The Bell System Technical Journal

October, 1933

Loudness, Its Definition, Measurement and Calculation*

By HARVEY FLETCHER and W. A. MUNSON

An empirical formula for calculating the loudness of any steady sound from an analysis of the intensity and frequency of its components is developed in this article. The development is based on fundamental properties of the hearing mechanism in such a way that a scale of loudness values results. In order to determine the form of the function representing this loudness scale and of the other factors entering into the loudness formula, measurements were made of the loudness levels of many sounds, both of pure tones and of complex wave forms. These tests are described and the method of measuring loudness levels is discussed in detail. Definitions are given endeavoring to clarify the terms used and the measurement of the physical quantities which determine the characteristics of a sound wave stimulating the auditory mechanism.

Introduction

LOUDNESS is a psychological term used to describe the magnitude of an auditory sensation. Although we use the terms "very loud," "loud," "moderately loud," "soft" and "very soft," corresponding to the musical notations *ff*, *f*, *mf*, *p*, and *pp*, to define the magnitude, it is evident that these terms are not at all precise and depend upon the experience, the auditory acuity, and the customs of the persons using them. If loudness depended only upon the intensity of the sound wave producing the loudness, then measurements of the physical intensity would definitely determine the loudness as sensed by a typical individual and therefore could be used as a precise means of defining it. However, no such simple relation exists.

The magnitude of an auditory sensation, that is, the loudness of the sound, is probably dependent upon the total number of nerve impulses that reach the brain per second along the auditory tract. It is evident that these auditory phenomena are dependent not alone upon the intensity of the sound but also upon their physical composition. For example, if a person listened to a flute and then to a bass drum placed at such distances that the sounds coming from the two instruments are judged to be equally loud, then the intensity of the sound at the ear produced by the bass drum would be many times that produced by the flute.

If the composition of the sound, that is, its wave form, is held constant, but its intensity at the ear of the listener varied, then the loud-

* Jour. Acous. Soc. Amer., October, 1933.

ness produced will be the same for the same intensity only if the same or an equivalent ear is receiving the sound and also only if the listener is in the same psychological and physiological conditions, with reference to fatigue, attention, alertness, etc. Therefore, in order to determine the loudness produced, it is necessary to define the intensity of the sound, its physical composition, the kind of ear receiving it, and the physiological and psychological conditions of the listener. In most engineering problems we are interested mainly in the effect upon a typical observer who is in a typical condition for listening.

In a paper during 1921 one of us suggested using the number of decibels above threshold as a measure of loudness and some experimental data were presented on this basis. As more data were accumulated it was evident that such a basis for defining loudness must be abandoned.

In 1924 in a paper by Steinberg and Fletcher [1] some data were given which showed the effects of eliminating certain frequency bands upon the loudness of the sound. By using such data as a basis, a mathematical formula was given for calculating the loudness losses of a sound being transmitted to the ear, due to changes in the transmission system. The formula was limited in its application to the particular sounds studied, namely, speech and a sound which was generated by an electrical buzzer and called the test tone.

In 1925 Steinberg [2] developed a formula for calculating the loudness of any complex sound. The results computed by this formula agreed with the data which were then available. However, as more data have accumulated it has been found to be inadequate. Since that time considerably more information concerning the mechanism of hearing has been discovered and the technique in making loudness measurements has advanced. Also more powerful methods for producing complex tones of any known composition are now available. For these reasons and because of the demand for a loudness formula of general application, especially in connection with noise measurements, the whole subject was reviewed by the Bell Telephone Laboratories and the work reported in the present paper undertaken. This work has resulted in better experimental methods for determining the loudness level of any sustained complex sound and a formula which gives calculated results in agreement with the great variety of loudness data which are now available.

[1] H. Fletcher and J. C. Steinberg, "Loudness of a Complex Sound," *Phys, Rev.* **24**, 306 (1924).

[2] J. C. Steinberg, "The Loudness of a Sound and Its Physical Stimulus," *Phys. Rev.* **26**, 507 (1925).

Definitions

The subject matter which follows necessitates the use of a number of terms which have often been applied in very inexact ways in the past. Because of the increase in interest and activity in this field, it became desirable to obtain a general agreement concerning the meaning of the terms which are most frequently used. The following definitions are taken from recent proposals of the sectional committee on Acoustical Measurements and Terminology of the American Standards Association and the terms have been used with these meanings throughout the paper.

Sound Intensity

The sound intensity of a sound field in a specified direction at a point is the sound energy transmitted per unit of time in the specified direction through a unit area normal to this direction at the point.

In the case of a plane or spherical free progressive wave having the effective sound pressure P (bars), the velocity of propagation c (cm. per sec.) in a medium of density ρ (grams per cubic cm.), the intensity in the direction of propagation is given by

$$J = P^2/\rho c \text{ (ergs per sec. per sq. cm.).} \qquad (1)$$

This same relation can often be used in practice with sufficient accuracy to calculate the intensity at a point near the source with only a pressure measurement. In more complicated sound fields the results given by this relation may differ greatly from the actual intensity.

When dealing with a plane or a spherical progressive wave it will be understood that the intensity is taken in the direction of propagation of the wave.

Reference Intensity

The reference intensity for intensity level comparisons shall be 10^{-16} watts per square centimeter. In a plane or spherical progressive sound wave in air, this intensity corresponds to a root-mean-square pressure p given by the formula

$$p = 0.000207[(H/76)(273/T)^{\frac{1}{2}}]^{\frac{1}{2}} \qquad (2)$$

where p is expressed in bars, H is the height of the barometer in centimeters, and T is the absolute temperature. At a temperature of 20° C. and a pressure of 76 cm. of Hg, $p = 0.000204$ bar.

Intensity Level

The intensity level of a sound is the number of db above the reference intensity.

Reference Tone

A plane or spherical sound wave having only a single frequency of 1,000 cycles per second shall be used as the reference for loudness comparisons.

Note: One practical way to obtain a plane or spherical wave is to use a small source, and to have the head of the observer at least one meter distant from the source, with the external conditions such that reflected waves are negligible as compared with the original wave at the head of the observer.

Loudness Level

The loudness level of any sound shall be the intensity level of the equally loud reference tone at the position where the listener's head is to be placed.

Manner of Listening to the Sound

In observing the loudness of the reference sound, the observer shall face the source, which should be small, and listen with both ears at a position so that the distance from the source to a line joining the two ears is one meter.

The value of the intensity level of the equally loud reference sound depends upon the manner of listening to the unknown sound and also to the standard of reference. The manner of listening to the unknown sound may be considered as part of the characteristics of that sound. The manner of listening to the reference sound is as specified above.

Loudness has been briefly defined as the magnitude of an auditory sensation, and more will be said about this later, but it will be seen from the above definitions that the *loudness level* of any sound is obtained by adjusting the intensity level of the reference tone until it sounds equally loud as judged by a typical listener. The only way of determining a typical listener is to use a number of observers who have normal hearing to make the judgment tests. The typical listener, as used in this sense, would then give the same results as the average obtained by a large number of such observers.

A pure tone having a frequency of 1000 cycles per second was chosen for the reference tone for the following reasons: (1) it is simple to define, (2) it is sometimes used as a standard of reference for pitch, (3) its use makes the mathematical formulae more simple, (4) its range of auditory intensities (from the threshold of hearing to the threshold of feeling) is as large and usually larger than for any other type of sound, and (5) its frequency is in the mid-range of audible frequencies.

There has been considerable discussion concerning the choice of the reference or zero for loudness levels. In many ways the threshold of hearing intensity for a 1000-cycle tone seems a logical choice. However, variations in this threshold intensity arise depending upon the individual, his age, the manner of listening, the method of presenting the tone to the listener, etc. For this reason no attempt was made to choose the reference intensity as equal to the average threshold of a given group listening in a prescribed way. Rather, an intensity of the reference tone in air of 10^{-16} watts per square centimeter was chosen as the reference intensity because it was a simple number which was convenient as a reference for computation work, and at the same time it is in the range of threshold measurements obtained when listening in the standard method described above. This reference intensity corresponds to the threshold intensity of an observer who might be designated a reference observer. An examination of a large series of measurements on the threshold of hearing indicates that such a reference observer has a hearing which is slightly more acute than the average of a large group. For those who have been thinking in terms of microwatts it is easy to remember that this reference level is 100 db below one microwatt per square centimeter. When using these definitions the intensity level β_r of the reference tone is the same as its loudness level L and is given by

$$\beta_r = L = 10 \log J_r + 100, \tag{3}$$

where J_r is its sound intensity in microwatts per square centimeter.

The intensity level of any other sound is given by

$$\beta = 10 \log J + 100, \tag{4}$$

where J is its sound intensity, but the loudness level of such a sound is a complicated function of the intensities and frequencies of its components. However, it will be seen from the experimental data given later that for a considerable range of frequencies and intensities the intensity level and loudness level for pure tones are approximately equal.

With the reference levels adopted here, all values of loudness level which are positive indicate a sound which can be heard by the reference observer and those which are negative indicate a sound which cannot be heard by such an observer.

It is frequently more convenient to use two matched head receivers for introducing the reference tone into the two ears. This can be done provided they are calibrated against the condition described above. This consists in finding by a series of listening tests by a number of observers the electrical power W_1 in the receivers which produces the same loudness as a level β_1 of the reference tone. The intensity level β_r of an open air reference tone equivalent to that produced in the receiver for any other power W_r in the receivers is then given by

$$\beta_r = \beta_1 + 10 \log (W_r/W_1). \tag{5}$$

Or, since the intensity level β_r of the reference tone is its loudness level L, we have

$$L = 10 \log W_r + C_r, \tag{6}$$

where C_r is a constant of the receivers.

In determining loudness levels by comparison with a reference tone there are two general classes of sound for which measurements are desired: (1) those which are steady, such as a musical tone, or the hum from machinery, (2) those which are varying in loudness such as the noise from the street, conversational speech, music, etc. In this paper we have confined our discussion to sources which are steady and the method of specifying such sources will now be given.

A steady sound can be represented by a finite number of pure tones called components. Since changes in phase produce only second order effects upon the loudness level it is only necessary to specify the magnitude and frequency of the components.[3] The magnitudes of the components at the listening position where the loudness level is desired are given by the intensity levels $\beta_1, \beta_2, \cdots \beta_k, \cdots \beta_n$ of each component at that position. In case the sound is conducted to the ears by telephone receivers or tubes, then a value W_k for each component must be known such that if this component were acting separately it would produce the same loudness for typical observers as a tone of the same pitch coming from a source at one meter's distance and producing an intensity level of β_k.

In addition to the frequency and magnitude of the components of a sound it is necessary to know the position and orientation of the head with respect to the source, and also whether one or two ears are used in listening. The monaural type of listening is important in telephone use and the binaural type when listening directly to a sound source in air. Unless otherwise stated, the discussion and data which follow apply to the condition where the listener faces the source and uses both ears, or uses head telephone receivers which produce an equivalent result.

[3] Recent work by Chapin and Firestone indicates that at very high levels these second order effects become large and cannot be neglected. K. E. Chapin and F. A. Firestone, "Interference of Subjective Harmonics," *Jour. Acous. Soc. Am.* **4**, 176*A* (1933).

Formulation of the Empirical Theory for Calculating the Loudness Level of a Steady Complex Tone

It is well known that the intensity of a complex tone is the sum of the intensities of the individual components. Similarly, in finding a method of calculating the loudness level of a complex tone one would naturally try to find numbers which could be related to each component in such a way that the sum of such numbers will be related in the same way to the equally loud reference tone. Such efforts have failed because the amount contributed by any component toward the total loudness sensation depends not only upon the properties of this component but also upon the properties of the other components in the combination. The answer to the problem of finding a method of calculating the loudness level lies in determining the nature of the ear and brain as measuring instruments in evaluating the magnitude of an auditory sensation.

One can readily estimate roughly the magnitude of an auditory sensation; for example, one can tell whether the sound is soft or loud. There have been many theories to account for this change in loudness. One that seems very reasonable to us is that the loudness experienced is dependent upon the total number of nerve impulses per second going to the brain along all the fibers that are excited. Although such an assumption is not necessary for deriving the formula for calculating loudness it aids in making the meaning of the quantities involved more definite.

Let us consider, then, a complex tone having n components each of which is specified by a value of intensity level β_k and of frequency f_k. Let N be a number which measures the magnitude of the auditory sensation produced when a typical individual listens to a pure tone. *Since by definition the magnitude of an auditory sensation is the loudness, then N is the loudness of this simple tone.* Loudness as used here must not be confused with loudness level. The latter is measured by the intensity of the equally loud reference tone and is expressed in decibels while the former will be expressed in units related to loudness levels in a manner to be developed. If we accept the assumption mentioned above, N is proportional to the number of nerve impulses per second reaching the brain along all the excited nerve fibers when the typical observer listens to a simple tone.

Let the dependency of the loudness N upon the frequency f and the intensity β for a simple tone be represented by

$$N = G(f, \beta), \tag{7}$$

where G is a function which is determined by any pair of values of f

and β. For the reference tone, f is 1000 and β is equal to the loudness level L, so a determination of the relation expressed in Eq. (7) for the reference tone gives the desired relation between loudness and loudness level.

If now a simple tone is put into combination with other simple tones to form a complex tone, its loudness contribution, that is, its contribution toward the total sensation, will in general be somewhat less because of the interference of the other components. For example, if the other components are much louder and in the same frequency region the loudness of the simple tone in such a combination will be zero. Let $1 - b$ be the fractional reduction in loudness because of its being in such a combination. Then bN is the contribution of this component toward the loudness of the complex tone. It will be seen that b by definition always remains between 0 and unity. It depends not only upon the frequency and intensity of the simple tone under discussion but also upon the frequencies and intensities of the other components. It will be shown later that this dependence can be determined from experimental measurements.

The subscript k will be used when f and β correspond to the frequency and intensity level of the kth component of the complex tone, and the subscript r used when f is 1000 cycles per second. The "loudness level" L by definition, is the intensity level of the reference tone when it is adjusted so it and the complex tone sound equally loud. Then

$$N_r = G(1000,\, L) = \sum_{k=1}^{k=n} b_k N_k = \sum_{k=1}^{k=n} b_k G(f_k,\, \beta_k). \tag{8}$$

Now let the reference tone be adjusted so that it sounds equally loud successively to simple tones corresponding in frequency and intensity to each component of the complex tone.

Designate the experimental values thus determined as $L_1, L_2, L_3, \cdots L_k, \cdots L_n$. Then from the definition of these values

$$N_k = G(1000,\, L_k) = G(f_k,\, \beta_k), \tag{9}$$

since for a single tone b_k is unity. On substituting the values from (9) into (8) there results the fundamental equation for calculating the loudness of a complex tone

$$G(1000,\, L) = \sum_{k=1}^{k=n} b_k G(1000,\, L_k). \tag{10}$$

This transformation looks simple but it is a very important one since instead of having to determine a different function for every component, we now have to determine a single function depending only upon the properties of the reference tone and as stated above this function is the relationship between loudness and loudness level. And since the frequency is always 1000 this function is dependent only upon the single variable, the intensity level.

This formula has no practical value unless we can determine b_k and G in terms of quantities which can be obtained by physical measurements. It will be shown that experimental measurements of the loudness levels L and L_k upon simple and complex tones of a properly chosen structure have yielded results which have enabled us to find the dependence of b and G upon the frequencies and intensities of the components. When b and G are known, then the more general function $G(f, \beta)$ can be obtained from Eq. (9), and the experimental values of L_k corresponding to f_k and β_k.

Determination of the Relation Between L_k, f_k and β_k

This relation can be obtained from experimental measurements of the loudness levels of pure tones. Such measurements were made by Kingsbury [4] which covered a range in frequency and intensity limited by instrumentalities then available. Using the experimental technique described in Appendix A, we have again obtained the loudness levels of pure tones, this time covering practically the whole audible range. (See Appendix B for a comparison with Kingsbury's results.)

All of the data on loudness levels both for pure and also complex tones taken in our laboratory which are discussed in this paper have been taken with telephone receivers on the ears. It has been explained previously how telephone receivers may be used to introduce the reference tone into the ears at known loudness levels to obtain the loudness levels of other sounds by a loudness balance. If the receivers are also used for producing the sounds whose loudness levels are being determined, then an additional calibration, which will be explained later, is necessary if it is desired to know the intensity levels of the sounds.

The experimental data for determining the relation between L_k and f_k are given in Table I in terms of voltage levels. (Voltage level $= 20 \log V$, where V is the e.m.f. across the receivers in volts.) The pairs of values in each double column give the voltage levels of the reference tone and the pure tone having the frequency indicated at the top of the column when the two tones coming from the head receivers were judged to be equally loud when using the technique

[4] B. A. Kingsbury, "A Direct Comparison of the Loudness of Pure Tones," *Phys. Rev.* **29**, 588 (1927).

TABLE I

Voltage Levels (db) for Loudness Equality

Reference	62 c.p.s.	Reference	125 c.p.s.	Reference	250 c.p.s.	Reference	500 c.p.s.	Reference	2000 c.p.s.	Reference	4000 c.p.s.	Reference	5650 c.p.s.	Reference	8000 c.p.s.	Reference	11,300 c.p.s.	Reference	16,000 c.p.s.
−12.2	+ 9.8	− 4.4	+ 9.8	− 2.9	+ 6.6	− 2.2	+ 5.8	− 2.2	− 1.2	− 1.7	− 0.7	− 3.7	+ 0.8	−10.9	− 4.3	−20.2	+ 1.7	−50.2	+ 1.8
−17.2	+ 5.8	−10.2	+ 7.0	− 3.7	+ 5.7	− 4.2	+ 6.8	− 1.7	− 1.5	− 1.2	− 1.5	− 7.2	− 1.2	−12.2	−12.0	−22.2	+ 1.8	−66.2	−13.2
−19.2	+ 2.8	−13.3	− 0.8	− 5.2	+ 5.8	− 6.2	− 2.0	− 3.2	− 1.3	−23.2	−17.3	−28.2	−19.2	−26.2	−24.0	−38.2	−20.2	−77.2	−28.2
−15.7	+ 2.6	−18.6	− 3.2	− 6.7	− 2.2	− 7.2	− 2.3	−18.2	−13.4	−24.7	−16.7	−30.2	−19.2	−27.1	−24.3	−38.7	−20.3	−85.2	−38.2
−21.2	+ 0.8	−23.2	− 5.2	−12.2	− 2.2	−12.2	− 8.2	−21.2	−17.2	−44.7	−35.2	−53.2	−38.2	−46.2	−38.3	−55.2	−34.2		
−27.2	− 0.2	−27.9	−12.3	−25.5	−18.3	−21.2	−22.3	−22.2	−18.3	−47.7	−35.1	−54.7	−39.1	−48.2	−38.2	−55.7	−34.2		
−32.2	− 7.2	−31.0	−14.2	−32.2	−22.2	−21.7	−21.9	−40.2	−35.2	−63.2	−54.2	−72.2	−58.2	−64.2	−52.2	−72.2	−52.2		
−33.2	− 7.2	−35.2	−15.2	−32.2	−23.2	−32.2	−30.2	−41.2	−35.3	−65.2	−54.5	−72.7	−58.1	−70.2	−56.2	−78.7	−58.1		
−41.2	−10.2	−40.7	−23.6	−52.5	−40.4	−34.2	−31.2	−42.2	−35.4	−77.7	−72.2	−85.2	−71.2	−76.2	−72.2	−88.2	−72.2		
−35.4	−10.4	−66.6	−35.0	−72.9	−56.3	−41.7	−41.9	−59.2	−54.2	−80.2	−72.5	−92.7	−78.1	−82.6	−76.2	−90.7	−77.1		
−56.2	−15.2	−88.8	−46.5	−00.2	−68.3	−43.7	−42.2	−61.2	−57.5										
−67.2	−20.2					−63.7	−61.0	−64.2	−57.3										
−68.7	−20.3					−64.2	−61.2	−78.2	−77.3										
−97.2	−30.3					−83.2	−80.2	−78.2	−80.2										
						−83.7	−78.0	−81.7	−77.3										
						−108.1	−102.6												
−108.1	−39.8					−108.3	−101.7	−108.3	−105.2	−108.3	−104.6								
−108.3	−39.5	−108.1	−62.8	−108.1	−86.7	−108.3	− 99.7	−108.3	−109.0	−108.3	−105.7	−108.3	−101.0	−108.3	−108.1	−108.3	−93.7		
−109.3	−42.4	−108.3	−60.7	−108.3	−86.4	−109.3	−103.4	−109.3	−108.9	−109.3	−102.0	−109.3	− 99.3	−109.3	−103.1	−100.3	−94.6		
−113.1	−38.5	−113.1	−63.5	−113.1	−86.3	−113.1	−103.0	−113.1	−111.4	−113.1	−108.1	−113.1	−102.3	−113.1	−106.6	−113.1	−93.7	−109.3	−57.3

described in Appendix A. For example, in the second column it will be seen that for the 125-cycle tone when the voltage is + 9.8 db above 1 volt then the voltage level for the reference tone must be 4.4 db below 1 volt for equality of loudness. The bottom set of numbers in each column gives the threshold values for this group of observers.

Each voltage level in Table I is the median of 297 observations representing the combined results of eleven observers. The method of obtaining these is explained in Appendix A also. The standard deviation was computed and it was found to be somewhat larger for tests in which the tone differed most in frequency from the reference tone. The probable error of the combined result as computed in the usual way was between 1 and 2 db. Since deviations of any one observer's results from his own average are less than the deviations of his average from the average of the group, it would be necessary to increase the size of the group if values more representative of the average normal ear were desired.

The data shown in Table I can be reduced to the number of decibels above threshold if we accept the values of this crew as the reference threshold values. However, we have already adopted a value for the 1000-cycle reference zero. As will be shown, our crew obtained a threshold for the reference tone which is 3 db above the reference level chosen.

It is not only more convenient but also more reliable to relate the data to a calibration of the receivers in terms of physical measurements of the sound intensity rather than to the threshold values. Except in experimental work where the intensity of the sound can be definitely controlled, it is obviously impractical to measure directly the threshold level by using a large group of observers having normal hearing. For most purposes it is more convenient to measure the intensity levels $\beta_1, \beta_2, \cdots \beta_k$, etc., directly rather than have them related in any way to the threshold of hearing.

In order to reduce the data in Table I to those which one would obtain if the observers were listening to a free wave and facing the source, we must obtain a field calibration of the telephone receivers used in the loudness comparisons. The calibration for the reference tone frequency has been explained previously and the equation

$$\beta_r = \beta_1 + 10 \log (W_r/W_1) \tag{5}$$

derived for the relation between the intensity β_r of the reference tone and the electrical power W_r in the receivers. The calibration consisted of finding by means of loudness balances a power W_1 in the receivers which produces a tone equal in loudness to that of a free wave having an intensity level β_1.

For sounds other than the 1000-cycle reference tone a relation similar to Eq. (5) can be derived, namely,

$$\beta = \beta_1 + 10 \log (W/W_1), \tag{11}$$

where β_1 and W_1 are corresponding values found from loudness balances for each frequency or complex wave form of interest. If, as is usually assumed, a linear relation exists between β and 10 log W, then determinations of β_1 and W_1 at one level are sufficient and it follows that a change in the power level of Δ decibels will produce a corresponding change of Δ decibels in the intensity of the sound generated. Obviously the receivers must not be overloaded or this assumption will not be valid. Rather than depend upon the existence of a linear relation between β and 10 log W with no confirming data, the receivers used in this investigation were calibrated at two widely separated levels.

Referring again to Table I, the data are expressed in terms of voltage levels instead of power levels. If, as was the case with our receivers, the electrical impedance is essentially a constant, Eq. (11) can be put in the form:

$$\beta = \beta_1 + 20 \log (V/V_1) \tag{12}$$

or

$$\beta = 20 \log V + C, \tag{13}$$

where V is the voltage across the receivers and C is a constant of the receivers to be determined from a calibration giving corresponding values of β_1 and 20 log V_1. The calibration will now be described.

By using the sound stage and the technique of measuring field pressures described by Sivian and White [5] and by using the technique for making loudness measurements described in Appendix A, the following measurements were made. An electrical voltage V_1 was placed across the two head receivers such that the loudness level produced was the same at each frequency. The observer listened to the tone in these head receivers and then after $1\frac{1}{2}$ seconds silence listened to the tone from the loud speaker producing a free wave of the same frequency. The voltage level across the loud speaker necessary to produce a tone equally loud to the tone from the head receivers was obtained using the procedure described in Appendix A. The free wave intensity level β_1 corresponding to this voltage level was measured in the manner described in Sivian and White's paper. Threshold values both for the head receivers and the loud speaker were also observed. In these tests eleven observers were used. The results obtained are given in Table II. In the second row values of 20 log V_1, the voltage

[5] L. J. Sivian and S. D. White, "Minimum Audible Sound Fields," *Jour. Acous. Soc. Am.* **4**, 288 (1933).

TABLE II

FIELD CALIBRATION OF TELEPHONE RECEIVERS

Frequency c.p.s.	60	120	240	480	960	1920	3850	5400	7800	10,500	15,000
Voltage level (20 log V_1)	−13.0	−26.2	−38.5	−47.0	−48.2	−42.3	−36.3	−34.0	−39.1	−32.4	− 6.4
Intensity level (β_1)	+79.3	+71.0	+67.4	+63.8	+65.3	+64.0	+62.2	+65.5	+74.0	+78.6	+75.0
$C_1 = \beta_1 = - 20 \log V_1$	92.3	97.2	105.9	110.8	113.5	106.3	98.5	99.5	113.1	111.0	81.4
Threshold voltage level (20 log V_0)	−48.0	−61.8	−86.2	−105.4	−110.7	−109.0	−104.0	−97.1	−100.5	−102.0	−74.0
Threshold intensity level (β_0)	+49.3	+33.7	+19.7	+8.4	+5.4	−0.9	−4.2	+2.7	+10.6	+16.1	+22.0
$C_0 = \beta_0 - 20 \log V_0$	97.3	95.5	105.9	113.8	116.1	108.1	99.8	99.8	111.1	118.1	96.0
Diff. = $C_1 - C_0$	−5.0	1.7	0	−3.0	−2.6	−1.8	−1.3	−0.3	+2.0	−7.1	−14.6

level, are given. The intensity levels, β_1, of the free wave which sounded equally loud are given in the third row. In the fourth row the values of the constant C, the calibration we are seeking, are given. The voltage level added to this constant gives the equivalent free wave intensity level. In the fifth, sixth and seventh rows, similar values are given which were determined at the threshold level. In the bottom row the differences in the constants determined at the two levels are given. The fact that the difference is no larger than the probable error is very significant. It means that throughout this wide range there is a linear relationship between the equivalent field intensity levels, β, and the voltage levels, 20 log V, so that the formula (13)

$$\beta = 20 \log V + C$$

can be applied to our receivers with considerable confidence.

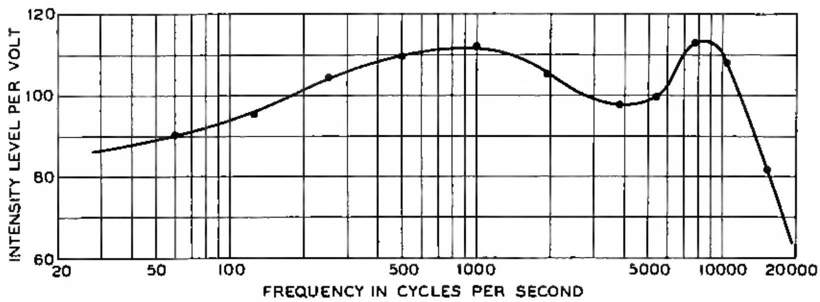


Fig. 1—Field calibration of loudness balance receivers.[6] (Calibration made at $L = 60$ db.)

The constant C determined at the high level was determined with greater accuracy than at the threshold. For this reason only the values for the higher level were used for the calibration curve. Also in these tests only four receivers were used while in the loudness tests eight receivers were used. The difference between the efficiency of the former four and the latter eight receivers was determined by measurements on an artificial ear. The figures given in Table II were corrected by this difference. The resulting calibration curve is that given in Fig. 1. It should be pointed out here that such a calibration curve on a single individual would show considerable deviations from this average curve. These deviations are real, that is, they are due to the sizes and shapes of the ear canals.

[6] The ordinates represent the intensity level in db of a free wave in air which, when listened to with both ears in the standard manner, is as loud as a tone of the same frequency heard from the two head receivers used in the tests when an e.m.f. of one volt is applied to the receiver terminals.

We can now express the data in Table I in terms of field intensity levels. To do this, the data in each double column were plotted and a smooth curve drawn through the observed points. The resulting curves give the relation between voltage levels of the pure tones for equality of loudness. From the calibration curve of the receivers these levels are converted to intensity levels by a simple shift in the axes of coordinates. Since the intensity level of the reference tone is by definition the "loudness level," these shifted curves will represent the loudness level of pure tones in terms of intensity levels. The resulting curves for the ten tones tested are given in Figs. 2A to 2J. Each point on these curves corresponds to a pair of values in Table I except for the threshold values. The results of separate determinations by the crew used in these loudness tests at different times are given by the circles. The points represented by (*) are the values adopted by Sivian and White. It will be seen that most of the

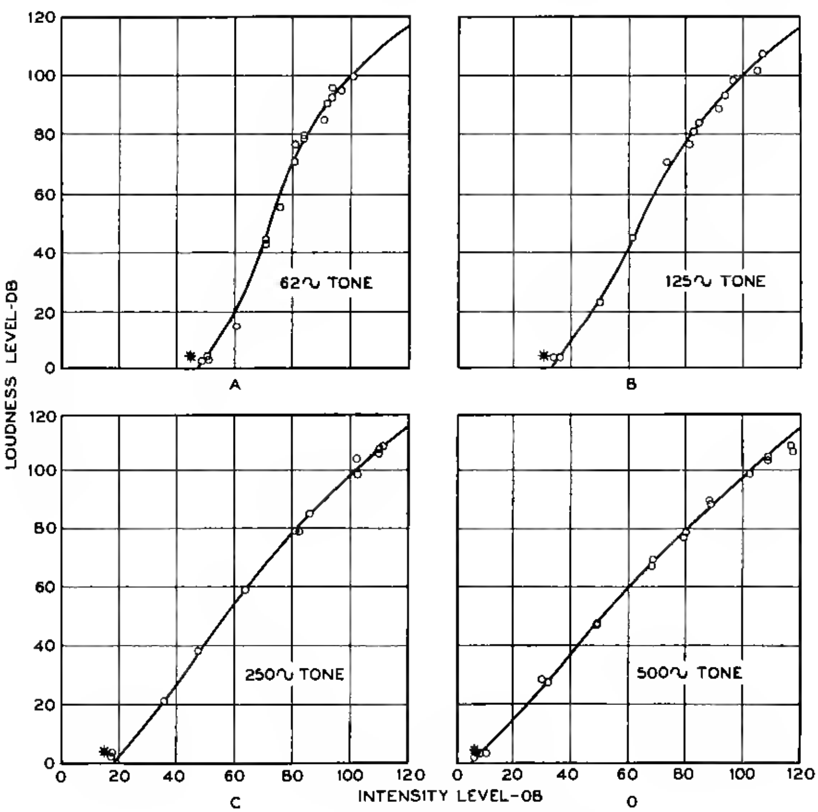


Fig. 2 (A to D)—Loudness levels of pure tones.

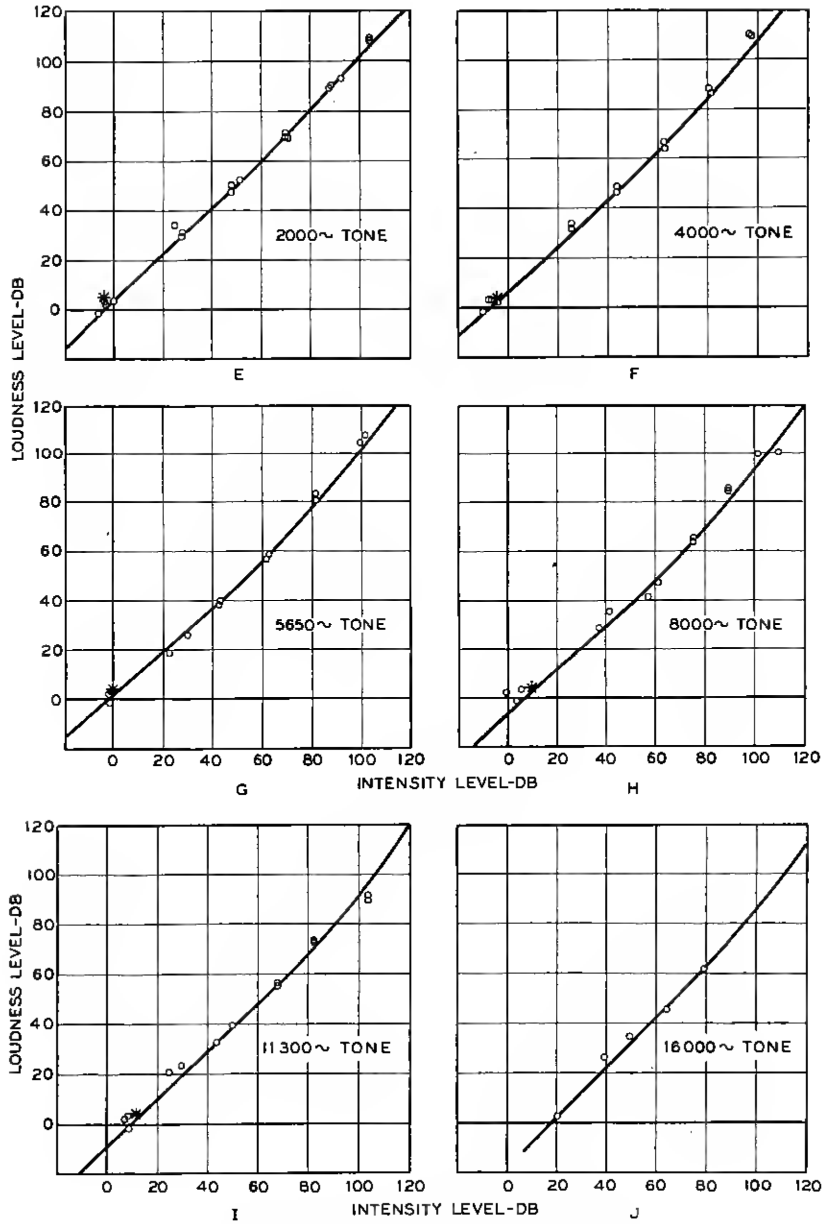


Fig. 2 (E to J)—Loudness levels of pure tones.

threshold points are slightly above the zero we have chosen. This means that our zero corresponds to the thresholds of observers who are slightly more acute than the average.

From these curves the loudness level contours can be drawn. The first set of loudness level contours are plotted with levels above reference threshold as ordinates. For example, the zero loudness level contour corresponds to points where the curves of Figs. 2A to 2J intersect the abscissa axis. The number of db above these points is plotted as the ordinate in the loudness level contours shown in Fig. 3. From a consideration of the nature of the hearing mechanism we believe that these curves should be smooth. These curves, therefore,

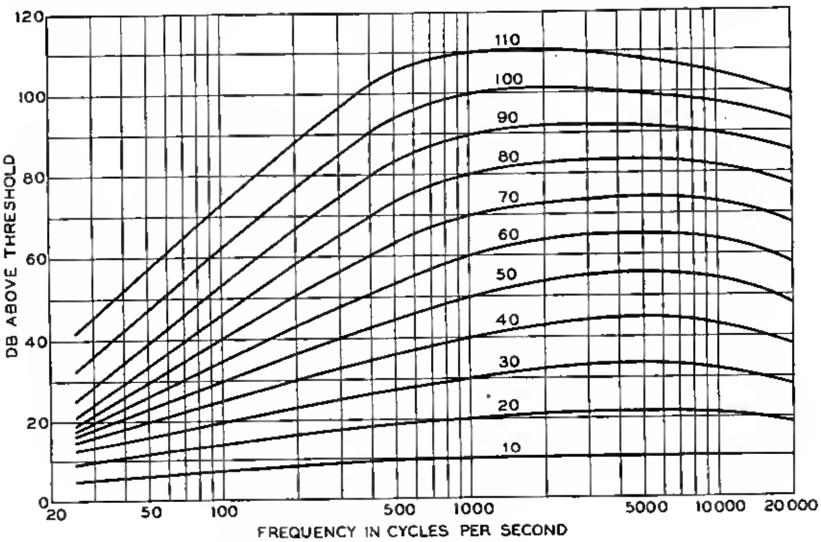


Fig. 3—Loudness level contours.

represent the best set of smooth curves which we could draw through the observed points. After the smoothing process, the curves in Figs. 2A to 2J were then adjusted to correspond. The curves shown in these figures are such adjusted curves.

In Fig. 4 a similar set of loudness level contours is shown using intensity levels as ordinates. There are good reasons [5] for believing that the peculiar shape of these contours for frequencies above 1000 c.p.s. is due to diffraction around the head of the observer as he faces the source of sound. It was for this reason that the smoothing process was done with the curves plotted with the level above threshold as the ordinate.

[5] Loc. cit.

From these loudness level contours, the curves shown in Figs. 5A and 5B were obtained. They show the loudness level *vs.* intensity level with frequency as a parameter. They are convenient to use for calculation purposes.

It is interesting to note that through a large part of the practical range for tones of frequencies from 300 c.p.s. to 4000 c.p.s. the loudness level is approximately equal to the intensity level. From these curves, it is possible to obtain any value of L_k in terms of β_k and f_k.

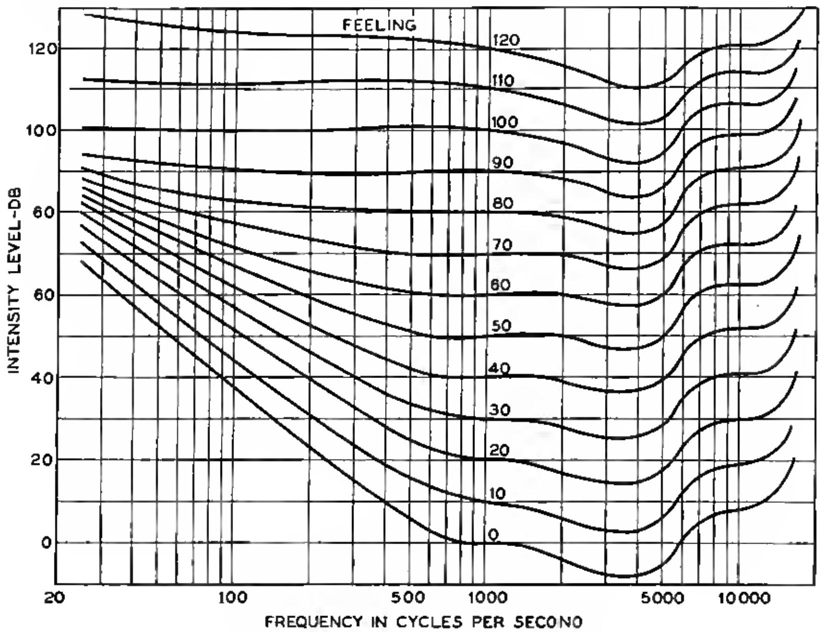


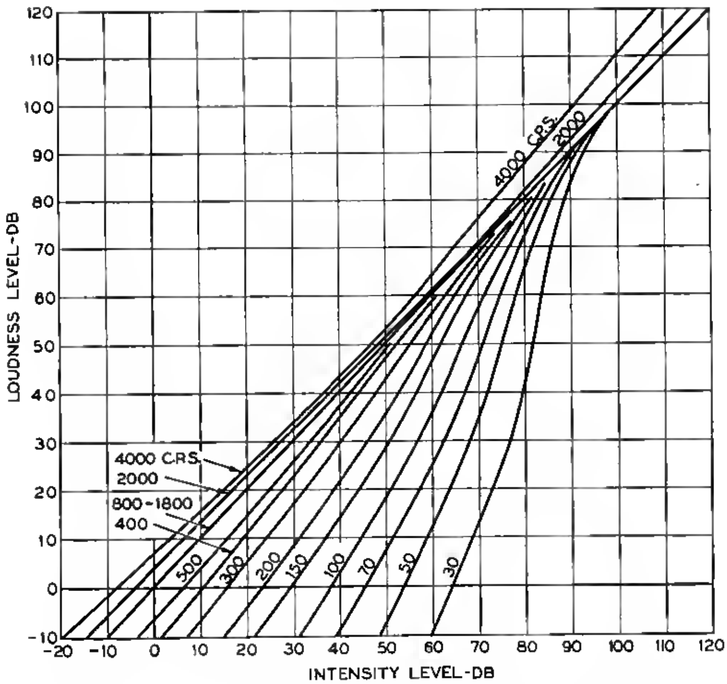
Fig. 4.—Loudness level contours.

On Fig. 4 the 120-db loudness level contour has been marked "Feeling." The data published by R. R. Riesz [7] on the threshold of feeling indicate that this contour is very close to the feeling point throughout the frequency range where data have been taken.

Determination of the Loudness Function G

In the section "Formulation of the Empirical Theory for Calculating the Loudness of a Steady Complex Tone," the fundamental equation for calculating the loudness level of a complex tone was derived,

[7] R. R. Riesz, "The Relationship Between Loudness and the Minimum Perceptible Increment of Intensity," *Jour. Acous. Soc. Am.* **4**, 211 (1933).



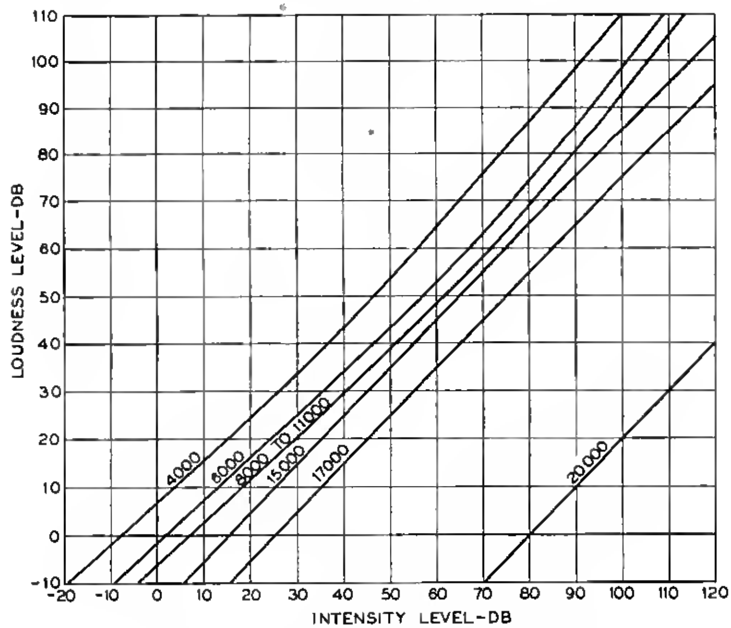


Fig. 5 (A and B)—Loudness levels of pure tones.

namely,

$$G(1000, L) = \sum_{k=1}^{k=n} b_k G(1000, L_k). \tag{10}$$

If the type of complex tone can be chosen so that b_k is unity and also so that the values of L_k for each component are equal, then the fundamental equation for calculating loudness becomes

$$G(L) = nG(L_k), \tag{14}$$

where n is the number of components. Since we are always dealing in this section with $G(1000, L)$ or $G(1000, L_k)$, the 1000 is left out in the above nomenclature. If experimental measurements of L corresponding to values of L_k are taken for a tone fulfilling the above conditions throughout the audible range, the function G can be determined. If we accept the theory that, when two simple tones widely separated in frequency act upon the ear, the nerve terminals stimulated by each are at different portions of the basilar membrane, then we would expect the interference of the loudness of one upon that of the other would be negligible. Consequently, for such a combination b is unity. Measurements were made upon two such tones, the two components being equally loud, the first having frequencies of 1000 and 2000 cycles and the second, frequencies of 125 and 1000 cycles. The observed points are shown along the second curve from the top of Fig. 6. The abscissae give the loudness level L_k of each component and the ordinates the loudness level L of the two components combined. The equation $G(y) = 2G(x)$ should represent these data. Similar measurements were made with a complex tone having 10 components, all equally loud. The method of generating such tones is described in Appendix C. The results are shown by the points along the top curve of Fig. 6. The equation $G(y) = 10G(x)$ should represent these data except at high levels where b_k is not unity.

There is probably a complete separation between stimulated patches of nerve endings when the first component is introduced into one ear and the second component into the other ear. In this case the same or different frequencies can be used. Since it is easier to make loudness balances when the same kind of sound is used, measurements were made (1) with 125-cycle tones (2) with 1000-cycle tones and (3) with 4000-cycle tones. The results are shown on Fig. 7. In this curve the ordinates give the loudness levels when one ear is used while the abscissae give the corresponding loudness levels for the same intensity level of the tone when both ears are used for listening. If binaural versus monaural loudness data actually fit into this scheme of calcula-

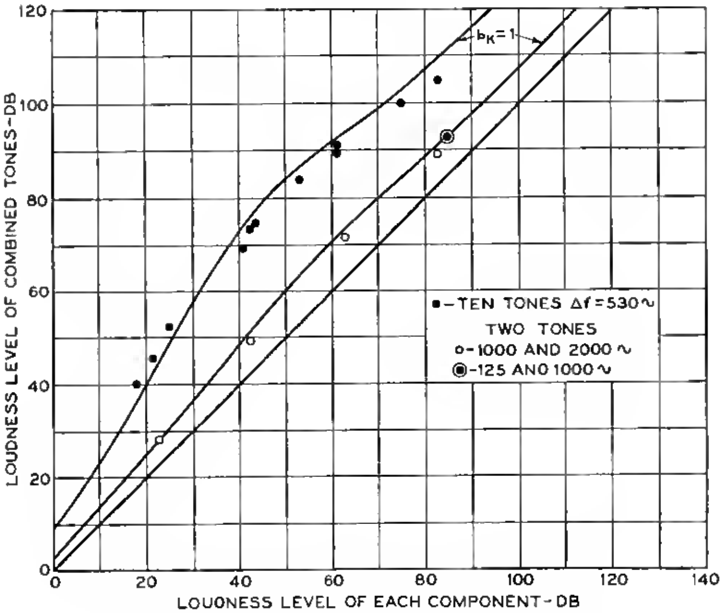


Fig. 6—Complex tones having components widely separated in frequency.

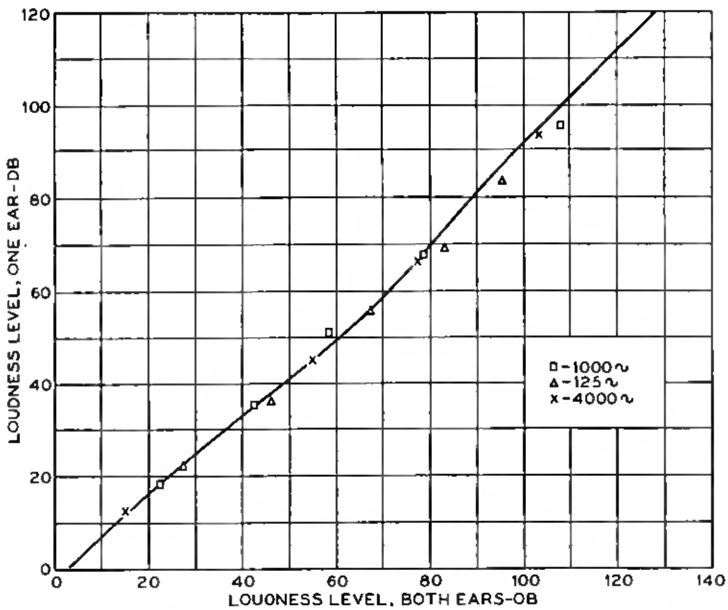


Fig. 7—Relation between loudness levels listening with one-ear and with both ears.

tion these points should be represented by

$$G(y) = \tfrac{1}{2}G(x).$$

Any one of these curves which was accurately determined would be sufficient to completely determine the function G.

For example, consider the curve for two tones. It is evident that it is only necessary to deal with relative values of G so that we can choose one value arbitrarily. The value of $G(0)$ was chosen equal to unity. Therefore,

$$\begin{aligned}
&G(0) = 1, && \\
&G(y_0) = 2G(0) = 2 && \text{where } y_0 \text{ corresponds to } x = 0, \\
&G(y_1) = 2G(x_1) = 2G(y_0) = 4 && \text{where } y_1 \text{ corresponds to } x_1 = y_0, \\
&G(y_2) = 2G(x_2) = 2G(y_1) = 8 && \text{where } y_2 \text{ corresponds to } x_2 = y_1, \\
&G(y_k) = 2G(x_k) = 2G(y_{k-1}) = 2^{k+1} && \text{where } y_k \text{ corresponds to } x_k = y_{k-1}.
\end{aligned}$$

In this way a set of values for G can be obtained. A smooth curve connecting all such calculated points will enable one to find any value of $G(x)$ for a given value of x. In a similar way sets of values can be obtained from the other two experimental curves. Instead of using any one of the curves alone the values of G were chosen to best fit all three sets of data, taking into account the fact that the observed points for the 10-tone data might be low at the higher levels where b would be less than unity. The values for the function which were finally adopted are given in Table III. From these values the three solid curves of Figs. 6 and 7 were calculated by the equations

$$G(y) = 10G(x), \qquad G(y) = 2G(x), \qquad G(y) = \tfrac{1}{2}G(x).$$

The fit of the three sets of data is sufficiently good, we think, to justify the point of view taken in developing the formula. The calculated points for the 10-component tones agree with the observed ones when the proper value of b_k is introduced into the formula. In this connection it is important to emphasize that in calculating the loudness level of a complex tone under the condition of listening with one ear instead of two, a factor of $\frac{1}{2}$ must be placed in front of the summation of Eq. (10). This will be explained in greater detail later. The values of G for negative values of L were chosen after considering all the data on the threshold values of the complex tones studied. These data will be given with the other loudness data on complex tones. It is interesting to note here that the threshold data show that 10 pure tones, which are below the threshold when sounded separately, will combine

TABLE III

VALUES OF $G(L_k)$.

L	0	1	2	3	4	5	6	7	8	9
−10	0.015	0.025	0.04	0.06	0.09	0.14	0.22	0.32	0.45	0.70
0	1.00	1.40	1.90	2.51	3.40	4.43	5.70	7.08	9.00	11.2
10	13.9	17.2	21.4	26.6	32.6	39.3	47.5	57.5	69.5	82.5
20	97.5	113	131	151	173	197	222	252	287	324
30	360	405	455	505	555	615	675	740	810	890
40	975	1060	1155	1250	1360	1500	1640	1780	1920	2070
50	2200	2350	2510	2680	2880	3080	3310	3560	3820	4070
60	4350	4640	4950	5250	5560	5870	6240	6620	7020	7440
70	7950	8510	9130	9850	10600	11400	12400	13500	14600	15800
80	17100	18400	19800	21400	23100	25000	27200	29600	32200	35000
90	38000	41500	45000	49000	53000	57000	62000	67500	74000	81000
100	88000	97000	106000	116000	126000	138000	150000	164000	180000	197000
110	215000	235000	260000	288000	316000	346000	380000	418000	460000	506000
120	556000	609000	668000	732000	800000	875000	956000	1047000	1150000	1266000

to give a tone which can be heard. When the components are all in the high pitch range and all equally loud, each component may be from 6 to 8 db below the threshold and the combination will still be audible. When they are all in the low pitch range they may be only 2 or 3 db below the threshold. The closeness of packing of the components also influences the threshold. For example, if the ten components are all within a 100-cycle band each one may be down 10 db. It will be shown that the formula proposed above can be made to take care of these variations in the threshold.

There is still another method which might be used for determining this loudness function $G(L)$, provided one's judgment as to the magnitude of an auditory sensation can be relied upon. If a person were asked to judge when the loudness of a sound was reduced to one half it might be expected that he would base his judgment on the experience of the decrease in loudness when going from the condition of listening with both ears to that of listening with one ear. Or, if the magnitude of the sensation is the number of nerve discharges reaching the brain per second, then when this has decreased to one half, he might be able to say that the loudness has decreased one half.

In any case, if it is assumed that an observer can judge when the magnitude of the auditory sensation, that is, the loudness, is reduced to one half, then the value of the loudness function G can be computed from such measurements.

Several different research workers have made such measurements. The measurements are somewhat in conflict at the present time so that they did not in any way influence the choice of the loudness function. Rather we used the loudness function given in Table III to calculate what such observations should give. A comparison of the calculated and observed results is given below. In Table IV is shown a comparison of calculated and observed results of data taken by Ham and Parkinson.[8] The observed values were taken from Tables 1a, 1b, 2a, 2b, 3a and 3b of their paper. The calculation is very simple. From the number of decibels above threshold S the loudness level L is determined from the curves of Fig. 3. The fractional reduction is just the fractional reduction in the loudness function for the corresponding values of L. The agreement between observed and calculated results is remarkably good. However, the agreement with the data of Laird, Taylor and Wille is very poor, as is shown by Table V. The calculation was made only for the 1024-cycle tone. The observed data were taken from Table VII of the paper by Laird,

[8] L. B. Ham and J. S. Parkinson, "Loudness and Intensity Relations," *Jour. Acous. Soc. Am.* 3, 511 (1932).

TABLE IV

COMPARISON OF CALCULATED AND OBSERVED FRACTIONAL LOUDNESS (HAM AND PARKINSON)

350 Cycles

S	L	G	Fractional Reduction in Loudness	
			Cal. %	Obs. %
74.0	85	25,000	100	100.0
70.4	82	19,800	79	83.0
67.7	79	15,800	63	67.0
64.0	75	11,400	46	49.0
59.0	70	7,950	32	35.0
54.0	65	5,870	24	26.0
44.0	53	2,680	11	15.0
34.0	41	1,100	4	8.0
59.5	71	8,510	100	100.0
57.7	69	7,440	87	92.0
55.0	66	6,240	73	77.0
49.0	59	4,070	48	57.0
44.0	53	2,680	31	38.0
39.0	47	1,780	21	25.0
34.0	41	1,060	12	13.0
24.0	29	324	4	6.0

1000 Cycles

S	L	G	Cal. %	Obs. %
86.0	86	27,200	100	100.0
82.4	82	19,800	73	68.0
79.7	80	17,100	63	53.0
76.0	76	12,400	46	41.0
71.0	71	8,510	31	26.0
66.0	66	6,420	24	20.0
56.0	56	3,310	12	13.0
46.0	46	1,640	6	8.0
56.0	56	3,310	100	100.0
54.2	54	2,880	87	93.4
51.5	52	2,510	76	74.6
48.8	49	2,070	62	55.0
46.0	46	1,640	49	40.9
41.0	41	1,060	32	24.5
36.0	36	675	20	10.8

2500 Cycles

S	L	G	Cal. %	Obs. %
74.0	69	7,440	100	100.0
70.4	64	5,560	75	86.4
67.7	62	4,950	67	68.1
64.0	58	3,820	51	49.5
59.0	53	2,680	36	32.8
54.0	48	1,920	26	23.3
44.0	39	890	12	13.0
34.0	30	360	5	6.7
44.0	39	890	100	100.0
42.2	37	740	83	94.6
39.5	36	675	76	82.2
36.8	33	505	57	61.1
34.0	30	360	41	46.0
29.0	26	222	25	27.8
24.0	21	113	13	14.9

TABLE V

Comparison of Calculated and Observed Fractional Loudness (Laird, Taylor and Wille)

Original Loudness Level	Level for ½ Loudness Reduction		Cal. Level for ¼ Loudness Reduction
	Cal.	Obs.	
100	92	76.0	84
90	82	68.0	73
80	71	60.0	60
70	58	49.5	48
60	50	40.5	41
50	42	31.0	34
40	33	21.0	27
30	25	14.9	20
20	16	6.5	13
10	7	5.0	4

Taylor and Wille.[9] As shown in Table V the calculation of the level for one fourth reduction in loudness agrees better with the observed data corresponding to one half reduction in loudness.

Firestone and Geiger reported some preliminary values which were in closer agreement with those obtained by Parkinson and Ham, but their completed paper has not yet been published.[10] Because of the lack of agreement of observed data of this sort we concluded that it could not be used for influencing the choice of the values of the loudness function adopted and shown in Table III. It is to be hoped that more data of this type will be taken until there is a better agreement between observed results of different observers. It should be emphasized here that changes of the level above threshold corresponding to any fixed increase or decrease in loudness will, according to the theory outlined in this paper, depend upon the frequency of the tone when using pure tones, or upon its structure when using complex tones.

Determination of the Formula for Calculating b_k

Having now determined the function G for all values of L or L_k we can proceed to find methods of calculating b_k. Its value is evidently dependent upon the frequency and intensity of all the other components present as well as upon the component being considered. For practical computations, simplifying assumptions can be made. In most cases the reduction of b_k from unity is principally due to the adjacent component on the side of the lower pitch. This is due to the fact that a tone masks another tone of higher pitch very much more

[9] Laird, Taylor and Wille, "The Apparent Reduction in Loudness," *Jour. Acous. Soc. Am.* **3**, 393 (1932).

[10] This paper is now available. P. H. Geiger and F. A. Firestone, "The Estimation of Fractional Loudness," *Jour. Acous. Soc. Am.* **5**, 25 (1933).

than one of lower pitch. For example, in most cases a tone which is 100 cycles higher than the masking tone would be masked when it is reduced 25 db below the level of the masking tone, whereas a tone 100 cycles lower in frequency will be masked only when it is reduced from 40 to 60 db below the level of the masking tone. It will therefore be assumed that the neighboring component on the side of lower pitch which causes the greatest masking will account for all the reduction in b_k. Designating this component with the subscript m, meaning the masking component, then we have b_k expressed as a function of the following variables.

$$b_k = B(f_k, f_m, S_k, S_m), \tag{15}$$

where f is the frequency and S is the level above threshold. For the case when the level of the kth component is T db below the level of the masking component, where T is just sufficient for the component to be masked, then the value of b would be equal to zero. Also, it is reasonable to assume that when the masking component is at a level somewhat less than T db below the kth component, the latter will have a value of b_k which is unity. It is thus seen that the fundamental of a series of tones will always have a value of b_k equal to unity.

For the case when the masking component and the kth component have the same loudness, the function representing b_k will be considerably simplified, particularly if it were also found to be independent of f_k and only dependent upon the difference between f_k and f_m. From the theory of hearing one would expect that this would be approximately true for the following reasons:

The distance in millimeters between the positions of maximum response on the basilar membrane for the two components is more nearly proportional to differences in pitch than to differences in frequency. However, the peaks are sharpest in the high frequency regions where the distances on the basilar membrane for a given Δf are smallest. Also, in the low frequency region where the distances for a given Δf are largest, these peaks are broadest. These two factors tend to make the interference between two components having a fixed difference in frequency approximately the same regardless of their position on the frequency scale. However, it would be extraordinary if these two factors just balanced. To test this point three complex tones having ten components with a common Δf of 50 cycles were tested for loudness. The first had frequencies of 50–100–150$\cdots$500, the second 1400–1450$\cdots$1900, and the third 3400–3450$\cdots$3900. The results of these tests are shown in Fig. 8. The abscissae give the loudness level of each component and the ordinates the measured loudness level of the combined tone. Similar results were obtained with a complex tone having ten components of equal loudness and a common frequency difference of 100 cycles. The results are shown in Fig. 9. It will be seen that although the points corresponding to the different

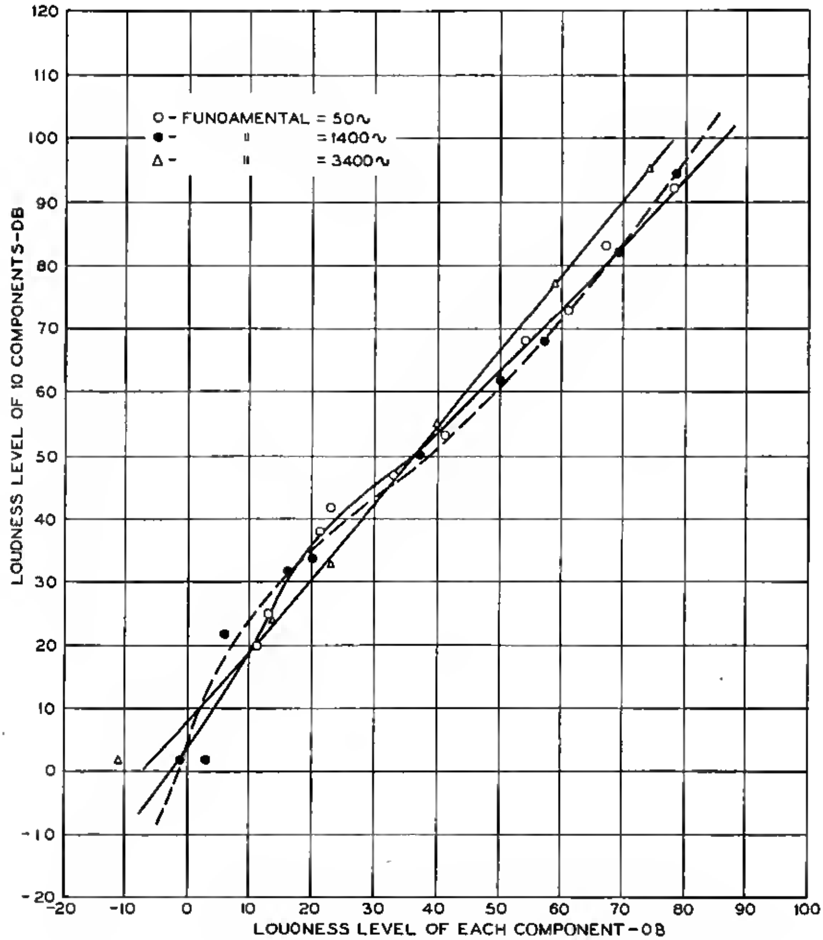


Fig. 8—Loudness levels of complex tones having ten equally loud components 50 cycles apart.

frequency ranges lie approximately upon the same curve through the middle range, there are consistent departures at both the high and low intensities. If we choose the frequency of the components largely in the middle range then this factor b will be dependent only upon Δf and L_k.

To determine the value of b for this range in terms of Δf and L_k, a series of loudness measurements was made upon complex tones having ten components with a common difference in frequency Δf and all having a common loudness level L_k. The values of Δf were 340, 230,

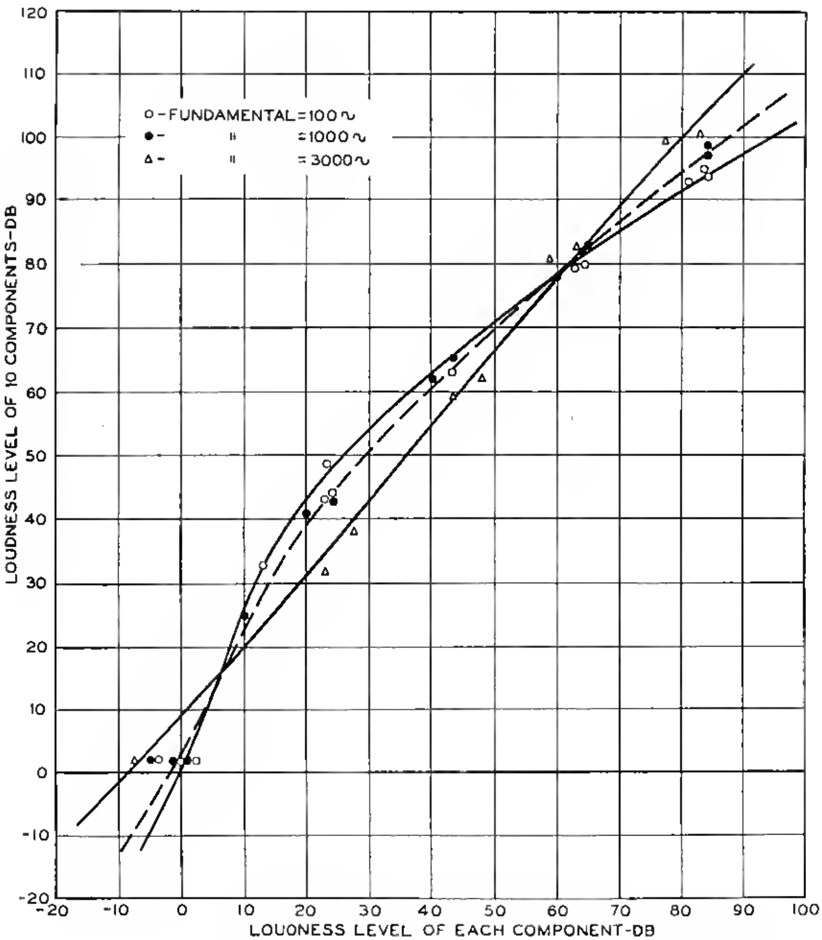


Fig. 9—Loudness levels of complex tones having ten equally loud components 100 cycles apart.

112 and 56 cycles per second. The fundamental for each tone was close to 1000 cycles. The ten-component tones having frequencies which are multiples of 530 was included in this series. The results of loudness balances are shown by the points in Fig. 10.

By taking all the data as a whole, the curves were considered to give the best fit. The values of b were calculated from these curves as follows:

According to the assumptions made above, the component of lowest pitch in the series of components always has a value of b_k equal to unity. Therefore for the series of 10 components having a common

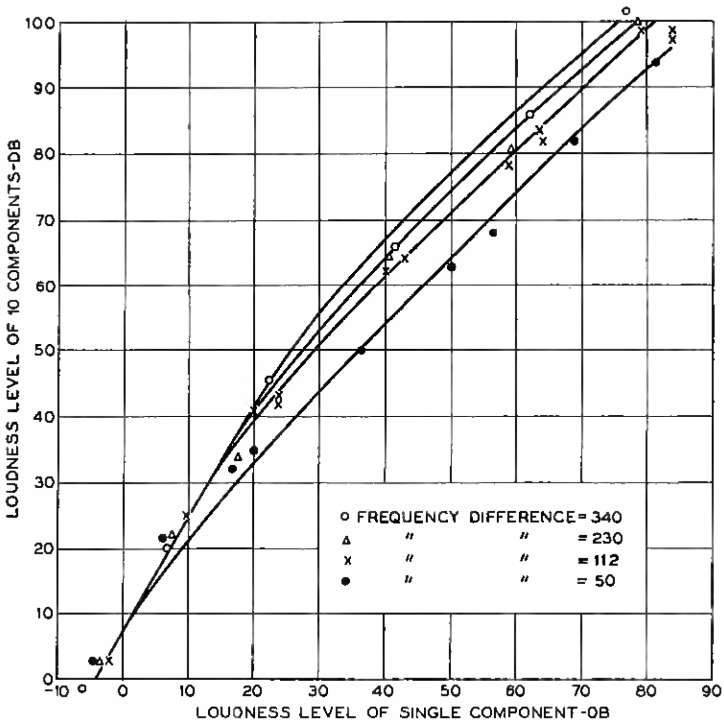


Fig. 10—Loudness levels of complex tones having ten equally loud components with a fundamental frequency of 1000 c.p.s.

loudness level L_k, the value of L is related to L_k by

$$G(L) = (1 + 9b_k)G(L_k)$$

or by solving for b_k

$$b_k = (1/9)[G(L)/G(L_k) - 1]. \tag{16}$$

The values of b_k can be computed from this equation from the observed values of L and L_k by using the values of G given in Table III. Because of the difficulty in obtaining accurate values of L and L_k such computed values of b_k will be rather inaccurate. Consequently, considerable freedom is left in choosing a simple formula which will represent the results. When the values of b_k derived in this way were plotted with b_k as ordinates and Δf as abscissae and L_k as a variable parameter then the resulting graphs were a series of straight lines going through the common point (− 250, 0) but having slopes depending upon L_k. Consequently the following formula

$$b_k = [(250 + \Delta f)/1000]Q(L_k) \tag{17}$$

will represent the results. The quantity Δf is the common difference in frequency between the components, L_k the loudness level of each component, and Q a function depending upon L_k. The results indicated that Q could be represented by the curve in Fig. 11.

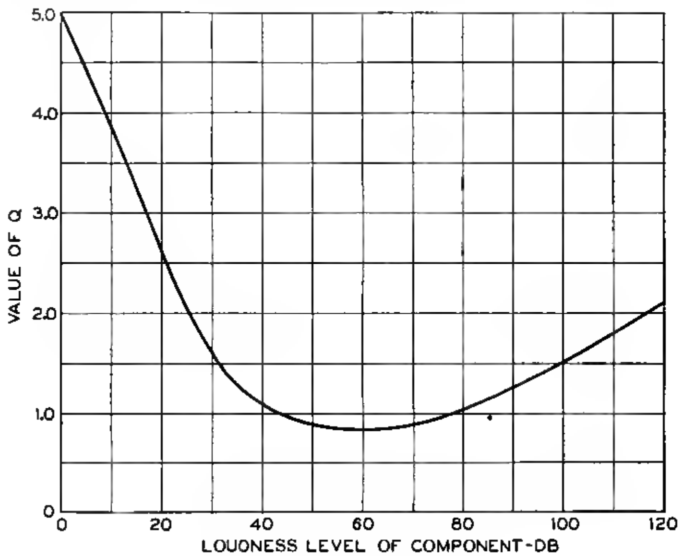


Fig. 11—Loudness factor Q.

Also the condition must be imposed upon this equation that b is always taken as unity whenever the calculation gives values greater than unity. The solid curves shown in Fig. 10 are actually calculated curves using these equations, so the comparison of these curves with the observed points gives an indication of how well this equation fits the data. For this series of tones Q could be made to depend upon β_k rather than L_k and approximately the same results would be obtained since β_k and L_k are nearly equal in this range of frequencies. However, for tones having low intensities and low frequencies, β_k will be much larger than L_k and consequently Q will be smaller and hence the calculated loudness smaller. The results in Figs. 8 and 9 are just contrary to this. To make the calculated and observed results agree with these two sets of data, Q was made to depend upon

$$x = \beta + 30 \log f - 95$$

instead of L_k.

It was found when using this function of β and f as an abscissa and the same ordinates as in Fig. 10, a value of Q was obtained which gives just as good a fit for the data of Fig. 10 and also gives a better fit for the data of Figs. 8 and 9. Other much more complicated factors were tried to make the observed and calculated results shown in these two figures come into better agreement but none were more satisfactory than the simple procedure outlined above. For purpose of calculation the values of Q are tabulated in Table VI.

TABLE VI

VALUES OF $Q(X)$

X	0	1	2	3	4	5	6	7	8	9
0	5.00	4.88	4.76	4.64	4.53	4.41	4.29	4.17	4.05	3.94
10	3.82	3.70	3.58	3.46	3.35	3.33	3.11	2.99	2.87	2.76
20	2.64	2.52	2.40	2.28	2.16	2.05	1.95	1.85	1.76	1.68
30	1.60	1.53	1.47	1.40	1.35	1.30	1.25	1.20	1.16	1.13
40	1.09	1.06	1.03	1.01	0.99	0.97	0.95	0.94	0.92	0.91
50	0.90	0.90	0.89	0.89	0.88	0.88	0.88	0.88	0.88	0.88
60	0.88	0.88	0.88	0.88	0.88	0.88	0.88	0.89	0.89	0.90
70	0.90	0.91	0.92	0.93	0.94	0.96	0.97	0.99	1.00	1.02
80	1.04	1.06	1.08	1.10	1.13	1.15	1.17	1.19	1.22	1.24
90	1.27	1.29	1.31	1.34	1.36	1.39	1.41	1.44	1.46	1.48
100	1.51	1.53	1.55	1.58	1.60	1.62	1.64	1.67	1.69	1.71

Note: $X = \beta_k + 30 \log f_k - 95$.

There are reasons based upon the mechanics of hearing for treating components which are very close together by a separate method. When they are close together the combination must act as though the energy were all in a single component, since the components act upon approximately the same set of nerve terminals. For this reason it seems logical to combine them by the energy law and treat the combination as a single frequency. That some such procedure is necessary is shown from the absurdities into which one is led when one tries to make Eq. (17) applicable to all cases. For example, if 100 components were crowded into a 1000-cycle space about a 1000-cycle tone, then it is obvious that the combination should sound about 20 db louder. But according to Eq. (10) to make this true for values of L_k greater than 45, b_k must be chosen as 0.036. Similarly, for 10 tones thus crowded together $L - L_k$ must be about 10 db and therefore $b_k = 0.13$ and then for two such tones $L - L_k$ must be 3 db and the corresponding value of $b_k = 0.26$. These three values must belong to the same condition $\Delta f = 10$. It is evident then that the formulae for b given by Eq. (17) will lead to very erroneous results for such components.

In order to cover such cases it was necessary to group together all components within a certain frequency band and treat them as a single component. Since there was no definite criterion for determining accurately what these limiting bands should be, several were tried and ones selected which gave the best agreement between computed and observed results. The following band widths were finally chosen:

For frequencies below 2000 cycles, the band width is 100 cycles; for frequencies between 2000 and 4000 cycles, the band width is 200 cycles; for frequencies between 4000 and 8000 cycles, the band width is 400 cycles; and for frequencies between 8000 and 16,000 cycles, the band width is 800 cycles. If there are k components within one of these limiting bands, the intensity I taken for the equivalent single frequency component is given by

$$I = \sum I_k = \sum 10^{\beta_k/10}. \tag{18}$$

A frequency must be assigned to the combination. It seems reasonable to assign a weighted value of f given by the equation

$$f = \sum f_k I_k / I = \sum f_k 10^{\beta_k/10} / \sum 10^{\beta/10}. \tag{19}$$

Only a small error will be introduced if the mid-frequency of such bands be taken as the frequency of an equivalent component except for the band of lowest frequency. Below 125 cycles it is important that the frequency and intensity of each component be known, since in this region the loudness level L_k changes very rapidly with both changes in intensity and frequency. However, if the intensity for this band is lower than that for other bands, it will contribute little to the total loudness so that only a small error will be introduced by a wrong choice of frequency for the band.

This then gives a method of calculating b_k when the adjacent components are equal in loudness. When they are not equal let us define the difference ΔL by

$$\Delta L = L_k - L_m. \tag{20}$$

Also let this difference be T when L_m is adjusted so that the masking component just masks the component k. Then the function for calculating b must satisfy the following conditions:

$$b_k = [(250 + \Delta f)/1000]Q \qquad \text{when } \Delta L = 0,$$

$$b_k = 0 \qquad \text{when } \Delta L = -T.$$

Also the following condition when L_k is larger than L_m must be satisfied, namely, $b_k = 1$ when ΔL = some value somewhat smaller than $+T$. The value of T can be obtained from masking curves. An examination of these data indicates that to a good approximation the value of T is dependent upon the single variable $f_k - 2f_m$. A curve showing the relation between T and this variable is shown in Fig. 12. It will be seen that for most practical cases the value of T is 25. It cannot be claimed that the curve of Fig. 12 is an accurate representation of the masking data, but it is sufficiently accurate for the purpose of loudness calculation since rather large changes in T will produce a very slight change in the final calculated loudness level.

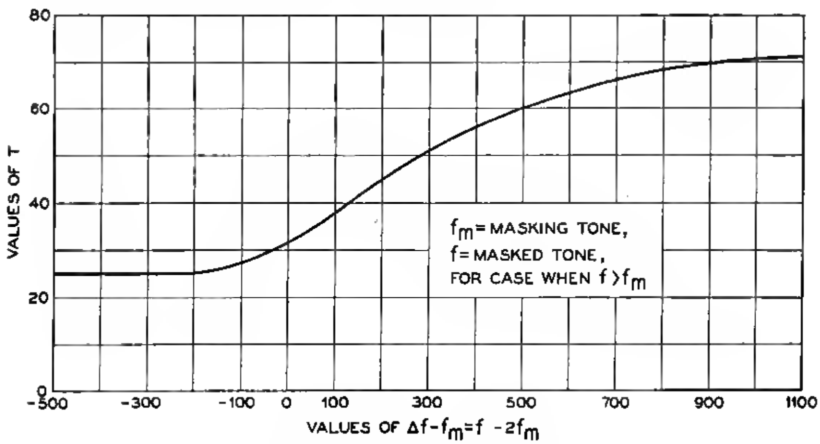


Fig. 12—Values of the masking T.

Data were taken in an effort to determine how this function depended upon ΔL but it was not possible to obtain sufficient accuracy in the experimental results. The difference between the resultant loudness level when half the tones are down so as not to contribute to loudness and when these are equal is not more than 4 or 5 db, which is not much more than the observational errors in such results.

A series of tests were made with tones similar to those used to obtain the results shown in Figs. 8 and 9 except that every other component was down in loudness level 5 db. Also a second series was made in which every other component was down 10 db. Although these data were not used in determining the function described above, it was useful as a check on the final equations derived for calculating the loudness of tones of this sort.

The factor finally chosen for representing the dependence of b_k upon ΔL is $10^{\Delta L/T}$. This factor is unity for $\Delta L = 0$, fulfilling the first condition mentioned above. It is 0.10 instead of zero for $\Delta L = -25$, the most probable value of T. For $\Delta f = 100$ and $Q = 0.88$ we will obtain the smallest value of b_k without applying the ΔL factor, namely, 0.31. Then when using this factor as given above, all values of b_k will be unity for values of ΔL greater than 12 db.

Several more complicated functions of ΔL were tried but none of them gave results showing a better agreement with the experimental values than the function chosen above.

The formula for calculation of b_k then becomes

$$b_k = [(250 + f_k - f_m)/1000]10^{(L_k - L_m)/T}Q(\beta_k + 30 \log f_k - 95) \quad (21)$$

where

f_k is the frequency of the component expressed in cycles per second,

f_m is the frequency of the masking component expressed in cycles per second,

L_k is the loudness level of the kth component when sounding alone,

L_m is the loudness level of the masking tone,

Q is a function depending upon the intensity level β_k and the frequency f_k of each component and is given in Table VI as a function of $x = \beta_k + 30 \log f_k - 95$,

T is the masking and is given by the curve of Fig. 12.

It is important to remember *that b_k can never be greater than unity so that all calculated values greater than this must be replaced with values equal to unity.* Also all components *within the limiting frequency bands must be grouped together as indicated above.* It is very helpful to remember that any component for which the loudness level is 12 db below the kth component, that is, the one for which b is being calculated, need not be considered as possibly being the masking component. If all the components preceding the kth are in this class then b_k is unity.

Recapitulation

With these limitations the formula for calculating the loudness level L of a steady complex tone having n components is

$$G(L) = \sum_{k=1}^{k=n} b_k G(L_k), \quad (10)$$

where b_k is given by Eq. (21). If the values of f_k and β_k are measured directly then corresponding values of L_k can be found from Fig. 5.

Having these values, the masking component can be found either by inspection or better by trial in Eq. (21). That component whose values of L_m, f_m and T introduced into this equation gives the smallest value of b_k is the masking component.

The values of G and Q can be found from Tables III and VI from the corresponding values of L_k, β_k, and f_k. If all these values are now introduced into Eq. (10), the resulting value of the summation is the *loudness* of the complex tone. The loudness level L corresponding to it is found from Table III.

If it is desired to know the loudness obtained if the typical listener used only one ear, the result will be obtained if the summation indicated in Eq. (10) is divided by 2. Practically the same result will be obtained in most instances if the loudness level L_k for each component when listened to with one ear instead of both ears is inserted in Eq. (10). ($G(L_k)$ for one ear listening is equal to one half $G(L_k)$ for listening with both ears for the same value of the intensity level of the component.) If two complex tones are listened to, one in one ear and one in the other, it would be expected that the combined loudness would be the sum of the two loudness values calculated for each ear as though no sound were in the opposite ear, although this has not been confirmed by experimental trial. In fact, the loudness reduction factor b_k has been derived from data taken with both ears only, so strictly speaking, its use is limited to this type of listening.

To illustrate the method of using the formula the loudness of two complex tones will be calculated. The first may represent the hum from a dynamo. Its components are given in the table of computations.

COMPUTATIONS

k	f_k	β_k	L_k	G_k	b_k	
1	60	50	3	3	1.0	
2	180	45	25	197	1.0	$\Sigma b_k G_k = 1009$
3	300	40	30	360	1.0	
4	540	30	27	252	1.0	$L = 40$
5	1200	25	25	197	1.0	

The first step is to find from Fig. 5 the values of L_k from f_k and β_k. Then the loudness values G_k are found from Table III. Since the values of L are low and the frequency separation fairly large, one familiar with these functions would readily see that the values of b would be unity and a computation would verify it so that the sum of the G values gives the total loudness 1009. This corresponds to a loudness level of 40.

The second tone calculated is this same hum amplified 30 db. It better illustrates the use of the formula.

COMPUTATIONS

k	f_k	β_k	L_k	G_k	f_m	L_m	$(30 \log f_k - 95)$	Q	b	$b \times G$
1	60	80	69	7440	—	—	—	—	1.00	7440
2	180	75	72	9130	60	69	−28	0.91	0.41	3740
3	300	70	69	7440	180	72	−21	0.91	0.27	2010
4	540	60	60	4350	300	69	−13	0.94	0.23	1000
5	1200	55	55	3080	540	60	− 3	0.89	0.61	1880
									loudness G =	16070
									loudness level L =	79 db

The loudness level of the combined tones is only 7 db above the loudness level of the second component. If only one ear is used in listening, the loudness of this tone is one half, corresponding to a loudness level of 70 db.

COMPARISON OF OBSERVED AND CALCULATED RESULTS ON THE LOUDNESS LEVELS OF COMPLEX TONES

In order to show the agreement between observed loudness levels and levels calculated by means of the formula developed in the preceding sections, the results of a large number of tests are given here, including those from which the formula was derived. In Tables VII to XIII, the first column shows the frequency range over which the components of the tones were distributed, the figures being the frequencies of the first and last components. Several tones having two components were tested, but as the tables indicate, the majority of the tones had ten components. Because of a misunderstanding in the

TABLE VII

TWO COMPONENT TONES ($\Delta L = 0$)

Frequency Range	Δf		Loudness Levels (db)				
1000–1100	100	L_k	83	63	43	23	2
		$L_{obs.}$	87	68	47	28	2
		$L_{calc.}$	87	68	47	28	4
1000–2000	1000	L_k	83	63	43	23	−1
		$L_{obs.}$	89	71	49	28	2
		$L_{calc.}$	91	74	52	28	1
125–1000	875	L_k	84				
		$L_{obs.}$	92				
		$L_{calc.}$	92				

TABLE VIII

TEN COMPONENT TONES ($\Delta L = 0$)

Frequency Range	Δf		Loudness Levels (db)									
50–500	50	L_k	67	54	33	21	11	−1				
		$L_{obs.}$	83	68	47	38	20	2				
		$L_{calc.}$	81	72	53	39	24	8				
50–500	50	L_k	78	61	41	23	13	−1				
		$L_{obs.}$	92	73	53	42	25	2				
		$L_{calc.}$	91	77	60	42	27	8				
1400–1895	55	L_k	78	69	50	16	6	−1				
		$L_{obs.}$	94	82	62	32	22	2				
		$L_{calc.}$	93	83	65	31	17	0				
1400–1895	55	L_k	57	37	20	3						
		$L_{obs.}$	68	50	34	2						
		$L_{calc.}$	73	52	36	5						
100–1000	100	L_k	84	64	43	24	2	84	64	43	24	2
		$L_{obs.}$	95	83	59	41	2	94	80	63	44	2
		$L_{calc.}$	100	83	68	47	12	100	83	68	47	12
100–1000	100	L_k	81	64	43	23	13	−4				
		$L_{obs.}$	93	82	65	49	33	2				
		$L_{calc.}$	98	83	68	45	27	3				
100–1000	100	L_k	83	63	43	23	0					
		$L_{obs.}$	95	79	59	43	2					
		$L_{calc.}$	99	82	68	45	9					
3100–3900	100	L_k	83	63	43	23	78	59	48	27	−7	
		$L_{obs.}$	100	82	59	32	99	81	62	38	2	
		$L_{calc.}$	100	80	60	38	95	77	65	42	0	
1100–3170	230	L_k	79	60	41	17	7	−4				
		$L_{obs.}$	100	81	65	33	22	2				
		$L_{calc.}$	100	83	64	34	18	3				
260–2600	260	L_k	79	62	42	23	13	−2				
		$L_{obs.}$	97	82	65	44	28	2				
		$L_{calc.}$	100	85	68	45	27	5				
530–5300	530	L_k	75	53	43	25	82	61	43	17	−2	
		$L_{obs.}$	100	83	73	52	105	90	73	40	2	
		$L_{calc.}$	101	82	72	48	108	89	72	34	5	
530–5300	530	L_k	61	41	21	−3						
		$L_{obs.}$	89	69	45	2						
		$L_{calc.}$	89	70	42	4						

design of the apparatus for generating the latter tones, a number of them contained eleven components, so for purposes of identification, these are placed in a separate group. In the second column of the tables, next to the frequency range of the tones, the frequency difference (Δf) between adjacent components is given. The remainder of

TABLE IX

ELEVEN COMPONENT TONES ($\Delta L = 0$)

Frequency Range	Δf	Loudness Levels (db)						
1000–2000	100	L_k $L_{obs.}$ $L_{calc.}$	84 97 103	64 83 84	43 65 64	24 43 45	−1 2 7	
1000–2000	100	L_k $L_{obs.}$ $L_{calc.}$	84 99 103	64 82 84	43 65 64	24 42 45	1 2 11	
1150–2270	112	L_k $L_{obs.}$ $L_{calc.}$	79 99 98	60 78 81	40 62 61	20 41 40	10 25 23	−5 2 1
1120–4520	340	L_k $L_{obs.}$ $L_{calc.}$	77 102 101	62 86 88	42 66 69	22 46 44	7 20 19	−7 2 −1

the data pertains to the loudness levels of the tones. Opposite L_k are given the common loudness levels to which all the components of the tone were adjusted for a particular test, and in the next line the results of the test, that is, the observed loudness levels ($L_{obs.}$), are given. Directly beneath each observed value, the calculated loudness levels ($L_{calc.}$) are shown. The three associated values of L_k, $L_{obs.}$, and $L_{calc.}$ in each column represent the data for one complete test. For example, in Table VIII, the first tone is described as having ten components, and for the first test shown each component was adjusted to have a loudness level (L_k) of 67 db. The results of the test gave an observed loudness level ($L_{obs.}$) of 83 db for the ten components acting together, and the calculated loudness level ($L_{calc.}$) of this same tone was 81 db. The probable error of the observed results in the tables is approximately $\pm$ 2 db.

TABLE X

TEN COMPONENT TONES ($\Delta L = 5$ db)

Frequency Range	Δf	Loudness Levels (db)						
1725–2220	55	L_k $L_{obs.}$ $L_{calc.}$	82 101 95	62 73 76	43 54 56	27 38 40	17 30 30	−6 2 −1
1725–2220	55	L_k $L_{obs.}$ $L_{calc.}$	80 94 93	62 66 76	42 50 54	22 33 35	12 22 22	−2 2 4

In the next series of data, adjacent components had a difference in loudness level of 5 db, that is, the first, third, fifth, etc., components had the loudness level given opposite L_k, and the even numbered components were 5 db lower. (Tables X and XI.)

TABLE XI

Eleven Component Tones (ΔL = 5 db).

Frequency Range	Δf	Loudness Levels (db)						
57–627	57	L_k	79	61	41	26	16	1
		$L_{obs.}$	91	73	56	41	28	2
		$L_{calc.}$	90	76	59	43	28	8
3420–4020	60	L_k	76	61	42	25	15	−9
		$L_{obs.}$	95	77	55	33	25	2
		$L_{calc.}$	89	75	54	36	26	−4

In the following set of tests (Tables XII and XIII) the difference in loudness level of adjacent components was 10 db.

TABLE XII

Ten Component Tones (ΔL = 10 db)

Frequency Range	Δf	Loudness Levels (db)						
1725–2220	55	L_k	79	59	40	19	9	−5
		$L_{obs.}$	95	71	54	33	22	2
		$L_{calc.}$	91	73	51	31	17	−1
1725–2220	55	L_k	79	61	41	27	17	−1
		$L_{obs.}$	89	67	48	37	27	2
		$L_{calc.}$	92	75	53	39	28	4

TABLE XIII

Eleven Component Tones (ΔL = 10 db)

Frequency Range	Δf	Loudness Levels (db)						
57–627	57	L_k	80	62	42	27	17	2
		$L_{obs.}$	88	70	53	40	27	2
		$L_{calc.}$	90	76	59	45	30	8
3420–4020	60	L_k	81	62	42	27	17	−4
		$L_{obs.}$	100	70	50	33	26	2
		$L_{calc.}$	94	75	53	37	27	0

The next data are the results of tests made on the complex tone generated by the Western Electric No. 3A audiometer. When analyzed, this tone was found to have the voltage level spectrum shown in Table XIV. When the r.m.s. voltage across the receivers used was unity, that is, zero voltage level, then the separate components had the voltage levels given in this table. Adding to the voltage levels the calibration constant for the receivers used in making the loudness tests gives the values of β for zero voltage level across the receivers. The values of β for any other voltage level are obtained by addition of the level desired.

TABLE XIV

VOLTAGE LEVEL SPECTRUM OF NO. 3A AUDIOMETER TONE

Frequency	Voltage Level	Frequency	Voltage Level
152	− 2.1	2128	−11.4
304	− 5.4	2280	−16.9
456	− 4.7	2432	−14.1
608	− 5.9	2584	−16.2
760	− 4.6	2736	−17.4
912	− 6.8	2880	−17.5
1064	− 6.0	3040	−20.0
1216	− 8.1	3192	−19.4
1368	− 7.6	3344	−22.7
1520	− 9.1	3496	−23.7
1672	−10.0	3648	−25.6
1824	− 9.9	3800	−24.6
1976	−14.1	3952	−26.8

Tests were made on the audiometer tone with the same receivers [11] that were used with the other complex tones, but in addition, data were available on tests made about six years ago using a different type of receiver. This latter type of receiver was recalibrated (Fig. 13) and computations made for both the old and new tests. In the older set of data, levels above threshold were given instead of voltage levels, so in utilizing it here, it was necessary to assume that the threshold levels of the new and old tests were the same.

Computations were made at the levels tested experimentally and a comparison of observed and calculated results is shown in Table XV.

The agreement of observed and calculated results is poor for some of the tests, but the close agreement in the recent data at low levels and in the previous data at high levels indicates that the observed results are not as accurate as could be desired. Because of the labor involved these tests have not been repeated.

At the time the tests were made several years ago on the No. 3A Audiometer tone, the reduction in loudness level which takes place when certain components are eliminated was also determined. As this

[11] See Calibration shown in Fig. 1.

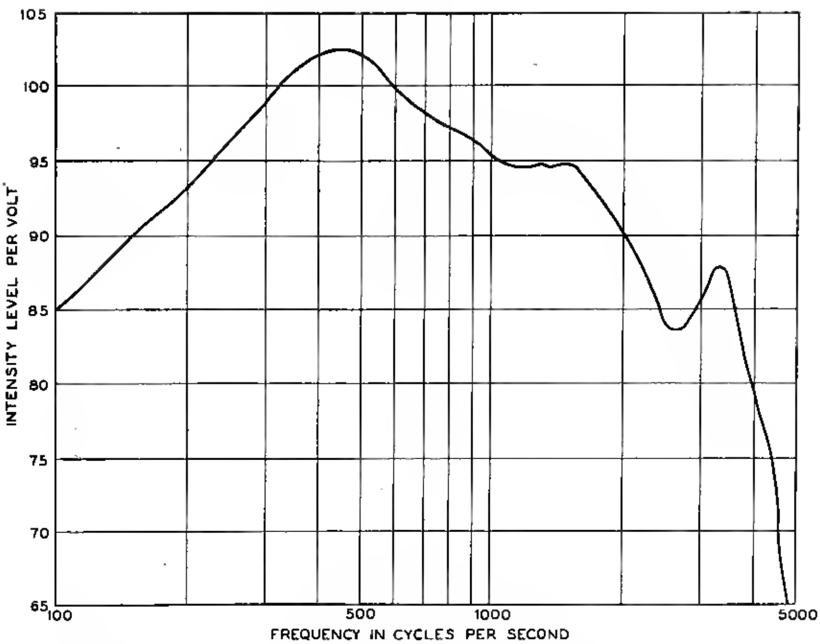


Fig. 13—Calibration of receivers for tests on the No. 3A audiometer tone

can be readily calculated with the formula developed here, a comparison of observed and calculated results will be shown. In Fig. 14A, the ordinate is the reduction in loudness level resulting when a No. 3A Audiometer tone having a loudness level of 42 db was changed by the insertion of a filter which eliminated all of the components above or below the frequency indicated on the abscissa. The observed data are the plotted points and the smooth curves are calculated results. A similar comparison is shown in Figs. 14B, C and D for other levels.

TABLE XV

A. Recent Tests on No. 3A Audiometer Tone

R.m.s. Volt. Level....	−38	−55	−59	−70	−75	−78	−80	−87	−89	−100	−102
L_{obs}.	95	85	79	61	56	41	42	28	22	2	2
L_{calc}.	89	74	71	57	49	44	40	28	25	7	4

B. Previous Tests on No. 3A Audiometer Tone

R.m.s. Volt. Level....	+10	−9	−40	−49	−60	−69	−91
L_{obs}.	118	103	77	69	61	50	2
L_{calc}.	119	103	82	73	56	41	6

This completes the data which are available on steady complex tones. It is to be hoped that others will find the field of sufficient importance to warrant obtaining additional data for improving and testing the method of measuring and calculating loudness levels.

In view of the complex nature of the problem this computation method cannot be considered fully developed in all its details and as more accurate data accumulates it may be necessary to change the formula for b. Also at the higher levels some attention must be given to phase differences between the components. However, we feel that the form of the equation is fundamentally correct and the loudness

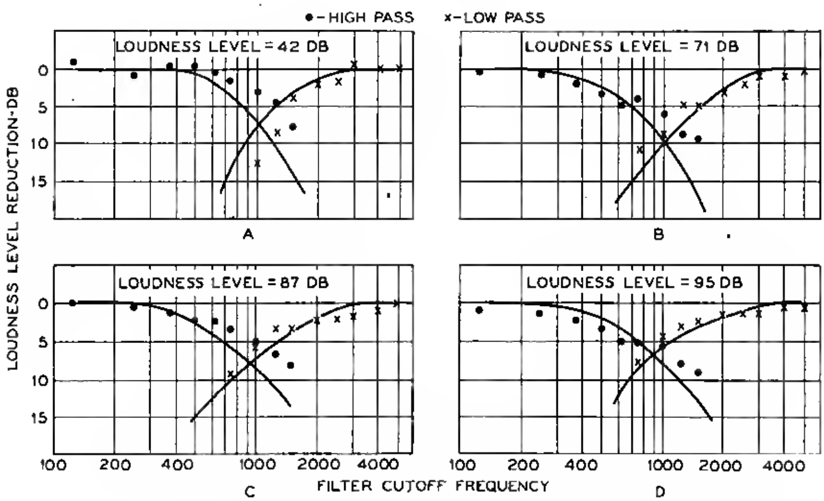


Fig. 14 (A to D)—Loudness level reduction tests on the No. 3A audiometer tone.

function, G, corresponds to something real in the mechanism of hearing. The present values given for G may be modified slightly, but we think that they will not be radically changed.

A study of the loudness of complex sounds which are not steady, such as speech and sounds of varying duration, is in progress at the present time and the results will be reported in a second paper on this subject.

Appendix A. Experimental Method of Measuring the Loudness Level of a Steady Sound

A measurement of the loudness level of a sound consists of listening alternately to the sound and to the 1000-cycle reference tone and adjusting the latter until the two are equally loud. If the intensity level of the reference tone is L decibels when this condition is reached, the sound is said to have a loudness level of L decibels. When the character of the sound being measured differs only slightly from that of the reference tone, the comparison is easily and quickly made, but for other sounds the numerous factors which enter into a judgment of equality of loudness become important, and an experimental method should be used which will yield results typical of the average normal ear and normal physiological and psychological conditions.

A variety of methods have been proposed to accomplish this, differing not only in general classification, that is, the method of average error, constant stimuli, etc., but also in important experimental details such as the control of noise conditions and fatigue effects. In some instances unique devices have been used to facilitate a ready comparison of sounds. One of these, the alternation phonometer,[12] introduces into the comparison important factors such as the duration time of the sounds and the effect of transient conditions. The merits of a particular method will depend upon the circumstances under which it is to be used. The one to be described here was developed for an extensive series of laboratory tests.

To determine when two sounds are equally loud it is necessary to rely upon the judgment of an observer, and this involves of course, not only the ear mechanism, but also associated mental processes, and effectively imbeds the problem in a variety of psychological factors. These difficulties are enhanced by the large variations found in the judgments of different observers, necessitating an investigation conducted on a statistical basis. The method of constant stimuli, wherein the observer listens to fixed levels of the two sounds and estimates which sound is the louder, seemed best adapted to control the many factors involved, when using several observers simultaneously. By means of this method, an observer's part in the test can be readily limited to an indication of his loudness judgment. This is essential as it was found that manipulation of apparatus controls, even though they were not calibrated, or participation in any way other than as a judge of loudness values, introduced undesirable factors which were aggravated by continued use of the same observers over a long period of time. Control of fatigue, memory effects, and the association of an observer's judgments with the results of the tests or with the judgments of other observers could be rigidly maintained with this method, as will be seen from the detailed explanation of the experimental procedure.

[12] D. Mackenzie, "Relative Sensitivity of the Ear at Different Levels of Loudness," *Phys. Rev.* **20**, 331 (1922).

The circuit shown in Fig. 15 was employed to generate and control the reference tone and the sounds to be measured. Vacuum tube oscillators were used to generate pure tones, and for complex tones and other sounds, suitable sources were substituted. By means of the voltage measuring circuit and the attenuator, the voltage level (voltage level = 20 log V) impressed upon the terminals of the receivers, could be determined. For example, the attenuator, which was calibrated in decibels, was set so that the voltage measuring set indicated 1 volt was being impressed upon the receiver. Then the difference between this setting and any other setting is the voltage level. To obtain the intensity level of the sound we must know the calibration of the receivers.

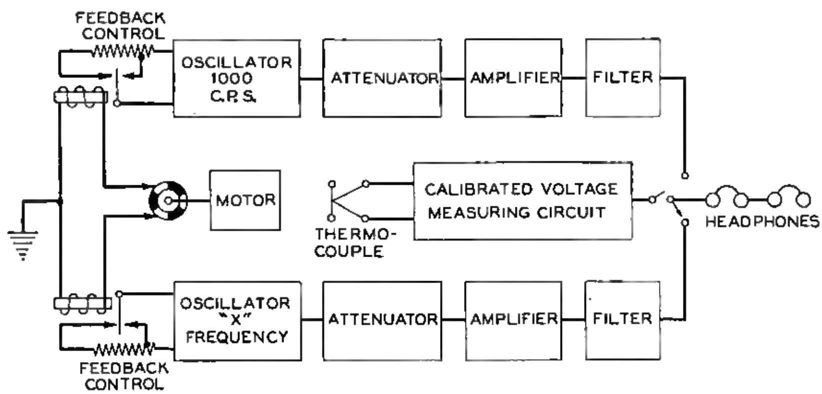


Fig. 15—Circuit for loudness balances.

The observers were seated in a sound-proof booth and were required only to listen and then operate a simple switch. These switches were provided at each position and were arranged so that the operations of one observer could not be seen by another. This was necessary to prevent the judgments of one observer from influencing those of another observer. First they heard the sound being tested, and immediately afterwards the reference tone, each for a period of one second. After a pause of one second this sequence was repeated, and then they were required to estimate whether the reference tone was louder or softer than the other sound and indicate their opinions by operating the switches. The levels were then changed and the procedure repeated. The results of the tests were recorded outside the booth.

The typical recording chart shown in Fig. 16 contains the results of three observers testing a 125-cycle tone at three different levels. Two

125 C.P.S. PURE TONE TEST NO. 4 CREW NO. 1. 1000 C.P.S. VOLTAGE LEVEL (DB)

Obs.		+6	+2	−2	−6	−10	−14	−18	−22	−26
125	CK	+	+	+	+	+	0	0	0	0
c.p.s.	AS	+	+	+	+	0	0	0	0	0
Volt.	DH	+	+	0	0	0	0	0	0	0
level =	CK	+	+	+	+	+	0	0	0	0
+ 9.8 db	AS	+	+	+	+	0	0	0	0	0
	DH	+	+	0	0	+	0	0	0	0
	CK	+	+	+	+	0	0	0	0	0
	AS	+	+	+	0	0	0	0	0	0
	DH	+	+	0	0	0	0	0	0	0
		0	−4	−8	−12	−16	−20	−24	−28	−32
125	CK	+	+	+	+	0	+	+	0	0
c.p.s.	AS	+	+	+	+	+	0	0	0	0
Volt.	DH	+	+	+	+	0	0	0	0	0
level =	CK	+	+	+	+	+	+	+	0	0
− 3.2 db	AS	+	+	+	+	+	+	0	0	0
	DH	+	+	+	0	+	0	+	0	0
	CK	+	+	+	+	+	+	0	0	0
	AS	+	+	+	+	+	0	0	0	0
	DH	+	+	+	0	+	0	0	0	0
		−15	−19	−23	−27	−31	−35	−39	−43	−47
125	CK	+	+	+	+	+	0	0	0	0
c.p.s.	AS	+	+	+	+	0	0	0	0	0
Volt.	DH	+	+	0	+	0	+	0	0	0
level =	CK	0	+	+	+	+	+	0	0	0
− 14.2	AS	+	+	+	+	0	+	0	0	0
db	DH	+	+	0	+	0	0	+	0	0
	CK	+	+	0	+	+	+	0	0	0
	AS	+	+	0	0	+	+	0	0	0
	DH	+	+	0	0	0	0	+	0	0

Fig. 16—Loudness balance data sheet.

marks were used for recording the observers' judgments, a cipher indicating the 125-cycle tone to be the louder, and a plus sign denoting the reference tone to be the louder of the two. No equal judgments were permitted. The figures at the head of each column give the voltage level of the reference tone impressed upon the receivers, that is, the number of decibels from 1 volt, plus if above and minus if below, and those at the side are similar values for the tone being tested. Successive tests were chosen at random from the twenty-seven possible combinations of levels shown, thus reducing the possibility of memory effects. The levels were selected so the observers listened to reference tones which were louder and softer than the tone being tested and the median of their judgments was taken as the point of equal loudness.

The data on this recording chart, combined with a similar number of observations by the rest of the crew, (a total of eleven observers) are shown in graphical form in Fig. 17. The arrow indicates the median

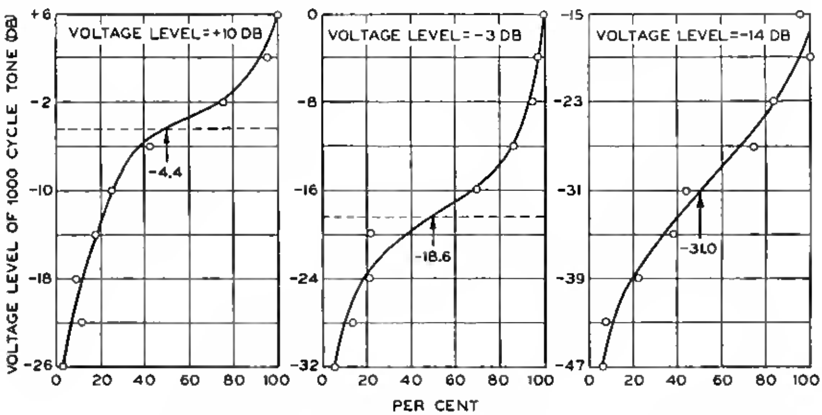


Fig. 17—Percent of observations estimating 1000-cycle tone to be louder than 125-cycle tone.

level at which the 1000-cycle reference, in the opinion of this group of observers, sounded equally loud to the 125-cycle tone.

The testing method adopted was influenced by efforts to minimize fatigue effects, both mental and physical. Mental fatigue and probable changes in the attitude of an observer during the progress of a long series of tests were detected by keeping a record of the spread of each observer's results. As long as the spread was normal it was assumed that the fatigue, if present, was small. The tests were conducted on a time schedule which limited the observers to five minutes of continuous testing, during which time approximately fifteen observations were made. The maximum number of observations permitted in one day was 150.

To avoid fatiguing the ear the sounds to which the observers listened were of short duration and in the sequence illustrated on Fig. 18. The

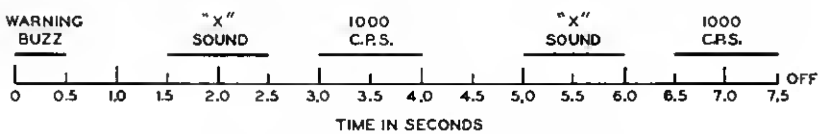


Fig. 18—Time sequence for loudness comparisons.

duration time of each sound had to be long enough to attain full loudness and yet not sufficiently long to fatigue the ear. The reference tone followed the x sound at a time interval short enough to permit a ready comparison, and yet not be subject to fatigue by prolonging the stimulation without an adequate rest period. At high levels it was found that a tone requires nearly 0.3 second to reach full loudness and if sustained for longer periods than one second, there is danger of fatiguing the ear.[13]

To avoid the objectionable transients which occur when sounds are interrupted suddenly at high levels, the controlling circuit was designed to start and stop the sounds gradually. Relays operating in the feedback circuits of the vacuum tube oscillators and in the grid circuits of amplifiers performed this operation. The period of growth and decay was approximately 0.1 second as shown on the typical oscillogram in Fig. 19. With these devices the transient effects were

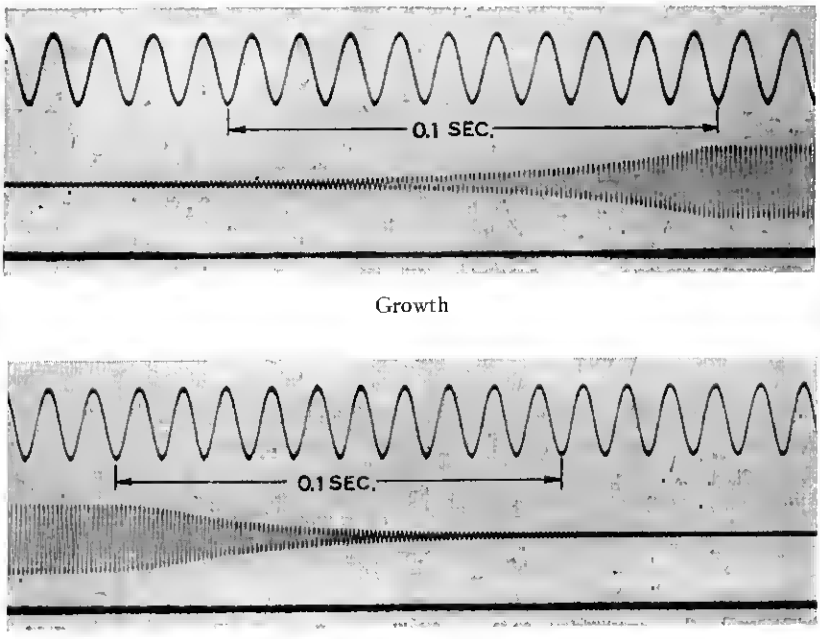


Fig. 19—Growth and decay of 1000-cycle reference tone.

reduced and yet the sounds seemed to start and stop instantaneously unless attention was called to the effect. A motor-driven commutator operated the relays which started and stopped the sounds in proper sequence, and switched the receivers from the reference tone circuit to the sound under test.

[13] G. v. Bekesy, "Theory of Hearing," *Phys. Zeits.* 30, 115 (1929).

The customary routine measurements to insure the proper voltage levels impressed upon the receivers were made with the measuring circuit shown schematically in Fig. 15. During the progress of the tests voltage measurements were made frequently and later correlated with measurements of the corresponding field sound pressures.

Threshold measurements were made before and after the loudness tests. They were taken on the same circuit used for the loudness tests (Fig. 15) by turning off the 1000-cycle oscillator and slowly attenuating the other tone below threshold and then raising the level until it again became audible. The observers signalled when they could no longer hear the tone and then again when it was just audible. The average of these two conditions was taken as the threshold.

An analysis of the harmonics generated by the receivers and other apparatus was made to be sure of the purity of the tones reaching the ear. The receivers were of the electrodynamic type and were found to produce overtones of the order of 50 decibels below the fundamental. At the very high levels, distortion from the filters was greater than from the receivers, but in all cases the loudness level of any overtone was 20 decibels or more below that of the fundamental. Experience with complex tones has shown that under these conditions the contribution of the overtones to the total loudness is insignificant.

The method of measuring loudness level which is described here has been used on a large variety of sounds and found to give satisfactory results.

Appendix B. Comparison of Data on the Loudness Levels of Pure Tones

A comparison of the present loudness data with that reported previously by B. A. Kingsbury [4] would be desirable and in the event of agreement, would lend support to the general application of the results as representative of the average ear. It will be remembered that the observers listened to the tones with both ears in the tests reported here, while a single receiver was used by Kingsbury.

Also, it is important to remember that the level of the tones used in the experiments was expressed as the number of db above the average threshold current obtained with a single receiver. For both of these reasons a direct comparison of the results cannot be made. However, in the course of our work two sets of experiments were made which give results that make it possible to reduce Kingsbury's data so that it may be compared directly with that reported in this paper.

In the first set of experiments it was found that if a typical observer listened with both ears and estimated that two tones, the reference tone and a tone of different frequency, appeared equally loud, then, making a similar comparison using one ear (the voltages on the receiver remaining unchanged) he would still estimate that the two tones were equally loud. The results upon which this conclusion is based are shown in Table XVI. In the first row are shown the frequencies of the tones tested. Under these frequencies are shown the differences in db of the voltage levels on the receivers obtained when listening by the two methods, the voltage level of the reference tone being constant at 32 db down from 1 volt. Under the caption "Other Reference Tone Levels" similar figures for frequencies of 62 c.p.s. and 2000 c.p.s. and for the levels of the reference tone indicated are given. It will be seen that these differences are well within the observational error. Consequently, the conclusion mentioned above seems to be justified. This is an important conclusion and although the data are confined to tests made with receivers on the ear it would be expected that a similar relation would hold when the sounds are coming directly to the ears from a free wave.

TABLE XVI

COMPARISON OF ONE AND TWO-EAR LOUDNESS BALANCES

A. Reference tone voltage level = − 32 db

Frequency, c.p.s.	62	125	250	500	2000	4000	6000	8000	10,000
Voltage level difference *	−0.5	0	+1.0	−1.0	−0.5	−0.5	+0.5	−3.0	−3.0

B. Other reference tone levels

62 c.p.s.		2000 c.p.s.	
Ref. Tone Volt. Level	Volt. Level Difference *	Ref. Tone Volt. Level	Volt. Level Difference *
−20	+0.5	− 3	0.0
−34	+0.2	−22	+0.3
−57	+2.0	−41	−0.8
−68	−0.5	−60	−0.8
		−79	−6.2

* Differences are in db, positive values indicating a higher voltage for the one ear balance.

This result is in agreement with the point of view adopted in developing the formula for calculating loudness. When listening with one instead of two ears, the loudness of the reference tone and also that of the tone being compared are reduced to oné half. Consequently, if they were equally loud when listening with two ears they must be equally loud when listening with one ear. The second set of data is concerned with differences in the threshold when listening with one ear *versus* listening with two ears.

It is well known that for any individual the two ears have different acuity. Consequently, when listening with both ears the threshold is determined principally by the better ear. The curve in Fig. 20 shows the difference in the threshold level between the average of the better of an observer's ears and the average of all the ears. The circles represent data taken on the observers used in our loudness tests while the crosses represent data taken from an analysis of 80 audiograms of persons with normal hearing. If the difference in acuity when listening with one ear *vs.* listening with two ears is determined entirely by the better ear, then the curve shown gives this difference. However, some experimental tests which we made on one ear acuity *vs.* two ear acuity showed the latter to be slightly greater than for the better ear alone, but the small magnitudes involved and the difficulty of avoiding

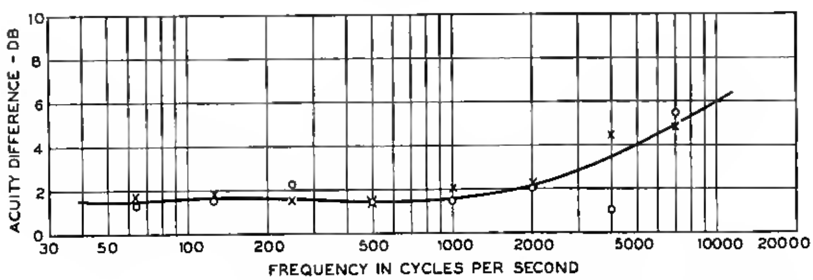


Fig. 20—Difference in acuity between the best ear and the average of both ears.

psychological effects caused a probable error of the same order of magnitude as the quality being measured. At the higher frequencies where large differences are usually present the acuity is determined entirely by the better ear.

From values of the loudness function G, one can readily calculate what the difference in acuity when using one *vs.* two ears should be. Such a calculation indicates that when the two ears have the same acuity, then when listening with both ears the threshold values are about 2 db lower than when listening with one ear. This small difference would account for the difficulty in trying to measure it.

We are now in a position to compare the data of Kingsbury with those shown in Table I. The data in Table I can be converted into decibels above threshold by subtracting the average threshold value in each column from any other number in the same column.

If now we add to the values for the level above threshold given by

Kingsbury an amount corresponding to the differences shown by the curve of Fig. 20, then the resulting values should be directly comparable to our data on the basis of decibels above threshold. Comparisons of his data on this basis with those reported in this paper are shown in Fig. 21. The solid contour lines are drawn through points taken from Table I and the dotted contour lines taken from Kingsbury's data. It will be seen that the two sets of data are in good agreement between 100 and 2000 cycles but diverge somewhat above and below these points. The discrepancies are slightly greater than would be expected from experimental errors, but might be explained

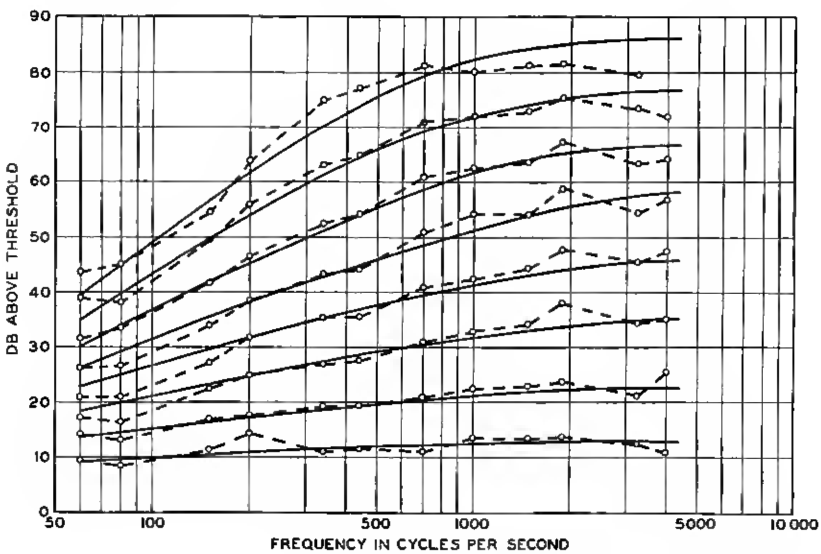


Fig. 21—Loudness levels of pure tones—A comparison with Kingsbury's data.

by the presence of a slight amount of noise during threshold determinations.

Appendix C. Optical Tone Generator of Complex Wave Forms

For the loudness tests in which the reference tone was compared with a complex tone having components of specified loudness levels and frequencies, the tones were listened to by means of head receivers as before; the circuit shown in Fig. 15 remaining the same excepting for the vacuum tube oscillator marked "x Frequency." This was replaced by a complex tone generator devised by E. C. Wente of the Bell Telephone Laboratories. The generator is shown schematically in Fig. 22.

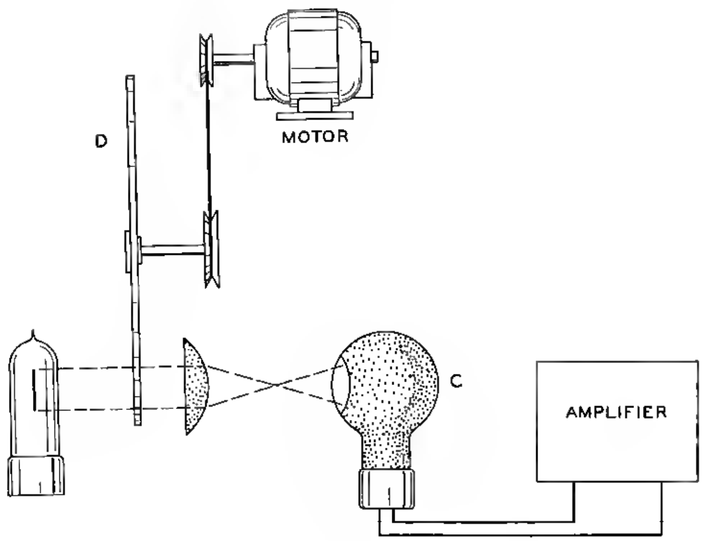


Fig. 22—Schematic of optical tone generator.

The desired wave form was accurately drawn on a large scale and then transferred photographically to the glass disk designated as *D* in the diagram. The disk, driven by a motor, rotated between the lamp *L* and a photoelectric cell *C*, producing a fluctuating light source which

Fig. 23—Ten disk optical tone generator.

was directed by a suitable optical system upon the plate of the cell. The voltage generated was amplified and attenuated as in the case of the pure tones.

The relative magnitudes of the components were of course fixed by the form of the wave inscribed upon the disk, but this was modified when desired, by the insertion of elements in the electrical circuit which gave the desired characteristic. Greater flexibility in the control of the amplitude of the components was obtained by inscribing each component on a separate disk with a complete optical system and cell for each. Frequency and phase relations were maintained by mounting all of the disks on a single shaft. Such a generator having ten disks is shown in Fig. 23.

An analysis of the voltage output of the optical tone generators showed an average error for the amplitude of the components of about $\pm$ 0.5 db, which was probably the limit of accuracy of the measuring instrument. Undesired harmonics due to the disk being off center or inaccuracies in the wave form were removed by filters in the electrical circuit.

All of the tests on complex tones described in this paper were made with the optical tone generator excepting the audiometer, and two tone tests. For the latter tests, two vacuum tube oscillators were used as a source.