The Minimal Retroreflective Microfacet Model

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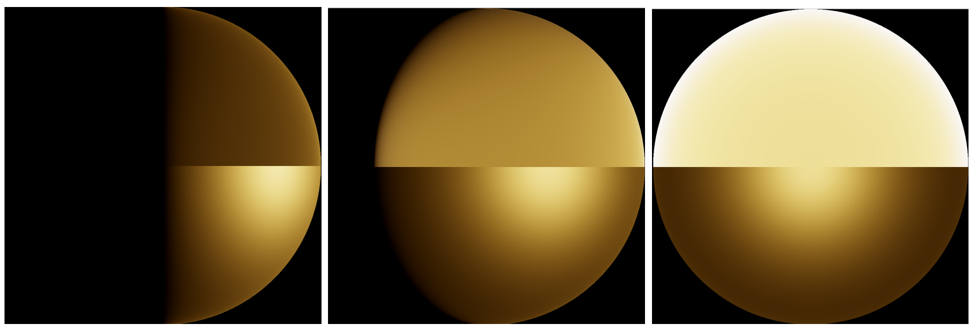

Figure 1. Comparison of our retroreflector model (top) obtained by an almost trivial modification of the equivalent roughness GGX microfacet model (bottom). The lighting varies (left to right) from side-on, to quarter-on, and straight-on, exhibiting the strong retro-reflective peak.

Abstract

We introduce the "Minimal Retroreflective Microfacet" (MRM) model, which provides a simple scheme for modifying a pre-existing microfacet BRDF implementation to produce a visually plausible, energy-conserving retroreflective result.

1. Introduction

Rendering of retro-reflective materials is useful in a variety of contexts. A number of CG models have been proposed. For practical purposes in visual effects, we are interested in a model which is a) visually plausible at least, b) provably energy-conserving, c) efficient and easy to implement, ideally a small modification to well-understood microfacet models. We present here a model based on the previously published "back-vector" formulation [Belcour et al. 2014] which meets these requirements. It is so simple a modification to a regular microfacet model, that we term it the "Minimal Retroreflective Microfacet" (MRM) model which achieves a plausible retro-reflective result.

In this document, we detail the correct usage and derivation of this retro-reflective BRDF model within the microfacet framework [Walter et al. 2007; Heitz 2014]. We first recall some basic principles from microfacet theory (Section 2) and show how to account for retro-reflective microfacets (Section 3) within this framework. We then

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$\mathbf{v}$	normalized vector to viewer
$\mathbf{l}$	normalized vector to light
$\mathbf{n}$	surface normal
$\mathbf{t_1}, \mathbf{t_2}$	tangent and bitangent (forming an orthonormal coordinate frame with the normal)
$\mathbf{h}$	half vector, $\mathbf{h} = \mathbf{h}(\mathbf{v}, \mathbf{l}) = \frac{\mathbf{v}+\mathbf{l}}{\Vert\mathbf{v}+\mathbf{l}\Vert}$
$\langle \mathbf{x}, \mathbf{y} \rangle$	dot product of $\mathbf{x}$ and $\mathbf{y}$
$\text{reflect}(\mathbf{x}, \mathbf{m})$	reflection of vector $\mathbf{x}$ on surface with normal $\mathbf{m}$, i.e. $\text{reflect}(\mathbf{x}, \mathbf{m}) = -\mathbf{x} + 2\langle \mathbf{x}, \mathbf{m} \rangle \mathbf{m}$
Ω	hemisphere of directions around the normal

Figure 2. Notations used.

detail how to implement this in practice (Section 5). Finally, we discuss how retro-reflection can occur at the microfacet level (Section 4).

2. Microfacet BRDF Models

Using the notations from figure 2, a microfacet BRDF [Walter et al. 2007; Heitz 2014] takes the form (Walter et al. [2007] Equation (20))

$$f(\mathbf{v}, \mathbf{l}) = \frac{D(\mathbf{h})\, G_2(\mathbf{v}, \mathbf{l})\, F(\mathbf{h}, \mathbf{v})}{4 \langle \mathbf{v}, \mathbf{n} \rangle \langle \mathbf{l}, \mathbf{n} \rangle} \tag{1}$$

where D is the distribution of microfacets, normalized to fulfill $\int_\Omega D(\mathbf{h}) \langle \mathbf{n}, \mathbf{h} \rangle \, d\mathbf{h} = 1$, G_2 is the shadowing-masking function, and F is the Fresnel term. The half-vector $\mathbf{h}$ is used as the *selector* for microfacets reflecting light from incident direction $\mathbf{l}$ to outgoing direction $\mathbf{v}$ (*i.e.* only microfacet with normals aligned with the half-vector will reflect light). In Section 4.1 of their work, Walter et al. [Walter et al. 2007] detail the effect of using the half vector as the microfacet selector and obtain Equation 1.

In this work, we show that we can use the back vector [Belcour et al. 2014] used to model retro-reflection as a microfacet selector. Before detailing the impact of such a change, we review some properties of microfacet models.

Shadowing-Masking. The shadowing-masking function combines the visibility masking function $G_1(\mathbf{v}, \mathbf{l})$ for view direction and $G_1(\mathbf{l}, \mathbf{v})$ for light direction:

$$G_2(\mathbf{v}, \mathbf{l}) = G_2\left(G_1(\mathbf{v}, \mathbf{l}), G_1(\mathbf{l}, \mathbf{v})\right) \tag{2}$$

Mathematically, the BRDF is plausible if G_2 is symmetric w.r.t. to exchanging $\mathbf{v}$ and $\mathbf{l}$, fulfills $G_2(\mathbf{v}, \mathbf{l}) \leq G_1(\mathbf{v}, \mathbf{l})$, and if G_1 further establishes the correct projected area of visible micro-surfaces [Heitz 2014]

$$\langle \mathbf{v}, \mathbf{n} \rangle = \int_{\{\mathbf{h} \in \Omega : \langle \mathbf{v}, \mathbf{h} \rangle \geq 0\}} D(\mathbf{h}) G_1(\mathbf{v}, \text{reflect}(\mathbf{v}, \mathbf{h})) \langle \mathbf{v}, \mathbf{h} \rangle \, d\mathbf{h}. \tag{3}$$

Choices for G_1 with this property are the Smith masking function

$$G_1(\mathbf{v},\mathbf{l}) = G_{1,\text{Smith}}(\mathbf{v}) = \frac{\langle \mathbf{v},\mathbf{n}\rangle}{\int_{\{\mathbf{h}\in\Omega:\langle \mathbf{v},\mathbf{h}\rangle\geq 0\}} D(\mathbf{h})\,\langle \mathbf{v},\mathbf{h}\rangle\, d\mathbf{h}}, \tag{4}$$

where analytic expressions are available for Beckmann and GGX distributions, and the v-cavity masking function

$$G_1(\mathbf{v},\mathbf{l}) = G_{1,\text{vc}}(\langle \mathbf{v},\mathbf{n}\rangle, \langle \mathbf{n},\mathbf{h}\rangle, \langle \mathbf{v},\mathbf{h}\rangle) = \min\left\{\frac{2\,\langle \mathbf{v},\mathbf{n}\rangle\,\langle \mathbf{n},\mathbf{h}\rangle}{\langle \mathbf{v},\mathbf{h}\rangle}, 1\right\}, \tag{5}$$

which is generally applicable for distributions with symmetry around the normal.

3. Back-vector Modification

If instead of half-vector $\mathbf{h}$, we use the back-vector $\mathbf{b}$ [Belcour et al. 2014] defined as

$$\mathbf{b}(\mathbf{v},\mathbf{l}) = \mathbf{h}(\mathbf{v}',\mathbf{l}) = \frac{\mathbf{v}'+\mathbf{l}}{\|\mathbf{v}'+\mathbf{l}\|}, \text{ with } \mathbf{v}' = \text{reflect}(\mathbf{v},\mathbf{n}), \tag{6}$$

to select the reflecting microfacet, we are to re-derive the microfacet model.

Jacobian of Back-Vector. The first step is to write the Jacobian of the change of variable from the back-vector to the light vector $\mathbf{l}$ (Equation (11) and (14) of Walter et al. [2007]). Fortunately, this jacobian has the same form as the half-vector one:

$$\left\|\frac{\partial\omega_{\mathbf{b}}}{\partial\omega_{\mathbf{l}}}\right\| = \left\|\frac{\partial\omega_{\mathbf{h}(\mathbf{v}',\mathbf{l})}}{\partial\omega_{\mathbf{l}}}\right\| = \frac{1}{4\,|\mathbf{l}\cdot\mathbf{h}(\mathbf{v}',\mathbf{l})|} = \frac{1}{4\,|\mathbf{l}\cdot\mathbf{b}|} \tag{7}$$

Microfacet reflection term. We do not know for sure how the retro-reflective microfacet interacts with incident light. However, we have experimental evidence [Belcour et al. 2014] that it behaves similarly to a Fresnel term. We opted to reuse the Fresnel as well in the $(\mathbf{v}',\mathbf{l})$ configuration. As such, we update the formulation of $\rho(\mathbf{v},\mathbf{m})$ (Equation (11) of Walter et al. [2007]) to use the reflected view vector instead. It follows that the reflection microsurface BRDF (Equation (15) of Walter et al. [2007]) becomes

$$f_r^m(\mathbf{v},\mathbf{l},\mathbf{m}) = F(\mathbf{v}',\mathbf{m})\frac{\delta(\mathbf{b},\mathbf{m})}{4\,|\mathbf{v}'\cdot\mathbf{b}|^2}. \tag{8}$$

We used the fact that $|\mathbf{l}\cdot\mathbf{b}| = |\mathbf{v}'\cdot\mathbf{b}|$.

Shadowing & Masking. **[TODO:** Laurent: I think we could extract the shadowing from Matthias derivation here. The other way to justify the derivation of the full model is to highlight that Smith's shadowing/masking does not depend on the microfacet (appart from the sideness). Hence, whatever the microfacet selector (as long as the sideness matches), the shadowing/masking is the same. Since $\mathbf{v}$ and $\mathbf{v}'$ share the same slope, we can exchange them in the equation.**]**

Final form.

$$f_R(\mathbf{v}, \mathbf{l}) = \frac{D(\mathbf{b})\, G_2(\mathbf{v}', \mathbf{l})\, F(\mathbf{b}, \mathbf{v}')}{4 \left\langle \mathbf{v}', \mathbf{n} \right\rangle \left\langle \mathbf{l}, \mathbf{n} \right\rangle} \tag{9}$$

4. Plausibility of the Back-vector Modification

In analogy to $\mathbf{v}'$ we let $\mathbf{l}' := \text{reflect}(\mathbf{l}, \mathbf{n})$. First we observe

$$\begin{aligned} \left\langle \mathbf{v}', \mathbf{n} \right\rangle &= \left\langle \mathbf{v}, \mathbf{n} \right\rangle \text{ and} \\ \left\langle \mathbf{l}, \mathbf{n} \right\rangle &= \left\langle \mathbf{l}', \mathbf{n} \right\rangle . \end{aligned} \tag{10}$$

For the back-vector we show $\mathbf{h}(\mathbf{v}', \mathbf{l}) = \text{reflect}\left(\mathbf{h}(\mathbf{l}', \mathbf{v}), \mathbf{n}\right)$, as

$$\begin{aligned} \mathbf{v}' + \mathbf{l} &= \left(-\mathbf{v} + 2 \left\langle \mathbf{v}, \mathbf{n} \right\rangle \mathbf{n}\right) + \mathbf{l} \\ &= \left(-\mathbf{v} + 2 \left\langle \mathbf{v}, \mathbf{n} \right\rangle \mathbf{n}\right) + \left(-\mathbf{l}' + 2 \left\langle \mathbf{l}', \mathbf{n} \right\rangle \mathbf{n}\right) \\ &= -(\mathbf{l}' + \mathbf{v}) + 2 \left\langle (\mathbf{l}' + \mathbf{v}), \mathbf{n} \right\rangle \mathbf{n} = \text{reflect}(\mathbf{l}' + \mathbf{v}, \mathbf{n}) \end{aligned} \tag{11}$$

and therefore $\|\mathbf{v}' + \mathbf{l}\| = \|\mathbf{l}' + \mathbf{v}\|$ as well. From that we can conclude

$$\begin{aligned} \left\langle \mathbf{n}, \mathbf{h}(\mathbf{v}', \mathbf{l}) \right\rangle &= \left\langle \mathbf{n}, \mathbf{h}(\mathbf{l}', \mathbf{v}) \right\rangle \\ \left\langle \mathbf{t_1}, \mathbf{h}(\mathbf{v}', \mathbf{l}) \right\rangle &= -\left\langle \mathbf{t_1}, \mathbf{h}(\mathbf{l}', \mathbf{v}) \right\rangle \\ \left\langle \mathbf{t_2}, \mathbf{h}(\mathbf{v}', \mathbf{l}) \right\rangle &= -\left\langle \mathbf{t_2}, \mathbf{h}(\mathbf{l}', \mathbf{v}) \right\rangle . \end{aligned} \tag{12}$$

As anisotropic Phong, Beckmann, and GGX distributions are reflection-symmetric around the normal, i.e. they take the form $D(\mathbf{h}) = D\left(\left\langle \mathbf{n}, \mathbf{h} \right\rangle, |\left\langle \mathbf{t_1}, \mathbf{h} \right\rangle|, |\left\langle \mathbf{t_2}, \mathbf{h} \right\rangle|\right)$, we can thus infer that using them with the back-vector obeys symmetry. Further, the reflection symmetry of D along with equation (10) gives

$$\begin{aligned} G_{1,\text{Smith}}(\mathbf{v}) &= \frac{\left\langle \mathbf{v}, \mathbf{n} \right\rangle}{\int_{\{\mathbf{h} \in \Omega : \langle \mathbf{v}, \mathbf{h} \rangle \geq 0\}} D(\mathbf{h}) \left\langle \mathbf{v}, \mathbf{h} \right\rangle d\mathbf{h}} \\ &= \frac{\left\langle \mathbf{v}', \mathbf{n} \right\rangle}{\int_{\{\mathbf{h} \in \Omega : \langle \mathbf{v}', \mathbf{h} \rangle \geq 0\}} D(\mathbf{h}) \left\langle \mathbf{v}', \mathbf{h} \right\rangle d\mathbf{h}} = G_{1,\text{Smith}}(\mathbf{v}') \end{aligned} \tag{13}$$

for Smith-masking. Next we show

$$\left\langle \mathbf{v}', \mathbf{l} \right\rangle = -\left\langle \mathbf{v}, \mathbf{l} \right\rangle + 2 \left\langle \mathbf{v}, \mathbf{n} \right\rangle \left\langle \mathbf{l}, \mathbf{n} \right\rangle = \left\langle \mathbf{v}, -\mathbf{l} + 2 \left\langle \mathbf{l}, \mathbf{n} \right\rangle \mathbf{n} \right\rangle = \left\langle \mathbf{v}, \mathbf{l}' \right\rangle \tag{14}$$

and therefore

$$\left\langle \mathbf{v}', \mathbf{h}(\mathbf{v}', \mathbf{l}) \right\rangle = \frac{1 + \left\langle \mathbf{l}, \mathbf{v}' \right\rangle}{\|\mathbf{v}' + \mathbf{l}\|} = \frac{1 + \left\langle \mathbf{l}', \mathbf{v} \right\rangle}{\|\mathbf{l}' + \mathbf{v}\|} = \left\langle \mathbf{l}', \mathbf{h}(\mathbf{l}', \mathbf{v}) \right\rangle . \tag{15}$$

Using this along with equations (12) and (10) we get

$$\begin{aligned} G_1(\mathbf{v}', \mathbf{l}) &= G_{1,\text{vc}}\left(\left\langle \mathbf{v}', \mathbf{n} \right\rangle, \left\langle \mathbf{n}, \mathbf{h}(\mathbf{v}', \mathbf{l}) \right\rangle, \left\langle \mathbf{v}', \mathbf{h}(\mathbf{v}', \mathbf{l}) \right\rangle\right) \\ &= G_{1,\text{vc}}\left(\left\langle \mathbf{v}, \mathbf{n} \right\rangle, \left\langle \mathbf{n}, \mathbf{h}(\mathbf{l}', \mathbf{v}) \right\rangle, \left\langle \mathbf{l}', \mathbf{h}(\mathbf{l}', \mathbf{v}) \right\rangle\right) \\ &= G_{1,\text{vc}}\left(\left\langle \mathbf{v}, \mathbf{n} \right\rangle, \left\langle \mathbf{n}, \mathbf{h}(\mathbf{v}, \mathbf{l}') \right\rangle, \left\langle \mathbf{l}', \mathbf{h}(\mathbf{v}, \mathbf{l}') \right\rangle\right) \\ &= G_1(\mathbf{v}, \mathbf{l}') \end{aligned} \tag{16}$$

and similarly $G_1(\mathbf{l},\mathbf{v}') = G_1(\mathbf{l}',\mathbf{v})$ for the v-cavity masking function. In summary, we can conclude the symmetry of the BRDF

$$\begin{aligned} f_{\text{retro}}(\mathbf{v},\mathbf{l}) = f(\mathbf{v}',\mathbf{l}) &= \frac{D(\mathbf{h}(\mathbf{v}',\mathbf{l}))G_2(G_1(\mathbf{v}',\mathbf{l}),G_1(\mathbf{l},\mathbf{v}'))}{4\langle\mathbf{v}',\mathbf{n}\rangle\langle\mathbf{l},\mathbf{n}\rangle} \\ &= \frac{D(\mathbf{h}(\mathbf{l}',\mathbf{v}))G_2(G_1(\mathbf{v},\mathbf{l}'),G_1(\mathbf{l}',\mathbf{v}))}{4\langle\mathbf{v},\mathbf{n}\rangle\langle\mathbf{l}',\mathbf{n}\rangle} = f(\mathbf{l}',\mathbf{v}) = f_{\text{retro}}(\mathbf{l},\mathbf{v}). \end{aligned} \tag{17}$$

For the albedo we simply have

$$\rho_{\text{retro}}(\mathbf{v}) = \int_\Omega f_{\text{retro}}(\mathbf{v},\mathbf{l})\langle\mathbf{l},\mathbf{n}\rangle\, d\mathbf{l} = \int_\Omega f(\mathbf{v}',\mathbf{l})\langle\mathbf{l},\mathbf{n}\rangle\, d\mathbf{l} = \rho(\mathbf{v}') \tag{18}$$

i.e. we get the albedo of the regular microfacet BRDF for the reflected direction. Which, using equation (3) and $\frac{d\mathbf{h}(\mathbf{v}',\mathbf{l})}{d\mathbf{l}} = \frac{1}{4\langle\mathbf{l},\mathbf{h}(\mathbf{v}',\mathbf{l})\rangle} = \frac{1}{4\langle\mathbf{v}',\mathbf{h}(\mathbf{v}',\mathbf{l})\rangle}$ [Walter et al. 2007], can be shown to fulfill energy conservation

$$\begin{aligned} \rho(\mathbf{v}') &= \int_\Omega \frac{D(\mathbf{h}(\mathbf{v}',\mathbf{l}))G_2(\mathbf{v}',\mathbf{l})}{4\langle\mathbf{v}',\mathbf{n}\rangle\langle\mathbf{l},\mathbf{n}\rangle}\langle\mathbf{l},\mathbf{n}\rangle\, d\mathbf{l} \\ &\le \int_{\{\mathbf{h}\in\Omega:\langle\mathbf{v}',\mathbf{h}\rangle\ge 0\}} \frac{D(\mathbf{h})G_1(\mathbf{v}',\text{reflect}(\mathbf{v}',\mathbf{h}))}{\langle\mathbf{v}',\mathbf{n}\rangle}\langle\mathbf{v}',\mathbf{h}\rangle\, d\mathbf{h} = 1. \end{aligned} \tag{19}$$

5. Implementation Notes

Given an implementation of a regular microfacet BRDF, extending it to retro-reflection is straightforward:

- Evaluation merely needs to replace $\mathbf{v}$ with $\mathbf{v}'$ upfront.
- Similarly, importance sampling of $\mathbf{l}$ given $\mathbf{v}$ can be realized by replacing $\mathbf{v}$ with $\mathbf{v}'$ upfront and then importance sampling the regular microfacet BRDF. This may include low variance sampling using the domain of visible microfacets [Heitz and d'Eon 2014].
- As the albedos of standard BRDF and retro-reflective BRDF are essentially identical, compensating for energy loss in the sense of [Kelemen and Szirmay-Kalos 2001] can be realized using the same data tables.
- Pseudo-code is provided in Listing 1.

```
vec3 brdf_retro(vec3 L, vec3 V, vec3 N, vec3 X, vec3 Y)
{
    vec3 Vp = -reflect(V,N); // jp - is this wrong? Why the minus sign

    // The following could all be replaced with:
    // We could give the simplified form of brdf_ggx, but not essential.
    return brdf_ggx(L, Vp, N, X, Y);

    /*
    float NdotL = dot(N, L);
    float NdotV = dot(N, Vp);
    if (NdotL < 0 || NdotV < 0) return vec3(0);

    vec3 B = normalize(L+Vp);
    float NdotH = dot(N, B);
    vec3 wB = vec3(dot(B,X),  dot(B,Y),  dot(B,N));
    vec3 wL = vec3(dot(L,X),  dot(L,Y),  dot(L,N));
    vec3 wV = vec3(dot(Vp,X), dot(Vp,Y), dot(Vp,N));

    float D = D_GGX(wB, ax, ay);
    float G = G2_GGX(wV, wL, ax, ay);

    float c = dot(L, B);
    float F = schlick(f0, c); // I don't think we need to give the form of
    the Fresnel

    float val = Ks * D * G * F / (4 * NdotL * NdotV);
    return vec3(val);
    */
}
```

Listing 1. Pseudo code to implement the retro-reflective microfacet model.

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