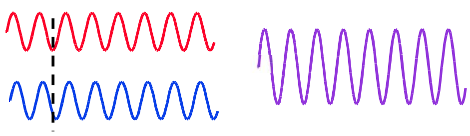
CONTENTS:

I. Interference: Introduction, Necessary conditions for interference, coherent waves and its production methods, thin film interference, interference in wedge shaped film, Newton’s rings and applications of interference.

II. Diffraction: Concept of diffraction, Fraunhoffer and Fresnel diffraction, Fraunhoffer diffraction at single slit, double slit, and multiple slits, Characteristics of diffraction grating and its applications.

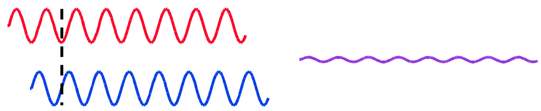
Part I: INTERFERENCE OF LIGHT

“When two light waves superimpose, then the resultant **amplitude/intensity** in the region of superposition is different than the amplitude of individual waves. **This modification in the distribution of intensity in the region of superposition is called interference.”**



CONSTRUCTIVE INTERFERENCE

(WAVES IN PHASE)



DESTRUCTIVE INTERFERENCE

(WAVES OUT OF PHASE)

<u>Conditions for sustained interference of light waves</u>

- Two sources should continuously emit waves of same wavelength or frequency.
- The amplitudes of the two interfering waves should be equal or approximately equal in order to reduce general illumination.
- **The sources of light must be coherent sources.**
- Two sources should be very narrow as a broad source is equivalent to large number of narrow sources lying side by side which causes loss of interference pattern resulting general illumination.
- Two sources emitting set of interfering beams must be placed very close to each other so that wavelength interact at very small angles.

<u>COHERENT SOURCES</u>

- **Two sources are said to be coherent if they emit light waves of the same frequency, nearly the same amplitude and always have a constant phase difference between them.**
- Therefore, two sources must emit radiations of the same wavelength/color.
- In practice, it is impossible to have two independent sources, which are coherent.
- For experimental purposes tow virtual sources formed from a single source can act as coherent sources.

- The two sources must be narrow and close to each other because the wavelengths of light waves are extremely small (of the order of 10^{-7} m).

GENERATION OF COHERENT SOURCES

In practice, Coherent sources are obtained by following two methods:

(1) Division of Wavefront

- The wavefront originating from a source of light is divided into two parts which serves the purpose of coherent sources.
- These two parts of the same wavefront travel unequal distances and reunite at some angle to produce interference bands.
- E.g. Youngs's double slit expt., Fresnel biprism.
- Path difference = Δ = xd/D, where x is the distance between two consecutive bright/dark fringes, d is the distance between two slits and D is the distance between source and screen.
- The spacing between any consecutive maxima or minima is expressed by fringe width (β).

$$\beta = \lambda D/d$$

(2) Division of Wavefront

- The amplitude of the beam is divided into two parts by partial reflection or refraction methods.
- The waves corresponding to the divided parts travel different paths and hence produce interference.
- E.g. Interference due to thin films, wedge shaped film interference, Newton's rings.
- The path difference, Δ = ***2μtcosr,*** where t is the thickness of thin film, r is the angle of reflection and μ is the refractive index of the material of film.

CONDITION FOR MAXIMA AND MINIMA:

- Maximum intensity of light is observed at a point where the phase difference between the two waves reaching the point is a whole number multiple of 2π or the path difference between the two waves is a whole number multiple of wavelength (λ).
- Minimum intensity of light is observed at a point where the phase difference between the two waves reaching the point is an odd number multiple of π or the path difference between the two waves is an odd number multiple of half wavelength ($\lambda/2$).

Relation between Phase Difference and Path Difference

- If the path difference between the two waves is λ, the corresponding phase difference is 2π
- Hence, for path difference x, the phase difference $\delta = 2\pi x/\lambda$
  Or

Phase difference, δ = (path difference) * ($2\pi/\lambda$)

Interference by Division of Wavefront

THIN FILM INTERFERENCE

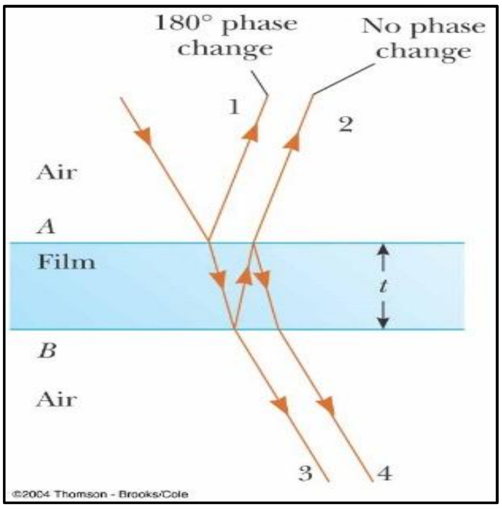
- The varied colors observed when white light is incident on thin films of soap or oil on surface of water result from the interference of waves reflected from the opposite surfaces of thin film.

- When light beam incidents on the first surface of a transparent film, a portion of the incident wave is partially reflected and partially transmitted.

- The transmitted portion is then reflected from a second surface and emerges back out of the film.

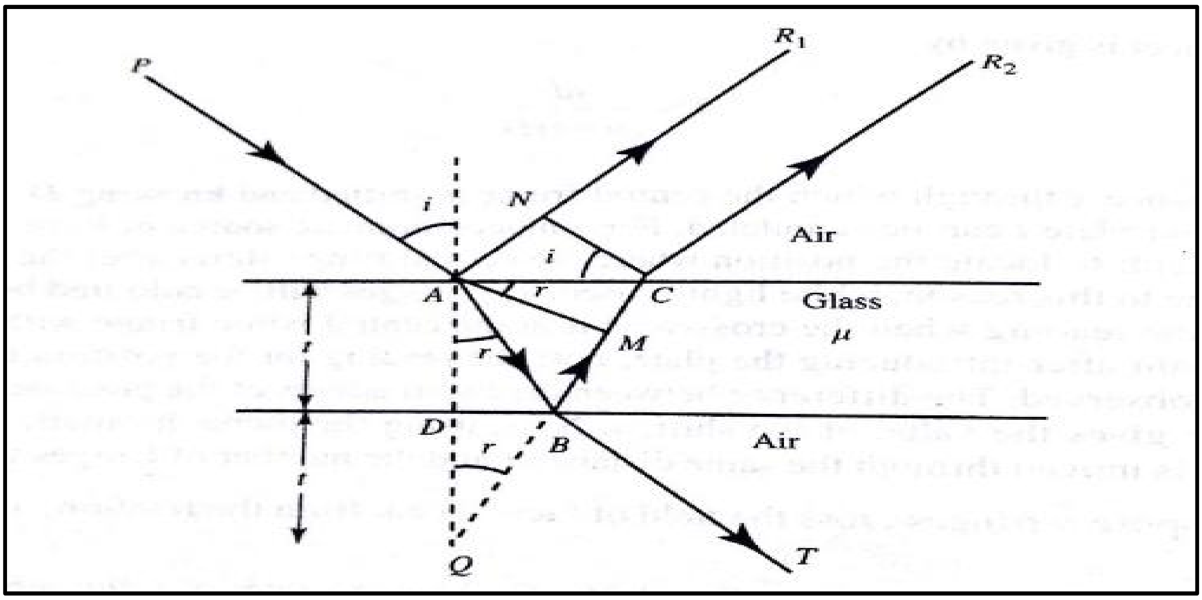
- Thus, from the thin film, two types of light rays we get:

  (1) rays reflected from front / top surface and
  (2) rays transmitted from rear / bottom surface.



- The two rays in both the cases have a different optical path length that is determined by the width of the film, and upon angle of incidence.
- When an observer see the film in reflected / transmitted light, the interference pattern will be observed.

INTERFERENCE FROM A THIN PARALLEL FILM IN REFLECTED LIGHT



- Consider a thin transparent film of thickness t and refractive index μ (Shown in fig.)
- A ray PA incident on the upper surface of the film is partly reflected along AR1 and partly refracted along AB. At the point B, a part of it is reflected along BC and finally after partial refraction it emerges out along CR2
- The difference in path between the two rays AR_1 and CR_2 can be calculated
- Draw CN normal to AR_1 and AM normal to BC.
- The angle of incidence is i and angle of refraction is r.
- Also, produce CB to meet AQ produced at Q. Here, ∟AQC = r.
- The optical path difference 'Δ' between the interfering rays AR_1 and CR_2 is ,

$$\Delta = \mu\ (AB + BC) - AN$$

Now from Snell's Law, $\mu = \sin i/\sin r = (AN/AC)/(CM/AC) = AN/CM$

$$AN = \mu.CM$$

Therefore, the optical path difference, Δ is

$$\Delta = \mu(AB + BC) - \mu CM$$

$$= \mu(AB + BC - CM)$$

$$= \mu(QB + BC - CM) \qquad [since,\ AB = QB]$$

$$= \mu(QC - CM)$$

$$= \mu.QM$$

In ΔAQM, $\cos r = QM/AQ$

Therefore, $QM = AQ\cos r = 2t\cos r$

$$QM = 2t\cos r$$

Hence, the path difference, $\Delta = \boldsymbol{2\mu t\cos r}$

But, using Stokes' Law in Optics, viz., "A light ray reflected from a denser medium suffers a phase change of π or path difference of λ/2**"**

Therefore, the effective path difference is $\Delta = 2\mu t\cos r \pm \lambda/2$

Condition for bright fringe

Path difference $= n\lambda$ *or* $2\mu t\cos r + (\lambda/2) = n\lambda$, *where* $n = 0,1,2\ldots$

$$2\mu t\cos r = (2n-1)(\lambda/2)\ \ \textbf{Or}\ 2\ \mu\ x\theta = 2\mu t = (2n-1)\ (\lambda/2)$$

Condition for dark fringe

Path difference $= (2n+1)\lambda/2$ *or* $2\mu t\cos r + (\lambda/2) = (2n+1)\lambda/2$, *where* $n = 0,1,2\ldots$

$$2\mu t\cos r = n\lambda\ \ \textbf{or}\ 2\ \mu\ x\theta = 2\mu t = n\lambda$$

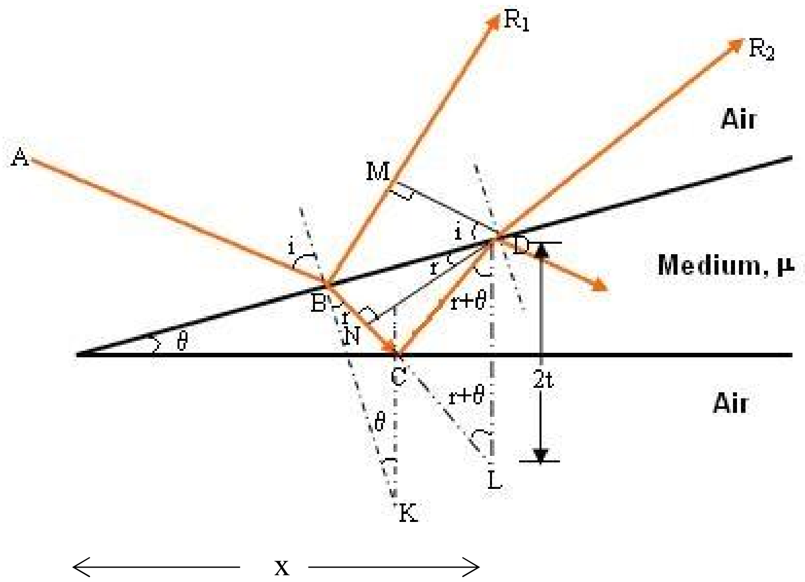
COLOURS IN THIN FILMS

- When a thin film is exposed to white light source and viewed in reflected light beam, multiple colored fringes/bands are observed.
- The incident white light beam on the film splits up in two parts, one part is reflected from the top and the other part is reflected from bottom surface of the film.
- These two rays get superimposed over each other and hence interfere to produce colored fringes or bands.
- The bright and dark appearance of the reflected light depends upon μ, t and r.
- For white light if t and r are made constant, μ varies with the wavelength. At a fixed point of the film and for a fixed position of the eye, the interfering rays of only certain wavelengths/colors will have a path difference satisfying the conditions of a maxima/ bright fringe. Hence only these wavelengths will be present.
- The wavelengths/colours for which the condition for minima/dark fringe is satisfied are absent.
- Similarly, if the same point is observed with the eye in different positions or different points of the film are observed with the eye in the same position, different set of colours are observed due to change in r.

<u>Two special cases of Thin Film Interference</u>

A) FRINGES PRODUCED BY WEDGE SHAPED THIN FILM

- Consider two glass slides placed with a piece of paper between them at one end.
- Fringes are seen from above. If we assume that the slides are coated so that no reflection occurs at the top of the top slide or bottom of the bottom slide, this is essentially like a thin film (of air) but of varying thickness.
- Let a light of wavelength λ incident from above.
- Some light reflects at the bottom of the top slide and some at the top of the bottom slide.
- Interference occurs between the rays reflected from the upper and lower surface of the film.



Following the approach of interference from a thin parallel film, we can find the total path difference Δ between the two interfering rays BR1 and DR2 as

The path difference, Δ = $\boldsymbol{2\mu t \cos (r+\theta)}$**,** where t be the thickness of the film at a distance x from the edge.

But if the rays are incident normally, $r = 0$

If θ is very small, $\theta = 0$

Also from the diagram $t = x\theta$

Hence, the effective **path difference =** $\boldsymbol{2\mu x\theta = 2\mu t}$

"A ray reflected from a denser medium suffers a phase change of π or path difference of $\lambda/2$**"** **[STOKES' Law in Optics]**

Condition for bright fringe	*Condition for dark fringe*
Path difference = $n\lambda$	*Path difference =* $(2n+1)\lambda/2$
$2\mu t \cos (r+\theta) + \boldsymbol{(\lambda/2)} = n\lambda,$	$2\mu t \cos (r+\theta) + \boldsymbol{(\lambda/2)} = (2n+1)\lambda/2$
where $n = 0,1,2\ldots$	*where* $n = 0,1,2\ldots$
$2\mu t \cos (r+\theta) = (2n-1)(\lambda/2)$	$2\mu t \cos (r+\theta) = n\lambda$
Or $2\mu x\theta = 2\mu t = (2n-1)(\lambda/2)$	***Or*** $2\mu x\theta = 2\mu t = n\lambda$

Fringe width (β)

If x_n is the distance of the n^{th} dark fringe from the edge and

x_m is the distance for $(n+m)^{th}$ dark fringe, then

$x_n = n\lambda/2\mu\theta$ *and* $x_{n+m} = (n+m)\lambda/2\mu\theta$

Therefore, Fringe width $\boldsymbol{\beta = x_{n+m} - x_n = m\lambda/2\mu\theta}$

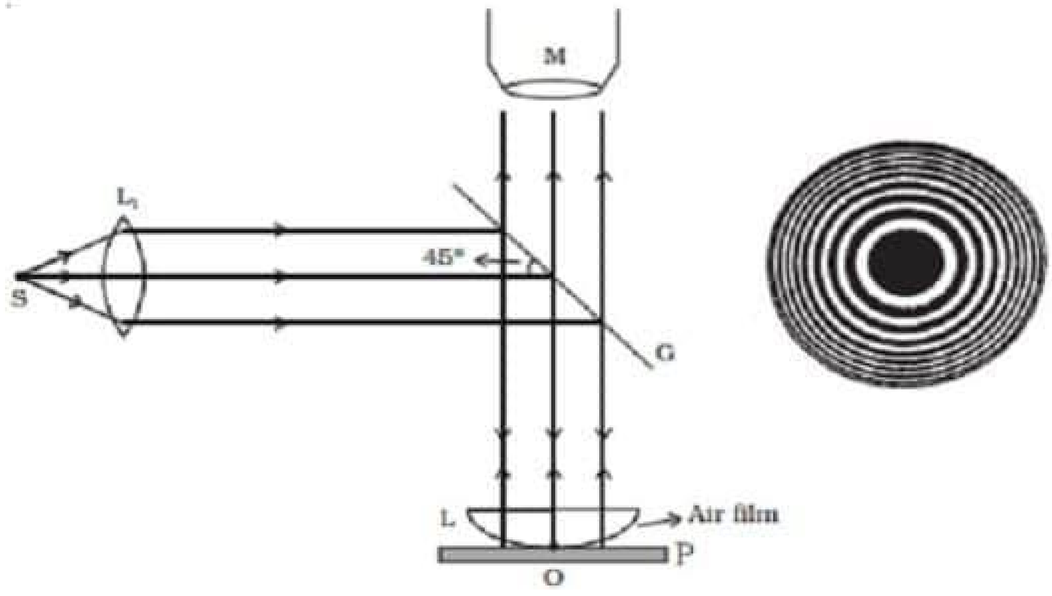
For m = 1, $\boldsymbol{\beta = \lambda/2\mu\theta}$

If the two surfaces forming the wedge-shaped film are optically plane, then the fringes of equal thickness will be straight.

(B) NEWTON'S RINGS

When two glass plates are kept inclined to each other, a wedge-shaped air film is formed and bright and dark fringes are observed when the plates are illuminated by monochromatic light.

If a glass plate and a plano convex lens is used, again a wedge-shaped air film is formed but with circular geometry.



The illumination by monochromatic light will produce bright and dark rings.

This phenomenon was explained by Newton and hence known as Newton's rings.

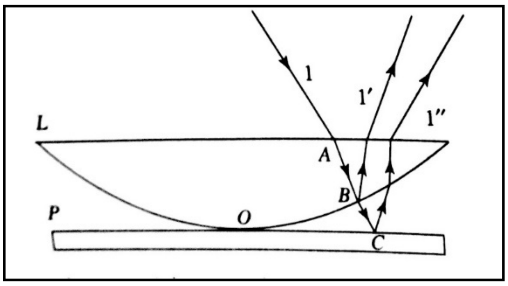
A plano convex lens with its convex surface is placed on a plane glass plate; an air film with gradually increasing thickness is produced between the two surfaces.

At the point of contact of the lens and plane surface, the thickness of the film is zero and becomes maximum at the edge of the lens and plane surfaces.

A monochromatic source, incident normally is used to obtain bright and dark concentric rings around a point of contact between lens and the glass plate.

EXPLANATION OF THE FORMATION OF THE RINGS

Newton's rings are formed due to interference between the two rays 1' and 1'' as a result of reflection from the top and bottom surfaces of the air film formed between the lens and the plate.



Monochromatic light ray 1 falls normally on the lens-plate setup at the point A.

At the point B on the glass-air boundary, the light gets partially reflected out as ray 1' without any phase change.

The remaining part is refracted along BC and reflected at the point C with a phase change of π radians and emerges out as ray 1''.

The two reflected rays are derived from the same ray 1 and hence produce interference.

For a very small wedge angle θ, and for normal incidence, $r = 0$, the total path difference Δ between the two reflected rays 1' and 1'' is

$\Delta = [2\mu t \cos(r+\theta) + \lambda/2]$, where $\lambda/2$ term is due to Stokes law.

At the point of contact $t = 0$ and for air film $\mu = 1$ so the path difference is $(\lambda/2)$.

This is the condition for minimum intensity and hence the central spot is dark.

The condition for n^{th} maxima is :

$$2t + \lambda/2 = n\lambda \quad \text{or} \quad 2t = (2n+1)\,\lambda/2, \text{ where } n = 1,2,3\ldots\ldots$$

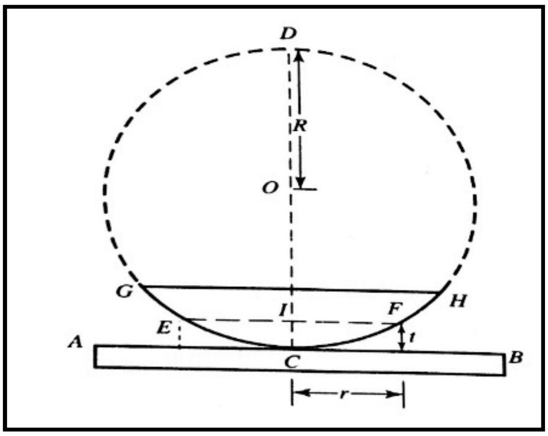
Similarly, we can show

The condition for n^{th} minima is :

$$2t = n\lambda, \text{ where } n = 0,1,2,3\ldots\ldots$$

Expressions of diameters of Newton's rings in reflected light

- To calculate the diameter of dark and bright rings, consider GCH to be the plano-convex lens placed on a glass plate AB.
- Let R be the radius of curvature of the lens.
- Point C is the point of contact between plate AB and lens GCH
- Regions GCA and HCB is the wedge-shape circular air film.
- Newton's rings are formed due to this air film.



- Let r be the radius of Newton's rings corresponding to constant film thickness t, the locus of which forms a locus of points of a circle with centre on point C.

- Now from the property of circle,

  IE x IF = IC x ID

- But $IE = IF = r$, radius of the rings,

  IC = t and $ID = 2R - t$

- Therefore, $r \times r = t \times (2R - t)$

  $r^2 = 2Rt - t^2$

  $r^2 = 2Rt$ *[t <<< R and hence t^2 can be neglected compared to 2Rt]*

- Now if D_n is the diameter of the nth ring, $D^n = 2r$
- Therefore, $\boldsymbol{D_n^2/4 = 2Rt}$ *Or* $\boldsymbol{2t = D_n^2/4R}$

For Bright Rings	For Dark Rings
$2t = (2n-1)\lambda/2$	$2t = n\lambda$
$D_n^2/4R = (2n-1)\lambda/2$	$D_n^2/4R = n\lambda$
$D_n^2 = (2n-1).2\lambda R$	$D_n^2 = 4n\lambda R$
$D_n \alpha \sqrt{2n-1}$	$D_n \alpha \sqrt{n}$
$D_n \alpha \sqrt{\lambda}$	$D_n \alpha \sqrt{\lambda}$
$D_n \alpha \sqrt{R}$	$D_n \alpha \sqrt{R}$

- This shows that the diameter of the rings is proportional to square root of λ and R.
- Also the diameter of bright and dark rings is proportional to $\sqrt{2n-1}$ and $\sqrt{n}$ respectively.
- Therefore the diameter of the bright rings reduces faster than dark rings.
- So as the order of rings increases, thinner and sharper rings are obtained.

Application of Newton's Ring Experiment

1) *TO DETERMINE WAVELENGTH OR RADIUS OF CURVATURE OF LENS:*

Let the diameter of n^{th} and $(n+m)^{th}$ dark rings are D_n and D_{n+m}

$$D_n^2 = 4n\lambda R \text{ and } D_{(n+m)}^2 = 4(n+m)\lambda R$$

$$D_{(n+m)}^2 - D_n^2 = 4(n+m)\lambda R - 4n\lambda R = 4m\lambda R$$

$$\boldsymbol{\lambda = [D_{(n+m)}^2 - D_n^2]/ 4m\lambda R} \quad \textit{and} \quad \boldsymbol{R = [D_{(n+m)}^2 - D_n^2]/ 4m\lambda}$$

2) *TO DETERMINE THE REFRACTIVE INDEX (μ) OF A LIQUID*

In air medium, let the diameter of nth and (n+m)th dark rings are D_n and D_{n+m}

$$D_n^2 = 4n\lambda R \text{ and } D_{(n+m)}^{\ 2} = 4(n+m)\lambda R$$

$$D_{(n+m)}^{\ 2} - D_n^2 = 4(n+m)\lambda R - 4n\lambda R = 4m\lambda R$$

Now, let the diameter of dark rings with liquid of refractive index μ be

$$d_n^2 = 4n\lambda R/\mu \quad \text{and } d_{(n+m)}^{\ 2} = 4(n+m)\lambda R/\mu$$

$$d_{(n+m)}^{\ 2} - d_n^2 = 4m\lambda R/\mu$$

$$\boldsymbol{\mu = [D_{(n+m)}^{\ 2} - D_n^2]/[d_{(n+m)}^{\ 2} - d_n^2]}$$

NEWTON'S RINGS WITH WHITE LIGHT

- If we use white light like a mercury source, colored rings are obtained.
- In this case, the diameters of different rings are different for different colors as it depends on $\sqrt{\lambda}$.
- So the first few colored rings are seen clearly, after which overlapping of colors occurs and the rings cannot be seen distinctly.

Part II: DIFFRACTION

- If we look clearly at the shadow cast by an opaque object, close to the region of geometrical shadow, there are alternate dark and bright regions just like in interference.
- This happens due to the phenomenon of **diffraction**.
- Diffraction is a general characteristic exhibited by all types of waves, be it sound waves, light waves, water waves or matter waves.
- Since the wavelength of light is much smaller than the dimensions of most obstacles; we do not encounter diffraction effects of light in everyday observations.
- However, the finite resolution of our eye or of optical instruments such as telescopes or microscopes is limited due to the phenomenon of diffraction.
- Indeed the colors that you see when a CD is viewed is due to diffraction effects.

Definition:

When light falls on obstacles or small apertures whose size is comparable with wavelength of light, there is a deviation from straight line, the light bends around the corner of the obstacle/aperture and enters in geometrical shadow. This bending of light is called DIFFRACTION.

OR

The bending of light waves around the edge of any obstacle/aperture whose size is comparable to the wavelength of light is called DIFFRACTION.

The necessary condition to observe a good diffraction pattern is that the size of the aperture (slit) should be comparable to the wavelength of the incident light

Classes of Diffraction

Diffraction phenomenon can be classified into following two general classes:

FRESNEL'S DIFFRACTION

- The source and/or the screen are placed at finite distances from the slit/aperture/object.
- The incident wavefronts are either spherical or cylindrical.
- The incoming rays/outgoing rays are not parallel.

FRAUNHOFER'S DIFFRACTION

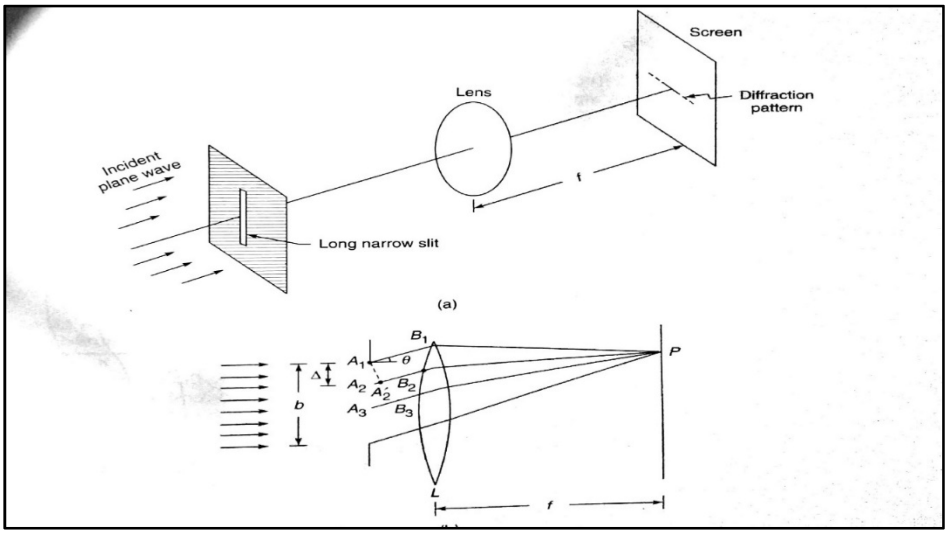
- The source and/or the screen are at infinite distances from the slit/aperture/object.
- The incident wavefronts are plane.
- The incoming rays/outgoing rays are parallel.

Difference between Interference and Diffraction

INTERFERENCE	DIFFRACTION
1. The interaction takes place between two separate wavefronts arising from two coherent sources. 2. In interference pattern the regions of minimum intensity are perfectly dark. 3. The interference fringes are equally spaced. 4. In interference pattern all bright fringes are of equal intensity.	1. The interaction takes place between the secondary wavelets originating from different points of the exposed parts of the same wavefront. 2. In diffraction pattern the regions of minimum intensity are not perfectly dark. 3. The diffraction fringes are never equally spaced. 4. In diffraction pattern only first maximum has maximum intensity and the intensity decreases fast as the order of maxima increases.

FRAUNHOFER DIFFRACTION AT SINGLE SLIT

- Let us assume that a plane wave is incident normally on the slit with width *b* and the intensity distribution on the focal plane of lens L is to be calculated.
- The slit is considered to have a large number of equally spaced point sources A1, A2, A3…..and each point on the slit is a source of Huygen's secondary wavelets which interfere with the wavelets emanating from other points.
- Let the distance between two consecutive points be Δ.



- Hence, for n number if point sources, $b = (n-1)\,\Delta$.
- Now let us calculate the resultant field produced by these *n* sources at the point P, P being an arbitary point receiving parallel rays making an angle θ with the normal to the slit. (figure b).

- For an incident plane wave, the points A1, A2….. are in phase and, therfore the additional path traversed by the disturbance emanating from the point A2 will be A2A2' where A2 is the foot of perpendicular drawn from A1 on A2B2.

- If the diffracted rays make an angle θ with the normal to the slit then the path difference would be

$$A2A2' = \Delta sin\ \theta$$

Corresponding Phase difference is, $\emptyset = \frac{2\pi}{\lambda}\Delta sin\theta$

- If the field at the point P due to the disturbance from the point A1 is $acos\omega t$ then the field due to the disturbance from A2 would be $\mathrm{acos}(\omega t - \emptyset)$.

- The resultant field at point P would be given by

$$E = a[cos\omega t + \cos(\omega t - \emptyset) + \cdots + \cos(\omega t - (n-1)\emptyset)]$$

- Using the method of vector addition

$$cos\omega t + \cos(\omega t - \emptyset) + \cdots + \cos(\omega t - (n-1)\emptyset)$$
$$= \frac{\sin n\emptyset/2}{\sin\emptyset/2}\cos[\omega t - \frac{1}{2}(n-1)\varphi]$$

- Thus, $E = E_0 \cos[\omega t - \frac{1}{2}(n-1)\varphi]$

Where the amplitude of the resultant field is given by

$$E_\theta = a\frac{\sin n\emptyset/2}{\sin\emptyset/2}$$

In the limit of (here $n \to \infty$, $\Delta \to 0$, but $n\Delta \to b$ we have

$$\frac{n\emptyset}{2} = \frac{\pi}{\lambda}n\Delta sin\theta \to \frac{\pi}{\lambda}bsin\theta$$

And

$$\emptyset = \frac{2\pi}{\lambda}\Delta sin\theta = \frac{2\pi}{\lambda}\frac{bsin\theta}{n} \to 0$$

Hence we can write,

$$E_\theta \approx \frac{a\sin(n\emptyset/2)}{\emptyset/2} = \mathrm{na}\frac{sin\frac{\pi bsin\theta}{\lambda}}{\frac{\pi bsin\theta}{\lambda}} = A\frac{sin\beta}{\beta}$$

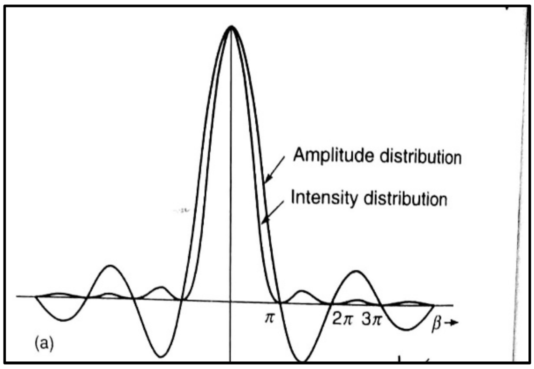
Where $A = na$ and $\beta = \frac{\pi bsin\theta}{\lambda}$

Hence E = $A\frac{sin\beta}{\beta}\cos(\omega t - \beta)$

The corresponding intensity distribution is given by

$$I = I_0 \frac{sin^2\beta}{\beta^2}$$

$where\ I_0\ represents\ the\ intensity\ at\ \theta = 0$



Positions of Maxima and Minima

The figure shows the variation of the intensity with β.

From the intensity equation, the intensity becomes zero when

$$\beta = m\pi, m \neq 0$$

Substituting the value of $\beta\ in\ \beta = \frac{\pi b sin\theta}{\boldsymbol{\lambda}}$

We obtain, $\boldsymbol{bsin\theta = m\ \lambda}$; m = $\pm 1, \pm 2, \pm 3 \ldots\ldots (minima)$

This is the condition for minima.

In order to obtain the condition for maxima, we can differentiate the intensity equation with respect to β and set it equal to zero

$$\frac{dI}{d\beta} = I_0[\frac{2sin\beta cos\beta}{\beta^2} - \frac{2sin^2\beta}{\beta^3}] = 0$$

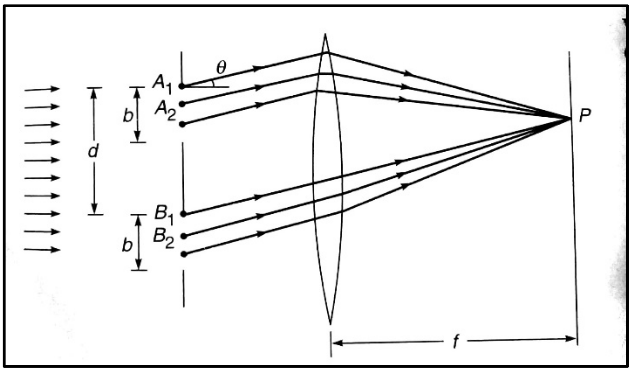
Or $\quad sin\beta[\beta - tan\beta] = 0$

The condition sinβ =0 or β = mπ corresponds to minima.

The condition for maxima are roots of the equation tanβ = β

FRAUNHOFER DIFFRACTION AT DOUBLE SLIT

- Consider the Fraunhoffer diffraction pattern produced by two parallel slits, each of width b and separated by distance d.
- The resultant intensity distribution is a product of the single slit diffraction pattern and the interference pattern produced by two-point sources separated by a distance d.



- The intensity distribution due to diffraction cab be calculated by similar method used for the case of single slit.
- Let athe point sources be A!, A2, A3… for the first slit and B1, B2, B3…. in the second slit.
- The field produced by the first slit will be

$$E_1 = A\frac{sin\beta}{\beta}\cos(\omega t-\beta)$$

- The field produced by the second slit will be

$$E_2 = A\frac{sin\beta}{\beta}\cos(\omega t-\beta-\emptyset_1)$$

where $\emptyset_1 = \frac{2\pi}{\lambda}dsin\theta$ represents the phase difference from two corresponding points on the slits which are separated by distance d.

- Hence the resultant field can be given as

$$E = E_1 + E_2$$

$$= A\frac{sin\beta}{\beta}[\cos(\omega t-\beta)-\cos(\omega t-\beta-\emptyset_1)$$

Which represents the interference of two waves, each of amplitude $A\frac{sin\beta}{\beta}$ and differing in phase by $\emptyset_1$

- Hence,

$$E = A\frac{sin\beta}{\beta}\cos\gamma\cos\left(\omega t-\frac{1}{2}\beta-\frac{1}{2}\emptyset_1\right)$$

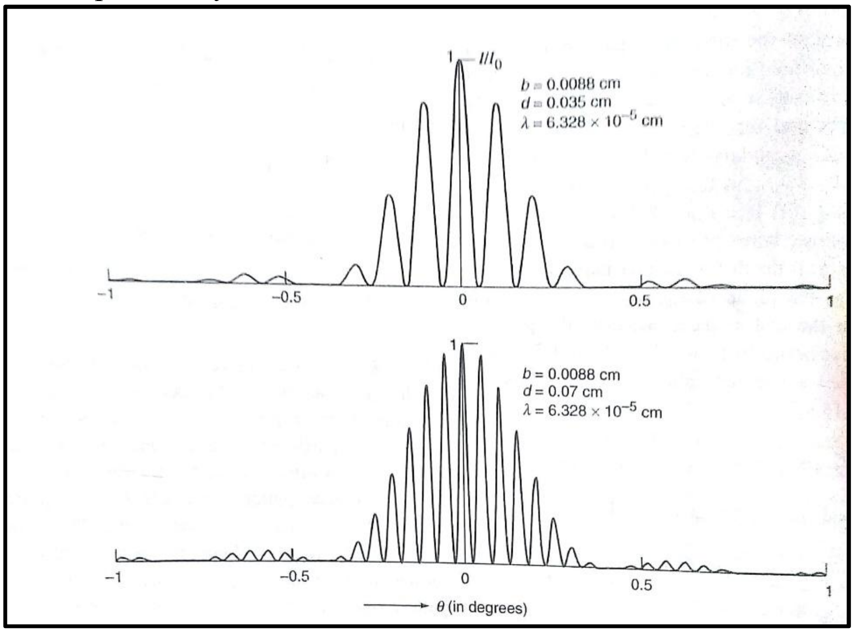
Where $\gamma = \frac{\emptyset_1}{2} = \frac{\pi}{\lambda}dsin\theta$

- The intensity distribution will be in the form

$$I = 4I_0\frac{sin^2\beta}{\beta^2}cos^2\gamma$$

where $I_0\frac{sin^2\beta}{\beta^2}$ represents the intensity, distribution produces by one of the slits.

- It can be seen from the equation that the intensity distribution is the product of two terms; the first term ($\frac{sin^2\beta}{\beta^2}$) represents the diffraction pattern produced by a single slit of width b and second term $cos^2\gamma$ represents the interference pattern produced by two point sources separated by a distance d.



<u>Positions for Maxima and Minima</u>

- The path difference between the two diffracted beamsemanating from the first slit is:

$$\Delta = bsin\theta$$

- If the path difference is odd number multiple of $\lambda/2$, then θ will give direction of diffraction maxima.
- So the condition for diffraction maxima is :

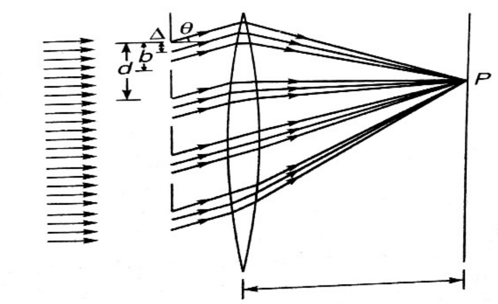
$$bsin\theta = (2m - 1)\lambda/2 \qquad \textit{where } m = 0,1,2,3\ldots\ldots$$

- If the path difference is whole number multiple of λ, then θ will give direction of diffraction minima.
- So the condition for diffraction minima is :

$$bsin\theta = m\lambda \qquad \textit{where } m = 1,2,3\ldots\ldots\ldots \textit{except zero.}$$

FRAUNHOFER DIFFRACTION AT N- SLITS

- Let us consider the diffraction pattern produced by N parallel slits, each of width b; the distance between two consecutive slits is assumed to be d.
- We assume that each slit consist of n equally spaced point sources with spacing Δ as shown in figure.



- Hence, the field at an arbitrary point P will essentially be a sum of N term:

$$E = A\frac{sin\beta}{\beta}[\cos(\omega t-\beta)-\cos(\omega t-\beta-\emptyset_1)+\cdots+ \\ +\cos(\omega t-\beta-(N-1)\emptyset_1)\,]$$

$$E = A\frac{sin\beta}{\beta}\frac{sinN\gamma}{sin\gamma}\cos\left(\omega t-\beta-\frac{1}{2}\,9N-1)\emptyset_1\right)$$

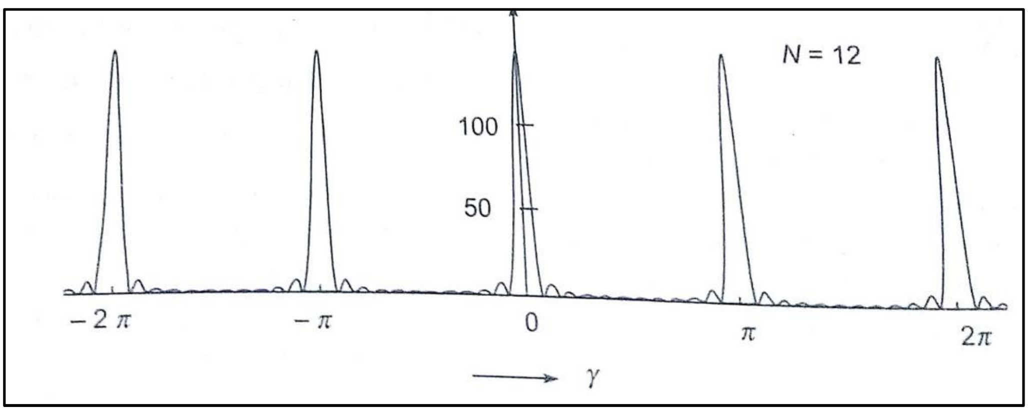
Where $\gamma = \frac{\emptyset_1}{2} = \frac{\pi}{\lambda}dsin\theta$

- The intensity distribution will be in the form

$$I = I_0\frac{sin^2\beta}{\beta^2}\frac{sin^2N\gamma}{sin^2\gamma}$$

where $I_0\frac{sin^2\beta}{\beta^2}$ represents the intensity, distribution produces by one of the slits.

- It can be seen from the equation that the intensity distribution is the product of two terms; the first term ($\frac{sin^2\beta}{\beta^2}$) represents the diffraction pattern produced by a single slit of width b and second term $\frac{sin^2N\gamma}{sin^2\gamma}$represents the interference pattern produced by N equally spaced point sources separated by a distance d.



Positions of Maxima amd Minima

Center Intense maxima is obtained at $\gamma \cong m\pi$ when the value of N is very large.

i.e when $dsin\theta = m\lambda$ ***(m = 0, 1,2…)***

Such maxima is known as Principal maxima.

The position of minima will be when $bsin\theta = n\lambda$ ***(n = 0, 1,2…)***

Between two principal maxima there are (N-1) minima.

Between two such consecutive minima the intensity has to have a maximum; thee maxima are known as secondary maxima.

DIFFRACTION GRATING

- **Diffraction Grating is an arrangement consisting of a large number of parallel slits of same width separated by equal opaque spaces.**
- Gratings are fabricated by ruling equidistant parallel lines on a glass plate with the help of a fine diamond point.
- The lines act as opaque spaces and the incident light cannot pass through them.
- The space between the two lines is transparent to light and acts as a slit.

CHARACTERISTICS OF GRATING

Grating Spectrum

- For N slit diffraction pattern we have seen that the positions of the principal maxima are given by
  $\boldsymbol{dsin\theta = m\lambda}$ ***(m = 0, 1,2…)***
- This relation is called as Grating Equation
- It can be used to study the dependence of the angle of diffraction θ on the wavelength λ
- The zeroth principal maximum occurs at θ = 0 irrespective of the wavelength.
- If we are using a polychromatic source (white light) then the central maximum will be of the same colour as the source itself.
- For m other than zero the angles of diffraction are differenct for different wavelengths and therefore, various spectral components appear at different positions.
- Thus by measuring the angles of diffraction for various colors one can determine the values of the wavelengths.

Dispersive power of the Grating

The dispersive power of grating is defined as:

"The rate at which the angle of diffraction varies with wavelength".

The factor (*dθ/dλ*) is called the dispersive power for that order.

- The diffraction of the n^{th} order principal maximum for a wavelength λ is:

$$(a+b)sin\theta = n\lambda$$

where: (a+b) is the grating element
n is the order of diffraction
θ is the angle of diffraction

- Differentiating the above equation w.r.t. λ, we get,

$$(a+b)cos\theta\, d\theta = nd\lambda$$

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)cos\theta}$$

Resolving Power of the Grating

The Resolving power of grating is defined as:

"The ratio of the wavelength of any spectral line to the difference in the wavelength between this line and a neighboring line such that the two lines appear to be just resolved".

Thus, ***the resolving power of grating = order of spectrum x total number of lines on grating***

Therefore, Resolving power, $$\frac{\lambda}{d\lambda} = n\lambda$$

APPLICATIONS OF DIFFRACTION GRATING

Diffraction gratings are useful whenever light needs to be separated into its separate frequencies (or wavelengths), for example in spectroscopy.

They are an essential item in spectroscopy in astronomy, where so much information is gained by analyzing spectra from stars, etc.

Diffraction gratings can be used to produce monochromatic light of a required wavelength.

Another use is "wavelength tuning" in lasers. The laser output can be varied using a diffraction grating.