

Probability Assignment -III

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Question: Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of X ?

Solution:

Let, H represent random variable representing the no of heads in 6 coin tosses, (PMF) of H is given by

$$\Pr(H = k) = {}^n C_k \times \left(\frac{1}{2}\right)^k \times \left(\frac{1}{2}\right)^{6-k}, 0 \leq H \leq 6 \quad (1)$$

Let, T represent random variable representing the no of tails in 6 coin tosses, (PMF) of T is given by

$$\Pr(T = k) = {}^n C_k \times \left(\frac{1}{2}\right)^k \times \left(\frac{1}{2}\right)^{6-k}, 0 \leq T \leq 6 \quad (2)$$

X is random variable representing the difference between no of tails and heads (PMF) of X is

$$X = H - T, X = -6, -4, -2, 0, 2, 4, 6 \quad (3)$$

$$H = k \Rightarrow T = 6 - k \Rightarrow X = 2k - 6 \quad (4)$$

$$\Pr(X = 2k - 6) = \Pr(H = k) = \Pr(T = 6 - k) \quad (5)$$

$$= {}^n C_k \times \left(\frac{1}{2}\right)^6 \quad (6)$$

$k=0$

$$\Pr(X = -6) = \left(\frac{1}{2}\right)^6 = \frac{1}{64} \quad (7)$$

$k=1$

$$\Pr(X = -4) = 6 \times \left(\frac{1}{2}\right)^6 = \frac{3}{32} \quad (8)$$

$k=2$

$$\Pr(X = -2) = 15 \times \left(\frac{1}{2}\right)^6 = \frac{15}{64} \quad (9)$$

$k=3$

$$\Pr(X = 0) = 20 \times \left(\frac{1}{2}\right)^6 = \frac{5}{16} \quad (10)$$

$k=4$

$$\Pr(X = 2) = 15 \times \left(\frac{1}{2}\right)^6 = \frac{15}{64} \quad (11)$$

$k=5$

$$\Pr(X = 4) = 6 \times \left(\frac{1}{2}\right)^6 = \frac{3}{32} \quad (12)$$

$k=6$

$$\Pr(X = 6) = \left(\frac{1}{2}\right)^6 = \frac{1}{64} \quad (13)$$

X	-6	-4	-2	0	2	4	6
Pr(X)	$\frac{1}{64}$	$\frac{3}{32}$	$\frac{15}{64}$	$\frac{5}{16}$	$\frac{15}{64}$	$\frac{3}{32}$	$\frac{1}{64}$

$$F_X(x) = \Pr(X \leq x)$$

$$F_X(-6) = \Pr(X \leq -6) = \frac{1}{64} \quad (14)$$

$$F_X(-4) = \Pr(X \leq -4) = \frac{1}{64} + \frac{3}{32} = \frac{7}{64} \quad (15)$$

$$F_X(-2) = \Pr(X \leq -2) = \frac{1}{64} + \frac{3}{32} + \frac{15}{64} \quad (16)$$

$$= \frac{11}{32} \quad (17)$$

$$F_X(0) = \Pr(X \leq 0) = \frac{1}{64} + \frac{3}{32} + \frac{15}{64} + \frac{5}{16} \quad (18)$$

$$= \frac{21}{32} \quad (19)$$

$$F_X(2) = \Pr(X \leq 2) = \frac{1}{64} + \frac{3}{32} + \frac{30}{64} + \frac{5}{16} \quad (20)$$

$$= \frac{57}{64} \quad (21)$$

$$F_X(4) = \Pr(X \leq 4) = \frac{1}{64} + \frac{6}{32} + \frac{30}{64} + \frac{5}{16} \quad (22)$$

$$= \frac{63}{64} \quad (23)$$

$$F_X(6) = \Pr(X \leq 6) = \frac{2}{64} + \frac{6}{32} + \frac{30}{64} + \frac{5}{16} \quad (24)$$

$$= \frac{64}{64} = 1 \quad (25)$$

X	-6	-4	-2	0	2	4	6
$F_X(x)$	$\frac{1}{64}$	$\frac{7}{64}$	$\frac{11}{32}$	$\frac{21}{32}$	$\frac{57}{64}$	$\frac{63}{64}$	1