Probability Assignment -III

Posa Harsha vardhan(EE22BTECH11214)*

Question: Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of X?

Solution:

Let, H represent random variable representing the no of heads in 6 coin tosses, (PMF) of H is given by

$$\Pr(H=k) = {}^{n}C_k \times \left(\frac{1}{2}\right)^{k} \times \left(\frac{1}{2}\right)^{6-k}, 0 \le H \le 6 \quad (1)$$

Let, H represent random variable representing the no of heads in 6 coin tosses, (PMF) of H is given by

$$\Pr(T=k) = {}^{n}C_k \times \left(\frac{1}{2}\right)^{k} \times \left(\frac{1}{2}\right)^{6-k}, 0 \le T \le 6 \quad (2)$$

X is random varible representing the difference between no of tails and heads (PMF) of X is

$$X = H - T, X = -6, -4, -2, 0, 2, 4, 6 \quad (3)$$

$$H = k \Rightarrow T = 6 - k \Rightarrow X = 2k - 6 \quad (4)$$

$$\Pr(X = 2k-6) = \Pr(H=k) = \Pr(T = 6-k) \quad (5)$$

$$= {}^{n}C_k \times \left(\frac{1}{2}\right)^{6} \quad (6)$$

k=0

$$\Pr(X=-6) = \left(\frac{1}{2}\right)^{6} = \frac{1}{64} \quad (7)$$

k=1

$$\Pr(X=-4) = 6 \times \left(\frac{1}{2}\right)^{6} = \frac{3}{32} \quad (8)$$

k=2

$$\Pr(X=-2) = 15 \times \left(\frac{1}{2}\right)^{6} = \frac{15}{64} \quad (9)$$

k=3

$$\Pr(X=0) = 20 \times \left(\frac{1}{2}\right)^{6} = \frac{5}{16} \quad (10)$$

k=4

$$\Pr(X=2) = 15 \times \left(\frac{1}{2}\right)^{6} = \frac{15}{64} \quad (11)$$

k=5

$$\Pr(X=4) = 6 \times \left(\frac{1}{2}\right)^{6} = \frac{3}{32} \quad (12)$$

k=6

$$\Pr(X=6) = \left(\frac{1}{2}\right)^{6} = \frac{1}{64} \quad (13)$$

X	-6	-4	-2	0	2	4	6
$\Pr(X)$	$\frac{1}{64}$	$\frac{3}{32}$	$\frac{15}{64}$	$\frac{5}{16}$	$\frac{15}{64}$	$\frac{3}{32}$	$\frac{1}{64}$

$$F_X(x) = \Pr(X \le x)$$

$$F_X(-6) = \Pr(X \le -6) = \frac{1}{64} \quad (14)$$

$$F_X(-4) = \Pr(X \le -4) = \frac{1}{64} + \frac{3}{32} = \frac{7}{64} \quad (15)$$

$$F_X(-2) = \Pr(X \le -2) = \frac{1}{64} + \frac{3}{32} + \frac{15}{64} \quad (16)$$

$$= \frac{11}{32} \quad (17)$$

$$F_X(0) = \Pr(X \le 0) = \frac{1}{64} + \frac{3}{32} + \frac{15}{64} + \frac{5}{16} \quad (18)$$

$$= \frac{21}{32} \quad (19)$$

$$F_X(2)\Pr(X \le 2) = \frac{1}{64} + \frac{3}{32} + \frac{30}{64} + \frac{5}{16} \quad (20)$$

$$= \frac{57}{64} \quad (21)$$

$$F_X(2)\Pr(X \le 2) = \frac{1}{64} + \frac{6}{32} + \frac{30}{64} + \frac{5}{16} \quad (22)$$

$$= \frac{63}{64} \quad (23)$$

$$F_X(2)\Pr(X \le 2) = \frac{2}{64} + \frac{6}{32} + \frac{30}{64} + \frac{5}{16} \quad (24)$$

$$= \frac{64}{64} = 1 \quad (25)$$

X	-6	-4	-2	0	2	4	6
$\Pr(X)$	$\frac{1}{64}$	$\frac{7}{64}$	$\frac{11}{32}$	$\frac{21}{32}$	$\frac{57}{64}$	$\frac{63}{64}$	1