Assignment

Part 1 – Circular membrane characterization

It is given a circular membrane with radius 0.15 m, and tension 10 N/m. The unit surface weight is σ=0.07 kg/m^2.

a) Compute the propagation speed in the membrane.

b) Compute the frequency of the first eighteen modes for this membrane (ordered by frequency) and draw in Matlab the modeshapes of the first six ones.

c) Limited to the frequency range where the first eighteen modes lie, and assuming that the drum is struck at coordinates r = 0.075m, ϕ=15°, and a displacement sensor is mounted at coordinates r=0.075m, ϕ=195°, derive the displacement time signal read by the sensor, assuming that all modes are characterized by the same quality factor, equal to 25. Moreover, the impact force signal of the hammer is

$$f(t) = 0.1\exp\,[-\frac{(t-0.03)^2}{0.01^2}]$$

Suggestion: use the modal superposition approach to derive the mobility of the membrane for the considered excitation and measurement points. Use then the convolution between the force signal and the inverse Fourier Transform of the mobility to obtain the response.

Part 2 – Circular plate characterization

Consider now a thin plate with clamped edges and with the same size of the membrane. The plate has a thickness of 1 mm, and it is made with aluminum (E=69 GPa, ρ=2700 kg/m^3, ν=0.334).

d) Compute the propagation speed of quasi-longitudinal and longitudinal waves;

e) plot the propagation speed of the bending waves as a function of the frequency for the considered plate;

f) find the modal frequencies of the first five bending modes of the plate.

Part 3 – Interaction between coupled systems

Consider now that a string is attached to the considered plate, and its fundamental mode is tuned to the frequency of the first mode of the plate. The string is made with iron (ρ=5000 kg/m^3), its cross section is circular with a radius of 0.001 m, and its length is L=0.4m.

Due to internal losses and sound radiation, the plate at the frequency of the considered mode dissipates energy, and the merit factor is 50.

g) Compute the tension of the string so that its fundamental mode is tuned with the first mode of the soundboard.

h) Compute now the frequencies of the first five modes of the string considering its stiffness. The Young modulus of the iron is E=200*10^9 Pa.

i) Neglecting the stiffness of the string, compute the frequencies of the modes of the string-soundboard system considering the coupling between the plate and the string.

Suggestion: use the diagram in the slides, which shows the relative frequency variation $\frac{\Omega_+ - \Omega_-}{\omega_0}$ *as a function of the coupling coefficient* $\frac{m}{n^2 M}$