

Probability Assignment -III

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Question: Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of X ?

Solution:

Let H be a random variable which represents the number of Heads obtained in 6 coin tosses.

And T be a random variable which represents the number of Tails obtained in 6 coin tosses.

Then,

$$H \in \{0, 1, 2, 3, 4, 5, 6\}$$

Similarly,

$$T \in \{0, 1, 2, 3, 4, 5, 6\}$$

$$H + T = 6 \quad (1)$$

$$X = |H - T| \quad (2)$$

$$X = |H - (6 - H)| \quad (3)$$

$$X = |2H - 6| \quad (4)$$

$$\Pr(X \leq 0) = \Pr(|2H - 6| \leq 0) \quad (5)$$

$$= \Pr(2H - 6 = 0) = \Pr(H = 3) \quad (6)$$

$$= {}^6C_3 \left(\frac{1}{2}\right)^6 \quad (7)$$

$$\Pr(X \leq 2) = \Pr(|2H - 6| \leq 2) \quad (8)$$

$$= \Pr(2H - 6 \leq 2) \text{ and } \Pr(2H - 6 \geq -2) \quad (9)$$

$$= \Pr(H \leq 4) \text{ and } \Pr(H \geq 2) \quad (10)$$

$$= \Pr(2 \leq H \leq 4) \quad (11)$$

$$= \sum_{i=2}^4 \Pr(H = i) \quad (12)$$

$$= {}^6C_2 \left(\frac{1}{2}\right)^6 + {}^6C_3 \left(\frac{1}{2}\right)^6 + {}^6C_4 \left(\frac{1}{2}\right)^6 \quad (13)$$

$$= \frac{25}{32} \quad (14)$$

$$\Pr(X \leq 4) = \Pr(|2H - 6| \leq 4) \quad (15)$$

$$= \Pr(2H - 6 \leq 4) \text{ and } \Pr(2H - 6 \geq -4) \quad (16)$$

$$= \Pr(H \leq 5) \text{ and } \Pr(H \geq 1) \quad (17)$$

$$= \Pr(1 \leq H \leq 5) \quad (18)$$

$$= \sum_{i=1}^5 \Pr(H = i) \quad (19)$$

$$= {}^6C_1 \left(\frac{1}{2}\right)^6 + {}^6C_2 \left(\frac{1}{2}\right)^6 + {}^6C_3 \left(\frac{1}{2}\right)^6 \quad (20)$$

$$+ {}^6C_4 \left(\frac{1}{2}\right)^6 + {}^6C_5 \left(\frac{1}{2}\right)^6 \quad (21)$$

$$= \frac{31}{32} \quad (22)$$

$$\Pr(X \leq 6) = \Pr(|2H - 6| \leq 6) \quad (23)$$

$$= \Pr(2H - 6 \leq 6) \text{ and } \Pr(2H - 6 \geq -6) \quad (24)$$

$$= \Pr(H \leq 6) \text{ and } \Pr(H \geq 0) \quad (25)$$

$$= \Pr(0 \leq H \leq 6) \quad (26)$$

$$= \sum_{i=0}^6 \Pr(H = i) \quad (27)$$

$$= {}^6C_0 \left(\frac{1}{2}\right)^6 + {}^6C_1 \left(\frac{1}{2}\right)^6 + {}^6C_2 \left(\frac{1}{2}\right)^6 + {}^6C_3 \left(\frac{1}{2}\right)^6$$

$$+ {}^6C_4 \left(\frac{1}{2}\right)^6 + {}^6C_5 \left(\frac{1}{2}\right)^6 + {}^6C_6 \left(\frac{1}{2}\right)^6 \quad (28)$$

$$= \frac{64}{64} = 1 \quad (29)$$

X	0	2	4	6
$F_X(x)$	$\frac{5}{16}$	$\frac{25}{32}$	$\frac{31}{32}$	1