading conditions.

Type of loading conditions.

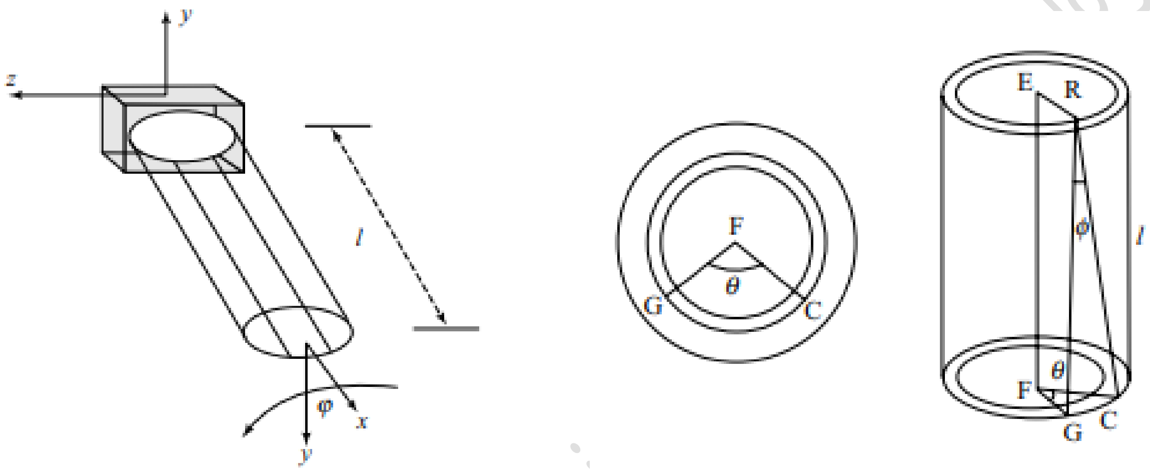
Effect of temperature, weather, chemical, radiation etc.

Safety of users.

1.15 TWISTING COUPLE ON A WIRE

Torsion is the twisting of an object due to an applied torque. Torque is a rotating force capable of turning a body. A stress is an internal resistance offered by a body per unit area of the cross section. For studying torsional stress, we may simply define it in terms of shearing stress which is produced when we apply the twisting moment to the end of a shaft about its axis. For example, when we turn a screw driver to produce torsion, our hand applies a torque 'T' to the handle and twists the shank of the screw driver.

Deformation means change in the shape or dimensions of a body as a result of stress and strain on a material. Let us understand this by taking an example of a shaft attached to the wall and rotating it, as shown.



A circular shaft remains undistorted because its axis is symmetric about the centre. A non-circular shaft, on the other hand, when subjected to torsion, will be distorted, because it is not having an axis that is symmetrical about its centre. For any type of circular shafts, whether it is a solid material or a hollow material, a circular shaft will remain undistorted due to torsion.

1.15.1 TWISTING COUPLE ON A WIRE

The upper end is clamped through a fixed support. The wire consists of a number of cylindrical tubes i.e., coaxial tubes. The radius of tubes ranges from zero to a. Consider such a cylindrical shell of radius r and thickness dr.

In elementary tube, the line AC is parallel to axis of the tube. Consider that a couple is applied at the bottom of wire. As a result, the wire is twisted through an angle θ. Thus, in this twisted state, the position of AC takes a new position as AG which is shown.

From figure, angle COG = θ and CG = r θ

It is known that the displacement is maximum for the points lying on the rim while it decreases as one move towards the centre of wire O. This results a maximum shearing strain at the rim and a minimum shearing strain at the axis or centre.

Consider the line AC which is flattened out from the hollow cylinder. Therefore, rectangle ACDB and AC1D1B are obtained before and after twisting. Hence, the shear and shearing strain are obtained i.e. CAG = ϕ

Or CG = l ϕ.

Comparing above two equations, l ϕ = r θ

$$Shearing\ strain\ \varphi = \frac{r\theta}{l}$$

And rigidity modulus

$$\eta = \frac{shear\ stress}{shear\ strain}$$

Thus, shear stress = η x shear strain

$$shear\ stress = \eta\frac{r\theta}{l}$$

Now let us consider its variation with radius of the wire. The cross-sectional area of the elementary tube is

$$A = \ \mathrm{\pi\, r^2}$$

$$dA = \ \mathrm{2\pi\, r\, dr}$$

The total shearing force on the wire = stress x area

$$Shearing\ force = \ \eta\frac{r\theta}{l} * \ \mathrm{2\pi\, r\, dr}$$

$$Shearing\ force = \ \frac{2\pi\,\eta\,\theta}{l} * \ r^2\ \mathrm{dr}$$

Also, the moment of force about the axis of the wire is

$$Shearing\ force = \ \mathrm{r\ x\ F}$$

$$Shearing\ force = \ \frac{2\pi\,\eta\,\theta}{l} * \ r^2\ \mathrm{dr} * \mathrm{r}$$

The above equation, the twisting couple applied to the wire, varies with the value of r as r changes from r = 0 to r = a. Thus the total twisting couple about the axis of the whole wire is

$$T = \tau = \int_0^a \frac{2\pi\,\eta\,\theta}{l} * \ r^3\ \mathrm{dr}$$

$$T = \tau = \frac{2\pi\,\eta\,\theta}{l} * \ \frac{a^4}{4}$$

When angular twist θ = 1 rad, we obtain twisting couple per unit angular twist as

$$\frac{\tau}{\theta} = C = \frac{1}{2}\frac{\pi\,\eta}{l}a^4$$

The above equation gives the couple per unit twist of wire.

1.15.2 TWISTING COUPLE ON A SOLID SHAFT

Consider a solid shaft with high rigidity modulus. It is used to transmit the power from the machine to load. The couple applied at one end of the shaft is transmitted to its other end without any appreciable twist in it. It is rotated on bearing about its own axis. Even for a large couple, the twist in the shaft is very small. The following assumptions are made to derive the relation between torque, angle of twist and shear stress in a solid shaft of diameter d

1. It contains a number of thin concentric tubes.
2. Each thin tube carries a shear force and is independent from its neighbours.
3. The shaft shift is within the elastic limit.

4. The radial lines before and after twist remain same.

Let dx be the thickness of an elementary thin tube and r be its radius. Consider that the shaft is twisted through an angle θ. The cross-sectional area of the elementary tube is $A = \pi r^2$

$$dA = 2\pi r\, dr$$

And

$$shear\ stress = \eta \frac{r\theta}{l}$$

The torque carried out by the tube

$$Shearing\ force = \frac{2\pi\,\eta\,\theta}{l} * r^3\, dr$$

The total torque or moment of the couple, carried out by solid shaft is the sum of all the elementary torques, i.e.,

$$T = \int_0^{d/2} \frac{2\pi\,\eta\,\theta}{l} * r^3\, dr$$

We know that the radial lines before and after twist remains same and hence, angle θ of the twist is constant. Therefore, the rigidity modulus η and the length of the shaft remain constant. Thus equation of torque can be written as

$$T = \frac{\eta\,\theta}{l}\int_0^{d/2} 2\pi\, r^3\, dr$$

$$T = \frac{\eta\,\theta}{l}\left[2\pi\left(\frac{d}{2}\right)^4 \frac{1}{4}\right]$$

Or torque T can be obtained as

$$T = \frac{\theta}{32}\left[\frac{\eta\,\pi\,(d)^4}{l}\right]$$

where d is the diameter of the shaft. The couple per twist is

$$\frac{\tau}{\theta} = C = \frac{1}{32}\frac{\pi\,\eta}{l}d^4 = \frac{1}{2}\frac{\pi\,\eta}{l}r^4$$

The above equation gives a couple per angular twist. Further, it is clear that the angle of twist is directly proportional to the length of the shaft.

1.15.3 TORSIONAL PENDULUM

Torsional pendulum is a pendulum which performs torsional oscillations. It is used to determine the moment of inertia and torsional rigidity of a given body. The compound and torsional pendulum differ from each other in terms of oscillations. The oscillations are twisted or torsional in a horizontal plane in torsional pendulum, while they are linear for a compound pendulum.

A torsional pendulum consists of a circular disc D as shown. The metal rod or wire is used to clamp the disc. The metal rod is suspended symmetrically by clamping at the torsion head H1. The length of torsion wire is adjusted employing the head H2. In order to measure the time period, a pointer is marked at the disc. The length of pendulum, i.e., the distance between head H2 and head H1, is adjusted by using the head H2. The head H1 is not disturbed.

The disc D is rotated through a small angle and left free. As a result, the elements in the wire undergo shearing strain due to the twisting of wire when it is rotated. Therefore, restoring couple which is acting on the wire brings back to the normal original position. Therefore, the restoring couple = - Cθ, where C is the couple per unit twist.

Therefore, angular acceleration in the wire is due to the restoring couple

$$a = \frac{d^2\theta}{dt^2}$$

If I is the moment of the wire about its axis,

$$I\,\frac{d^2\theta}{dt^2} = -C\,\theta$$

Above equation indicates that the angular acceleration $\frac{d^2\theta}{dt^2}$ is directly proportional to angular displacement θ. The negative sign indicates a decrease in twist on the wire with couple and directed towards the mean position. Therefore, torsional oscillations made by torsional pendulum are simple harmonic motion and the period of oscillation is controlled by the momentum of inertia of the suspended mass about the axis of suspension and a couple per unit twist produced in the wire. Therefore, time period of oscillator is

$$T = 2\pi\sqrt{\frac{displacement}{acceleration}} = 2\pi\sqrt{\frac{1}{C/I}} = 2\pi\sqrt{\frac{I}{C}}$$

The above equation gives the time period of torsional oscillation of torsional pendulum. From the above equation, it can be derived that if the torsional pendulum is allowed to oscillate, the angular velocity and the time period only depends on moment of inertia of wire as well on torsional rigidity.

1.16 BENDING OF BEAM

A beam is defined as a rod or a bar made of homogenous and isotropic elastic material with uniform cross-section, either circular or rectangular. The length of a beam is very large compared to its other dimensions, viz breadth and thickness. Thus, the shearing stress at any point of the beam is very small and negligible. Generally, beams are used to carry heavy loads in applications like roofs, bridges, etc.

In order to study flexural rigidity of the beam for heavy loading applications, a simple theory is developed to study the bending of a beam. The following are the important points to be considered

1. The weight of the beam should be low.
2. The beam should have small curvature.
3. The minimum deflection of the beam is small compared to its length.
4. There are no shearing forces.
5. The cross-section of the beam remains unaltered during bending. Hence, the geometrical moment of inertia of the beam remains constant.

One end of the beam is fixed while a load is applied at the other end. As a result, longitudinal filament is extended at the convex side, while a contraction is obtained at the concave side. However, there is no change in the filament at the centre i.e., in between the two sides. The central filament is in the neutral stage and hence, it is known as neutral filament. The corresponding central axis is known as neutral axis.

1.16.1 BENDING MOMENT OF A BEAM

The beam ABEF is fixed at one end AE, while a load is applied at the other end BF, as shown. Consider a small portion of the beam PP'BF. Let CD be the neutral axis of the beam. It is clear that the filament PB which lies above the neutral axis gets extended and experiences an inward pressure. On the other hand, the filament P'F which lies below the neutral axis gets shortened and experiences an outward pressure. Tensile and compressive stresses are developed in the respective halves of the beam due to inward and outward pressure in the upper and lower halves of the beam. The above stresses generate a couple and it opposes the bending of the beam. When the beam is at equilibrium, the strength of the bending moment is equal to the restoring couple or moment of resistance to the bending.

Consider a small portion of the beam ABCD as shown to obtain the expression for the moment of restoring couple i.e. bending moment. Consider that R is the radius of curvature of the beam and ϕ, the angle subtended by it at the centre of curvature. Let GH be the neutral filament at a distance r from the upper filament AB. R ϕ is the length of the filament under zero applied strain. When the strain is applied, the length of the filament is (R + r) ϕ.

The strain at the filament GH is

$$Strain = \frac{Change\ in\ length}{Original\ length}$$

Thus,

$$Strain = \frac{(\mathrm{R} + \mathrm{r})\varphi - \mathrm{R}\,\varphi}{\mathrm{R}\,\varphi} = \frac{r}{\mathrm{R}}$$

So the strain is directly proportional to the distance r from the neutral axis. Also, Young's modulus is

$$Y = \frac{Stress}{Strain}$$

$$Stress = Y * \frac{r}{R}$$

Thus, force is given by

$$F = Y\frac{r}{\mathrm{R}}a$$

Thus, the moment of force about GH

$$F = Y\frac{r}{\mathrm{R}}a\,r$$

But as we know that the moment of the force acting on the upper and lower halves of the beam are same, so the total moment of the force is

$$= \sum Y\,\frac{a\,r^2}{R} = \frac{Y}{R}\sum a\,r^2 = \frac{Y}{R}\,I_g$$

where I_g is the geometrical moment of inertia and is equal to A k^2. A is total area of the cross section and k is the radius of gyration. Under equilibrium conditions, when a load is applied at one, the bending moment of the beam is equal and opposite to the moment of bending couple. The term Y I_g is called flexural rigidity of the beam. The flexural rigidity is defined as bending moment of the beam for unit radius of curvature.

1.16.2 CANTILEVER

A cantilever is a beam fixed horizontally at one end and loaded at the other end. If a load 'W' is applied at the free end, a couple is created between two forces i.e., (a) force (W) applied at the free end towards downward direction and (b) reaction (R) acting in the upward direction at the supporting end as shown

Due to the load applied at the free end of the cantilever, an external couple is created between the load W at A and the force of reaction Q. Here the arm of the couple (distance between the two equal and opposite forces) is l – *x*.

The external bending moment = W (l – *x*)

We know that the internal bending moment under equilibrium condition = $\frac{Y}{R}\ I_g$

Under the situation where External bending moment = Internal bending moment

W (l – *x*) = $\frac{Y}{R}\ I_g = \frac{Y\ a\ k^2}{R}$

We know that the moment of a couple increases as one moves from U towards the fixed end P. The radius of curvature is different at different points and it decreases as one approaches the point P. Consider a point V at a distance dx from U. The radius of curvature at V is same as that of U, since the point V is very close to the point U. Thus, the equation

$$RV = R\ d\theta = dx$$

$$R = \frac{dx}{d\theta}$$

Thus, the above equation gives

$$W\ (l - x) = Y\ a\ k^2\ \frac{d\theta}{dx}$$

And

$$d\theta = \frac{W\ (l - x)\ dx}{Y\ a\ k^2}$$

The observed depression dy of V below U is

$$dy = (l - x) d\theta$$

$$\therefore\ d\theta = \frac{W\ (l - x)^2\ dx}{Y\ a\ k^2}$$

The total depression of the beam can be calculated taking the limit from zero to *l.*

$$= \sum_0^l \frac{W\ (l - x)^2\ dx}{Y\ a\ k^2}$$

$$= \frac{W}{Y\ a\ k^2}\left[l^2 x - 2\frac{l\ x^2}{2} + \frac{x^3}{3}\right]_0^l$$

$$= \frac{W}{Y\ a\ k^2} * \frac{l^3}{3} = \frac{W}{Y\ I_g} * \frac{l^3}{3}$$

Above equation gives the depression value at the free end of cantilever.

1.16.2.1 CANTILEVER-YOUNG'S MODULUS

Experimental arrangements used to determine the Young's modulus of a rod employing the cantilever. The given rod or bar whose Young's modulus is to be determined is clamped

UNIFORM BENDING – YOUNG'S MODULUS

NON – UNIFORM BENDING – YOUNG'S MODULUS

I – SHAPED GIRDERS

The girders with upper and lower sections broadened and middle section tapered, so that it can withstand heavy loads over it is called as I-shaped girders. Since the girder looks like the letter I, they are known as I-shaped girders.

We know that the depression in the case of a rectangular section is given as,

$$y = \frac{W}{4\,Y} * \frac{l^3}{b\,d^3}$$

Depression y can be minimized by either decreasing the load (W) or the length of the girder (l) or by increasing the Young's modulus or the breadth (b) or the thickness (d) of the girder. Since length l is a fixed quantity, it cannot be decreased. Therefore, breadth and thickness may be adjusted by increasing the depth and decreasing the breadth (since thickness increases by d^3). Thus, volume of girder is increased and hence depression produced is reduced. Depression can also be reduced by properly choosing materials of high Young's modulus.

Applications of I-shaped Girders

1. They are used in the construction of bridges over rivers.

2. They are very much useful in the production of iron nails which are used in railway tracks.

3. They are used in supporting beams for ceilings in the construction of buildings.

4. They are used in construction of dams.

Advantages

1. More stability

2. Stronger

3. High durability

1.17 VISCOSITY

Viscosity is a property of the fluid which describes the frictional resistance to flow of the fluid and it measures the resistance of fluid to deform under shear stress. If the viscosity of the fluid is high, it implies that particles have more affinity with each other and vice versa.

Consider a fluid is flowing over a fixed horizontal surface with streamline flow. The layer which is in contact with fixed horizontal surface remains stationary and the velocity of the other layer increase gradually with the distance from the fixed surface. So, the upper most layer has highest velocity and gradually bottom layers have decreasing velocity which develops velocity gradient.

When force is applied on the upper most layer, it tries to drag the lower layer along with it by exerting force F. However, the lower layer tries to retard the motion of upper layer by exerting equal and opposite force. These two equal and opposite forces develop shear stress which is proportional to velocity gradient. This is called Newton's law of viscosity.

$$\tau \propto \frac{du}{dy} = \mu \frac{du}{dy}$$

M is coefficient of viscosity, absolute viscosity or dynamic viscosity. The SI unit is Ns/m^2 or poise. (Ns/m^2 = 10 poise).

The measurement of dynamic viscosity is most useful for liquids which change their apparent characteristics as force or pressure is applied. Another type of viscosity is kinematic viscosity. It is the ratio of dynamic viscosity to the density of the fluid.

$$\vartheta = \frac{Dynamic\ viscosity}{Mass\ density} = \frac{\mu}{\rho} \quad \left(\frac{m^2}{s}\right)$$

The viscosity of liquids decrease when the temperature increases, while the viscosity of gases increases with the increase of temperature. This is due to reason that the viscous forces in a fluid are due to cohesive forces

2.1 Introduction:

2.1.1 WAVES

The wave is a form of disturbance which transmits energy from one place to another without the actual flow of matter as a whole. Wave motion is a form of disturbance which is due to the repeated periodic vibrations of the particles of the medium about their mean positions and the motion is handed over from one particle to the another without any net transport of the medium. Water waves or sound waves are called mechanical or elastic waves as they require a material medium for their propagation which possess elasticity as well as inertia. Light waves are electromagnetic waves or non-mechanical waves which can propagate through vacuum i.e. don't require any material medium.

Matter waves are associated with moving electrons, protons, neutrons and other fundamental particles and even atoms and molecules and all these constitute the matter, so are called matter waves. Matter waves arise in quantum mechanical description of nature. Thus, waves are of three types: Mechanical waves, electromagnetic waves and matter waves.

Mechanical waves are of two types; transverse waves and longitudinal waves.

A wave is said to be progressive or travelling wave if it travels from one point of the medium to another.

Waves on the surface of water are of two types; capillary waves and gravity waves. The restoring force that produces capillary waves is the surface tension of water. The restoring force that produces gravity waves is the pull of gravity which tends to keep the water surface at its lowest level. The oscillations of the particles in the gravity waves are not confined to the surface only, but extend with diminishing amplitude to the very bottom.

The particle motion in water waves involves a complicated motion, they not only move up and down but also back and forth. The waves in an ocean are a combination of both longitudinal waves travel with different speeds in the same medium. Speed of transverse waves in a string is determined by two factors; linear mass density and tension in the string.

2.1.2 Definitions:

1. Phase:
   Phase is the position of a point in time on a waveform cycle.
2. Wavelength:
   It is equal to the distance travelled by a wave during the time in which any one particle of the medium completes one vibration about its mean position. Distance between two consecutive crests or troughs.
3. Wave number:
   Wave number k and wavelength are related as $k = \frac{2\pi}{\lambda}$. Its unit is m^{-1}.
4. Wave velocity:
   Velocity = distance travelled / time taken
   Velocity of wave = length of pulse / time period for pulse
   $$v = \frac{\lambda}{T}$$
5. Frequency:
   The number of waves passing through a given point during the interval of one second. The higher the frequency, the shorter the wavelength. Its unit is Hz
6. Time period:

The time required for completing one cycle measured is termed as time period. Its unit is second.

7. Transverse waves:
   A wave in which the particles of medium vibrates up and down in a direction perpendicular to the direction of propagation of wave motion are defined as transverse waves. It travels in the form of crests and troughs.
   Movement of string of a sitar or violin, movement of the membrane of tabla or Dholak, movement of kink on a rope, wave set up on the surface of water.
8. Longitudinal waves
   If the particles of a medium vibrate in the direction of wave propagation, the wave is called longitudinal waves. It travels in the form of compression and rarefaction.
   Sound waves are example of longitudinal waves.

2.2 MOTION

For simple harmonic motion, the force is proportional to the mean position. We can write

$$F = m\,a = -k\,y$$

But, acceleration for SHM can be written as

$$a = -\omega^2\,y$$

So, on comparing two equations, we get,

$$a = \frac{k}{m}y = \omega^2\,y$$

$$\therefore\ \frac{k}{m} = \omega^2$$

$$\omega = \sqrt{\frac{k}{m}}$$

Pendulum and spring under force due to suspended mass are the examples of SHM. In this type of motion, rate of acceleration is directly proportional to its displacement from mean position. Further, SHM can be represented by means of vector of magnitude A rotating at constant angular velocity as shown. This type of motion is represented by a sinusoidal curve as shown in figure and hence the instantaneous displacement at any time can be expressed by following equation

$$y = A\sin(\omega t + \theta)$$

where A is the displacement from the mean position at time ‘t’.

If phase angle is assumed to be zero, then the equation becomes

$$y = A\sin(\omega t)$$

Therefore, velocity at any instant ‘t’ is given by

$$v = \frac{dy}{dt} = A\omega\cos(\omega t)$$

Further, acceleration at any instant 't' is given by

$$a = \frac{d^2y}{dt^2} = -A\omega^2\sin(\omega t)$$

$$= -\omega^2 y$$

$$\therefore \frac{d^2y}{dt^2} + \omega^2 y = 0$$

SHM is characterized by the fact that the acceleration is proportional to the displacement and negative sign indicates that the motion is towards the mean position.

MAXIMUM ACCELERATION

Acceleration for SHM is given by

$$a = -\omega^2 y$$

Acceleration will be maximum when, displacement becomes amplitude

$$a_{max} = -\omega^2 A$$

MAXIMUM VELOCITY

Velocity for SHM is given by

$$v = \frac{dy}{dt} = A\omega\cos(\omega t)$$

Velocity will be max when value of $\cos(\omega t) = 1$

$$v_{max} = A\omega$$

TIME PERIOD FOR SPRING MASS SYSTEM SHOWING SHM

For this system, time period (T) is given by,

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

where k is spring stiffness constant (force constant of a spring).

TIME PERIOD FOR PENDULUM SYSTEM SHOWING SHM

For this system, time period (T) is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

2.2.1 TOTAL MECHANICAL ENERGY

Total mechanical energy is summation of potential energy at any instant and kinetic energy. Now, potential energy at displacement y is the work done against the restoring force in moving the body from the mean position to this position

Potential Energy PE

$$PE = dW = -\int_0^y F\,dy = -\int_0^y (-ky)dy$$

$$\therefore PE = \frac{1}{2}k\,y^2$$

But $k = m\,\omega^2$ and $y = A\sin(\omega t)$

$$\therefore PE = \frac{1}{2}(m\,\omega^2)(A\sin\omega t)^2$$

$$\therefore PE = \frac{1}{2}\,m\,\omega^2\,(A)^2 sin^2\omega t$$

At x = 0; the potential energy is minimum i.e. zero and at x = A, potential energy is maximum,

i.e. $PE = \frac{1}{2}\,m\,\omega^2\,(A)^2$

Kinetic energy of this system = $KE = \frac{1}{2}\,m\,v^2$ but $v = A\omega\cos(\omega t)$

$$\therefore KE = \frac{1}{2}m\,(A\omega\cos(\omega t))^2$$

$$= \frac{1}{2}m\,A^2\,\omega^2\,cos^2\omega t$$

$$= \frac{1}{2}m\,A^2\,\omega^2\,(1 - sin^2\omega t)$$

$$= \frac{1}{2}m\,A^2\,\omega^2 - \frac{1}{2}m\,A^2\,\omega^2 sin^2\omega t$$

$$= \frac{1}{2}m\,\omega^2\,(A^2 - y^2)$$

From above equation, it is very clear that when, y = A, kinetic energy will become zero and on the other hand kinetic energy will be maximum when y = 0. So, total mechanical energy of the system at any extreme condition is given by,

$$E = KE + PE$$

$$E = \frac{1}{2}m\,\omega^2\,(A^2)$$

From the graph it is clear that, at the mean position, the total energy in simple harmonic motion is purely kinetic and at the extreme position, the total energy in simple harmonic motion is purely potential energy.

2.2.2 TYPES OF VIBRATION

Vibration can be classified in several ways.

2.2.2.1 Free and forced vibrations

If a system vibrates on its own, without any external force, once an initial disturbance is given, such vibration is called free vibration. Here, no external force is acting on the system. For example, oscillation of a simple pendulum is an example of free vibration. Free vibrations decline when a non-conservative force (frictional and air resistance) is present.

If a system vibrates under the effect of external force, then such vibration is called forced vibration. Forced vibrations can be deterministic or random. Behavior of system under forced vibration is depending on the type of excitations, which may be periodic, aperiodic, steady state or transient. For example, shaking motion experienced by a person seating in transporting vehicle is an example of forced vibration.

2.2.2.2 Undamped and Damped vibrations

If no energy is lost or dissipated in friction (or in any other form of resistance) during vibrations, then such vibrations are called as undamped vibrations. Practically very few examples exist, where damping is very small and there is possibility to have undamped vibrations. Free vibration theories are limited to the area of celestial mechanics, space dynamics and structural dynamics, where the amount of damping is so small that the system can be treated as undamped system. In undamped vibration, the amplitude of free vibration remains constant over time.

If energy is lost in friction, then such vibrations are called damped vibrations. In real life, all examples of vibrations are damped vibrations where the amplitude of free vibrations diminishes gradually over time due to resistance offered by the surrounding medium. For example, oscillation of a swing stops after certain time under condition of free vibrations.

2.2.2.3 Longitudinal, Transverse and Torsional Vibrations

Consider a weightless spindle hanging at one fixed end and carries mass m at the other end. If the mass m moves up a d down parallel to the spindle axis, such vibrations are called longitudinal vibrations.

If the body moves approximately perpendicular to the axis of the shaft, the vibrations are called transverse vibrations.

If the spindle gets alternately twisted and untwisted on account of vibratory motion of the suspended disc, it is called to be undergoing torsional vibrations.

2.2.3 DAMPED HARMONIC MOTION

Ideally in free vibrating system, it would carry on oscillations forever because the energy put into the system by the initial disturbance cannot get out of the system as there is no friction present but in reality, the oscillation always ends with time because some form of friction is present.

Friction dissipates the energy as heat. Such vibrations are called free and damped. The type of friction that is easiest to deal with mathematically is that created by a dashpot (damper). There were two kinds of dashpots available oil and air based.

Damping can be classified on the basis of resistance offering to the vibrations:

1. Viscous damping
2. Coulomb damping (Dry friction Damping)
3. Hysteresis damping

1. Viscous damping

   This is the most common used damping mechanism to reduce the amplitude of vibrations. Here the energy is dissipating in the form of heat conducted by air or liquid in concern medium. In viscous damping, damping resistance depends upon relative velocity of vibrating body and viscosity of damping of fluid (air or oil). Here, two fluid film surfaces having relative motion and resist each other which is known as damping resistance or damping force.
   According to Newtonian method for fluid dynamics,

$$\mathrm{F} = \mathrm{c\,x} = \frac{\mu A V}{y}$$

   where F = damping resistance in newton
   V = x = relative velocity between two surfaces in m/s
   c = damping coefficient of damping force per unit velocity in N s/m

   The damping coefficient depends upon the viscosity of fluid and geometric dimensions of dampers.

2. Coulomb damping

When two metal parts sliding over each other, they are acted upon by a frictional force F which is proportional to the normal reaction R. The constant of proportionality μ is expressed as the coefficient of friction.

$$F = \mu R$$

For smooth surfaces, the dry friction is practically constant and the damping which results because of this frictional force, is called as dry friction damping or coulomb damping.

3. Hysteresis damping
   This type of damping occurs in all vibrating system due to elasticity of material. When a body is subjected to vibrations, the stress – strain for a vibrating body is not obtained as a straight line but forms a hysteresis loop. The area under hysteresis loop represents the energy dissipated due to molecular friction per cycle per unit volume of the body. This develops damping to the vibrations and hence such vibration is called hysteresis damping.
   The size of hysteresis loop depends upon the type of material of the vibrating body. The amount of such damping is very small.

2.2.4 GENERAL SOLUTION OF FREE LONGITUDINAL VIBRATION WITH DAMPING

The diminishing vibration with time is called damping vibrations and they are developed by using dashpot or dampers. A dashpot has a piston which moves in a cylinder filled with some fluid.

DIAGRAM

From diagram we can write,

$$m\frac{d^2y}{dt^2} = -c\frac{dx}{dt} - kx$$

$$\therefore m\frac{d^2y}{dt^2} + c\frac{dx}{dt} + kx = 0$$

Hence, the above equation is differential equation for the free damped vibration.

2.2.5 SOLUTION OF FREE DAMPED VIBRATION EQUATION

The vibration of spring is decreasing with time 't'. Hence, we should select the x as a function of time 't' in such a way that it must be decreasing but increasing with time 't'.

Hence take $x = e^{\lambda t}$ which is decreasing with time 't'.

General solution to the equation $x = e^{\lambda t}$

$$\therefore \frac{dx}{dt} = \lambda\, e^{\lambda t}$$

$$\therefore \frac{d(\frac{dx}{dt})}{dt} = \lambda^2\, e^{\lambda t}$$

Putting the values in equation $mx + cx + kx = 0$ (of free damped vibration), we get

$$m\left(\lambda^2\, e^{\lambda t}\right) + c\left(\lambda\, e^{\lambda t}\right) + k\, e^{\lambda t} = 0$$

$$\therefore\ e^{\lambda t}[\, m\lambda^2 + c\lambda + k] = 0$$

$$\therefore\ m\lambda^2 + c\lambda + k = 0$$

Solving for above equation, its roots can be found as

$$\lambda_{1,2} = \frac{-b \pm \Delta}{2a}$$

where $\Delta = \sqrt{b^2 - 4ac}$ and $a = 1, b = \frac{c}{m}, c = \frac{k}{m}$

Thus,

$$\Delta\ = \sqrt{\left(\frac{c}{m}\right)^2 - 4\left(\frac{k}{m}\right)}$$

Thus, the roots of the above equation are

$$\lambda_{1,2} = \frac{\frac{c}{m} \pm \sqrt{\left(\frac{c}{m}\right)^2 - 4\left(\frac{k}{m}\right)}}{2\ (1)}$$

$$\lambda_{1,2} = \frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \left(\frac{k}{m}\right)}$$

These roots give general solution as $x = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$ where c_1 and c_2 are arbitrary constants which are required to be determined from the initial conditions of the system.

2.2.6 MEASUREMENT OF DAMPING

The ratio of $\left(\frac{c}{2m}\right)^2$ *and* $\left(\frac{k}{m}\right)$ measures the degree of dampness in the system and its square root is known as damping factor.

Damping factor

$$\xi = \sqrt{\frac{\left(\frac{c}{2m}\right)^2}{\left(\frac{k}{m}\right)}}$$

$$\xi = \sqrt{\frac{c^2}{4m^2}\,\frac{m}{k}}$$

$$\xi = \frac{c}{2\sqrt{m\,k}}$$

Thus the damping coefficient $c = 2\xi\sqrt{m\,k}$

but natural angular frequency $\omega_0^2 = \frac{k}{m}$

$$\therefore c\ =\ 2\xi\ \sqrt{m\omega_0^2\ m} =\ c\ =\ 2\xi\ \frac{k}{\omega_0}$$

The roots of above equation in terms of damping factor are

$$\lambda_{1,2} = -\left(\xi\ \pm \sqrt{\xi^2 - 1}\right)\omega_0$$

Thus the general solution can be written as

$$x(t) =\ c_1 e^{\left(-\xi + \sqrt{\xi^2 - 1}\right)\omega_0 t} +\ c_2 e^{\left(-\xi - \sqrt{\xi^2 - 1}\right)\omega_0 t}$$

Velocity calculation based for above equation

$$v(t) =\ \omega_0 c_1\left(-\xi + \sqrt{\xi^2 - 1}\right) e^{\left(-\xi + \sqrt{\xi^2 - 1}\right)\omega_0 t} +\ \omega_0\, c_2\left(-\xi - \sqrt{\xi^2 - 1}\right) e^{\left(-\xi - \sqrt{\xi^2 - 1}\right)\omega_0 t}$$

Condition 1: when $\xi > 1$; i.e. overdamping condition

Overdamping occurs when natural frequency of the system is much less than the frictional forces. Thus, in such a situation, the equation of displacement will be

$y \cong y_0 e^{-\beta t}$

We can observe that there is an exponential decay of the amplitude with time. The frictional forces will overcome the natural frequency of the damped oscillator with time. Hence overdamped oscillator is identified with the one, where amplitude decays exponentially with time and finally dies out. The motion is therefore not periodic. The particle just returns to its equillibrium position when released from its initial displacement.

Condition 2: when $\xi < 1$; i.e. underdamping condition

Underdamping occurs when the amplitude goes on reducing with time and at the end, the body stops vibrating. This occurs when natural frequency of a system is much greater than the frictional forces.

2.2.7 FORCED VIBRATION

Consider a damped oscillator subjected to a periodic force of frequency ω and force -λϑ. The negative sign shows the force is directed to its velocity, λ is the damping force per unit speed of oscillator (or damping constant). The time dependent force applied to the particle is assumed to be a complex function. The real part of the time dependent force is cosine function of frequency and time.

$$F(t) = F_0 e^{i\omega t} = F_0 \cos \omega t$$

$$m\frac{d^2y}{dt^2} + ky + \lambda\vartheta = F_0 \cos \omega t$$

$$\therefore \frac{d^2y}{dt^2} + \frac{k}{m}y + \frac{\lambda\vartheta}{m} = \frac{F_0}{m}\cos \omega t$$

We know

$$\omega_0^2 = \frac{k}{m}$$

$$\frac{1}{\tau} = \frac{\lambda}{m} = \frac{1}{relaxation\ time}$$

$$\vartheta = \frac{dy}{dt}$$

$$\therefore \frac{d^2y}{dt^2} + \omega_0^2 y + \frac{1}{\tau}\frac{dy}{dt} = \frac{F_0}{m}\cos \omega t$$

This is second order differential equation hence its solution will be of the form that represents the displacement as shown

$$y = A\cos \omega t + B \sin \omega t$$

where A and B are constants. Its first and second order derivatives are obtained as

$$\frac{dy}{dt} = -\omega A \sin \omega t + \omega B \cos \omega t$$

$$\frac{d^2y}{dt^2} = -[\omega^2 A \cos \omega t + \ \omega^2 B \sin \omega t]$$

$$-[\omega^2 A \cos \omega t + \ \omega^2 B \sin \omega t] + \omega_0^2\ [A\cos \omega t + B \sin \omega t] + \frac{1}{\tau} - [\omega^2 A \cos \omega t + \ \omega^2 B \sin \omega t]$$
$$= \frac{F_0}{m}\cos \omega t$$

On simplification

$$\cos \omega t\left[-\omega^2 A\ +\ \omega_0^2\ A\ +\ \frac{\omega B}{\tau}\right] + \sin \omega t\left[-\omega^2 B\ +\ \omega_0^2\ B - \ \frac{\omega A}{\tau}\right] = \frac{F_0}{m}\cos \omega t \ldots\ldots\ldots\ldots\ldots\ldots..(1)$$

Taking ω = ω_0 (for forced oscillation), the above equation reduces to

$$2\frac{\omega B}{\tau}\cos\omega t - \frac{\omega A}{\tau}\sin\omega t = \frac{F_0}{m}\cos\omega t \ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

Comparing coefficients of cosine and sine on both sides of equation (1) gives

$$-\omega^2 A + \omega_0^2 A + \frac{\omega B}{\tau} = \frac{F_0}{m} \; and \; -\omega^2 B + \omega_0^2 B - \frac{\omega A}{\tau} = 0$$

$$\left[-\omega^2 B + \omega_0^2 B - \frac{\omega A}{\tau}\right]^2 = 0$$

$$\therefore B = \left(\frac{\omega A}{\tau}\right) \Big/ (\omega_0^2 - \omega^2)$$

Above equation gives us value of coefficient B but it is in terms of A. To eliminate A, we use value of B and substitute in above equation

$$-\omega^2 A + \omega_0^2 A + \frac{\omega}{\tau}\left(\frac{\omega A}{\tau}\right) \Big/ (\omega_0^2 - \omega^2) = \frac{F_0}{m}$$

$$A\left[-\omega^2 + \omega_0^2 + \left[\frac{\omega^2}{\tau^2}\right] \Big/ (\omega_0^2 - \omega^2)\right] = \frac{F_0}{m}$$

$$A = \frac{\frac{F_0(\omega_0^2 - \omega^2)}{m}}{(\omega_0^2 - \omega^2)^2 + \left[\frac{\omega^2}{\tau^2}\right]}$$

Similarly, the value of B can be found as

$$\therefore B = \left(\frac{\omega A}{\tau}\right) \Big/ (\omega_0^2 - \omega^2)$$

$$\therefore B = \left(\frac{\omega}{\tau}\right) \Big/ (\omega_0^2 - \omega^2) \; x \; \frac{\frac{F_0(\omega_0^2 - \omega^2)}{m}}{(\omega_0^2 - \omega^2)^2 + \left[\frac{\omega^2}{\tau^2}\right]}$$

$$\therefore B = \frac{\frac{F_0}{m}\frac{\omega}{\tau}}{(\omega_0^2 - \omega^2)^2 + \left[\frac{\omega^2}{\tau^2}\right]}$$

The above equations give value of coefficient A and B independently. The force is time dependent complex hence the amplitude will also be complex by nature. If the amplitude of oscillation is given by A_0 then we can write

$$A = A_0 \cos\varphi \; and \; B = A_0 \sin\varphi$$

Squaring and adding

$$A^2 + B^2 = A_0^2(cos\,^2\varphi + sin^2\varphi) = A_0^2$$

Thus we obtain

$$A_0^2 = \left(\frac{\frac{F_0(\omega_0^2 - \omega^2)}{m}}{(\omega_0^2 - \omega^2)^2 + \left[\frac{\omega^2}{\tau^2}\right]}\right)^2 + \left(\frac{\frac{F_0}{m}\frac{\omega}{\tau}}{(\omega_0^2 - \omega^2)^2 + \left[\frac{\omega^2}{\tau^2}\right]}\right)^2$$

Solving further,

$$= \left(\frac{F_0}{m}\right)^2 \left[\frac{(\omega_0^2 - \omega^2)^2 + \left[\frac{\omega^2}{\tau^2}\right]}{\left((\omega_0^2 - \omega^2)^2 + \left[\frac{\omega^2}{\tau^2}\right]\right)^2}\right]$$

$$= \left(\frac{F_0}{m}\right)^2 \left[\frac{1}{(\omega_0^2-\omega^2)^2 + \left[\frac{\omega^2}{\tau^2}\right]}\right]$$

$$\therefore A_0 = \left(\frac{F_0}{m}\right) \sqrt{\frac{1}{(\omega_0^2-\omega^2)^2 + \left[\frac{\omega^2}{\tau^2}\right]}}$$

Also we can find tanϕ = (B / A). Thus, phase of forced vibration will be obtained as

$$\varphi = \tan^{-1}\left(\frac{B}{A}\right)$$

$$\varphi = \tan^{-1}\left(\frac{\frac{\frac{F_0}{m}\frac{\omega}{\tau}}{(\omega_0^2-\omega^2)^2 + \left[\frac{\omega^2}{\tau^2}\right]}}{\frac{\frac{F_0(\omega_0^2-\omega^2)}{m}}{(\omega_0^2-\omega^2)^2 + \left[\frac{\omega^2}{\tau^2}\right]}}\right)$$

$$\varphi = \tan^{-1}\left(\frac{\frac{\omega}{\tau}}{\omega_0^2-\omega^2}\right)$$

Thus, the equation for displacement of forced vibration,

$$y = A_0 \cos(\omega t - \varphi)$$

$$y = \left(\frac{F_0}{m}\right) \sqrt{\frac{1}{(\omega_0^2-\omega^2)^2 + \left[\frac{\omega^2}{\tau^2}\right]}}\ x\ \cos(\omega t - \varphi)$$

AMPLITUDE RESONANCE

The displacement will be maximum under the condition that can be obtained by equating its first order derivative with zero i.e. condition for maximum.

$$\frac{d}{d\omega}\left[(\omega_0^2-\omega^2)^2 + \left[\frac{\omega^2}{\tau^2}\right]\right] = 0$$

$$\therefore 2(\omega_0^2-\omega^2)\,(-2\omega) + \frac{2\omega}{\tau^2} = 0$$

$$\therefore -2\omega\left[2(\omega_0^2-\omega^2) - \frac{1}{\tau^2}\right] = 0$$

Maximum is very close to ω = ω_0, and the resonance frequency is obtained as per the above equation. If the damping is very small, relaxation time will provide the condition

$$\frac{1}{2\tau} \ll \omega_0$$

The corresponding maximum amplitude is given by the equation of amplitude of forced oscillation as

$$(A_0)_{max} = \left(\frac{F_0}{m}\right) \sqrt{\frac{1}{(\omega_0^2-\omega^2)^2 + \left[\frac{\omega^2}{\tau^2}\right]}}$$

$$(A_0)_{max} = \left(\frac{F_0}{m}\right) \frac{\tau}{\omega_0}$$

2.2.8 VELOCITY RESONANCE AND ENERGY INTAKE

While describing forced vibration, we have considered external periodic force. Let us consider the force

$$F(t) = F_0 \sin(\omega t - \varphi)$$

The displacement with this force is expressed as

$$y = \left(\frac{F_0}{m}\right)\sqrt{\frac{1}{(\omega_0^2 - \omega^2)^2 + \left[\frac{\omega^2}{\tau^2}\right]}}\; x\; \sin\,(\omega t - \varphi)$$

$$\therefore v = \frac{dy}{dt} = \left(\frac{F_0}{m}\right)\sqrt{\frac{1}{(\omega_0^2 - \omega^2)^2 + \left[\frac{\omega^2}{\tau^2}\right]}}\; x\; \omega \cos\,(\omega t - \varphi)$$

Let us consider ϑ_0 as amplitude of velocity

$$\vartheta_0 = \left(\frac{F_0\omega}{m}\right)\sqrt{\frac{1}{(\omega_0^2 - \omega^2)^2 + \left[\frac{\omega^2}{\tau^2}\right]}}$$

$$\vartheta_0 = \left(\frac{F_0}{m}\right)\sqrt{\frac{1}{(\omega_0^2 - \omega^2)^2 + \left[\frac{1}{\tau^2}\right]}}$$

2.2.9 Condition of velocity resonance

The velocity amplitude is maximum when $\omega = \omega_0$ i.e. velocity amplitude of the oscillator will be maximum when frequency of applied force matches the natural frequency of undamped oscillation.

The amplitude resonance and velocity resonance occur at frequencies different from each other.

Whenever an oscillator is driven by an external force, energy is absorbed by the oscillator. The energy absorbed by the oscillator is equal to the energy dissipated due to damping.

The rate of energy absorption or power absorbed is function of driving force. It is maximum at resonance i.e. when the frequency of the periodic force is equal to that of the natural frequency of the oscillator.

The power which is defined as rate at which work is done. Here the applied force is source and the oscillator is the place at which work is done. The maximum power transferred is

$$P_{max} = \frac{F_0\tau}{2m}$$

2.3 ACOUSTICS OF BUILDING

Acoustic is the science of sound which deals with the properties of sound waves and its effect of the construction of auditorium, buildings, halls, etc. Sound is classified into infrasonic, audible and ultrasonic. Audible sound is classified into musical sound and noise. Musical sound is one that causes pleasing effect on ear while noise produces a jarring effect. W.C. Sabine first of all scientifically solved the acoustical problems faced in musical hall or in auditorium.

The minimum sound intensity which a human ear can sense is called the threshold intensity. Its value is 10^{-12} watt/m^2 If the intensity is less than this value then our ear cannot hear the sound. This minimum intensity is also known as zero or standard intensity. The intensity of a sound is measured with reference to the standard intensity.

The reverberation time always has to be maintained at an optimum value. This is because, if the reverberation time is too small the loudness will be inadequate (not in proper amount) and the sound will die away instantaneously. Such a hall is said to have dead effect. On the other hand, if the reverberation time is too large, the greater will be the confusion due to mixing or overlapping of sound. Therefore, the reverberation time has to be maintained at an optimum value.

2.3.1. Definitions:

1. Intensity of sound wave: Intensity of sound wave at a point is defined as the amount of sound energy Q flowing per unit area in unit time when the surface is held normal to the direction of propagation of sound wave.

   $$I = \frac{Q}{At}$$

   $$I = 2\pi^2 f^2 a^2 \rho v$$

   Its unit is W / m^2.

2. Echo: An echo is produced when the reflected sound waves reach the ear just when the original sound from the same source has already been heard. The sensation of sound persists on human ear for 1 / 10th of a second after the sound source has stopped. So, to hear an echo, the reflected sound must reach to the listener 1 / 10th of a second later than the original sound.
3. Reverberation time: The time gap between the initial direct sound and the reflected sound upto a minimum audibility level is called reverberation time. It is the interval of time taken by a sustained or continuous sound to fall to an intensity level equal to one millionth of its original value. Its unit is second.
4. Absorption of sound: Sound wave that strikes a surface may have part of its energy transmitted, part of its energy absorbed or the remaining energy is reflected. The effectiveness of surface in absorbing sound energy is expressed by absorption coefficient a as per equation

   $$a = \frac{sound\ energy\ absorbed\ by\ the\ surface}{total\ sound\ energy\ incident\ on\ the\ surface}$$

   Its unit is Sabine or OWU (Open Window Unit).
5. Sabine: The absorption coefficient of a material is defined as the reciprocal of its area which absorbs the same sound energy as absorbed by unit area of open window. Effective absorbing area A is product of total area S and absorption coefficient a

   $$A = S\,a$$

   If there are n surfaces, the average value of absorption co-efficient is

   $$a = \frac{a_1S_1 + a_2S_2 + a_3S_3 + \cdots a_nS_n}{S_1 + S_2 + S_3 + \cdots S_n}$$

   $$\therefore a = \frac{\sum_{i=1}^{n} a_iS_i}{S}$$

6. Pitch: It depends upon the frequency. It is a sensation which separates two sounds having same intensities and different frequencies produced by same musical instrument. (Pitch helps in distinguishing a note of high frequency and low frequency sound of the same intensity produced by the same musical instruments.)

   The voice produced by ladies and children are of high pitch type as frequency is high. Thus, greater the frequency of a sound the higher is the pitch and vice versa. The pitch of sound changes due to Doppler's principle. When either the source or the observer or both are in relative motion.
7. Loudness: Loudness is the characteristics which is common to all sounds whether classified as musical sound or noise. Loudness is a degree of sensation (awareness) produced on ear. The loudness depends upon intensity and also upon the sensitiveness of the ear. Loudness varies from one listener to another. Loudness is a physiological quantity. Units of loudness are phon and sone.

Loudness and intensity are related to each other by the relation.

$L \propto \log I \; or \; L = k \log I \;\; where\ k\ is\ constant$

From this relation it is seen that, loudness is directly proportional to the logarithm of intensity, and is known as Weber – Fechner law. From the above equation,

$$\frac{dL}{dI} = \frac{k}{I} \; where \; \frac{dL}{dI} \; is\ called\ sensitiveness\ of\ ear$$

Therefore, sensitiveness decrease with increase of intensity.

8. Timbre: It is a sensation which separates two sounds having same intensities and same frequencies produced by different musical instrument. (It is the quality of sound which separates two sounds having the same loudness and pitch.). It helps us to distinguish between musical notes emitted by different musical instruments and voices of different persons. Sound of same intensity and same frequency coming from tabla and violin can be separated due to timbre.
9. Intensity level I_L: The intensity level or relative intensity of a sound is defined as the logarithmic ratio of intensity I of a sound to the standard intensity I_0.

$$I_L = k \log \frac{I}{I_0}$$

Let and represent intensities of two sounds of a particular frequency; and L_1 and L_0 be their corresponding measures of loudness, Then, according to Weber-Fechner law,

$L_1 = k \log I \; and \; L_0 = k \log I_0$

Therefore, the intensity level or relative intensity is $I_L = L_1 - L_0$

$I_L = k \log I - k \log I_0$

$I_L = k (\log I - \log I_0)$

$I_L = k \log_{10} \frac{I}{I_o}$

If k = 1, then I_L is expressed in a unit called bel. From above equation we get, 10 times increase in intensity i.e., $I = 10\, I_0$ corresponds to 1 bel. Therefore, bel is the intensity level of a sound whose intensity is 10 times the standard intensity. Similarly, 100 times increase in intensity, i.e., $I = 100\, I_0$ corresponds to 2 bel and 1000 times increase in intensity, i.e., $I = 1000\, I_0$ corresponds to 3 bel and so on.

2.3.2. SABINE FORMULA

Sabine defined the reverberation time as, the time taken by the sound intensity to fall to one millionth of its original intensity after the source stopped emitting sound.

The reverberation time is given by

$$T = \frac{0.167\, V}{A}$$

where V is the volume of the hall and A is total absorption in the room.

$A = a_1S_1 + a_2S_2 + a_3S_3 \ldots$, where ‘a’ is the absorption coefficient and S is the surface area.

The absorption coefficient “a” of a sound absorbing material can be determined by measuring the reverberation time. The reverberation time T_1 in a hall is measured without placing the sound absorbing material in the hall. Thus

$$T_1 = \frac{0.167\ V}{A}$$

where A is total absorption. The sound absorbing material of absorption coefficient a_1 having surface area S is placed in the hall and the reverberation time T_2 is measured. Thus,

$$T_2 = \frac{0.167\ V}{A + a_1S_1}$$

From above equations,

$$\frac{1}{T_2} - \frac{1}{T_1} = \frac{a_1 S_1}{0.167V}$$

and

$$a_1 = \frac{0.167V}{S_1}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

.

Hence, knowing the values of reverberation time, surface area of material, and volume absorption coefficient of the material can be determined.

2.3.3. SOUND ABSORBING MATERIALS

2.3.3.1 Properties of sound absorbing materials

1. They should be highly porous.
2. They should be cheap and easily available.
3. They should be easy to fix and good looking.
4. They should be light in weight and durable.
5. They should be water proof and fire proof.
6. They should be efficient over a wide range of frequencies.

2.3.3.2 Porous absorbent

When sound wave strike a porous material, a small part of it gets reflected, while major part of sound enters into the porous material and gets converted into heat energy hence it becomes inaudible and do not create interference.

e.g. fiber board, soft plastic, rock wood, wool wood, glass silk (Textiles-fine spun glass massed into a wool-like bulk, used in insulation, filtering, etc) etc.

2.3.3.3 Cavity resonator

A chamber with a small opening is known as a cavity resonator. The sound waves which enter the cavity gets multiple reflections become inaudible. These materials are used in air condition plants.

2.3.3.4 Resonant absorbent/panel absorbent

When sound energy is incident on these materials, it gets converted into heat due to flexural vibration of the panel.

E.g. gypsum board, hard board panels, wood board, suspended plasters, rigid plastic board panels.

2.3.3.5 Composite absorbent

When the functions of all the three types described above are combined in a single unit, it is known as Composite type absorbent. When the sound energy strikes the panel, it passes through it and is damped by resonance of air in the cavity.

E.g. empty jars and bottles, perforated cardboard etc.

3.1 Introduction

The human ear is sensitive to sound wave in the range of 20 Hz to 20 kHz. Waves are coming in the range of greater than 20 kHz are called ultrasonic waves. And which are coming in the range less than 20Hz frequencies are coming in the range of infrasonic (subsonic) range.

We can describe the sound waves as the vibrations or oscillations of molecules (particles) in solids, liquids and gases.

Hz (hertz) is the unit of frequencies.

1 Hz = 1 Cycle per second

1 kHz = 1 thousand Hz

1 MHz = 1 million Hz

Humans cannot sense the ultrasonic waves, but dogs and other animals has ability of hear the high frequency sounds. The wavelengths of ultrasonic waves are very small that's why they have very wide applications.

They are widely used in marine applications, medical diagnostics and non-destructive testing of new products.

Dolphins and Bats can generate ultrasonic waves and find their path.

3.2 Properties of Ultrasonic waves

Ultrasonic waves have very high frequency.

Ultrasonic waves have very low wavelength.

They have very high penetration power because of higher energy as compared to audible waves. Ultrasonic waves can penetrate through metals and other materials which are opaque to e.m. waves.

They have high energy content.

They can travel over long distances as a high directional ray and without loss of energy.

With increasing of frequency, the speed of propagation in medium of ultrasonic waves is increases.

In ultrasonic waves the diffraction effect is negligible because of the wavelength of ultrasonic is very small

Optical laws like reflection, refraction, diffraction etc. are observed with ultrasonic waves.

Ultrasonic waves produce 'cavitation' effect when made to pass through some solids.

The velocity of the ultrasonic waves depends on the temperature of the medium through which waves propagate.

3.3 Generation of Ultrasonic waves

There are two methods for generation of ultrasonic waves.

Mechanical generator (Galton whistle generator)

Electrical generator (Magnetostriction and piezoelectric generator).

3.4 Magnetostriction Oscillator (Pierce Oscillator)

3.4.1 Principle: Magnetostriction Effect

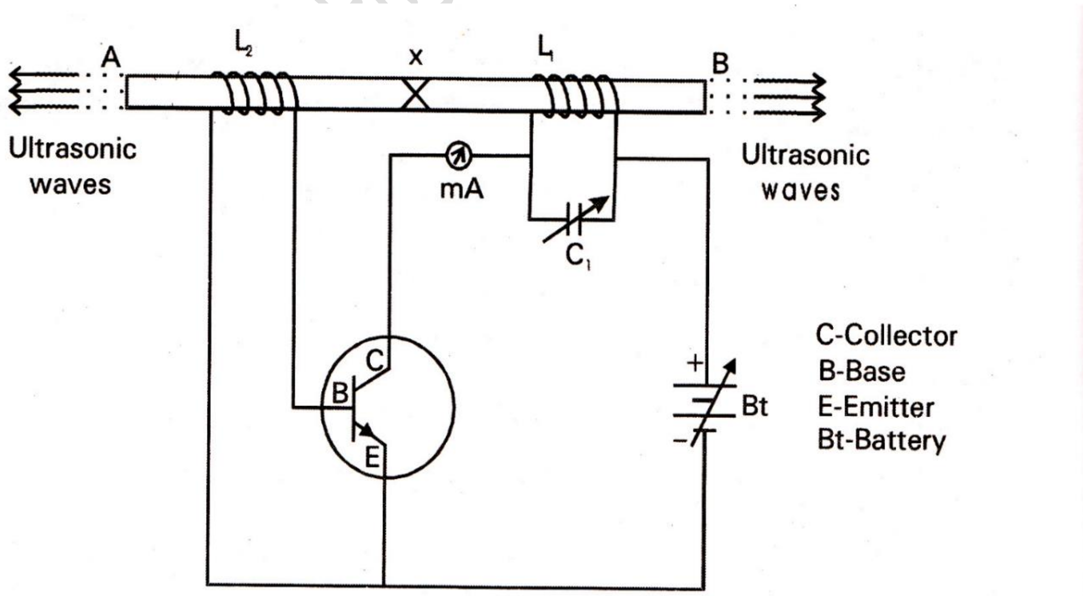
When we put a rod of ferromagnetic material like nickel or iron, in a magnetic field parallel to its length, there is change in length of rod and it depends on the magnitude of the field and nature of the ferromagnetic material. This phenomenon is called magnetostriction effect. This method was discovered by Joule in 1842. Nickel produces the very large magnetostriction effect in compare to other ferromagnetic materials.

If we put a rod in an alternating field of frequency f, in each half cycle rod changes its length. As a result, there is some setup of vibrations in the rod whose frequency is twice the magnetic field frequency. The amplitude of the vibration is small, but when the frequency of the field is set equal to the natural frequency of the rod, resonance occurs and amplitude of the vibrations will be larger. If the frequency of the field comes in ultrasonic range, an ultrasound of frequency 2f will be generated in the medium that is in contact with the ends of the rod.

The frequency of rod is given by

$$f = \frac{1}{2l}\sqrt{\frac{Y}{\rho}}$$ where L is length of the rod, Y is the Young's modulus, ρ the density if the rod.

3.4.2 Construction:



G. W. Pierce was the first to design an ultrasonic oscillator basing on the phenomenon of magnetostriction. Schematic diagram is shown in Figure.

A nickel rod is clamped from the center. Two coils L_1 and L_2 are wound at the ends of the rod. Coil L_1 is connected to variable capacitor and form tank circuit. One end of tank circuit is connected to collector of transistor via milliammeter and other end is connected to emitter of transistor via battery. Ends of L_2 are connected to base and emitter of transistor and form feedback loop.

3.4.3 Working:

When battery is switched on, due to tank circuit alternating current (AC) is generated with frequency

$$f = \frac{1}{2\,\pi\,\sqrt{L_1\,C_1}}$$

Due to this current rod gets magnetized and induce emf in coil L_2. This emf is given back to tank circuit to maintain oscillation.

Also due to this current rod vibrates with frequency

$$f = \frac{1}{2\,l}\sqrt{\frac{Y}{\rho}} \quad \text{where l is length of the rod, Y is the Young's modulus, } \rho \text{ the density if the rod.}$$

With the help of variable capacitor, we can vary the frequency of AC. When frequency of AC becomes equal to natural frequency of the rod i.e., resonance condition is achieved, rod vibrates vigorously and ultrasonic waves are generated. During this milliammeter shows maximum reading. The frequency of generated ultrasonic waves is

$$f = \frac{1}{2\,\pi\,\sqrt{L_1 C_1}} = \frac{1}{2\,l}\sqrt{\frac{Y}{\rho}}$$

3.4.4 Advantages:

The construction is very simple.

The cost of the generator is low.

At low ultrasonic frequencies, a large power output is possible without the any risk of damage of the oscillator circuit.

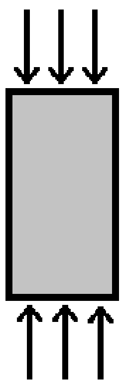
It can generate ultrasonic of frequency till 3 MHz.

3.4.5 Disadvantages:

It cannot generate ultrasonic of frequency above 3 MHz.

The frequency of oscillations depends on temperature. At higher temperatures, the o/p from the oscillator will not be very stable.

3.5 Piezoelectric Effect



When pressure is applied to a pair of opposite faces of crystals like quartz, tourmaline, Rochelle salt etc., cut with their faces perpendicular to its optic axis, equal and opposite charges appears on the other faces as shown in figure. This phenomenon is known as piezoelectric effect.

The sign of charges gets reversed if the crystal is subjected to tension instead of pressure.

3.6 Piezoelectric Oscillator

3.6.1 Principle: Inverse piezoelectric effect

When an alternating voltage is applied on a pair of opposite faces of piezoelectric crystal (quartz), it starts vibrating at the frequency of the applied voltage.

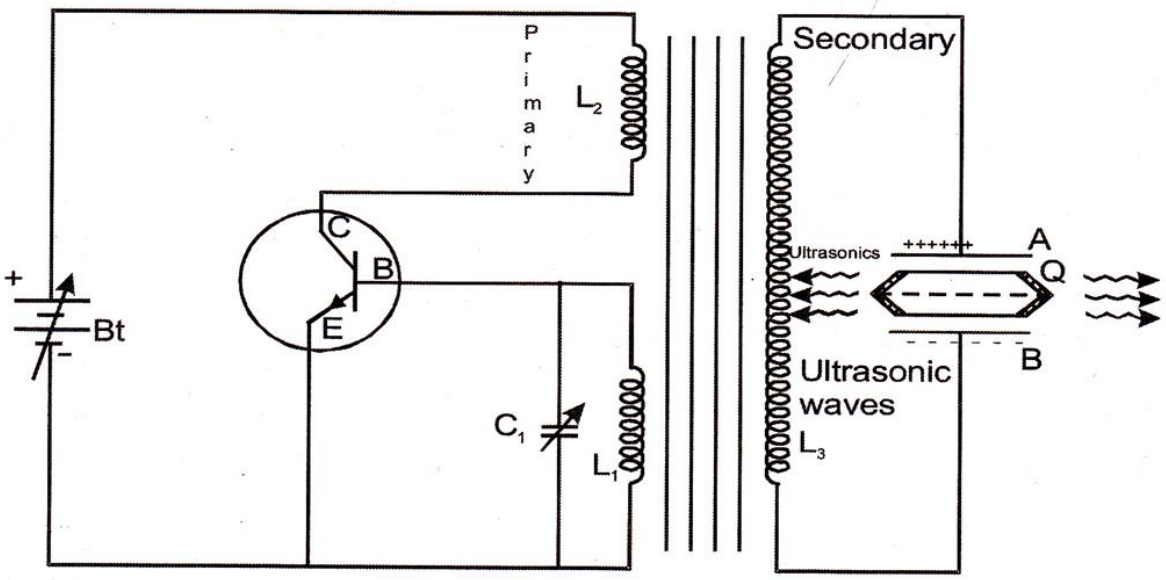
Vibrations occur at maximum amplitude at the natural resonant frequency of the piezoelectric crystal, which is determined by physical dimensions. The frequency of the vibrations is calculated by equation given below

$$f = \frac{n}{2l}\sqrt{\frac{Y}{\rho}}$$

where L is the length of crystal plate, Y is the Young's modulus along the appropriate direction, ρ is the density of the crystal and n (number of modes) = 1, 2, 3, …..

3.6.2 Construction:

Paul Langevin developed a method for producing ultrasonic waves using the piezoelectric effect in 1917. It was first observed by Paul-Jacques Curie and Pierre Curie in 1880



The X-cut quartz crystal is placed between two metallic plates A and B which are connected to the coil L_3. Coils L_1, L_2 and L_3 are inductively coupled. Coil L_1 is connected with variable capacitor C to form tank circuit. One end of tank circuit is connected to emitter of transistor and other end is connected to base of the transistor. One end of L_2 is connected to emitter of transistor via battery and other end to collector.

3.6.3 Working:

When battery is switched on, due to tank circuit alternating current (AC) or alternating voltage is generated with frequency

$$f = \frac{1}{2\pi\sqrt{L_1 C_1}}$$

Due to this AC, an alternating emf is induced in coil L_3 by transformer action. This emf is imparted on metallic plates. Due to this quartz starts vibrating with frequency,

$$f = \frac{n}{2t}\sqrt{\frac{Y}{\rho}}$$

where, n = 1, 2, 3.. etc. for fundamental mode (first mode), first overtone (second mode), etc. E = Young's modulus of the material, ρ = density of the material and t = thickness of the crystal plate.

With the help of variable capacitor, we can vary the frequency of AC. When frequency of AC becomes equal to natural frequency of the quartz i.e., resonance condition is achieved, quartz vibrates in one of the modes of vibration and ultrasonic waves are generated. The frequency of generated ultrasonic waves is

$$f = \frac{1}{2\,\pi\sqrt{L_1\,C_1}} = \frac{n}{2\,t}\sqrt{\frac{Y}{\rho}}$$

3.6.4 Advantages:

Through this oscillator we can generate ultrasonic waves up to 500 MHz.

It is more efficient then magnetostriction generator.

The ultrasonic generator output (o/p) is very high.

Output does not depend on environmental changes such as temperature and humidity.

3.4.6 Disadvantages:

The cost of the natural piezoelectric crystals is high.

The cutting and shaping of piezoelectric materials is very complex and involve tedious mechanical processes.

3.7 Detection of Ultrasonic waves

The presence of ultrasonic waves is detected in a room by the phenomena of stationary wave method using the Kundt's tube, or the sensitive flame method or the thermal detectors or the quartz method as was done by Langevin. As ultrasound is beyond audibility, the above indirect methods are used.

3.7.1 Kundt's tube method

The Kundt's tube method for the detection of ultrasonic waves is limited to low frequency waves. When ultrasonic waves go through the glass tube filled with light weight power (lycopodium powder) spread in the tube, the powder gets blown off at antinodes and forms heaps at nodes.

The distance between any two nodes is equal to half of the wavelength, i.e. $d=\lambda_{us}/2$. From this equation the wavelength of ultrasonic is estimated. This method, however, fails as the wavelength of ultrasonic is very small as equal to few mm.

3.7.2 Thermal detector method

This method is also used to detect the ultrasonic waves in which a fine platinum wire is moved through the medium. The temperature changes at the nodes and thereby there is a change in the resistance of the platinum wire, whereas the resistance remains constant at the antinodes as the temperature remains constant there. With the help of bridge circuit, the change in resistance of the platinum wire is measured.

The distance between any two nodes is equal to half of the wavelength, i.e. $d=\lambda_{us}/2$. From this equation the wavelength of ultrasonic is estimated.

3.7.3 Sensitive flame method

Ultrasonic waves are detected by moving a sensitive flame in the medium. The stationary positions of flame at antinodes and flicks at nodes will help in finding the exact location of nodes and antinodes and by knowing the mean distance between two consecutive nodes, the ultrasonic wavelength can be determined as usual. However, if the frequency of the ultrasonic wave is known, its velocity through the medium can also be estimated. The distance between any two nodes is equal to half of the wavelength, i.e. $d=\lambda_{us}/2$. From this equation the wavelength of ultrasonic is estimated.

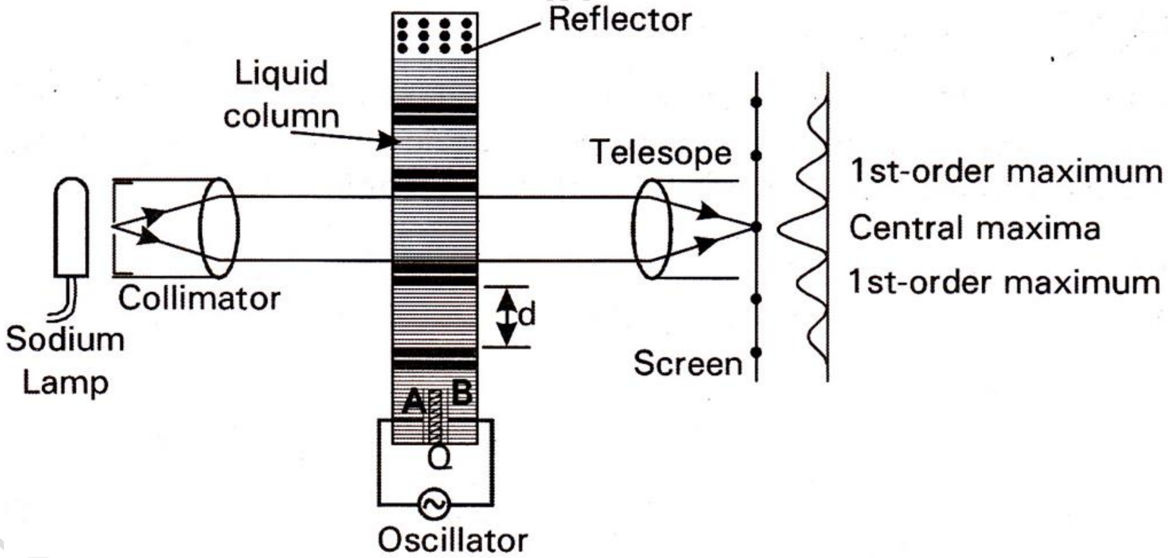
3.7.4 Quartz crystal method

This method of detecting ultrasonic waves depends upon the piezoelectric effect. When one pair of faces of a quartz crystal is exposed to ultrasonic waves, electric charges are developed on the other pair of opposite faces which are perpendicular to the previous one. These charges are amplified and detected by using electronic circuits.

3.8 Determination of Velocity of Ultrasonic waves (Acoustic Grating Method)

When we propagate ultrasonic waves in a transparent liquid medium, after reflection it will superimpose with incident wave and forms stationary waves. At nodes density of the medium becomes maximum and at antinodes density of the medium becomes minimum. Due to this there is periodic variation in density of liquid. It leads to a periodic variation of refractive index of the liquid. Such a liquid column subjected to ultrasonic waves constitutes an acoustical grating. If monochromatic light is passed through the liquid at right angles to the waves, the liquid causes the diffraction of light. Such liquid column subjected to ultrasonic waves is called acoustic grating.

Figure below shows experimental setup for determination of the velocity of ultrasonic waves. Ultrasonic waves are produced in a liquid contained in a glass tube.



The density and hence the refractive index of the liquid is maximum at nodal points and minimum at antinodal points. Therefore, the nodal areas act as opaque regions while antinodal areas act as transparent regions for light. The liquid column thus resembles a ruled grating.

When the crystal is at rest, a single image of the slit is formed in the screen. When the crystal is excited a diffraction pattern is produced. It consist of a central maxima followed is the grating element which is given by $d = \lambda_u / 2$.

$$\therefore \lambda_u = \frac{2\,n\,\lambda}{\sin\theta}$$

where λ is the wavelength of monochromatic light beam, n is the order of the maxima.

By using above equation λ_u can be determined. The frequency f of the waves is known from the frequency of the oscillator. The velocity of waves in the liquid can be found from the equation,

$$v = f\lambda_u$$

The above method of determining λ_u and v of ultrasonic waves is known as 'acoustic diffraction method'.

3.9 Applications of Ultrasonics

Ultrasonic is widely used in industry, medicine and marine applications. Here some of the applications is given below,

3.9.1 Echo sounder

Ultrasonic waves can be generated in the form of directed beam like light beam. Further, the ultrasonic waves can travel over long distances in water. As a result, ultrasound is widely used in marine applications. The depth of the ocean can be found by an echo sounder. The ship is equipped at its bottom with a source and a receiver of ultrasound of a specific frequency. The source sends out short pulses of ultrasonic waves and receiver receives reflected pulses. Measuring the time interval between the pulse sent and the pulse received, the depth of the ocean can be computed with the help of the formula

$$d = \frac{v\,t}{2}$$

where, d is the depth, t is the time interval and v is the velocity of ultrasonic wave.

3.9.2 SONAR

The word sonar means Sound Navigation and Ranging. The ultrasonic waves which are highly directional can be used for locating objects and determining their distance in the seas. Sonar can also measure depth of sea, find underwater obstacles like rocks and shipwrecks, locate shoals of fish and trace their movement, help divers and military applications such as finding submarines, mines, enemy ships etc.

3.9.3 Flow measurement

There are various ways to measure the flow velocity in a pipe by measuring its effect on the passage of ultrasonic waves transmitted and received at the outside of the pipe, without disturbing the flow by inserting anything into the pipe.

3.9.4 Material characterization

Ultrasonic velocity and reduction are dependent on material properties which in turn may be related to other characteristics that can be of special interest to the engineer (e.g. breaking strength of cast iron, grain structure of steel).

3.9.5 Detection of cracks or flaw in metals

It there is any hidden flaw or crack in metal, ultrasonic waves can be used to identify these defects by non-destructive testing.

3.9.6 Medical diagnostics

Medical diagnostic is another field of application of ultrasonic waves. This field, on the one hand, mainly consist of examining and anatomical structure of soft tissue or its functioning, while on the other hand it is used for checking the normality of blood flow in various vessels. All these examinations are noninvasive.

3.10 Non Destructive Testing (NDT)

By Non Destructive Testing(NDT), one can detect all microscopic flaws(defects).

Definition: NDT is the method of testing material without causing any damage to it or decreasing their service life.

3.10.1 Difference between DT and NDT

Sr. No.	Destructive Testing	Non Destructive Testing
1.	Measurements are direct and reliable.	Measurements are indirect and reliability has to be verified.
2.	Tests are not made on the objects directly.	Tests are made on objects directly.
3.	Time consumption is generally high.	Time consumption is generally low.
4.	Usually quantitative measurements.	Usually qualitative measurements. But, quantitative measurements can also be done.
5.	In service testing is not possible.	In service testing is possible.
6.	More Costly	Less Costly
7.	It is carried to find properties and behaviour of specimen under different load	It is used to find properties of material and to find out defects.
8.	Specimen is damaged during test	Specimen is not damaged during test
9.	Defects cannot be found using Destructive testing	Defects are found using NDT
10.	Bending Test, Tensile Testing, Compression Testing, Impact testing etc.	Ultrasonic testing, Liquid die penetrant method, eddy current testing.

3.10.2 Objective of NDT

To increase serviceability

To improve production

To increase safety

One can detect defects in crystalline structure

One can test materials properties

Measure a size of cracks & holes of material

To find the composition & their percentage in any material

One can detect Internal stresses & strain with the help of NDT

3.10.3 Methods of NDT

Liquid (Dye) Penetrant Method

Radiography Method

X-ray radiography (OR)

gamma ray radiography

Ultrasonic Inspection Method

Eddy Current Method

Magnetic Particle Method

Thermography

3.10.4 Ultrasonic Inspection Method

It is a form of non-destructive testing used in many industries including aerospace, automotive and other transportation sectors.

Ultrasonic testing is often performed on steel and other metals and alloys, and it can also be used on concrete, wood composites and also rail road, water or oil pipes, aircraft parts.

3.10.4.1 Principle

Ultrasonic is used because of its property to reflect whenever medium is changed.

3.10.4.2 Basic equipment's used in ultrasonic methods:

An electronic signal generator to generate alternating voltage (electrical signal).

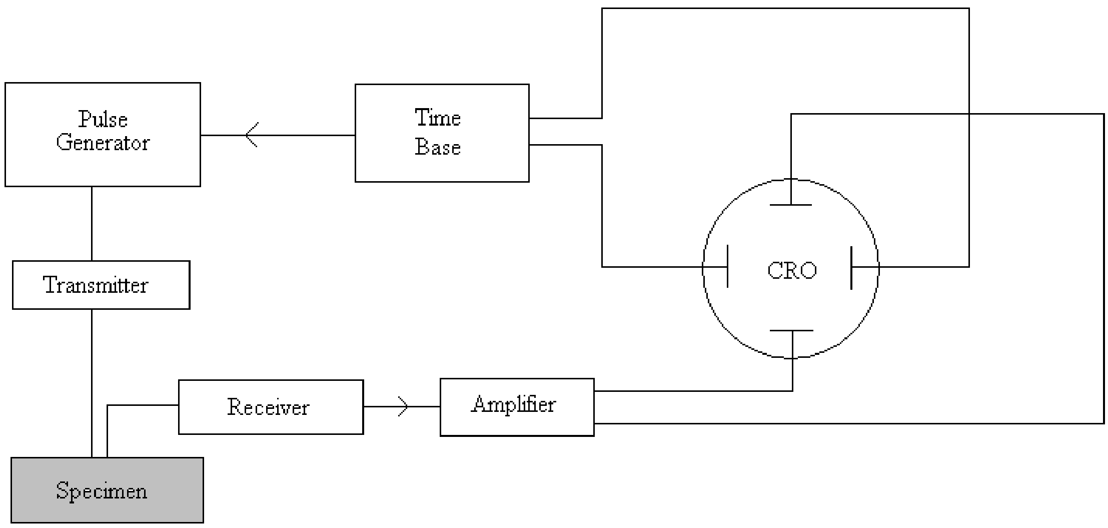
A transmitter (sending transducer) that emits a beam of ultrasonic waves when alternating voltage is applied.

A receiver (receiving transducer) that accepts the ultrasonic waves from the specimen and convert it into electrical signal.

An amplifier to amplify the signal which is received by the receiver.

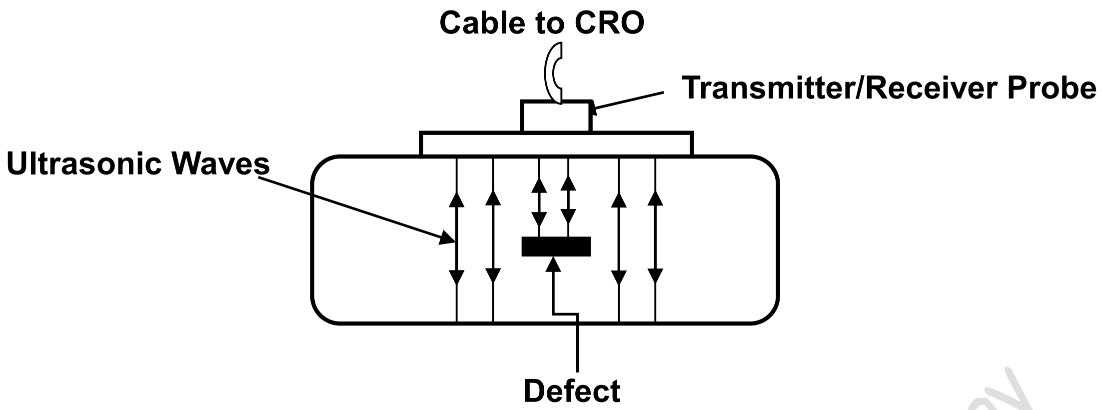
An electronic clock or timer to measure the minute(small) time interval between the sending and receiving of the signal.

3.10.4.3 Block diagram of ultrasonic pulse echo system is shown below.



3.10.4.4 Pulse Echo System:

Only one transducer is used as both transmitter and receiver.



3.10.4.4.1 Working:

In ultrasonic testing, an ultrasound transducer connected to a CRO.

For better contact between the transducer and specimen thin oil and glycerin is used.

Ultrasonic waves can be produced by Piezoelectric effect.

In this method reflection (Pulse-echo) mode is used.

In reflection mode the transducer performs both sending & receiving of the pulse.

When ultrasonic waves are transmitted they will reflect from the surface of the material so we will get transmitted pulse as shown in figure.

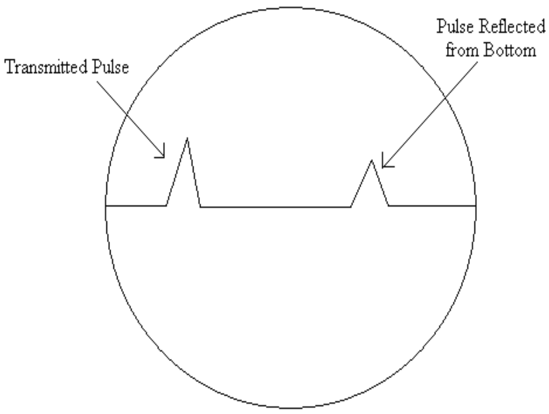


Figure. 1

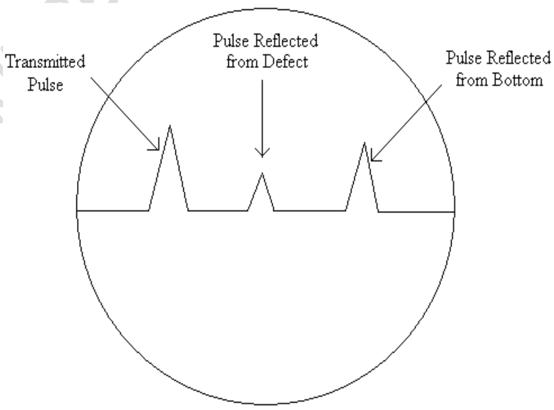


Figure. 2

When no defect is present then we will get transmitted pulse and pulse reflected from bottom as shown in figure 1.

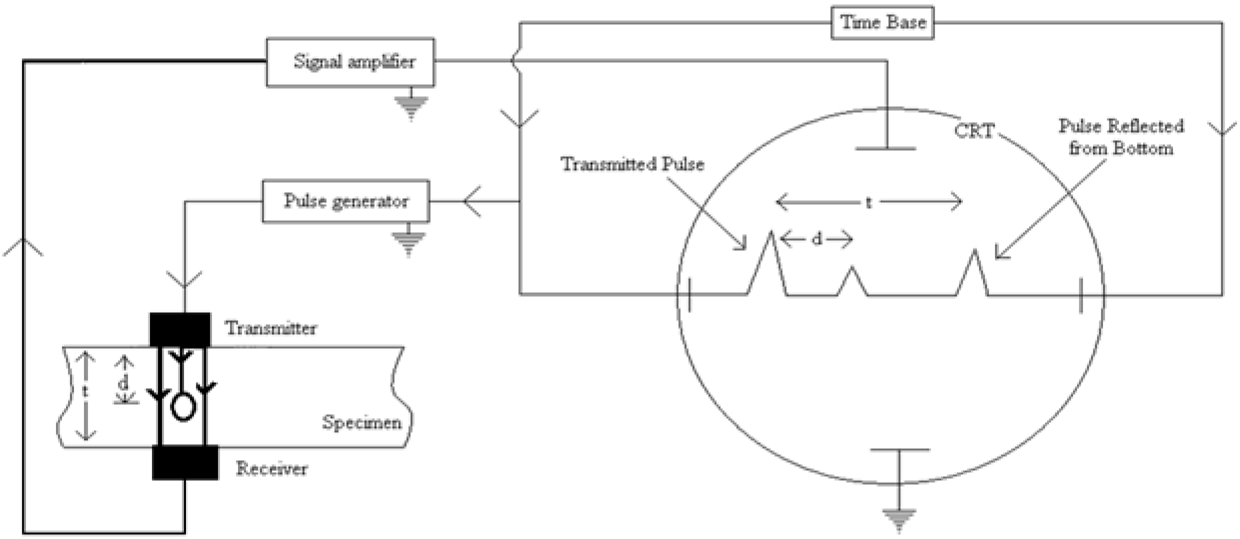
And if there is any defect we will get pulse reflected from the defect along with the other two as shown in figure 2.

If the velocity v of the ultrasonic waves in the specimen is known, the position of the crack can be located by $d=vt/2$.

The exact size and shape of the cavity or crack can be found out by examining the specimen from all directions.

3.10.4.5 Ultrasonic Flaw Detector (Through Transmission System):

Two transducers are used separately as transmitter and receiver.



3.10.4.5.1 Working:

In ultrasonic testing, an ultrasound transducer connected to a CRO.

For better contact between the transducer and specimen thin oil and glycerin is used.

Ultrasonic waves can be produced by Piezoelectric effect.

In this method attenuation (through transmission) mode is used.

In attenuation mode a transmitter sends ultrasound through one surface & separate receiver detect the pulse.

When ultrasonic waves are transmitted, they will reflect from the surface of the material so we will get transmitted pulse as shown in figure.

When no defect is present then we will get transmitted pulse and pulse reflected from bottom.

And if there is any defect we will get pulse reflected from the defect along with the other two as shown in figure.

If the velocity v of the ultrasonic waves in the specimen is known, the position of the crack can be located by d=vt/2.

The exact size and shape of the cavity or crack can be found out by examining the specimen from all directions.

3.10.4.6 Advantages

High penetrating power so we can detect small size defects and deeply seated defects.

It is very cheap and it has high speed of operation.

Greater accuracy.

Capability of estimating the size, orientation & shape of the defects

Not harmful.

Large size specimen can be inspected in a very short duration.

3.10.4.7 Disadvantages

No permanent record of the defect can be obtained.

Manual operation requires careful attention.

Extensive technical knowledge required so only skilled and well trained technicians are required.

Inspected items must be water resistant.

Discontinuities that are present immediately beneath (below) the surface may not be detectable.

3.11 Other applications

1. Science and engineering

Used to detect flaws or defects in metals.

Used to detect ships, submarines, iceberg etc., in the ocean.

Used for soldering aluminium coil capacitors, aluminium wires and plates without using any fluxes.

Used to weld some metals which cannot be welded by electric or gas welding.

Used for cutting and drilling holes in metals.

Used as catalyst to accelerate the reaction.

2. Medicine

Used to remove kidney stones and brain tumours without shedding any blood.

Used to remove broken teeth.

Used for sterilizing milk and for killing bacteria.

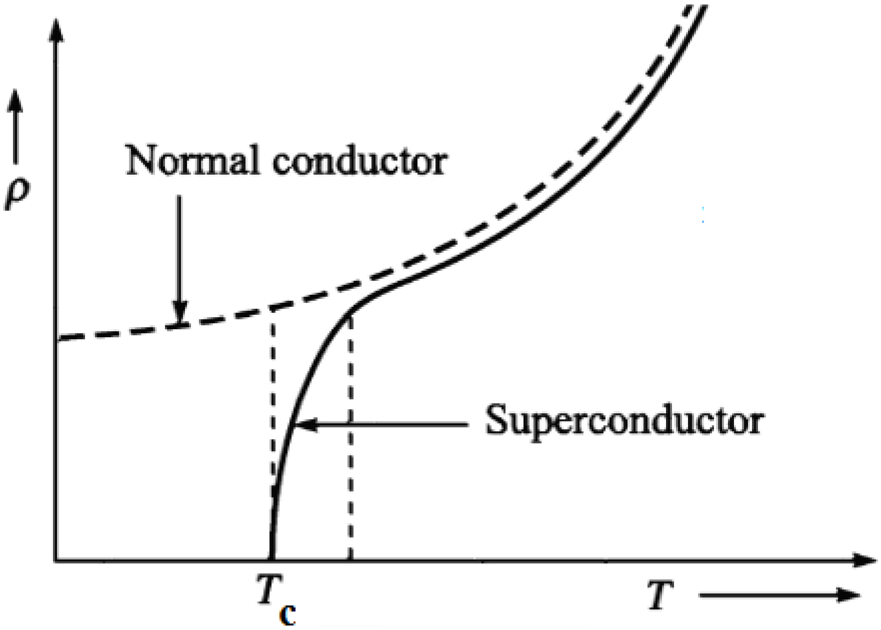
Used to study the blood flow velocities in blood vessels of our body.

Used as a diagnostic tool to detect tumours, and also the growth of foetus.

4.1 Introduction:

Helium gas - was liquefied at 4.2 K by Dutch Physicist Heike Kamerlingh Onnes in 1908. Superconductivity was discovered by K. Onnes in 1911. Liquid Helium has a temperature of 4.2 K. It was observed that the resistance of mercury dropped from 0.08 Ω at about 4.3 K to less than 3 * 10^{-6} Ω at 4.2 K. A large number and wide variety of metals, alloys, binary and ternary chemical compounds have been found to show superconductivity at various temperatures.

Superconductivity is a state in which quantum mechanics operates on a macroscopic scale of the order of many atomic distances rather than the usual atomic and subatomic scale. The superconducting state is influenced by the temperature, magnetic field and current. All these three parameters have critical values, above which material enters into normal state. Every superconductor has its own transition temperature.



Good electrical conductors such as silver, gold, and copper are not good superconductors because the resistivity of these conductors at low temperatures is limited to low resistivity i.e. residual resistivity, value due to scattering of electrons from crystal defects and impurities. Similarly, good superconducting materials like zinc and lead are not good electrical conductors.

4.2 Definitions

4.2.1 Superconductor:

It is a material that loses all its resistance (offers zero resistance) to the flow of electric current when it is cooled below a certain temperature called the critical temperature or transition temperature T_c.

Eg: Mercury, Zinc, Niobium, etc

4.2.2 Critical temperature:

The temperature at which a material's electrical resistivity drops to absolute zero is called the critical temperature or transition temperature. It is denoted by T_c.

4.2.3 Superconductivity:

It is a phenomenon in which certain metals, alloys and ceramics conduct electricity without resistance when it is cooled below a certain temperature called the critical temperature.

4.3 Properties of Superconductors

4.3.1 Electrical Resistance:

The electrical resistance of a superconducting material is very low and is of the order of $10^{-7}\ \Omega$.

4.3.2 Effect of impurities:

When impurities are added to superconducting elements, the superconducting property is not lost but the T_c value is lowered

4.3.3 Isotope Effect:

The critical temperature of a superconductor is found to vary with its isotopic mass. The atomic mass of Hg varies from 199.5 to 203.4. Due to this variation in atomic mass, the transition temperature of isotopes of Hg varies from 4.185 to 4.146 K. They are related as $T_c \propto \frac{1}{\sqrt{M}}$ where M is the isotopic mass.

4.3.4 Magnetic field effect:

If a sufficiently strong magnetic field is applied to a superconductor at any temperature below critical temperature T_c, the superconductor is found to undergo a transition from the superconducting state to the normal state. OR This minimum magnetic field required to destroy the superconducting state is called critical magnetic field H_c. $H_c = H_0\left[1-\left(\frac{T}{T_c}\right)^2\right]$ where H_0 is the applied magnetic field at 0 K.

4.3.5 Effect of pressure and stress:

Certain materials are found to exhibit the superconductivity phenomena on increasing the pressure over them. For eg. Cs shows superconductivity at T_c = 1.5 K and 110 kbar

Also, in Superconductors, an increase in stress results in increase of the T_c value.

4.3.6 Critical current density:

It is defined as the maximum current that can be permitted in a superconducting material without destroying its superconductivity state. OR Minimum current required to destroy the superconducting state is called critical current density J_c and this current is called critical current I_c. The equation relating

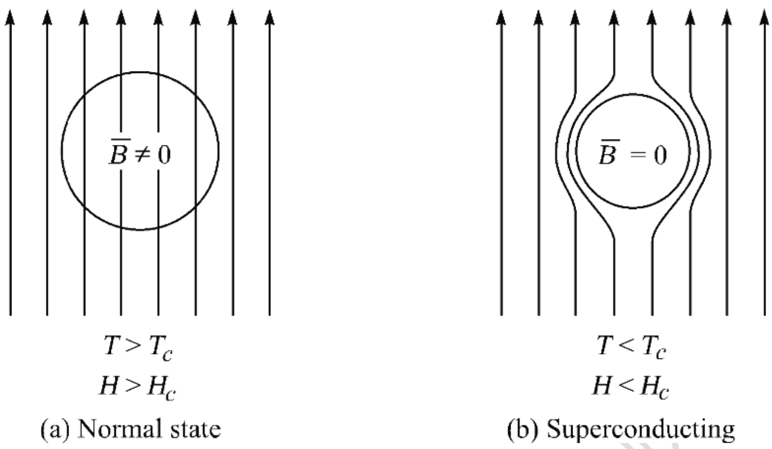
$J_c = \frac{I_c}{A}$ and $I_c = 2\pi r H_c$.

4.3.7 Persistent current:

If current is made to flow through a superconducting ring then it is observed that the current flows through the material without any significant loss. This steady flow of current in a superconducting ring without any potential deriving is called the persistent current.

4.3.8 Meissner effect:

The complete expulsion of all the magnetic field lines by a superconductor material is called Meissner effect. The Meissner effect is a distinct characteristic of a superconductor from a normal perfect conductor. In addition, this effect is exhibited by the superconducting materials only when the applied field is less than the critical field H_c.



To Prove that $\chi_m = -1$ for superconductors

We know that for a magnetic material the magnetic induction or magnetic flux density B is given by the equation $B = \mu_o(M + H)$ where μ_o is the permeability of free space; M is the intensity of magnetization; H is the applied magnetic field.

But for the superconductors, we know that B = 0, thus the above equation can be written as

i.e. $0 = \mu_o(M + H)$

i.e. $0 = (M + H)$ since $\mu_0 \neq 0$

i.e. M = - H

OR $\frac{M}{H} = -1 = \chi_m$

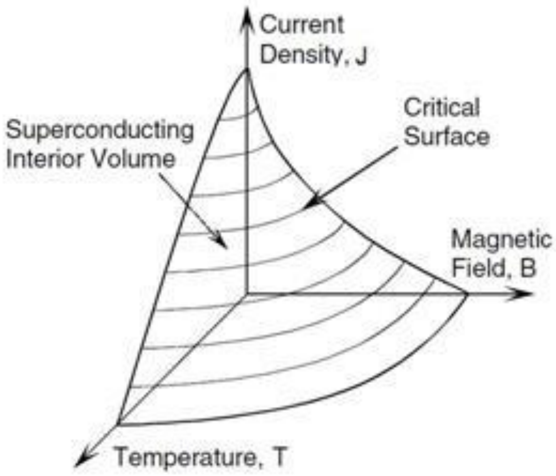
where χ_m is called as the magnetic susceptibility. This means that for a superconductor, the susceptibility is negative and maximum, i.e., a superconductor exhibits perfect diamagnetism. For all other magnetic materials, the susceptibility values are positive.

4.4 Factors to define a Superconducting State

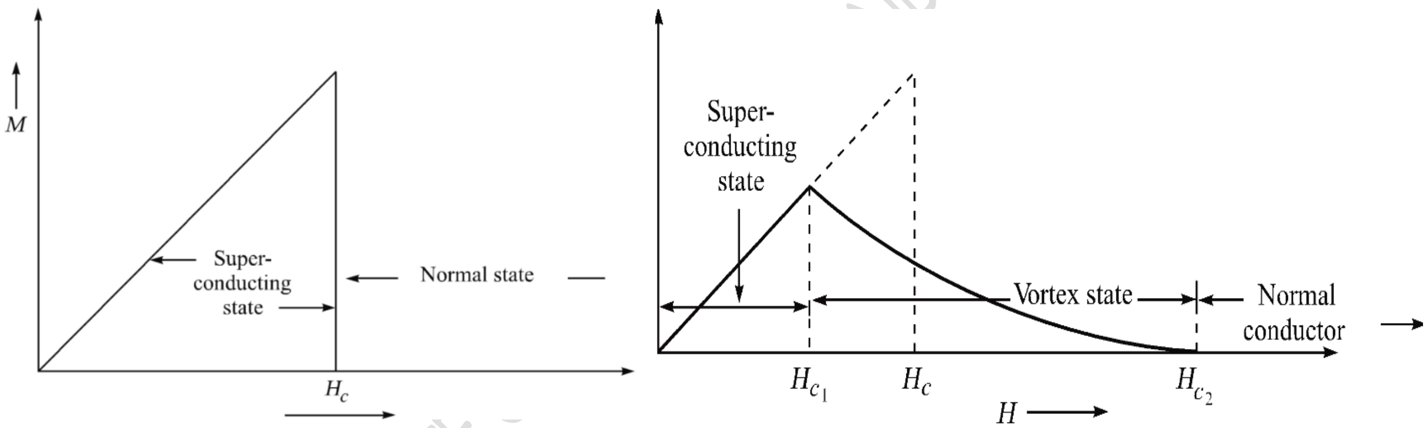
Critical temperature

Critical current density

Critical magnetic field



Each of the above three parameters is very dependent on the other two properties. The highest values for H_c and J_c occur at 0 K; while the highest value for T_c occurs when H and J are zero. Thus the plot of all these three parameters represents a critical surface. Within the surface, the material is superconducting and outside the surface, the material is said to be in the normal state.



Graph of –M (Magnetization) vs H (Applied Magnetic Field) for Type I and II superconductors

Sr. No.	Type I superconductor	Type II superconductor
1	These superconductors are called as soft superconductors.	These superconductors are called as hard superconductors.
2	Only one critical field exists for these superconductors.	Two critical fields H_{c1} (lower critical field) and H_{c2} (upper critical field) exists for these superconductors.
3	The critical field value is very low.	The critical field value is very high.
4	These superconductors exhibit perfect and complete Meissner effect.	These do not exhibit a perfect and complete Meissner effect.
5	These materials have limited technical applications because of very low field strength value.	These materials have wider technological applications because of very high field strength value.
	Examples: Pb, Hg, Zn, etc.	Examples: Nb_3Ge, Nb_3Si, $Y_1Ba_2Cu_3O_7$, etc.

4.5 Applications

4.5.1 Magnetic Levitation

Magnetic levitation or maglev is the process by which an object is suspended above another object with no other support but magnetic fields. We know that a diamagnetic substance repels a magnetic field. Thus, the perfect diamagnetic property of superconductors make them suitable for achieving frictionless motion in motors and bearing. The phenomena of magnetic levitation is based on Meissner effect.

4.5.1.1 How to achieve magnetic levitation?

The magnetic levitation is brought about by enormous repulsion between two highly powerful magnetic fields. If a small magnet is brought near a superconductor, it will be repelled. This repulsion takes place due to the induced currents in the superconductor which is being generated by the magnetic field of the magnet. Because of zero resistance property of the superconductor this current persists, and thus the field due to this induced current repels the field due to the magnet. As a result, the magnet floats freely above the superconductor. Maglev demonstrates zero resistance and Meissner effect properties for superconductors.

Magnetically levitated vehicles are called maglev vehicles. The utility of such levitation in vehicles is that in the absence of contact between the moving and stationary systems, the friction is eliminated. With such an arrangement, great speeds could be achieved with very low energy consumption. This levitation is based on:

Electromagnetic suspension (EMS) - In attractive EMS, the electromagnets installed on the train bogies attract the iron rails. The vehicle magnets wrap around the iron guide-ways and the attractive upward force lifts the train.

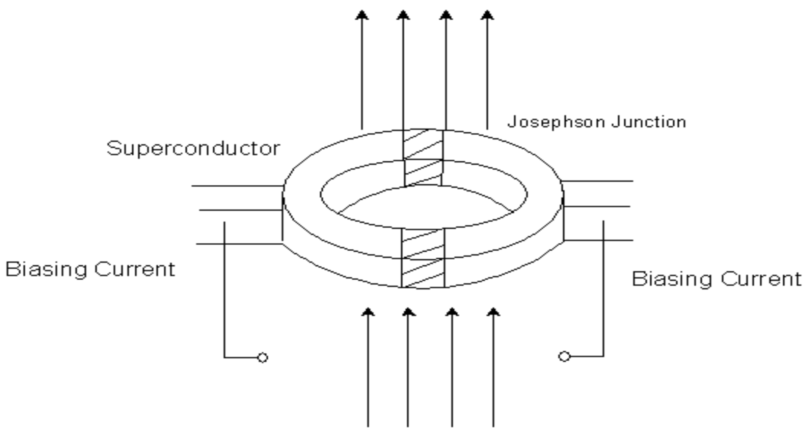
Electrodynamic suspension (EDS) - In EDS, levitation is achieved by creating a repulsive force between the train and guide-ways.

The basic idea of maglev train is to levitate it with magnetic fields so that there is no physical contact between the train and the rails. Consequently the maglev train can travel at a very high speed. Maglev trains travel at speed of about 500 kmph. A similar magnetic propulsion system is being used to launch the satellite into orbits directly from the earth without the use of rockets.

4.5.2 Josephson Effect

Two superconductors separated by a very thin strip of an insulator forms a Josephson junction.

The wave nature of moving particles makes the electrons to tunnel through the barrier (insulator) i.e. the electrons can tunnel from one superconductor to the other forming a current through the junction. As a consequence of the tunneling of electrons across the insulator, there is a net current across the junction. This is called as dc Josephson Effect. This current flows even in the absence of a potential difference.



On the other hand, when a potential difference is applied between the two sides of the junction, there will be an oscillation of the tunneling current with angular frequency f = 2eV/h. This is called ac Josephson Effect.

4.5.2.1 Application of Josephson junction

Josephson junctions are used in sensitive magnetometers called SQUID – Superconducting Quantum Interference Device. A SQUID is formed by connecting two Josephson junctions in parallel. When current is passed into this arrangement, it splits flowing across the two opposite arc. The current through the circuit will have a periodicity which is very sensitive to the magnetic flux passing normally through the closed circuit. As a result, extremely small magnetic flux can be detected with this device. This device can also be used to detect voltages as small as 10^{-15} V. Magnetic field changes as small as 10^{-21} T can be detected. Weak magnetic fields produced by biological currents such as those in the brain can also be detected using SQUIDs. SQUID detectors are used to measure the levels of iron in liver – so that iron built up can be treated before much harm is done to the body.

4.6 Other Applications

To transmit electrical power over very large distances without any power loss or voltage drop.

Superconducting generators has the benefit of small size and low energy consumption than the conventional generators.

Used in NMR for scanning the whole body to diagnose medical problems.

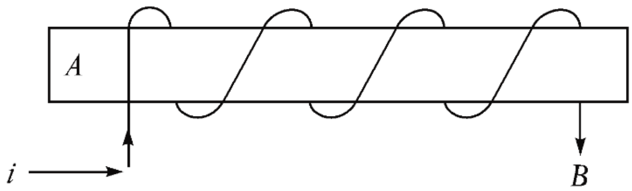
To generate very strong magnetic fields.

Superconductors can act as relay or switching system in a computer and also as a memory or storage element.

Very fast and accurate computer can be generated that consumes low power.

Ore separation can be done efficiently.

Cryotron: A small size relay or switch made of superconductors. They consume very less current. Let the critical field of the material A be H_{cA} and that of B be H_{cB} and also $H_{cA} < H_{cB}$. If a current I is passed through the material B, the current induces a magnetic field H. If this induced field H happens to be greater than H_{cA} then the superconducting property of the material A gets destroyed. Hence the resistivity increases and the contact is broken. Thus, the current in A can be controlled by the current in B and hence this system can act as a relay or switch element.



4.7 London Penetration Depth

Maxwell's equations could not explain the zero resistance and perfect diamagnetism observed as main characteristics of superconductivity.

F. London and H. London (London brothers) suggested that motion of superconducting electrons in the presence of applied electric field E.

If a magnetic field is applied to a superconductor which is initially in zero field, the magnetic field is a function of time.

According to the Maxwell equation,

$$\nabla \times E = -\frac{1}{c}\frac{\partial B}{\partial t}$$

The time-varying magnetic field gives rise to an electric field.

In a normal metal this will induce eddy currents, but in a superconductor the electric field will give rise to persistent currents (i.e. supercurrents).

The induced supercurrents will in turn generate a magnetic field of their own which opposes the applied magnetic field.

If the applied magnetic field is weak, the flux is totally screened from the bulk of the superconductor.

This phenomenon is often described as ``perfect diamagnetism''.

From Newton's law, the equation of motion for a superconducting carrier with mass m and charge -e in the presence of an electric field E is

$$F = m\frac{dvs}{dt} = -eE$$

where v_s is the velocity of the superconducting carrier. The field-induced supercurrent density is given by

$$Js = -\,ns\;e\;vs$$

where n_s is the local density of the superconducting carriers.

Thus we get

$$vs = -\,\frac{Js}{ns\;e}$$

Thus the substitution gives

$$\frac{dJs}{dt} = \frac{nse2}{m}E$$

which is called as the first London equation.

Taking curl on both sides, we get

$$\frac{m}{nse2}\left(\nabla \times \frac{dJs}{dt}\right) = \nabla \times E$$

which can be rewritten using the Maxwell's equation as

$$\frac{mc}{nse2}\left(\nabla \times \frac{dJs}{dt}\right) + \frac{dB}{dt} = 0$$

In order to obtain the Meissner effect, they removed the time derivative. Thus the equation becomes

$$\frac{mc}{nse2}(\nabla \times Js) + B = 0$$

which is called as second London equation.

The supercurrent density and magnetic field B are related as per Maxwell's equation

$$Js = \frac{c}{4\pi}(\nabla \times B)$$

This substitution gives

$$\frac{mc}{nse2} \times \frac{c}{4\pi}\big(\nabla \times (\nabla \times B)\big) + B = 0$$

which gives

$$\lambda_L^2 \, (\nabla \times \nabla \times B) + B = 0$$

Where

$$\frac{1}{\lambda_L^2} = \frac{4\pi n_s e^2}{mc^2}$$

λ_L is called the London penetration depth. It is a measure of magnetic field penetration inside the superconductor.

OR

$$\nabla \times J = \frac{ne^2}{m} B$$

These equations are called first and second London equations which explains Meissner effect or Perfect diamagnetism.

Their equation also predicts the penetration of super current and magnetic flux in a superconductor.

The penetration depth is the depth where the current drops to 1/e times of its value at the surface.

5.1 Introduction

LASER is an acronym for Light Amplification by Stimulated Emission of Radiation. In laser, the intensity of light is amplified by the process called stimulated emission. Lasers are optical phenomena which find major applications in various fields such as medicine, engineering, fiber optic communication, industries etc. Light emitted by Laser is more powerful than ordinary light source.

5.2 Characteristics (Properties) of LASER

Laser light is highly powerful, is capable of propagating over long distances and is not easily absorbed by water.

The following characteristics distinguish a laser beam from an ordinary light.

5.1 Coherence

Waves having same phase, direction and frequency are called coherent waves. This property is called coherence. Waves emitted by laser have same phase, direction and frequency (same energy and momentum), hence laser is highly coherent source.

5.2 High directionality

Due to coherence property of laser, waves emitted by it will travel in same direction. Hence laser beam is highly directional beam. An ordinary light spreads in all directions, but since laser travels as a parallel beam, it can travel over a long distance without spreading. Spreading of a laser beam is 1mm/meter.

5.3 High intensity

Due to the coherence property (or high directionality) of laser, all energy emitted by laser is focused over a small area of 10^{-6} cm^2. Hence intensity of laser is high.

5.4 High monochromaticity

Monochromatic means single colour. Laser emits light of single colour (wavelength) only. So it is highly monochromatic. For normal monochromatic source, spreading of wavelength is of the order 100 nm. While for laser, spreading of wavelength is of the order of 1 nm. (Spreading of wavelength = maximum wavelength emitted - minimum wavelength emitted by the source)

5.5 LASER Speckles

Diffused reflections takes place when laser beam is reflected from a rough surface.

When these reflected wave interfere they produce speckles.

Speckle pattern can change by changing direction of beam.

Speckle pattern can severely degrade the quality of image of laser display.

Speckle can be avoided by using a light source with large optical bandwidth or short coherence length.

5.3 Einstein's theory

Einstein explained the action of laser beam based on quantum theory of light. Production of laser light is a particular effect due to interaction of radiation with matter. Radiation interacts with matter under appropriate conditions and may lead to the transition of an atom or a molecule from one energy state to another.

If the transition is from a higher state to a lower state, the system gives part of its energy. But, if the transition is from lower to higher state, then it absorbs the incident energy.

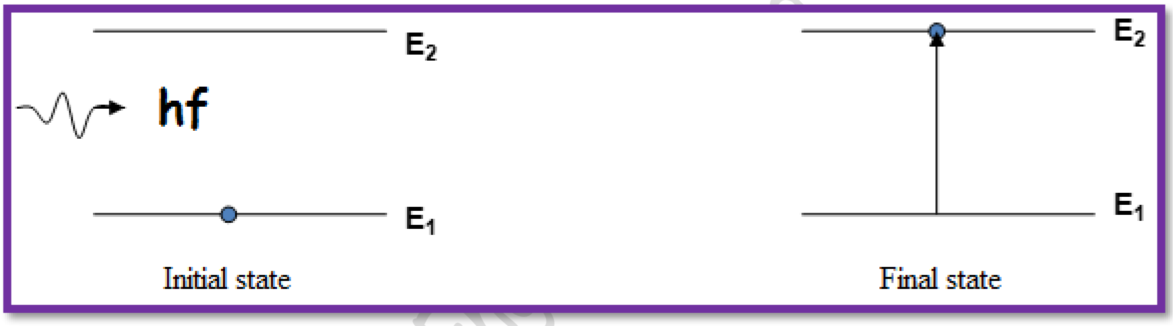
There are three possible ways through which interaction of radiation and matter can take place. Among the three types, one is absorption also known as induced absorption and the other two are emissions. The emission of radiation can occur in two ways as suggested by Einstein. They are spontaneous emission and stimulated emission.

The interpretation of the interaction is done on the basis of ideas related to energy levels of the concerned system for which light is to be obtained. All the three processes are described by considering an atom having only two energy levels E_1 and E_2.

5.3.1 Induced absorption

An atom has infinite number of quantized energy states. Initially an atom is in the ground state, i.e. all of its electrons possess the lowest possible energy states. If an atom transits from ground state (lowest energy level) to higher energy level by absorbing energy of incident photons, then the process is called absorption (induced absorption or stimulated absorption).

For absorption: Atom + Photon → Atom*.

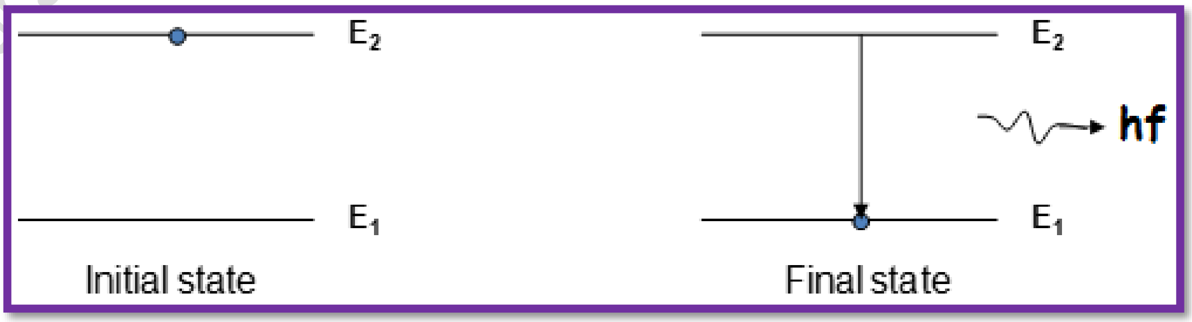


If E_1 and E_2 are the energies of an electron in the initial and final states respectively and f the frequency of absorbed radiation, then $E_2 - E_1 = hf$ or $f = (E_2 - E_1)/h$, where h is Planck's constant. The rate of absorption depends on the properties of states 1 and 2 and is proportional to the photon density (ρ) and to the number of atoms per unit volume N_1 in the ground state. Thus $R_{12} \propto N_1\rho$ OR $R_{12} = B_{12}N_1\rho$, where B_{12} is called the (probability of absorption per unit time) Einstein's coefficient for absorption of radiation.

5.3.2 Spontaneous emission (Natural emission)

The process of emission of photons when an atom transits from higher energy level to lower energy level without use of external agency (source) is called spontaneous emission.

For spontaneous emission: Atom* → Atom + Photon.



If E_1 and E_2 are the energies of an electron in the initial and final states respectively and f the frequency of emitted photons, then $E_2 - E_1 = hf$ or $f = (E_2 - E_1)/h$, where h is Planck's constant. The rate of spontaneous emission is proportional to the number of atoms per unit volume in the excited state, i.e. $R_{21(sp)} \propto N_2$ or $R_{21(sp)} = A_{21}N_2$, where A_{21} is called the (probability of spontaneous emission per unit time) Einstein's coefficient of spontaneous emission of radiation.

5.3.3 Stimulated emission (Forced emission)

The process of emission of photons when an atom transits from higher energy level to lower energy level, with use of external agency (source) is called stimulated emission. (When the atom is in the excited state, then an incident photon of correct energy may cause the atom to transit to lower energy state, emitting an additional photon of the same frequency. Thus, now two photons of the same frequency are present.)

For stimulated emission: Atom* + Photon → Atom + Photon + Photon

If E_1 and E_2 are the energies of an electron in the initial and final states respectively and f the frequency of emitted photons, then $E_2 - E_1 = hf$ or $f = (E_2 - E_1)/h$, where h is Planck's constant. The rate of stimulated emission depends both on the external radiation and on the number of atoms per unit volume in the upper level E_2, i.e. $R_{21(st)} \propto N_2\rho$ or $R_{21(st)} = B_{21}N_2\rho$, where B_{21} is called the (probability of stimulated emission per unit time) Einstein's coefficient of stimulated emission of radiation.

5.3.4 Difference between Spontaneous and Stimulated Emission

Sr. No.	Spontaneous Emission	Stimulated Emission
1.	Definition	Definition
2.	$R_{21(sp)} = A_{21}N_2$	$R_{21(st)} = B_{21}N_2\rho$
3.	It is natural emission	It is forced emission
4.	It is a random process	It is not a random process
5.	Emission takes place in all directions	Emission takes place in specific direction
6.	Intensity is low	Intensity is high
7.	Polychromatic light is emitted	Monochromatic light is emitted
8.	Photons will not be multiplied by chain reaction	Photons will be multiplied by chain reaction

5.4 Basics Definitions:

5.4.1 Population inversion

Population inversion is a state of achieving more number of atoms in the excited state compared to that in ground state. i.e. $N_2 > N_1$.

If this condition is satisfied, then there is more chance for stimulated emission to take place.

Population inversion is an essential condition for producing laser.

Population inversion can be achieved by a process called pumping.

5.4.2 Pumping

Pumping is the process of exciting atoms from the lower energy state to a higher energy state by supplying energy from an external source.

The most commonly used pumping processes are:

5.4.3 Optical pumping:

In this type of pumping atoms are excited by means of an external optical (light) source. This type of pumping is used in solid state lasers such as ruby laser and Nd: YAG laser.

5.4.4 Electrical pumping (Direct electron excitation):

In this type of pumping atoms are excited by means of high external electric field. The electrons are accelerated to a high velocity by a strong electric field with help of discharge tube. These moving electrons collide with the natural gas atoms and ionize the medium. Thus, due to ionization they transit to a higher energy level. This type of pumping is used in gas laser such as CO_2 laser.

5.4.5 Direct conversion:

In this type of pumping the direct conversion of electric energy into light takes place. This type of pumping is used in semiconductor laser.

Pumping by collision and chemical methods:

In addition to the above three, the other types of pumping are, inelastic collision between atoms and chemical methods which are respectively adopted in He-Ne gas laser and in dye and chemical lasers.

5.4.6 Lasing

The process which leads to emission of stimulated photons after achieving population inversion is called lasing. (OR the process due to which help to get stimulated photons after achieving population inversion is called lasing.)

5.4.7 Active medium

The medium in which population inversion is achieved for laser action is called active medium.

5.4.8 Life time

The time interval for which an atom can stay in higher energy level is called lifetime (10^{-9} s).

5.4.9 Metastable state

The higher energy level in which atom can stay for longer time (10^{-3} to 10^{-2} s) are called metastable states. This property helps in achieving the population inversion.

5.4.10 Optical resonator (Resonator cavity)

It is a pair of reflecting mirrors (surfaces) of which one is a perfect reflector and other is a partial reflector. It is used for amplification of photons.

5.5 Principle of Laser

Laser is based on principle of stimulated emission of radiation with light amplification.

For stimulated emission of radiation, more number of atoms are required in higher energy level then lower energy level. This is done with the help of pumping.

Light amplification is obtained by photon multiplication within an optical resonator.

5.6 Ruby LASER (Solid State LASER)

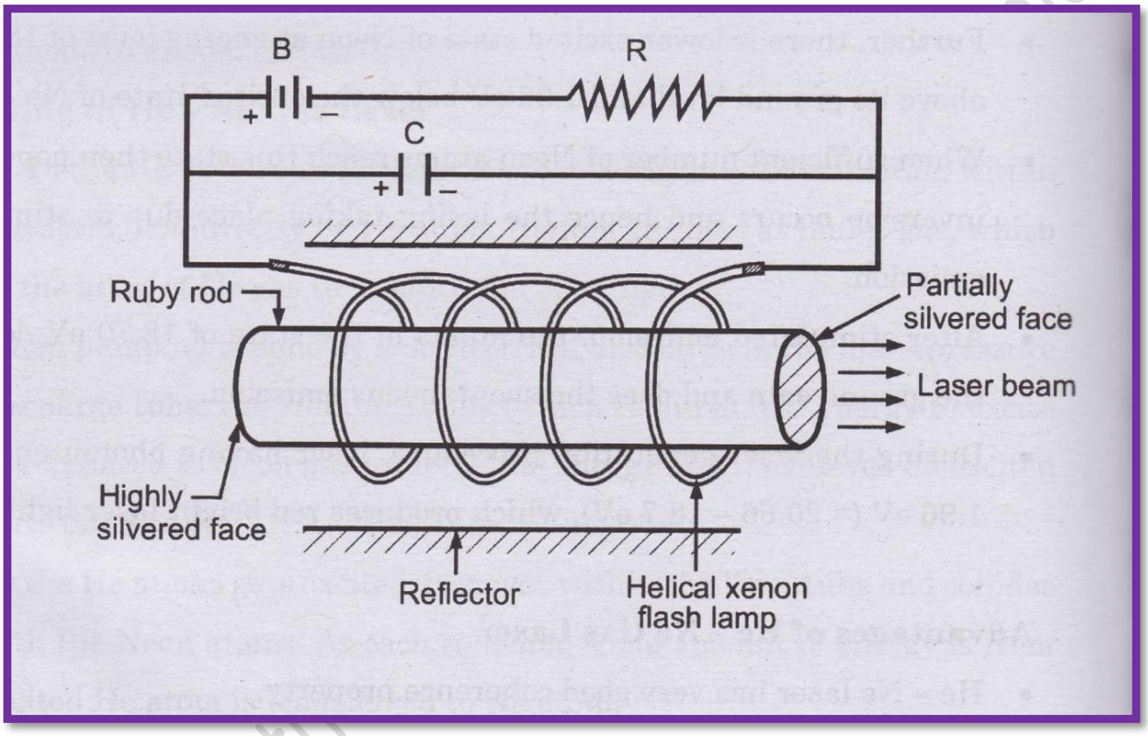
Ruby laser is the first successful laser developed by Theodore Maiman in 1960.

Ruby laser is one of the few solid-state lasers that produce visible light. (deep red light of wavelength 694.3 nm)

It is 3 level solid state laser.

It is a pulsed laser.

5.6.1 Construction



5.6.1.1 Active Medium:

A single crystal of ruby ($Al_2O_3{:}Cr^{+3}$) in form of cylinder (4cm long and 5mm diameter) acts as laser medium or active medium. Ruby is made of host sapphire (Al_2O_3) which is doped with small amount (0.05%) of chromium ions (Cr^{+3}). Chromium ions are active centers.

5.6.1.2 Pumping Source:

Helical Flash tube filled with xenon flash lamp is used as a pumping source.

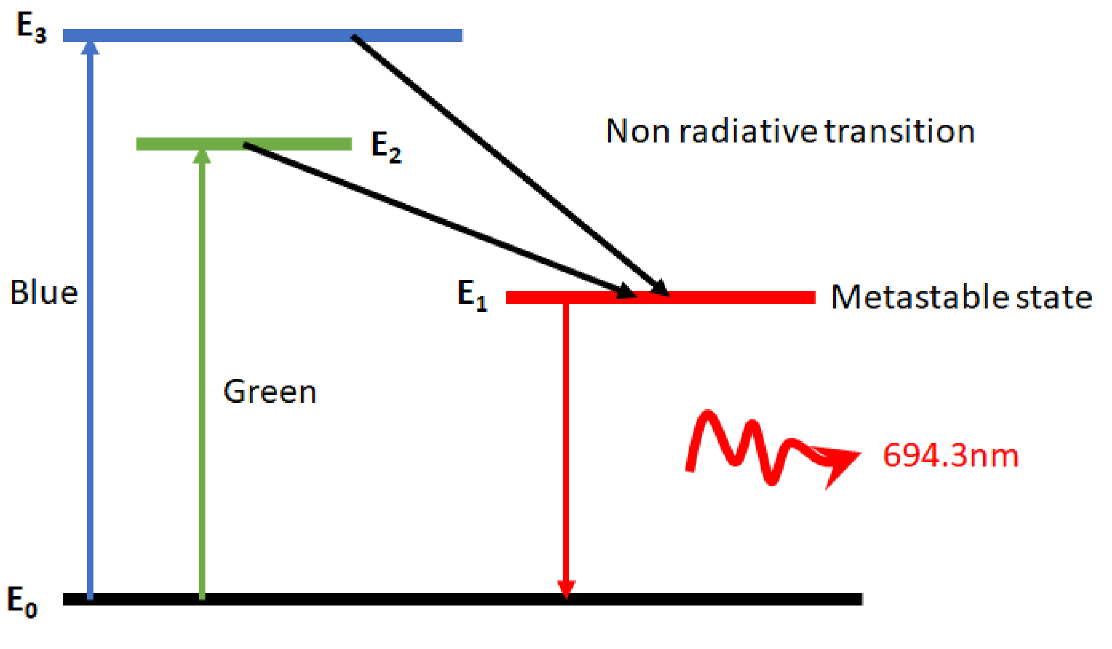
5.6.1.3 Resonator cavity (Optical resonator):

A pair of mirrors of which one is total reflector and other is partial reflector is called optical resonator. (OR The end faces of the RUBY rod are ground polished and silvered to act as the optical resonator mirrors.)

5.6.1.4 Reflective cylinder:

Reflective cylinder is used to focused light on Ruby.

5.6.2 Working



Energy level diagram of Chromium ion

When flash tube is switched on, light will get focused on the rod, chromium ions absorbs energy of Green (5500 Å) and Blue (4200 Å) light and transit to higher energy level E_2 and E_3.

Transition of chromium ions from E_3 to E_1 and E_2 to E_1 is not radiative transition.

E2 behaves as metastable state.

Population inversion can be achieved here.

Transition of chromium ions from E_1 to E_0 due to external photons, gives stimulated emission. These stimulated photons after reflection from two mirrors produce intense laser beam of wavelength 6943Å.

Only a part of the energy emitted by the flash lamp is used to excite the Cr^{3+} ions, while the rest heats up the crystal. Thus, the system can be cooled by either air or water circulation.

5.6.3 Applications

One of the first applications for the ruby laser was in range finding.

The ruby laser was the first laser used to optically pump tunable dye lasers and is particularly well suited to excite laser dyes emitting in the near infrared.

Ruby lasers are rarely used in industry, mainly due to low efficiency and low repetition rates. One of the main industrial uses is drilling holes through diamond, because ruby's high-powered beam closely matches diamond's broad absorption band (the GR1 band) in the red.

Ruby lasers are in optical photography

Ruby lasers are used to remove the melanin of the skin.

Ruby laser can be used for recording of holograms.

Ruby lasers can be used for measurement of plasma properties such as electron density and temperature.

5.7 Helium neon gas LASER (gaseous LASER)

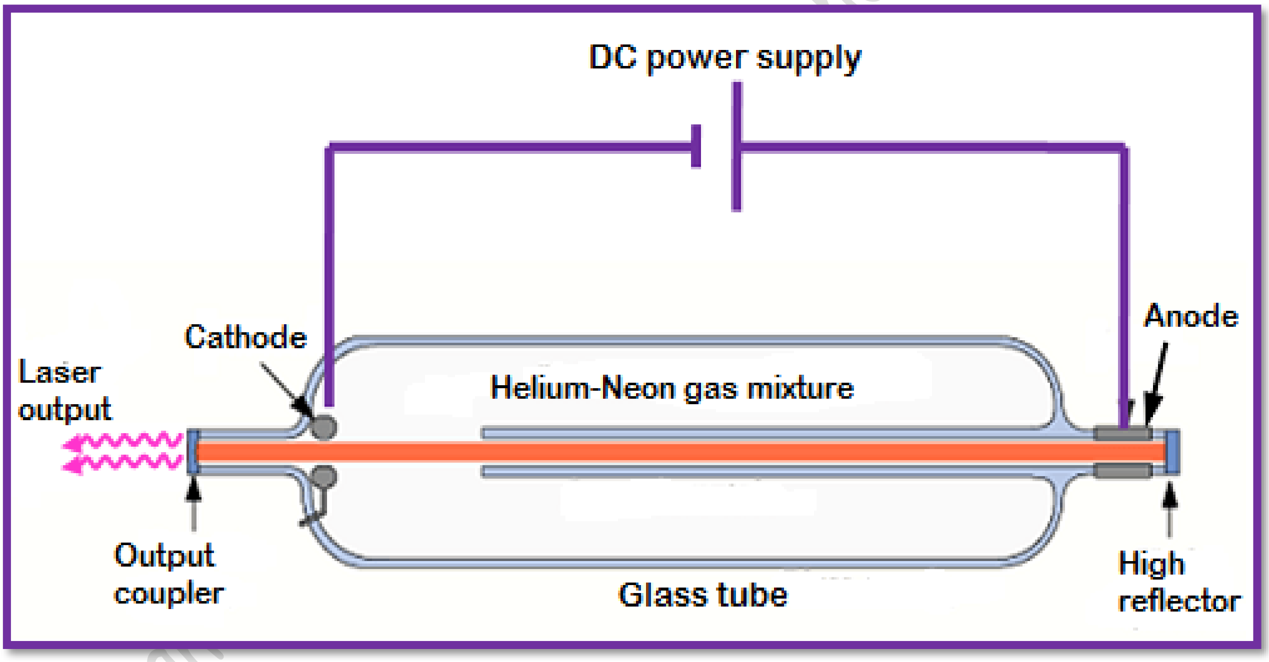
The first gas laser, the Helium–neon laser (HeNe), was co-invented by Iranian-American physicist Ali Javan and American physicist William R. Bennett, Jr. in 1960.

He-Ne laser is a four-level laser.

Its usual operation wavelength is 632.8 nm, in the red portion of the visible spectrum. It operates in Continuous Working (CW) mode.

A helium-neon laser, usually called a He-Ne laser, is a type of small gas laser. He-Ne lasers have many industrial and scientific uses, and are often used in laboratory demonstrations of optics.

5.7.1 Construction



5.7.1.1 Active Medium:

The active medium of the laser, as suggested by its name, is a mixture of helium and neon gases, in a 5:1 to 20:1 ratio, contained at low pressure (an average 50 Pa per cm of cavity length) in a glass envelope. Generally, it is taken as 8:1.

5.7.1.2 Pumping Source:

A discharge tube of length 80.6cm and bore diameter of 11.5cm.

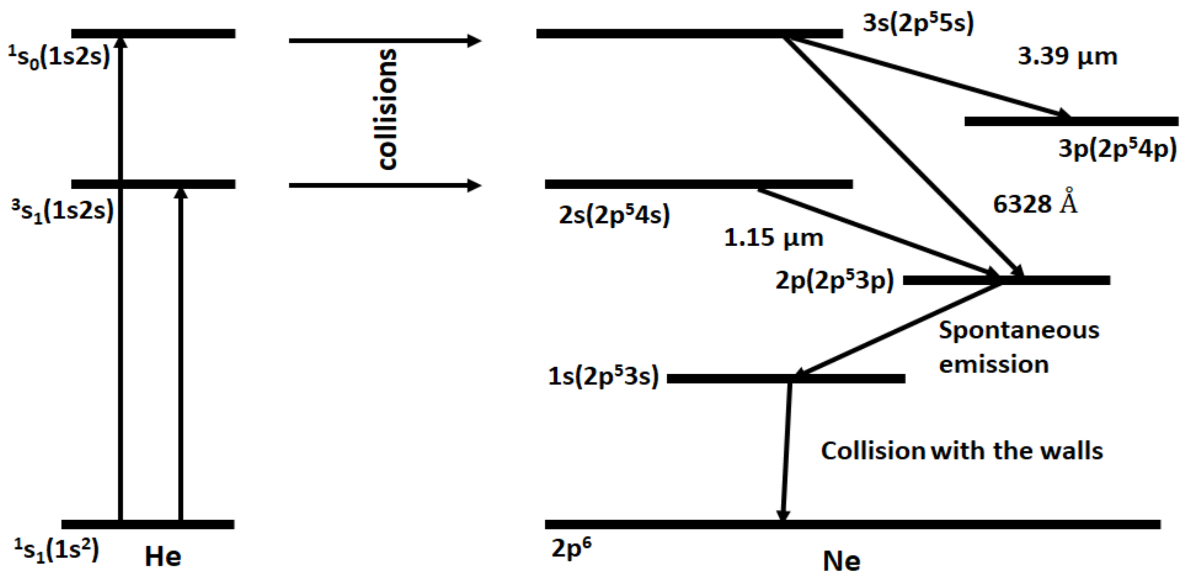
The energy or pump source of the laser is provided by an electrical discharge of around 1000 volts through an anode and cathode at each end of the glass tube. A current of 5 to 100 mA is typical for CW operation.

5.7.1.3 Resonator cavity (Optical resonator):

A pair of mirrors of which one is total reflector and other is partial reflector is called optical resonator.

5.7.2 Working

When high voltage is applied to discharge tube electrons will get accelerated. Due to collision of these



Energy level diagram

Energy level are represented according to paschen's notations and electronic configuration is given in bracket

electrons with the mixture (He:Ne) Helium will be excited to higher levels.

These level have nearly same energy as that of higher energy level of neon. These levels behaves as metastable states.

The excited He atom (He*) collides with an unexcited Ne atom. Due to this collision Ne atom gets excited to higher level 3s and 2s.

There are three transitions giving stimulated emission from higher levels.

From 2s to 2p, From 3s to 3p, From 3s to 2p

These transitions give stimulated emission of light with wavelength 11523 Å, 33912 Å and 6328 Å.

First two are in infrared region and third is in visible region.

3p line is suppressed to get maximum power output at 6328 Å.

Transition from 2p to 1s gives spontaneous emission.

Transition from 1s to ground state is due to collision with the walls.

5.7.3 Applications

The Narrow red beam of He-Ne laser is used in supermarkets to read bar codes.

The He- Ne Laser is used in Holography in producing the 3D images of objects.

He-Ne lasers have many industrial and scientific uses, and are often used in laboratory demonstrations of optics.

Low-level He-Ne laser is commonly used in therapy and has advantages of having beneficial effects on tissue healing and pain relief.

5.8 Application of LASER

5.8.1 In Industries

For welding and melting.

For cutting and drilling holes.

To test the quality of the material.

For the heat treatment of metallic and non-metallic materials.

5.8.2 In Medicine

For removing eye defects such as myopia.

For treatment of detached retina.

In performing micro and bloodless surgery.

For the treatment of human and animal cancers and skin tumors.

5.8.3 Military application

For targeting.

The laser beam can serve as a war weapon, i.e. a powerful laser beam can be used to destroy in few seconds, the big size objects like airplanes, missiles etc., by pointing the laser beam onto them. For this reason, it can be even called as death ray.

The laser beam can be used to determine precisely the distance, velocity and directions as well as the size and form of distant objects of the reflected signal. It is known as LIDAR (Light Detection and Ranging).

5.8.4 Science and engineering applications

In fiber optic communication, Communication between planets is possible with laser.

In holography.

In underwater communication between submarines, as they are not easily absorbed by water.

To accelerate some chemical reactions.

To create new chemical compound by destroying atomic bonds between molecules.

To drill minute holes in cell wall without damaging the cell.

5.9 LASER in Material Processing

Laser Material Processing

- Involving change of phase/state
  - Solid → Vapour
    - Machining(cuting, drilling)
    - Depositing/coating
    - Laser spectrocopy
    - Laser-assisted purification
  - Solid → Liquid
    - Joining (welding/brazing)
    - Surface alloying/cladding
    - Rapid prototyping
    - Reclamation
- Involving no phase chage
  - Solid → Solid
    - Surface hardening
    - Bending or forming
    - Semiconductor annealing
    - Shocking/shot-peening

Classification of Laser Material processing

- Forming
  - Bending/straightening
  - Manufacturing
  - Colouring/deposition
  - Rapid prototyping
- Joining
  - Welding
  - Brasing
  - Soldering/sintering
  - Repair/reclamation
- Machining
  - Cutting
  - Drilling
  - Scribing/marking
  - Cleaning
- Surface Engineering
  - Surface alloying/cladding
  - Surface melting/remelting
  - Surface amorphization
  - Surface hardening/shocking