Lecture 10

10 September 2021 17:02

- Recap
- Congestion control / Utility Maximization

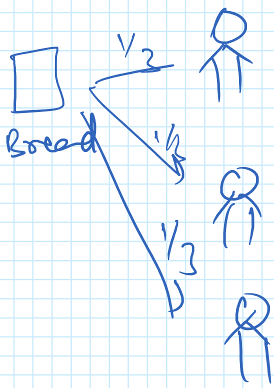
Mathematical model

Reliable Transport layer principles

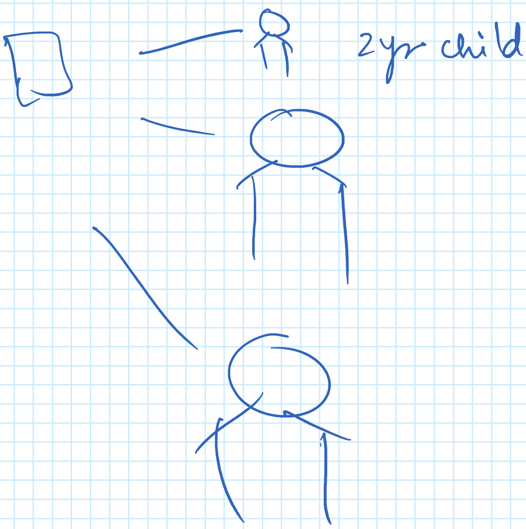
- Stop & Go ✓
- Sliding window algo ✓
  - Go Back N (TX window) Rx No buffering
  - Selective Repeat: Both TX & RX maintains window; Rx. accepts even out-of-order Pckts

- congestion control / Fairness / optimal way of sharing resources

Scenario-1



Modified Scenario-1



Requirements need to be taken into account!

Utility Maximization framework:

Properties:

$U_i(x_i)$: → Increasing fn ✓

→ concave fn ✓

→ differentiability

U1: 100 Mbps Speed $\xrightarrow{+100}$ 200 Mbps Speed

U2: 1000 Mbps $\xrightarrow{+100}$ 1100 Mbps Speed

Concave fn

$x_j > x_i$

$x_i \rightarrow x_i + \delta$ $\qquad$ $x_j \rightarrow x_j + \delta$

$\underbrace{f(x_i+\delta) - f(x_i)}$ >

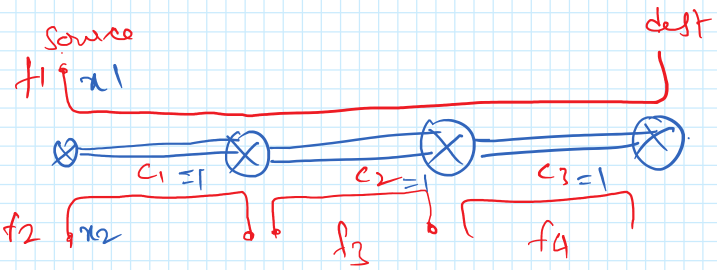
Example:

$x_i \geq 0$

$x_1 + x_2 \leq 1$

$x_1 + x_3 \leq 1$

$x_1 + x_4 \leq 1$



$x_1 \quad x_2 \quad x_3 \quad x_4$ → flow rates

flow1 = $\{l_1, l_2, l_3\}$

$f_2 = \{l_1\}$

$f_3 = \{l_2\}$

$f_4 = \{l_3\}$

$$\sum_{\substack{i:\, l \in i}} x_i \leq C_l \qquad \forall l \text{ (links)} \quad -(1)$$

(Link l is being used by flow i)

$$x_i \geq 0 \qquad -(2)$$

Allocation Strategy - I x (Maximizing worst case user throughput) ✓

$x_i = 0.5 \quad \forall i$

$x_1 + x_2 \leq 1$

$0.5 + 0.5 \leq 1$

$0.25 + 0.75 = 1$

0.5 α

0.5

↓ to each

0.5 α
0.5
0.5
0.5
2

$x_i = 0.5 \quad \forall i$

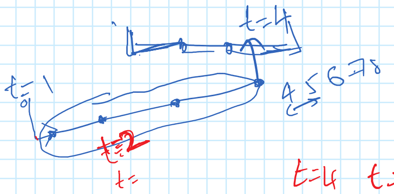
same rate to each user/flow

$0.5 + 0.5 = 1$
$0.25 + 0.75 = 1$

Allocation - II

$x_1 = 3/4 \,;\, x_i = 1/4 \quad i \neq 1$

Intention is to have same delay for links.



1 byte $\frac{1}{3/4} = 4/3$ s

$1/\frac{1}{4} = 4$

5 sec 15 pckt $f_1 \Rightarrow$ t=4 1P t=5 4P t=6 7P ......

$f_2 \Rightarrow$

Allocation - 3 Max. Revenue max. $\sum x_i$

$\Rightarrow \boxed{x_1 = 0 \\ x_i = 1} \quad \forall i \neq 1 \longrightarrow \boxed{x_i = 0.5}$

Allocation - 4 (Proportional Fairness)

$x_1 = 1/4 \quad x_i = 3/4, \; i \neq 1$

Allocation 3 (sum rate maximization)

$U_i(x_i) = x_i \qquad \max. \sum_i U_i(x_i) \;$ s.t x is feasible

Allocation (Max-min fairness) $\frac{1}{x_i}$ Transmit time $\quad \min \sum_i \frac{1}{x_i}$

$U_i(x_i) = \frac{1}{x_i}$

Max-Min

Prop fairness $\Rightarrow U_i(x_i) = \log x_i \qquad \sum_i \log_2 x_i \qquad \log_2 \frac{1}{2}$

not allowing any x_i $\quad -\infty$

$x_i = 0.5 \Rightarrow -1 \times 4$

$\checkmark\ x_i = 0.5 \Rightarrow -1 \times 4$
$= -4 \quad (\checkmark)$

$\left.\begin{array}{l} x_1 = 1/4 \\ x_2 = x_3 = x_4 = 3/4 \end{array}\right\}$ utility $-2 - (0.4)3$
$= -3.2 \quad (\checkmark)$

$U_i = x_i (\checkmark)$ (Max. Throughput

$U_i = 1/x_i$ (Min. Transmit delay)

$U_i = \log x_i$ (Trade off b/w max-min & Max-rate)

<u>α-fairness</u>

$$\max \sum_i U_i(x_i)$$

$$U_i = \frac{x_i^{1-\alpha}}{1-\alpha}, \quad \alpha > 0$$

$\alpha = 0 \Rightarrow$ Sum-rate Maximization

$\alpha = 2 \Rightarrow$ Min. Transit delay

$\left(\min \sum \frac{1}{x_i} \equiv \max \sum -\frac{1}{x_i} \right)$

$\alpha = 1 \Rightarrow$ Proportional fairness.

$$U_i = \frac{x_i^{1-\alpha} - 1}{1-\alpha}$$

$$\lim_{\alpha \to 1} U_i(x_i) = \log x_i$$

$\alpha \Rightarrow \infty \Rightarrow$ max-min fairness
(Max. worst case throughput)

Practical issues

- It is not feasible to have a central entity which can gather the information about which flow is using which links.
- Too Complex to solve such an optimization

- TOO Complex to solve such an optimization problem involving millions of users & links in the Internet

Sol^n – WE NEED TO COME UP WITH A SIMPLE SOLUTION WHICH CAN BE IMPLEMENTED IN A DISTRIBUTED FASHION.

Reference: Chapter 2 in the book by "R. SRIKANT" in the 3 reference listed in course handout.