UNIT 3- Application of Integrals

Application of definite integration

1. Find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.
Ans. 12π
2. Find the area of the region bounded by the curve $y = x^2$ and the line y = 4.
Ans. $\frac{32}{3}$
3. Find the area of the region in the first quadrant enclosed by the x-axis, the line y = x, and the circle $x^2 + y^2 = 32$.
Ans. 4π
4. Find the area of the region bounded by the curve $y^2 = x$ and the lines x = 1, x = 4 and the x-axis in the first quadrant.
Ans. $\frac{14}{3}$
5. Find the area lying above x-axis and included between the circle $x^2 + y^2 = 8x$ and inside of the parabola $y^2 = 4x$.
Ans. $\frac{4}{3}(8 + 3\pi)$

Additional Problems

1.

Find the area enclosed by the circle $x^2 + y^2 = a^2$.

2.

Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

3.

1. Find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.
2. Find the area of the region bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

4. Find the area of the region bounded by the line y = 3x+2, the X-axis and the ordinates x = -1 and x = 1.
5. Find the area bounded by the curve y = cos x between x = 0 and x = 2\pi .
6.

Find the area under the given curves and given lines:

(i) $y = x^2$, $x = 1$, $x = 2$ and x-axis

(ii) $y = x^4$, $x = 1$, $x = 5$ and x-axis

7.

Sketch the graph of $y = |x+3|$ and evaluate $\int_{-6}^{0} |x+3|\, dx$.

8.

Find the area bounded by the curve $y = \sin x$ between $x = 0$ and $x = 2\pi$.

Summary

- The area of the region bounded by the curve $y = f(x)$, x-axis and the lines $x = a$ and $x = b$ $(b > a)$ is given by the formula: $\text{Area} = \int_a^b y\,dx = \int_a^b f(x)\,dx$.
- The area of the region bounded by the curve $x = \phi(y)$, y-axis and the lines $y = c$, $y = d$ is given by the formula: $\text{Area} = \int_c^d x\,dy = \int_c^d \phi(y)\,dy$.