Unit IV – Permutations and Combinations

Introduction

In our day to day life we come across many situations where we make use of permutations and combinations knowingly or unknowingly. It is based on some basic counting techniques. These techniques will be useful in determining the number of different ways of arranging and selecting objects without actually listing them. As a first step, we shall examine a principle which is most fundamental to the learning of these techniques.

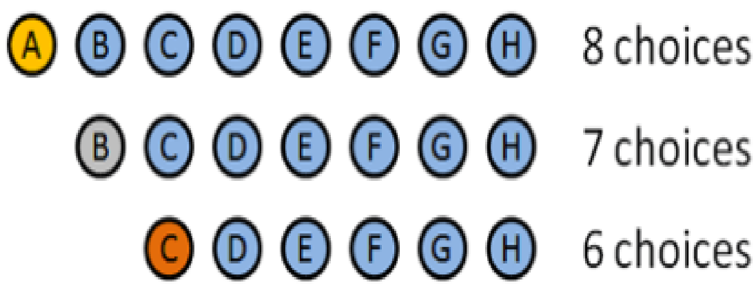
Fundamental principle of counting (Multiplication principal or product rule)

"If an event can occur in m different ways, following which another event can occur in n different ways, then the total number of occurrence of the events in the given order is $m \times n$."

Example:

Example

Suppose we have eight contestants battling for a First, Second and Third position in a 100m race. How many ways can we award a 1st, 2nd and 3rd place prize among eight contestants? (Gold / Silver / Bronze)



Gold medal: 8 choices: A B C D E F G H (Anyone can get the gold, lets mark A as the person who wins the gold)

Silver medal: 7 choices: B C D E F G H. Let's say B wins the silver.

Bronze medal: 6 choices: C D E F G H. Let's say… C wins the bronze.

So we had 8 choices at first, then 7 choices for the second, then 6 choices for the third. The total number of options are $8 \times 7 \times 6 = 336$. ____

Note:

- The word ***and*** suggests multiplying the possibilities.
- The word ***or*** suggests adding the possibilities.
- Problems with restrictions - when there are restrictions, the restrictions should be considered first
- Problems without restrictions - when there are no restrictions involved, the problems can be solved in any order.

Problems

1. Find the number of 4 letter words, with or without meaning which can be formed out of the letters of the word ROSE assuming that

   a. Repetition of the letters is not allowed.
   b. Repetition of the letters is allowed.

Solution:

a) Repetition of the letters is not allowed

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The first place can be filled in 4 different ways by any letter.

The second place in 3 different ways, third place is 2 ways and the fourth place in 1 ways. Therefore by multiplication rule, the number of words formed are

$4 \times 3 \times 2 \times 1 = 24$

b) Repetition of the letters is allowed

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The first place can be filled in 4 different ways by any letter.

The second place in 4 different ways, third and fourth place also in 4 ways Therefore by multiplication rule, the number of words formed are

$4 \times 4 \times 4 \times 4 = 256$

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2. How many 3 digit even numbers can be formed from the digits $1, 2, 3, 4, 5, 6$ if the digits can be repeated.

Solution:

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For the number to be even, the last placeholder should an even number. Therefore there are 3 ways for the third placeholder. As the numbers can be repeated, the first and second place can be filled in 6 ways. Therefore by multiplication rule, the number of even digits formed are

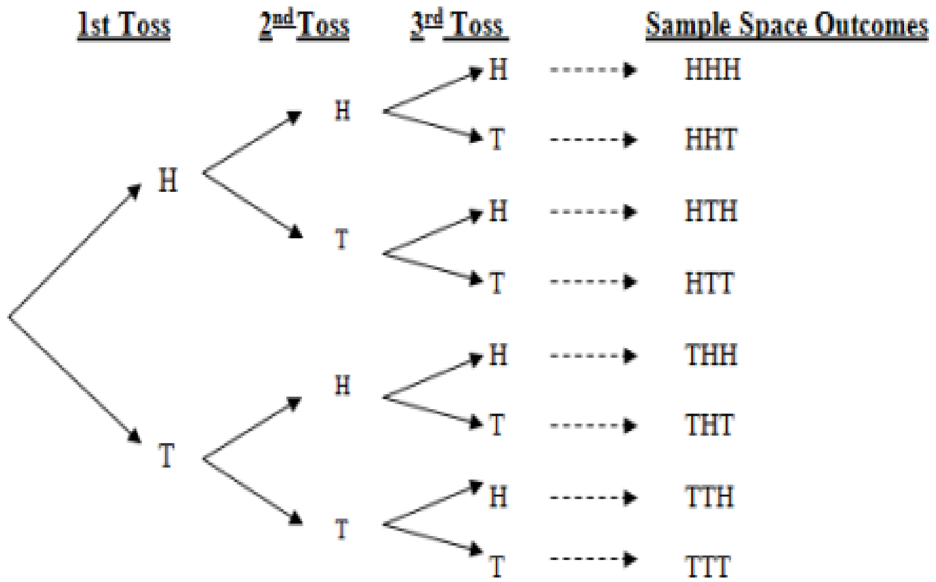
$6 \times 6 \times 3 = 108$

3. A coin is tossed 3 times and the outcomes are recorded. How many possible outcomes are there?

Solution:

Sometimes drawing tree diagram helps in understanding the problems in a better way.

Consider the first toss and record the outcomes, the second toss and its outcomes, third toss and the outcomes.



Thus we can say that when we toss a coin thrice, in all we have eight outcomes.

In other words you can also say that it has 2^3 outcomes.

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4. Find the number of different signals that can be generated by arranging at least 2 flags in order (one below the other) on a vertical staff, if five different flags are available.

Solution:

The following are the different ways to generate signals (by arranging atleast 2 or 3 or 4 or 5)

----- ----- (1)

----- ----- ----- (2)

----- ----- ----- ------ (3)

----- ------ ----- ------ ----- (4)

For (1) The first place can be filled in 5 ways. The second place holder can be filled in 4 ways. Therefore by multiplication rule, the number of different signals that can be generated is $5 \times 4 = 20$

For (2)

$5 \times 4 \times 3 = 60$

For (3)

$5 \times 4 \times 3 \times 2 = 120$

For (4)

$5 \times 4 \times 3 \times 2 \times 1 = 120$

Adding them, we get $20 + 60 + 120 + 120 = 320$

Summary

Many problems in probability and statistics require careful analysis of complex events. Combinatorics basic roots are to develop systematic ways of counting. These systematic counting methods will allow the solving of complex counting problems that are useful in all facets of life.

- Counting objects with restrictions We will continue to use the fundamental counting principle and use "blanks" instead of the tree method, but it is important that we count the restricted value first!
- If the problem is without a restriction then you can start with any blank or in any order.
- If data is small then the tree method helps visualize problems better.
- 'AND' refers to multiplication whereas 'OR' refers to addition.

Unit III – Homework 1

1. How many 3-digit numbers can be formed with the digits 1,4,7,8 and 9 if the digits are not repeated.
2. In a school there are 5 English teachers, 7 Hindi teachers and 3 French teachers. A three member committee is to be formed with one teacher representing each language. In how many ways can this be done?
3. How many three digit numbers greater than 600 can be formed using the digits 1,2,5,6,8 without repeating the digits?
4. A person wants to make a time table for 4 periods. He has to fix one period each for English, Mathematics, Economics and Commerce. How many different time tables can he make?

Factorial notation

It is denoted by the product of first n natural numbers.

$n! = 1 \times 2 \times 3 \times \cdots \times (n-1) \times n$ or

$n! = n \times (n-1) \ldots \times 3 \times 2 \times 1$. We read the symbol as n factorial.

$1! = 1$,
$2! = 2 \times 1$,
$3! = 3 \times 2 \times 1$,
$4! = 4 \times 3 \times 2 \times 1$
$n! = n \times (n-1)!$
We define $\mathbf{0}! = \mathbf{1}$
$n! = n \times (n-1)!$
$5! = 5 \times 4!$

Problems

1. Evaluate
   a. $8!$
   b. $4! - 3!$

Solution:

a) $8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40320$

b) $4! = 4 \times 3 \times 2 \times 1 = 24$, $3! = 3 \times 2 \times 1 = 6$

Therefore $4! - 3! = 24 - 6 = 18$

Alternatively, $4! - 3! = 4 \times 3! - 3! = 3!\,(4-1) = 3! \times 3 = 3 \times 2 \times 1 \times 3 = 18$

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2. Compute $\frac{8!}{6! \times 2!}$

Solution:

$\frac{8!}{6! \times 2!} = \frac{8 \times 7 \times 6!}{6! \times 2 \times 1} = \frac{8 \times 7 \times 6!}{6! \times 2 \times 1} = \frac{8 \times 7}{2 \times 1} = 28$

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3. If $\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$ find x.

Solution:

Given $\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!} \Rightarrow \frac{1}{6!} + \frac{1}{7 \times 6!} = \frac{x}{8 \times 7 \times 6!} \quad \Rightarrow \frac{1}{6!}\left(1 + \frac{1}{7}\right) = \frac{x}{6!(8 \times 7)}$

$\Rightarrow \left(1 + \frac{1}{7}\right) = \frac{x}{(56)} \Rightarrow \left(\frac{8}{7}\right) = \frac{x}{(56)} \Rightarrow \frac{8 \times 56}{7} = x \Rightarrow x = 64$

Permutations

Permutation is an arrangement of objects in a specific order, where either all the objects or some of them can be considered at a time.

Permutations when all the objects are distinct:

- The number of permutations of n different things taken r at a time, where repetition is not allowed, is denoted by ${}^{n}P_{r}$ and is given by ${}^{n}P_{r} = \frac{n!}{(n-r)!}$ where $0 \leq r \leq n$.
- The number of permutations of n different things, taken r at a time, where repetition is allowed, is n^{r}.

Permutations when all the objects are not distinct objects:

- The number of permutations of n objects, where p objects are of the same kind and rest are all different $=\frac{n!}{p!}$
- The number of permutations of n objects taken all at a time, where p_1 objects are of first kind, p_2 objects are of the second kind, ..., p_k objects are of the k^th kind and rest, if any, are all different $=\frac{n!}{p_1!p_2!p_3!..p_k!}$

Problems

1. How many 4 digit numbers can be formed by using the digits 1 to 9 if repetition of the digits is not allowed.

Solution:

The number of permutations of $n$different objects taken r at a time is $^{n}P_{r}$

The required 4-digit numbers are $^{9}P_{4} = \frac{9!}{(9-4)!} = \frac{9!}{5!} = \frac{9\times8\times7\times6\times5!}{5!}$

$= 9 \times 8 \times 7 \times 6 = 3024$

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2. How many 4 digit numbers are there with no digit repeated?

Solution:

Method 1

The 4digit numbers have to be made from 0 to 9. Zero cannot be included in the first place holder (in the beginning) as the numbers will be 3 digit number.

Required 4 digit number = total 4 digit number – 4 digit number having zero in the beginning

Total 4 digit number

n=10, r=4. Therefore $^{10}P_{4} = \frac{10!}{(10-4)!} = 10 \times 9 \times 8 \times 7$=5040

4 digit number having zero in the beginning

n=9, r=3. Therefore $^{9}P_{3} = \frac{9!}{(9-3)!} = 9 \times 8 \times 7$=504

Therefore,

Required 4 digit number =5040-504=4536

Method 2

The 4digit numbers have to be made from 0 to 9. Hence the number of digits are 10

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The first place holder cannot have zero. Therefore it can be filled in 9 ways. The second place holder is also filled in 9 ways, third place holder in 8 ways and for the last place holder in 7 ways. By multiplication rule, the required 4-digit numbers are = $9 \times 9 \times 8 \times 7 = 4536$

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3. Find the numbers of different 8 letter arrangements that can be made from the letters of the word DAUGHTER so that
   a. All vowels occur together
   b. All vowels do not occur together

Solution:

There are 8 different letters in DAUGHTER, and there are 3 vowels.

a) All vowels occur together.

Consider the three vowels as one object. Therefore there are 6 letters

--Vowels--- -------- -------- -------- ------- --------

n=6, r=6. Therefore ${}^{6}P_{6} = \frac{6!}{(6-6)!} = \frac{6!}{0!} = 6 \times 5 \times 4 \times 3 \times 2 \times 1$=720

Arranging the vowels

n=3, r=3. Therefore ${}^{3}P_{3} = \frac{3!}{(3-3)!} = \frac{3!}{0!} = 3 \times 2 \times 1$=6

Therefore total number of arrangements are 720×6=4320

b) All vowels do not occur together.

Total number of arrangements=number of arrangements-number of arrangements of all vowels together

Number of arrangements

n=8, r=8. Therefore ${}^{8}P_{8} = \frac{8!}{(8-8)!} = \frac{8!}{0!} = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$=40320

Total number of arrangements of all vowels not together =

40320-4320=36000

4. Find $r$if
   a. ${}^{5}P_{r} = 2\ {}^{6}P_{r-1}$ b. ${}^{5}P_{r} = {}^{6}P_{r-1}$

Solution:

a) $\quad {}^{5}P_r = 2\ {}^{6}P_{r-1}, \Rightarrow \frac{5!}{(5-r)!} = 2\left(\frac{6!}{(6-r+1)!}\right)$

$\Rightarrow \frac{5!}{2\times 6!} = \frac{(5-r)!}{(7-r)!} \Rightarrow \frac{5!}{2\times 6\times 5!} = \frac{(5-r)!}{(7-r)\times(6-r)\times(5-r)!}$

$\Rightarrow \frac{1}{12} = \frac{1}{(7-r)\times(6-r)} \Rightarrow (7-r)\times(6-r) = 12 \Rightarrow r^2 - 13r + 42 = 12$

$\Rightarrow r^2 - 13r + 30 = 0 \Rightarrow (r-3)(r-10) = 0 \Rightarrow r = 3,10$. Rejecting $r = 10$ as ${}^{n}P_r$ is defined only for $0 \le r \le n$. Therefore $r = 3$

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5. In how many ways can 4red, 3yellow and 2green discs be arranged in a row if the discs of the same color are indistinguishable?

Solution:

There are 9 discs. As the colors are repeating, we have

$\frac{9!}{4!3!2!} = \frac{9\times 8\times 7\times 6\times 5\times 4!}{4!\times 3\times 2\times 1\times 2\times 1} = \frac{9\times 8\times 7\times 6\times 5}{3\times 2\times 1\times 2\times 1} = \frac{15120}{12} = 1260$

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6. Find the numbers of arrangements of the letters of the word INDEPENDENCE. In how many of these arrangements,
   a. Do the words start with P.
   b. Do all the vowels always occur together.
   c. Do the vowels never occur together.
   d. Do the words begin with I and end in P.

Solution:

a) The words start with P.

Total letters are $n = 12$, N repeated 3 times $p_1 = 3$, D repeated 2 times $p_2 = 2$, E repeated 4 times $p_3 = 4$

Total arrangements: $\frac{n!}{p_1!p_2!p_3!} \Rightarrow \frac{12!}{3!2!4!} = 1663320$

Words starting with P.

Fix the first position with P. Number of letters are $n = 11$, $p_1 = 3$, $p_2 = 2$, $p_3 = 4$

Required words are $\frac{11!}{3!2!4!} = 138600$

b) All vowels occur together

There 5 vowels. Consider them as one object. Therefore there are 8 objects.
i.e. $n = 8, p_1 = 3, p_2 = 2$

$$\frac{8!}{3!\,2!}$$

Arranging the vowels

$n = 5, p_1 = 4 \Rightarrow \frac{5!}{4!}$. Therefore required arrangements are

$$\frac{8!}{3!\,2!} \times \frac{5!}{4!} = 16800$$

c) Vowels do not occur together

The required arrangement=total arrangement-arrangement with the vowels together

$$1663320 - 16800 = 1646400$$

d) Words begin with I and end in P

Fix the position of the first placeholder and the last place holder. The number of letters are 10

$n = 10, p_1 = 3, p_2 = 2, p_3 = 4$

Required arrangement is $\frac{10!}{3!2!4!} = 12600$

Unit III – Homework 2

1. 5 students are staying in a dormitory. In how many ways can you allot 5 beds to them? Ans: 120
2. In how many ways can the letters of the word 'TRIANGLE' be arranged? Ans: 40320
3. How many four digit numbers can be formed with digits 1, 2, 3 and 4 and with distinct digits? Ans: 24
4. Mr. Gupta with Ms. Gupta and their four children are travelling by train. Two lower berths, two middle berths and 2 upper berths have been allotted to them. Mr. Gupta has undergone a knee surgery and needs a lower berth while Ms. Gupta wants to rest during the journey and needs an upper berth. In how many ways can the berths be shared by the family? Ans: 96

2. Consider the word UNBIASED. How many words can be formed with the letters of the word in which no two vowels are together? Ans: 1152

3. There are 4 books on Mathematics, 5 books on English and 6 books on Science. In how many ways can you arrange them so that books on the same subject are together and they are arranged in the order Mathematics, English, Science. Ans: 2073600

5. 4 boys and 3 girls are to be seated in 7 chairs such that no two boys are together. In how many ways can this be done? Ans: 144

6. Find the number of permutations of the letters of the word 'TENDULKAR', in each of the following cases:
(i) beginning with T and ending with R. (ii) vowels are always together.
(iii) vowels are never together. Ans: (i) 5040 (ii) 30240 (iii) 332640

Combinations

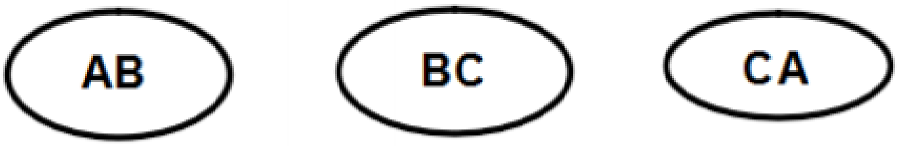
Combinations is an arrangement of objects in which order is not important, where either all the objects or some of them can be considered at a time.

Combinations is merely counting the number of ways in which some or all objects at a time are selected.

Examples:

- Suppose there are 3 lawn Tennis players A, B, C. A team consisting of 2 players is to be formed. In how many ways can we do so? Is the team of A and B different from the team of B and A? Here, order is not important.

In fact, there are only 3 possible ways in which the team could be constructed.



- Suppose there are 12 children who have gone for a group project to a different work place. How many handshakes will take place in the room with these 12 students?

Twelve persons meet in a room and each shakes hand with all the others. How do we determine the number of hand shakes. X shaking hands with Y and Y with X will not be two different hand shakes. Here, order is not important. There will be as many hand shakes as there are combinations of 12 different things taken 2 at a time.

Combinations formulae

The number of combinations of n different things taken r at a time, denoted by ${}^{n}C_{r}$, is given by ${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$, $0 \le r \le n$.

We define ${}^{n}C_{0} = 1$, ${}^{n}C_{n} = 1$.

Note:

- ${}^{n}C_{r} = \frac{{}^{n}P_{r}}{r!}$
- ${}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$

Problems

1. What is the number of ways of choosing 4from a pack of 52 playing cards? In how many ways of these
    a. 4cards are of the same suit
    b. 4cards belong to 4 different suits
    c. Are face cards
    d. 2are red cards and 2 are black cards
    e. Cards are of the same color

Solution:

a) 4 cards are of the same suit

There are 4 suits - Diamond, Spade, Heart, Club and 13 cards of each suit

	Total	No. of card to be chosen	Number of ways
Diamond	13	4	${}^{13}C_{4}$
Spade	13	4	${}^{13}C_{4}$
Heart	13	4	${}^{13}C_{4}$
Club	13	4	${}^{13}C_{4}$

4 cards can be chosen from diamond or spade or heart or club, so we add the number of ways. The required number of ways of choosing four cards of the same suit

$= {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4$

$= 4 \times {}^{13}C_4$

$= 4 \times \frac{13!}{4!(13-4)}$

$= 4 \times \frac{13!}{4!\,9!}$

$= 4 \times \frac{13 \times 12 \times 11 \times 10 \times 9!}{4! \times 3 \times 2 \times 1 \times 9!}$

$= 4 \times \frac{13 \times 12 \times 11 \times 10}{4 \times 3 \times 2 \times 1}$

= **2860 ways**

b) 4 cards belong to 4 different suits

	Total	No. of card to be chosen	Number of ways
Diamond	13	1	${}^{13}C_1$
Spade	13	1	${}^{13}C_1$
Heart	13	1	${}^{13}C_1$
Club	13	1	${}^{13}C_1$

Since one card is to be chosen from diamond, spade, heart and club, therefore the required number of ways of choosing four cards from each suit is

$= {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1$

$= ({}^{13}C_1)^4$

$= \left(\frac{13!}{1!(13-1)!}\right)^4$

$$= \left(\frac{13!}{1!12!}\right)^4$$

$$= \left(\frac{13 \times 12!}{12!}\right)^4$$

$$= (13)^4$$

$$= 13 \times 13 \times 13 \times 13$$

$$= \mathbf{28561 \text{ ways}}$$

c) Are face cards

King, Queen and Jack are face cards. Therefore the number of face cards in one suit is 3 and the total number of face cards in 4 suits is 12.

Thus $n = 12$and $r = 4$. The required number of ways of choosing face cards is ${}^{12}C_4$

$$= \frac{12!}{4!(12-4)!}$$

$$= \frac{12!}{4!\,8!}$$

$$= \frac{12 \times 11 \times 10 \times 9 \times 8!}{4 \times 3 \times 2 \times 1 \times 8!}$$

$$= \frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1}$$

$$= \mathbf{495 \text{ ways}}$$

d) 2are red cards and 2 are black cards

	Total	No. of card to be chosen	Number of ways
Red	26	2	${}^{26}C_2$
Black	26	2	${}^{26}C_2$

Total number of ways choosing 2 red & 2 black cards

$= {}^{26}C_2 \times {}^{26}C_2$

$= ({}^{26}C_2)^2$

$= \left(\frac{26!}{2!\ (26-2)!}\right)^2$

$= \left(\frac{26!}{2!\ 24!}\right)^2$

$= \left(\frac{26 \times 25 \times 24!}{2 \times 1 \times 24!}\right)^2$

$= \left(\frac{26 \times 25}{2 \times 1}\right)^2$

$= (13 \times 25)^2$

$= (325)^2$

= **105625**

e) Cards are of the same color

	Total	No. of card to be chosen	Number of ways
Red	26	4	${}^{26}C_4$
Black	26	4	${}^{26}C_4$

The required number of ways are

$= {}^{26}C_4 + {}^{26}C_4$

$= 2 \times \frac{26!}{4!(26-4)!}$

$= 2 \times \frac{26!}{4!\ 22!}$

$= 2 \times \frac{26 \times 25 \times 24 \times 23 \times 22!}{4 \times 3 \times 2 \times 1 \times 22!}$

$= 2 \times \frac{26 \times 25 \times 24 \times 23}{4 \times 3 \times 2 \times 1}$

= 29900

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2. Determine n if ${}^{2n}C_3 : {}^{n}C_3 = 12:1$

Solution:

${}^{2n}C_3 : {}^{n}C_3 = 12:1 \Rightarrow \frac{{}^{2n}C_3}{{}^{n}C_3} = \frac{12}{1} \Rightarrow \frac{2n\times(2n-1)\times(2n-2)}{n\times(n-1)\times(n-2)} = \frac{12}{1}$

$\Rightarrow \frac{2\times(2n-1)\times 2(n-1)}{(n-1)\times(n-2)} = \frac{12}{1} \Rightarrow \frac{4\times(2n-1)}{(n-2)} = \frac{12}{1}$

$\Rightarrow \frac{(2n-1)}{(n-2)} = \frac{3}{1} \Rightarrow 2n-1 = 3n-6 \Rightarrow 5 = n$

Therefore $n = 5$

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3. Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each color.

Solution:

Number of ways of selecting 3 red balls from 6 red balls is

${}^{6}C_3 = \frac{6!}{3!(6-3)!} = \frac{6\times5\times4}{3\times2\times1} = 20$

Number of ways of selecting 3 white balls from 5 white balls is

$5C_3 = \frac{5!}{3!(5-3)!} = \frac{5\times4\times3}{3\times2\times1} = 10$

Number of ways of selecting 3 blue balls from 5 blue balls is

$5C_3 = \frac{5!}{3!(5-3)!} = \frac{5\times4\times3}{3\times2\times1} = 10$. Thus, the number of ways of selecting 3 balls of each color

= (No. of ways selecting 3 red balls)

× (No. of ways selecting 3 white balls)

× (No. of ways selecting 3 blue balls)

= 20 × 10 × 10

= **2000**

4. In how many ways can a student choose a programme of 5 courses if 9 courses are available and 2 specific courses are compulsory for every student?

Solution:

	Total number	Number to be chosen
Compulsory Courses	2	2
Other courses	7	3
	9	5

The student has to select 3 courses from the remaining 7 courses. This can be done in $7C_3 = \frac{7!}{3!(7-3)!} = \frac{7\times6\times5}{3\times2\times1} = 35$ ways.

5. How many chords can be drawn through 21 points on a circle?

Solution:

A chord is formed by joining 2 points. The number of chords drawn through 21 points is $21C_2 = \frac{21!}{2!(21-2)!} = \frac{21\times20}{2\times1} = 210$

6. In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers?

Solution:

	Total number	Number to be chosen	Number of ways to choose
Bowler	5	4	${}^{5}C_4$
Other	12	7	${}^{12}C_7$
	17	11	

Total number ways = ${}^{5}C_4 \times {}^{12}C_7$

$$= \frac{5!}{4!(5-4)!} \times \frac{12!}{7!(12-7)!}$$

$$= \frac{5!}{4!1!} \times \frac{12!}{7!5!}$$

$$= \frac{12!}{4! \times 7!}$$

$$= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7!}{(4 \times 3 \times 2 \times 1) \times 7!}$$

$$= 3960$$

Application problems

1. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has
   a. No girl
   b. At least one boy and one girl
   c. At least 3 girls

Solution:

a) no girl to be chosen

	Total number	Number to be chosen	Number of ways to choose
Girls	4	0	${}^{4}C_0$
Boys	7	5	${}^{7}C_5$

Total number of ways = $^4C_0 \times {}^7C_5$

$$= \frac{4!}{0!(4-0)!} \times \frac{7!}{5!(7-5)!}$$

$$= \frac{4!}{1 \times (4)!} \times \frac{7!}{5!2!}$$

$$= 1 \times \frac{7 \times 6 \times 5!}{5! \times 2 \times 1} = \frac{7 \times 6}{2} = 21$$

b) At least one boy and one girl

Option 1 - 1 boys, and 4 girls

Option 2 - 2 boys, and 3 girls

Option 3 - 3 boys, and 2 girls

Option 4 - 4 boys, and 1 girls

We have to calculate all these combinations separately and then add it

Option 1

Number of ways selecting 1 boy & 4 girls

$$= {}^7C_1 \times {}^4C_4$$

$$= \frac{7!}{1!(7-1)!} \times \frac{4!}{4!(4-4)!}$$

$$= \frac{7!}{1!\,6!} \times \frac{4!}{4!\,0!}$$

$$= \frac{7 \times 6!}{6!} \times \frac{4!}{4!}$$

$$= 7 \times 1$$

$$= \mathbf{7}$$

Option 2

Number of ways selecting 2 boys & 3 girls

$$= {}^{7}C_{2} \times {}^{4}C_{3}$$

$$= \frac{7!}{2!(7-2)!} \times \frac{4!}{3!(4-3)!}$$

$$= \frac{7!}{2!\,5!} \times \frac{4!}{3!\,1!}$$

$$= \frac{7 \times 6 \times 5!}{2!\,5!} \times \frac{4 \times 3!}{3!}$$

$$= 21 \times 4$$

$$= \mathbf{84}$$

Option 3

Number of ways selecting 3 boys & 2 girls

$$= {}^{7}C_{3} \times {}^{4}C_{2}$$

$$= \frac{7!}{3!(7-3)!} \times \frac{4!}{2!(4-2)!}$$

$$= \frac{7!}{3!\,4!} \times \frac{4!}{2!\,2!}$$

$$= \frac{7 \times 6 \times 5 \times 4!}{3!\,4!} \times \frac{4 \times 3 \times 2!}{2!\,2!}$$

$$= 7 \times 5 \times 2 \times 3$$

$$= \mathbf{210}$$

Option 4

Number of ways selecting 4 boys & 1 girl

$$= {}^7C_4 \times {}^4C_1$$

$$= \frac{7!}{4!(7-4)!} \times \frac{4!}{1!(4-1)!}$$

$$= \frac{7!}{4!\,3!} \times \frac{4!}{1!\,3!}$$

$$= \frac{7 \times 6 \times 5 \times 4!}{4!\,3!} \times \frac{4 \times 3!}{3!}$$

$$= 7 \times 5 \times 4$$

$$= \mathbf{140}$$

Hence,

Total number of ways = 7 + 84 + 210 + 140

= 441 ways

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2. How many words, with or without meaning can be formed using all the letters of the word EQUATION at a time so that the vowels and consonants occur together?

Solution:

There are 5 vowels and 3 consonants.

Number of ways vowels can be arranged is ${}^5P_5 = \frac{5!}{(5-5)!} = 5 \times 4 \times 3 \times 2 \times 1 = 120$

Number of ways consonants can be arranged is

$${}^3P_3 = \frac{3!}{(3-3)!} = 3 \times 2 \times 1 = 6$$

The number of ways the vowels and consonants can be arranged is

$${}^2P_2 = \frac{2!}{(2-2)!} = 2 \times 1 = 2$$

Total number of ways in which vowels & consonants occur together

= 2 × (Number of ways vowel arrange)

× (Number of ways consonants arrange)

= 2 × (120 × 6)

= **1440**

3. If the different permutations of all the letter of the word EXAMINATION are listed as in a dictionary, how many words are there in this list before the first word starting with E?

Solution:

Words are given alphabetically in the dictionary. Therefore, we need to find words starting before E – starting with A, B, C or D. The word EXAMINATION does not have B, C or D.

The number of words in the list before the word starting with E= number of words starting with letter A.

The first letter is A. The number of letters are 10, where I is repeated 2 times, N is repeated 2 times.

Therefore the required words starting with letter A is

$$\frac{n!}{p_1!\,p_2!} = \frac{10!}{2!\,2!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2!}{2!\,2!}$$

$$= \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3}{2!} = 907200$$

4. It is required to seat 5 men and 4women in a row so that the women occupy the even places. How many such arrangements are possible?

Solution:

There are total 9 people

Since women occupy even places

Places 2, 4, 6, 8 will be occupied by women

And Rest will be occupied by men

The seating arrangement will be follows

I	II	III	IV	V	VI	VII	VIII	IX
M	**W**	M	**W**	M	**W**	M	**W**	M

Now, we have to arrange 5 women and 4 men

Arrangements of 4 women

4 women can sit in 4 positions

Total number of ways

$= {}^4P_4$

$= \frac{4!}{(4-4)!}$

$= \frac{4!}{0!} = \frac{4!}{1} = 4 \times 3 \times 2 \times 1$

= 24 ways

Arrangements of 5 men

5 men can sit in 5 positions

Total number of ways

$= {}^5P_5$

$= \frac{5!}{(5-5)!}$

$= \frac{5!}{0!} = \frac{5!}{1} = 5 \times 4 \times 3 \times 2 \times 1$

= 120

Thus,

Total number of arrangements = 24 × 120

= **2880**

5. In an examination, a question paper consists of 12 questions divided into 2 parts, i.e. Part I and Part II, containing 5 and 7 question respectively. A student is required to attempt 8 questions in all, selecting at least 3 from each part. In how many ways can a student select the questions?

Solution:

Student is required to attempt total 8 questions and selecting atleast 3 question from part 1 and part 2.

So, she can select

Option 1	3 from Part 1 & 5 from Part 2	*We have to calculate all these combinations separately and then add it*
Option 2	4 from Part 1 & 4 from Part 2	
Option 3	5 from Part 1 & 3 from Part 2	

Option 1 – 3 questions from Part 1 and 5 Questions from Part 2

	Total number	Number to be chosen	Number of ways to choose
Part 1	5	3	${}^{5}C_{3}$
Part 2	7	5	${}^{7}C_{5}$

Number of ways selecting = ${}^{5}C_{3} \times {}^{7}C_{5}$

$$= \frac{5!}{3!(5-3)!} \times \frac{7!}{5!(7-5)!}$$

$$= \frac{5!}{3!\,2!} \times \frac{7!}{5!\,2!}$$

$$= \frac{5 \times 4 \times 3!}{3!\,2!} \times \frac{7 \times 6 \times 5!}{5!\,2!}$$

$$= 5 \times 2 \times 7 \times 3$$

$$= \mathbf{210}$$

Option 2 – 4 questions from Part 1 and 4 Questions from Part 2

	Total number	Number to be chosen	Number of ways to choose
Part 1	5	4	$^5C_4$
Part 2	7	4	$^7C_4$

Number of ways selecting = $^5C_4 \times {^7C_4}$

$$= \frac{5!}{4!(5-4)!} \times \frac{7!}{4!(7-4)!}$$

$$= \frac{5!}{4!\,1!} \times \frac{7!}{4!\,3!}$$

$$= \frac{5 \times 4!}{4!} \times \frac{7 \times 6 \times 5 \times 4!}{4!\,3!}$$

$$= 5 \times 7 \times 5$$

$$= \mathbf{175}$$

Option 3 – 5 questions from Part 1 and 3 Questions from Part 2

	Total number	Number to be chosen	Number of ways to choose
Part 1	5	5	$^5C_5$
Part 2	7	3	$^7C_3$

Number of ways selecting = $^5C_5 \times {}^7C_3$

$$= \frac{5!}{5!(5-5)!} \times \frac{7!}{3!(7-3)!}$$

$$= \frac{5!}{5!\,0!} \times \frac{7!}{3!\,4!}$$

$$= \frac{5!}{5!} \times \frac{7 \times 6 \times 5 \times 4!}{3!\,4!}$$

$$= 1 \times 7 \times 5$$

$$= \mathbf{35}$$

Hence,

Total ways = 210 + 175 + 35

= **420 ways**
