Linear Equation.

If each term in a D.E including the derivative is linear in terms of dependent Variable then the equation is called linear.

A differential Eq. of the form

$$\frac{dy}{dx} + Py = Q$$

where P and Q are fn. of x is called linear differential Equation

Here $I.F = e^{\int P dx}$

The solution of D.E is

$$\boxed{y \cdot (I.F) = \int (I.F) Q dx + C}$$

For

$$\frac{dx}{dy} + P(y) x = Q(y)$$

$$I.F = e^{\int P(y) dy}$$

and solution is given by

$$\boxed{x \cdot (I.F) = \int (I.F) Q(y) dy + c}$$

classmate
Date
Page

1. Solve $\frac{dy}{dx} + y\sin x = e^{\cos x}$.

$\Rightarrow$ Equation is linear in y.
Here $P = \sin x$ $\quad Q = e^{\cos x}$.

$$I.F = e^{\int P dx} = e^{\int \sin x\, dx} = e^{-\cos x}$$

The general solution is

$$y\cdot(I.F) = \int Q\cdot(I.F)\,dx + C.$$

$$ye^{-\cos x} = \int e^{\cos x}\cdot e^{-\cos x}\,dx + C$$

$$\boxed{ye^{-\cos x} = x + C}$$

2) $\frac{dy}{dx} + 2y\tan x = \sin x$.

$\Rightarrow$ Equation is linear in y.

Here $P = 2\tan x$ $\quad Q = \sin x$

$$I.F = e^{\int P dx} = e^{2\int \tan x\, dx} = e^{2\log \sec x}$$

$$= \sec^2 x$$

The general solution is

$$y\cdot(I.F) = \int Q\cdot(I.F)\,dx + C$$

$$y\cdot\sec^2 x = \int \sin x\cdot\sec^2 x\,dx + C$$

Scanned by CamScanner

$$= \int \sec x \cdot \frac{\sin x}{\cos x} \, dx + c$$

$$= \int \sec x \cdot \tan x \, dx + c.$$

$$\boxed{\int \sec^2 x = \sec x + c.}$$

3. $\frac{dy}{dx} + y \cot x = 2\cos x.$

→ The equation is linear in y

Here $P = \cot x$, $Q = 2\cos x.$

$I.F = e^{\int \cot x}.$

$= e^{\log \sin x} = \sin x.$

The general solution is

$y(I.F) = \int Q \cdot (I.F) dx + c.$

$y \cdot \sin x = \int 2\cos x \cdot \sin x \, dx + c.$

$= \int \sin 2x + c.$

$y \sin x = -\frac{\cos 2x}{2} + c.$

$$\boxed{2y \sin x + \cos 2x = C_1}$$

4. Solve $\frac{dy}{dx} + \tan x \cdot y = \sin 2x$, $y(0) = 0$

$\Rightarrow$ Here equation is linear in y.

$P = \tan x$, $Q = \sin 2x$.

$$I.F = e^{\int P dx}$$

$$= e^{\int \tan x dx}$$

$$= e^{\log \sec x}$$

$$= \sec x.$$

The general solution is

$$y \cdot (I.F) = \int Q \cdot (I.F) dx + C.$$

$$y \cdot \sec x = \int \sin 2x \cdot \sec x dx + C.$$

$$= 2 \int \sin x \cdot \cos x \cdot \frac{1}{\cos x} dx + C$$

$$= 2 \int \sin x \, dx + C$$

$$= -2 \cos x + C.$$

$$\Rightarrow \boxed{y \sec x + 2 \cos x = C}.$$

Now $y(0) = 0$

$$\Rightarrow 0 + 2 = C \Rightarrow \boxed{C = 2}$$

Hence Particular Solution is

$$\boxed{y \sec x + 2 \cos x = 2}$$

5. $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$

$\Rightarrow$ Here $\frac{dy}{dx} - \frac{1}{x+1}y = e^{3x}(x+1)$

$P(x) = \frac{-1}{x+1}$ $\quad Q(x) = e^{3x}(x+1)$

$I.F = e^{\int P dx} = e^{-\int \frac{1}{x+1}dx} = e^{-\log(1+x)}$

$= \frac{1}{1+x}$.

General Solution.

$$y(I.F) = \int Q.(I.F)\,dx + c.$$

$$y\left(\frac{1}{1+x}\right) = \int e^{3x}\frac{(x+1)}{x+1}\,dx + C.$$

$$\frac{y}{1+x} = \frac{e^{3x}}{3} + C.$$

$$\boxed{3y = e^{3x}(1+x) + C_1(1+x)}$$

6. Solve $\frac{dy}{dx} + \frac{1}{x^2}y = 6e^{1/x}$.

$\Rightarrow$ $P(x) = \frac{1}{x^2}$ $\quad Q(x) = 6e^{1/x}$.

$I.F = e^{\int P dx} = e^{\int \frac{1}{x^2}dx} = e^{-\frac{1}{x}}$

G.S is

$$y(I.F) = \int Q(x)(I.F)\,dx + C.$$

$$y.e^{-1/x} = \int 6e^{1/x}.e^{-1/x}\,dx + C$$

$$\boxed{y.e^{-1/x} = 6x + C}$$

Scanned by CamScanner

7. $\frac{dy}{dx} + \frac{4x}{1+x^2} y = \frac{1}{(x^2+1)^3}$

$\Rightarrow$ $P(x) = \frac{4x}{1+x^2}$, $Q(x) = \frac{1}{(x^2+1)^3}$

$I.F = e^{\int P(x) dx} = e^{2\int \frac{2x}{1+x^2}} = e^{2\log(1+x^2)}$

$= (1+x^2)^2$

$\therefore$ General Solution is

$y(I.F) = \int Q \cdot (I.F) \, dx + c$

$y(1+x^2)^2 = \int \frac{1}{(x^2+1)^3} (x^2+1)^2 \, dx + c$

$y(1+x^2)^2 = \int \frac{1}{(1+x^2)} dx + c$

$$\boxed{y(1+x^2)^2 = \tan^{-1}x + c}$$

8. Solve. $(1+y^2) dx = (\tan^{-1}y - x) \, dy$

$\Rightarrow$ $\frac{dx}{dy} = \frac{\tan^{-1}y - x}{(1+y^2)}$

$\frac{dx}{dx} + \frac{1}{(1+y^2)} x = \frac{\tan^{-1}y}{1+y^2}$

Here $P(y) = \frac{1}{1+y^2}$, $Q(y) = \frac{\tan^{-1}y}{1+y^2}$

$I.F = e^{\int P(y) dy} = e^{\int \frac{1}{1+y^2} dy}$

$= e^{\tan^{-1}y}$

Hence General Solution is

$$x \cdot e^{\tan^{-1}y} = \int e^{\tan^{-1}y}\left(\frac{\tan^{-1}y}{1+y^2}\right) dy + C.$$

Let $\tan^{-1}y = t \Rightarrow \frac{1}{1+y^2}\, dy = dt$

$$\therefore x \cdot e^{\tan^{-1}y} = \int e^t\, t \cdot dt + C$$

$$x \cdot e^{\tan^{-1}y} = te^t - e^t + C$$

$$x \cdot e^{\tan^{-1}y} = \tan^{-1}y\, e^{\tan^{-1}y} - e^{\tan^{-1}y} + C.$$

9. Show that current in a circuit containing Resistance R, Inductance L and constant emf E is given by

$$I(t) = \frac{E}{R}\left(1 - e^{-Rt/L}\right).$$

$\Rightarrow$ The modelling of RL circuit by Kirchhoff's voltage law is $L\frac{dI}{dt} + RI = E$

$$\therefore \frac{dI}{dt} + \frac{R}{L} I = \frac{E}{L}$$

Here $P = \frac{R}{L}$ & $Q = \frac{E}{L}$

$$\therefore I.F = e^{\int P dt} = e^{\frac{R}{L}\int dt} = e^{\frac{Rt}{L}}$$

General Solution is

$$I \cdot (I.F) = \int Q \cdot (I.F)\, dt + C.$$

$$I e^{Rt/L} = \int \frac{E}{L} \cdot e^{Rt/L}\, dt + C$$

$$\Rightarrow I \cdot e^{Rt/L} = \frac{E}{L} \int e^{Rt/L} dt + c$$

$$\Rightarrow I \cdot e^{Rt/L} = \frac{E}{L} \times \frac{L}{R} e^{Rt/L} + c$$

$$\Rightarrow I = \frac{E}{R} + c \cdot e^{-Rt/L}$$

Here initial condition is $I(0) = 0$, then.

$$0 = \frac{E}{R} + c e^{0}$$

$$\boxed{c = -\frac{E}{R}}$$

$\therefore$ Particular Solution is

$$I = \frac{E}{R} - \frac{E}{R} e^{-Rt/L}$$

$$I = \frac{E}{R}\left(1 - e^{-Rt/L}\right)$$

10 $\frac{dy}{dx} - (1+3x^{-1}) y = x+2$ where $y(1) = e-1$

$\Rightarrow$ Here $P(x) = -(1+\frac{3}{x})$, $Q(x) = x+2$

$$I.F = e^{\int P(x)dx}$$

$$= e^{-\int (1+\frac{3}{x})dx}$$

$$= e^{-(x+3\log x)}$$

$$= e^{-x} \cdot \frac{1}{x^3}$$

$\therefore$ General Solution is

classmate
Date
Page

$$y(I.F) = \int Q \cdot I.F \, dx + c$$

$$y \cdot x^{-3} e^{-x} = \int \frac{e^{-x}}{x^3} \cdot (x+2)\, dx + c$$

$$y \frac{e^{-x}}{x^3} = \int \left[\frac{1}{x^2} + \frac{2}{x^3} \right] e^{-x} dx + c$$

Let $-x = v \Rightarrow -dx = dv$

$$y \cdot \frac{e^{-x}}{x^3} = \int \left[\frac{1}{v^2} - \frac{2}{v^3} \right] e^{v} (-dv) + c$$

$$y \frac{e^{-x}}{x^3} = -e^{v} \frac{1}{v^2} + c$$

$$y \frac{e^{-x}}{x^3} = -e^{-x} \frac{1}{x^2} + c$$

$$\boxed{y = -x + c x^3 e^{x}}$$

11) Solve $(1+y^2)dx = (e^{-\tan^{-1} y} - x)\, dy$

$\Rightarrow$ Here $\frac{dx}{dy} = \frac{1}{1+y^2} (e^{-\tan^{-1} y} - x)$

$$\Rightarrow \frac{dx}{dy} + \frac{1}{1+y^2} x = \frac{e^{-\tan^{-1} y}}{1+y^2}$$

Here $P(y) = \frac{1}{1+y^2}$, $Q(x) = \frac{e^{-\tan^{-1} y}}{1+y^2}$

$$I.F = e^{\int P(y) dy} = e^{\int \frac{1}{1+y^2} dx} = e^{\tan^{-1} y}$$

Scanned by CamScanner

Date
Page

Its solution is

$$x(I.F) = \int Q.(I.F)\,dy + c.$$

$$x\,e^{\tan^{-1}y} = \int \frac{1}{1+y^2} e^{-\tan^{-1}y} . e^{\tan^{-1}y}\,dy + c$$

$$x.e^{\tan^{-1}y} = \int \frac{1}{1+y^2}\,dy + c$$

$$\boxed{x.e^{\tan^{-1}y} = \tan^{-1}y + c}$$

12:) Solve $y' - \frac{1}{x}y = x^2$

$\Rightarrow$ $y' - \frac{1}{x}y = x^2$

Here $P(x) = \frac{-1}{x}$, $Q(x) = x^2$

$I.F = e^{\int P(x)dx}$

$= e^{\int -\frac{1}{x}dx} = \frac{1}{x}.$

$\therefore$ General Solution is

$$y.(I.F) = \int Q.(I.F)\,dx + c.$$

$$y/x = \int x^2/x\,dx + c$$

$$\boxed{y/x = \frac{x^2}{2} + c}$$

13) $\frac{dy}{dx} + y \cdot \tan x = \cos x$, $y(0)=2$

$\Rightarrow$ $\frac{dy}{dx} + \tan x \, y = \cos x$

$P(x) = \tan x$, $Q(x) = \cos x$.

$I.F = e^{\int P(x)dx}$

$= e^{\int \tan x dx} = e^{[\log \sec x]} = \sec x$.

G.S $y \cdot (I.F) = \int Q \cdot (I.F) dx + c$.

$y \sec x = \int \cos x \cdot \sec x \, dx + c$.

$$\boxed{y \sec x = x + C}$$

$\Rightarrow$

14) $x \frac{dy}{dx} + (1+x)y = x^3$

$\frac{dy}{dx} + \left(\frac{1+x}{x}\right)y = x^2$

$P(x) = \frac{1+x}{x}$ $Q(x) = x^2$.

$I.F = e^{\int P(x)dx} = e^{\int(\frac{1}{x}+1)dx} = e^{\log x + x}$

$= x \cdot e^x$.

$\therefore$ G.S $\Rightarrow$ $y(I.F) = \int Q(I.F) dx + c$.

$y \cdot x \cdot e^x = \int x^2 \cdot x \cdot e^x dx + c$

$xye^x = x^3e^x - 3x^2e^x + 6xe^x - 6e^x + c$

$$\boxed{xy = x^3 - 3x^2 + 6x - 6 + ce^{-x}}$$

15 ⇒ $y' + 6x^2 y = \frac{e^{-2x^3}}{x^2}$, $y(1) = 0$

Here $P(x) = 6x^2$, $Q(x) = \frac{e^{-2x^3}}{x^2}$

$I.F = e^{\int P(x)dx}$
$= e^{6\int x^2 dx} = e^{\frac{6x^3}{3}} = e^{2x^3}$

∴ Its solution is

$y\cdot(I\cdot F) = \int Q\cdot(I\cdot F)\,dx + c$

$y\cdot e^{2x^3} = \int \frac{e^{-2x^3}}{x^2}\cdot e^{2x^3}\,dx + c$

$y e^{2x^3} = \int \frac{1}{x^2}\,dx + c$

$\boxed{y e^{2x^3} = \frac{-1}{x} + c}$

Here $x=1$, $y=0$

$0 = -1 + c \Rightarrow \boxed{c=1}$

∴ $\boxed{y e^{2x^3} = \frac{-1}{x} + 1}$

Scanned by CamScanner

16. $\frac{dy}{dx} - y = e^{2x}$.

Here $P(x) = -1$, $Q(x) = e^{2x}$.

$$I.F = e^{\int P(x)dx} = e^{\int -1 dx} = e^{-x}.$$

General solution $\Rightarrow$ $y(I.F) = \int Q.(I.F)\,dx + C$.

$$y \cdot e^{-x} = \int e^{2x} \cdot e^{-x} dx + C$$

$$y \cdot e^{-x} = \int e^{x} dx + C$$

$$y e^{-x} = e^{x} + C$$

$$\boxed{y = e^{2x} + Ce^{x}}$$

Scanned by CamScanner

Non Linear Equation Reducing Into Linear Form.

Bernoulli's Equation.

The first-order differential equation of the form

$$y' + p(x)y = Q(x)y^n \quad \text{—} (1)$$

where P and Q are functions of x or constants is a nonlinear Eq, known as Bernoulli's Eq.

This Equation can be converted to linear form by following method.

Divide Eq (1) by y^n

$$\boxed{\frac{1}{y^n}\frac{dy}{dx} + \frac{1}{y^{n-1}}P(x) = Q(x)} \quad \text{—} (2)$$

Let $\frac{1}{y^{n-1}} = t$

then $\frac{-(n-1)}{y^n}\frac{dy}{dx} = dt$

$$\frac{1}{y^n}\frac{dy}{dx} = \frac{1}{1-n}\frac{dt}{dx}$$

$\therefore$ Eq (2) becomes

$$\frac{1}{1-n}\frac{dt}{dx} + t \cdot P(x) = Q(x)$$

$$\therefore \frac{dt}{dx} + (1-n)t \cdot P(x) = Q(x)$$

which is linear Equation

classmate

Date

Page

1) $\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}$

$\Rightarrow$ The given eq is Bernoulli's eq.

Here $y^{-2}\frac{dy}{dx} + \frac{1}{xy} = \frac{1}{x^2}$

Let $\frac{1}{y} = t \Rightarrow \frac{-1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}$.

$\Rightarrow -\frac{dt}{dx} + t.\frac{1}{x} = \frac{1}{x^2} \Rightarrow \frac{dt}{dx} - t.\frac{1}{x} = -\frac{1}{x^2}$

Here $P(x) = \frac{-1}{x}$, $Q(x) = \frac{-1}{x^2}$

$I.F = e^{\int P(x)dx} = e^{-\int \frac{1}{x}dx} = \frac{1}{x}$

Solution is

$t.(I.F) = \int Q.(I.F)\,dx + c$

$t.\frac{1}{x} = \int \frac{-1}{x^2}.\frac{1}{x}\,dx + c$

$t\frac{1}{x} = -\left(\frac{1}{-2x^2}\right) + c$

$\rightarrow \boxed{\frac{1}{xy} = \frac{1}{2x^2} + c}$

Scanned by CamScanner

2. $y' + (x+1)y = e^{x^2} y^3, \quad y(0) = 0.5$

$\Rightarrow \frac{dy}{dx} + y(x+1) = e^{x^2} y^3$ (Bernoulli's Eq)

$\frac{1}{y^3}\frac{dy}{dx} + \frac{1}{y^2}(x+1) = e^{x^2}$ — (1)

Let $\frac{1}{y^2} = t \Rightarrow \frac{-2}{y^3}\frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{1}{y^3}\frac{dy}{dx} = \frac{-1}{2}\frac{dt}{dx}$

$\therefore \frac{-1}{2}\frac{dt}{dx} + t\cdot(x+1) = e^{x^2}$

$\frac{dt}{dx} - 2t(x+1) = -2e^{x^2}$

Here $P = -2(x+1)$, $Q = -2e^{x^2}$

$I.F = e^{\int P(x)dx} = e^{-2\int(x+1)dx} = e^{-2\left(\frac{x^2}{2}+x\right)}$

$= e^{-x^2-2x}$

Its solution is

$t\cdot(I.F) = \int Q\cdot(I.F)\,dx + C.$

$t\cdot e^{-x^2-2x} = \int -2e^{x^2}\cdot e^{-x^2-2x}\,dx + C.$

$= -2\int e^{-2x}dx + C$

$\frac{e^{-x^2-2x}}{y^2} = -2\frac{e^{-2x}}{-2} + C$

$\frac{1}{y^2} = e^{x^2} + C\cdot e^{x^2+2x}$

Put $x=0$ & $y=0.5$

$$\Rightarrow \frac{1}{(0.5)^2} = (1+ce^0)e^0$$

$$\Rightarrow \frac{1}{1/4} = 1+c \Rightarrow \boxed{c=3}$$

$\therefore$ Particular Solution is $\frac{1}{y^2} = (1+3e^x)e^{x^2}$.

3. $\frac{dy}{dx} + \frac{1}{x}y = x^3y^3$

$\Rightarrow$ Given Equation is bernoulli's Equation.

$$\Rightarrow \frac{1}{y^3}\frac{dy}{dx} + \frac{1}{y^2}\frac{1}{x} = x^3$$

Let $\frac{1}{y^2} = t \Rightarrow \frac{-2}{y^3}\frac{dy}{\cancel{dx}} = \frac{dt}{dx}$

$$\Rightarrow \frac{1}{y^3}\frac{dy}{dx} = \frac{-1}{2}\frac{dt}{dx}.$$

$$\therefore \frac{-1}{2}\frac{dt}{dx} + t\cdot\frac{1}{x} = x^3$$

$$\Rightarrow \frac{dt}{dx} - 2\frac{1}{x}t = -2x^3$$

Here $P = \frac{-2}{x}$ $\quad Q = -2x^3$

$$\therefore I.F = e^{-2\int \frac{1}{x}dx} = \frac{1}{x^2}$$

Its solution is

$$t\cdot(I.F) = \int Q\cdot(I.F)\,dx + c$$

$$t \cdot \frac{1}{x^2} = -2\int \frac{1}{x^2} x^3 dx + c$$

$$\frac{1}{x^2y^2} = -2\int x\, dx + c$$

$$\frac{1}{x^2y^2} = -x^2 + c$$

$$\boxed{\frac{1}{y^2} = (c - x^2)x^2}$$

4) $x \cdot \frac{dy}{dx} = y^2 + y$

$\Rightarrow \frac{dy}{dx} = \frac{y^2}{x} + \frac{y}{x}$

$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = \frac{y^2}{x}$

$\Rightarrow \frac{1}{y^2}\frac{dy}{dx} - \frac{1}{y}\frac{1}{x} = \frac{1}{x}$.

which is Bernoulli's Equation.

Let $\frac{1}{y} = t \Rightarrow -\frac{1}{y^2}\frac{dy}{dx} = \frac{dt}{dx} \Rightarrow \frac{1}{y^2}\frac{dy}{dx} = -\frac{dt}{dx}$

$\therefore -\frac{dt}{dx} - t \cdot \frac{1}{x} = \frac{1}{x}$

$\Rightarrow \frac{dt}{dx} + \frac{t}{x} = \frac{-1}{x}$.

Here $P = 1/x$, $Q = -1/x$

classmate

Date

Page

$I.F = e^{\int P dx} = e^{\int \frac{1}{x} dx} = x.$

It solution is

$$t \cdot I.F = \int Q \cdot (I.F)\, dx + C.$$

$$t \cdot x = \int \left(\frac{-1}{x}\right)(x)\, dx + C.$$

$$\frac{1}{y} \cdot x = -\int dx + C$$

$$\frac{x}{y} = -x + C.$$

$$\boxed{x = (c - x)\, y}$$

5. $\frac{dy}{dx} + y = \frac{-x}{y}$

$\Rightarrow y \frac{dy}{dx} + y^2 = -x.$

Let $y^2 = t \Rightarrow 2y \frac{dy}{dx} = \frac{dt}{dx} \Rightarrow y \frac{dy}{dx} = \frac{1}{2} \frac{dt}{dx}.$

$\therefore \frac{1}{2} \frac{dt}{dx} + t = -x.$

$$\frac{dt}{dx} + 2t = -2x$$

Here $P = 2 \quad Q = -2x.$

$\Rightarrow I.F = e^{2\int dx} = e^{2x}.$

Its solution is

$$t \cdot (I \cdot F) = \int Q \cdot (I \cdot F)\, dx + C$$

$$t \cdot e^{2x} = \int (-2x)\, e^{2x}\, dx + C$$

$$y^2 \cdot e^{2x} = -2\left[\frac{x e^{2x}}{2} - \frac{e^{2x}}{4}\right] + C$$

$$y^2 e^{2x} = -x e^{2x} + \frac{e^{2x}}{2} + C$$

$$\boxed{y^2 = -x + \frac{1}{2} + C e^{-2x}}$$

Type 2 The equation of the form

$$f'(y)\frac{dy}{dx} + P f(y) = Q$$

where P and Q are functions of x or constants & can be reduced to linear form by putting $f(y) = t$.

classmate
Date
Page

1) $\frac{dy}{dx} + \frac{1}{x} = \frac{e^y}{x^2}$

$\Rightarrow \frac{1}{e^y}\frac{dy}{dx} + \frac{1}{e^y}\frac{1}{x} = \frac{1}{x^2}.$

Let $e^{-y} = t \Rightarrow -e^{-y}\frac{dy}{dx} = \frac{dt}{dx}.$

$\Rightarrow e^{-y}\frac{dy}{dx} = -\frac{dt}{dx}.$

$\therefore -\frac{dt}{dx} + t\cdot\frac{1}{x} = \frac{1}{x^2}$

$\frac{dt}{dx} - \frac{1}{x} = \frac{-1}{x^2}$

Here $P = \frac{-1}{x}$ $Q = \frac{-1}{x^2}$

$\therefore I.F = e^{\int P dx} = e^{-\int \frac{1}{x}dx} = e^{-\log x} = e^{\log x^{-1}} = \frac{1}{x}$

$\therefore$ Its Solution is

$t(I.F) = \int Q(I.F)\,dx + c$

$t\cdot\frac{1}{x} = \int \frac{-1}{x^2}\cdot\frac{1}{x}\,dx + c.$

$\frac{e^{-y}}{x} = -\int \frac{1}{x^3}\,dx + c.$

$\frac{e^{-y}}{x} = -\left(\frac{-1}{2x^2}\right) + c.$

$$\boxed{e^{-y} = \frac{1}{2x} + cx}$$

Scanned by CamScanner

2. $2y\frac{dy}{dx} + e^x = e^{-e^x} e^{-y^2}$

$\Rightarrow \frac{2y}{e^{-y^2}}\frac{dy}{dx} + \frac{1}{e^{-y^2}} e^x = e^{-e^x}$

$2ye^{y^2}\frac{dy}{dx} + e^{y^2}e^x = e^{-e^x}$

Let $e^{y^2} = t \Rightarrow e^{y^2}(2y)\frac{dy}{dx} = \frac{dt}{dx}$

$\therefore \frac{dt}{dx} + t\cdot e^x = e^{-e^x}$

Here $P(x) = e^x$ & $Q(x) = e^{-e^x}$

$I.F = e^{\int e^x dx} = e^{e^x}$

Its solution is

$t\cdot(I.F) = \int Q\cdot(I.F)\, dx + c$

$t\cdot e^{e^x} = \int e^{-e^x}\cdot e^{e^x}\, dx + c$

$e^{y^2}\cdot e^{e^x} = \int dx + c$

$\boxed{e^{y^2}\cdot e^{e^x} = x + c}$