Probability Assignment -II

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Problem:

1) One card is drawn at random from a well shuffled deck of 52 cards. In which of the following cases are the events E and F independent ?
   a) E : 'the card drawn is a spade'
      F : 'the card drawn is an ace'
   b) E: 'the card drawn is black'
      F : 'the card drawn is a king'
   c) E : 'the card drawn is a king or queen'
      F : 'the card drawn is a queen or jack'.

Solution:

1) E denotes the event that the card drawn is spade

$$\Pr(E) = \frac{{}^{13}C_1}{{}^{52}C_1} = \frac{13}{52} = \frac{1}{4} \tag{1}$$

F denotes the event that card drawn is ace

$$\Pr(F) = \frac{{}^{4}C_1}{{}^{52}C_1} = \frac{4}{52} = \frac{1}{13} \tag{2}$$

EF denotes the event the card is spade and ace

$$\Pr(EF) = \frac{{}^{1}C_1}{{}^{52}C_1} = \frac{1}{52} \tag{3}$$

$$\Pr(E)\Pr(F) = \frac{1}{4} \times \frac{1}{13} = \frac{1}{52} \tag{4}$$

$$\therefore \Pr(EF) = \Pr(E)\Pr(F) \tag{5}$$

$\therefore$ E and F are independent events.

2) E denotes the event that the card drawn is black

$$\Pr(E) = \frac{{}^{26}C_1}{{}^{52}C_1} = \frac{26}{52} = \frac{1}{2} \tag{6}$$

F denotes the event that card drawn is a king

$$\Pr(F) = \frac{{}^{4}C_1}{{}^{52}C_1} = \frac{4}{52} = \frac{1}{13} \tag{7}$$

EF denotes the event the card is black and king

$$\Pr(EF) = \frac{{}^{2}C_1}{{}^{52}C_1} = \frac{2}{52} = \frac{1}{26} \tag{8}$$

$$\Pr(E)\Pr(F) = \frac{1}{2} \times \frac{1}{13} = \frac{1}{26} \tag{9}$$

$$\therefore \Pr(EF) = \Pr(E)\Pr(F) \tag{10}$$

$\therefore$ E and F are independent events.

3) E denotes the event that the card drawn is king or queen

$$\Pr(E) = \frac{{}^{8}C_1}{{}^{52}C_1} = \frac{8}{52} = \frac{2}{13} \tag{11}$$

F denotes the event that card drawn is a queen or jack

$$\Pr(F) = \frac{{}^{8}C_1}{{}^{52}C_1} = \frac{8}{52} = \frac{2}{13} \tag{12}$$

EF denotes the event the card is either a king or queen and either queen or jack

$$\Pr(EF) = \frac{{}^{4}C_1}{{}^{52}C_1} = \frac{4}{52} = \frac{1}{13} \tag{13}$$

$$\Pr(E)\Pr(F) = \frac{2}{13} \times \frac{2}{13} = \frac{4}{169} \tag{14}$$

$$\therefore \quad \Pr(EF) \neq \Pr(E)\Pr(F) \tag{15}$$

$\therefore$ E and F are not independent events.