Musical Acoustics

First Lab Homework

Report

Glass Harp

Students

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Introduction

In this assignment it is requested to perform the modeling and the eigenfrequencies study of a Wine Glass. The report will focus on two exercise: the first is the study based on the 3D the model while the second is based on the study of the model considered as axysimmetric.

Ex 1) 3D wine glass modeling

a) building 3D model of the Wineglass

The building of the glass has been achieved thanks to its axial symmetry. As a matter of fact the principal strategy to build it has been the rotation of its radial section around the vertical axis. The procedure adopted in order to reach the goal has been:

- find a reference glass (figure 1)
- take the measures
- in COMSOL create a 3D component
- create a 2D work plane
- inside the 2D work plane, create a Node Group with the measures taken from the model (figure 2a)
- create another node group and interpolate the measures with cubic bezier curves (figure 2b)
- convert the shape into a solid material
- into the 3D component revolve the solid shape (figure 2c)
- build all

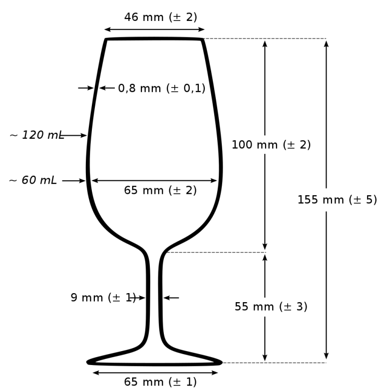
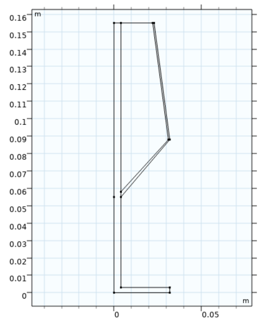
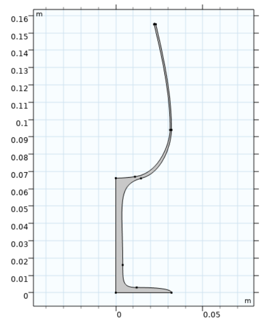


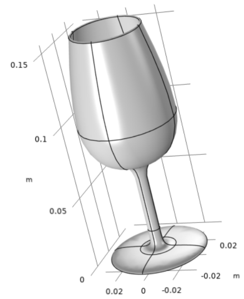
Figure 1: Reference Glass - ISO wine tasting glass



(a) Measures points



(b) Cross section



(c) 3D Model

Figure 2: Glass building process

b) Eigenfrequencies simulation

Once the shape has been set up, for the eigenfrequencies simulations it is necessary to specify the *material*, the *physics* and the *mesh* to adopt. The material used is a custom blank material given by the problem formulation, with the following specifications:

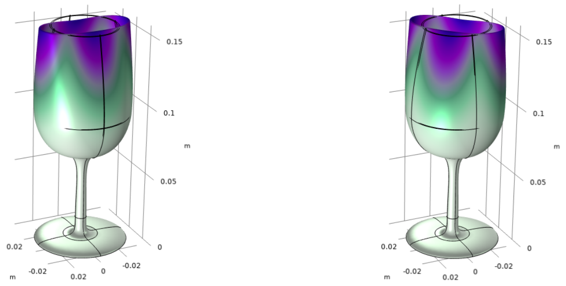
- Young Modulus: $E = 73.1\ GPa$
- Poisson ratio: $\nu = 0.17$
- Density: $\rho = 2203\ kg/m^3$

The physics used for the eigenfrequencies simulation is the *solid mechanics*, which inherits properties from the structural mechanics physics.

Finally, after a few tests on mesh construction it has been realized that a *user defined mesh* is not the best choice for the problem. Since the irregular shape of the glass, visible in figure 2, imposing shape and sizes of the mesh to the system does not lead to a better result than the one obtained exploiting the physics-controlled sequence, which is based on heuristics and knowledge built-in by application experts. A *user-defined mesh* can be useful in same cases to simplify the meshing for complex objects exploiting the simple geometric shape of some components of them.
For the study of eigenfrequencies another important step needs to be taken into account, and it is the definition of the boundary conditions. In the physics section has been defined a fixed constrain in the boundaries surfaces that represent the bottom part of the glass base.
Once the whole problem is set up the eigenfrequencies study is computed. The simulation has been done setting the computation of 20 eigenfrequencies around 1 kHz. The results present the modes visible in table 1. We can appreciate that some eigenmodes have the same value, and this is because they're the axial specular version of each other, an example is visible in figure 3.

EigenModes [Hz]							
1	153.29	**6**	1473.7	**11**	6025.6	**16**	9635.6
2	153.29	**7**	1473.8	**12**	6025.9	**17**	10242
3	413.29	**8**	3284.0	**13**	8637.6	**18**	10242
4	1178.9	**9**	3284.0	**14**	8637.6	**19**	11056
5	1178.9	**10**	4331.0	**15**	9635.5	**20**	11057

Table 1: First 20 modes of the glass

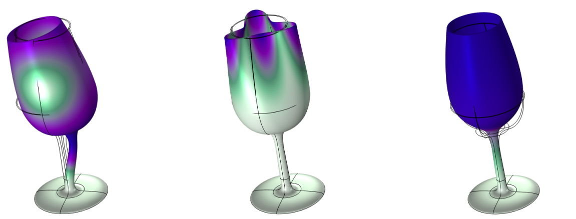


(a) 4th mode - 1178.9 Hz (b) 5th mode - 1178.9 Hz

Figure 3: Specular modes

The first six modes (except for the specular ones) are represented below in figure 4.

(a) First mode 153.3 Hz (b) Third mode 413.3 Hz (c) Fourth mode 1178.9 Hz



(d) Sixth mode 1473.7 Hz (e) Eighth mode 3284 Hz (f) Tenth mode 4331 Hz

Figure 4: First 6 modes of eigenfrequencies study

We can notice the variety of the modeshapes, in which are visible both the displacement of cup and the stem of the glass, characteristic of the tridimensional analysis. As a matter of fact this behaviour is not achievable through the axisymmetric study of eigenfrequencies. The first mode shows the oscillations of the entire glass back and forth. The third mode instead shows a pulsating behavior of the cup. The fourth is the compression of the cup with two nodes, while the eighth has three nodes. The sixth mode shows the glass vibrating off-axis, the tenth instead behaves like a piston, pushing inward and outward the cup of the glass.

Moving to a broader perspective, we can see that the behaviour of the glass is given by the interaction between the stem and the cup. Indeed, we note that in some ways the glass tends to behave like a beam (due to the stem geometry), while in others it vaguely resembles the behaviour of a membrane (due to the cup geometry). The first natural frequency in this case does not exhibit axisymmetrical behaviour, nor does it resemble that of a membrane. Instead, the modeshape closely remember the fundamental mode of a beam characterised by free-fixed boundary conditions. As a matter of fact, if we consider the glass as an equivalent beam with the same mechanical properties and the total mass distributed along the length we obtain approximately the same result. The procedure to verify these properties will follow.
The goal is to build an equivalent beam model of the glass. In order to do this it has been approximated, like it is composed by three cylinders, representing respectively the cup, the stem and the base, as depicted in figure 5.

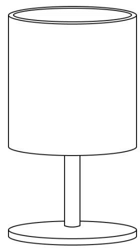


Figure 5: Glass cylindrical approximation

With the measures of:

- Base Radius $r_b = 32\ mm$
- Stem Radius $r_s = 4.3\ mm$
- Cup external Radius $r_{c_{ext}} = 32\ mm$
- Cup internal Radius $r_{c_{int}} = 31\ mm$
- Base length $L_b = 3\ mm$
- Stem length $L_s = 63\ mm$
- Cup length $L_c = 89\ mm$

Which are helpful to compute the total mass of the glass as:

$$M = \rho\pi(r_b^2 L_s + (r_{c_{ext}}^2 - r_{c_{int}}^2)L_c + r_b^2 L_b) \qquad (1)$$

And from which we can compute the mass per unit length:

$$\mu = \frac{M}{L_b + L_s + Lc} \qquad (2)$$

Now we have to introduce the wavenumber, that for a cantilever beam (free-fixed ends), is defined as:

$$k_n L = n \cdot 1.8751 \quad \text{with } n = 1, 2, 3, \ldots$$

So, since we're looking for the fundamental mode we impose $n = 1$ and we derive the wavenumber $k_1 = 1.8751/L$. The last quantity we need in order to compute the fundamental frequency is the area moment of inertia of the beam, that for a circular cross section beam has to be computed as:

$$I_x = \frac{1}{4}\pi r^4 \tag{3}$$

Where as radius r has been considered the radius of the stem. Doing so the glass has been condensed into an equivalent beam with circular cross section of length equal to the glass length $L = L_b + L_s + L_c$, of radius equal to the stem radius r_s and with linear mass density distributed along the length μ.

Now we can put all these quantities into the formula to compute the bending waves fundamental frequency of the equivalent cantilever beam:

$$f_1 = \frac{k_1^2}{2\pi}\sqrt{\frac{EI_x}{\mu}} \approx 155.64\ Hz \tag{4}$$

This result shows that with a very good approximation the first resonance frequency of the glass presents a behaviour that is comparable with an equivalent cantilever beam affected by bending waves.

Ex 2) Axisymmetrical Model of the Wineglass

a) Wineglass geometry

As discussed in the previous exercise the axisymmetrical property of the object to be modelled allows to exploit this property represent the 3d mesh of it using only the cross section and performing the studies on it, to be then expanded to represent the whole 3d mesh. This type of configuration considers only constant constraints on the whole circumference and by default, only the radial and axial displacements, are solved in 2D axisymmetry. It has been decided to export the cross section created in the previous section, visible in figure 2b, using the *Export* and *Import* functions implemented by *Comsol*, in order to keep full coherence on the shape of the different studies performed.

b) Eigenfrequency study

In order to compute the required study, and to keep consistent results with respect to the previous study the material has been exported from the previous settinds and imported in the current file. As before the physics used for the object is *Solid Mechanics.* For the creation of the mesh the decision made was to exploit *physics controlled geometry* with *extra fine* element size in order to optimize the capabilities of the system to compute the study. As a matter of fact it can be seen in figure 6 how the mesh is finer in areas with more complicated shapes or where the shape is narrow. An example of the importance of the size of the mesh used is analisys of the upper border, which requires a higher density of elements to correctly approximate the shape modelled. The *Eigenfrequencies study* is then computed, imposing boundary conditions: *Fixed Constraint* on the bottom edge. In table 2 the first 20 eigenfrequencies found with the axisymmetrical study method are listed, while in figure 7 it is possible to visualize the first six 3d modes computed through the same study with the respective frequencies associated.

EigenModes [Hz]							
1	4331	**6**	31554	**11**	38552	**16**	54391
2	19806	**7**	33063	**12**	39812	**17**	56492
3	24284	**8**	34593	**13**	41970	**18**	58265
4	29008	**9**	35629	**14**	45539	**19**	62044
5	30312	**10**	37126	**15**	49837	**20**	67639

Table 2: First 20 modes of the glass computed through axisymmetrical studies

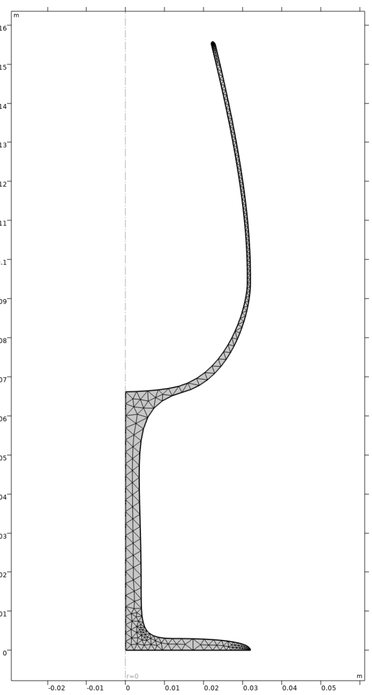
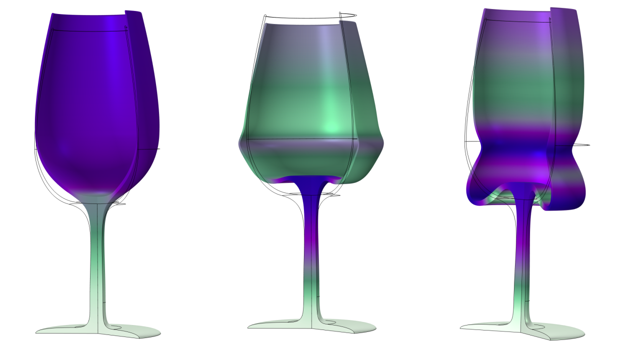


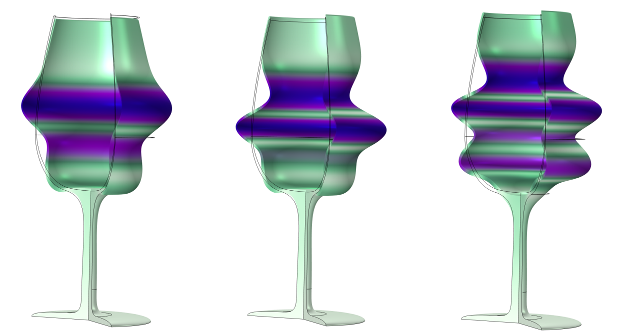
Figure 6: Cross Section of the axisymmetrical glass meshed for study

c) Eigenfrequency study circumferential mode extension

The last study is performed on the axisymmetrical model, using the same parameters considered for the previous eigenfrequency study. The major change performed is the extension of the study for circumferential modes. This feature of the software allows to consider circumferential wave numbers to be used in eigenfrequency or frequency domain studies. In figure 8 the



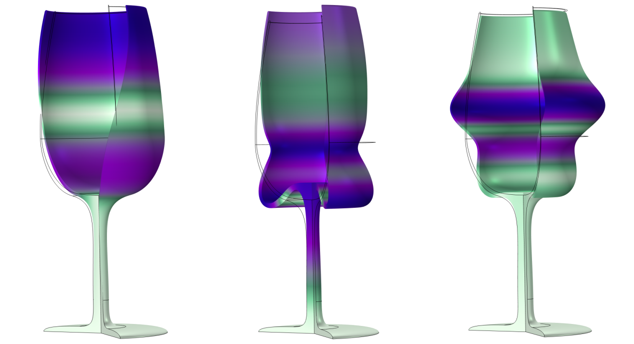
(a) First mode 4331 Hz
(b) Second mode 19806 Hz
(c) Third mode 24284 Hz



(d) Fourth mode 29008 Hz
(e) Fifth mode 30312 Hz
(f) Sixth mode 31554 Hz

Figure 7: AxiSymmetrical modes glass study

(a) First mode 413.3 Hz
(b) Second mode 4331 Hz
(c) Third mode 19806 Hz



(d) Fourth mode 21822 Hz
(e) Fifth mode 24284 Hz
(f) Sixth mode 29008 Hz

Figure 8: AxiSymmetrical modes study circumferential mode extension

first six modes obtained are listed with the specified eigenfrequency while in table 2 the first 20 eigenfrequencies found are listed. We can notice, comparing the results with figure 7 how the first and fourth mode are obtained thanks to the circumferential mode extension. The other modes are the same obtained previously redistributed in the correct abscending order

EigenModes [Hz]							
1	413.3	**6**	29008	**11**	35629	**16**	41967
2	4331	**7**	30312	**12**	37126	**17**	41970
3	19806	**8**	31554	**13**	37874	**18**	45539
4	21822	**9**	33063	**14**	38552	**19**	49837
5	24284	**10**	34593	**15**	39812	**20**	54391

Table 3: First 20 modes of the glass computed through axisymmetrical studies employing circumferential mode extension

d) Results check

Evaluating the values obtained from the study in exercise 1 and comparing them with the ones obtained in the two studies of exercise 2 we noticed how the second study, can be used as complementary to the first one. As a matter of fact the axisymmetrical method, for the object considered in the specific conditions, allows to retrieve the modes of the glass where the cup, oscillates exhibiting only nodal circles. This allows to simplify calculations focusing the study only to the behaviour of the cross-section of the object and then extending it to the whole 3d shape. In a complex object as the one considered, the modes are caused by the interaction between the movements of the stem and the cup. Therefore the main drawback is that such a method does not take into account many of the modes achieved by the 3d studies. As an example considering the first ten modes, as presented in figure 4, only the third (eigenfrequency 413.3) and tenth (eigenfrequency 4331) are computed by axisymmetrical study considering circumferential mode extension. Only the tenth is computed employing standard axisymmetrical study. Therefore for a general case study of the modes the axisymmetrical study is not completely suitable. A study on the 3d model is more precise although requires longer computation times with respect to the axisymmetrical study.