ResearchGate

See discussions, stats, and author profiles for this publication at: https://www.researchgate.net/publication/349393088

Optimal Method for Determination of Rayleigh Damping Coefficients for Different Materials using Modal Analysis

Article *in* International Journal of Vehicle Structures and Systems · January 2021

DOI: 10.4273/ijvss.13.1.20

CITATIONS
2

READS
2,733

3 authors, including:

Rahul .B
MVJ College of Engineering
9 PUBLICATIONS **5** CITATIONS

SEE PROFILE

Dharani .J
GKM
10 PUBLICATIONS **3** CITATIONS

SEE PROFILE

All content following this page was uploaded by Rahul .B on 18 February 2021.

The user has requested enhancement of the downloaded file.

Rahul et al. 2021. *Int. J. Vehicle Structures & Systems*, 13(1), 102-111
ISSN: 0975-3060 (Print), 0975-3540 (Online)
doi: 10.4273/ijvss.13.1.20
© 2021. MechAero Foundation for Technical Research & Education Excellence

International Journal of
Vehicle Structures & Systems
Available online at www.maftree.org/eja

Optimal Method for Determination of Rayleigh Damping Coefficients for Different Materials using Modal Analysis

B. Rahul[a], J. Dharani[b] and R. Balaji[c]

[a]*Dept. of Aerospace Engg., MVJ College of Engg., Bengaluru, Karnataka, India*
Corresponding Author, Email: mailto2rahul@gmail.com
[b]*Dept. of Aeronautical Engg., GKM College of Engg. & Tech., Chennai, Tamil Nadu, India*
[c]*Dept. of Aeronautical Engg., Rajalakshmi Engg. College, Chennai, Tamil Nadu, India*

ABSTRACT:

Rayleigh damping coefficients are the essential parameters to determine the damping matrix of a system in dynamic analysis. For the systems with multiple degrees of freedom, it is difficult to arrive for suitable Rayleigh damping coefficients. This paper presents a simple and effective method for the determination of Rayleigh coefficients α and β for the system with multiple degrees of freedom. An unrealistic constant damping ratio for all modes is assumed to get rational value of α and β, which leads the determination of progressively varying damping ratio for all modes. By comparing the damping ratio arrived from assumed α and β with assumed unrealistic damping ratio, the suitable and most precise values are determined. This method is implemented for different materials with different boundary conditions by considering significant modes and the effect of above parameters on α and β values are also discussed.

KEYWORDS:

Rayleigh damping coefficients; Finite element analysis; Modal analysis; Natural frequency

CITATION:

B. Rahul, J. Dharani and R. Balaji. 2021. Optimal Method for Determination of Rayleigh Damping Coefficients for Different Materials using Modal Analysis, *Int. J. Vehicle Structures & Systems*, 13(1), 102-111. doi:10.4273/ijvss.13.1.20.

1. Introduction

Vibration is an undesirable phenomenon in dynamics, which occurs in many platforms such as wing of aircraft, building structure during earth quake, automobiles and machine parts. Damping is a method to reduce the vibration by dissipating the energy of it [4]. The characterization of damping is an active area of research in structural dynamics, present in the system depends on the energy dissipating mechanism of the structure. Generally, damping mechanism can be classified into three types for convenience called viscous damping, coulomb damping and hysteretic damping for single degree of freedom, continuous and multiple-degree of freedom systems. In viscous damping, damping force is assumed to be proportional and in-phase to the velocity of the structure which is a dependent parameter of the frequency and in coulomb damping, the damping force will be in opposite sign to the velocity, generated by the friction between structure and over which it is sliding. Hysteretic damping, damping force will be in-phase with velocity and proportional to displacement of structure.

Viscous and hysteresis damping are the most widely used mechanism for the dynamic analysis of the structures [3]. Primary source for dissipating the vibration energy is the structural joints rather than the material [1]. Hence, representing all these parameters in a unified manner is so difficult and requires linearization. Since, viscous damping is dependent parameter of frequency rather than displacement which can be used to model the dynamic structural behaviour linearly and the sources of non-linearity can be ignored for convenience. A well suitable method to avoid these kinds of problems is introduced by Rayleigh through the representation of dissipation function as a quadratic equation with symmetric matrix of coefficients called damping matrix [2]. This damping is represented as a linear combination of mass and stiffness matrices and this model led the modal analysis developed for un-damped system used for damped systems [5].

Pan et al [6] proposed an optimization method to determine Rayleigh coefficients for dynamic analysis of complex structures. Zhe et al [7] introduced an advance method to calculate Rayleigh coefficients through the 2 consideration of spectrum and frequency characteristics of the structure. Hongshi [8] analyzed various methods to determine Rayleigh damping coefficients and proposed the least square method as an appropriate one. Liu et al [3] studied the extension formulation in Rayleigh damping to determine the damping matrix and shown good correlation with the experimental results. Mohammed et al [9] carried out an analytical work to determine Rayleigh damping coefficients in order to obtain damping ratio and compared it with theoretical results. Ju et al [10] determined the Rayleigh damping parameters using least square method for FEA of wave propagation in soils.

Wang et al [11] determined Rayleigh damping parameters with minimized error from the peak response of soil in seismic response. Yusuf et al [12] analyzed a rubber plate with different boundary conditions to determine Rayleigh damping coefficients through linear interpolation method. Chowdhry et al [13] estimated the Rayleigh damping parameters rationally and checked them experimentally for various real-life systems. Kyriazoglou et al [14] determined the dynamic properties of laminated plates experimentally as well as analytically using hybrid formulation. In the present work, Rayleigh coefficients α and β for different materials with different boundary conditions are found using finite element modal analysis. From the results of modal analysis, natural frequencies and mass participation behaviour of dynamic structure are extracted and utilized for assuming unrealistic damping ratios. The assumed damping ratios are used to guide in the determination of damping ratio for the required modes with good precision to estimate the dynamic behaviour of the system.

2. Rayleigh damping

Rayleigh damping is also called as proportional damping or viscous damping is used to solve damping problems numerically and in simple form it is,

$$[C] = \alpha[M] + \beta[K] \tag{1}$$

Where, [C], [M] and [K] are the damping, mass and stiffness matrices respectively. α and β are the Rayleigh damping coefficients related to the mass and stiffness of the system respectively. Considering a dynamic system under an exciting force F (t), the equation of equilibrium is given by,

$$\{F(t)\} = [M][\ddot{x}(t)] + [C][(\dot{x})] + [K]\{x(t)\} \tag{2}$$

Where, $\{\ddot{x}(t)\}$, $\{\dot{x}(t)\}$ and $\{x(t)\}$ are the acceleration, velocity and displacement vectors respectively. The displacement vector $\{x(t)\}$ could be,

$$\{x(t)\} = [X]\{p(t)\} \tag{3}$$

Where, $[X]$ is the normalized Eigen vector and $\{p(t)\}$ is displacement vector in transformed co-ordinate. Hence, Eqn. (2) can be re-written as,

$$\{F(t)\} = [M][X]\{\ddot{p}(t)\} + [C][X]\{\dot{x}(t)\}[K][X]\{x(t)\} \tag{4}$$

By multiplying Eqn. (4) with $[X]^T$,

$$[X]^T\{F(t)\} = [X]^T[M][X]\{\ddot{p}(t)\} + [X]^T[C][X]\{\dot{x}(t)\} + [X]^T[K][X]\{x(t)\} \tag{5}$$

Eqn. (5) can be subsequently reduced to uncoupled form,

$$\{\ddot{p}_j(t)\} + 2\zeta_j\omega_j\{\dot{p}_j(t)\} + \omega_j^2\{p_j(t)\} = \{F_j(t)\} \tag{6}$$

Where, ζ is the damping ratio and ω is the natural frequency.

The transformation Eqn. (6) is valid only when damping matrix is proportional. Hence, the damping term in the Eqn. can be written as,

$$[X^T][C][X] = \begin{bmatrix} \alpha+\beta\omega_1^2 & 0 & \dots & 0 \\ 0 & \alpha+\beta\omega_2^2 & \dots & 0 \\ \vdots & \vdots & \vdots & \cdots \\ 0 & 0 & \cdots & \alpha+\beta\omega_n^2 \end{bmatrix} \tag{7}$$

When the system has multiple degrees of freedom, the damping matrix can be written in the following form from Eqn. (7),

$$2\zeta_j\omega_j = \alpha + \beta\omega_j^2 \tag{8}$$

On simplification, Eqn. (8) becomes,

$$\zeta_j = \frac{\alpha}{2\omega_j} + \frac{\beta\omega_j}{2} \tag{9}$$

In Eqn. (9), the damping ratio is a dependent parameter of natural frequency for constant values of α and β. The variation of damping ratio with respect to natural frequency for a typical system in graphical form is given in Fig. 1. When ω is small, the term $\frac{\alpha}{2\omega}$ dominates in the Eqn. (9) and makes the curve non-linear. When the natural frequency, ω, increases, the term $\frac{\beta\omega}{2}$ in the Eqn. (9) tends to dominate and $\frac{\alpha}{2\omega}$ approaches to zero. Hence, the structure is to have high fundamental frequencies to avoid non-linearity of the damping ratio variation.

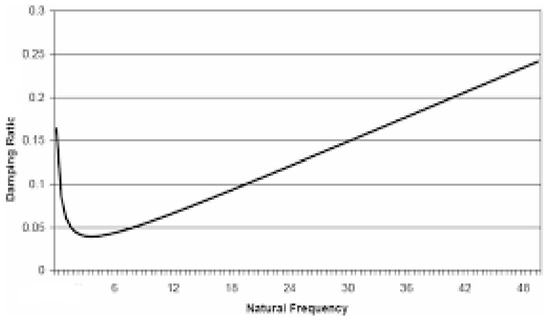

Fig. 1: Damping ratio vs. Natural frequency

3. Methodology

To demonstrate the methodology, a square plate of 500×500×5 mm^3 is taken for consideration having symmetry about both x and y axis. The finite element model of aluminium plate is shown in Fig. 2. The model has a total number of nodes as 17300 and elements as 2401. Aluminium, Titanium, GFRP and CFRP material properties as given in Table are considered. Four boundary conditions - One end is fixed, two ends are fixed, three ends are fixed and all four ends are fixed four end conditions are considered. The boundary conditions for one end is fixed case are applied such a way to arrest the displacement in x axis and rotation about y axis. The mass matrices are calculated with maximum wave font as 144 and Block Lanczos algorithm is used to extract the Eigen values, Eigen vectors for the first 25 modes using Ansys 15.0. For the considered cases as a plate with specified end conditions, first 9 modes are sufficient for description of dynamic properties in lateral and vertical directions due to 90% of the accumulated mass participation. But modal participation ratios are identified in different modes in different directions. This identified mode numbers will change depending upon the boundary conditions. Hence, all the first 9 modes are considered for better solution.

Table 1: Material properties

Material	ρ (kg/m^3)	E (GPa) Direction			ν Plane			G (GPa) Plane		
		X	Y	Z	XY	YZ	XZ	XY	YZ	XZ
Aluminium	2770	71			0.33			26.69		
Titanium	4620	96			0.36			35.29		
GFRP	1814	50	8	8	0.3	0.4	0.3	5	3.8	5
CFRP	1351	121	8.6	8.6	0.3	4	0.3	4.7	3.1	4.7

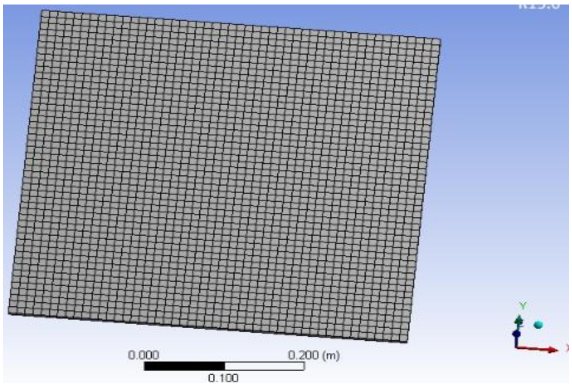

Fig. 2: Finite element model of aluminium plate

The mode shapes for the one end fixed case of aluminium plate are shown in Fig. 3. It is observed that the effective mass participation occurs at first 3 modes. The analysis has restricted to 3 times the effective mass participation modes called as consideration modes and beyond that the damping results has no physical consequence [12]. The damping ratio varies from 2% to 8% for effective mass participation modes. For determination of Rayleigh damping coefficients two reference modes i and j are taken such that (i, j) = (1, 1), (i, j) = (1, 2), (i, j) = (1, 3), etc., From natural frequencies of consideration modes and damping ratio of minimum as well as maximum effective mass participation modes, the damping ratio for remaining consideration modes has been calculated through linear interpolation technique as,

$$\zeta_j = \frac{(\zeta_m - \zeta_1)}{(\omega_m - \omega_1)}\left(\omega_j - \omega_1\right) + \zeta_1 \qquad (10)$$

Where, ζ_j, ζ_m and ζ_1 are the damping ratios of consideration mode, maximum effective mass participation mode and first mode respectively and ω_j, ω_m and ω_1 are their corresponding natural frequencies.

From the obtained values of natural frequency and damping ratio, the value of β for each consideration mode is calculated using,

$$\beta = (2\zeta_1\omega_1 - 2\zeta_n\omega_n)/(\omega_1^2 - \omega_n^2) \qquad (11)$$

Where, ζ_n and ω_n are the damping ratio and natural frequency of the required approximation mode respectively. From obtained β, the value of α is calculated using,

$$\alpha = 2\zeta_1\omega_1 - \beta\omega_1^2 \qquad (12)$$

The value of damping ratio of materials with different end conditions for varying approximation modes are listed from Table 2 to Table 17. The results show that there is a progressive increment in the damping ratio with respect to an increase in the natural frequency.

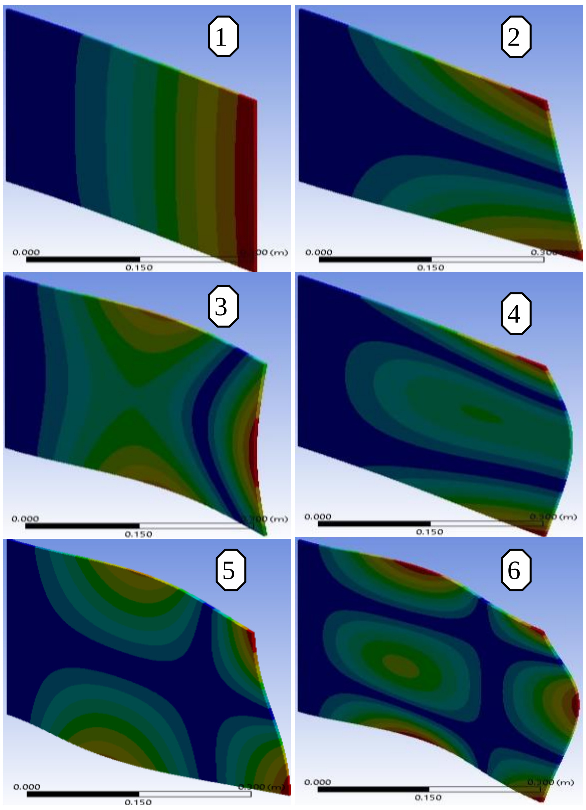

Fig. 3: The first 6 mode shapes of one end fixed aluminium plate

Table 2: α, β and damping ratio of one end fixed aluminium plate with different mode of approximation

Mode	Natural frequency in rad/s	Damping through linear interpolation	Damping up to 3rd mode approx.	Damping up to 4th mode approx.	Damping up to 5th mode approx.	Damping up to 6th mode approx.	Damping up to 7th mode approx.	Damping up to 8th mode approx.	Damping up to 9th mode approx.
1	107.43609	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
2	258.89215	0.0366068	0.0341095	0.0336623	0.0334701	0.0327979	0.0326776	0.0326469	0.0325661
3	654.64452	0.08	0.08	0.0786705	0.0780994	0.0761011	0.0757438	0.0756523	0.0754122
4	837.48506	0.100048	0.1017671	0.100048	0.0993096	0.0967256	0.0962635	0.0961453	0.0958348
5	946.05841	0.1119528	0.114739	0.11279	0.1119528	0.1090233	0.1084994	0.1083654	0.1080134
6	1654.2985	0.1896095	0.1996915	0.1962536	0.1947767	0.1896091	0.188685	0.1884486	0.1878275
7	1895.1956	0.2160232	0.2286453	0.2247028	0.2230092	0.2170831	0.2160233	0.2157522	0.21504
8	1967.7035	0.2239735	0.237363	0.2332686	0.2315098	0.2253555	0.2242549	0.2239734	0.2232338
9	2186.1696	0.2479278	0.2636351	0.2590836	0.2571284	0.2502869	0.2490635	0.2487505	0.2479283
	α		1.5172323	1.5654114	1.5861071	1.6585249	1.6714756	1.6747883	1.6834966
	β		0.0002409	0.0002367	0.0002349	0.0002286	0.0002275	0.0002272	0.0002265

Table 3: α, β and damping ratio of two ends fixed aluminium plate with different mode of approximation

Mode	Natural frequency in rad/s	Damping through linear interpolation	Damping up to 3rd mode approx.	Damping up to 4th mode approx.	Damping up to 5th mode approx.	Damping up to 6th mode approx.	Damping up to 7th mode approx.	Damping up to 8th mode approx.	Damping up to 9th mode approx.
1	212.41547	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
2	731.92764	0.0710636	0.0711182	0.0713353	0.0714125	0.0714245	0.0714802	0.0714865	0.0715381
3	822.84525	0.08	0.0799998	0.0802487	0.080337	0.0803508	0.0804146	0.0804218	0.080481

4	1462.0332	0.1428267	0.1423627	0.1428264	0.142991	0.1430168	0.1431356	0.1431491	0.1432593
5	1930.6956	0.1888922	0.188055	0.188673	0.1888923	0.1889266	0.1890849	0.1891031	0.1892499
6	2029.2158	0.1985759	0.1976588	0.198309	0.1985398	0.198576	0.1987425	0.1987616	0.1989161
7	2633.4692	0.2579688	0.2565555	0.2574032	0.2577041	0.2577512	0.2579683	0.2579932	0.2581946
8	2720.3656	0.2665099	0.2650247	0.2659007	0.2662117	0.2662603	0.2664847	0.2665104	0.2667186
9	3740.6284	0.366793	0.3644565	0.3656646	0.3660934	0.3661605	0.3664699	0.3665054	0.3667925
	α		-0.296657	-0.325895	-0.3362726	-0.3378969	-0.3453869	-0.3462442	-0.3532053
	β		0.0001949	0.0001955	0.0001958	0.0001958	0.000196	0.000196	0.0001961

Table 4: α, β and damping ratio of three ends fixed aluminium plate with different mode of approximation

Mode	Natural frequency in rad/s	Damping through linear interpolation	Damping up to 3rd mode approx.	Damping up to 4th mode approx.	Damping up to 5th mode approx.	Damping up to 6th mode approx.	Damping up to 7th mode approx.	Damping up to 8th mode approx.	Damping up to 9th mode approx.
1	740.7241	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
2	1236.153	0.0444004	0.046169	0.0468125	0.046966	0.0480002	0.0481209	0.0483408	0.0484354
3	1958.97	0.08	0.08	0.0813635	0.0816904	0.0838803	0.0841362	0.084602	0.0848025
4	2373.22	0.1004023	0.0986627	0.1004023	0.1008194	0.1036133	0.1039399	0.1045342	0.1047899
5	2492.161	0.1062603	0.1039727	0.1058179	0.1062602	0.1092237	0.1095701	0.1102004	0.1104717
6	3603.655	0.1610026	0.1530258	0.1558288	0.1565008	0.1610027	0.1615288	0.1624864	0.1628985
7	3785.553	0.1699613	0.1609921	0.1639488	0.1646577	0.1694063	0.1699613	0.1709714	0.1714061
8	4158.9	0.1883491	0.1773107	0.1805812	0.1813652	0.1866178	0.1872317	0.188349	0.1888298
9	4339.164	0.1972273	0.1851769	0.1885983	0.1894185	0.1949133	0.1955556	0.1967244	0.1972274
	α		-17.71727	-18.608484	-18.82213	-20.25345	-20.42074	-20.72520	-20.85621
	β		8.63E-05	8.79E-05	8.83E-05	9.09E-05	9.12E-05	9.18E-05	9.20E-05

Table 5: α, β and damping ratio of four ends fixed aluminium plate with different mode of approximation

Mode	Natural frequency in rad/s	Damping through linear interpolation	Damping up to 3rd mode approx.	Damping up to 4th mode approx.	Damping up to 5th mode approx.	Damping up to 6th mode approx.	Damping up to 7th mode approx.	Damping up to 8th mode approx.	Damping up to 9th mode approx.
1	1119.537	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.0200008
2	2281.4855	0.08	0.0799999	0.0846403	0.0866741	0.0867213	0.088759	0.088759	0.0906919
3	2281.4855	0.08	0.0799999	0.0846403	0.0866741	0.0867213	0.088759	0.088759	0.0906919
4	3361.25	0.1357562	0.1277504	0.1357562	0.139265	0.1393464	0.142862	0.142862	0.1461963
5	4085.5121	0.1731552	0.1585952	0.1687184	0.1731552	0.1732582	0.1777035	0.1777035	0.1819197
6	4105.1157	0.1741675	0.1594227	0.1696024	0.1740639	0.1741675	0.1786376	0.1786376	0.1828772
7	5119.2837	0.2265365	0.2018873	0.214946	0.2206693	0.2208022	0.2265365	0.2265365	0.2319751
8	5119.2837	0.2265365	0.2018873	0.214946	0.2206693	0.2208022	0.2265365	0.2265365	0.2319751
9	6526.9673	0.2992257	0.2601232	0.2770945	0.2845326	0.2847053	0.2921577	0.2921577	0.2992257
	α		-56.79144	-63.50696	-66.45023	-66.518543	-69.46745	-69.46745	-72.264237
	β		8.10E-05	8.64E-05	8.87E-05	8.88E-05	9.12E-05	9.12E-05	9.34E-05

Table 6: α, β and damping ratio of one end fixed titanium plate with different mode of approximation

Mode	Natural frequency in rad/s	Damping through linear interpolation	Damping up to 3rd mode approx.	Damping up to 4th mode approx.	Damping up to 5th mode approx.	Damping up to 6th mode approx.	Damping up to 7th mode approx.	Damping up to 8th mode approx.	Damping up to 9th mode approx.
1	97.6280508	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02000001
2	232.006422	0.03633635	0.03388055	0.03343761	0.03326234	0.03260451	0.03247907	0.03245444	0.03237254
3	591.17184	0.08	0.08000006	0.07866594	0.07813805	0.07615671	0.07577891	0.07570473	0.07545803
4	759.636462	0.1004802	0.10221343	0.10048019	0.09979436	0.09722026	0.09672942	0.09663305	0.09631255
5	851.496554	0.11164761	0.11436851	0.11241901	0.11164761	0.10875235	0.10820027	0.10809187	0.10773138
6	1490.05614	0.18927715	0.19919349	0.1957514	0.1943894	0.18927745	0.18830269	0.1881113	0.18747481
7	1723.66477	0.21767689	0.23028907	0.226303	0.22472575	0.21880591	0.2176771	0.21745546	0.21671838
8	1777.70012	0.22424596	0.23748402	0.23337219	0.23174519	0.2256386	0.22447417	0.22424554	0.2234852
9	1983.78842	0.24930006	0.26493123	0.26033999	0.25852328	0.25170469	0.2504045	0.25014921	0.24930023
	α		1.36266475	1.40688968	1.42438903	1.49006878	1.50259283	1.50505189	1.51323126
	β		0.00026675	0.00026211	0.00026027	0.00025338	0.00025207	0.00025181	0.00025095

Table 7: α, β and damping ratio of two ends fixed titanium plate with different mode of approximation

Mode	Natural frequency in rad/s	Damping through linear interpolation	Damping up to 3rd mode approx.	Damping up to 4th mode approx.	Damping up to 5th mode approx.	Damping up to 6th mode approx.	Damping up to 7th mode approx.	Damping up to 8th mode approx.	Damping up to 9th mode approx.
1	193.364865	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
2	664.823276	0.07034498	0.07038888	0.0705462	0.07060341	0.07061315	0.07065301	0.07065788	0.07069591
3	755.238236	0.08	0.08000007	0.0801825	0.08024884	0.08026014	0.08030636	0.08031201	0.08035611
4	1333.03947	0.14170087	0.14136325	0.14170059	0.14182326	0.14184414	0.14192961	0.14194005	0.14202162

5	1764.25411	0.18774839	0.18713429	0.18758487	0.18774872	0.18777661	0.18789078	0.18790473	0.18801367
6	1865.09915	0.19851719	0.19783721	0.19831416	0.19848759	0.19851711	0.19863797	0.19865273	0.19876804
7	2407.21192	0.25640705	0.25536852	0.25598677	0.25621159	0.25624985	0.25640651	0.25642564	0.25657512
8	2490.58972	0.2653106	0.26421646	0.2648564	0.2650891	0.26512871	0.26529086	0.26531066	0.26546539
9	3427.78885	0.36538999	0.36366612	0.36454939	0.36487057	0.36492524	0.36514905	0.36517638	0.36538994
	α		-0.1997067	-0.2190374	-0.2260667	-0.2272631	-0.2321612	-0.2327595	-0.2374344
	β		0.0002122	0.00021272	0.00021291	0.00021294	0.00021307	0.00021309	0.00021321

Table 8: α, β and damping ratio of three ends fixed titanium plate with different mode of approximation

Mode	Natural frequency in rad/s	Damping through linear interpolation	Damping up to 3rd mode approx.	Damping up to 4th mode approx.	Damping up to 5th mode approx.	Damping up to 6th mode approx.	Damping up to 7th mode approx.	Damping up to 8th mode approx.	Damping up to 9th mode approx.
1	672.8657	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02000006
2	1123.118	0.04436172	0.04612665	0.0467697	0.04691966	0.04794904	0.04807036	0.04829176	0.0483836
3	1781.784	0.08	0.08000001	0.0813645	0.08168261	0.08386674	0.08412416	0.08459392	0.08478872
4	2160.409	0.10048615	0.09874367	0.1004861	0.10089233	0.10368145	0.10401017	0.10461005	0.1048588
5	2266.092	0.10620432	0.10392845	0.1057740	0.10620428	0.10915846	0.10950664	0.11014202	0.11040549
6	3277.872	0.1609485	0.15300153	0.1558059	0.15645965	0.16094855	0.1614776	0.16244307	0.1628434
7	3445.193	0.1700017	0.16105511	0.1640149	0.16470491	0.16944262	0.17000099	0.17101998	0.17144249
8	3789.323	0.18862145	0.17758598	0.1808638	0.18162794	0.18687472	0.18749308	0.18862156	0.18908948
9	3949.544	0.1972905	0.18526991	0.1886952	0.18949368	0.19497646	0.19562264	0.19680187	0.19729083
	α		-16.026646	-16.83543	-17.02396	-18.318548	-18.471124	-18.749564	-18.864938
	β		9.48E-05	9.66E-05	9.70E-05	9.99E-05	0.00010025	0.00010086	0.00010112

Table 9: α, β and damping ratio of four ends fixed titanium plate with different mode of approximation

Mode	Natural frequency in rad/s	Damping through linear interpolation	Damping up to 3rd mode approx.	Damping up to 4th mode approx.	Damping up to 5th mode approx.	Damping up to 6th mode approx.	Damping up to 7th mode approx.	Damping up to 8th mode approx.	Damping up to 9th mode approx.
1	1021.7079	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.01999995
2	2082.05736	0.08	0.0800000	0.08464006	0.08667393	0.08672135	0.08875893	0.08875893	0.09069169
3	2082.05736	0.08	0.0800000	0.08464006	0.08667393	0.08672135	0.08875893	0.08875893	0.09069169
4	3067.38564	0.13575492	0.1277491	0.13575431	0.13926323	0.13934504	0.14286036	0.14286036	0.14619487
5	3728.18769	0.17314648	0.1585876	0.16870964	0.17314643	0.17324988	0.17769476	0.17769476	0.18191102
6	3746.15758	0.17416331	0.1594189	0.16959758	0.17405922	0.17416324	0.17863303	0.17863303	0.1828729
7	4671.4815	0.22652287	0.2018755	0.21493248	0.22065575	0.22078919	0.22652291	0.22652291	0.23196172
8	4671.4815	0.22652287	0.2018755	0.21493248	0.22065575	0.22078919	0.22652291	0.22652291	0.23196172
9	5956.14048	0.29921545	0.2601136	0.27708309	0.28452135	0.28469477	0.29214662	0.29214662	0.29921518
	α		-51.83313	-57.961688	-60.648028	-60.710660	-63.401906	-63.401906	-65.954829
	β		8.88E-05	9.47E-05	9.72E-05	9.73E-05	9.99E-05	9.99E-05	0.00010233

Table 10: α, β and damping ratio of one end fixed GFRP plate with different mode of approximation

Mode	Natural frequency in rad/s	Damping through linear interpolation	Damping up to 3rd mode approx.	Damping up to 4th mode approx.	Damping up to 5th mode approx.	Damping up to 6th mode approx.	Damping up to 7th mode approx.	Damping up to 8th mode approx.	Damping up to 9th mode approx.
1	40.8500948	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02000001
2	122.974399	0.04294513	0.0403797	0.03922931	0.03850986	0.03844378	0.03821545	0.03814429	0.03786148
3	255.599762	0.08	0.07999997	0.07738099	0.0757431	0.07559266	0.07507284	0.07491082	0.07426698
4	424.554473	0.12720511	0.13162797	0.12720511	0.12443909	0.12418503	0.12330719	0.12303357	0.12194626
5	682.290516	0.1992153	0.21083436	0.2036858	0.19921514	0.19880451	0.19738568	0.19694344	0.19518604
6	721.057737	0.21004667	0.22276479	0.21520721	0.21048075	0.21004663	0.20854661	0.20807907	0.20622112
7	894.222178	0.25842796	0.27608056	0.26669743	0.2608293	0.26029032	0.25842797	0.25784749	0.25554075
8	965.536271	0.27835277	0.29804599	0.28791155	0.28157355	0.28099141	0.27897994	0.27835299	0.27586154
9	1402.97126	0.40056995	0.43283578	0.41809605	0.40887793	0.40803125	0.40510574	0.40419388	0.40057027
	α		0.60486448	0.63995787	0.66190501	0.66392084	0.67088612	0.67305714	0.68168524
	β		0.00061672	0.00059569	0.00058254	0.00058133	0.00057716	0.00057586	0.00057069

Table 11: α, β and damping ratio of two ends fixed GFRP plate with different mode of approximation

Mode	Natural frequency in rad/s	Damping through linear interpolation	Damping up to 3rd mode approx.	Damping up to 4th mode approx.	Damping up to 5th mode approx.	Damping up to 6th mode approx.	Damping up to 7th mode approx.	Damping up to 8th mode approx.	Damping up to 9th mode approx.
1	133.1532	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.01999998
2	339.4865	0.04323295	0.04198955	0.04176161	0.04160419	0.04118967	0.04109057	0.04093056	0.04087914
3	666.0171	0.08	0.08000004	0.07949268	0.07914229	0.07821964	0.07799904	0.0776429	0.07752844
4	781.8789	0.09304594	0.09364827	0.09304585	0.0926298	0.09153427	0.09127235	0.09084947	0.09071357
5	883.7293	0.1045142	0.10567276	0.10498744	0.10451416	0.10326788	0.10296992	0.10248886	0.10233426

6	1305.393	0.15199321	0.15559058	0.15456553	0.15385762	0.15199353	0.15154786	0.15082832	0.15059708
7	1463.667	0.16981464	0.17435763	0.17320582	0.17241037	0.17031578	0.16981499	0.16900648	0.16874665
8	1808.299	0.20861994	0.21524984	0.21382274	0.21283716	0.21024194	0.20962146	0.20861969	0.20829776
9	1953.064	0.22492029	0.23243443	0.23089187	0.22982657	0.2270214	0.22635072	0.22526792	0.22491995
	α		1.11124755	1.13938468	1.1588165	1.20998459	1.22221813	1.24196908	1.24831009
	β		0.00023773	0.00023614	0.00023505	0.00023216	0.00023147	0.00023036	0.00023

Table 12: α, β and damping ratio of three ends fixed GFRP plate with different mode of approximation

Mode	Natural frequency in rad/s	Damping through linear interpolation	Damping up to 3rd mode approx.	Damping up to 4th mode approx.	Damping up to 5th mode approx.	Damping up to 6th mode approx.	Damping up to 7th mode approx.	Damping up to 8th mode approx.	Damping up to 9th mode approx.
1	661.7445	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02000004
2	788.8532	0.0361517	0.0381360	0.04025783	0.0403915	0.0406969	0.04130099	0.04191011	0.04209819
3	1133.925	0.08	0.0799998	0.08678744	0.0872151	0.08819207	0.09012461	0.09207322	0.0926748
4	1746.41	0.15782834	0.1442513	0.15782822	0.1586837	0.16063782	0.16450337	0.16840106	0.16960434
5	1799.314	0.1645509	0.1495365	0.16366064	0.1645506	0.16658347	0.1706048	0.17465958	0.17591135
6	1929.313	0.18106986	0.1624187	0.17787169	0.1788454	0.18106954	0.18546923	0.18990552	0.19127507
7	2231.597	0.21948104	0.1919215	0.21039757	0.2115618	0.21422101	0.21948141	0.22478558	0.22642305
8	2617.447	0.26851098	0.2289493	0.25119058	0.2525920	0.25579317	0.26212557	0.26851065	0.2704818
9	2758.19	0.28639521	0.2423325	0.26592881	0.2674156	0.27081183	0.27753004	0.28430414	0.28639539
	α		-53.561446	-61.511626	-62.012590	-63.156838	-65.420373	-67.702738	-68.407272
	β		0.0001827	0.00020091	0.0002020	0.00020467	0.00020984	0.00021505	0.00021666

Table 13: α, β and damping ratio of four ends fixed GFRP plate with different mode of approximation

Mode	Natural frequency in rad/s	Damping through linear interpolation	Damping up to 3rd mode approx.	Damping up to 4th mode approx.	Damping up to 5th mode approx.	Damping up to 6th mode approx.	Damping up to 7th mode approx.	Damping up to 8th mode approx.	Damping up to 9th mode approx.
1	742.4834	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.01999987
2	1080.958	0.0411767	0.0432798	0.04366777	0.04420876	0.044917	0.04500383	0.04584001	0.0459524
3	1701.485	0.08	0.07999992	0.08093592	0.08224109	0.08395	0.08415924	0.08617656	0.08644787
4	1865.225	0.09024438	0.08917747	0.09024406	0.09173132	0.093678	0.09391708	0.09621586	0.09652504
5	2133.894	0.10705366	0.10400169	0.10527613	0.10705321	0.10938	0.10966491	0.11241165	0.11278109
6	2582.324	0.13510974	0.12831673	0.12992636	0.13217083	0.135109	0.13546945	0.13893861	0.13940524
7	2647.041	0.13915875	0.1317947	0.13345185	0.13576259	0.138788	0.13915859	0.14273018	0.14321059
8	3429.862	0.18813602	0.1734777	0.17569908	0.17879658	0.182852	0.18334885	0.18813649	0.18878048
9	3563.883	0.196521	0.18056197	0.18287851	0.18610873	0.190338	0.19085604	0.1958488	0.19652039
	α		-27.348384	-28.097575	-29.142254	-30.509984	-30.677573	-32.292277	-32.509670
	β		0.00010348	0.00010484	0.00010674	0.00010922	0.00010952	0.00011245	0.00011284

Table 14: α, β and damping ratio of one end fixed CFRP plate with different mode of approximation

Mode	Natural frequency in rad/s	Damping through linear interpolation	Damping up to 3rd mode approx.	Damping up to 4th mode approx.	Damping up to 5th mode approx.	Damping up to 6th mode approx.	Damping up to 7th mode approx.	Damping up to 8th mode approx.	Damping up to 9th mode approx.
1	48.9202112	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
2	140.278277	0.04129372	0.03867639	0.03765429	0.03686367	0.03665765	0.03654773	0.03641515	0.03630728
3	306.342724	0.08	0.08000007	0.07752376	0.07560824	0.0751091	0.07484281	0.07452159	0.07426023
4	491.60857	0.12318174	0.12721929	0.12318178	0.12005861	0.11924478	0.11881061	0.11828687	0.11786073
5	857.088584	0.20836776	0.22093576	0.21384938	0.20836778	0.2069394	0.20617737	0.20525815	0.20451021
6	1050.86186	0.25353241	0.27071192	0.26201391	0.25528567	0.25353243	0.2525971	0.25146882	0.25055078
7	1192.48473	0.28654184	0.30710781	0.29723279	0.28959409	0.28760361	0.28654171	0.28526075	0.28421849
8	1420.81549	0.33976114	0.3658041	0.3540324	0.34492655	0.34255376	0.3412879	0.33976091	0.33851846
9	1679.55685	0.40006854	0.432334	0.4184139	0.40764617	0.40484033	0.40334344	0.40153777	0.40006856
	α		-16.026646	-16.835435	0.72536533	0.7650683	0.79578007	0.80378289	0.80805233
	β		9.48E-05	9.66E-05	0.00051456	0.00049797	0.00048514	0.0004818	0.00048001

Table 15: α, β and damping ratio of two ends fixed CFRP plate with different mode of approximation

Mode	Natural frequency in rad/s	Damping through linear interpolation	Damping up to 3rd mode approx.	Damping up to 4th mode approx.	Damping up to 5th mode approx.	Damping up to 6th mode approx.	Damping up to 7th mode approx.	Damping up to 8th mode approx.	Damping up to 9th mode approx.
1	208.15547	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.01999997
2	425.698011	0.03775068	0.03705786	0.03692116	0.03684067	0.03671224	0.03670382	0.03657652	0.03653034
3	943.482309	0.08	0.0799999	0.07962113	0.07939809	0.0790422	0.07901887	0.07866613	0.07853822
4	1164.83874	0.09806188	0.09853775	0.09806189	0.09778167	0.09733456	0.09730524	0.09686208	0.09670138
5	1340.20229	0.11237091	0.11324814	0.11269622	0.11237122	0.11185264	0.11181864	0.11130465	0.11111827
6	1735.9798	0.14466492	0.14649088	0.14576883	0.14534364	0.14466522	0.14462073	0.1439483	0.14370447

7	1768.58951	0.14732575	0.14923162	0.14849562	0.14806221	0.14737068	0.14732533	0.14663991	0.14639137
8	2454.83843	0.20332117	0.20694246	0.20591396	0.20530832	0.20434198	0.20427861	0.2033208	0.20297349
9	2838.23807	0.23460519	0.23920197	0.23801068	0.23730917	0.23618986	0.23611646	0.23500703	0.23460476
	α		1.02840923	1.06497866	1.08651302	1.12087268	1.12312577	1.15718213	1.16951762
	β		0.00016843	0.00016759	0.00016709	0.0001663	0.00016624	0.00016546	0.00016517

Table 16: α, β and damping ratio of three ends fixed CFRP plate with different mode of approximation

Mode	Natural frequency in rad/s	Damping through linear interpolation	Damping up to 3rd mode approx.	Damping up to 4th mode approx.	Damping up to 5th mode approx.	Damping up to 6th mode approx.	Damping up to 7th mode approx.	Damping up to 8th mode approx.	Damping up to 9th mode approx.
1	1170.6821	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.01999987
2	1271.21298	0.03428359	0.03563096	0.03758148	0.03937886	0.03944558	0.03951848	0.03984774	0.04039895
3	1592.97463	0.08	0.08000017	0.08740016	0.09421918	0.0944723	0.09474887	0.09599802	0.0980896
4	2244.47756	0.17256658	0.15606334	0.17256623	0.18777347	0.18833797	0.18895476	0.19174051	0.19640514
5	3192.1696	0.30721619	0.25359483	0.28150088	0.307216	0.30817056	0.30921354	0.31392418	0.32181203
6	3237.59699	0.31367059	0.25807793	0.28650367	0.31269768	0.31367001	0.31473241	0.31953077	0.32756552
7	3288.30225	0.32087487	0.26306773	0.29207155	0.31879825	0.31979036	0.32087436	0.32577031	0.33396846
8	3532.65512	0.35559292	0.2869247	0.31868779	0.34795712	0.34904362	0.35023074	0.35559246	0.36457055
9	4007.85203	0.42310966	0.33259453	0.36962195	0.4037423	0.40500887	0.40639275	0.4126431	0.42310921
	α		-197.48565	-225.17105	-250.68285	-251.62986	-252.66459	-257.33798	-265.16382
	β		0.00017827	0.00019847	0.00021708	0.00021777	0.00021853	0.00022194	0.00022765

Table 17: α, β and damping ratio of four ends fixed CFRP plate with different mode of approximation

Mode	Natural frequency in rad/s	Damping through linear interpolation	Damping up to 3rd mode approx.	Damping up to 4th mode approx.	Damping up to 5th mode approx.	Damping up to 6th mode approx.	Damping up to 7th mode approx.	Damping up to 8th mode approx.	Damping up to 9th mode approx.
1	1233.95372	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.01999997
2	1541.64105	0.03928459	0.04145503	0.04353805	0.04362624	0.04394865	0.0445461	0.04517707	0.04534739
3	2191.25903	0.08	0.08	0.08562672	0.08586495	0.08673585	0.0883497	0.09005409	0.09051421
4	3183.49881	0.14218955	0.13201749	0.14218961	0.14262028	0.14419471	0.14711227	0.1501935	0.15102534
5	3238.35097	0.14562746	0.13477799	0.14518683	0.14562752	0.14723859	0.15022405	0.15337698	0.15422818
6	3451.09945	0.15896167	0.14541239	0.1567302	0.15720937	0.15896113	0.1622073	0.16563557	0.1665611
7	3903.6769	0.18732738	0.16772242	0.18093435	0.18149372	0.18353865	0.18732809	0.19133011	0.19241053
8	4487.76132	0.22393542	0.19606182	0.21166092	0.21232135	0.21473576	0.21920989	0.223935	0.22521065
9	4668.84256	0.23528485	0.20477109	0.22110057	0.22179193	0.22431939	0.229003	0.23394936	0.23528473
	α		-90.528779	-101.97965	-102.46446	-104.23681	-107.52115	-110.98973	-111.92623
	β		9.19E-05	9.94E-05	9.97E-05	0.00010087	0.00010303	0.00010531	0.00010592

4. Results and discussion

The damping ratio is calculated for all consideration modes from the obtained values of α and β by varying the approximation mode. From Figs. 4 to 19, the damping ratio variation depends on the approximation mode. The values of Rayleigh damping coefficients will vary based on the selection of (i, j) and will have relative errors with linear damping. The relative errors are function of many variables and one should select the appropriate modes for the determination of damping values to reduce the damping analysis errors. The base shear errors should be made always lesser for the pair of damping coefficient because of the inconsistent selection of modes will lead more error solutions. So, the selection of approximation modes should be based on material behaviour for given load and past time histories. The damping ratios thus obtained were compared with damping ratio from linear interpolation and then the corresponding errors are estimated. The errors are plotted in the graphs with datum line as the damping ratio through linear interpolation method in Figs. 4 to 19.

The lower approximation modes perform well with datum line in significant mode range and deviate significantly in higher consideration modes. One could take approximation modes which results less than 3% errors with linear damping for appropriate modelling of the damping. Fig. 4 shows the approximation modes 4, 5 it will lead better models and the modes such as 1, 2 & higher modes than 6 will yield erroneous models. Also, the best approximation mode for this case j = 5 and one could see that few reference modes will yield better results but some modes not satisfactory. For example, the 3rd mode approximation gives better value in mode 3 than the 5th mode approximation but not for all the cases and the error of 3rd mode approximation is around 6% in mode 2, which is a significant participation mode in vibration of this plate.

Similarly, from Fig. 5, one could notice that 9th mode approximation gives very good results in mode 8 having lesser than 1% and yields 7% error for the mode 2. So, the selection of the approximation mode should give satisfactory results with minimum allowable error for all the primary modes which are possible and thus requires a detailed modelling to understand it. The 5th, 6th and 7th mode approximations match well in entire range of consideration mode. There is a slight deviation in results of higher consideration modes but in reality, 90% mass participation takes place in first six consideration modes itself [13]. Hence, the higher modes are not of any consequence. The 6th mode approximation match significantly among all with datum line and it is taken as the design value of α and β for respective cases [12].

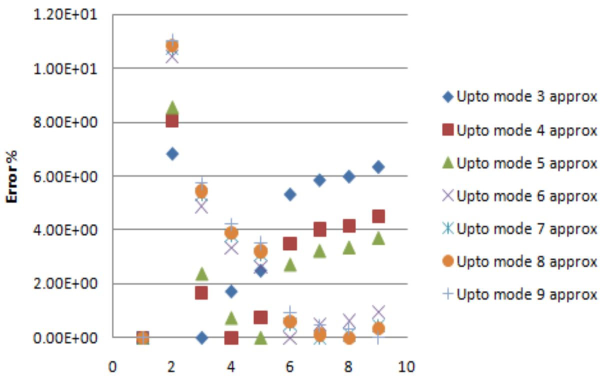

Fig. 4: Error in damping ratio vs. Approximation mode, linear damping for one end fixed aluminium plate

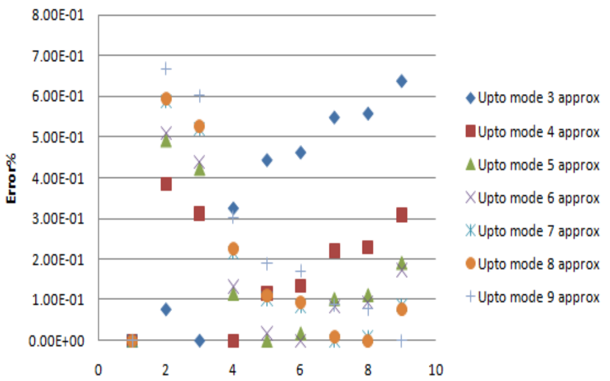

Fig. 5: Error in damping ratio vs. Approximation mode, linear damping for two ends fixed aluminium plate

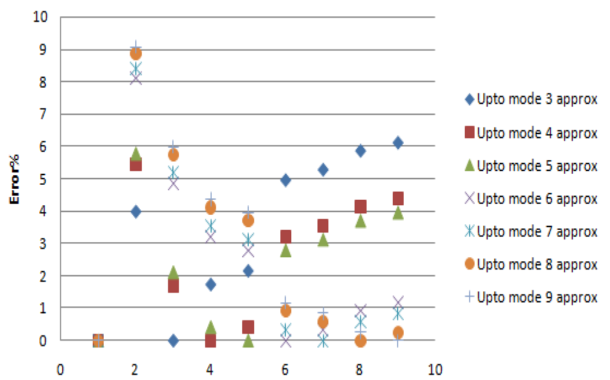

Fig. 6: Error in damping ratio vs. Approximation mode, linear damping for three ends fixed aluminium plate

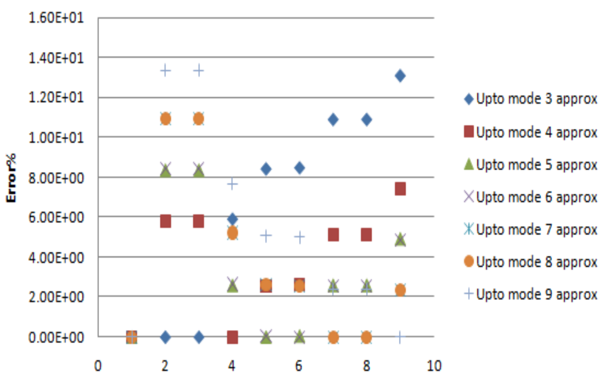

Fig. 7: Error in damping ratio vs. Approximation mode, linear damping for four ends fixed aluminium plate

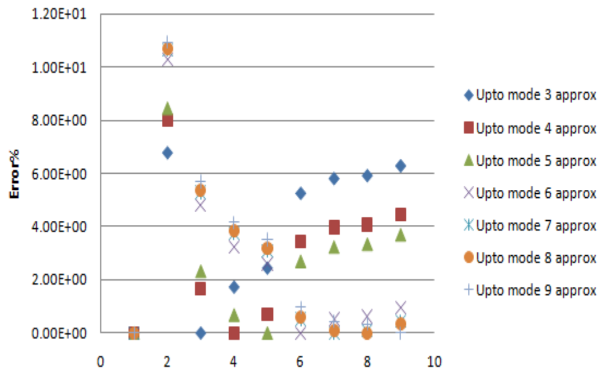

Fig. 8: Error in damping ratio vs. Approximation mode, linear damping for one end fixed titanium plate

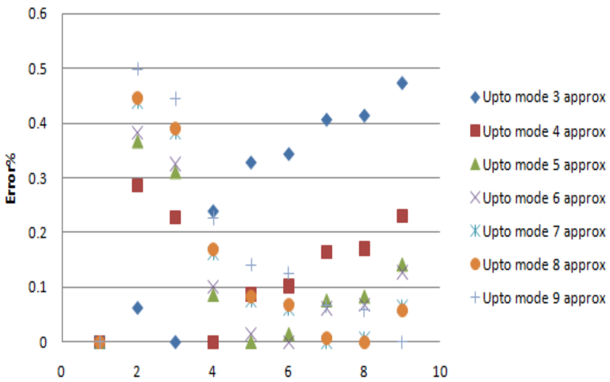

Fig. 9: Error in damping ratio vs. Approximation mode, linear damping for two ends fixed titanium plate

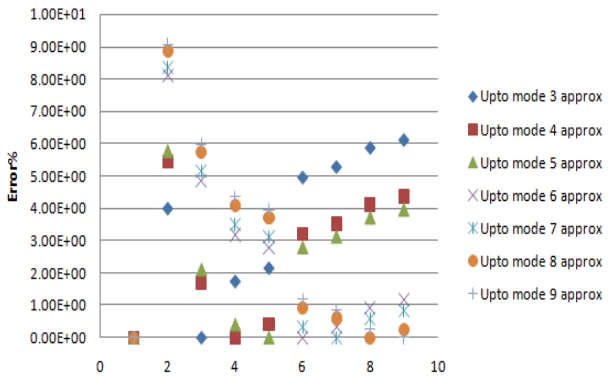

Fig. 10: Error in damping ratio vs. Approximation mode, linear damping for three ends fixed titanium plate

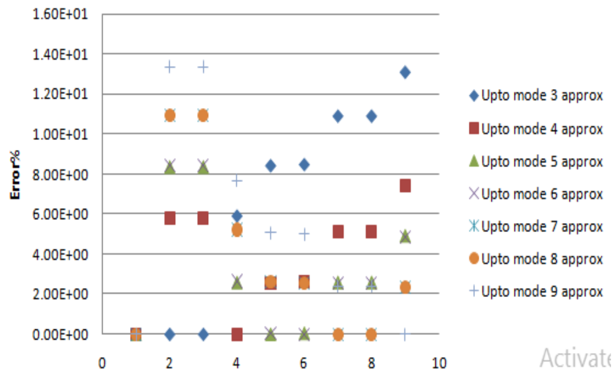

Fig. 11: Error in damping ratio vs. Approximation mode, linear damping for four ends fixed titanium plate

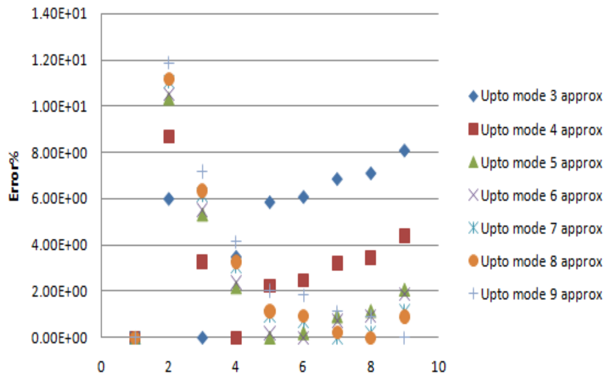

Fig. 12: Error in damping ratio vs. Approximation mode, linear damping for one end fixed GFRP plate

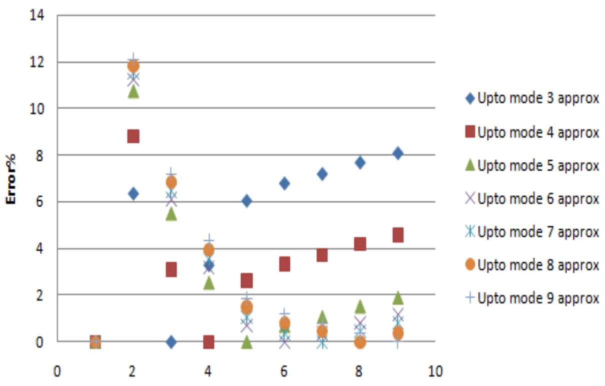

Fig. 16: Error in damping ratio vs. Approximation mode, linear damping for one end fixed CFRP plate

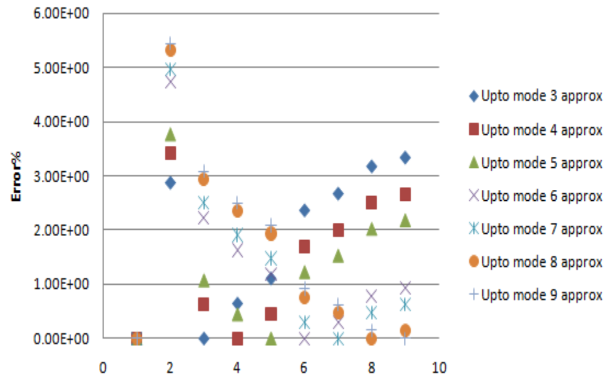

Fig. 13: Error in damping ratio vs. Approximation mode, linear damping for two ends fixed GFRP plate

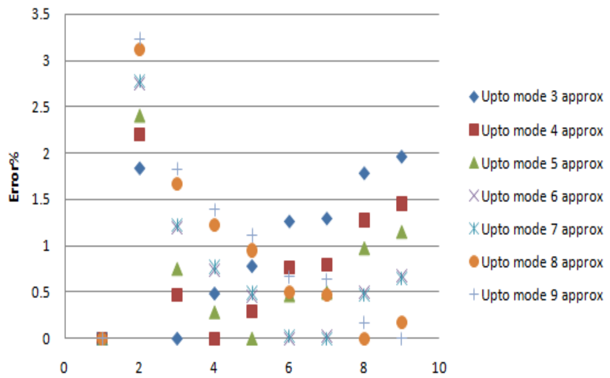

Fig. 17: Error in damping ratio vs. Approximation mode, linear damping for two ends fixed CFRP plate

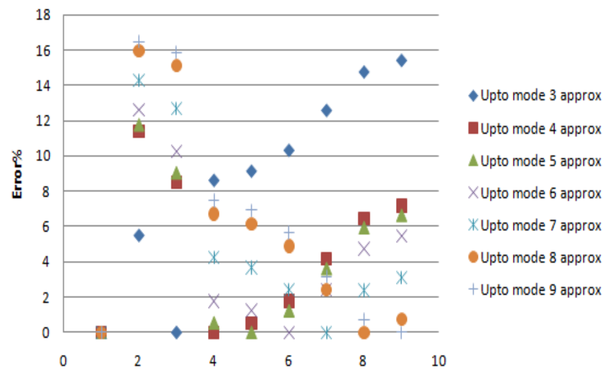

Fig. 14: Error in damping ratio vs. Approximation mode, linear damping for three ends fixed GFRP plate

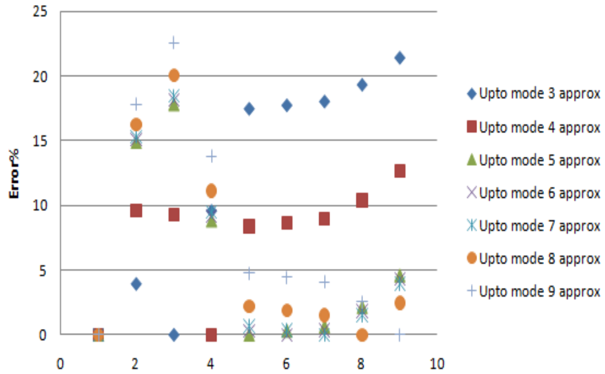

Fig. 18: Error in damping ratio vs. Approximation mode, linear damping for three ends fixed CFRP plate

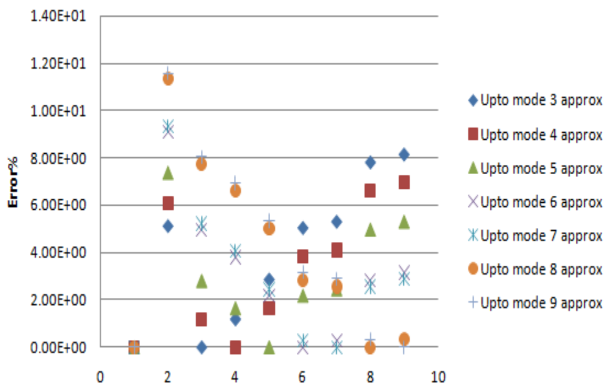

Fig. 15: Error in damping ratio vs. Approximation mode, linear damping for four ends fixed GFRP plate

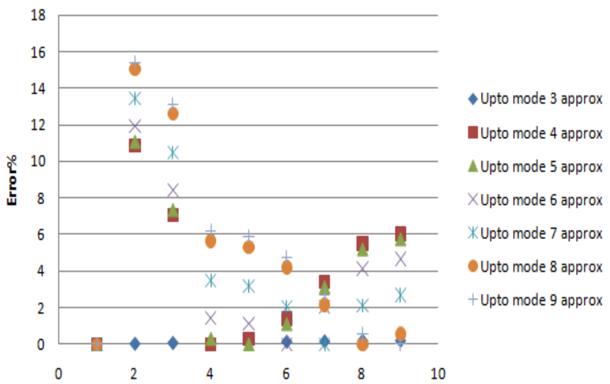

Fig. 19: Error in damping ratio vs. Approximation mode, linear damping for four ends fixed CFRP plate

The design values α and β are listed in Table 18 and from that one could observe that α tends to increase and β tend to decrease when the natural frequency of the mode increases. But it is not obvious in all cases (β of four ends fixed aluminium plate, α of four ends fixed GFRP plate, α and β of four ends fixed CFRP plate) [14]. From the analytical results, it is evident that the variation found to be non-linear and a dependent factor of first mode natural frequency, first mode ratio, approximation mode natural frequency as well as approximation mode damping ratio.

Table 18: Design values of α and β

Material	No. of fixed ends	α	β
Aluminium	1	1.65852488	0.000228626
	2	-0.337896903	0.000195799
	3	-20.2534577	9.09E-05
	4	-66.51854345	8.88E-05
Titanium	1	1.490068783	0.000253383
	2	-0.227263197	0.000212941
	3	-18.31854855	9.99E-05
	4	-60.71066092	9.73E-05
GFRP	1	0.663920839	0.00058133
	2	1.209984592	0.00023216
	3	-63.15683863	0.000204671
	4	-30.50998406	0.000109217
CFRP	1	0.803782887	0.0004818
	2	1.120872678	0.000166295
	3	-251.6298664	0.000217773
	4	-104.2368185	0.000100874

5. Conclusion

Rayleigh damping coefficients α and β were determined for different materials with different boundary conditions using finite element modal analysis by assuming that the damping occurs only through material properties but not through other external factors. Damping ratio was extracted from the mass participation results of modal analysis. The obtained damping ratio from α and β varies progressively with the mode and gives a realistic picture about damping than the conventionally assumed values.

REFERENCES:

[1] S.W.E. Earls. 1966. Theoretical estimation of frictional energy dissipation in a simple lap joint, *J. Mech. Engg. Sci.*, 8(2), 207-214. https://doi.org/10.1243/JMES_JOUR_1966_008_025_02.

[2] L. Rayleigh. 1877. *Theory of Sound*, Dover Pub., New York.

[3] M. Liu and D.G. Corman. 1995. Formulation of Rayleigh damping and its extensions, *Comput. Struct*., 57(2), 277-285. https://doi.org/10.1016/0045-7949(94)00611-6.

[4] J.P. Talbot and J. Woodhouse. 1997. The vibration damping of laminated plates, *Compos. A Appl. Sci. Manuf.*, 28(12), 1007-1012. https://doi.org/10.1016/S1359-835X(97)00056-0.

[5] J. Wang. 2015. Rayleigh coefficients for series infrastructure systems with multiple damping properties, *J. Vibration and Control*, 21(6), 1234-1248. https://doi.org/10.1177/1077546313496832.

[6] G.D. Pan, G.D. Chen and L.L. Gao. 2017. A constrained optimal Rayleigh damping coefficients for structures with closely spaced natural frequencies in seismic analysis, *Advances in Structural Engg.*, 20(1), 81-95. https://doi.org/10.1177/1369433216646007.

[7] L. Zhe, W. Gongxian and H. Yong. 2015. Application of improved calculation method of Rayleigh damping coefficients to seismic response analysis on quay crane structure, *J. South China University of Tech. Natural Sci. Ed.*, 43(6), 103-109.

[8] L. Hongshi. 2001. Relative errors and determination of Rayleigh damping scale co-efficient, *J. Hunan Institute of Engg.*, 11(3-4), 36-38.

[9] D.R.A. Mohammad, N.U. Khan and V. Ramamurti. 1995. On the role of Rayleigh damping, *J. Sound Vib.*, 185(2), 207-218. https://doi.org/10.1006/jsvi.1995.0376.

[10] S.H. Ju and S.H. Ni. 2007. Determining Rayleigh damping parameters of soils for finite element analysis, *Int. J. Numerical and Analytical Methods in Geomechanics*, 31(10), 1239-1255. https://doi.org/10.1002/nag.598.

[11] H.F. Wang, M.L. Lou and R.L. Zhang. 2017. Selection of Rayleigh damping coefficients for seismic response analysis of soil layers, *Int. J. Environmental, Chemical, Ecological, Geological and Geophysical Engg.*, 11(2), 134-139.

[12] A.I. Yusuf and N.M. Amin. 2014. Determination of Rayleigh damping co-efficient for natural damping rubber plate using finite element modal analysis, *Proc. Int. Civil and Infrastructure Engg. Conf.*, 713-725. https://doi.org/10.1007/978-981-287-290-6_62.

[13] I. Chowdhury and S. P. Dasgupta. 2003. Computation of Rayleigh damping coefficients for large systems, *Electron. J. Geotech. Engg*., 8, 1-11.

[14] C. Kyriazoglou and F. Guild. 2006. Finite element prediction of damping of composite GFRP and CFRP laminates - A hybrid formulation - Vibration damping experiments and Rayleigh damping, *Compos. Sci. Tech.*, 66(3-4), 487-498. https://doi.org/10.1016/j.compscitech.2005.06.011.

View publication stats