

## NCERT solutions for class 11 maths chapter 5 Complex Numbers and Quadratic Equations-Exercise: 5.1

**Question:1** Express each of the complex number in the form  $a + ib$ .

$$(5i) \left( -\frac{3}{5}i \right)$$

**Answer:**

On solving

$$(5i) \left( -\frac{3}{5}i \right)$$

we will get

$$\begin{aligned}(5i) \left( -\frac{3}{5}i \right) &= 5 \times \left( -\frac{3}{5} \right) \times i \times i \\&= -3 \times i^2 (\because i^2 = -1) \\&= -3 \times -1 \\&= 3\end{aligned}$$

Now, in the form of  $a + ib$  we can write it as

$$= 3 + 0i$$

**Question:2** Express each of the complex number in the form  $a + ib$ .

$$i^9 + i^{19}$$

**Answer:**

We know that  $i^4 = 1$

Now, we will reduce  $i^9 + i^{19}$  into

$$\begin{aligned} i^9 + i^{19} &= (i^4)^2 \cdot i + (i^4)^3 \cdot i^3 \\ &= (1)^2 \cdot i + (1)^3 \cdot (-i) \quad (\because i^4 = 1, i^3 = -i \text{ and } i^2 = -1) \\ &= i - i = 0 \end{aligned}$$

Now, in the form of  $a + ib$  we can write it as

$$0 + i0$$

Therefore, the answer is  $0 + i0$

**Question:3** Express each of the complex number in the form  $a+ib$ .

$$i^{-39}$$

**Answer:**

We know that  $i^4 = 1$

Now, we will reduce  $i^{-39}$  into

$$\begin{aligned} i^{-39} &= (i^4)^{-9} \cdot i^{-3} \\ &= (1)^{-9} \cdot (-i)^{-1} \quad (\because i^4 = 1, i^3 = -i) \\ &= \frac{1}{-i} \\ &= \frac{1}{-i} \times \frac{i}{i} \\ &= \frac{i}{-i^2} \quad (\because i^2 = -1) \\ &= \frac{i}{-(-1)} \\ &= i \end{aligned}$$

Now, in the form of  $a + ib$  we can write it as

$$0 + i1$$

Therefore, the answer is  $0 + i1$

**Question:4** Express each of the complex number in the form  $a+ib$ .

$$3(7 + 7i) + i(7 + 7i)$$

**Answer:**

Given problem is

$$3(7 + 7i) + i(7 + 7i)$$

Now, we will reduce it into

$$\begin{aligned} 3(7 + 7i) + i(7 + 7i) &= 21 + 21i + 7i + 7i^2 \\ &= 21 + 21i + 7i + 7(-1) \quad (\because i^2 = -1) \\ &= 21 + 21i + 7i - 7 \\ &= 14 + 28i \end{aligned}$$

Therefore, the answer is  $14 + i28$

**Question:5** Express each of the complex number in the form  $a + ib$ .

$$(1 - i) - (-1 + 6i)$$

**Answer:**

Given problem is

$$(1 - i) - (-1 + 6i)$$

Now, we will reduce it into

$$(1 - i) - (-1 + 6i) = 1 - i + 1 - 6i$$

$$= 2 - 7i$$

Therefore, the answer is  $2 - 7i$

**Question:6** Express each of the complex number in the form  $a + ib$ .

$$\left(\frac{1}{5} + i\frac{2}{5}\right) - \left(4 + i\frac{5}{2}\right)$$

**Answer:**

Given problem is

$$\left(\frac{1}{5} + i\frac{2}{5}\right) - \left(4 + i\frac{5}{2}\right)$$

Now, we will reduce it into

$$= \frac{1 - 20}{5} + i\frac{(4 - 25)}{10}$$

$$= -\frac{19}{5} - i\frac{21}{10}$$

Therefore, the answer is  $-\frac{19}{5} - i\frac{21}{10}$

**Question:7** Express each of the complex number in the form  $a + ib$ .

**Answer:**

Given problem is

Now, we will reduce it into

$$= \frac{1 + 4 + 12}{3} + i \frac{(7 + 1 - 3)}{3}$$

$$= \frac{17}{3} + i \frac{5}{3}$$

Therefore, the answer is  $\frac{17}{3} + i \frac{5}{3}$

**Question:8** Express each of the complex number in the form  $a + ib$ .

$$(1 - i)^4$$

**Answer:**

The given problem is

$$(1 - i)^4$$

Now, we will reduce it into

$$(1 - i)^4 = ((1 - i)^2)^2$$

$$= (1^2 + i^2 - 2 \cdot 1 \cdot i)^2 \text{ (using } (a - b)^2 = a^2 + b^2 - 2ab \text{)}$$

$$= (1 - 1 - 2i)^2 \text{ (}\because i^2 = -1 \text{)}$$

$$= (-2i)^2$$

$$= 4i^2$$

$$= -4$$

Therefore, the answer is  $-4 + i0$

**Question:9** Express each of the complex number in the form  $a + ib$ .

$$\left(\frac{1}{3} + 3i\right)^3$$

**Answer:**

Given problem is

$$\left(\frac{1}{3} + 3i\right)^3$$

Now, we will reduce it into

$$\begin{aligned} & (\text{using } (a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2) \\ &= \frac{1}{27} + 27i^3 + i + 9i^2 \\ &= \frac{1}{27} + 27(-i) + i + 9(-1) \quad (\because i^3 = -i \text{ and } i^2 = -1) \\ &= \frac{1}{27} - 27i + i - 9 \\ &= \frac{1 - 243}{27} - 26i \\ &= -\frac{242}{27} - 26i \end{aligned}$$

Therefore, the answer is

$$-\frac{242}{27} - 26i$$

**Question:10** Express each of the complex number in the form  $a + ib$ .

$$\left(-2 - \frac{1}{3}i\right)^3$$

**Answer:**

Given problem is

$$\left(-2 - \frac{1}{3}i\right)^3$$

Now, we will reduce it into

$$\begin{aligned} & (\text{using } (a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2) \\ &= - \left( 8 + \frac{1}{27}i^3 + 3 \cdot 4 \cdot \frac{1}{3}i + 3 \cdot \frac{1}{9}i^2 \cdot 2 \right) \\ &= - \left( 8 + \frac{1}{27}(-i) + 4i + \frac{2}{3}(-1) \right) (\because i^3 = -i \text{ and } i^2 = -1) \\ &= - \left( 8 - \frac{1}{27}i + 4i - \frac{2}{3} \right) \\ &= - \left( \frac{(-1 + 108)}{27}i + \frac{24 - 2}{3} \right) \\ &= -\frac{22}{3} - i\frac{107}{27} \end{aligned}$$

Therefore, the answer is  $-\frac{22}{3} - i\frac{107}{27}$

**Question:11** Find the multiplicative inverse of each of the complex numbers.

$$4 - 3i$$

**Answer:**

$$\text{Let } z = 4 - 3i$$

Then,

$$\bar{z} = 4 + 3i$$

And

$$|z|^2 = 4^2 + (-3)^2 = 16 + 9 = 25$$

Now, the multiplicative inverse is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{4 + 3i}{25} = \frac{4}{25} + i\frac{3}{25}$$

Therefore, the multiplicative inverse is

$$\frac{4}{25} + i\frac{3}{25}$$

**Question:12** Find the multiplicative inverse of each of the complex numbers.

$$\sqrt{5} + 3i$$

**Answer:**

$$\text{Let } z = \sqrt{5} + 3i$$

Then,

$$\bar{z} = \sqrt{5} - 3i$$

And

$$|z|^2 = (\sqrt{5})^2 + (3)^2 = 5 + 9 = 14$$

Now, the multiplicative inverse is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{\sqrt{5} - 3i}{14} = \frac{\sqrt{5}}{14} - i\frac{3}{14}$$

Therefore, the multiplicative inverse is  $\frac{\sqrt{5}}{14} - i\frac{3}{14}$

**Question:13** Find the multiplicative inverse of each of the complex numbers.

$$-i$$

**Answer:**



Let  $z = -i$

Then,

$$\bar{z} = i$$

And

$$|z|^2 = (0)^2 + (1)^2 = 0 + 1 = 1$$

Now, the multiplicative inverse is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{i}{1} = 0 + i$$

Therefore, the multiplicative inverse is  $0 + i1$

**Question:14** Express the following expression in the form of  $a + ib$  :

$$\frac{(3 + i\sqrt{5})(3 - i\sqrt{5})}{(\sqrt{3} + \sqrt{2}i) - (\sqrt{3} - i\sqrt{2})}$$

**Answer:**

Given problem is

$$\frac{(3 + i\sqrt{5})(3 - i\sqrt{5})}{(\sqrt{3} + \sqrt{2}i) - (\sqrt{3} - i\sqrt{2})}$$

Now, we will reduce it into

$$\begin{aligned} & \text{(using } (a - b)(a + b) = a^2 - b^2) \\ &= \frac{9 - 5i^2}{\sqrt{3} + \sqrt{2}i - \sqrt{3} + \sqrt{2}i} \\ &= \frac{9 - 5(-1)}{2\sqrt{2}i} \quad (\because i^2 = -1) \\ &= \frac{14}{2\sqrt{2}i} \times \frac{\sqrt{2}i}{\sqrt{2}i} \\ &= \frac{7\sqrt{2}i}{2i^2} \end{aligned}$$

$$= -\frac{7\sqrt{2}i}{2}$$

Therefore, answer is  $0 - i\frac{7\sqrt{2}}{2}$

## NCERT solutions for class 11 maths chapter 5 Complex Numbers and Quadratic Equations-Exercise: 5.2

**Question:1** Find the modulus and the arguments of each of the complex numbers.

$$z = -1 - i\sqrt{3}$$

**Answer:**

Given the problem is

$$z = -1 - i\sqrt{3}$$

Now, let

$$r \cos \theta = -1 \quad \text{and} \quad r \sin \theta = -\sqrt{3}$$

Square and add both the sides

$$r^2(\cos^2 \theta + \sin^2 \theta) = (-1)^2 + (-\sqrt{3})^2 \quad (\because \cos^2 \theta + \sin^2 \theta = 1)$$

$$r^2 = 1 + 3$$

$$r^2 = 4$$

$$r = 2 \quad (\because r > 0)$$

Therefore, the modulus is 2

Now,

$$2 \cos \theta = -1 \quad \text{and} \quad 2 \sin \theta = -\sqrt{3}$$

$$\cos \theta = -\frac{1}{2} \quad \text{and} \quad \sin \theta = -\frac{\sqrt{3}}{2}$$

Since, both the values of  $\cos \theta$  and  $\sin \theta$  is negative and we know that they are

negative in III quadrant

Therefore,

$$\text{Argument} = -\left(\pi - \frac{\pi}{3}\right) = -\frac{2\pi}{3}$$

Therefore, the argument is

$$-\frac{2\pi}{3}$$

**Question:2** Find the modulus and the arguments of each of the complex numbers.

$$z = -\sqrt{3} + i$$

**Answer:**

Given the problem is

$$z = -\sqrt{3} + i$$

Now, let

$$r \cos \theta = -\sqrt{3} \quad \text{and} \quad r \sin \theta = 1$$

Square and add both the sides

$$r^2(\cos^2 \theta + \sin^2 \theta) = (-\sqrt{3})^2 + (1)^2 \quad (\because \cos^2 \theta + \sin^2 \theta = 1)$$

$$r^2 = 1 + 3$$

$$r^2 = 4$$

$$r = 2 \quad (\because r > 0)$$

Therefore, the modulus is 2

Now,

$$2 \cos \theta = -\sqrt{3} \quad \text{and} \quad 2 \sin \theta = 1$$

$$\cos \theta = -\frac{\sqrt{3}}{2} \quad \text{and} \quad \sin \theta = \frac{1}{2}$$

Since values of  $\cos \theta$  is negative and value  $\sin \theta$  is positive and we know that this is the case in II quadrant

Therefore,

$$\text{Argument} = \left( \pi - \frac{\pi}{6} \right) = \frac{5\pi}{6}$$

Therefore, the argument is

$$\frac{5\pi}{6}$$

**Question:3** Convert each of the complex numbers in the polar form:

$$1 - i$$

**Answer:**

Given problem is

$$z = 1 - i$$

Now, let

$$r \cos \theta = 1 \quad \text{and} \quad r \sin \theta = -1$$

Square and add both the sides

$$r^2(\cos^2 \theta + \sin^2 \theta) = (1)^2 + (-1)^2 (\because \cos^2 \theta + \sin^2 \theta = 1)$$

$$r^2 = 1 + 1$$

$$r^2 = 2$$

$$r = \sqrt{2} (\because r > 0)$$

Therefore, the modulus is  $\sqrt{2}$

Now,

$$\sqrt{2} \cos \theta = 1 \quad \text{and} \quad \sqrt{2} \sin \theta = -1$$

$$\cos \theta = \frac{1}{\sqrt{2}} \quad \text{and} \quad \sin \theta = -\frac{1}{\sqrt{2}}$$

Since values of  $\sin \theta$  is negative and value  $\cos \theta$  is positive and we know that this is the case in the IV quadrant

Therefore,

$$\theta = -\frac{\pi}{4} \quad (\text{lies in IV quadrant})$$

Therefore,

$$\begin{aligned} 1 - i &= r \cos \theta + ir \sin \theta \\ &= \sqrt{2} \cos \left( -\frac{\pi}{4} \right) + i\sqrt{2} \sin \left( -\frac{\pi}{4} \right) \\ &= \sqrt{2} \left( \cos \left( -\frac{\pi}{4} \right) + i \sin \left( -\frac{\pi}{4} \right) \right) \end{aligned}$$

Therefore, the required polar form is  $\sqrt{2} \left( \cos \left( -\frac{\pi}{4} \right) + i \sin \left( -\frac{\pi}{4} \right) \right)$

**Question:4** Convert each of the complex numbers in the polar form:

$$-1 + i$$

**Answer:**

Given the problem is

$$z = -1 + i$$

Now, let

$$r \cos \theta = -1 \quad \text{and} \quad r \sin \theta = 1$$

Square and add both the sides

$$r^2(\cos^2 \theta + \sin^2 \theta) = (-1)^2 + (1)^2 \quad (\because \cos^2 \theta + \sin^2 \theta = 1)$$

$$r^2 = 1 + 1$$

$$r^2 = 2$$

$$r = \sqrt{2} \quad (\because r > 0)$$

Therefore, the modulus is  $\sqrt{2}$

Now,

$$\sqrt{2} \cos \theta = -1 \quad \text{and} \quad \sqrt{2} \sin \theta = 1$$

$$\cos \theta = -\frac{1}{\sqrt{2}} \quad \text{and} \quad \sin \theta = \frac{1}{\sqrt{2}}$$

Since values of  $\cos \theta$  is negative and value  $\sin \theta$  is positive and we know that this is the case in II quadrant

Therefore,

Therefore,

$$\begin{aligned}-1 + i &= r \cos \theta + ir \sin \theta \\&= \sqrt{2} \cos \left( \frac{3\pi}{4} \right) + i\sqrt{2} \sin \left( \frac{3\pi}{4} \right) \\&= \sqrt{2} \left( \cos \left( \frac{3\pi}{4} \right) + i \sin \left( \frac{3\pi}{4} \right) \right)\end{aligned}$$

Therefore, the required polar form is  $\sqrt{2} \left( \cos \left( \frac{3\pi}{4} \right) + i \sin \left( \frac{3\pi}{4} \right) \right)$

**Question:5** Convert each of the complex numbers in the polar form:

$$-1 - i$$

**Answer:**

Given problem is

$$z = -1 - i$$

Now, let

$$r \cos \theta = -1 \quad \text{and} \quad r \sin \theta = -1$$

Square and add both the sides

$$r^2(\cos^2 \theta + \sin^2 \theta) = (-1)^2 + (-1)^2 \quad (\because \cos^2 \theta + \sin^2 \theta = 1)$$

$$r^2 = 1 + 1$$

$$r^2 = 2$$

$$r = \sqrt{2} \quad (\because r > 0)$$

Therefore, the modulus is  $\sqrt{2}$

Now,

$$\sqrt{2} \cos \theta = -1 \quad \text{and} \quad \sqrt{2} \sin \theta = -1$$
$$\cos \theta = -\frac{1}{\sqrt{2}} \quad \text{and} \quad \sin \theta = -\frac{1}{\sqrt{2}}$$

Since values of both  $\cos \theta$  and  $\sin \theta$  is negative and we know that this is the case in III quadrant

Therefore,

Therefore,

$$\begin{aligned} -1 - i &= r \cos \theta + ir \sin \theta \\ &= \sqrt{2} \cos \left( -\frac{3\pi}{4} \right) + i \sqrt{2} \sin \left( -\frac{3\pi}{4} \right) \\ &= \sqrt{2} \left( \cos \left( -\frac{3\pi}{4} \right) + i \sin \left( -\frac{3\pi}{4} \right) \right) \end{aligned}$$

Therefore, the required polar form is  $\sqrt{2} \left( \cos \left( -\frac{3\pi}{4} \right) + i \sin \left( -\frac{3\pi}{4} \right) \right)$

**Question:6** Convert each of the complex numbers in the polar form:

$$-3$$

**Answer:**

Given problem is

$$z = -3$$

Now, let

$$r \cos \theta = -3 \quad \text{and} \quad r \sin \theta = 0$$

Square and add both the sides

$$r^2(\cos^2 \theta + \sin^2 \theta) = (-3)^2 + (0)^2 (\because \cos^2 \theta + \sin^2 \theta = 1)$$

$$r^2 = 9 + 0$$

$$r^2 = 9$$

$$r = 3 (\because r > 0)$$

Therefore, the modulus is **3**

Now,

$$3 \cos \theta = -3 \quad \text{and} \quad 3 \sin \theta = 0$$

$$\cos \theta = -1 \quad \text{and} \quad \sin \theta = 0$$

Since values of  $\cos \theta$  is negative and  $\sin \theta$  is Positive and we know that this is the case in II quadrant

Therefore,

$$\theta = \pi \quad (\text{lies in II quadrant})$$

Therefore,

$$-3 = r \cos \theta + ir \sin \theta$$

$$= 3 \cos (\pi) + i3 \sin (\pi)$$

$$= 3 (\cos \pi + i \sin \pi)$$

Therefore, the required polar form is  $3 (\cos \pi + i \sin \pi)$

**Question:7** Convert each of the complex numbers in the polar form:

$$\sqrt{3} + i$$

**Answer:**

Given problem is

$$z = \sqrt{3} + i$$

Now, let

$$r \cos \theta = \sqrt{3} \quad \text{and} \quad r \sin \theta = 1$$



Square and add both the sides

$$r^2(\cos^2 \theta + \sin^2 \theta) = (\sqrt{3})^2 + (1)^2 (\because \cos^2 \theta + \sin^2 \theta = 1)$$

$$r^2 = 3 + 1$$

$$r^2 = 4$$

$$r = 2 (\because r > 0)$$

Therefore, the modulus is **2**

Now,

$$2 \cos \theta = \sqrt{3} \quad \text{and} \quad 2 \sin \theta = 1$$

$$\cos \theta = \frac{\sqrt{3}}{2} \quad \text{and} \quad \sin \theta = \frac{1}{2}$$

Since values of Both  $\cos \theta$  and  $\sin \theta$  is Positive and we know that this is the case in I quadrant

Therefore,

$$\theta = \frac{\pi}{6} \quad (\text{lies in I quadrant})$$

Therefore,

$$\begin{aligned} \sqrt{3} + i &= r \cos \theta + ir \sin \theta \\ &= 2 \cos \left( \frac{\pi}{6} \right) + i 2 \sin \left( \frac{\pi}{6} \right) \\ &= 2 \left( \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right) \end{aligned}$$

Therefore, the required polar form is  $2 \left( \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)$

**Question:8** Convert each of the complex numbers in the polar form:

$$i$$

**Answer:**

Given problem is

$$z = i$$

Now, let

$$r \cos \theta = 0 \quad \text{and} \quad r \sin \theta = 1$$

Square and add both the sides

$$r^2(\cos^2 \theta + \sin^2 \theta) = (0)^2 + (1)^2 \quad (\because \cos^2 \theta + \sin^2 \theta = 1)$$

$$r^2 = 0 + 1$$

$$r^2 = 1$$

$$r = 1 \quad (\because r > 0)$$

Therefore, the modulus is **1**

Now,

$$1 \cos \theta = 0 \quad \text{and} \quad 1 \sin \theta = 1$$

$$\cos \theta = 0 \quad \text{and} \quad \sin \theta = 1$$

Since values of Both  $\cos \theta$  and  $\sin \theta$  is Positive and we know that this is the case in I quadrant

Therefore,

$$\theta = \frac{\pi}{2} \quad (\text{lies in I quadrant})$$

Therefore,

$$\begin{aligned} i &= r \cos \theta + ir \sin \theta \\ &= 1 \cos \left( \frac{\pi}{2} \right) + i1 \sin \left( \frac{\pi}{2} \right) \\ &= \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \end{aligned}$$

Therefore, the required polar form is  $\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$

## NCERT solutions for class 11 maths chapter 5 Complex Numbers and Quadratic Equations-Exercise: 5.3

**Question:1** Solve each of the following equations:  $x^2 + 3 = 0$

**Answer:**

Given equation is

$$x^2 + 3 = 0$$

Now, we know that the roots of the quadratic equation is given by the formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case value of **a = 1** , **b = 0** and **c = 3**

Therefore,

$$\frac{-0 \pm \sqrt{0^2 - 4.1.(3)}}{2.1} = \frac{\pm \sqrt{-12}}{2} = \frac{\pm 2\sqrt{3}i}{2} = \pm \sqrt{3}i$$

Therefore, the solutions of requires equation are  $\pm \sqrt{3}i$

**Question:2** Solve each of the following equations:  $2x^2 + x + 1 = 0$

**Answer:**

Given equation is

$$2x^2 + x + 1 = 0$$

Now, we know that the roots of the quadratic equation are given by the formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case value of **a = 2** , **b = 1** and **c = 1**

Therefore,

$$\frac{-1 \pm \sqrt{1^2 - 4 \cdot 2 \cdot 1}}{2 \cdot 2} = \frac{-1 \pm \sqrt{1 - 8}}{4} = \frac{-1 \pm \sqrt{-7}}{4} = \frac{-1 \pm \sqrt{7}i}{4}$$

Therefore, the solutions of requires equation are

$$\frac{-1 \pm \sqrt{7}i}{4}$$

**Question:3** Solve each of the following equations:  $x^2 + 3x + 9 = 0$

**Answer:**

Given equation is

$$x^2 + 3x + 9 = 0$$

Now, we know that the roots of the quadratic equation are given by the formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case value of **a = 1** , **b = 3** and **c = 9**

Therefore,

$$\frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot 9}}{2 \cdot 1} = \frac{-3 \pm \sqrt{9 - 36}}{2} = \frac{-3 \pm \sqrt{-27}}{2} = \frac{-3 \pm 3\sqrt{3}i}{2}$$

Therefore, the solutions of requires equation are

$$\frac{-3 \pm 3\sqrt{3}i}{2}$$

**Question:4** Solve each of the following equations:  $-x^2 + x - 2 = 0$

**Answer:**

Given equation is

$$-x^2 + x - 2 = 0$$

Now, we know that the roots of the quadratic equation is given by the formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case value of **a = -1** , **b = 1** and **c = -2**

Therefore,

Therefore, the solutions of equation are

$$\frac{-1 \pm \sqrt{7}i}{-2}$$

**Question:5** Solve each of the following equations:  $x^2 + 3x + 5 = 0$

**Answer:**

Given equation is

$$x^2 + 3x + 5 = 0$$

Now, we know that the roots of the quadratic equation are given by the formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case value of **a = 1** , **b = 3** and **c = 5**

Therefore,

$$\frac{-3 \pm \sqrt{3^2 - 4.1.5}}{2.1} = \frac{-3 \pm \sqrt{9 - 20}}{2} = \frac{-3 \pm \sqrt{-11}}{2} = \frac{-3 \pm \sqrt{11}i}{2}$$

Therefore, the solutions of the equation are

**Question:6** Solve each of the following equations:  $x^2 - x + 2 = 0$

**Answer:**

Given equation is

$$x^2 - x + 2 = 0$$

Now, we know that the roots of the quadratic equation are given by the formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case value of **a = 1** , **b = -1** and **c = 2**

Therefore,

$$\frac{1 \pm \sqrt{7}i}{2}$$

Therefore, the solutions of equation are

**Question:7** Solve each of the following equations:  $\sqrt{2}x^2 + x + \sqrt{2} = 0$

**Answer:**

Given equation is

$$\sqrt{2}x^2 + x + \sqrt{2} = 0$$

Now, we know that the roots of the quadratic equation is given by the formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case the value of  $a = \sqrt{2}$ ,  $b = 1$  and  $c = \sqrt{2}$

Therefore,

$$\frac{-1 \pm \sqrt{7}i}{2\sqrt{2}}$$

Therefore, the solutions of the equation are

**Question:8** Solve each of the following equations:  $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

**Answer:**

Given equation is

$$\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$$

Now, we know that the roots of the quadratic equation are given by the formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case the value of  $a = \sqrt{3}$ ,  $b = -\sqrt{2}$  and  $c = 3\sqrt{3}$

Therefore,  

$$= \frac{\sqrt{2} \pm \sqrt{34}i}{2\sqrt{3}}$$

Therefore, the solutions of the equation are  $\frac{\sqrt{2} \pm \sqrt{34}i}{2\sqrt{3}}$

$$x^2 + x + \frac{1}{\sqrt{2}} = 0$$

**Question:9** Solve each of the following equations:

**Answer:**

Given equation is  

$$x^2 + x + \frac{1}{\sqrt{2}} = 0$$

Now, we know that the roots of the quadratic equation is given by the formula  

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case the value of  $a = 1$ ,  $b = 1$  and  $c = \frac{1}{\sqrt{2}}$

Therefore,  

$$\frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot \frac{1}{\sqrt{2}}}}{2 \cdot 1} = \frac{-1 \pm \sqrt{1 - 2\sqrt{2}}}{2} = \frac{-1 \pm \sqrt{-(2\sqrt{2} - 1)}}{2}$$

$$= \frac{-1 \pm \sqrt{(2\sqrt{2} - 1)}i}{2}$$

Therefore, the solutions of the equation are

$$\frac{-1 \pm \sqrt{(2\sqrt{2} - 1)}i}{2}$$

**Question:10** Solve each of the following equations:

$$x^2 + \frac{x}{\sqrt{2}} + 1 = 0$$

**Answer:**

Given equation is

$$x^2 + \frac{x}{\sqrt{2}} + 1 = 0$$

Now, we know that the roots of the quadratic equation are given by the formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case the value of  $a = 1, b = \frac{1}{\sqrt{2}}$  and  $c = 1$

Therefore,

$$= \frac{-1 \pm \sqrt{7}i}{2\sqrt{2}}$$

Therefore, the solutions of the equation are

$$\frac{-1 \pm \sqrt{7}i}{2\sqrt{2}}$$

### NCERT solutions for class 11 maths chapter 5 Complex Numbers and Quadratic Equations-Miscellaneous Exercise

**Question:1** Evaluate  $\left[ i^{18} + \left( \frac{1}{i} \right)^{25} \right]^3$ .

**Answer:**

The given problem is

$$\left[ i^{18} + \left( \frac{1}{i} \right)^{25} \right]^3$$

Now, we will reduce it into



$$\begin{aligned}
&= \left[ 1^4 \cdot (-1) + \frac{1}{16 \cdot i} \right]^3 \quad (\because i^4 = 1, i^2 = -1) \\
&= \left[ -1 + \frac{1}{i} \right]^3 \\
&= \left[ -1 + \frac{1}{i} \times \frac{i}{i} \right]^3 \\
&= \left[ -1 + \frac{i}{i^2} \right]^3 \\
&= \left[ -1 + \frac{i}{-1} \right]^3 = [-1 - i]^3
\end{aligned}$$

Now,

$$\begin{aligned}
-(1 + i)^3 &= -(1^3 + i^3 + 3 \cdot 1^2 \cdot i + 3 \cdot 1 \cdot i^2) \\
&\text{(using } (a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2) \\
&= -(1 - i + 3i + 3(-1)) \quad (\because i^3 = -i, i^2 = -1) \\
&= -(1 - i + 3i - 3) = -(-2 + 2i) \\
&= 2 - 2i
\end{aligned}$$

Therefore, answer is  $2 - 2i$

**Question:2** For any two complex numbers  $z_1$  and  $z_2$ , prove that  $\operatorname{Re}(z_1 z_2) = \operatorname{Re} z_1 \operatorname{Re} z_2 - \operatorname{Im} z_1 \operatorname{Im} z_2$

**Answer:**

Let two complex numbers are

$$z_1 = x_1 + iy_1$$

$$z_2 = x_2 + iy_2$$

Now,

$$\begin{aligned}
z_1 \cdot z_2 &= (x_1 + iy_1) \cdot (x_2 + iy_2) \\
&= x_1 x_2 + ix_1 y_2 + iy_1 x_2 + i^2 y_1 y_2
\end{aligned}$$

$$= x_1x_2 + ix_1y_2 + iy_1x_2 - y_1y_2 (\because i^2 = -1)$$

$$= x_1x_2 - y_1y_2 + i(x_1y_2 + y_1x_2)$$

$$\operatorname{Re}(z_1z_2) = x_1x_2 - y_1y_2$$

$$= \operatorname{Re}(z_1z_2) - \operatorname{Im}(z_1z_2)$$

**Hence proved**

**Question:3** Reduce  $\left(\frac{1}{1-4i} - \frac{2}{1+i}\right) \left(\frac{3-4i}{5+i}\right)$  to the standard form.

**Answer:**

Given problem is

$$\left(\frac{1}{1-4i} - \frac{2}{1+i}\right) \left(\frac{3-4i}{5+i}\right)$$

Now, we will reduce it into

$$\begin{aligned} &= \left(\frac{1+i-2+8i}{1-4i+i-4i^2}\right) \left(\frac{3-4i}{5+i}\right) \\ &= \left(\frac{-1+9i}{1-3i-4(-1)}\right) \left(\frac{3-4i}{5+i}\right) \\ &= \left(\frac{-1+9i}{5-3i}\right) \left(\frac{3-4i}{5+i}\right) \end{aligned}$$

Now, multiply numerator and denominator by  $(14+5i)$

$$\begin{aligned} &\Rightarrow \frac{33+31i}{2(14-5i)} \times \frac{14+5i}{14+5i} \\ &\Rightarrow \frac{462+165i+434i+155i^2}{2(14^2-(5i)^2)} \quad (\text{using } (a-b)(a+b) = a^2 - b^2) \end{aligned}$$

$$\Rightarrow \frac{462 + 599i - 155}{2(196 - 25i^2)}$$

$$\Rightarrow \frac{307 + 599i}{2(196 + 25)} = \frac{307 + 599i}{2 \times 221} = \frac{307 + 599i}{442} = \frac{307}{442} + i\frac{599}{442}$$

Therefore, answer is  $\frac{307}{442} + i\frac{599}{442}$

**Question:4** If  $x - iy = \sqrt{\frac{a - ib}{c - id}}$ , prove that  $(x^2 + y^2)^2 = \frac{a^2 + b^2}{c^2 + d^2}$ .

**Answer:**

the given problem is

$$x - iy = \sqrt{\frac{a - ib}{c - id}}$$

Now, multiply the numerator and denominator by

$$\sqrt{c + id}$$

$$x - iy = \sqrt{\frac{a - ib}{c - id} \times \frac{c + id}{c + id}}$$

$$= \sqrt{\frac{(ac + bd) + i(ad - bc)}{c^2 - i^2d^2}} = \sqrt{\frac{(ac + bd) + i(ad - bc)}{c^2 + d^2}}$$

Now, square both the sides

$$(x - iy)^2 = \left( \sqrt{\frac{(ac + bd) + i(ad - bc)}{c^2 + d^2}} \right)^2$$

$$= \frac{(ac + bd) + i(ad - bc)}{c^2 + d^2}$$

$$x^2 - y^2 - 2ixy = \frac{(ac + bd) + i(ad - bc)}{c^2 + d^2}$$

On comparing the real and imaginary part, we obtain

$$x^2 - y^2 = \frac{ac + bd}{c^2 + d^2} \text{ and } -2xy = \frac{ad - bc}{c^2 + d^2} \quad - (i)$$

Now,

$$\begin{aligned} (x^2 + y^2)^2 &= (x^2 - y^2)^2 + 4x^2y^2 \\ &= \left( \frac{ac + bd}{c^2 + d^2} \right)^2 + \left( \frac{ad - bc}{c^2 + d^2} \right)^2 \quad (\text{using (i)}) \\ &= \frac{a^2c^2 + b^2d^2 + 2acbd + a^2d^2 + b^2c^2 - 2adbcd}{(c^2 + d^2)^2} \\ &= \frac{a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2}{(c^2 + d^2)^2} \\ &= \frac{a^2(c^2 + d^2) + b^2(c^2 + d^2)}{(c^2 + d^2)^2} \\ &= \frac{(a^2 + b^2)(c^2 + d^2)}{(c^2 + d^2)^2} \\ &= \frac{(a^2 + b^2)}{(c^2 + d^2)} \end{aligned}$$

Hence proved

**Question:5(i)** Convert the following in the polar form:

$$\frac{1 + 7i}{(2 - i)^2}$$

**Answer:**

Let

$$z = \frac{1 + 7i}{(2 - i)^2} = \frac{1 + 7i}{4 + i^2 - 4i} = \frac{1 + 7i}{4 - 1 - 4i} = \frac{1 + 7i}{3 - 4i}$$

Now, multiply the numerator and denominator by  $3 + 4i$

$$\Rightarrow z = \frac{1+7i}{3-4i} \times \frac{3+4i}{3+4i} = \frac{3+4i+21i+28i^2}{3^2+4^2} = \frac{-25+25i}{25} = -1+i$$

Now,

let

$$r \cos \theta = -1 \text{ and } r \sin \theta = 1$$

On squaring both and then add

$$r^2(\cos^2 \theta + \sin^2 \theta) = (-1)^2 + 1^2$$

$$r^2 = 2$$

$$r = \sqrt{2} \quad (\because r > 0)$$

Now,

$$\sqrt{2} \cos \theta = -1 \text{ and } \sqrt{2} \sin \theta = 1$$

$$\cos \theta = -\frac{1}{\sqrt{2}} \text{ and } \sin \theta = \frac{1}{\sqrt{2}}$$

Since the value of  $\cos \theta$  is negative and  $\sin \theta$  is positive this is the case in II quadrant

Therefore,

$$\begin{aligned} z &= r \cos \theta + ir \sin \theta \\ &= \sqrt{2} \cos \frac{3\pi}{4} + i\sqrt{2} \sin \frac{3\pi}{4} \\ &= \sqrt{2} \left( \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right) \end{aligned}$$

Therefore, the required polar form is

$$\sqrt{2} \left( \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)$$

**Question:5(ii)** Convert the following in the polar form:

$$\frac{1+3i}{1-2i}$$

**Answer:**

Let

$$z = \frac{1 + 3i}{1 - 2i}$$

Now, multiply the numerator and denominator by  $1 + 2i$

$$\Rightarrow z = \frac{1 + 3i}{1 - 2i} \times \frac{1 + 2i}{1 + 2i} = \frac{1 + 2i + 3i - 6}{1 + 4} = \frac{-5 + 5i}{5} = -1 + i$$

Now,

let

$$r \cos \theta = -1 \text{ and } r \sin \theta = 1$$

On squaring both and then add

$$r^2(\cos^2 \theta + \sin^2 \theta) = (-1)^2 + 1^2$$

$$r^2 = 2$$

$$r = \sqrt{2} \quad (\because r > 0)$$

Now,

$$\sqrt{2} \cos \theta = -1 \text{ and } \sqrt{2} \sin \theta = 1$$

$$\cos \theta = -\frac{1}{\sqrt{2}} \text{ and } \sin \theta = \frac{1}{\sqrt{2}}$$

Since the value of  $\cos \theta$  is negative and  $\sin \theta$  is positive this is the case in II quadrant

Therefore,

$$\begin{aligned} z &= r \cos \theta + ir \sin \theta \\ &= \sqrt{2} \cos \frac{3\pi}{4} + i\sqrt{2} \sin \frac{3\pi}{4} \\ &= \sqrt{2} \left( \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right) \end{aligned}$$

Therefore, the required polar form is

$$\sqrt{2} \left( \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)$$

**Question:6** Solve each of the equation:  $3x^2 - 4x + \frac{20}{3} = 0$

**Answer:**

Given equation is

$$3x^2 - 4x + \frac{20}{3} = 0$$

Now, we know that the roots of the quadratic equation are given by the formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case the value of

$$a = 3, b = -4 \text{ and } c = \frac{20}{3}$$

Therefore,

$$\frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 3 \cdot \frac{20}{3}}}{2 \cdot 3} = \frac{4 \pm \sqrt{16 - 80}}{6} = \frac{4 \pm \sqrt{-64}}{6} = \frac{4 \pm 8i}{6} = \frac{2}{3} \pm i \frac{4}{3}$$

Therefore, the solutions of requires equation are

$$\frac{2}{3} \pm i \frac{4}{3}$$

**Question:7** Solve each of the equation:  $x^2 - 2x + \frac{3}{2} = 0$

**Answer:**

Given equation is

$$x^2 - 2x + \frac{3}{2} = 0$$

Now, we know that the roots of the quadratic equation are given by the formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case the value of  $a = 1, b = -2 \text{ and } c = \frac{3}{2}$

Therefore,

$$\frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot \frac{3}{2}}}{2 \cdot 1} = \frac{2 \pm \sqrt{4 - 6}}{2} = \frac{2 \pm \sqrt{-2}}{2} = \frac{2 \pm i\sqrt{2}}{2} = 1 \pm i\frac{\sqrt{2}}{2}$$

Therefore, the solutions of requires equation are

$$1 \pm i\frac{\sqrt{2}}{2}$$

**Question:8** Solve each of the equation:  $27x^2 - 10x + 1 = 0$  .

**Answer:**

Given equation is

$$27x^2 - 10x + 1 = 0$$

Now, we know that the roots of the quadratic equation are given by the formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case the value of  $a = 27, b = -10$  and  $c = 1$

Therefore,

$$\begin{aligned} \frac{-(-10) \pm \sqrt{(-10)^2 - 4 \cdot 27 \cdot 1}}{2 \cdot 27} &= \frac{10 \pm \sqrt{100 - 108}}{54} = \frac{10 \pm \sqrt{-8}}{54} \\ &= \frac{10 \pm i2\sqrt{2}}{54} = \frac{5}{27} \pm i\frac{\sqrt{2}}{27} \end{aligned}$$

Therefore, the solutions of requires equation are  $\frac{5}{27} \pm i\frac{\sqrt{2}}{27}$

**Question:9** Solve each of the equation:  $21x^2 - 28x + 10 = 0$

**Answer:**

Given equation is

$$21x^2 - 28x + 10 = 0$$

Now, we know that the roots of the quadratic equation are given by the formula



$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case the value of  $a = 21, b = -28$  and  $c = 10$

Therefore,

$$\begin{aligned} \frac{-(-28) \pm \sqrt{(-28)^2 - 4 \cdot 21 \cdot 10}}{2 \cdot 21} &= \frac{28 \pm \sqrt{784 - 840}}{42} = \frac{28 \pm \sqrt{-56}}{42} \\ &= \frac{28 \pm i2\sqrt{14}}{42} = \frac{2}{3} \pm i\frac{\sqrt{14}}{21} \end{aligned}$$

Therefore, the solutions of requires equation are

$$\frac{2}{3} \pm i\frac{\sqrt{14}}{21}$$

**Question:10** If  $z_1 = 2 - i, z_2 = 1 + i$ , find  $\left| \frac{z_1 + z_2 + 1}{z_1 - z_2 + 1} \right|$ .

**Answer:**

It is given that

$$z_1 = 2 - i, z_2 = 1 + i$$

Then,

Now, multiply the numerator and denominator by  $1 + i$

Now,

$$|1 + i| = \sqrt{1^2 + 1^2} = \sqrt{1 + 1} = \sqrt{2}$$

Therefore, the value of

$$\left| \frac{z_1 + z_2 + 1}{z_1 - z_2 + 1} \right| \text{ is } \sqrt{2}$$

**Question:11** If  $a + ib = \frac{(x + i)^2}{2x^2 + 1}$ , prove that  $a^2 + b^2 = \frac{(x^2 + 1)^2}{(2x^2 + 1)^2}$ .

**Answer:**

It is given that

$$a + ib = \frac{(x + i)^2}{2x^2 + 1}$$

Now, we will reduce it into

On comparing real and imaginary part. we will get

$$a = \frac{x^2 - 1}{2x^2 + 1} \text{ and } b = \frac{2x}{2x^2 + 1}$$

Now,

$$\begin{aligned} a^2 + b^2 &= \left( \frac{x^2 - 1}{2x^2 + 1} \right)^2 + \left( \frac{2x}{2x^2 + 1} \right)^2 \\ &= \frac{x^4 + 1 - 2x^2 + 4x^2}{(2x^2 + 1)^2} \\ &= \frac{x^4 + 1 + 2x^2}{(2x^2 + 1)^2} \\ &= \frac{(x^2 + 1)^2}{(2x^2 + 1)^2} \end{aligned}$$

**Hence proved**

**Question:12(i)** Let  $z_1 = 2 - i$ ,  $z_2 = -2 + i$ . Find

$$\operatorname{Re} \left( \frac{z_1 z_2}{\bar{z}_1} \right)$$

**Answer:**

It is given that

$$z_1 = 2 - i \text{ and } z_2 = -2 + i$$

Now,

$$z_1 z_2 = (2 - i)(-2 + i) = -4 + 2i + 2i - i^2 = -4 + 4i + 1 = -3 + 4i$$

And

$$\bar{z}_1 = 2 + i$$

Now,

$$= \frac{-2 + 11i}{5} = -\frac{2}{5} + i\frac{11}{5}$$

Now,

$$\operatorname{Re}\left(\frac{z_1 z_2}{z_1}\right) = -\frac{2}{5}$$

Therefore, the answer is

$$-\frac{2}{5}$$

**Question:12(ii)** Let  $z_1 = 2 - i$ ,  $z_2 = -2 + i$ . Find

$$\operatorname{Im}\left(\frac{1}{z_1 z_1}\right)$$

**Answer:**

It is given that

$$z_1 = 2 - i$$

Therefore,

$$\bar{z}_1 = 2 + i$$

Now,

$$z_1 \bar{z}_1 = (2 - i)(2 + i) = 2^2 - i^2 = 4 + 1 = 5 \text{ (using } (a - b)(a + b) = a^2 - b^2 \text{)}$$

Now,

$$\frac{1}{z_1 \bar{z}_1} = \frac{1}{5}$$

Therefore,

$$\operatorname{Im}\left(\frac{1}{z_1 \bar{z}_1}\right) = 0$$

Therefore, the answer is **0**

**Question:13** Find the modulus and argument of the complex number  $\frac{1+2i}{1-3i}$ .

**Answer:**

Let

$$z = \frac{1+2i}{1-3i}$$

Now, multiply the numerator and denominator by  $(1+3i)$

$$= -\frac{1}{2} + i\frac{1}{2}$$

Therefore,

$$r \cos \theta = -\frac{1}{2} \text{ and } r \sin \theta = \frac{1}{2}$$

Square and add both the sides

$$r^2 \cos^2 \theta + r^2 \sin^2 \theta = \left(-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$r^2(\cos^2 \theta + \sin^2 \theta) = \left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)$$

$$r = \frac{1}{\sqrt{2}} \quad (\because r > 0)$$

Therefore, the modulus is  $\frac{1}{\sqrt{2}}$

Now,

$$\frac{1}{\sqrt{2}} \cos \theta = -\frac{1}{2} \text{ and } \frac{1}{\sqrt{2}} \sin \theta = \frac{1}{2}$$

$$\cos \theta = -\frac{1}{\sqrt{2}} \text{ and } \sin \theta = \frac{1}{\sqrt{2}}$$

Since the value of  $\cos \theta$  is negative and the value of  $\sin \theta$  is positive and we know that it is the case in II quadrant

Therefore,

$$\text{Argument} = \left( \pi - \frac{\pi}{4} \right) = \frac{3\pi}{4}$$

Therefore, Argument and modulus are  $\frac{3\pi}{4}$  and  $\frac{1}{\sqrt{2}}$  respectively

**Question:14** Find the real numbers x and y if  $(x - iy)(3 + 5i)$  is the conjugate of  $-6 - 24i$ .

**Answer:**

Let

$$z = (x - iy)(3 + 5i) = 3x + 5xi - 3yi - 5yi^2 = 3x + 5y + i(5x - 3y)$$

Therefore,

$$\bar{z} = (3x + 5y) - i(5x - 3y) \quad \text{--- (i)}$$

Now, it is given that

$$\bar{z} = -6 - 24i \quad \text{--- (ii)}$$

Compare (i) and (ii) we will get

$$(3x + 5y) - i(5x - 3y) = -6 - 24i$$

On comparing real and imaginary part. we will get

$$3x + 5y = -6 \quad \text{and} \quad 5x - 3y = 24$$

On solving these we will get

$$x = 3 \quad \text{and} \quad y = -3$$

Therefore, the value of x and y are 3 and -3 respectively

**Question:15** Find the modulus of  $\frac{1+i}{1-i} - \frac{1-i}{1+i}$ .

**Answer:**

Let

$$z = \frac{1+i}{1-i} - \frac{1-i}{1+i}$$

Now, we will reduce it into

$$\begin{aligned} z &= \frac{1+i}{1-i} - \frac{1-i}{1+i} = \frac{(1+i)^2 - (1-i)^2}{(1+i)(1-i)} = \frac{1^2 + i^2 + 2i - 1^2 - i^2 + 2i}{1^2 - i^2} \\ &= \frac{4i}{1+1} = \frac{4i}{2} = 2i \end{aligned}$$

Now,

$$r \cos \theta = 0 \quad \text{and} \quad r \sin \theta = 2$$

square and add both the sides. we will get,

$$r^2 \cos^2 \theta + r^2 \sin^2 \theta = 0^2 + 2^2$$

$$r^2 (\cos^2 \theta + \sin^2 \theta) = 4$$

$$r^2 = 4 \quad (\because \cos^2 \theta + \sin^2 \theta = 1)$$

$$r = 2 \quad (\because r > 0)$$

Therefore, modulus of

$$\frac{1+i}{1-i} - \frac{1-i}{1+i} \text{ is } 2$$

**Question:16** If  $(x + iy)^3 = u + iv$ , then show that  $\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$ .

**Answer:**

it is given that

$$(x + iy)^3 = u + iv$$

Now, expand the Left-hand side

$$x^3 + (iy)^3 + 3.(x)^2.iy + 3.x.(iy)^2 = u + iv$$

$$x^3 + i^3y^3 + 3x^2iy + 3xi^2y^2 = u + iv$$

$$x^3 - iy^3 + 3x^2iy - 3xy^2 = u + iv \quad (\because i^3 = -i \text{ and } i^2 = -1)$$

$$x^3 - 3xy^2 + i(3x^2y - y^3) = u + iv$$

On comparing real and imaginary part. we will get,

$$u = x^3 - 3xy^2 \quad \text{and} \quad v = 3x^2y - y^3$$

Now,

$$\frac{u}{x} + \frac{v}{y} = \frac{x(x^2 - 3y^2)}{x} + \frac{y(3x^2 - y^2)}{y}$$

$$= x^2 - 3y^2 + 3x^2 - y^2$$

$$= 4x^2 - 4y^2$$

$$= 4(x^2 - y^2)$$

Hence proved

**Question:17** If  $\alpha$  and  $\beta$  are different complex numbers with  $|\beta| = 1$ , then find  $\left| \frac{\beta - \alpha}{1 - \bar{\alpha}\beta} \right|$ .

**Answer:**

Let

$$\alpha = a + ib \text{ and } \beta = x + iy$$

It is given that

$$|\beta| = 1 \Rightarrow \sqrt{x^2 + y^2} = 1 \Rightarrow x^2 + y^2 = 1$$

and

$$\bar{\alpha} = a - ib$$

Now,

$$\begin{aligned}
&= \left| \frac{(x-a) + i(y-b)}{(1-ax-yb) - i(bx-ay)} \right| \\
&= \frac{\sqrt{(x-a)^2 + (y-b)^2}}{\sqrt{(1-ax-yb)^2 + (bx-ay)^2}} \\
&= \frac{\sqrt{x^2 + a^2 - 2xa + y^2 + b^2 - yb}}{\sqrt{1 + a^2x^2 + b^2y^2 - 2ax + 2abxy - by + b^2x^2 + a^2y^2 - 2abxy}} \\
&= \frac{\sqrt{(x^2 + y^2) + a^2 - 2xa + b^2 - yb}}{\sqrt{1 + a^2(x^2 + y^2) + b^2(x^2 + y^2) - 2ax + 2abxy - by - 2abxy}} \\
&= \frac{\sqrt{1 + a^2 - 2xa + b^2 - yb}}{\sqrt{1 + a^2 + b^2 - 2ax - by}} \quad (\because x^2 + y^2 = 1 \text{ given}) \\
&= 1
\end{aligned}$$

Therefore, value of  $\left| \frac{\beta - \alpha}{1 - \bar{\alpha}\beta} \right|$  is **1**

**Question:18** Find the number of non-zero integral solutions of the equation  $|1 - i|^x = 2^x$ .

**Answer:**

Given problem is

$$|1 - i|^x = 2^x$$

Now,

$$(\sqrt{1^2 + (-1)^2})^x = 2^x$$

$$(\sqrt{1+1})^x = 2^x$$

$$(\sqrt{2})^x = 2^x$$

$$2^{\frac{x}{2}} = 2^x$$

$$\frac{x}{2} = x$$

$$\frac{x}{2} = 0$$

**x = 0** is the only possible solution to the given problem



Therefore, there are **0** number of non-zero integral solutions of the equation  $|1 - i|^x = 2^x$

**Question:19** If  $(a + ib)(c + id)(e + if)(g + ih) = A + iB$ , then show that  $(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$

**Answer:**

It is given that

$$(a + ib)(c + id)(e + if)(g + ih) = A + iB,$$

Now, take mod on both sides

$$\begin{aligned} |(a + ib)(c + id)(e + if)(g + ih)| &= |A + iB| \\ |(a + ib)||c + id||e + if||g + ih| &= |A + iB| (\because |z_1 z_2| = |z_1||z_2|) \\ (\sqrt{a^2 + b^2})(\sqrt{c^2 + d^2})(\sqrt{e^2 + f^2})(\sqrt{g^2 + h^2}) &= (\sqrt{A^2 + B^2}) \end{aligned}$$

Square both the sides. we will get

$$(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = (A^2 + B^2)$$

**Hence proved**

**Question:20** If  $\left(\frac{1+i}{1-i}\right)^m = 1$ , then find the least positive integral value of  $m$ .

**Answer:**

Let

$$z = \left(\frac{1+i}{1-i}\right)^m$$

Now, multiply both numerator and denominator by  $(1 + i)$

We will get,

$$\begin{aligned} z &= \left( \frac{1+i}{1-i} \times \frac{1+i}{1+i} \right)^m \\ &= \left( \frac{(1+i)^2}{1^2 - i^2} \right)^m \\ &= \left( \frac{1^2 + i^2 + 2i}{1+1} \right)^m \\ &= \left( \frac{1-1+2i}{2} \right)^m \quad (\because i^2 = -1) \\ &= \left( \frac{2i}{2} \right)^m \\ &= i^m \end{aligned}$$

We know that  $i^4 = 1$

Therefore, the least positive integral value of  $m$  is **4**