Musical Acoustics

Fourth Lab Homework

Report
Brass instrument simulation

Students

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Introduction

The report provided for the assignment summarises the steps followed to implement the model of a trumpet in *COMSOL MULTIPHYSICS* in order to simulate its acoustic behaviour. In particular the input impedance is analyzed in the various components that compose the trumpet in order to distinguish its contribution to the behaviour of the trumpet.

Simulation setup

In the section all the steps needed to setup the simulation to correctly compute the required response for the trumpet are discussed.

Axisymmetric simulation

Figure 1: Trumpet example

The trumpet simulated for the assignment, similar to the one represented in figure 1, is a semplification of the modern natural trumpet. The three main components that will be studied are:

- the mouthpiece
- the tube
- the bell

At first they will be studied singularly and then put together to compose the trumpet. The main advantage given by the shape of the trumpet considered is the axial symmetry of the model to be created. This simmetry allows to simplify both the modelling of the trumpet and the computation cost for the simulation. The *axisymmetric modelling* function provided by *COMSOL* allows to model and study a two-dimensional model and expand its study to 3d model created through a complete rotation around the main symmetry axis.

Physics

In order to be able to study the acoustic behaviour of the trumpet the *Physics* component chosen in *COMSOL* is the *Pressure acoustics, Frequency Domain* one. This component allows to compute the pressure variations for the propagation of acoustic waves in fluids at quiescent background conditions. It is suited for all frequency-domain simulations with harmonic variations of the pressure field.

Study

Given the necessity to study the input impedance for the trumpet on the frequency range the study performed is a *frequency domain study*, where all the parameters needed to retrieve the input impedance are extracted, saved in *COMSOL* variables, and then processed.

Ex 1) Impedance of a tube

In this section the tube, first component for the trumpet is modelled and studied.

Simulation model

In order to create the model for the acoustic simulation, the geometry comprises both the model to be studied and the air volume used to simulate the air response to the pressure perturbation caused by the trumpet. All the parameters used to model the tube and the air volume are listed in figure 2. The air volume is modelled as an inner semicircle with radius rS, and an outer *Perfectly matched layer* with thickness λ_{max}, used to simulate open field conditions. This component of the model will be discussed in detail in the next section. As for the tube, it is modelled as a segment with constant distance rT from the symmetry axis. At the input end of the tube is positioned a discontinuity in the air volume, as shown in figure 4. This discontinuity is useful to apply the pressure generator to the bottom boundary of the tube, as will be discussed in the next sections, and to avoid the propagation of a wave on the bottom part of the tube. The height of the discontinuity *Lspace* is specified in the table in figure 2. In order to create the 3d components needed a rotation around the symmetry axis is then performed.

Name	Expression	Value	Description
rT	0.6[cm]	0.006 m	Tube Radius
rS	2[m]	2 m	Air Sphere Radius
N	255	255	Number of Frequencies
Lt	1.37[m]	1.37 m	Tube Length
Lspace	20[mm]	0.02 m	Length empty Space
LambdaMax	c0/fMax	0.28583 m	Wave Length Max Frequency
fMin	50[Hz]	50 Hz	Min Frequency
fMax	1200[Hz]	1200 Hz	Max Frequency
c0	343[m/s]	343 m/s	Wave speed
rSMP	rS/2	1 m	AirSphere Radius MouthPiece

Figure 2: Simulation Parameters

Mesh

The mesh chosen for the 2d model is the *Free triangular* for the air volume, and *Mapped* for the *Perfectly Matched Layer* as shown in figure 5. In order to satisfy the required condition for the maximum mesh dimensions of *5 points per wavelenght max frequency* the mesh dimensions are set as shown in figure 6.

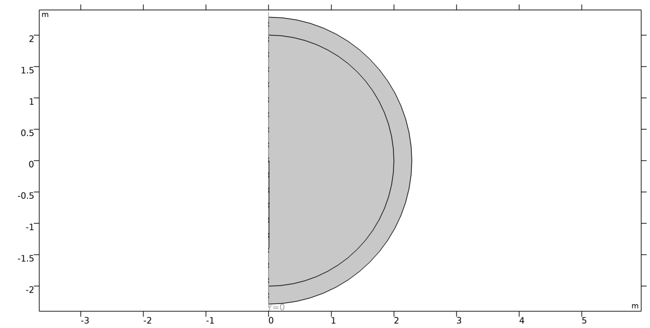


Figure 3: Tube and air volume

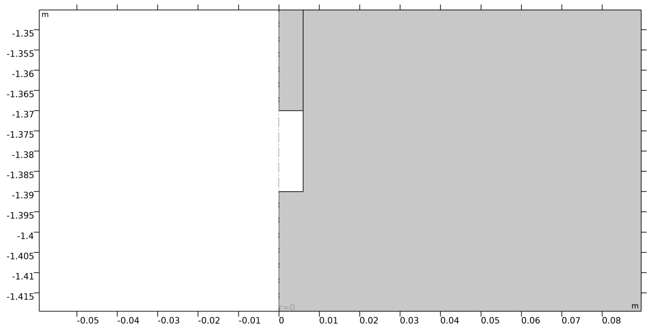


Figure 4: Air discontinuity

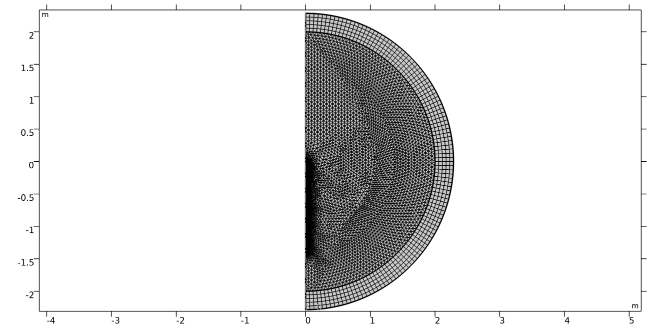


Figure 5: Meshed tube simulation

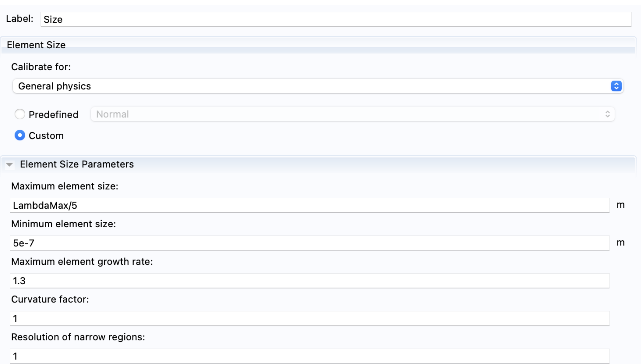


Figure 6: Mesh size

PML conditions

The perfectly matched layer is a domain added to the simulation to mimic an open and nonreflecting infinite domain. It sets up a perfectly absorbing domain as an alternative to nonreflecting boundary conditions. This layer can assume various thickness values and the *COMSOL* engine is able to adapt the conditions to respect the needed behaviour. Nevertheless some conditions have to be complied with in order to make the system work at its better possibilities:

- The PML must be at least at a distance $\lambda/8$ from the propagating waves source. In the specific, considering the parameters specified for all the simulations the distance is fully satisfied.
- The Layer is better meshed with a mapped mesh with at least 5 or 6 mesh layers. Therefore, in order to keep the correct dimensions for the mesh and suggested conditions, the *Perfectly Matched Layer*'s thickness is chosen to be *LambdaMax*.

Physics

In order to obtain the right results for the study to be computed the first component to be setup is the pressure generator. This is represented as a *Port*, which allows to excite acoustic waves that enter or leave waveguide structures, like a duct or channel, in an acoustic model. In the specific a circular port is implemented with the characteristics listed in figure 7 and positioned in the model as shown in figure 8. For the simulation purposes the behaviour of the trumpet is simplified, assuming its components as rigid boundaries for the sorrounding air volume. In *COMSOL* this behaviour can be imposed setting the elements of the 2d object as *Interior Sound Hard Boundaries*.

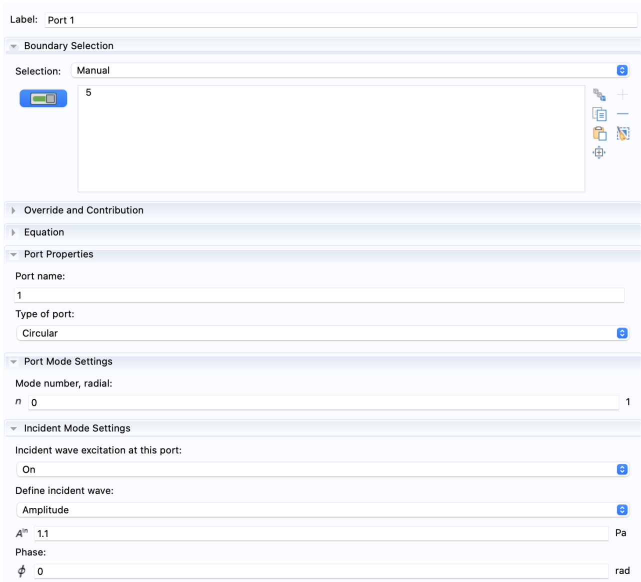


Figure 7: Port Parameters

Simulation Results

After the simulation has been computed, in a frequency range from f_{min} to f_{max}, as specified by the parameters

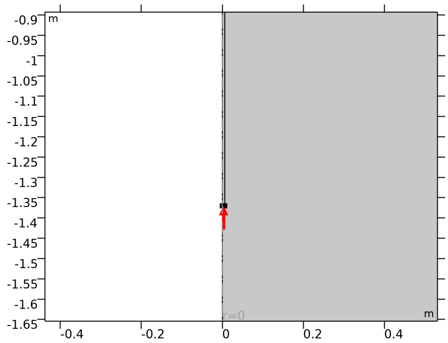


Figure 8: Port geometry

in figure 2. The frequency steps are specified as

$$range(fMin, (fMax - fMin)/N, fMax) \quad (1)$$

the input impedance is calculated as a derived result from the simulation results. As a matter of fact the impedance on the input cavity can be calculated as:

$$|Z_{in_{Tube}}| = 10 \cdot \log_{10} \frac{|\int Press_{in_{Tube}}|}{|\int Vel_{in_{Tube}}|} \quad (2)$$

In *COMSOL* this equation is defined through the command shown in figure 9. Where *intop1()* is the command for the spatial integration on the input boundary enlightened in figure 10. The *integration* node is part of the *Nonlocal couplings* functions defined by *COMSOL*. While $Press_{Tube}$ and Vel_{Tube} are two *COMSOL* variables which store respectively the pressure and the velocity results of the simulation, as shown in figure 11. The acoustic pressure $Press_{tube}$ from the tube at the maximum of the frequency range is shown in figure 13. As we can notice from the results of input impedance of the tube, plotted in figure 12, the results obtained respect what we expect from the theory, the minima presented are positioned in an even number of quarter wavelength harmonics of an open tube. Using the equation

$$f_{min}^{open} = \frac{nc}{2Lt} \quad (3)$$

we can calculate the minima locations, assuming $c = 343[m/s]$ and that the pipe is open ended. The results for the numerical minima in range f_{min}:f_{max} can be found in second column of table 1. Although open ends are unfeasible because some of the energy in the pipe is radiated outside, and therefore $p(Lt, t) \neq 0$, we can notice how the results obtained from the simulation are very similar to those obtained using the numerical equation. The maxima instead, are calculated recalling the equation for calculating the input impedance of an open pipe:

$$f_{max}^{open} = \frac{(2n-1)c}{4Lt} \quad (4)$$

This equation allows to calculate all the multiple numbers to an odd number of quarter wavelengths in the pipe. As noticeable in the third column of table 1.

Expression	Unit	Description
10*log10(abs(intop1(Press_Tube)/intop1(Vel_Tube)))		|Z_in Tube|

Figure 9: Input impedance Equation

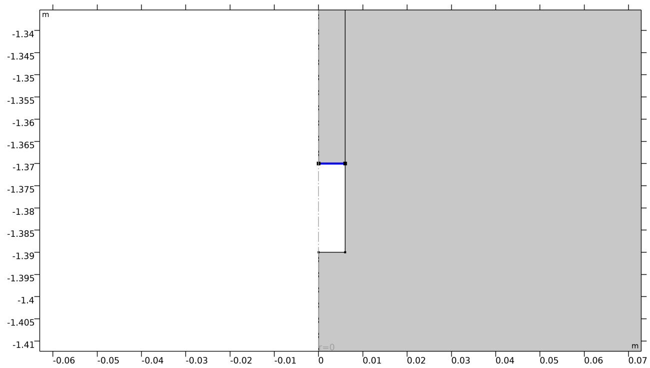


Figure 10: Input boundary

Name	Expression	Unit	Description
Press_Tube	acpr.p_t	Pa	
Vel_Tube	acpr.vz	m/s	

Figure 11: Variables

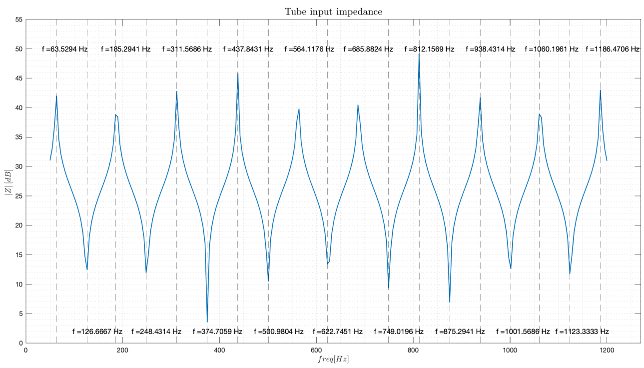


Figure 12: Input impedance tube

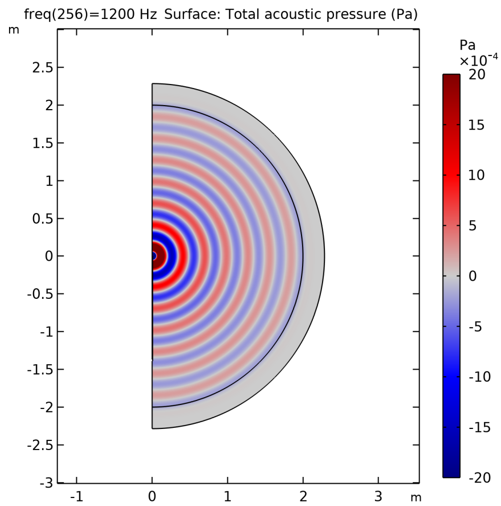


Figure 13: Tube pressure 1200 Hz

Numerical Impedance minima and maxima locations [Hz]		
	minima	maxima
1	125.2	62.6
2	250.4	187.8
3	375.5	313.0
4	500.7	438.1
5	626.0	563.3
6	751.1	688.5
7	876.3	813.7
8	1001.5	938.9
9	1126.6	1064.1
10		1189.2

Table 1: Numerical Impedance minima locations [Hz]

Ex 2) Tube with bell

In this section the analysis is made coupling the tube with a bell at the radiating end. The simulation results are presented below.

Simulation model

The model created previously is now extended adding an exponential bell.
The curve that determines the shape of the bell is described by the equation:

$$r = \sqrt{\frac{St}{\pi} e^{mz}} \tag{5}$$

where St is the surface of the tube, calculated as $\pi r T^2$. m is the exponent that determines the bell shape, z is the vertical independent variable. In order to model the exponential bell the *parametric curve* component is exploited. The node setup is shown in figure 14 and the costructing parameters are shown in figure 16. While the modelled shape for the horn is shown in figure 15.

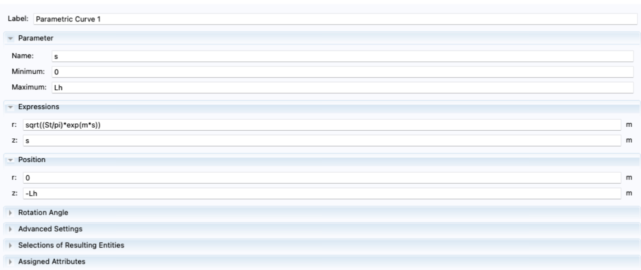


Figure 14: Parametric curve parameters

All the other parameters for the model, the mesh, the PML conditions and the simulation are the same as the previous section discussed. As the tube before, the bell is defined as an *Interior Sound Hard Boundary* that drives the air volume.

Simulation Results

After the simulation is run, and the results needed for the following computations are stored in in two variables:

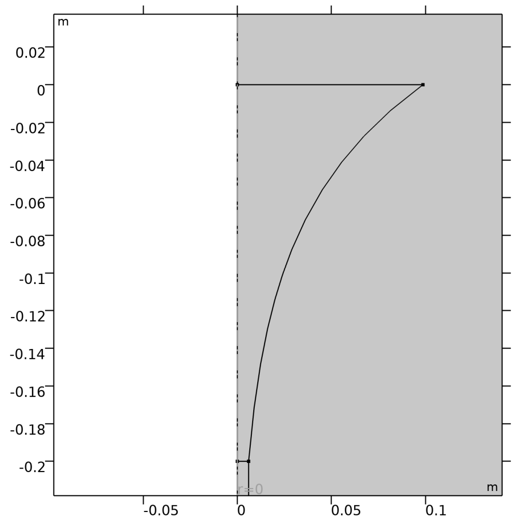


Figure 15: Shape of the exponential Bell

Name	Expression	Value	Description
Lh	0.2[m]	0.2 m	Bell Length
m	28	28	Bell Exponent
St	rT^2*pi	1.131E-4 m²	Surface Tube

Figure 16: Parameters for the bell shape

$Press_{TubeBell}$ and $Vel_{TubeBell}$, the process is similar to the one followed in the previous section. The results are then combined, as in the previous section, in order to compute the *input impedance* shown in figure 17. Comparing the impedance plot to the one computed in the previous section we can notice how the peaks in the higher side of the frequency spectrum are smoother and the overall radiation of the trumpet is improved.

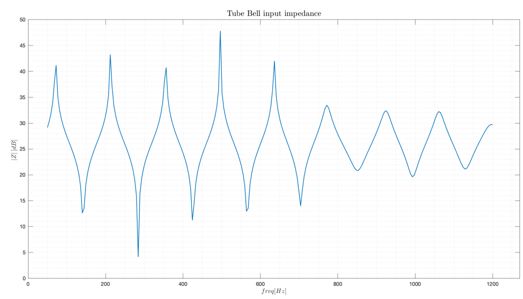


Figure 17: Input impedance of the tube and bell system

Ex 3) Mouthpiece

In this section the Mouthpiece for the trumpet is modelled and analyzed.

Simulation model

The mouthpiece is modelled as a combination between a circular cup and a truncated cone, which in 2D axisymmetric geometry are modelled as a segment and

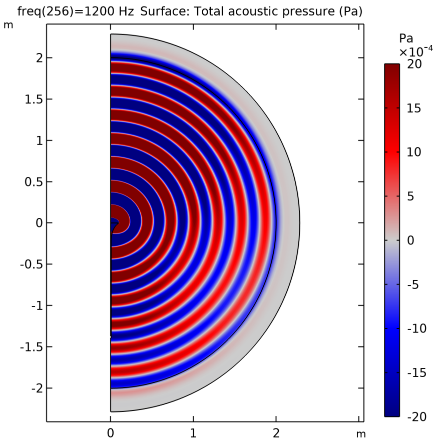
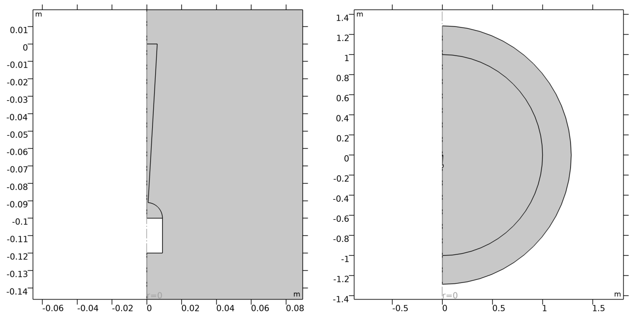


Figure 18: Tube and bell pressure 1200 Hz

a circular arc, as shown in figure 20a. The building parameters are shown in figure 19, in particular the parameter MPH specifies the height of the attachment point from the center of the cup on the z-axis in order obtain a distance from the symmetry axis of $\frac{rT}{8}$. The air volume is resized given the smaller dimension of the piece with a dimension $rSMP = \frac{rS}{2}$, shown in table in figure 2. The *Perfectly matched layer* has the same characteristics declared in the previous sections. The complete model for the mouthpiece simulation is shown in figure 20b.

Name	Expression	Value	Description
Lm	10[cm]	0.1 m	lengthMouthpiece
MPH	sqrt(rM^2-(rTEi)^2)	0.0089687 m	height of the backbore to the cup
rM	rT+0.3[cm]	0.009 m	MouthPieceRadius
rTEi	rT/8	7.5E-4 m	Radius of the backbore at the cup

Figure 19: Mouthpiece building parameters



(a) Shape of the mouthpiece (b) Air Volume Mouthpiece

Figure 20: Mouthpiece Model

All the simulation settings defined in the previous sections are kept the same. This allows us to guarantee homogeneity between the simulation sessions.

Physics

The port defined before is now positioned on the mouthhole of the mouthpiece in order to setup a frequency study on the mouthpiece.

Simulation Results

After the simulation is run, and the results needed for the following computations are stored in in two variables: $Press_{MouthPiece}$ and $Vel_{MouthPiece}$, the process is similar to the one followed in the previous sections. The results are then combined, as in the previous section, in order to compute the *input impedance*. The results in figure 21 show how the mouthpiece can be considered as an helmotz resonator. Its input impedance therefore presents a single resonance frequency, that in this case is at $523Hz$. The mouthpiece can be tuned modifying the constructing parameters of the mouthpiece, in order to control the tonal quality of the instrument to which it is coupled. In order to validate the results obtained through the resonance frequency of the mouthpiece is now computed through the numerical method. As a matter of fact the mouthpiece can be considered as an Helmoltz resonator. And therefore its resonance frequency can be computed as:

$$f_0 = \frac{c}{2\pi}\sqrt{\frac{S_c}{Vl_c}} \tag{6}$$

where S_c is the mean cross section of the backbore, l_c is the length of the backbore and V is the volume of the cup. The volume V is calculated as $V = \frac{4}{3}\pi r_M^3$, l_c is considered with the applied neck correction for the Helmoltz resonator. And $S_c = \pi(\frac{(rT/8)+rT}{2})^2$. The calculations lead to a resonance frequency $f_0 = 598[Hz]$. The result obtained is sufficiently close to the simulation results considering the approximations made for the numerical calculation and therefore the numerical result confirms correctly the one obtained through the simulation.

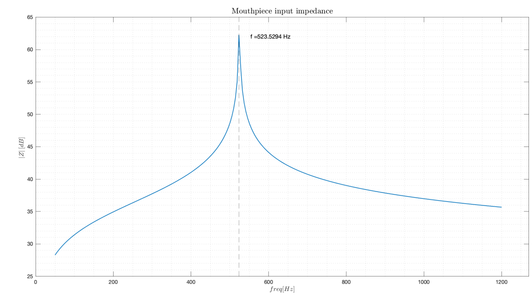


Figure 21: Input impedance of the Mouthpiece

Ex 4) Complete model

In the section the complete trumpet body is modelled and analyzed.

Simulation model

The model for the trumpet is built composing all the pieces modelled in the previous sections, hence we compose the *Mouthpiece*, the *Tube*, and the *exponential horn*, the resulting model is therefore shown in pictures

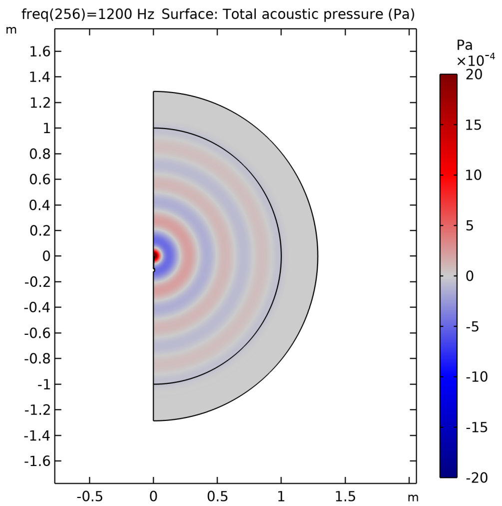
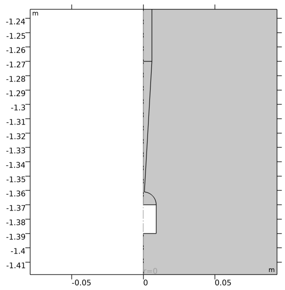
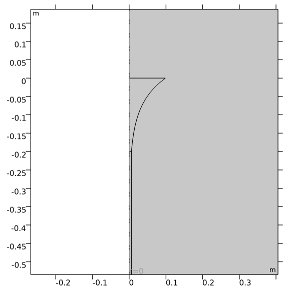


Figure 22: Mouthpiece acoustic pressure 1200 Hz



(a) Bottom shape of the Trumpet



(b) Top shape of the Trumpet

Figure 23: Shape of the Trumpet

23a and 23b. The simulation parameters are the same used in the previous sections, using as input boundary for the *port* node the *mouth-hole* of the mouthpiece.

Simulation Results

After the simulation is run, and the results needed for the following computations are stored in in two variables: $Press_{Trumpet}$ and $Vel_{Trumpet}$, the process is similar to the one followed in the previous sections. In figure 25 the acoustic pressure generated by the full trumpet is shown at 1200 Hz. The higher limit frequency has been selected through all the previous steps of this design and simulation because it is the frequency which allows to better visualize the directional behaviour of the components considered. The tube in figure 13 doesn't present a defined directional behaviour. The tube with the bell in series, in figure 18, presents a more directional behaviour with a higher pressure produced. And the full trumpet produces a lower pressure as cause of the addition of the mouthpiece to the other components. Considering the acoustic pressure generated by the mouthpiece, shown in figure 22, we can notice it geometry is not design for radiation and therefore the pressure produced is not comparable with the one produced by other components. The results of pressure and velocity are then combined, as in the previous section, in order to compute the *input impedance* shown in figure 24. We can notice how the mouthpiece impedance combined with the one derived from the trumpet and the bell allows to improve the impedance in the mid of the frequency range, enhancing the impedance in correspondence of the resonant frequency of the mouthpiece and smoothing the behaviour on the trumpet at high frequency and enhancing the peaks at the low-end of the spectrum considered. A comparison between all the impedances presented is shown in figure 26.

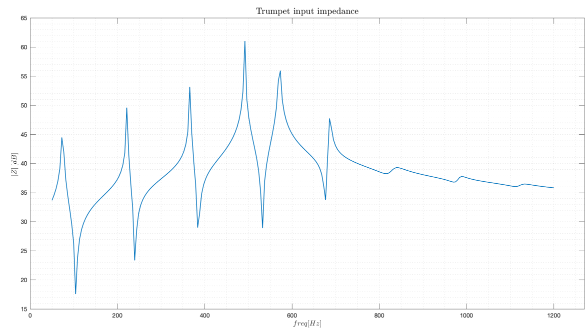


Figure 24: Input impedance of the Trumpet

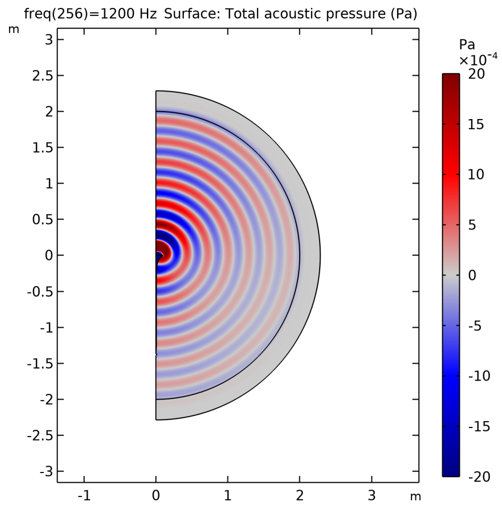


Figure 25: Trumpet acoustic pressure 1200 Hz

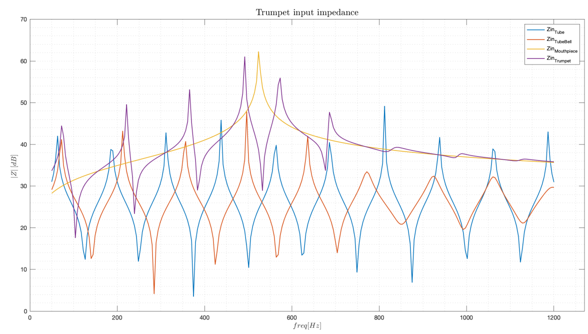


Figure 26: Input impedances comparison