Portfolio Optimization

Introduction

Daniel P. Palomar (2025). *Portfolio Optimization: Theory and Application.* Cambridge University Press.

portfoliooptimizationbook.com

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Outline

Abstract

This course offers a deep dive into practical algorithms, departing from conventional Gaussian assumptions and exploring a wide range of portfolio formulations. A must for anyone interested in financial data modeling and portfolio design, it is suitable as portfolio optimization course and financial data modeling course (Palomar 2025, chap. 1).

Outline

Introduction

- **Introduction to Modern Portfolio Theory:**
  - Originated with Harry Markowitz's 1952 paper "Portfolio Selection" (Markowitz 1952).
  - Awarded the Nobel Prize in Economic Sciences in 1990.
  - Core concept: Investors optimize portfolios based on expected return and risk.
  - Real-world application issues led to various enhancements and alternatives.
- **Evolution of portfolio formulations:**
  - 70 years of research and practice revisiting Markowitz's formulation.
  - Developments include robust optimization, alternative risk measures, and regularization.
  - Incorporation of factor models, volatility clustering, and risk parity approaches.

Introduction

- **Scope of the book "Portfolio Optimization" (Palomar 2025):**
  - Focus on practical financial data modeling and portfolio optimization.
  - From mathematical formulations to practical numerical algorithms with code examples.
  - Transition from Gaussian assumptions to heavy-tailed distribution models.
  - Extensive use of Kalman filtering and advanced techniques for financial graphs.
- **Portfolio formulations explored:**
  - From Markowitz's mean-variance (1952) to maximum Sharpe ratio (1966) portfolios.
  - Advanced formulations: Kelly-based, utility-based, high-order, downside risk portfolios.
  - Coverage of semivariance, CVaR, drawdown, risk parity, and graph-based portfolios.
  - Includes robust, bootstrapped, bagged, pairs trading, and statistical arbitrage portfolios.
  - Exploration of deep learning applications in portfolio optimization.
- **Primary focus:**
  - Theoretical understanding and practical algorithms for portfolio optimization.

Outline

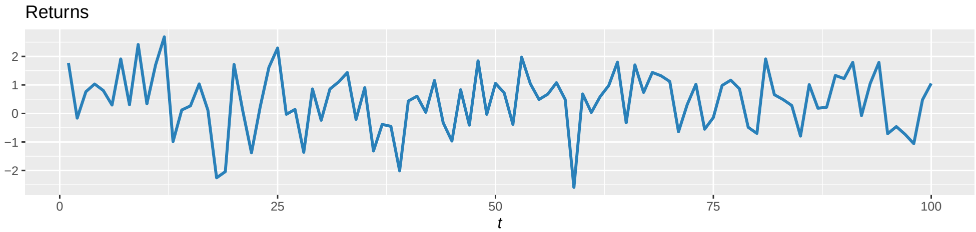
What is portfolio optimization?

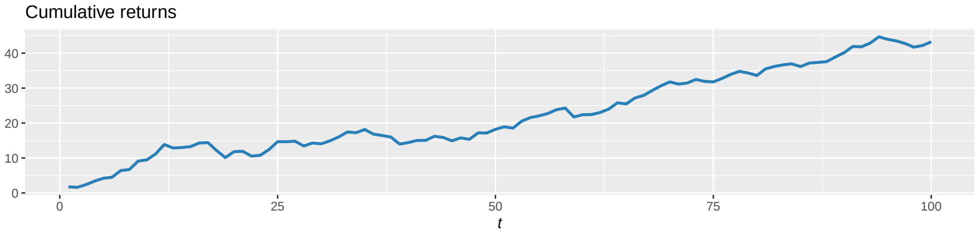
- **Random variables in finance:**
  - Random variable X with mean $\mu = \mathbb{E}[X]$ and variance $\sigma^2 = \mathbb{E}[(X-\mu)^2]$.
  - Example: Normal (Gaussian) random variable $X \sim \mathcal{N}(\mu, \sigma^2)$.
  - Mean μ is the expected value, whereas variance σ^2 is the variability around μ.
  - Ratio μ/σ: Measure of deterministic-to-random balance.
    - In finance: Known as *Sharpe ratio*.
    - In signal processing: *Signal-to-noise ratio (SNR)* defined as μ^2/σ^2.

- **Random processes in finance:**
  - Investment returns as independent values observed over time: $X_t \sim \mathcal{N}(\mu, \sigma^2)$.
  - Cumulative returns reflects accumulated wealth.
  - Sharpe ratio μ/σ influences cumulative returns' growth and fluctuations.

What is portfolio optimization?

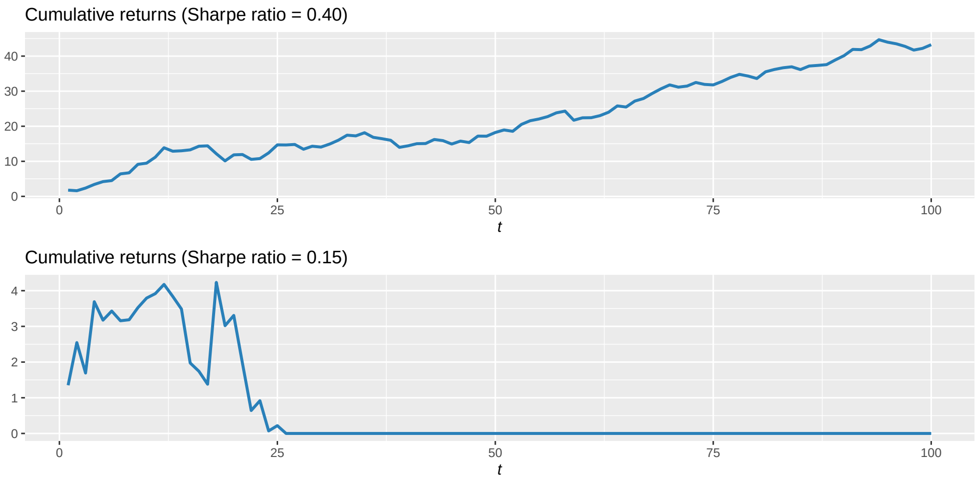
Illustration of random returns and cumulative returns:





What is portfolio optimization?

Illustration of cumulative returns with different values of Sharpe ratio:



What is portfolio optimization?

Improving cumulative returns: Investors can't change the random nature of assets but can exploit:

- **Temporal dimension:**
  - Distribution of X_t may change over time (μ_t, σ_t^2).
  - Adapting investment size to current μ_t/σ_t can be beneficial.
  - Requires time series modeling: Data model at time t given past observations.
- **Asset dimension:**
  - Choice of N assets, $X_i \sim \mathcal{N}(\mu, \sigma^2)$, potentially to invest in.
  - Average returns $\frac{1}{N}\sum_{i=1}^{N} X_i$ preserve μ but reduce variance to σ^2/N.
  - $1/N$ portfolio: Distributes capital equally over N assets.
  - Real-world challenge: Returns X_i are correlated, affecting variance reduction.
  - Portfolio optimization: allocate capital over correlated assets to minimize risk/variance.

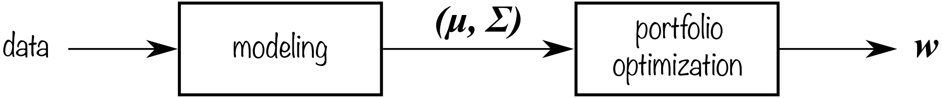
Components:

- **Financial data modeling** for adapting investments over time.
- **Portfolio optimization** to properly allocate budget among correlated assets.

Outline

Big picture

Block diagram of data modeling and portfolio optimization:



- **Components:**
  - **Data modeling:** Characterizes statistical distribution of future returns.
    - Focuses on first- and second-order moments: $\boldsymbol{\mu}$ (mean vector) and $\boldsymbol{\Sigma}$ (covariance matrix).
    - Foundation for portfolio optimization.
  - **Portfolio optimization:** Utilizes $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ to produce optimal portfolio weights $\boldsymbol{w}$.
    - Explores various formulations for optimizing portfolios.

Big picture: Taxonomy of portfolios

- **According to data used:**
  - *Second-order Portfolios:* Based on mean and variance. Examples include Markowitz mean-variance portfolio, maximum Sharpe ratio portfolio, index tracking portfolios, and volatility-based risk parity portfolios.
  - *High-order Portfolios:* Utilize high-order moments and utility-based portfolio approximations.
  - *Raw-data Portfolios:* Require raw data for construction. Include downside risk portfolios, semivariance portfolios, CVaR portfolios, drawdown portfolios, graph-based portfolios, and deep learning portfolios.
- **According to Efficient-Market Hypothesis (EMH):**
  - *Active Portfolios:* Aim to beat the market through selection or timing.
  - *Passive Portfolios:* Track the market, minimizing frequent rebalancing.
- **According to portfolio formulation nature:**
  - *Single-period portfolios:* Based on a single future step.
  - *Multi-period portfolios:* Consider multiple future steps for long-term planning (Boyd et al. 2017).

Outline

Outline of the book "Portfolio Optimization" (Palomar 2025)

Part I: Financial Data - Focuses on financial data modeling for portfolio design:

- Chapter 2 overviews financial data characteristics like non-stationarity, volatility clustering, heavy-tailed distributions, and asset correlation.
- Chapter 3 covers i.i.d. modeling for financial data, including robust estimators, prior information incorporation, and non-Gaussian distribution challenges.
- Chapter 4 discusses time series models for financial data, focusing on GARCH and stochastic volatility models, and Kalman filtering for efficient approximation.
- Chapter 5 explores graph modeling techniques for financial assets, such as sparse Gaussian, low-rank, and heavy-tailed models, showcasing their analytical value.

Outline of the book "Portfolio Optimization" (Palomar 2025)

Part II: Portfolio Optimization - Covers a wide variety of portfolio formulations:

- Chapter 6 introduces portfolio basics, covering notation, returns, transaction costs, rebalancing, constraints, performance measures, heuristic portfolios, and risk-based portfolios.
- Chapter 7 explores modern portfolio theory, focusing on mean-variance, maximum Sharpe ratio, Kelly, and utility-based portfolios, along with a universal algorithm.
- Chapter 8 discusses portfolio backtesting challenges, pitfalls, and solutions, including multiple randomized backtests and stress testing with resampled data.
- Chapter 9 covers high-order portfolios, introducing recent advances that make this approach practical despite past difficulties in parameter estimation, memory requirements, and optimization complexity.
- Chapter 10 considers portfolios with alternative risk measures, such as downside risk, semivariance, CVaR, and drawdown, formulated in convex form for efficient optimization.

Outline of the book "Portfolio Optimization" (Palomar 2025)

- Chapter 11 presents risk parity portfolios, which diversify risk allocation using granular asset risk contributions, with emphasis on practical numerical algorithms.
- Chapter 12 overviews graph-based portfolios that utilize graphical representations of asset relationships such as in hierarchical clustering portfolios.
- Chapter 13 covers index tracking portfolios, including sparse index tracking, providing a state-of-the-art overview and introducing new formulations and algorithms for automatic sparsity selection.
- Chapter 14 gives an overview of robust portfolios, addressing parameter estimation errors using robust optimization and resampling methods.
- Chapter 15 explores pairs trading or statistical arbitrage portfolios, covering basics and sophisticated Kalman filtering techniques.
- Chapter 16 presents deep learning portfolios, utilizing deep learning for financial time series analysis and portfolio optimization, acknowledging challenges and providing a starting point.

Outline

Other books

- **Financial data modeling**: Many excellent textbooks cover financial data modeling (Campbell, Lo, and MacKinlay 1997; Meucci 2005; Tsay 2010, 2013; Ruppert and Matteson 2015; Lütkepohl 2007; Fabozzi 2007; Fabozzi, Focardi, and Kolm 2010; Feng and Palomar 2016). The book (Palomar 2025, chaps. 3–4) provides a succinct overview of i.i.d. and temporal structure models, emphasizing heavy-tailed models, stochastic volatility models, and state-space models with Kalman filtering.
- **Modern portfolio theoy**: Traditional books that focus primarily on portfolio foundations and mean–variance portfolios include (Grinold and Kahn 2000; Meucci 2005; Cornuejols and Tütüncü 2006; Fabozzi 2007; Prigent 2007; Michaud and Michaud 2008; Bacon 2008; Fabozzi, Focardi, and Kolm 2010). The book (Palomar 2025, chaps. 6–7) covers this material with an optimization perspective.
- **Risk parity portfolios**: Some standard references include (Roncalli 2013; Qian 2016). The book (Palomar 2025, chap. 11) covers risk parity portfolios from an optimization perspective, focusing on numerical algorithms.

Other books

- **Backtesting**: The book (López de Prado 2018) covers backtesting and its dangers in great detail from the perspective of machine learning, while (Pardo 2008) describes the walk-forward backtest. The book (Palomar 2025, chap. 8) presents the many dangers of backtesting and the different forms of executing backtesting based on market data, as well as synthetic data.
- **Index tracking**: The topic of index tracking is treated in detail in (Prigent 2007; Benidis, Feng, and Palomar 2018), with shorter treatments in (Cornuejols and Tütüncü 2006; Feng and Palomar 2016). The book (Palomar 2025, chap. 13) provides a concise, state-of-the-art exposure, offering new formulations and an algorithm for automatic sparsity selection.
- **Robust portfolios**: Robust optimization is widely explored within the context of portfolio design, with standard references including (Fabozzi 2007; Cornuejols and Tütüncü 2006). This book (Palomar 2025, chap. 14) gives a concise presentation of these techniques for obtaining robust portfolios with illustrative numerical experiments.

Other books

- **Pairs trading**: The standard reference to this topic is (Vidyamurthy 2004) (see also (Feng and Palomar 2016)). This book (Palomar 2025, chap. 15) covers the basics and presents a sophisticated use of Kalman filtering for better adaptability.

- **Machine learning in finance**: Recent textbooks that give a broad account of the use of machine learning in financial systems include (López de Prado 2018; Dixon, Halperin, and Bilokon 2020). This book (Palomar 2025, chap. 16) briefly discusses machine learning and deep learning techniques in portfolio design.

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