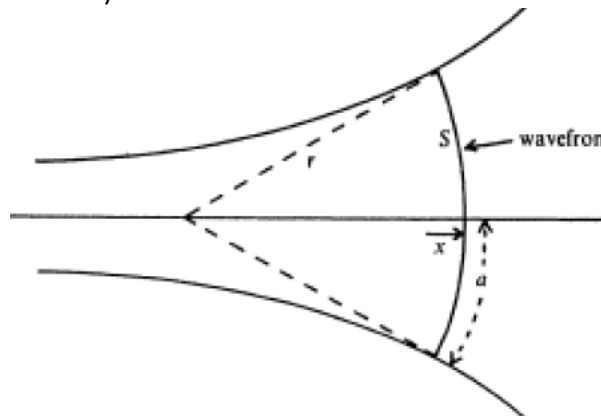


Part 1: design of the exponential section

It is given an exponential horn. The equivalent radius (see diagram below) is $a = a_0 e^{mx}$, with $a_0 = 0.008$ m and $m = 4.2$ m⁻¹. The length of the horn is $L = 0.35$ m. We intend to approximate the exponential horn as a concatenation of short sections of conical horns. In particular, we assume that all the sections are of the same length δ .

Recall that in a Salmon horn the wavefronts can be approximated as planar (and therefore the equivalent radius coincides with...).




We aim at computing, in Matlab, the input impedance of the concatenation and compare it, in the frequency range $[0, 2\text{kHz}]$, with the analytical expression of the impedance of the exponential horn. In a first stage, we neglect the impedance radiation at the mouth of the horn. We measure the similarity between the impedances $Z_1(\omega)$ and $Z_2(\omega)$, which are the input impedance of the approximated and analytical model, respectively, through two metrics:

- $e_1 = \frac{1}{(\omega_{max} - \omega_{min})} \int_{\omega_{min}}^{\omega_{max}} |Z_1(\omega) - Z_2(\omega)|^2 d\omega$, which evaluates the mean squared error between the two impedances;
 - $e_2 = \sum_{i=1}^5 \min |\arg \max_{\omega} \text{abs}(Z_1(\omega)) - \arg \max_{\omega} \text{abs}(Z_2(\omega))|$, which evaluates the difference in frequency between the first five maxima of the two impedances.
- a) Evaluate the error e_1 as a function of the length δ of the conical sections and plot it in Matlab.
- b) Evaluate the error e_2 as a function of the length δ of the conical sections and plot it in Matlab.

In the rest of the assignment, use the length δ that guarantees the minimum value of the error function e_2 .

In the approximated horn the adaptation to the external air is modeled through an additional section whose impedance is $Z_L(\omega) = Z_{L0}(\omega) \frac{S_p}{S_s}$, where $Z_{L0}(\omega)$ is the impedance of an unflanged cylindrical pipe of radius a , given by $Z_{L0}(\omega) = 0.25 \frac{\omega^2 \rho}{\pi c} + 0.61j \frac{\rho \omega}{\pi a}$, S_p is the cross-sectional area of the cylinder, and S_s the spherical wave front area at the open end of the cone. It can be approximated by $S_s = \frac{2S_p}{1 + \cos \theta}$, where θ is the flaring angle of the last conical section.

 POLITECNICO MILANO 1863	051483 Musical Acoustics Module 1: Modeling of musical instruments Academic Year 2022/2023	Homework 3: Horn design
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- c) **Compute the impedance of the approximated model when the radiation load is kept into account and evaluate the error brought by neglecting it in terms of the metric e_1 .**

Part 2: design of the compound horn

Consider now a compound horn, composed by a cylindrical pipe, whose length is 0.85 m, followed by the exponential horn. For the exponential horn use the impedance obtained before, including the radiation load.

- d) **Compute the impedance of the compound horn and list in a table the frequencies of the first ten maxima of the impedance.**
- e) **Evaluate the inharmonicity of the succession of impedance maxima for the different setups: exponential horn, exponential horn with impedance radiation and compound horn with impedance radiation.**

Upload instructions

- Upload the report on WeBeep by Nov. 18.
- If two students collaborated to the same report, all the students must upload the report on WeBeep.
- On the front page of the report specify the name, surname and ID of all the students participating to the HW.

Tip...

This assignment is a (small) research challenge and not all the answers are found in the course material. Some degrees of freedom are left to students: different solutions to the same problem can be found. Use an engineering approach!