5 Cauchy's Homogeneous Linear Differential Equations

Ex.1 Solve $x^2\dfrac{d^2y}{dx^2}-3x\dfrac{dy}{dx}+5y=\sin(\log x)$ **[M.U. 1991, 92, 2005]**

Solution: Putting $z=\log x$ and $x=e^z$, we get

$[D(D-1)-3D+5]y=\sin z \qquad \therefore \qquad (D^2-4D+5)y=\sin z$

$\therefore$ The A.E. is $(D^2-4D+5)=0 \qquad \therefore \qquad D=\dfrac{4\pm 2i}{2}=2\pm i$

$\therefore$ The C.F. is $y=e^{2z}(c_1\cos z+c_2\sin z)$

P.I. $=\dfrac{1}{D^2-4D+5}\sin z=\dfrac{1}{-4D+4}.\sin z$

$=\dfrac{1}{-4}.\dfrac{D+1}{D^2-1}.\sin z=\dfrac{1}{8}(D+1)\sin z=\dfrac{1}{8}(\cos z+\sin z)$

$\therefore$ The complete solution is

$$y=e^{2z}(c_1\cos z+c_2\sin z)+\frac{1}{8}(\cos z+\sin z)$$

Ex.2 Solve $x^3\dfrac{d^3y}{dx^3}-x^2\dfrac{d^2y}{dx^2}+2x\dfrac{dy}{dx}-2y=x^3+3x$ **[M.U. 1991]**

Solution: Putting $z=\log x$ and $x=e^z$, we get

$[D(D-1)(D-2)-D(D-1)+2D-2]y=e^{3z}+3e^z$

$\therefore \quad (D^3-3D^2+2D-D^2+D+2D-2)y=e^{3z}+3e^z$

$\therefore \quad (D^3-4D^2+5D-2)y=e^{3z}+3e^z$

$\therefore$ The A.E. is $D^3-4D^2+5D-2=0$

$\therefore \quad D^3-D^2-3D^2+3D+2D-2=0$

$\therefore \quad (D-1)(D^2-3D+2)=0 \qquad \therefore \qquad (D-1)(D-1)(D-2)=0$

$\therefore \quad D=1,1,2.$

$\therefore$ The C.F. is $y=(c_1+c_2z)e^z+c_3e^{2z}$

$\therefore$ P.I. $=\dfrac{1}{(D-1)^2(D-2)}e^{3z}+\dfrac{1}{(D-1)^2(D-2)}3e^z$

$=\dfrac{1}{(3-1)^2(3-2)}e^{3z}+\dfrac{z^2}{2}.\dfrac{1}{(1-2)}3e^z$

$=\dfrac{e^{3z}}{4}-\dfrac{z^2}{2}3e^z$

$\therefore$ The complete solution is

$$y=(c_1+c_2z)e^z+c_3e^{2z}+\frac{e^{3z}}{4}-\frac{3z^2}{2}e^z$$

$\therefore$ $$y=(c_1+c_2\log x)x+c_3x^2+\frac{x^3}{4}-\frac{3x}{2}(\log x)^2$$

Ex.3 Solve $x^2\dfrac{d^2y}{dx^2}-3x\dfrac{dy}{dx}+5y=x^2\sin\log x$ **[M.U. 1988, 91, 94]**

Solution: C.F. $=e^{2z}(c_1\cos z+c_2\sin z)$

P.I. $=\dfrac{1}{D^2-4D+5}e^{2z}.\sin z$

$=e^{2z}.\dfrac{1}{(D+2)^2-4(D+2)+5}\sin z$

$=e^{2z}.\dfrac{1}{D^2+1}\sin z$

$\therefore$ P.I. $=e^{2z}.\left(\dfrac{-z}{2}\right)\cos z$

$\therefore$ The complete solution is

$$y=e^{2z}(c_1\cos z+c_2\sin z)-\frac{1}{2}e^{2z}.z\cos z$$

$\therefore$ $$y=x^2(c_1\cos\log x+c_2\sin\log x)-\frac{1}{2}x^2\log x\cos\log x$$

Ex.4 Solve $x^2\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}+4y=\cos\log x+x\sin\log x$ **[M.U. 1998, 2010]**

Solution: Putting $z=\log x$ and $x=e^z$, we get

$$[D(D-1)-D+4]y=\cos z+e^z\sin z$$

$\therefore$ $$(D^2-2D+4)y=\cos z+e^z\sin z$$

$\therefore$ The A.E. is $D^2-2D+4=0$

$\therefore$ $D=1\pm\sqrt{3}.i$

$\therefore$ The C.F. is $y=e^z\left(c_1\cos\sqrt{3}.z+c_2\sin\sqrt{3}.z\right)$

P.I. for $\cos z=\dfrac{1}{D^2-2D+4}\cos z=\dfrac{1}{3-2D}\cos z$

$=\dfrac{1}{9-4D^2}(3+2D)\cos z=\dfrac{1}{13}(3+2D)\cos z$

$=\dfrac{1}{13}(3\cos z-2\sin z)$

P.I. for $e^z\sin z=\dfrac{1}{D^2-2D+4}e^z\sin z$

$$= e^z \frac{1}{(D+1)^2 - 2(D+1) + 4} . \sin z$$

$$= e^z \frac{1}{D^2+3} \sin z = e^z . \frac{1}{2} \sin z$$

$\therefore$ The complete solution is

$$y = e^z \left(c_1 \cos\sqrt{3}.z + c_2 \sin\sqrt{3}.z\right) + \frac{1}{13}(3\cos z - 2\sin z) + e^z . \frac{1}{2}\sin z$$

i.e. $y = x\left[c_1 \cos\left(\sqrt{3}\log x\right) + c_2 \sin\left(\sqrt{3}\log x\right)\right]$

$$+\frac{1}{13}(3\cos\log x - 2\sin\log x) + \frac{x}{2}\sin(\log x)$$

Ex.5 Solve $x^2 \dfrac{d^2y}{dx^2} - 3x\dfrac{dy}{dx} + y = \dfrac{(\sin\log x)+1}{x}$ **[M.U. 2002, 09]**

Solution: Putting $z = \log x$ and $x = e^z$, we get

$$\left[D(D-1) - 3D + 1\right]y = (\sin z + 1).e^{-z}$$

$\therefore$ $\left(D^2 - 4D + 1\right)y = e^{-z}\sin z + e^{-z}$

$\therefore$ The A.E. is $D^2 - 4D + 1 = 0$ $\therefore$ $D = 2 \pm \sqrt{3}$

$\therefore$ The C.F. is $y = Ae^{\left(2+\sqrt{3}\right)z} + Be^{\left(2-\sqrt{3}\right)z}$

$\therefore$ $y = e^{2z}\left(Ae^{\sqrt{3}.z} + Be^{-\sqrt{3}.z}\right)$ which can be expressed as

$$y = e^{2z}\left(c_1 \cosh\sqrt{3}.z + c_2 \sinh\sqrt{3}.z\right)$$

P.I. for $e^{-z} = \dfrac{1}{D^2 - 4D + 1}e^{-z} = \dfrac{1}{6}e^{-z}$

P.I. for $e^{-z}\sin z = e^{-z}.\dfrac{1}{(D-1)^2 - 4(D-1) + 1}\sin z$

$$= e^{-z}.\frac{1}{D^2 - 6D + 6}\sin z = e^{-z}.\frac{1}{5-6D}\sin z$$

$$= e^{-z}.\frac{5+6D}{25-36D^2}\sin z = e^{-z}\frac{(5\sin z + 6\cos z)}{61}$$

$\therefore$ The complete solution is

$$y = e^{2z}\left(c_1 \cosh\sqrt{3}.z + c_2 \sinh\sqrt{3}.z\right) + \frac{1}{6}e^{-z} + \frac{e^{-z}}{61}(5\sin z + 6\cos z)$$

$\therefore$ $y = x^2\left[c_1 \cosh\left(\sqrt{3}\log x\right) + c_2 \sinh\left(\sqrt{3}\log x\right)\right] + \dfrac{1}{6x}$

$$+\frac{1}{61x}\left[5\sin(\log x) + 6\cos(\log x)\right]$$

Ex.6 Solve $\left(x^2D^2 + 5xD + 3\right)y = \left(1 + \dfrac{1}{x}\right)^2 \log x$ **[M.U. 2003, 06]**

Solution: Putting $z = \log x$ and $x = e^z$, we get

$$\left[D(D-1)+5D+3\right]y = \left(1+e^{-z}\right)^2 .z$$

$$\therefore \quad \left[D^2+4D+3\right]y = \left(1+e^{-z}\right)^2 z$$

$\therefore$ The A.E. is $D^2+4D+3=0$

$\therefore$ $(D+1)(D+3)=0 \quad \therefore \quad D=-1,-3$

$\therefore$ The C.F. is $y = c_1e^{-z}+c_2e^{-3z}$

$$\text{P.I.} = \frac{1}{D^2+4D+3}\left(z+2e^{-z}z+e^{-2z}z\right)$$

Now, $$\frac{1}{D^2+4D+3}z = \frac{1}{3}\left[1\frac{4D+D^2}{3}\right]^{-1} z$$

$$= \frac{1}{3}\left[1-\frac{4D}{3}....\right]z = \frac{1}{3}\left[z-\frac{4}{3}....\right]$$

$$\frac{1}{D^2+4D+3}2e^{-z}z = 2e^{-z}.\frac{1}{(D-1)^2+4(D-1)+3}z$$

$$= 2.\frac{e^{-z}}{D^2+2D}z = 2.\frac{e^{-z}}{2D}\left[1+\frac{D}{2}....\right]^{-1} z$$

$$= \frac{e^{-z}}{D}\left[z-\frac{1}{2}\right] = e^{-z}\int\left[z-\frac{1}{2}\right]dz = e^{-z}\left(\frac{z^2}{2}-\frac{z}{2}\right)$$

$$\frac{1}{D^2+4D+3}e^{-2z}.z = e^{-2z}.\frac{1}{(D-2)^2+4(D-2)+3}z$$

$$= \frac{e^{-2z}}{D^2-1}z = e^{-2z}.(-1)\left[1-D^2\right]^{-1} z$$

$$= -e^{-2z}\left[1+D^2+....\right]z = -e^{-2z}z$$

$$\therefore \quad \text{P.I.} = \frac{z}{3}-\frac{4}{9}+e^{-z}\left(\frac{z^2}{2}-\frac{z}{2}\right)-e^{-2z}.z$$

$\therefore$ The complete solution is

$$y = c_1e^{-z}+c_2e^{-3z}+\frac{z}{3}-\frac{4}{9}+e^{-z}\left(\frac{z^2}{2}-\frac{z}{2}\right)-e^{-2z}.z$$

$$y = \frac{c_1}{x}+\frac{c_2}{x^3}+\frac{\log x}{3}-\frac{4}{9}-\frac{1}{x}\left[\frac{(\log x)^2}{2}-\frac{(\log x)}{2}\right]-\frac{1}{x^2}.\log x$$

Ex.7 Solve $x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}+y = \log x.\sin(\log x)$ **[M.U. 1987]**

Solution: Putting $z = \log x$ and $x = e^z$, we get

$$\left[D(D-1)+D+1\right]y = z\sin z$$

$\therefore$ $$\left[D^2+1\right]y = z\sin z$$

$\therefore$ The A.E. is $D^2+1=0$ $\quad\therefore\quad$ $D = i, -i$

$\therefore$ The C.F. is $y = c_1\cos z + c_2\sin z$

$$\text{P.I.} = \frac{1}{D^2+1}z\sin z = \text{I.P. of } \frac{1}{D^2+1}e^{iz}.z$$

$$= \text{I.P. of } e^{iz}\frac{1}{(D+i)^2+1}z = \text{I.P. of } e^{iz}\frac{1}{D^2+2iD}.z$$

$\therefore$ $$\text{P.I.} = \text{I.P. of } e^{iz}\frac{1}{D^2+2iD}.z$$

$$= \text{I.P. of } e^{iz}.\frac{1}{2iD}\left[1+\frac{D}{2i}\right]^{-1}.z$$

$$= \text{I.P. of } e^{iz}.\frac{1}{2iD}\left[1-\frac{D}{2i}+....\right]z$$

$$= \text{I.P. of } e^{iz}.\frac{1}{2iD}\left[z-\frac{1}{2i}\right]$$

$$= \text{I.P. of } e^{iz}.\frac{1}{2i}\int\left(z-\frac{1}{2i}\right)dz$$

$$= \text{I.P. of } e^{iz}.\frac{1}{2i}\left[\frac{z^2}{2}-\frac{z}{2i}\right]$$

$$= \text{I.P. of } (\cos z + i\sin z)\frac{1}{2i}\left(\frac{z^2}{2}-\frac{z}{2i}\right)$$

$$= \text{I.P. of } (\cos z + i\sin z)\left(-\frac{i}{2}\right)\left(\frac{z^2}{2}+\frac{zi}{2}\right)$$

$$= -\frac{z^2}{4}\cos z + \frac{z}{4}\sin z$$

$\therefore$ The complete solution is

$$y = c_1\cos z + c_2\sin z - \frac{z^2}{4}\cos z + \frac{z}{4}\sin z$$

$\therefore$ $$y = c_1\cos(\log x) + c_2\sin(\log x) - \frac{(\log x)^2}{4}\cos(\log x) + \frac{(\log x)}{4}\sin(\log x)$$

Ex.8 Solve $\left(\frac{d}{dx}+\frac{1}{x}\right)y = \frac{1}{x^4}$ **[M.U. 2003, 2007]**

Solution: We have $\left(\frac{d}{dx}+\frac{1}{x}\right)\left(\frac{dy}{dx}+\frac{y}{x}\right) = \frac{1}{x^4}$

$\therefore$ $$\frac{d}{dx}\left(\frac{dy}{dx}+\frac{y}{x}\right) + \frac{1}{x}\left(\frac{dy}{dx}+\frac{y}{x}\right) = \frac{1}{x^4}$$

$\therefore \quad \dfrac{d^2y}{dx^2}+\dfrac{1}{x}.\dfrac{dy}{dx}-\dfrac{y}{x^2}+\dfrac{1}{x}\dfrac{dy}{dx}+\dfrac{y}{x^2}=\dfrac{1}{x^4}$

$\therefore \quad \dfrac{d^2y}{dx^2}+\dfrac{2}{x}\dfrac{dy}{dx}=\dfrac{1}{x^4}$

Multiplying by x^2, we get,

$$x^2\frac{d^2y}{dx^2}+2x\frac{dy}{dx}=\frac{1}{x^2}$$

Putting $z=\log x$ and $x=e^z$, we get

$$\left[D(D-1)+2D\right]y=e^{-2z}$$

$\therefore \quad$ The A.E. is $D^2+D=0$

$\therefore \quad D(D+1)=0 \qquad \therefore \quad D=0,-1$

$\therefore \quad$ The C.F. is $y=c_1+c_2e^{-z}$

P.I. $=\dfrac{1}{D(D+1)}e^{-2z}=\dfrac{1}{-2(-2+1)}e^{-2z}=\dfrac{1}{2}e^{-2z}$

$\therefore \quad$ The complete solution is

$$y=c_1+c_2e^{-z}+\frac{1}{2}e^{-2z}$$

$\therefore \quad y=c_1+\dfrac{c_2}{x}+\dfrac{1}{2x^2}$

EXERCISE

Solve the following differential equations :

- $x^2\dfrac{d^2y}{dx^2}-2x\dfrac{dy}{dx}+2y=x^{-1}$ **[M.U. 1994]**

Ans. $y=c_1x+c_2x^2+\dfrac{1}{6x}$

- $x^3\dfrac{d^3y}{dx^3}+2x^2\dfrac{d^2y}{dx^2}+2y=10\left(x+\dfrac{1}{x}\right)$ **[M.U. 1989, 95]**

Ans. $y=\dfrac{c_1}{x}+x\left(c_2\cos\log x+c_3\sin\log x\right)+5x+\dfrac{2}{x}\log x$

- $x^2\dfrac{d^3y}{dx^3}+3x\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}=x^2\log x$ **[M.U. 2002]**

Ans. $y=c_1+c_2\log x+c_3(\log x)^2+\dfrac{1}{27}x^3(\log x-1)$

- $x^3\dfrac{d^2y}{dx^2}+3x^2.\dfrac{dy}{dx}+xy=\sin\log x$ **[M.U. 1989]**

Ans. $y=\dfrac{1}{x}\left[c_1+c_2\log x-\sin\log x\right]$

- $x^2\dfrac{d^3y}{dx^3}+3x\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}+\dfrac{y}{x}=4\log x$ **[M.U. 2007]**

Ans. $y=\dfrac{c_1}{x}+\sqrt{x}\left[c_2\cos\left(\dfrac{\sqrt{3}}{2}\log x\right)+c_3\sin\left(\dfrac{\sqrt{3}}{2}\log x\right)\right]+x(2\log x-3)$

- The radial displacement 'u' in a rotating disc at a distance 'r' from the axis is given by $\dfrac{d^2u}{dr^2}+\dfrac{1}{r}\dfrac{du}{dr}-\dfrac{u}{r^2}+kr=0$. Find the displacement if u = 0 when r = 0 and r = a.

[M.U. 1996, 2005, 08]

Ans. **Hint :** $ur=c_1r^2+c_2-\dfrac{k}{8}r^4$ $\quad\therefore\quad$ $u=\dfrac{k}{8}r\left(a^2-r^2\right)$

- Find the equation of the curve which satisfies the differential equation $4x^2\dfrac{d^2y}{dx^2}-4x\dfrac{dy}{dx}+y=0$ and crosses the x-axis at an angle of 60° at x = 1.

[M.U. 2004]

Ans. $y=x^{\left(2+\sqrt{3}\right)/2}-x^{\left(2-\sqrt{3}\right)/2}$

Ex.9 Solve $(x+2)^2\dfrac{d^2y}{dx^2}-(x+2)\dfrac{dy}{dx}+y=3x+4$ **[M.U. 1988, 90, 2002, 03]**

Solution: Put $x+2=v$ $\quad\therefore\quad$ $\dfrac{dv}{dx}=1$

$$\therefore\quad \frac{dy}{dx}=\frac{dy}{dv}\cdot\frac{dv}{dx}=\frac{dy}{dv}$$

$$\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d}{dx}\left(\frac{dy}{dv}\right)=\frac{d}{dv}\left(\frac{dy}{dv}\right)\frac{dv}{dx}=\frac{d^2y}{dv^2}.1$$

$\therefore$ The given equation changes to

$$v^2\frac{d^2y}{dv^2}-v\frac{dy}{dv}+y=3(v-2)+4=3v-2 \qquad \ldots(1)$$

Now, put $z=\log v$ and $v=e^z$

Then as in 2, $v\dfrac{dy}{dv}=Dy, v^2\dfrac{d^2y}{dv^2}=D(D-1)y$

The equation (1) then changes to

$$\left[D(D-1)-D+1\right]y=3e^z-2$$

i.e. $\left(D^2-2D+1\right)y=3e^z-2$

The A.E. is $(D-1)^2=0$ $\quad\therefore\quad$ $D=1,1$

The C.F. is $y=(c_1+c_2z)e^z$

$$\text{P.I.} = \frac{1}{(D-1)^2}\left(3e^z - 2\right) = 3.\frac{1}{(D-1)^2}e^z - 2\frac{1}{(D-1)^2}e^{0z}$$

$$\text{P.I.} = 3.\frac{z^2}{2}.e^z - 2$$

$\therefore$ The complete solution is

$$y = (c_1 + c_2 z)e^z + \frac{3}{2}z^2 e^z - 2$$

Putting $z = \log v = \log(x+2)$ and $e^z = v = x+2$, we get the solution as

$$y = \left[c_1 + c_2\log(x+2)\right](x+2) + \frac{3}{2}\left[\log(x+2)\right]^2(x+2) - 2$$

Ex.10 Solve $\left(1+x^2\right)\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos\log(1+x)$ **[M.U. 1997, 2007, 08, 11]**

Solution: Put $x+1 = v \quad \therefore \quad \frac{dv}{dx} = 1$

$\therefore \quad \frac{dy}{dx} = \frac{dy}{dv}, \frac{d^2y}{dx^2} = \frac{d^2y}{dv^2}$

$\therefore$ The given equation changes to

$$v^2\frac{d^2y}{dv^2} + v\frac{dy}{dv} + y = 4\cos\log v$$

Now put $\log v = z, v = e^z$

$\therefore \quad \left[D(D-1) + D + 1\right]y = 4\cos z \qquad \therefore \quad \left(D^2+1\right)y = 4\cos z$

$\therefore$ The A.E. is $D^2 + 1 = 0 \qquad \therefore \quad D = i, -i$

$\therefore$ The C.F. is $y = c_1\cos z + c_2\sin z$

$\therefore$ P.I. $= \frac{4}{D^2+1}\cos z = 4\frac{z}{2}.\sin z = 2z\sin z$

$\therefore$ The complete solution is

$$y = c_1\cos z + c_2\sin z + 2z\sin z$$

Putting $z = \log v = \log(1+x)$, we get,

$$y = c_1\cos\left[\log(1+x)\right] + c_2\sin\left[\log(1+x)\right] + 2\log(1+x)\sin\left[\log(1+x)\right]$$

Ex.11 Solve $(5+2x)^2\frac{d^2y}{dx^2} - 6(5+2x)\frac{dy}{dx} + 8y = 6x$ **[M.U. 2004, 08]**

Solution: Put $5+2x = v \quad \therefore \quad \frac{dv}{dx} = 2$

$$\frac{dy}{dx} = \frac{dy}{dv}.\frac{dv}{dx} = 2\frac{dy}{dv}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(2\frac{dy}{dv}\right) = \frac{d}{dv}\left(2\frac{dy}{dv}\right).\frac{dv}{dx}$$

$$= 2.\frac{d^2y}{dv^2}.2 = 4\frac{d^2y}{dv^2}$$

The given equation then changes to

$$v^2.4\frac{d^2y}{dv^2} - 6.v.2\frac{dy}{dv} + 8y = 6.\left(\frac{v-5}{2}\right)$$

$$4v^2\frac{d^2y}{dv^2} - 12v\frac{dy}{dv} + 8y = 3(v-5)$$

Putting $z = \log v$ and $v = e^z$

$$\left[4D(D-1) - 12D + 8\right]y = 3\left(e^z - 5\right)$$

$\therefore \quad \left(4D^2 - 16D + 8\right)y = 3\left(e^z - 5\right)$

$\therefore \quad \left(D^2 - 4D + 2\right)y = \frac{3}{4}\left(e^z - 5\right)$

$\therefore$ The A.E. is $D^2 - 4D + 2 = 0$ $\quad \therefore \quad D = 2 \pm \sqrt{2}$

$\therefore$ The C.F. is $y = e^{2z}\left(c_1 e^{\sqrt{2}z} + c_2 e^{-\sqrt{2}z}\right)$

$\therefore \quad y = e^{2\log v}\left(c_1 e^{\sqrt{2}\log v} + c_2 e^{-\sqrt{2}\log v}\right)$

$\therefore \quad y = v^2\left(c_1 v^{\sqrt{2}} + c_2 v^{-\sqrt{2}}\right)$

$$= (5+2x)^2\left[c_1(5+2x)^{\sqrt{2}} + c_2(5+2x)^{-\sqrt{2}}\right]$$

$\therefore$ P.I. $= \frac{1}{D^2 - 4D + 2}.\frac{3}{4}\left(e^z - 5\right)$

$$= \frac{3}{4}\left[\frac{1}{D^2 - 4D + 2}\left(e^z - 5e^{0z}\right)\right]$$

$$= \frac{3}{4}\left[\frac{e^z}{-1} - \frac{5e^{0z}}{2}\right] = \frac{3}{4}\left[-e^z - \frac{5}{2}\right]$$

$$= \frac{3}{4}\left(-v - \frac{5}{2}\right) = -\frac{3}{4}(5+2x) - \frac{15}{8}$$

$\therefore$ P.I. $= -\frac{3}{2}x - \frac{45}{8}$

$\therefore$ The complete solution is

$$y = (5+2x)^2\left[c_1(5+2x)^{\sqrt{2}} + c_2(5+2x)^{-\sqrt{2}}\right] - \frac{3}{2}x - \frac{45}{8}$$

EXERCISE

Solve the following differential equations :

- $(1+2x)^2\frac{d^2y}{dx^2} - 6(1+2x)\frac{dy}{dx} + 16y = 8(1+2x)^2$ **[M.U. 2011, 12]**

Ans. $y=(1+2x)^2\left[c_1+\{\log(1+2x)\}(c_2+\log(1+2x))\right]$

- $(x+a)^2\dfrac{d^2y}{dx^2}-4(x+a)\dfrac{dy}{dx}+6y=x$ **[M.U. 1997]**

Ans. $y=c_1(x+a)^2+c_2(x+a)^3+\dfrac{1}{2}(x+a)-\dfrac{a}{6}$

- $(3x+2)^2\dfrac{d^2y}{dx^2}+5(3x+2)\dfrac{dy}{dx}-3y=x^2+x-1$ **[M.U. 1998, 2006]**

Ans. $y=c_1(3x+2)^{1/3}+c_2(3x+2)^{-1}+\dfrac{1}{27}\left[\dfrac{1}{15}(3x+2)^2+\dfrac{1}{4}(3x+2)-7\right]$

- $(1+x)^2\dfrac{d^2y}{dx^2}+(1+x)\dfrac{dy}{dx}+y=2\cos\log(1+x)$ **[M.U. 2002]**

Ans. $y=c_1\cos\log(1+x)+c_2\sin\log(1+x)+2\log(1+x).\sin\log(1+x)$

- $(1+x)^2\dfrac{d^2y}{dx^2}+(1+x)\dfrac{dy}{dx}+y=2\sin\log(1+x)$ **[M.U. 1992]**

Ans. $y=c_1\cos\log(1+x)+c_2\sin\log(1+x)-\log(1+x)\cos\log(1+x)$

- $(3x+1)^2\dfrac{d^2y}{dx^2}-3(3x+1)\dfrac{dy}{dx}-12y=9x$ **[M.U. 1999]**

Ans. $y=(3x+1)\left[c_1(3x+1)^{\sqrt{7/3}}+c_2(3x+1)^{-\sqrt{7/3}}\right]-3\left[\dfrac{3x+1}{7}-\dfrac{1}{4}\right]$

- $(2x+1)^2\dfrac{d^2y}{dx^2}-2(2x+1)\dfrac{dy}{dx}-12y=6x$ **[M.U. 2001, 02, 09]**

Ans. $y=c_1(2x+1)^3+c_2(2x+1)^{-1}-\dfrac{3}{4}\left[\dfrac{(2x+1)}{4}-\dfrac{1}{3}\right]$

- $(x+1)^2\dfrac{d^2y}{dx^2}-3(x+1)\dfrac{dy}{dx}+4y=x^2$ **[M.U. 1993]**

Ans. $y=\left[c_1+c_2\log(x+1)\right](x+1)+\left[\dfrac{\log(x+1)}{2}\right]^2(x+1)+2(x+1)+\dfrac{1}{4}$

- $(3x+2)^2\dfrac{d^2y}{dx^2}+3(3x+2)\dfrac{dy}{dx}-36y=3x^2+4x+1$ **[M.U. 1988, 2002, 10]**

Ans. $y=c_1(3x+2)^2+\dfrac{c_2}{(3x+2)^2}+\dfrac{1}{108}(3x+2)^2\log(3x+2)+\dfrac{1}{108}$

- $(2x+1)^2\dfrac{d^2y}{dx^2}-2(2x+1)\dfrac{dy}{dx}-12y=x^2$ **[M.U. 2004]**

Ans. $y=c_1(2x+1)^3+c_2(2x+1)^{-1}+\dfrac{1}{16}\left[-\dfrac{1}{3}(2x+1)^2+\dfrac{1}{2}(2x+1)-\dfrac{1}{3}\right]$