

Probability Assignment -III

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Question: Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of X ?

Solution:

Let H be a random variable which represents the number of Heads obtained in 6 coin tosses.

And T be a random variable which represents the number of Tails obtained in 6 coin tosses.

Then,

$$H \in \{0, 1, 2, 3, 4, 5, 6\}$$

Similarly,

$$T \in \{0, 1, 2, 3, 4, 5, 6\}$$

$$H + T = 6 \quad (1)$$

$$X = |H - T| \quad (2)$$

$$X = |H - (6 - H)| \quad (3)$$

$$X = |2H - 6| \quad (4)$$

$$F_X(k) = \Pr(X \leq 0) = \Pr(|2H - 6| \leq k) \quad (5)$$

$$= \Pr(2H - 6 \leq k) \text{ and } \Pr(2H - 6 \geq -k) \quad (6)$$

$$= \Pr\left(H \leq \frac{k+6}{2}\right) \text{ and } \Pr\left(H \geq \frac{6-k}{2}\right) \quad (7)$$

$$= \Pr\left(\frac{6-k}{2} \leq H \leq \frac{k+6}{2}\right) \quad (8)$$

$$= F_H\left(\frac{6+k}{2}\right) - F_H\left(\frac{6-k}{2} - 1\right) \quad (9)$$

$$= F_H\left(\frac{6+k}{2}\right) - F_H\left(\frac{4-k}{2}\right) \quad (10)$$

$$F_X(0) = F_H(3) - F_H(2) \quad (11)$$

$$= {}^6C_3 \left(\frac{1}{2}\right)^6 \quad (12)$$

$$F_X(2) = F_H(4) - F_H(1) \quad (13)$$

$$= {}^6C_2 \left(\frac{1}{2}\right)^6 + {}^6C_3 \left(\frac{1}{2}\right)^6 + {}^6C_4 \left(\frac{1}{2}\right)^6 \quad (14)$$

$$= \frac{25}{32} \quad (15)$$

$$F_X(4) = F_H(5) - F_H(0) \quad (16)$$

$$= {}^6C_1 \left(\frac{1}{2}\right)^6 + {}^6C_2 \left(\frac{1}{2}\right)^6 + {}^6C_3 \left(\frac{1}{2}\right)^6 \quad (17)$$

$$+ {}^6C_4 \left(\frac{1}{2}\right)^6 + {}^6C_5 \left(\frac{1}{2}\right)^6 \quad (18)$$

$$= \frac{31}{32} \quad (19)$$

$$F_X(6) = F_H(6) \quad (20)$$

$$= {}^6C_0 \left(\frac{1}{2}\right)^6 + {}^6C_1 \left(\frac{1}{2}\right)^6 + {}^6C_2 \left(\frac{1}{2}\right)^6 + {}^6C_3 \left(\frac{1}{2}\right)^6$$

$$+ {}^6C_4 \left(\frac{1}{2}\right)^6 + {}^6C_5 \left(\frac{1}{2}\right)^6 + {}^6C_6 \left(\frac{1}{2}\right)^6 \quad (21)$$

$$= \frac{64}{64} = 1 \quad (22)$$