

Vector Algebra

Class work

Vector Algebra

- Find the values of x, y and z so that the vectors $\vec{a} = xi + 2j + zk$ and $\vec{b} = 2i + yj + k$ are equal. Ans. $x = 2, y = 2, z = 1$.
- If $\vec{a} = 2i - k$ and $\vec{b} = -i + 3k$ and $\vec{c} = i + 2k$ then find scalars x and y such that $x\vec{a} + y\vec{b} = \vec{c}$ Ans: $x = 1, y = 1$
- Let $\vec{a} = i + 2j$ and $\vec{b} = 2i + j$. Is $|\vec{a}| = |\vec{b}|$? Are the vectors \vec{a} and \vec{b} equal? Ans. Yes $|\vec{a}| = |\vec{b}| = \sqrt{5}$, two vectors are not equal.
- Find unit vector in the direction of vector $\vec{a} = 2i + 3j + k$. Ans. $\vec{a} = \frac{1}{\sqrt{14}}(2i + 3j + k)$.
- Find the unit vector in the direction of the sum of the vectors, $\vec{a} = 2i + 2j - 5k$ and $\vec{b} = 2i + j + 3k$. Ans. $\frac{1}{\sqrt{29}}(4i + 3j - 2k)$.
- If A(-4,-7,1) and B(-1,-6,2) are any two points the find unit vector along \vec{AB}
Ans: $\frac{3i+j+k}{\sqrt{11}}$
- If $\vec{a} = 2i - 3j + k$ and $\vec{b} = i + 2j - 3k$ are position vectors of point A and B respectively then find \vec{AB} , $|\vec{AB}|$ and a vector of magnitude 7 along \vec{AB}
Ans: $|\vec{AB}| = \sqrt{42}, \frac{7}{\sqrt{42}}(i + 5j - 4k)$
- Find the direction ratios and direction cosines of the line passing through the points A(-4,2,3) and B(1,3,-2) Ans: 5,1,-5 and $\pm \frac{5}{\sqrt{51}}, \pm \frac{1}{\sqrt{51}}, \pm \frac{5}{\sqrt{51}}$
- If a line has direction ratios -18,12,-4 the find its direction cosines.
(Ans: $\frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$)
- Find the vector joining the points P(2, 3, 0) and Q(-1, -2, -4) directed from P to Q.
Ans. $\vec{PQ} = -3i - 5j - 4k$.
- Show that the points A(2,-1,4), B(3,2,5) and C(5,8,7) are collinear using vectors
- If \vec{a}, \vec{b} and \vec{c} are position vectors of the vertices of a triangle where $\vec{a} = 4i + j + 3k$ and $\vec{b} = i + 3j + 2k$ and $\vec{c} = 2i + 7k$ then prove that the triangle is a right angled triangle

13. If A(1,1,1), B(-2,4,3) and C(-1,5,5) and D(2,2,6) are four points find the vectors of magnitude 4 units perpendicular to both \overline{AB} and \overline{CD} Ans: $4\left(\frac{i+j}{\sqrt{2}}\right)$
14. If $\vec{a} = 5i - j - 3k$ and $\vec{b} = i + 3j - 5k$, then show that the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular.
15. If A(2,-1,5), B(2,1,4) and C(1,1,0), D(0,0,1) then find unit vector parallel to $\overline{AB} - \overline{CD}$ Ans: $\frac{i+3j-2k}{\sqrt{14}}$
16. Find unit vector perpendicular to both \vec{a} and \vec{b} where $\vec{a} = 3i - 2j + 5k$ and $\vec{b} = 2i + 3j - 7k$ Ans: $\frac{\pm(-i+31j+13k)}{\sqrt{1131}}$
17. For vectors \vec{a}, \vec{b} and \vec{c} prove that $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b}) = 0$
18. Find the projection of \vec{a} on \vec{b} if $\vec{a} \cdot \vec{b} = 8$ and $\vec{b} = 2i + 6j + 3k$
19. Find sine of the angle between the vectors $\vec{a} = 4i - j + 3k$ and $\vec{b} = -2i + j - 2k$ Ans: $\sin \theta = \frac{1}{\sqrt{26}}$
20. Find the magnitude of two vectors \vec{a} and \vec{b} having same magnitude such that the angle between them is 60° and their scalar product is $1/2$.
21. Show that no line in space can make angles $\frac{\pi}{6}$ and $\frac{\pi}{4}$ with the X-axis and Y-axis
22. Find the angle between the lines whose direction ratios are 5,12,-13 and 3,-4,5 Ans: $\cos^{-1}\left(\frac{49}{65}\right)$
23. Find unit vectors perpendicular to \vec{a} and \vec{b} if $\vec{a} = 2i - j + 3k$ and $\vec{b} = 3i + j - 4k$ Ans: $\frac{i+17j+5k}{3\sqrt{35}}$
24. Find the projection of \vec{b} on \vec{a} where $\vec{a} = 2i + 3j - 4k$ and $\vec{b} = i - j - k$ Ans: $\frac{3}{\sqrt{29}}$
25. Find sine of the angle between the vectors $\vec{a} = i - 2k$ and $\vec{b} = j - 4k$ Ans: $\sin \theta = \sqrt{\frac{21}{85}}$
26. Find angle ' θ ' between the vectors $\vec{a} = i + j - k$ and $\vec{b} = i - j + k$. Ans: $\theta = \cos^{-1}\frac{1}{3}$.
27. Find the angle between the lines whose direction ratios are 5,12,-13 and 3,-4,5 Ans: $\cos^{-1}\left(\frac{49}{65}\right)$
28. Find $|\vec{a} \times \vec{b}|$, if $\vec{a} = 2i + j + 3k$ and $\vec{b} = 3i + 5j - 2k$. Ans: $\sqrt{507}$

29. If a line makes angles 90° , 135° and 45° with the X, Y and Z axis respectively the find its direction cosines. (Ans: $0, \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$)
30. Find the area of a triangle having points $A(1,1,1)$, $B(1,2,3)$, $C(2,3,1)$ as its vertices. Ans. $\frac{1}{2}\sqrt{21}$.
31. Find the area of a triangle ABC if the position vectors of the points A, B, C are respectively $i - 2j + k$, $2i - j + 2k$, $-i - j - k$. Ans. $\frac{3}{\sqrt{2}}$
32. Find the area of a parallelogram whose adjacent sides are given by the vectors $\vec{a} = 3i + j + 4k$ and $\vec{b} = i - j + k$. Ans. $\sqrt{42}$.