Tutorial 7-9: Vector Algebra

1. Find the values of $x, y \text{ and } z$ so that the vectors $\bar{a} = xi + 2j + zk \text{ and } \bar{b} = 2i + yj + k$ are equal.
2. Let $\bar{a} = i + 2j \text{ and } \bar{b} = 2i + j$. Is $|\bar{a}| = |\bar{b}|$? Are the vectors $\bar{a} \text{ and } \bar{b}$ equal?
3. Find unit vector in the direction of vector $\bar{a} = 2i + 3j + k$.
4. Find the unit vector in the direction of the sum of the vectors, $\bar{a} = 2i + 2j - 5k \text{ and } \bar{b} = 2i + j + 3k$.
5. Find the vector joining the points $P(2, 3, 0)$ and $Q(-1, -2, -4)$ directed from P to Q.
6. Find a vector in the direction of vector $\vec{a} = \hat{\imath} - 2\hat{\jmath}$ that has magnitude 7 units.
7. Show that the points A (2, -1, 1), B(1, -3, -5) C(3, -4, -4) are the vertices of a right angled triangle.
8. Find the unit vector parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$ where $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{b} = 2\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\vec{c} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$.
9. Show that the points with position vectors $5\vec{a} + 6\vec{b}$, $7\vec{a} - 8\vec{b}$ and $3\vec{a} + 20\vec{b}$ are collinear.
10. Find the value of x for which $x(\hat{\imath} + \hat{\jmath} + \hat{k})$ is a unit vector.
11. Find the position vector of the midpoint of the vector joining the points P(2, 3, 4) and Q(4, 1, –2).
12. Write the direction ratios of the vector $(\hat{\imath} + 2\hat{\jmath} + \widehat{3k})$ and find its direction cosines.
13. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes 1 and 2 respectively and when $\vec{a} \cdot \vec{b} = 1$.
14. If $\vec{a}$ is a unit vector and $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 8$, , then find $|\vec{x}|$.
15. Find the angle between the vectors , $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$, $\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$.
16. Find the projection of the vector on the vector $\hat{\imath} - \hat{\jmath}$ on $\hat{\imath} + \hat{\jmath}$.
17. Evaluate the product $(3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + 7\vec{b})$.
18. If $\vec{a} = 2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, $\vec{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{c} = 3\hat{\imath} + \hat{\jmath}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ.
19. If $\vec{a}$ is a nonzero vector of magnitude 'a', and λ is a nonzero scalar such that $\lambda\vec{a}$ is a unit vector, then find the value of λ.

20. Show that the points A, B and C with position vectors, $\vec{a} = 3\hat{\imath} - 4\hat{\jmath} - 4\hat{k}$, $\vec{b} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\vec{c} = \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$. respectively form the vertices of a right angled triangle.
21. Find $|\vec{a} \times \vec{b}|$, if $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$, $\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$.
22. Find the unit vector perpendicular to each of the vector if $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$, where $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$, $\vec{b} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$.
23. Find λ and μ if $(2\hat{\imath} + 6\hat{\jmath} + 27\widehat{k}) \times (\hat{\imath} + \lambda\hat{\jmath} + \mu\widehat{k}) = 0$.
24. Find the area of the triangle with vertices A(1, 1, 2), B(2, 3, 5) and C(1, 5, 5).
25. Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{\imath} - \hat{\jmath} + 3\hat{k}$, $\vec{b} = 2\hat{\imath} - 7\hat{\jmath} + \hat{k}$.
26. Let the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}| = 3$ and $|\vec{b}| = \frac{\sqrt{2}}{3}$, then $\vec{a} \times \vec{b}$ is a unit vector, then what is the angle between $\vec{a}$ and $\vec{b}$?
27. Find sine of the angle between the vectors $\vec{a} = 3\hat{\imath} - \hat{\jmath} + 4\hat{k}$, $\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$.