

Unit – 5 : Electricity and Magnetism

Lorentz Force law

The Lorentz Force is the force on a charged particle due to electric and magnetic fields. Let us suppose that there is a point charge q (moving with a velocity \mathbf{v} and, located at \mathbf{r} at a given time t) in presence of both the electric field $\mathbf{E}(\mathbf{r})$ and the magnetic field $\mathbf{B}(\mathbf{r})$. The force on an electric charge q due to both of them, known as the Lorentz force, can be written as

$$\begin{aligned}\mathbf{F} &= q [\mathbf{E}(\mathbf{r}) + \mathbf{v} \times \mathbf{B}(\mathbf{r})] \\ &= F_{\text{electric}} + F_{\text{magnetic}}\end{aligned}$$

The main features of the Lorentz force equation are

- (i) It depends on q , v and B (charge of the particle, the velocity and the magnetic field). Force on a negative charge is opposite to that on a positive charge.
- (ii) The magnetic force $q[\mathbf{v} \times \mathbf{B}]$ includes a vector product of velocity and magnetic field. The vector product makes the force due to magnetic field vanish (become zero) if velocity and magnetic field are parallel or anti-parallel.
- (iii) The magnetic force is zero if charge is not moving (as then $|\mathbf{v}| = 0$). Only a moving charge feels the magnetic force.

The magnitude of the Lorentz force is given by $F = q(E + v B \sin \theta)$, where θ is the angle between the magnetic field and the velocity of the charged particle, measured in radians.

Significance of Lorentz force equation

Lorentz's force explains the mathematical equations along with the physical importance of forces acting on the charged particles that are travelling through the space containing electric as well as the magnetic field. This is the significance of the Lorentz force.

Biot-Savart law

If a conductor of arbitrary shape is carrying a steady current, I , then a small element AB of the conductor of length dl produce a magnetic field dB at point P . Let r be the distance of P from the current element dl and θ be the angle between dl and r . According to Biot-Savart law, the magnitude of magnetic field at point P will be given as

$$dB \propto I \quad (1)$$

$$dB \propto dl \quad (2)$$

$$dB \propto \sin \theta \quad (3)$$

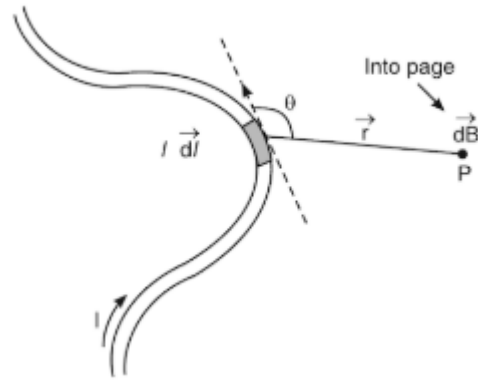
$$dB \propto \frac{1}{r^2} \quad (4)$$

$$dB \propto \frac{I dl \sin \theta}{r^2} \quad (5)$$

$$B = k \frac{Idl \sin \theta}{r^2} \quad (6)$$

where k is constant of proportionality. The value of k depends on medium in which conductor is situated and the system of units adopted. In general,

$k = \mu_r \mu_o / 4\pi$ where $\mu_o = 4\pi \times 10^{-7} \text{ H/m}$ = free space permeability and μ_r = relative permeability.



Ampere's law of electromagnetism

Ampere's law states that the line integral of the tangential component of the magnetic field over any closed path is equal to the amount of the current enclosed by the loop. Thus,

$$\oint B \cdot dl = \mu_o I \quad (1)$$

Both Ampere's law and the Biot- Savart law are relations between a current distribution and the magnetic field that it generates. We can apply Biot-Savart law to calculate the magnetic field caused by any current distribution. On the other hand, Ampere's law allows us to calculate magnetic field with ease in case of symmetry.

Let us consider an infinite long wire along the z-axis carrying a current I. The magnetic flux density due to this wire is directed everywhere circular to the wire and its magnitude is dependent only on the distance from the wire. Let us consider a circular path C of radius r in the plane normal to the wire and centred at the wire. The current enclosed by an arbitrary closed path C is given by the surface integral of the current density over any surface S bounded by the closed path C.

The total current flowing through the surface area S is given by,

$$I = \int J \cdot ds$$

Multiplying above equation by μ_o

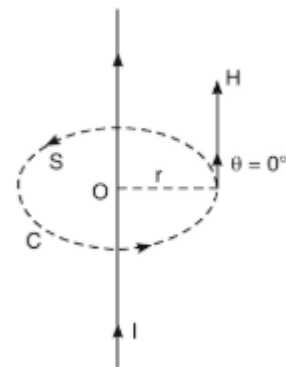
$$\mu_o I = \mu_o \int J \cdot ds \quad (2)$$

Where, J is the volume current density.

Combining equations (1) and (2), we get

$$\oint B \cdot dl = \mu_o \int J \cdot ds$$

This is known as *Ampere's circuital law*.



Magnetism and Magnetic materials

Magnetic materials

Magnetic material are the substances which get magnetised in the magnetic field. They are widely used in industrial electronics, entertainment electronics and computer industry.

Magnetic Field: The space around a magnet in which a magnetic force is exerted

–The shape of a magnetic field is revealed by magnetic field lines.

–Directed away from north poles and toward south poles.

Magnetic field intensity

The magnetic field intensity (H) at any point around a magnetic pole is defined as the force experienced by unit pole placed at that point. The S.I. unit of magnetic field intensity is A/m.

Magnetization

Magnetic moment (m) per unit volume (V) is known as magnetization (M). Units: [A/m]

$$M = \frac{m}{V}$$

M measures the materials response to the applied field H. M is the magnetization induced by the applied external field H.

Magnetic flux density

Magnetic flux density (B) is defined as the amount of magnetic flux passing normally through a unit area. If ϕ be the flux incident normally on an area A, then flux density $B = \phi/A$. Its S.I. unit is Weber/meter² or Tesla.

Magnetic susceptibility and permeability

Magnetic susceptibility (χ_m) is defined as the magnetization per unit applied magnetic field. It is expressed as $\chi_m = \mathbf{M}/\mathbf{H}$ where \mathbf{M} is the magnetization and \mathbf{H} is the applied magnetic field.

In electromagnetism, **permeability** (μ) is the measure of the ability of a material to support the formation of a magnetic field within itself. Thus, it is the degree of magnetization that a material obtains in response to an applied magnetic field.

Relationship between B and H

When a material is kept in a magnetic field two types of induction arise: one due to magnetic field H and the other as a consequence of magnetization M of the material. The magnetic induction B produced inside the material is given by

$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M})$$

Where, μ_0 is the magnetic permeability of vacuum = $4\pi \times 10^{-7}$ H/m.

Origin of Magnetization using Atomic Theory

The magnetization in a material originates from the magnetic moments of individual atoms. These atomic magnetic moments arise from three sources,

- (i) **The orbital motion of electron:** Each electron orbit can be considered as a current loop, generating a small magnetic field and having a magnetic moment along its axis.
- (ii) **The spin of electron:** Each electron spins about its own axis. This gives rise to spin magnetic moment along the axis of spin.
- (iii) **The spin of the nucleus:** The nucleus of the atom also spins about its own axis. Due to its heavy mass, the magnetic moment is very small about 10^{-3} times that of an electron. Hence, it is negligible as compared to electron magnetic moment.

Thus for an electron,

The total magnetic moment = orbital magnetic moment + spin magnetic moment.

Classification of magnetic materials

There are mainly three types of magnetic materials,

(i) Diamagnetic materials:

- Materials which don't have any permanent magnetic dipole moment are diamagnetic materials.
- In these substances the individual magnetic moment vectors are randomly oriented and the resultant magnetic moment is zero.
- An external magnetic field can cause rotation of the individual magnetic moment vector. This field rise to an induced magnetic field opposite to the applied field in such a manner that the magnetic induction drops to zero. We know that

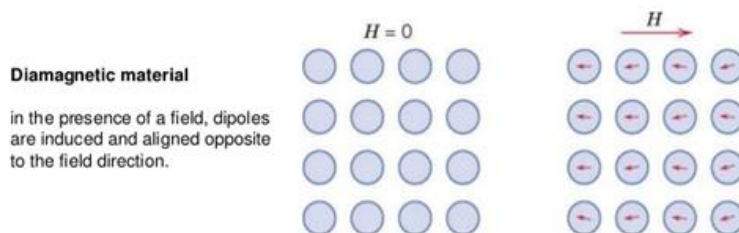
$$B = \mu_0 (H + M)$$

For diamagnetic materials $B = 0$. Thus we have

$$0 = \mu_0 (H + M)$$

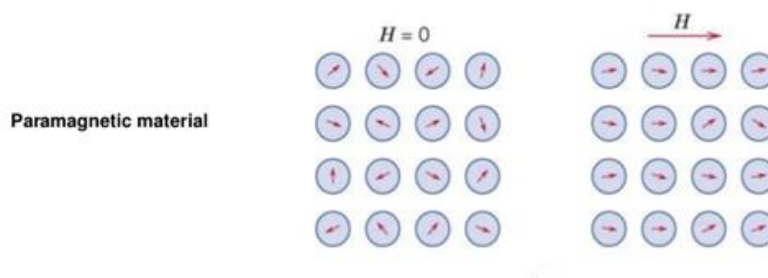
$$\text{Or } \chi_m = - (M / H)$$

- Thus diamagnetic materials have negative susceptibility.
- Examples of diamagnetic materials are Gold, Germanium, Silicon etc.



(ii) Paramagnetic materials:

- In these substances the magnetic dipole moment vectors are oriented in such a manner that there exists a small permanent magnetic moment even in the absence of an external magnetic field.
- When placed in a magnetic field, the dipoles tend to get aligned with the field which leads to an increase in the magnetic flux density inside it as shown in above figure.
- Paramagnetic materials have very small but positive susceptibility χ ranging from 10^{-5} to 10^{-2} .
- Examples: Aluminium, Chromium, Sodium etc.



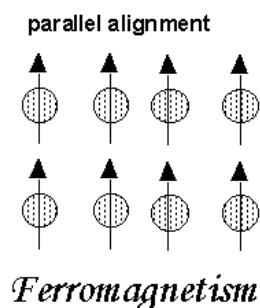
(iii) Ferromagnetic materials:

- Materials which are having a permanent magnetic dipole moment are ferromagnetic materials.
- In these substances the magnetic dipole moment vectors align themselves parallel to each other as shown in figure. This gives rise to permanent magnetic moment.
- As a result of this on the application of even small external magnetic field H a large magnetization μ is produced.
- The magnetic susceptibility, χ is written as

$$B = \chi_m M$$

and can be as high as 10^6 .

- Examples: Iron, Nickel, Cobalt.



Soft and Hard Magnetic Materials and their uses

Soft Magnetic Materials:

- Magnetic materials which are easily magnetized or demagnetized are called soft magnetic materials.
- These materials offer a small resistance to magnetization and hence the domains expand and shrink very easily to orient the magnetic moment vectors.
- A small magnetic field is required to magnetize soft magnetic materials.
- Examples: Iron, Silicon alloys, Nickel- iron alloys, Iron-Cobalt alloys.

Hard Magnetic Materials:

- These materials show high resistance to magnetization and demagnetization.
- In the microstructure of these materials, the domains are very rigid to orient the magnetic moments along the applied field.
- Hence, a sufficiently strong magnetic field is required to magnetize these materials.
- Examples: Carbon steels, tungsten steel, chromium steel, etc.