Probability Assignment -III

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Question: Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of X?

Solution:
Let H be a random variable which represents the number of Heads obtained in 6 coin tosses.
And T be a random variable which represents the number of Tails obtained in 6 coin tosses.
Then,
$H \in \{0,1,2,3,4,5,6\}$
Similarly,
$T \in \{0,1,2,3,4,5,6\}$

$$H + T = 6 \tag{1}$$
$$X = |H - T| \tag{2}$$
$$X = |H - (6 - H)| \tag{3}$$
$$X = |2H - 6| \tag{4}$$

$$F_X(0) = \Pr(X \le 0) = \Pr(|2H - 6| \le 0) \tag{5}$$
$$= \Pr(2H - 6 = 0) = \Pr(H = 3) \tag{6}$$
$$= {}^6C_3 \left(\frac{1}{2}\right)^6 \tag{7}$$

$$F_X(2) = \Pr(X \le 2) = \Pr(|2H - 6| \le 2) \tag{8}$$
$$= \Pr(2H - 6 \le 2) \text{ and } \Pr(2H - 6 \ge -2) \tag{9}$$
$$= \Pr(H \le 4) \text{ and } \Pr(H \ge 2) \tag{10}$$
$$= \Pr(2 \le H \le 4) \tag{11}$$
$$= \sum_{i=2}^{4} \Pr(H = i) \tag{12}$$
$$= {}^6C_2 \left(\frac{1}{2}\right)^6 + {}^6C_3 \left(\frac{1}{2}\right)^6 + {}^6C_4 \left(\frac{1}{2}\right)^6 \tag{13}$$
$$= \frac{25}{32} \tag{14}$$

$$F_X(4) = \Pr(X \le 4) = \Pr(|2H - 6| \le 4) \tag{15}$$
$$= \Pr(2H - 6 \le 4) \text{ and } \Pr(2H - 6 \ge -4) \tag{16}$$
$$= \Pr(H \le 5) \text{ and } \Pr(H \ge 1) \tag{17}$$
$$= \Pr(1 \le H \le 5) \tag{18}$$
$$= \sum_{i=1}^{5} \Pr(H = i) \tag{19}$$
$$= {}^6C_1 \left(\frac{1}{2}\right)^6 + {}^6C_2 \left(\frac{1}{2}\right)^6 + {}^6C_3 \left(\frac{1}{2}\right)^6 \tag{20}$$
$$+ {}^6C_4 \left(\frac{1}{2}\right)^6 + {}^6C_5 \left(\frac{1}{2}\right)^6 \tag{21}$$
$$= \frac{31}{32} \tag{22}$$

$$F_X(6) = \Pr(X \le 6) = \Pr(|2H - 6| \le 6) \tag{23}$$
$$= \Pr(2H - 6 \le 6) \text{ and } \Pr(2H - 6 \ge -6) \tag{24}$$
$$= \Pr(H \le 6) \text{ and } \Pr(H \ge 0) \tag{25}$$
$$= \Pr(0 \le H \le 6) \tag{26}$$
$$= \sum_{i=0}^{6} \Pr(H = i) \tag{27}$$
$$= {}^6C_0 \left(\frac{1}{2}\right)^6 + {}^6C_1 \left(\frac{1}{2}\right)^6 + {}^6C_2 \left(\frac{1}{2}\right)^6 + {}^6C_3 \left(\frac{1}{2}\right)^6$$
$$+ {}^6C_4 \left(\frac{1}{2}\right)^6 + {}^6C_5 \left(\frac{1}{2}\right)^6 + {}^6C_6 \left(\frac{1}{2}\right)^6 \tag{28}$$
$$= \frac{64}{64} = 1 \tag{29}$$

X	0	2	4	6
$F_X(x)$	$\frac{5}{16}$	$\frac{25}{32}$	$\frac{31}{32}$	1