Portfolio Optimization

Introduction

Daniel P. Palomar (2025). *Portfolio Optimization: Theory and Application.* Cambridge University Press.

portfoliooptimizationbook.com

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Outline

1. Introduction
2. What is Portfolio Optimization?
3. Big Picture
4. Outline of the Book
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Executive Summary

The book (Palomar 2025) offers:

- deep dive into **practical algorithms** for portfolio optimization
- moves beyond conventional Gaussian assumptions
- explores diverse portfolio formulations and frameworks
- essential for financial data modeling and portfolio design practitioners
- suitable for both portfolio optimization and financial data modeling courses
- suitable for a portfolio optimization course and financial data modeling course (Palomar 2025, chap. 1)

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Introduction

Introduction to Modern Portfolio Theory:

- Originated with Harry Markowitz's 1952 paper "Portfolio Selection" (Markowitz 1952).
- Awarded the Nobel Prize in Economic Sciences in 1990.
- Core concept: Investors optimize portfolios based on expected return and risk.
- Real-world application issues led to various enhancements and alternatives.

Evolution of Portfolio Formulations:

- 70 years of research and practice revisiting Markowitz's formulation.
- Developments include robust optimization, alternative risk measures, and regularization.
- Incorporation of factor models, volatility clustering, and risk parity approaches.

Introduction

Scope of the Book "Portfolio Optimization" (Palomar 2025):

- Focus on practical financial data modeling and portfolio optimization.
- From mathematical formulations to practical numerical algorithms with code examples.
- Transition from Gaussian assumptions to heavy-tailed distribution models.
- Extensive use of Kalman filtering and advanced techniques for financial graphs.

Portfolio Formulations Explored:

- From Markowitz's mean-variance (1952) to maximum Sharpe ratio (1966) portfolios.
- Advanced formulations: Kelly-based, utility-based, high-order, downside risk portfolios.
- Coverage of semivariance, CVaR, drawdown, risk parity, and graph-based portfolios.
- Includes robust, bootstrapped, bagged, pairs trading, and statistical arbitrage portfolios.
- Exploration of deep learning applications in portfolio optimization.

Primary Focus:

- Theoretical understanding and practical algorithms for portfolio optimization.

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What is Portfolio Optimization?

Random Variables in Finance:

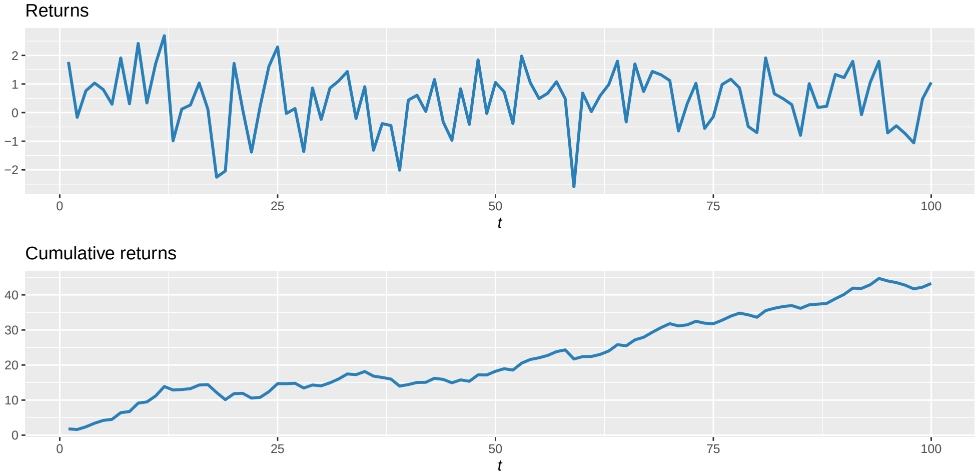
- Random variable X with mean $\mu = \mathbb{E}[X]$ and variance $\sigma^2 = \mathbb{E}[(X-\mu)^2]$.
- Example: normal (Gaussian) random variable $X \sim \mathcal{N}(\mu, \sigma^2)$.
- Mean μ is the expected value, whereas variance σ^2 is the variability around μ.
- Ratio μ/σ: measure of deterministic-to-random balance.
    - In finance: known as *Sharpe ratio*.
    - In signal processing: *signal-to-noise ratio (SNR)* defined as μ^2/σ^2.

Random Processes in Finance:

- Investment returns as independent values observed over time: $X_t \sim \mathcal{N}(\mu, \sigma^2)$.
- Cumulative returns reflects accumulated wealth.
- Sharpe ratio μ/σ influences cumulative returns' growth and fluctuations.

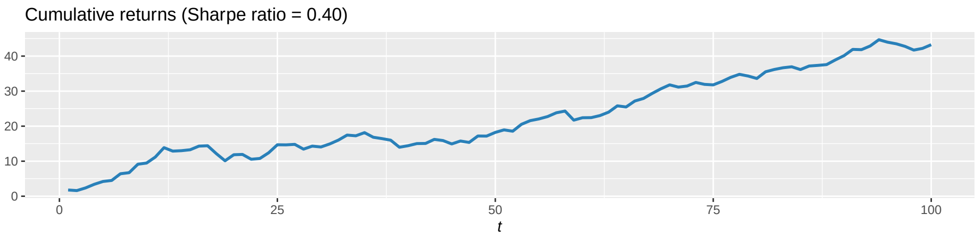
What is Portfolio Optimization?

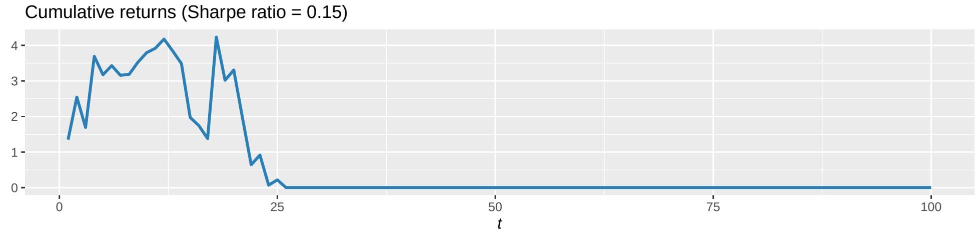
Illustration of random returns and cumulative returns:



What is Portfolio Optimization?

Illustration of cumulative returns with different values of Sharpe ratio:





What is Portfolio Optimization?

Improving Cumulative Returns: Investors can't change the random nature of assets but can exploit other dimensions:

Temporal Dimension:

- Distribution of X_t may change over time (μ_t, σ_t^2).
- Adapting investment size to current μ_t/σ_t can be beneficial.
- Requires time series modeling: data model at time t given past observations.

Asset Dimension:

- Choice of N assets, $X_i \sim \mathcal{N}(\mu, \sigma^2)$, potentially to invest in.
- Average returns $\frac{1}{N}\sum_{i=1}^{N} X_i$ preserve μ but reduce variance to σ^2/N.
- $1/N$ portfolio: distributes capital equally over N assets.
- Real-world challenge: returns X_i are correlated, affecting variance reduction.
- Portfolio optimization: allocate capital over correlated assets to minimize risk/variance.

Components:

- Financial data modeling for adapting investments over time.
- Portfolio optimization to properly allocate budget among correlated assets.

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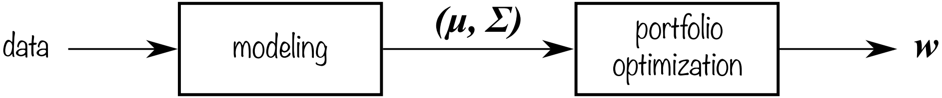
Big Picture

Block diagram of data modeling and portfolio optimization:



Data Modeling:

- Characterizes statistical distribution of future returns.
- Focuses on first- and second-order moments: $\boldsymbol{\mu}$ (mean vector) and $\boldsymbol{\Sigma}$ (covariance matrix).
- Foundation for portfolio optimization.

Portfolio Optimization:

- Utilizes $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ to produce optimal portfolio weights $\boldsymbol{w}$.
- Explores various formulations for optimizing portfolios.

Big Picture: Taxonomy of Portfolios

According to Data Used:

- *Second-order portfolios:* based on mean and variance. Examples include Markowitz mean-variance portfolio, maximum Sharpe ratio portfolio, index tracking portfolios, and volatility-based risk parity portfolios.
- *High-order portfolios:* utilize high-order moments and utility-based portfolio approximations.
- *Raw-data portfolios:* require raw data for construction. Include downside risk portfolios, semivariance portfolios, CVaR portfolios, drawdown portfolios, graph-based portfolios, and deep learning portfolios.

According to Efficient-Market Hypothesis (EMH):

- *Active portfolios:* aim to beat the market through selection or timing.
- *Passive portfolios:* track the market, minimizing frequent rebalancing.

According to Portfolio Formulation Nature:

- *Single-period portfolios:* based on a single future step.
- *Multi-period portfolios:* consider multiple future steps for long-term planning (Boyd et al. 2017).

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Outline of the Book “Portfolio Optimization” (Palomar 2025)

Part I: Financial Data – Focuses on financial data modeling for portfolio design:

- **Chapter 2:** Overviews financial data characteristics like non-stationarity, volatility clustering, heavy-tailed distributions, and asset correlation.
- **Chapter 3:** Covers i.i.d. modeling for financial data, including robust estimators, prior information incorporation, and non-Gaussian distribution challenges.
- **Chapter 4:** Discusses time series models for financial data, focusing on GARCH and stochastic volatility models, and Kalman filtering for efficient approximation.
- **Chapter 5:** Explores graph modeling techniques for financial assets, such as sparse Gaussian, low-rank, and heavy-tailed models, showcasing their analytical value.

Outline of the Book “Portfolio Optimization” (Palomar 2025)

Part II: Portfolio Optimization – Covers a wide variety of portfolio formulations:

- **Chapter 6:** Introduces portfolio basics, covering notation, returns, transaction costs, rebalancing, constraints, performance measures, heuristic portfolios, and risk-based portfolios.
- **Chapter 7:** Explores modern portfolio theory, focusing on mean-variance, maximum Sharpe ratio, Kelly, and utility-based portfolios, along with a universal algorithm.
- **Chapter 8:** Discusses portfolio backtesting challenges, pitfalls, and solutions, including multiple randomized backtests and stress testing with resampled data.
- **Chapter 9:** Covers high-order portfolios, introducing recent advances that make this approach practical despite past difficulties in parameter estimation, memory requirements, and optimization complexity.
- **Chapter 10:** Considers portfolios with alternative risk measures, such as downside risk, semivariance, CVaR, and drawdown, formulated in convex form for efficient optimization.

Outline of the Book “Portfolio Optimization” (Palomar 2025)

- **Chapter 11:** Presents risk parity portfolios, which diversify risk allocation using granular asset risk contributions, with emphasis on practical numerical algorithms.
- **Chapter 12:** Overviews graph-based portfolios that utilize graphical representations of asset relationships such as in hierarchical clustering portfolios.
- **Chapter 13:** Covers index tracking portfolios, including sparse index tracking, providing a state-of-the-art overview and introducing new formulations and algorithms for automatic sparsity selection.
- **Chapter 14:** Gives an overview of robust portfolios, addressing parameter estimation errors using robust optimization and resampling methods.
- **Chapter 15:** Explores pairs trading or statistical arbitrage portfolios, covering basics and sophisticated Kalman filtering techniques.
- **Chapter 16:** Presents deep learning portfolios, utilizing deep learning for financial time series analysis and portfolio optimization, acknowledging challenges and providing a starting point.

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Other Books

- **Financial Data Modeling:** Many excellent textbooks cover financial data modeling (Campbell, Lo, and MacKinlay 1997; Meucci 2005; Tsay 2010, 2013; Ruppert and Matteson 2015; Lütkepohl 2007; Fabozzi 2007; Fabozzi, Focardi, and Kolm 2010; Feng and Palomar 2016). The book (Palomar 2025, chaps. 3–4) provides a succinct overview of i.i.d. and temporal structure models, emphasizing heavy-tailed models, stochastic volatility models, and state-space models with Kalman filtering.
- **Modern Portfolio Theory:** Traditional books that focus primarily on portfolio foundations and mean–variance portfolios include (Grinold and Kahn 2000; Meucci 2005; Cornuejols and Tütüncü 2006; Fabozzi 2007; Prigent 2007; Michaud and Michaud 2008; Bacon 2008; Fabozzi, Focardi, and Kolm 2010). The book (Palomar 2025, chaps. 6–7) covers this material with an optimization perspective.
- **Risk Parity Portfolios:** Some standard references include (Roncalli 2013; Qian 2016). The book (Palomar 2025, chap. 11) covers risk parity portfolios from an optimization perspective, focusing on numerical algorithms.

Other Books

- **Backtesting:** The book (López de Prado 2018) covers backtesting and its dangers in great detail from the perspective of machine learning, while (Pardo 2008) describes the walk-forward backtest. The book (Palomar 2025, chap. 8) presents the many dangers of backtesting and the different forms of executing backtesting based on market data, as well as synthetic data.
- **Index Tracking:** The topic of index tracking is treated in detail in (Prigent 2007; Benidis, Feng, and Palomar 2018), with shorter treatments in (Cornuejols and Tütüncü 2006; Feng and Palomar 2016). The book (Palomar 2025, chap. 13) provides a concise, state-of-the-art exposure, offering new formulations and an algorithm for automatic sparsity selection.
- **Robust Portfolios:** Robust optimization is widely explored within the context of portfolio design, with standard references including (Fabozzi 2007; Cornuejols and Tütüncü 2006). This book (Palomar 2025, chap. 14) gives a concise presentation of these techniques for obtaining robust portfolios with illustrative numerical experiments.

Other Books

- **Pairs trading:** The standard reference to this topic is (Vidyamurthy 2004) (see also (Feng and Palomar 2016)). This book (Palomar 2025, chap. 15) covers the basics and presents a sophisticated use of Kalman filtering for better adaptability.

- **Machine Learning in Finance:** Recent textbooks that give a broad account of the use of machine learning in financial systems include (López de Prado 2018; Dixon, Halperin, and Bilokon 2020). This book (Palomar 2025, chap. 16) briefly discusses machine learning and deep learning techniques in portfolio design.

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