G.S. is.

$y(x) = C_1 y_1 + C_2 y_2$

$= C_1 a_0 \{1 + 4x + 9x^2 + \dots\}$

$+ C_2 \{ a_0 [\ln x \{1 + 4x + 9x^2 + \dots\}$
$- 2 \{2x + 6x^2 + \dots\}] \}$

$= A(1 + 4x + 9x^2 + \dots) + B(\ln x \{1 + 4x + 9x^2 + \dots\}$
$- 2\{2x + 6x^2 + \dots\})$

where $A = C_1 a_0$
$B = C_2 a_0$

Greense Theorem :-

→ If $M(x,y)$, $N(x,y)$, $\frac{\partial M}{\partial y}$ & $\frac{\partial N}{\partial x}$ be Continuous everywhere in a Region R of x, y plane bounded by a closed curve C then

$$\oint_C (M\,dx + N\,dy) = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$$

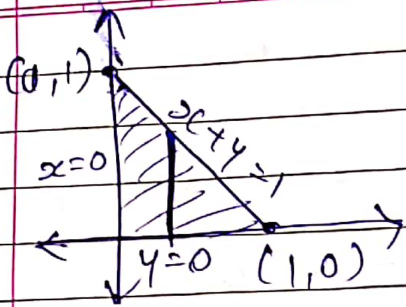
Q1. use ~~green~~ Greense theorem evaluate

$\oint_C [(3x^2 - 8y^2)\,dx + (4y - 6xy)\,dy]$ where C is the boundary of the triangle vertices $(0,0)$, $(1,0)$, $(0,1)$

→

$M = 3x^2 - 8y^2$ | $N = 4y - 6xy$

$\frac{\partial M}{\partial y} = -16y$ | $\frac{\partial N}{\partial x} = 0 - 6y$



$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1}$$

$$\frac{x-1}{0-1} = \frac{y-0}{1-0}$$

$$x-1 = -y$$

$$\Rightarrow \boxed{x+y=1}$$

$\rightarrow$ $0 \leq x \leq 1$
$0 \leq y \leq 1-x$

$$\Rightarrow \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}$$

$$= -6y + 16y$$

$$\Rightarrow \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = 10y$$

By Green's theorem...

$$\oint_C (M\,dx + N\,dy) = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$$

$$= \int_0^1 \int_0^{1-x} 10y \; dy\, dx.$$

$$= \int_0^1 \left\{ \left[\frac{10y^2}{2} \right]_0^{1-x} \right\} dx$$

$$= 5 \int_0^1 (1-x)^2 \, dx$$

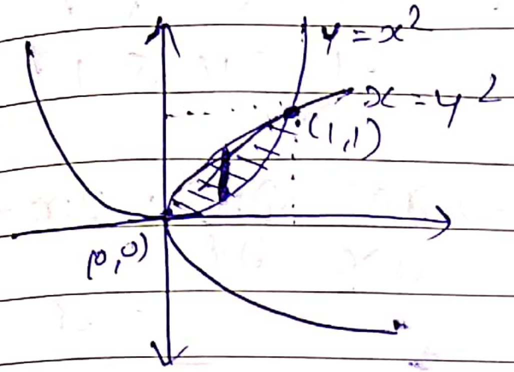
$$= 5 \left[\frac{(1-x)^3}{-3} \right]_0^1 \cancel{dx}$$

$$= -\frac{5}{3} \left[0 - 1 \right]$$

$$= \frac{5}{3}$$

Q-2 $\oint_C (3x^2-8y^2)\,dx + (4y-6xy)\,dy$.

$y^2 = x \ , \ x^2 = y$



$0 \leq x \leq 1$

$x^2 \leq y \leq \sqrt{x}$.

$\rightarrow M = 3x^2 - 8y^2$

$\frac{\partial M}{\partial y} = -16y$

$\rightarrow N = 4y - 6xy$

$\Rightarrow \frac{\partial N}{\partial x} = -6y$

$\Rightarrow \frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = -10y$

$\Rightarrow \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = 10y$

$\rightarrow \oint_C (M\,dx + N\,dy) = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$

$= \int_0^1 \int_{x^2}^{\sqrt{x}} 10y \, dy \, dx$

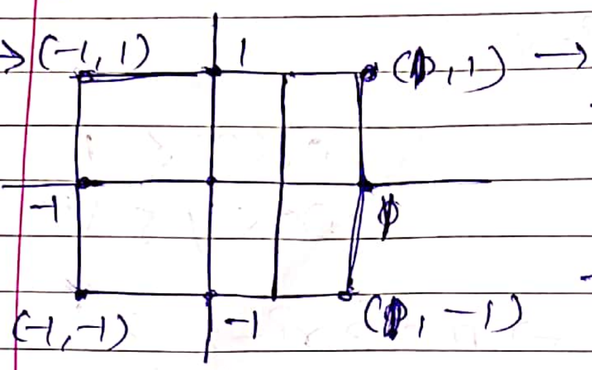
$= 10 \int_0^1 \left[\frac{y^2}{2}\right]_{x^2}^{\sqrt{x}} dx$

$= 10 \int_0^1 \left[\frac{x - x^4}{2}\right] dx$

$= 5 \left[\frac{x^2}{2} - \frac{x^5}{5}\right]_0^1$

$\Rightarrow 5\left[\frac{1}{2} - \frac{1}{5}\right] \Rightarrow \frac{5}{2} - 1 = \frac{3}{2}$

Ex. 3 $\oint (x^2+xy)\,dx + (x^2+y^2)\,dy$, $x=\pm 1, y=\pm 1$



$\rightarrow$ $-1 \leq x \leq 1$

$-1 \leq y \leq 1$

$\rightarrow \dfrac{\partial N}{\partial x} - \dfrac{\partial M}{\partial y} = 2x - x$

$= x$

$$\rightarrow \oint_C (M\,dx + N\,dy) = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$$

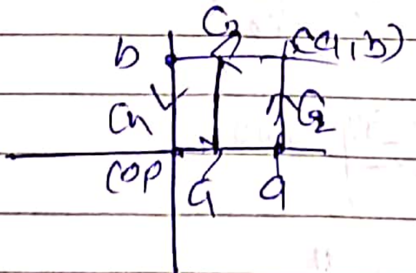
$$= \int_{-1}^{1}\int_{-1}^{1} x\,dx\,dy$$

$$= \int_{-1}^{1} [0]\,dy$$

$$= 0$$

Ex. 4 $F = (x+y)\hat{i} + 2xy\hat{j}$, $x=0, y=0, x=a, y=$

$\rightarrow \dfrac{\partial N}{\partial x} - \dfrac{\partial M}{\partial y} = 2y - 1$



$\rightarrow$ $0 \leq x \leq a$

$0 \leq y \leq b$

$$\Rightarrow \oint_C (M\,dx + N\,dy) = \int_0^a\int_0^b (2y-1)\,dy\,dx$$

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$$= \int_0^a \left[y^2 - y \right]_0^b \, dx$$

$$= \int_0^a (b^2 - b) \, dx$$

$$= a(b^2 - b)$$

$$\Rightarrow ab(b-1).$$

→ To verify Green's therom.

$$\oint_C (Mdx + Ndy) = \int_{C_1} (Mdx + Ndy) + \int_{C_2} (Mdx + Ndy)$$

$$+ \int_{C_3} (Mdx + Ndy) + \int_{C_4} (Mdx + Ndy)$$

- $\int_C \vec{F} \cdot d\vec{r}$ for C_1, $y = 0$, $dy = 0$ $\boxed{0 \leq x \leq a}$

$$\Rightarrow \int_{C_1} (Mdx + Ndy) = \int_0^a x \, dx$$

$$= \left[\frac{x^2}{2} \right]_0^a$$

$$= \frac{a^2}{2}$$

→ $\oint_{C_2}$ for C_2, $x = a$, $dx = 0$ $\boxed{0 \leq y \leq b}$

$$\Rightarrow \int_{C_2} (Mdx + Ndy) = \int_0^b 2ay \, dy \Rightarrow ab^2$$

for C_3, $y = b$; $a \le x \le 0$
$dy = 0$,

$$\int_{C_3} (Mdx + Ndy) = \int_a^0 (x+b)\,dx$$

$$= \left[\frac{x^2}{2} + bx\right]_a^0$$

$$\Rightarrow -\frac{a^2}{2} - ab$$

for C_4, $x = 0$, $b \le y \le 0$
$dx = 0$

$$\int_{C_4} (Mdx + Ndy) = \int_b^0 0\,dy$$

$$= 0$$

$$\rightarrow \oint_C (Mdx + Ndy) = \cancel{\frac{a^2}{2}} + ab^2 - \cancel{\frac{a^2}{2}} - ab$$

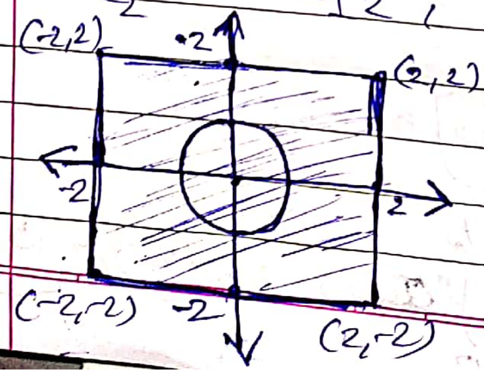
$$= ab(b-1)$$

Ex:-5 $\displaystyle\oint_C \left[\frac{-y}{y^2+x^2}\,dx + \frac{x}{x^2+y^2}\,dy\right]$

where $C = C_1 \cup C_2$

$C_1 = x^2 + y^2 = 1$

$C_2 = x = \pm 2$, $y = \pm 2$

$\rightarrow$ 

$\rightarrow$ $-2 \le x \le 2$
$-2 \le y \le 2$

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$\rightarrow$ $M = \frac{-y}{y^2+x^2} \Rightarrow \frac{\partial M}{\partial y} = \frac{(y^2+x^2)(-1) - (-y)(2y)}{(y^2+x^2)^2}$

$= \frac{y^2-x^2}{(y^2+x^2)^2}$

$\rightarrow$ $N = \frac{x}{x^2+y^2} \Rightarrow \frac{\partial N}{\partial x} = \frac{(x^2+y^2)(1) - (x)(2x)}{(y^2+x^2)^2}$

$= \frac{y^2-x^2}{(y^2+x^2)^2}$

$\rightarrow$ $\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = \frac{y^2-x^2-y^2+x^2}{(y^2+x^2)^2} = 0.$

$\Rightarrow$ $\oint_C (Mdx+Ndy) = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$

$= \int_{-2}^{2}\int_{-2}^{2} 0\, dx\, dy$

$= \int_{-2}^{2} 0\, dy$

$= \underline{\underline{0}}$

$\rightarrow$ To verify Green's theorem.

$\oint_C (Mdx+Ndy) = \int_{C_1} (Mdx+Ndy) + \int_{C_2} (Mdx+Ndy)$

$= \int_0^1 0\, dy + \int_{-2}^{2} 0\, dy$

$\Rightarrow$ for $C_1 \Rightarrow x=0,\ dx=0 \Rightarrow 0 \le y \le 1$

for $C_2 \Rightarrow x=-2,\ dx=0 \Rightarrow -2 \le y \le 2$

$= 0+0$

$= \underline{\underline{0}}$

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