Probability Assignment -III

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Question: Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of X?

Solution:
Let H be a random variable which represents the number of Heads obtained in 6 coin tosses.
And T be a random variable which represents the number of Tails obtained in 6 coin tosses.
Then,
$H \in \{0,1,2,3,4,5,6\}$
Similarly,
$T \in \{0,1,2,3,4,5,6\}$

$$H + T = 6 \tag{1}$$
$$X = |H - T| \tag{2}$$
$$X = |H - (6 - H)| \tag{3}$$
$$X = |2H - 6| \tag{4}$$

$$F_H(k) = \Pr(H \le k) \tag{5}$$
$$= \sum_{i=0}^{k} \Pr(H = i) \tag{6}$$
$$= \sum_{i=0}^{k} {}^{n}C_i \times \left(\frac{1}{2}\right)^6 \tag{7}$$

$$F_X(k) = \Pr(X \le k) = \Pr(|2H - 6| \le k) \tag{8}$$
$$= \Pr(2H - 6 \le k) \text{ and } \Pr(2H - 6 \ge -k) \tag{9}$$
$$= \Pr\left(H \le \frac{k+6}{2}\right) \text{and} \Pr\left(H \ge \frac{6-k}{2}\right) \tag{10}$$
$$= \Pr\left(\frac{6-k}{2} \le H \le \frac{k+6}{2}\right) \tag{11}$$
$$= \sum_{i=\frac{6-k}{2}}^{\frac{k+6}{2}} \Pr(H = i) \tag{12}$$
$$= \sum_{i=\frac{6-k}{2}}^{\frac{k+6}{2}} {}^{n}C_i \times \left(\frac{1}{2}\right)^6 \tag{13}$$

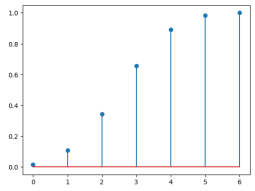

Fig. 0. Stem plot for the distribution $F_H(k)$

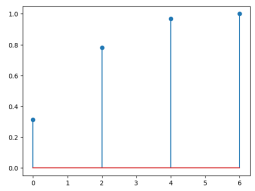

Fig. 0. Stem plot for the distribution $F_X(k)$

$$F_X(0) = \sum_{i=3}^{3} \Pr(H = i) \tag{14}$$
$$= \frac{5}{16} \tag{15}$$

$$F_X(2) = \sum_{i=2}^{4} \Pr(H = i) \tag{16}$$
$$= \frac{25}{32} \tag{17}$$

$$F_X(4) = \sum_{i=1}^{5} \Pr(H = i) \tag{18}$$
$$= \frac{31}{32} \tag{19}$$

$$F_X(6) = \sum_{i=0}^{6} \Pr(H = i) \tag{20}$$
$$= \frac{64}{64} = 1 \tag{21}$$