Probability Assignment -III

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Question: Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of X?

Solution:
Let H be a random variable which represents the number of Heads obtained in 6 coin tosses.
And T be a random variable which represents the number of Tails obtained in 6 coin tosses.
Then,
$H \in \{0, 1, 2, 3, 4, 5, 6\}$
Similarly,
$T \in \{0, 1, 2, 3, 4, 5, 6\}$

$$H + T = 6 \tag{1}$$
$$X = |H - T| \tag{2}$$
$$X = |H - (6 - H)| \tag{3}$$
$$X = |2H - 6| \tag{4}$$

$$F_H(k) = \Pr(H \leq k) \tag{5}$$
$$= \sum_{i=0}^{k} \Pr(H = i) \tag{6}$$

$$F_X(k) = \Pr(X \leq k) = \Pr(|2H - 6| \leq k) \tag{7}$$
$$= \Pr(2H - 6 \leq k) \text{ and } \Pr(2H - 6 \geq -k) \tag{8}$$
$$= \Pr\left(H \leq \frac{k+6}{2}\right) \text{and} \Pr\left(H \geq \frac{6-k}{2}\right) \tag{9}$$
$$= \Pr\left(\frac{6-k}{2} \leq H \leq \frac{k+6}{2}\right) \tag{10}$$
$$= F_H\left(\frac{6+k}{2}\right) - F_H\left(\frac{6-k}{2} - 1\right) \tag{11}$$
$$= F_H\left(\frac{6+k}{2}\right) - F_H\left(\frac{4-k}{2}\right) \tag{12}$$
$$= \sum_{i=\frac{6-k}{2}}^{\frac{k+6}{2}} \Pr(H = i) \tag{13}$$

$$F_X(0) = F_H(3) - F_H(2) \tag{14}$$
$$= \sum_{i=3}^{3} \Pr(H = i) \tag{15}$$
$$=^6 C_3 \left(\frac{1}{2}\right)^6 \tag{16}$$

$$F_X(2) = F_H(4) - F_H(1) \tag{17}$$
$$= \sum_{i=2}^{4} \Pr(H = i) \tag{18}$$
$$=^6 C_2 \left(\frac{1}{2}\right)^6 +^6 C_3 \left(\frac{1}{2}\right)^6 +^6 C_4 \left(\frac{1}{2}\right)^6 \tag{19}$$
$$= \frac{25}{32} \tag{20}$$

$$F_X(4) = F_H(5) - F_H(0) \tag{21}$$
$$= \sum_{i=1}^{5} \Pr(H = i) \tag{22}$$
$$=^6 C_1 \left(\frac{1}{2}\right)^6 +^6 C_2 \left(\frac{1}{2}\right)^6 +^6 C_3 \left(\frac{1}{2}\right)^6 +^6 C_4 \left(\frac{1}{2}\right)^6 +^6 C_5 \left(\frac{1}{2}\right)^6 \tag{23}$$
$$= \frac{31}{32} \tag{24}$$

$$F_X(6) = F_H(6) \tag{25}$$
$$= \sum_{i=0}^{6} \Pr(H = i) \tag{26}$$
$$=^6 C_0 \left(\frac{1}{2}\right)^6 +^6 C_1 \left(\frac{1}{2}\right)^6 +^6 C_2 \left(\frac{1}{2}\right)^6 +^6 C_3 \left(\frac{1}{2}\right)^6 +^6 C_4 \left(\frac{1}{2}\right)^6 +^6 C_5 \left(\frac{1}{2}\right)^6 +^6 C_6 \left(\frac{1}{2}\right)^6 \tag{27}$$
$$= \frac{64}{64} = 1 \tag{28}$$

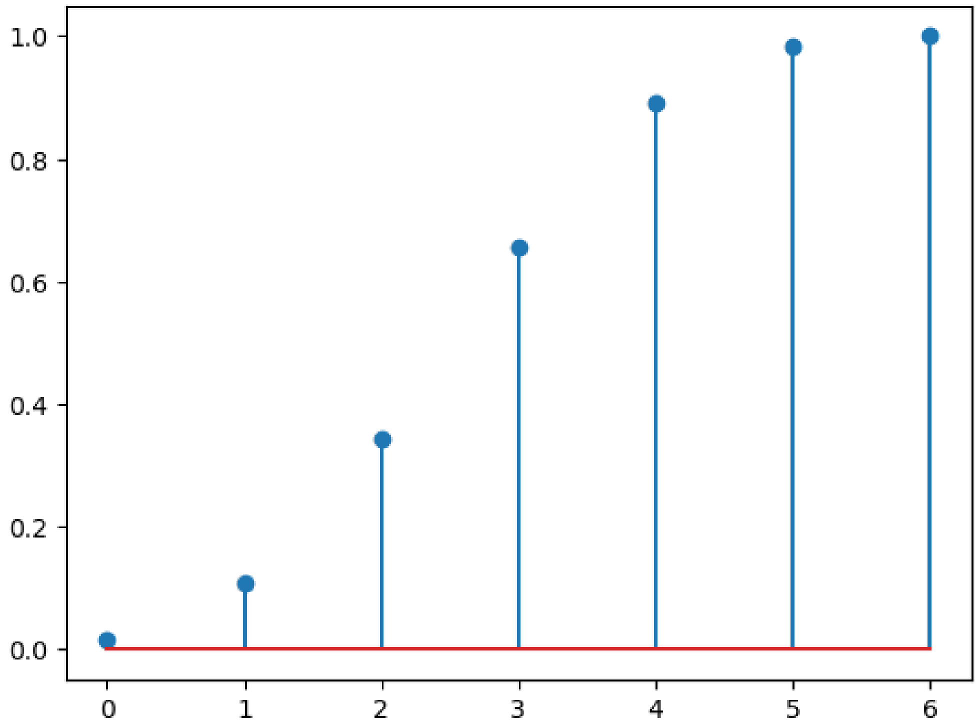

Fig. 0. Stem plot for the distribution $F_H(k)$

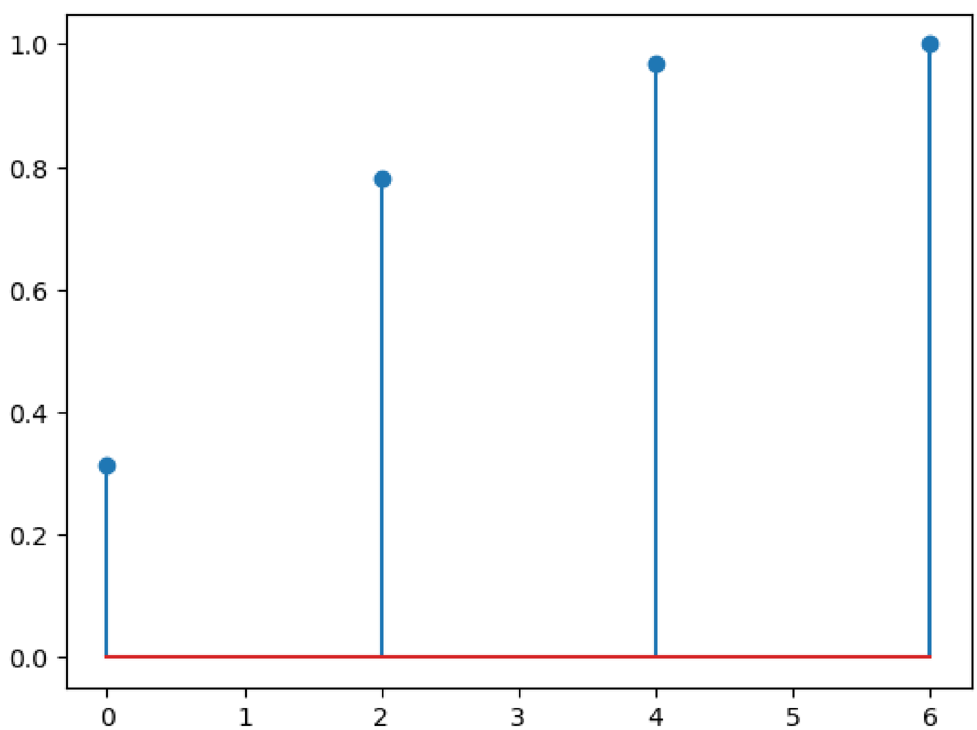

Fig. 0. Stem plot for the distribution $F_X(k)$