

# Basics of Engineering Economics

## Chapter-1

### Introduction with Economics :

Economics is a social science which deals with human wants and their satisfaction. It is mainly concerned with the way in which a society chooses to employ its scarce resource which have alternative uses, for the good production for present and future consumption. All the scarce goods which satisfy our product wants are known as wealth. So, in economics, we study about the production of wealth, exchange of wealth, distribution of wealth and consumption of wealth. As wealth is produced to promote human welfare, we study the relationship between wealth and welfare.

→ By Alfred Marshall, Economics is a study of mankind in the ordinary business of life. Marshall agrees that economic studies about wealth. But he does not accept the view that economics studies about wealth alone. In other words of Marshall, 'Economics is one of one the one side a study of wealth, and on the other and more important side, a part of the study of man. Man is the centre of his study.'

→ Lionel Robbins has defined economics as "the science which studies human behaviour as a relationship between ends and scarce means which have alternative uses".  
1. The term ends means wants, Human wants are unlimited.  
2. Means are limited ie time, money, resources  
3. We can put time and money to alternative uses.  
4. All wants are not of equal importance.

→ Samuelson's definition of economics is known as modern definition of economics. ie. "Economics is a social science concerned chiefly with the way society chooses to employ its resources, which have alternative uses, to produce goods and services for present and future consumption".  
Samuelson's definition tells us that economics is a social science and it is mainly concerned with the way how society employs its limited resources for alternative uses.

→ There are four main divisions of economy

- Consumption
- Production
- Exchange
- distribution

In modern times, economists add one more division and that is public finance (study about the economics of government)

### Microeconomics and Macroeconomics

Economics theory can be broadly divided into Microeconomics and Macroeconomics. The term micro means small and macro means large.

In Micro-economics, we deal with problems such as the output of a single firm or industry, price of a single commodity and spending on goods by a single household.

Macro-economics studies the economic system as whole. In it, we get a complete picture of the working of the economy. It is a study of the relations between broad-economics aggregates such as total employment, saving and investment.

Macroeconomics is the theory of income, employment, prices and money.

## # Demand

- Desire with ability to pay and willingness to pay
- Various quantities of item that a buyer is willing to buy at alternative prices, other thing being equal

Demand for a commodity refers to the desire backed by ability to pay and willingness ( $\text{S} \neq \text{G} \neq \text{T}$ ) to buy it. If a person below a poverty line wants to buy a car, it is only a desire ( $\text{S} \neq \text{G}$ ) but not a demand ( $\text{G} \neq \text{T}$ ) as he cannot pay for the car. If a rich man wants to buy a car then it is demand as he has will and ability to pay. Thus, desire backed by purchasing power is demand.

The demand for any commodity ( $\text{Q}_D$ ) mainly depends on the price of that commodity. The other determinants include: price of related commodities, the income of consumers, tastes and preferences of consumers, and wealth ( $\text{E}_M$ ) of consumers ( $\text{Z}_M$ ). Hence, demand function can be written as:

$$D_x = F(P_x, P_s, Y, T, W)$$

where  $x$  is a commodity

$D_x$ : demand for  $x$  good

$P_x$ : Price of good  $X$

$P_s$ : Price of alternate goods.

$Y$ : income of consumer

$T$ : Tastes and Preferences of the consumer

$W$ : Wealth of consumer

## # Law of demand

The law of demand states that there is negative (inverse) relationship between the price and quantity demanded of a commodity over period of time.

Definition: Alfred Marshall stated that, "The greater the amount of sold, the smaller must be the price at which it is offered, in order that it may find purchasers". In other word, the amount increases demanded increases with a fall in price and diminishes with rise in price.

Alc Ferguson, the law of demand is that the quantity demanded varies inversely with price.

Thus, The law of demand states that people will buy more at lower prices and buy less at higher prices, other things remaining the same. i.e

- No change in consumers income

- No change in consumer's taste and preference

- No change in the prices of other goods.

- No new substitutes for the goods have been discovered.

- People do not feel that the present fall in price is prelude to a further decline in price

## Demand schedule and Demand curve

- Demand schedule is a tabular statement showing how much ~~no~~ of a commodity is demanded at different prices
- The demand schedule can be converted into a demand curve by measuring price on vertical axis and quantity on horizontal axis.

Price (Rs)	Qty of demands (unit)
5	10
4	20
3	30
2	40
1	50

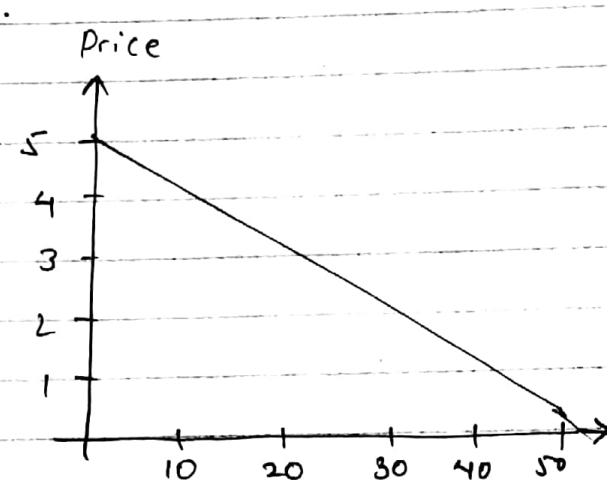


fig: Demand table (schedule)

fig: Demand curve

The curve slopes downward from left to right showing that, when price rises, less is demanded and vice-versa. Thus the demand curve represents the inverse relationship between the price and quantity demanded, other things remain constant.

The demand curve slopes downward mainly due to the law of diminishing marginal utility. The law of diminishing marginal utility states that an additional unit of a commodity gives a lesser satisfaction. Therefore, the consumer will buy more at a lower price. The demand curve slopes downward because the marginal utility curve also slopes downward.

→ The demand curve is graphical representation of demand schedule and always slopes downward.

## \* factors influencing demand.

1. Price of commodity
2. Income of consumer
3. Price of related goods
4. Weather
5. Custom and fashion
6. Size of population
7. Future expectation of price change
8. Tastes and preferences of the consumer

## # Supply

Supply means the good offered for sale at a price during a specific period of time. It is the capacity and intention of the producers to produce goods and services for sale at a specific price.

The supply of a commodity at a given price may be defined as the amount of it which is actually offered for sale per unit of time at that price.

## \* Law of Supply:

The law of supply establishes a direct relationship between price and supply. Firms will supply less at lower prices and more at higher prices.

"Other things remains same, as the price of commodity rises, its supply extent expands and as the price falls, its supply contracts"

## Supply schedule and supply curve

A supply schedule is a statement of the various quantities of given commodity offered for sale at various prices per unit time. With the help of the supply schedule, a supply curve can be drawn.

Price (in Rs)	Qty supplied (unit)
4	3
6	6
8	9
10	12

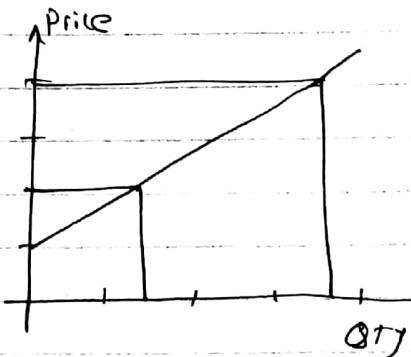


Fig: Supply schedule

Fig: Supply curve

It has positive slope. It moves upward to the right. The price of the product and quantity supplied are directly related to each other.

### \* Factors influencing supply

1. Production Technology
2. Price of factors
3. Prices of other products
4. Number of producers or firms
5. Future price expectations
6. Taxes and subsidies
7. Non-economic factors

## # Concept of Utility: Marshallian approach

There are two basic approaches to the study of consumer demand theory. They are:

i) Utility approach: use of measurable (cardinal) utility to study consumer behaviour. Marshall is the chief exponent of it. It is known as **Cardinal utility or marginal utility analysis or Marshallian utility analysis**

ii) indifference curve approach: uses the idea of comparable utility (ordinal utility)

Utility means 'usefulness'. In economics utility is defined as the power of a commodity or a service to satisfy a human want. Utility is a subjective or psychological concept.

- For a vegetarian, mutton has no utility.
- warm clothes have little utility for people living in hot countries.

So, utility depends on the consumer and his need for the commodity.

### 1. Total utility

Total utility refers to the sum of utilities of all units of a commodity consumed. For example, if a consumer consumes 10 biscuits, then the total utility is the sum of satisfaction of consuming all the ten biscuits.

## 2. Marginal Utility:

Marginal utility is the addition made to the total utility by consuming one more unit of commodity.

Example: If a consumer consumes 10 biscuits, the marginal utility is the utility derived from the 10<sup>th</sup> unit. It is nothing but the result of difference of total utility of 10 biscuits and 9 biscuits.

Thus

$$MUn = TUn - TUn-1$$

where

$MUn$  = Marginal utility of  $n^{\text{th}}$  commodity.

$TUn$  = Total utility of  $n^{\text{th}}$  commodity.

$TUn-1$  = Total utility of  $n-1$  commodity.

## \* Law of Diminishing Marginal Utility:

The law of diminishing marginal utility explains an ordinary experience of a consumer: if a consumer takes more and more units of commodity, the additional utility he derives from an extra unit of the commodity goes on falling.

Alc. Marshall, The additional benefit which a person derives from a given increase of his stock of a thing diminishes with every increase in the stock that he already has.

It states that in the process of fulfilling human wants, when a consumer consumes additional units of commodity, the utility derived from each successive unit of commodity goes decreasing.

This law is based on certain assumptions:

- utility is measurable. The measurement unit is util.
- Consumer taste and preferences unchanged.
- Suitable and similar units of commodity.
- Rational consumer
- continuous consumption etc.

<u>Units of Apple</u>	<u>Total Utility</u>	<u>Marginal Utility</u>
1	10	10
2	18	8
3	24	6
4	28	4
5	30	2
6	30	0
7	28	-2

	MU	TU
(i)	Decreases	
(ii)	Reaches zero	Increases
(iii)	Becomes negative	Reaches maximum Decline

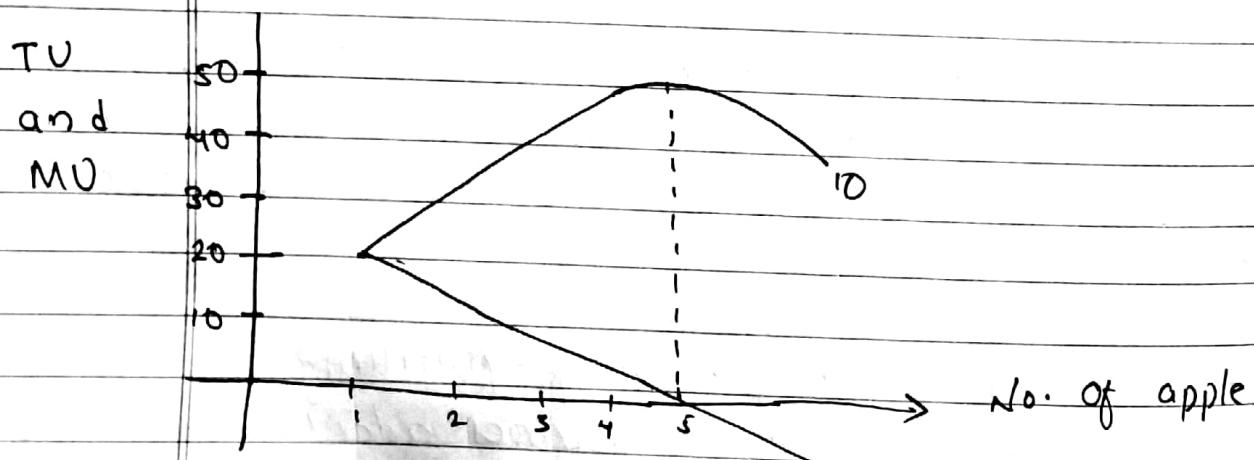


fig: law of DMO

## # Engineering Economics :

Engineering economics is the application of economic techniques to the evaluation of engineering alternatives. The role of engineering economics is to assess the appropriateness of the given project, estimates its value and justify it from an engineering standpoint.

It deals with the concepts and techniques of analysis useful in evaluating the worth of systems, products, and services in relation to their costs.

Common types of Engineering Economic Decision / Applications of EE are :

1. Equipment or Process selection
2. Equipment replacement
3. New product or product replacement expansion
4. Cost reduction, and
5. Service or quality improvement

## # Principles of Engineering Economics

Engineering Economics is focused on the principles and procedures engineers used to make sound economic decisions.

According to Parle There are four fundamental Principles of EE decisions

1. The time value of money
2. differential (incremental) cost and revenue
3. Marginal cost and revenue
4. The trade-off between risk and reward

## 1. The time value of money :

A fundamental concept in engineering economics is that money has a time value associated with it. Because we can earn interest on money received today, it is better to receive money earlier than later. This concept will be the basic foundation for all engineering project evaluation.

Ex: if you receive \$100, you can invest it and have more money available six months from now. (A nearby penny is worth than a distant dollar)

## 2. Differential (Incremental) cost and revenue

- All that counts are the differences among alternatives
- An economic decision should be based on the differences among the alternatives considered. All that is common is irrelevant to the decision.
- The economic decision should be based on the objective of making the best use of limited resources
- Whenever choice is made, something is given up. The opportunity cost of a choice is the value of the best alternative given up.

Comparing Buy versus Lease

Whatever you decide, you need to spend the same amount of money on fuel and maintenance

~~3rd qtr 2nd qtr~~  
~~1st qtr 2nd qtr~~

Option	Monthly fuel cost	Monthly maintenance	Cash outlay at signing	Monthly Payment	Salvage value at end of year 3
Buy	\$960	\$550	\$6,500	\$350	\$9000
lease	\$960	\$550	\$2400	\$550	0

Irrelevant items in decision making

3 Marginal revenue must exceed marginal cost.

- Marginal revenue means the additional revenue made possible by increasing the activity by one unit (or small unit). Marginal cost has an analogous definition.

- Effective decision making requires comparing the the additional benefits.

- Any increased economic activity must be justified on the basis of fundamental economic principle that marginal revenue must exceed marginal cost.

- Here, Marginal revenue means the additional revenue made possible by increasing the activity by one unit (or small unit).

- Marginal cost has an analogous definition

- Productive resources - the natural resources, human resources, and capital goods available to make goods and services are limited.

- Therefore, people cannot have all the goods and service they want; as a result, they must choose somethings and give up others.

- Marginal revenue must ~~not~~ exceed marginal cost

<u>Cost of Good sold</u>	\$2	per unit	Marginal revenue
<u>Gross Revenue</u>	\$4	per unit	Marginal cost

#### 4. The trade-off between risk and reward.

- Additional risk is not taken without the expected additional return.
- For delaying consumption, investors demand a minimum return that must be greater than the anticipated rate of inflation or perceived risk.
- If they didn't receive enough to compensate for anticipated inflation and the perceived investment risk, investors would purchase whatever goods they desired ahead of time or invest in assets that would provide a sufficient return to compensate for any loss from inflation or potential risk.

Expected returns and rewards from bonds and stocks are normally higher than the expected return from a savings account.

<u>Investment Class</u>	<u>Potential Risk</u>	<u>Expected Return</u>
Saving account (cash)	Low/ None	1.5 - 1.
Bond <del>equity</del> (debt) <del>equity</del>	Moderate	4.8 - 1.
Stock (equity)	High	11.5 - 1.

(According to W. Sullivan)

The principles of Engineering Economics are:

1. Develop the alternative
2. Focus on the difference
3. Use a consistent viewpoint (Economic viewpoint)
4. Use common unit of measure.
5. Consider all relevant criteria.
6. Make uncertainty explicit.
7. Revisit your decision (self evaluation)

## # Why engineering student need to study engineering economics?

- In any organization, commonly engineers are called upon to participate in a variety of strategic business decisions ranging from product design to marketing
- In manufacturing, engineering is involved in every detail of products production, from conceptual design to shipping.
- Decisions made during the engineering design phase of product development determine the majority of the costs associated with the manufacturing of that product (some say that this value may be as high as 85%).
- With more increasing GDP (Gross Domestic Product) provided by the service sector worldwide, engineers work on various economic decision problems in the service sector as well.
- As design and manufacturing processes become more complex

→ Already done in Assignment copy

### Mr. Cost Terminology

#### Manufacturing

cost (product cost)

- direct raw material cost
- direct labor cost
- manufacturing overhead

#### Non-Manufacturing

cost (

- Marketing or selling cost
- Administrative cost

### All cost of Business Decision

- ① Differential cost: A difference in cost between any two alternatives.
- ② Differential revenue: A difference in revenue between any two alternatives.
- ③ Opportunity cost: Potential benefit that is given up as you select an alternative course of action. In fact, virtually every alternative has some opportunity cost associated with it.
- ④ Sunk cost: Cost that has already been incurred (过去) by past action.
- ⑤ Marginal cost: Added cost that would result from increasing the rate of op by a single unit.
- ⑥ Marginal revenue: The revenue that can be obtained from selling one more unit of product.

# Depreciation and corporate income tax

classmate

Date \_\_\_\_\_

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## Depreciation (折舊)

Organization must deal with and account for different types of fixed asset like plant and machinery, buildings, equipment, furniture, vehicles etc. These assets lose their value due to use and the lapses of the time. This loss of value is called as depreciation.

Depreciation can be defined as a gradual decrease in the utility of fixed assets with use and time  
L chan S. parik.

- Depreciation is the gradual and permanent decrease in the value of an asset from any cause. The reduction in value of such capital asset is known as depreciation.
- Organization consider those decreased value of capital assets for accounting purpose.
- This non cash expense (loss) will collected as depreciation fund for replacement of assets and get fair financial position of firm

Depreciation accounting is to account for the cost of fixed assets in a pattern that matches their decline in value over time.

On a project level, engineers must be able to assess how the practice of depreciating fixed assets influences the investment value of a given project.

To do this, the engineer need to estimate the allocation of capital cost over the life of project, which requires an understanding of the conventions and techniques that accountants use to depreciate assets.

## Depreciation

### Economic depreciation

The gradual decrease in utility in an asset with use and time

### Accounting depreciation

The systematic allocation of an asset's value in portions over its depreciable life - often used in engineering economic analysis

→ Physical depreciation

→ Functional depreciation

→ Book depreciation

→ Tax depreciation

### \* Cause of depreciation

Economic depreciation = purchase price - market value

1. Physical depreciation - reduction in an asset's capacity to perform its intended service due to physical impairment. Physical depreciation can occur in any fixed asset in the form of
  - a. deterioration from interaction with the environment including such agents as corrosion, rotting and other chemical changes
  - b. wear and tear from use.
  - c. accidental causes due to natural disaster or by man made disaster.

Physical depreciation leads to a decline in performance and high maintenance costs

Nothing in this world is certain but death and taxes  
—Benjamin Franklin

2. functional depreciation - occurs as a result of changes in the organization or in technology that decrease or eliminate the need for an asset.

- a. Obsolescence (उत्तराधिकारी) attributable to advances in technology
- b. declining need for the services performed by an asset
- c. the inability to meet increased quantity or quality demands
- d. Time: Decline in market due to passage of time.

### Accounting depreciation

- Accounting depreciation is the systematic allocation of the initial cost of an asset in parts over a time, known as the asset's depreciable life. Because accounting depreciation is the standard of the business world, we sometimes refer to it more generally as asset depreciation.  
- In engineering economic analysis, we use the concept of accounting depreciation exclusively. This is because accounting depreciation provides a basis for determining the income taxes associated with any project undertaken.

### \* Factors affecting Depreciation amount

#### 1. Depreciable property:

1. must be used in business or must be held for the production of income
2. It must have a definite service life, and that life must be longer than 1 year
3. It must be something that wears out, decays, gets used up, becomes obsolete or loses value from natural cause

- Depreciable property includes buildings, machinery, vehicles
- Inventories are not depreciable property because they are held primarily for sale to customers in the ordinary course of business
- If an asset has no definite service life, the asset cannot be depreciate.  
Eg: you can never depreciate land.

## 2. Total cost of property (cost basis)

- The cost basis of an asset represents the total cost that is claimed as an expense over the asset's life. ie - sum of the annual depreciation expenses
- The cost basis generally includes the actual cost of the asset and all the other incidental expenses such as freight, site preparation, and installation.
- This total cost, rather than the cost of the asset only, must be the depreciation basis charged as an expense over the asset's life.

## 3. Useful life

Historically, depreciation accounting included choosing a depreciable life that was based on the service life of an asset.

## 4. Salvage value (অবস্থা মূল্য)

The Salvage value is an asset's estimated value at the end of its life - the amount eventually recovered through sale, trade-in or Salvage. The estimate salvage value must be estimated when depreciation schedule is made, if estimate prove to be inaccurate, adjustment is done.

5. Depreciation methods: Book and Tax depreciation

(i) Intended for financial reports

(book depreciation method) such as for the balance sheet or income statement

(ii) Internal revenue service (IRS) for the purpose of determining taxes (tax depreciation method)

## # Book depreciation Method

### (i) Straight line method (SL)

- Straight line method of depreciation assumes that the asset provides an equal amount of service in each year of its useful life. Thus, this method charges an equal fraction or same or fixed amount expenses as depreciation each year.

→ It is also known as fixed ~~is~~ installment method, and simplest method for charging depreciation

$$\text{Annual Depreciation} = \frac{\text{Original cost} - \frac{\text{Estimated salvage value}}{\text{Estimated life of asset}}}{\text{Estimated life of asset}}$$

If rate is given,

$$\text{Annual Depreciation} = \text{Total depreciable value} * (\text{Rate} / \cdot) / 100$$

$$\text{Rate of depreciation} = \frac{1}{N} * 100 \cdot \quad (N = \text{life of assets})$$

### Example

Consider the following data on an automobile: cost basis of the asset,  $I = \$10000$ , useful life,  $N = 5 \text{ yrs}$ . Estimated salvage value ( $s$ ) =  $\$2000$ . Use the straight line depreciation method to compute the annual depreciation and the resulting book values.

→ Solution:

Given cost of asset ( $I$ ) = \$10,000

Salvage value ( $S$ ) = \$2000

Useful life ( $N$ ) = 5 years

Annual depreciation = ?

book value for 1 to 5 years = ?

Now,

Depreciation =  $\frac{\text{Original cost of asset} - \text{Salvage value}}{\text{Expected life of asset}}$

$$= \frac{I - S}{N} = \frac{10000 - 2000}{5}$$

$$= \$1,600$$

The asset would then have the following book values during its useful life

Year	Book value at beginning	Depreciation	Book value at end
0	-	-	10,000
1	10000	1600	8400
2	8400	1600	6800
3	6800	1600	5200
4	5200	1600	3600
5	3600	1600	2000

## (ii) Declining Balance method (constant percentage method)

- Some assets value may decrease greatest in the first year of an asset's service life and least in its last year.

This pattern may occur because the mechanical efficiency of an asset tends to decline with age, because of the increasing likelihood that better equipment will become available and make the original asset obsolete or higher maintenance cost of original asset.

A depreciation method, in which double the st. line depreciation amount is taken the first year, and then that same percentage is applied to the undepreciated amount in subsequent years.

The most commonly used multipliers are:

1.5 (called 150% or DB) and

2.0 (called 200% or double-declining balance, ~~DB~~)

Then, Declining Balance Rate ( $R$ ) =  $\frac{1}{N} * 100 * 2$   
or 1.5

As  $N$  increases, depreciation expense decreases.

Depreciation high in 1<sup>st</sup> year, decreases over depreciable life.

→ If question is asked to calculate declining balance method without giving any rate, then calculate rate of depreciation first using following formula

$$\text{Rate of depreciation } (R) = 1 - \sqrt[N]{S/I}$$

where

$N$  = useful life

$S$  = salvage value

$I$  = initial cost of asset

Example:

# Use the double-declining depreciation method to compute the annual depreciation allowance and the resulting book values of the following information.

→ Initial cost of assets (I) = \$10,000

useful life (N) = 5 years

Estimated salvage value (SV) = \$778 (S Park 9.4)  
(Salvage value = book value)

Depreciation value = ?

Book value = ?

Now

$$\text{Declining Balance rate (R)} = \frac{1}{N} * 100 * 2$$

$$= \frac{1}{5} * 100 * 2 = 40\%$$

Year	Book value at B(2)	Depreciation <sup>d</sup> (40% of 2)	Bv
0	-	-	10,000
1	10000	4000	6000
2	6000	2400	3600
3	3600	1440	2160
4	2160	864	1296
5	1296	512	778 (salvage value)

- Qn. A new electric saw for cutting small pieces of lumber in the furniture manufacturing plant has a cost basis of \$4,000 and a 10 year depreciable life. The estimated salvage value SV of the saw is zero at the end of 10 years. Use declining balance (DB) method to calculate the annual depreciation amount when

$$(a) R = 2/N \quad (200\% \text{ DB method})$$

$$(b) R = 1.5/N \quad (150\% \text{ DB method})$$

Tabulate the annual depreciation amount and BV for each year.

→ The following relationship hold true for DB method.

$$d_i = B(R)$$

$$d_k = B(1-R)^{k-1}(R) \quad \text{--- } (i)$$

$$d_k^* = B[1 - (1-R)^k] \quad \text{--- } (ii)$$

$$BV_k = B(1-R)^k \quad \text{--- } (iii)$$

we define

$$B = I$$

$$R = r [0-1]$$

$$k = N (\text{time})$$

d = depreciation

Annual depreciation, cumulative depreciation, and BV are determined by using equation (i), (ii), (iii) respectively. Simple calculation for year six are as follows

$$(a) R = 2/10 = 0.2$$

$$d_6 = (\text{depreciation at year 6}) = B(1-R)^{k-1}(R) \\ = 4000(1-0.2)^6(0.2) = 262.14$$

$$d_6^* = B[1 - (1-R)^k] = 4000[1 - (1-0.2)^6] = 2951.42$$

$$BV_6 = 4000(1-0.2)^6 = \$1048.576$$

$$(b) R = 1.5/10 = 0.15$$

$$d_6 = B(1-R)^{k-1}(R) = 4000(1-0.15)^5(0.15) = 266.23$$

$$d_6^* = B[1 - (1-R)^k] = \$4000[1 - (1-0.15)^6] = \$2491.4$$

$$BV_6 = B(1-R)^k = 4000(1-0.15)^6 = \$1508.50$$

YEAR, k	0	1	2	3	4	5	6	7
$d_k$	-	800	640	512	409.6	327.68	262.144	209.7
$BV_k$	4000	3200	2560	2048	1638.4	1310.72	1048.576	838.8

8	9	10
167	134.21	107.37
671.08	536.87	429.50

$$A = 0.2 * x : B = 4000$$

Note: In last year, previously calculate depreciation and adjust depreciation.

Qn calculate BV for each year for given data using declining balance method

$$\rightarrow I = \$10,000$$

$$S = \$1000$$

$$N = 5 \text{ years}$$

Rate of depreciation not given ( $R = ?$ )

Depreciation (each year) = ?

Book value (each year) = ?

Now

$$\text{Rate of depreciation } (R) = 1 - \sqrt[5]{S/I}$$

$$= 1 - \sqrt[5]{1000/10,000} = 0.369$$

~~COT, K~~  
~~dk~~  
BVIC

$$(x) \quad A = 0.369 * X \quad B = X - A$$

COT, K	Book value at B	depreciation	BV at E
0	<del>\$10,000</del>	→	\$10,000
1	\$10,000	\$3690	\$6310
2	\$6310	\$2328.39	\$3981
3	\$3981	\$1461.21	\$2512.39
4	\$2512.39	\$927.07	\$1585.32
5	\$1585.32	\$585.32	\$1000

## # Declining Balance with conversion to straight-line depreciation:

→ Because declining balance method never reaches zero a BV of zero salvage value, it is permissible to switch from this method to the SL method so that an asset's BV will be zero (or some other determined amount, such as SVR). Also this method, is used in calculating the MACRS recovery rates. This the switchover occurs in the year when larger or equal depreciation amount is obtained from the straight line method in comparison to declining DB method.

### Example:

# Declining Balance with conversion to SL depreciation.  
Use the double-declining depreciation method to compute the annual depreciation allowances and the resulting book values of following information.  
Cost basis of the asset ( $I$ ) = \$10,000; Useful life ( $N$ ) = 5 years; Estimated Salvage value,  $S=0$  (Salvage value = 0)

→ Given. Initial Asset Value ( $I$ ) = \$10,000  
useful life ( $N$ ) = 5 years  
salvage value ( $S$ ) = 0

Depreciation and book value = ?

Declining Balance Rate ( $R$ ) =  $(\frac{1}{N}) * 100 * 2$

$$= \frac{200}{5} = 40\%$$

Year	Book value at beginning <sup>(2)</sup>	DB depreciation <sup>(2)</sup>	Switchover decision <sup>(3)</sup>	SL Dep	Selected dep	single choice					
						(1)	(2)	40% of (2)	(4)	(5)	(6)
0	-	-									
1	10,000	4,000	>	2,000	4,000	10,000					
2	6000	2400	>	1500	2400	6000					
3	3600	1440	>	1200	1440	3600					
4	2160	864	<	1080	1080	2160					
5	1080	432	<	1080	1080	1080					0

- Here 1 is year, 2 is book value at beginning
- DB depreciation = DB rate + column 2
- Swoch over  $\Rightarrow$  column 1 - estimated SVN decision
- SL Dep  $\Rightarrow$  remaining life time decision
- selected dep = ~~which one will select first~~ select ~~first~~
- Book value at end = column 2 - selected dep

Qn2. Given :

$$I = \$10,000, S = 200, N = 5 \text{ years}$$

$$\text{Declining Balance rate (R)} = \left(\frac{1}{N}\right) * 100 * 2 = 0.2 * 200 \\ = 40\%$$

Year	Book value at B	DB dep	Switchover decision	SL Dep	Selected dep	Book value at end
0	-	-				10,000
1	10,000	4000	>	1960	4000	6000
2	6000	2400	>	1450	2400	3600
3	3600	1440	>	1133.3	1440	2160
4	2160	864	<	980	980	1180
5	1180	472		980	980	200

$$\text{SL Dep} = \frac{\text{Book value}}{\text{remaining year}}$$

## \* Sinking Fund Method

- This methods consider time value of money principle while calculating depreciation amount of certain assets.
- This method calculate fixed annual depreciation amounts (fixed installment = annuity; considering time value of money) for each year as well as net depreciation amount based on compound interest rate.
- The amount with compound interest earned over the life will be equal to original cost of asset.
- This method shows the book value of assets decreases at increasing rate with respect to the life of the asset.

• Fixed annual depreciation  $Amt(A) = (I-S) \times (A/F, i\%, n)$   
 Net depreciation charges in year  $k = A \times (F/P, i\%, k-1)$   
 Book value at the end of the year,  $K = I - (A \times (F/A, i\%, k))$

Example: Compute depreciation charge and book value of each year by using sinking fund method with the following information. Salvage value = Rs 20000, Initial Cost of Asset = Rs 100,000. Useful life of asset = 8 yrs Interest = 12%.

Solution: Fixed Annual depreciation

$$\text{ie } A = \frac{(I-S)}{(A/F, 12\%, 8)} \rightarrow \text{look from table}$$

$$= \frac{(100,000 - 20,000)}{6.504} = 0.081303$$

$$= 6504.22$$

$$(A/F, 12\%, 8) = \frac{i}{(1+i)^8 - 1} = \frac{0.12}{(1+0.12)^8 - 1} = 0.081303$$

$$(1 + 0.12)^{x-1}$$

Year (1)	Book value at beginning of the year (2)	fixed Depre ciation	Int. factor (FIP, i=12%, N=8)	Net Dep (5)	Book value at the end of year (2-5)
0	-				100,000
1	100,000	6504	1	6504	93,496
2	93,496	6504	1.12	7284.48	86,211.52
3	86,211	6504	1.2544	8158.61	78,052.96
4	78052.90	6504	1.4049	9137.46	68915.25
5	68915.25	6504	1.5735	10234.17	58681.27
6	58681.27	6504	1.7623	11461.99	47219.28
7	47219.28	6504	1.9738	12837.59	34381.69
8	34381.69	6504	2.2107	14378.39	20,003.3

## # factor formula

Factor	Factor name	Formula	Purpose
FIP, i, N	single Payment compound amt factor	$(1 + i)^N$	Moves a single payment to N periods later in time
AIF, i, N	Sinking fund factor	$\frac{i}{(1+i)^N - 1}$	-
FIA, i, N	Uniform series compound Amt factor	$\frac{(1+i)^N - 1}{i}$	-
PIF, i, N		$\frac{1}{(1+i)^N}$	-

Net Dep = Fixed \* int. factor

## Sum of Year digit Method (SOYD)

- According to this method, per year depreciation charge is calculated from the ratio of the sum of the year digit for the total useful life & remaining useful life at the beginning of particular time year.
- SOYD depreciation = Remaining useful life at the beginning of the particular year  $\times \frac{(I-S)}{\text{SOYD for the total useful life}}$
- This method gives larger depreciation amount during the beginning years of assets and smaller depreciation amount as assets getting old.

Example: we have just purchased a minicomputer at a cost of Rs. 20,000 with salvage value of Rs. 1,000 and a projected useful life of 6 years.

Determine SOYD depreciation

[CPU 2013]

$$\rightarrow \text{Initial investment } (I) = 20,000$$

$$\text{Salvage value } (S) = 1000$$

$$\text{Useful life } (N) = 6 \text{ yrs}$$

$$\text{SOYD} = \frac{6+5+4+3+2+1}{21} = \frac{21}{21} = \frac{6+7}{2} = \frac{13}{2}$$

Depreciation proportion for each year = 6:5:4:3:2:1

Year	Dep proportion	SOYD calculation	Depreciation	Book Value
0	-	-	-	20,000
1	6	$6 \times (20,000 - 1000) / 21$	5428.57	14571.43
2	5	$5 \times (20,000 - 1000) / 21$	4523.81	10047.61
3	4	$4 \times (20,000 - 1000) / 21$	3619.04	6428.57
4	3	$3 \times (20,000 - 1000) / 21$	2714.28	3714.28
5	2	$2 \times (20,000 - 1000) / 21$	1809.52	1904.76
6	1	$1 \times (20,000 - 1000) / 21$	904.76	1000
		Total	18,450.00	

## \* Unit of Production method

- straight-line depreciation can be defended only if the machine is used for exactly the same amount of time each year.
- When the decrease in value is mostly a function of use, depreciation may be based on a method not expressed in terms of years.
- What happens when a punch press machine runs 1670 hours one year and 780 the next or when some of its output is shifted to a new machining center?

Depreciation =  $\frac{\text{Units of production used}}{\text{total working hours or production unit}} \times (\text{initial SV} - \text{salvage value})$

### Example

A truck for hauling coal has an estimated net cost of \$55,000 and is expected to give service for 250,000 miles, resulting in a \$5,000 salvage value. Compute the allowed depreciation amount for a truck usage of 30,000 miles.

→ Given I = \$5,000

total service units = 250,000 miles

$$S = \$5,000$$

$$\text{usage year} = 30,000$$

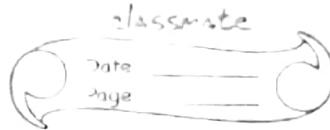
Depreciation amount in this year = ?

Now

Depreciation =  $\frac{\text{Unit of Production} - \text{Salvage value}}{\text{Total working hours}}$

$$= \frac{30,000 * (\$55,000 - \$5,000)}{250,000} = \$6000$$

BV = Book value



A piece of equipment used in a business has a basis of \$50,000 and is expected to have a \$10,000 SV when replaced after 30,000 hours of use. Find its depreciation rate per hour of use, and find its BV after 10,000 hours of operation.

$$\rightarrow \text{Initial cost (I)} = \$50,000$$

$$\text{Salvage value (J)} = \$10,000$$

$$\text{Service unit (E)} = 30,000 \text{ hours}$$

Now

$$\text{Depreciation} = \frac{\$50,000 - \$10,000}{30,000 \text{ hours}} = \$1.33 \text{ per hour}$$

$$\begin{aligned} \text{After 10,000 hours, BV} &= \$50,000 - \left( \frac{\$1.33}{\text{hour}} \right) * 10,000 \text{ hours} \\ &= \$36,700 \end{aligned}$$

Some Term :

\* Book value (BV) : The worth of a depreciable property as shown on the accounting records of company. It is the original cost basis of the property, including any adjustments, less all allowable depreciation deductions.

$$(BV)_k = \text{adjusted cost basis} - \sum_{j=1}^k (\text{depreciation deduction})$$

\* Market value (MV) : The amount that will be paid by a willing buyer to a willing seller for a property.

\* Salvage value (SV) : The estimated value of a property at the end of its useful life. It is the expected selling price of a property when the asset can no longer be used productively by its owner.

## Modified Accelerated cost recovery system ( MACRS )

- The MACRS was created by TRA 86 and is now the principal method for computing depreciation deductions for property in engineering project.
- Under the MACRS, the SV of property is always treated as zero. And useful life estimates are not used directly in calculating depreciation amount
- MACRS consist of two systems for computing depreciation deductions: The main system is called the General Depreciation System ( GDS ) and Alternative Depreciation System ( ADS )
- In general, ADS provides a longer recovery period and uses only the SL method of depreciation. Property that is placed in any tax-exempt use and property used predominantly outside the US

### example

A taxpayer wants to place in service a \$10,000 asset that is assigned to the five year class. Compute the MACRS percentages and depreciation amounts for the asset.

→ Solution :

Life of Asset ( $N$ ) = MACRS 5 year class

Cost ( $I$ ) = \$10,000

$$\text{DB rate} = \frac{1}{N} * 2 * 100 = 40\%$$

↳ Declining balance

Then MACRS percentage and Depreciation amounts are as follows:

(1)

(2)

2/accrued

Date \_\_\_\_\_

Type \_\_\_\_\_

0.20

Year	MACRS Percentage (%)	Depreciation	Depreciation (1x2)
		Basis	Amount (Rs)
1	20	\$10,000	2000
2	32	\$10,000	3200
3	19.20	\$10,000	1920
4	11.52	10,000	1152
5	11.52	10,000	1152
6	5.76	10,000	576

To calculate MACRS %, we have

Year	Calculation	MACRS %
1	$\frac{1}{2}$ year ADB depr = $0.5 * 0.40 * 100\% = 20\%$	20%
2	ADB dep = $0.40 * (100 - 20\%)$ SL dep = $100 - 20 / (5 - \frac{1}{2}) = \frac{1}{4} * 80\% = 17.78\%$	13.78%
3	ADB dep = $0.40 * (100 - 52\%)$ SL dep = $(100 - 52) / (5 - 1.5) = 13.71\%$	13.71
4	ADB dep = $0.40 * (100 - 71.2\%)$ SL dep = $(100 - 71.2) * (1/2 \cdot 5) = 11.52\%$	11.52
5	SL depreciation = $0.40 * 100 - 82.72 / 1.5 = 11.52$	11.52
6	$\frac{1}{2}$ year SL dep = $100 - (0.5)(11.52\%) = 15.76\%$ or $\frac{100 - 94.24}{2} = 15.76\%$	15.76

## # Tax depreciation method

\* Depreciation rates in Nepal:

Income tax ACT 2058 provisioned depreciation rates for different types of assets in Nepal.

<u>Block</u>	<u>Assets</u>	<u>Rate (on base amount)</u>
A	Building, structures and similar asset of permanent nature	5% per year
B	Office equipment, fixtures, furniture computer and data processing equipment	25% per year
C	Automobiles, Buses or transport assets	20% per year
D	Construction and earthmoving equipment and any other depreciable asset that are not included in any other block	15% per year
E	All intangible assets including software	cost price - <u>salvage value</u> <u>useful life</u>

For first accounting year, asset in different dates

Purchase from Shrawan 1 - Posh 30 = 100% of value  
 Magh 1 - Chaitra 31 = 2/3 of value  
 Baishakh 1 - Aasar 32 = 1/3 of value

## \* Introduction to Corporate Income tax

- Any individual and corporation have to pay income tax. Corporation income tax is tax levied (मुद्रा) by government to organization for their taxable income.
- The corporate income tax law allows deductions of the cost of goods sold, salaries and wages, rent, interest, advertising, depreciation (वृद्धि), amortization (उत्तरार्थ) { The extinction of debt, usually by sinking fund }, depletion (प्रकोप) etc as expenses
- The corporate tax rate in US is simple. There are → 4 basic rates [15%, 25%, 34% , 35%]
  - 2 surtax rates [5% and 3%]
- US. tax rates are progressive ; that is business with lower taxable income are taxed at lower rates than those with higher rates

Taxable Income	Tax rate	Tax computation formula
\$0 - \$50,000	15%	\$0 + 0.15x
50,001 - 75K	25%	7500 + 0.25(x - \$50,000)
75,001 - 100K	34%	13,750 + 0.34(x - 75,000)
100,001 - 335K	34% + 5%	22,250 + 0.39(x - 100,000)
335,001 - 10M	34%	113,900 + 0.34(x - 335,000)
10,000,001 - 15M	35%	3,400,000 + 0.35(x - 10,000,000)
15,000,001 - 18,333,333	35% + 3%	5,150,000 + 0.38(x - 15,000,000)
18,333,334 and up	35%	6,416,666 + 0.35(x - 18,333,333)

Effective tax rate =  $\frac{\text{Total tax paid}}{\text{total taxable income}} \times 100$

Example

- \* calculate effective tax rate from the following information (C.S Park Ex - 9.13)
- Rent expenses = \$ 20,000  
 Gross income = \$ 12,50,000  
 Depreciation = \$ 58,000  
 Supplies and operating expense = \$ 840,000  
 $\rightarrow$  Taxable income = Gross income - All allowable reduction (expenses)  
 $= \$12,50,000 - (\$840,000 + \$20,000 + \$58,000)$   
 $= \$ 3,32,000$

As, income is between \$100,001 to 335,000 range

$$\begin{aligned} \text{Total tax amount} &= \$22,250 + 0.39(X - 100,000) \\ &= \$22,250 + 0.39(332,000 - 100,000) \\ &= \$112,730 \end{aligned}$$

Then

$$\begin{aligned} \text{effective tax rate} &= \frac{\$112,730}{\$3,32,000} * 100 \\ &= 33.95\% \end{aligned}$$

- \* Taxation Law in Nepal

## \* Types of Tax Revenue

### (i) Direct Tax:

charging directly to person and reduces the wealth, directly paid by person who are taxed.

- Income tax (Personal and corporate)
- Property tax (E.G.JJT)
- vehicle tax (bike, car)

### (ii) Indirect Tax

Charging to person but payment burden shifted to another person / seller

- Value added tax
- Excise Duty (Bharat Bhutan)
- Custom Duty (T.S.J.T)

- The duty that is levied for goods manufactured inside the state is called excise duty.  
The duty that is levied on goods imported from a foreign country is called custom duty.

## \* Income Tax in Nepal

Income tax made up of

i) Normal Corporate tax: at 25%, certain sectors like hydropower are taxed at concessional rate of 20% and other sectors like banking are taxed at 30%.

ii) dividend: at 5%.

iii) Capital gains: For the gain from the disposition of the shares of non-listed company are subjected to withholding tax at 10% for a person and 15% for others.

## \* Corporate tax in Nepal

### corporation (Entity)

Tax rate

- The income from export and special industry 20%.
- The income from petroleum industry, banks and financial institution 30%.
- Income from industrial enterprises and related with infrastructure projects 20%.
- Other general corporate organization 25%.

## \* Personal tax in Nepal

- The taxable income of resident individual for an income will be taxed at the following rates.
  - upto Rs. 250,000 - @ 1%.
  - Next Rs 100,000 @ 15%.
  - Next Rs. 350,001 - Rs 25,00,000 - @ 25%.
  - Balance exceeding Rs 25,00,000 - @ 35%.

- The taxable income of a couple :

Upto Rs 300,000 - 1%.

Next Rs 100,000 @ 15%.

~~Balance exceeding~~

Next Rs 400,001 to 25,00,000 @ 25%.

~~Balance exceeding~~ Rs 25,00,000 @ 35%.

\* General procedure for making After Tax Economic Analysis :

→ After tax economic analysis refers to the profitability measurement of any project including all income taxes. In another word, it is the analysis of after tax cash flow estimates and profitability analysis of the project.

→ General procedure for ATCF estimates

1. Find gross income before depreciation expenses also known as BTCF.
2. calculate depreciation expenses for each year.
3. find Taxable income (Deduct depreciation from BTCF / gross income)
4. find taxes for each period
5. Deduct tax amount from BTCF to get ATCF for each year
6. Find NPV / NFW (consider time value of money) and make economic analysis.

\* After Tax cash flow (ATCF) Estimate :

Qn. Suppose an asset has been purchased for Rs. 200,000. It will be expected to produce net cash inflows of Rs. 60,000 per year during 6 years. The effective tax rate is 25% as per the Income-Tax Act 2018. Depreciation charges for the asset for next six years will be Rs 20,000; Rs 40,000; Rs 40,000; Rs 40,000; Rs 40,000; Rs 20,000 respectively. Calculate after tax cash flow (ATCF) and do an economic analysis based on PW at 10% interest rate.

Solution

calculation of NPW (10%) of ATCF ~~20~~

$$\begin{aligned}
 &= 200000 + 50,000 * (0.9091) + 55,000 * (0.8264) + \\
 &\quad 55,000 * (0.7513) + 55,000 * (0.6830) + \\
 &\quad 55,000 * (0.6209) + 50,000 * (0.5645) \\
 &= 432171.51
 \end{aligned}$$

Thus

Year	BTCF (Given)	Depreciation Given	Taxable Income BTCF - Depr	Income Tax (25%)	ATCF ATCF - Tax
0	-Rs 2,00,000	-	-	-	2,00,000
1	60,000	20,000	40,000	10,000	50,000
2	60,000	40,000	20,000	5,000	55,000
3	60,000	40,000	20,000	5,000	55,000
4	60,000	40,000	20,000	5,000	55,000
5	60,000	40,000	20,000	5,000	55,000
6	60,000	20,000	40,000	10,000	50,000

# Time Value of Money

classmate

Date \_\_\_\_\_

Page \_\_\_\_\_

## # Interest

- Money left in a saving account earns interest, so that the balance over time is greater than the sum of deposits
- Borrowing money to buy a car means repaying an amount over time, that amount includes interest (ie greater than amount borrowed)
- Cost of money is established and measured by a market interest rate, a percentage that is periodically applied and added to an amount (or varying amounts) of money over a specified length of time.

Interest, then defined as the cost of having money available for use.

Time value of money is the idea that a "dollar today is worth more than a dollar in the future" because dollar received today can earn interest.

→ Many types of transaction (eg: borrowing or investing money or purchasing machinery on credit) involve interest, but certain elements are common

\* An initial amount of money in transactions involving debt or investment

\* The interest rate measures the cost of price of money and is expressed as a percentage period of time.

\* Interest period: A period of time the ~~cost~~ determines how frequently interest is calculated.

## Methods of calculating interest

At the end of each interest period, the interest earned on the principal amount is calculated according to a specified interest rate

### → Simple Interest

(only principal earned extra amount)

$$I = (\gamma * P * N)$$

↓ Time  
 ↓ Principal  
 ↓ rate

The total amount after  $N$  periods

$$\begin{aligned} F &= P + I \\ &= P + \gamma * P * N \\ &= P(1 + \gamma N) \end{aligned}$$

### → Compound Interest

The interest earned in each period is calculated on the basis of the total amount at the end of the previous period

$$F = P(1 + \gamma)^N$$

### \* Nominal vs Effective interest Rate

Effective interest rate is the one which caters the compounding period during a payment plan. It is used to compare the annual interest between loans with different compounding periods like week, month, year etc. In general, nominal interest rate is less than the effective one. Example: A nominal interest rate of 12%. Then effective be 13% interest rate.

Assuming that the nominal interest rate is  $r$  and  $M$  compounding periods occur during the year.

$$(i_{\text{eff}}) = \left(1 + \frac{x}{M}\right)^M - 1$$

18.1. compounded monthly

$$I_{\text{eff}} = \left(1 + \frac{0.18}{12}\right)^{12} - 1 = 0.1956 = 19.56\%$$

for continuously,

$$I_{\text{eff}} = e^r - 1$$

\* What is the effective interest rate if the nominal rate is 9.1% per year. A 365 day year is used and compounding period is:

- a. yearly      c. quarterly      e. weekly      g. hourly
- b. semiannually      d. monthly      f. day      h. continuously

a. Yearly:

$$I_{\text{eff}} = \left(1 + \frac{x}{m}\right)^m - 1 = \left(1 + \frac{0.09}{1}\right)^1 - 1 = 9\%$$

b. Semiannually

$$\left(1 + \frac{0.09}{2}\right)^2 - 1$$

$$= 9.2$$

c. Quarterly

$$\left(1 + \frac{0.09}{4}\right)^4$$

$$= 9.3\%$$

d. monthly

$$\left(1 + \frac{0.09}{12}\right)^{12} - 1 = 9.38\%$$

(e) weekly :  $\left(1 + \frac{0.09}{52}\right)^{52} - 1 = 9.40\%$

f. daily :  $\left(1 + \frac{0.09}{365}\right)^{365} - 1 = 9.41\%$

g. hourly :  $\left(1 + \frac{0.09}{365 \times 24}\right)^{365 \times 24} - 1 = 9.417\%$

h. Continuously ( $I_{eff}$ ) =  $e^r - 1$   
 $= e^{0.09} - 1$   
 $= 0.09417$   
 $= 9.417\%$

### \* Cash flow

Engineering projects generally have economic consequences that occur over an extended period of time.

If an expensive piece of machinery is installed in a plant were brought on credit, the simple process of paying for it may take several years

Each project is described as cash receipts or disbursement (expenses) at different points in time

→ First cost: expense to build or to buy and install

→ Operations and Maintenance (O and M): annual expenses, such as electricity, labor and minor repair

- Salvage value: receipt at project termination for sale or transfer of the equipment (can be a salvage cost)
- Revenues: annual receipts due to sale of products or services
- Overhaul: major capital expenditure that occur during the asset's life
- Cash flow diagram (CFD) illustrates the size, size, sign and timing of individual cash flows, and forms the basis for engineering economic analysis.

CFD is created by first drawing a segmented time-based horizontal line, divided into appropriate time units. Each time when there is a cash flow, a vertical arrow is added - pointing down for costs and up for revenues or benefits. drawn to relative scale.

- Beginning of period cash flows are: rent, lease and insurance payments
- End of period cash flows are: O&M, salvage revenues

An example of cash flow diagram (CFD)

Qn. A man borrowed \$1000 from a bank at 8% interest. Two end-of-year payments:

- At the end of 1<sup>st</sup> year; he will repay half of the principal plus the interest that is due
- At the end of 2<sup>nd</sup> year, he will repay the remaining half plus the interest for the second year

Develop cash flow diagram for this problem

→ End of year      cash flow

0                        \$1000

1                        - \$580 (-\$500 - \$80)

2                        - \$540 (-\$500 - \$40)

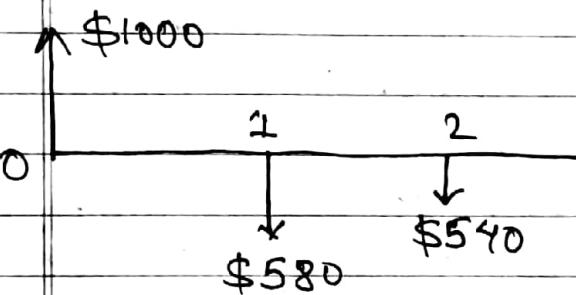
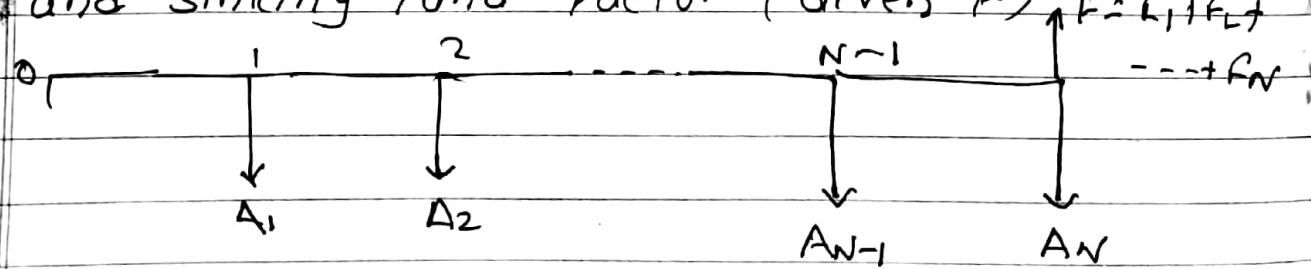


Fig: Cash flow diagram

\* Economic Equivalence: Annual worth; capital fund / Annuity factor (Given  $i$ ) and sinking fund factor (Given  $i^2$ )



# Development of formulas for equivalence calculation

Flow type	Factor notation	Formula	<del>Explain</del>
✓ Single	Compound Amt ( $F/P, i, N$ )	$F = P(1+i)^N$	
	Present worth ( $P/F, i, N$ )	$P = F(1+i)^{-N}$	
✓ Equal Payment Series	Compound Amount ( $F/A, i, N$ )	$F = A \left[ \frac{(1+i)^N - 1}{i} \right]$	
	Sinking fund ( $A/F, i, N$ )	$A = F \left[ \frac{i}{(1+i)^N - 1} \right]$	
	Present worth ( $P/A, i, N$ )	$P = A \left[ \frac{(1+i)^N - 1}{i(1+i)^N} \right]$	
	Capital Recovery ( $A/P, i, N$ )	$A = P \left[ \frac{i(1+i)^N}{(1+i)^N - 1} \right]$	
Gradient Series	Linear Gradient ( $P/G, i, N$ )	$P = G \left[ \frac{(1+i)^N - iN - 1}{i^2(1+i)^N} \right]$	
	Conversion factor ( $A/G, i, N$ )	$A = G \left[ \frac{(1+i)^N - iN - 1}{i[(1+i)^N - 1]} \right]$	

\* Given the choice of these two plans, which would you choose

Year	0	1	2	3	4	5	Total
Plan 1	\$1000	\$1000	\$1000	\$1000	\$1000	\$1000	\$1000
Plan 2	\$500						

→ Defining Cash flow diagram

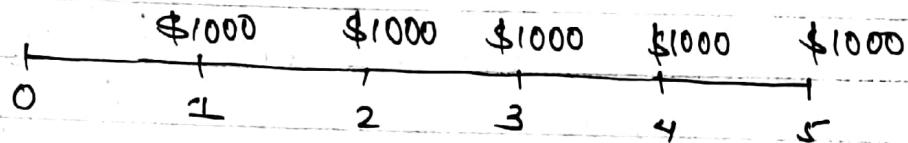
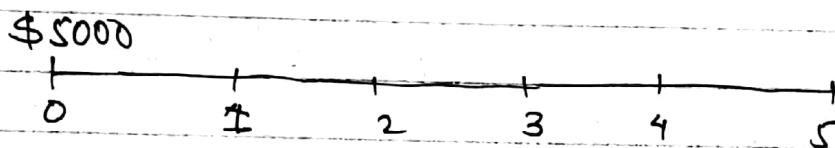


Fig: CFD for Plan 1



$$\text{(Plan 1) Present value} = \frac{1000}{1.1^1} + \frac{1000}{1.1^2} + \frac{1000}{1.1^3} + \frac{1000}{1.1^4} + \frac{1000}{1.1^5}$$

or  $1000 \times (P/A, i, N)$

$$= 1000 \times \left[ \frac{(1+i)^N - 1}{i(1+i)^N} \right] = 1000 \frac{1.1^5 - 1}{0.1 \times 1.1^5}$$

$$= 3790.78$$

$$\text{(Plan 2)} = \$5000$$

Alternative 2 is better than alternative 1 since it has greater present value

\* An Example of future value:

If \$500 were deposited in a bank saving account, how much would be in the account three years hence if the bank paid 6% interest compounded annually?

Given  $P = 500$ ,  $i = 6\% = 0.06$ ,  $N = 3$  years  
Now

future worth ( $F$ ) = ? when given  $P$   
Now

$$F = P(1+i)^N = \cancel{500} \quad P(F/P, i, N)$$

$$= 500 * (1+0.06)^3 = 595.508$$

\* An Example of Present Value

If you wished to have \$800 in a saving account at end of 4 years and 5% interest we paid annually, how much should you put into the saving account?

→ Future worth ( $F$ ) = \$800

rate ( $i$ ) = 5% = 0.05

time period ( $N$ ) = 4 years

Present worth ( $P$ ) = ? when given  $F$

Now

$$P = \cancel{0}(1 - \frac{1}{(1+i)^N}) = \frac{F}{(1+i)^N}$$

$$P = 800 * (1+0.05)^{-4} = \$658.16$$

Example :

① calculate effective rate of interest when nominal rate is 7% and compounding is:

- (i) Monthly    (ii) Daily    (iii) Quarterly    (iv) Continuously

→ Given

$$\text{Nominal rate } (\gamma) = 0.07$$

$$\text{Effective rate } (I_{\text{eff}}) = \left(1 + \frac{\gamma}{M}\right)^M - 1$$

$$\text{for continuously, } I_{\text{eff}} = e^{\gamma} - 1$$

Now

a. Monthly ( $M = 12$ )

$$I_{\text{eff}} = \left(1 + \frac{0.07}{12}\right)^{12} - 1 = 0.0722\% = 7.22\%$$

b. Daily ( $M = 365$ )

$$I_{\text{eff}} = \left(1 + \frac{0.07}{365}\right)^{365} - 1 = 7.25\%$$

c. Quarterly ( $M = 4$ )

$$I_{\text{eff}} = \left(1 + \frac{0.07}{4}\right)^4 - 1 = 7.18\%$$

d. Continuously

$$= e^{\gamma} - 1 = e^{0.07} - 1$$

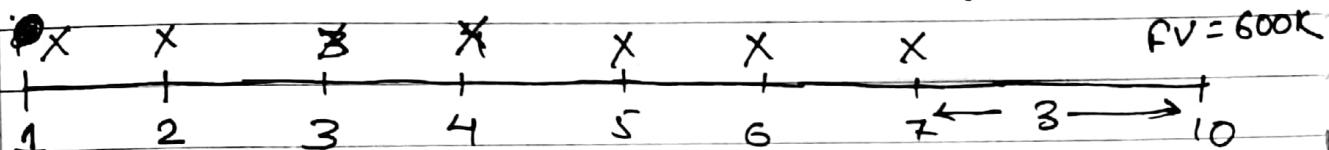
$$= 7.25$$

11) Your sister needs Rs. 600,000 at the end of 10<sup>th</sup> year for the study of +2 level. How much money should be deposited in the bank account at the end of each year for 7 continuous years from this year. If bank provides 6% rate of interest per year? Make also cash flow diagram.

$$\text{Future value (FV)} = 6,00,000$$

$$\text{Rate} = 6\% = 0.06$$

$$PV_7$$



Here, first we need to calculate the present value ( $PV_7$ ) when future worth as

~~$PV_7$~~   $PV_7 = F * (1+i)^{-N}$

$$PV_7 = 600,000 * 1.06^{-3}$$

$$PV_7 = 503771.5698$$

Here, if we pay X for 7 years and at the end of 7 years, we will get

$$PV_7$$

Here  $FV_7 = 503771.5698$  (for this it became future value)

$$A = A(F/A, i, N) \quad \text{Eqn 2}$$

$$X = (F/A, 0.06, 7) = 5,03,771.56 * \left[ \frac{1}{(1+0.06)^{7-1}} \right]$$

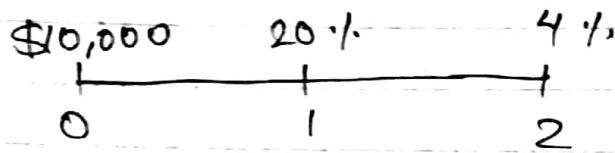
2

$$= 5,03,771.56 * \frac{0.06}{1.06^7 - 1}$$

$$= 60,016.83$$

Hence, My sister needs to deposit Rs 60,016 at the end of each of 7 years so that --

(iii) You invest \$10,000. During 1<sup>st</sup> year, you earned 20%. & second year you earned only 4%. for that year. How much is your original worth at the end of two years.



$$\begin{aligned}
 F &= PV(1+i_1) * (1+i_2) * \dots * (1+i_n) \\
 &= PV(1+i_1) * (1+i_2) \\
 &= 10,000 * 1.2 * 1.04 = \$12,480
 \end{aligned}$$

(iv) why is  $(1+i)$  called an interest factor  
 → factoring the expression  $\$10,000 + 10,000 \times i^0$   
 $= 10,000 \times (1+i)$

Thus  $(1+i)$  is an interest factor

or

we know  $I = P \times i^0 \times N \rightarrow \text{Time}$   
 and

$$\text{Total amount} = P + I$$

$$= P + P \times i^0 \times N$$

$$\therefore = P(1+i^0N)$$

WRONG  
ATTEMPT

# Nepal Engineering college (NEC) is considering to purchase a new generator costing of RS. 4,00,000 having salvage value RS. 1,00,000 at the end of 5<sup>th</sup> year. The use of generator will increase RS. 1,50,000 that needs fuel cost of RS 30,000 per year. Find the following when Minimum attractive rate of return (MARR) is 10%.

- (i) PW, AW and FW
- (ii) IRR (internal rate of return)  
(Also develop investment balance diagram and table.)
- (iii) BIC (Benefit cost ratio by PW, FW, Formulae Using conventional and modified method)
- (iv) Simple and discounted payback period
- (v) MIRR, if re-investment rate is 20%.



Given

Initial Investment (I) = RS. 4,00,000

Salvage value (S) = RS. 1,00,000

Annual Revenue (AR) = RS 1,50,000 p.a

Annual Expenses (AE) = RS 30,000 p.a

$$\text{MARR} = 10\% = 0.1$$

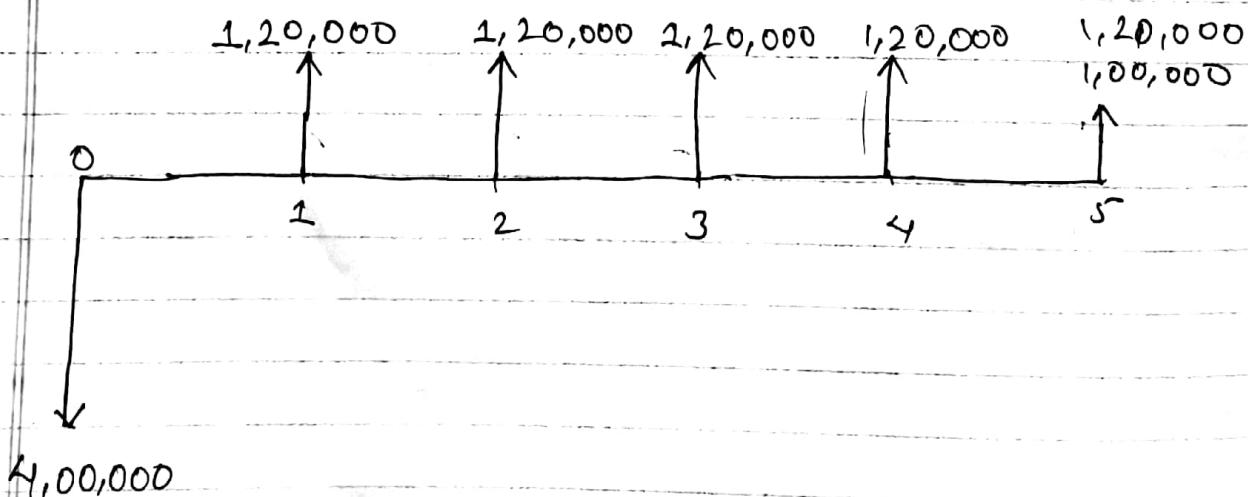


fig: Cash flow Diagram

Annual revenue = 1,50,000

Annual ~~over~~ expenses = - 30,000

Net profit 1,20,000 p.a

→ Present worth (PW / NPW) =

Present value of cash inflow - Present value of cash outflow  
which is :

$$= 1,20,000 (P/A, i, N) + 1,00,000 (P/F, i, N) - 4,00,000$$

$$= 1,20,000 \left[ \frac{(1+i)^N - 1}{i(1+i)^N} \right] + 1,00,000 \left[ \frac{1}{(1+i)^N} \right] - 4,00,000$$

$$= 1,20,000 * \left[ \frac{1 \cdot 1^5 - 1}{0 \cdot 1 * 1 \cdot 1^5} \right] + 1,00,000 \left[ \frac{1}{1 \cdot 1^5} \right] - 4,00,000$$

$$= \text{Rs. } 1,16,986.54$$

→ Annual worth (Aw or NAW)

Annual worth of cash inflow - Annual worth of cash outflow  
which is

$$= 1,20,000 \cancel{\text{NPV}} + 1,00,000 (A/F, i, N) - 4,00,000 (A/P, i, N)$$

$$= 1,20,000 + 1,00,000 \left[ \frac{i}{(1+i)^N - 1} \right] - 4,00,000 \left[ \frac{i(1+i)^N}{(1+i)^N - 1} \right]$$

$$= 1,20,000 + 1,00,000 * \left[ \frac{0 \cdot 1}{1 \cdot 1^5 - 1} \right] - 4,00,000 * \left[ \frac{0 \cdot 1 * 1 \cdot 1^5}{1 \cdot 1^5 - 1} \right]$$

$$= \text{Rs. } 30,860.76$$

→ Future worth (FW) or NFW

future value of cash inflow - future value of cash out flow

$$= 1,20,000(F/A, i, N) + 100000 \quad (1+i)^N - 4,00,000(F/P, i, N)$$

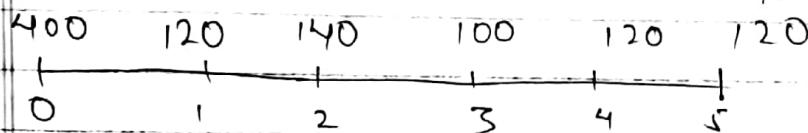
$$= 1,20,000 \left[ \frac{(1+i)^N - 1}{i} \right] + 100000 - 4,00,000 \left[ (1+i)^N \right]$$

$$= 1,20,000 * \left[ \frac{1.1^5 - 1}{0.1} \right] + 100000 - 4,00,000 * (1.1)^5$$

$$= \cancel{1,20,000} \quad 1,88408$$

\* What if, <sup>Annuity</sup> ~~is~~ is not constant through out the year.

Suppose,



$$NPW = \frac{120}{1.1^1} + \frac{140}{1.1^2} + \frac{100}{1.1^3} + \frac{120}{1.1^4} + \frac{120}{1.1^5} - 400$$

co.e, have:

$$NAW = NPW(F/P, i, N)$$

$$NAW = NFW(A/F, i, N)$$

Decision rule,

$NPW, NFW, NAW > 0$  Accept

$NPW = 0$  Indecision = indifferent

$NPW < 0$  Reject

$\therefore$  Decisions, Hence, NPW, NFW, NAW are positive so, we accept project.

### internal rate of return (IRR) (using Trial method)

(i) Trial at 10%:

$$\text{we have } NPW = 116986$$

IRR is when there is "cash inflow is equal to cash outflow"

$$\begin{aligned} \text{ie } NPW &= \text{cash inflow} - \text{cash outflow} \\ &= X - X \\ &= 0 \end{aligned}$$

But we have 116986.54

We increase rate

ii) Trial at 20%:

~~trial~~

$$NPW = 120,000(P/A, i, N) + 100,000(P/F, i, N) - 400,000$$

$$= 120,000 \times \left[ \frac{(1+i)^N - 1}{i(1+i)^N} \right] + \frac{100,000}{(1+i)^N} - 400,000$$

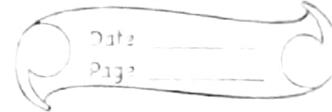
$$= 120,000 \times \left[ \frac{1.2^5 - 1}{0.2 \times 1.2^5} \right] + \frac{100,000}{1.2^5} - 400,000$$

$$= -938.786 - 938.786$$

So, rate of return must be in between  
10% to 20%.

$$Y: X: 0.1992 : A = X * 1.1992 : M: B = A + M$$

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Now,

### Interpolation

$$\text{IRR} = \frac{\text{lower rate} + \text{Amount at L.R} (\text{H.R} - \text{L.R})}{\text{Amount at L.R} + \text{Amt at H.R}}$$

$$= 0.1 + \frac{116986.54 * (0.2 - 0.1)}{116986.54 + 938.76}$$

$$116986.54 + 938.76$$

$$= 0.19920$$

$$= 19.92\%$$

### Decision Rule

If  $\text{IRR} > \text{MARR}$ , Accept

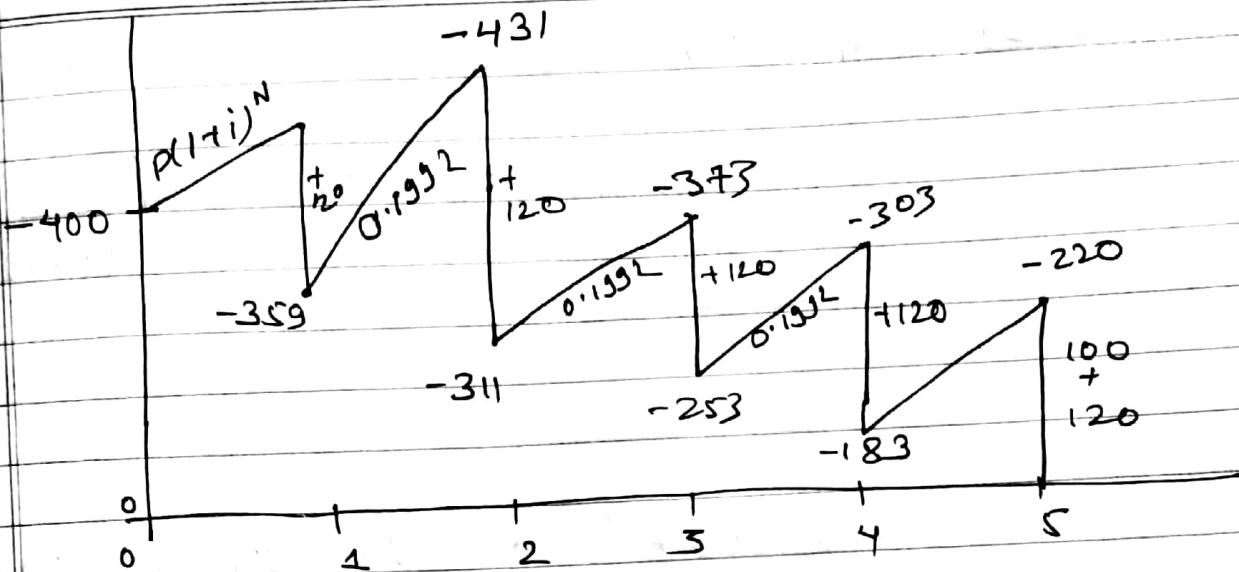
$\text{IRR} = \text{MARR}$ , indecision

$\text{IRR} < \text{MARR}$ , Reject

Hence, the  $\text{IRR}$  (ie 19.92%) which is greater than  $\text{MARR}$  (ie 10%), So we accept the project.

### Investment balance table

Year (N)	Net (Beg) cash flow (P)	Internal factor	Amount $P(1+i)^N$	Annual Revenue	Net cash flow
0	-400000	0.1992			-400000
1	-400,000	0.1992	-479680	+120000	-359680
2	-359680	0.1992	-517248	+120000	-397248
3	-397248.8	0.1992			
4					
5					



(iii) Benefit-cost ratio (BCR) ratio by Using

a. PW formulation

$$BCR \text{ (Modified)} = \frac{PW(B) - PW(OCM)}{PW(I) - PW(S)}$$

$$BCR \text{ (Modified conventional)} = \frac{PW(B)}{PW(I) - PW(S) + PW(OCM)}$$

where  $\nearrow$  Revenue

$\searrow$  expenses

B = Benefit, I = Initial, S = Salvage OCM = Operation & maint.

and

$PW(B) = PW(B)$  = present value worth benefit

$\therefore PW(B)$  is 120000 which is annuity thus  
need to convert to present value)

$$PW(B) = 120000(P/A, i, N) = 120,000 \times \left[ \frac{(1+i)^N - 1}{i(1+i)^N} \right]$$

$$= 120000 \times \frac{1.1^5 - 1}{0.1 \times 1.1^5}$$

$$= 568618.01$$

$$PW(I) = 4,00,000$$

$$PW(S) = 100,000 * (F/P, i, N) = 1,00,000 * \left( \frac{1}{(1+i)^N} \right)$$

$$= \frac{100000}{1.1^5}$$

$$= 62092.13$$

$$PW(O&M) = 30,000 * (P/A, i, N) = 30,000 * \left( \frac{(1+i)^N - 1}{i(1+i)^N} \right)$$

$$= 30,000 * \frac{1.1^5 - 1}{0.1 * 1.1^5}$$

$$= 113723.60$$

~~\* B/C conventional = PW(B)~~

$$PW(I) = PW(S) + PW(O&M)$$

$$= 568618.01$$

$$400000 - 62092.13 + 113723.60$$

$$= 1.259$$

$$* B/C (modified) = \frac{568618.01 - 113723.60}{400,000 - 62092.13}$$

$$= 1.346$$

(b) AW formulation

$$AW(\underline{I}) = 150000 * (A/I_p)$$

$$AW(I) = 400000 * (A/I_p, i, N) = 400000 * \left( \frac{i * (1+i)^N}{(1+i)^N - 1} \right)$$

$$= 400000 * \left[ \frac{0.1 + 1.1^5}{1.1^5 - 1} \right] = 400,000 * (0.2637)$$

$$= 105518.99$$

$$AW(B) = 150,000$$

$$AW(O&M) = 30,000$$

$$AW(S) = 100,000 * (A/F, i, N) = 100,000 * \left[ \frac{1}{(1+i)^N - 1} \right]$$

$$= 100,000 * \frac{0.1}{1.15 - 1}$$

$$= 16379.74$$

Now,

$$B/C (\text{Conventional}) = \frac{AW(B) - AW(O&M)}{AW(I) - AW(S) + AW(O&M)}$$

$$= \frac{150,000}{105518.99 - 16379.74 + 30,000}$$

$$= 1.259$$

$$B/C (\text{Modified}) = \frac{AW(B) - AW(O&M)}{AW(J) - AW(S)}$$

$$= \frac{150,000 - 30,000}{105518.99 - 16379.74}$$

$$= 1.346$$

Here,  $B/C > 1$ , accept

$B/C = 1$ , indifferent

$B/C < 1$ , reject

Here, using conventional,  $B/C > 1$ , accept the project  
 and using modified  $B/C > 1$ , so accept the project

## (iv) Simple and discounted payback method

* Year	Net cash flow	Cumulative cash flow	Interest factor 10%	Discount cash flow	Cumulative cash flow
0	-4,00,000	-4,00,000			
1	1,20,000	-280,000			
2	1,20,000	-160,000			
3	1,20,000	-40,000			
4	1,20,000	80,000			
5	2,20,000	3,00,000			

fig: simple payback table

Simple payback period

= minimum + Amount to be recovered

Year                      Upcoming year cash flow

$$= 3 + \frac{40,000}{1,20,000} = 3.3333$$

* Year	Net cash flow	Interest factor 10%	Discount cash flow	Cumulative cash flow
0	-4,00,000	<del>(1.1)</del> <sup>0</sup>	-4,00,000	-4,00,000
1	120000	<del>(1.1)</del> <sup>-1</sup>	109090.9	-290909.09
2	120000	<del>(1.1)</del> <sup>-2</sup>	99173.55	-191735.53
3	120000	<del>(1.1)</del> <sup>-3</sup>	90157.77	-101577.77
4	120000	<del>(1.1)</del> <sup>-4</sup>	81,961.61	-19,616.08
5	220000	<del>(1.1)</del> <sup>-5</sup>	<del>74510.55</del>	116986.61

$$\text{Simple payback period} = 4 + \frac{19,616.08}{136,602.69}$$

$$= 4.14 \text{ years}$$

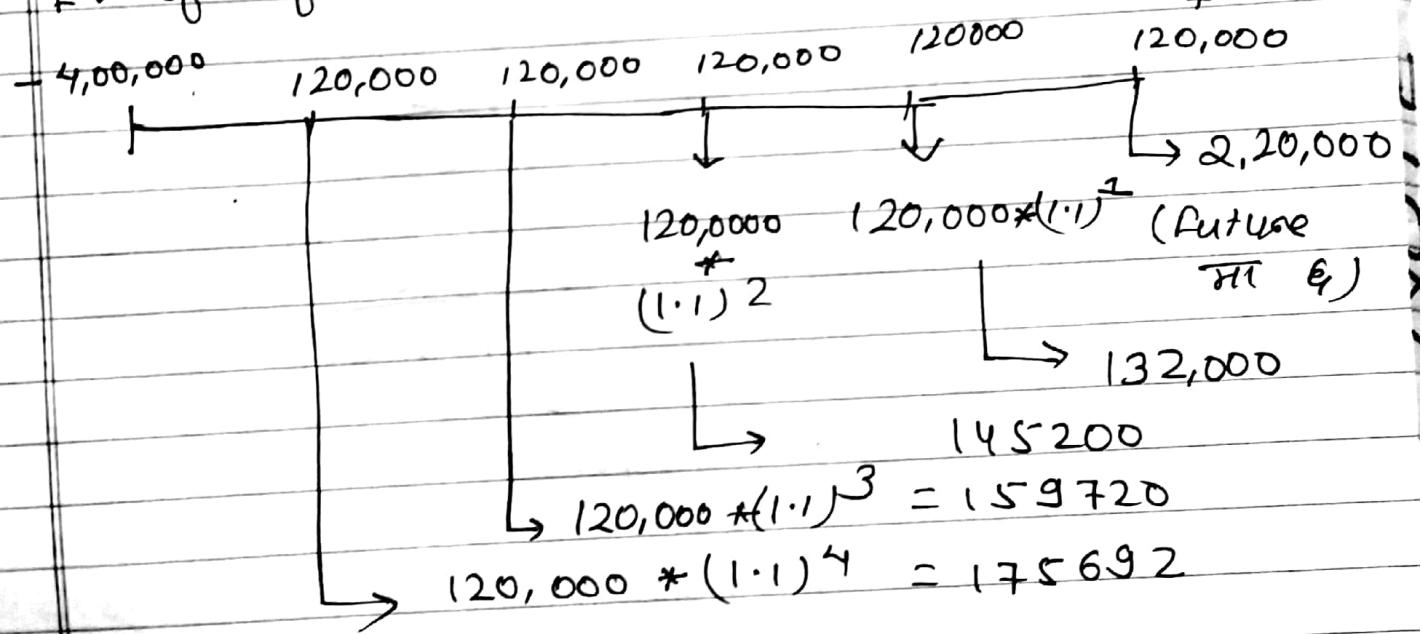
(v) Using MIRR (if re-investment rate is 10%)

$$PV_{outflow} = \frac{\text{future value of inflow}}{(1 + MIRR)^n}$$

The O

PV of outflow = -4,00,000

$$FV \text{ of inflow} = (\frac{\$100,000}{1.05}) \text{ future value of } \$100,000$$



$$\therefore \text{fv of inflow} = 220000 + 132000 + 145200 + 159720 \\ + 175692 \\ = 832612$$

allow

$$4,000,000 = \frac{832,612}{(1 + MIRR)^5}$$

$$(1 + MIRR)^5 = 2.08153$$

$\leftarrow \log 1 + MIRR = \sqrt[5]{2.08153}$

$$\text{MIRR} = 0.1579 \\ = 15.80\%$$

If re-investment rate is 20%.

$$PV \text{ of outflow} = -4,00,000$$

$$\begin{aligned} TV \text{ of inflow} &= 2,20,000 + (1.2)^0 + 1,20,000 \times 1.2^1 \\ &\quad + 120000 \times 1.2^2 + 120000 \times 1.2^3 + 120000 \times 1.2^4 \end{aligned}$$

$$\begin{aligned} &= 2,20,000 + 144000 + 172800 + 207360 + 248832 \\ &= 992992 \end{aligned}$$

Then

$$PV \text{ of outflow} = \frac{IV \text{ of inflow}}{(1 + MIRR)^n}$$

$$\frac{9929 - 4,00,000}{(1 + MIRR)^5} = \frac{992992}{4,00,000}$$

$$(1 + MIRR)^5 = 992992$$

$$4,00,000$$

$$MIRR = 0.1994$$

$$= 19.94\%$$

As MIRR (19.94%) > MARR (10%), so I accept the project without considering the other factors.

## \* Method of financing

In economic analysis, two broad decision are taken: investment decisions  
finance decisions

first the investment project is selected, and then the source of financing is considered. After the source is chosen, appropriate modifications to the investment decision are made

Under finance decision

- equity financing
- debt financing

### a. Equity financing

Equity financing can take one of two forms

1. use of retained earnings otherwise paid to stockholders, or

2. the issuance of stock

Both forms of equity financing are funds invested by the current or new owners of the stock company.

#### Use of retained earnings:

until now, most of our economic analysis presumed that companies had cash on hand to make capital investments; implicitly, we were dealing with cases of financing by retained earnings.

If a company had not reinvested these earnings, it might have paid them to the company's owner - the stockholders - in the form of a dividend, or it might have kept these earnings on hand for future needs.

## The issuance of stock:

- If a company does not have sufficient cash on hand to make an investment and does not wish to borrow in order to fund the investment, financing can be arranged by selling common stock to raise the required funds.
- Many small biotechnology and computer firms raise capital by going public and selling common stock.
- To do this, the company has to decide how much money to raise, the type of securities to issue (common stock or preferred stock), and the basis for pricing the issues.
- Once the company has decided to issue the common stock, it must estimate 'floatation costs' - the expenses it will incur in connection with the issues, such as investment banker's fees, lawyer's fees, accountants' costs and cost of printing and engraving.
- Usually, an investment banker will buy the issue from the company at a discount, below the price at which the stock is to be offered to the public, the discount usually represents the floatation costs.
- If the company is already publicly owned, the offering price will commonly be based on the existing market price of the stock.
- If the company is going public for the first time, no established price will exist, so investment bankers have to estimate the expected market price at which the stock will sell after the stock issue.

- Company should analyze how the floatation cost affects the cost of issuing common stock.
- ~~so~~ floatation cost are higher for small issues than for large ones due to the existence of fixed costs. Certain costs must be incurred regardless of the size of the issue, so the per percentage of floatation cost increases as the size of issue gets smaller

\* The company expects to acquire considerable market penetration. To produce, the company which will cost \$10 million. The company decided to raise this \$10 million by selling common stock. The firm's current stock price is \$30 per share. Investment bankers informed that public issue must be priced at \$28 per share because of decreasing demand, which will occur as more shares become available on the market. The floatation cost will be 6% of issue price, so SSI will net \$26.32 per share. How many shares must SSI sell to net \$10 million after floatation expenses.

→ Let  $X$  be no. of shares to be sold. Then total floatation cost will be

$$0.06 * \$28 * X =$$

$$= 1.68X$$

To net \$10 million, we must have

$$\text{Sales proceeds} - \text{floatation cost} = \text{Net proceeds}$$

$$28 * X - 1.68 * X = 10,000,000$$

$$26.32 * X = 10M$$

$$X = 379,940 \text{ shares}$$

$$\therefore \text{floatation cost} = \$638297.872$$

## Debt financing

- The second major type of financing a company can select is debt financing, which includes both short-term borrowing from financial institutions and the sale of long-term bonds wherein money is borrowed from investors for a fixed period.
- With debt financing, the interest paid on the loans or bonds is treated as an expense or income-tax purposes.

### a. Bond financing:

- does not involve the partial payment of principal - only interest is paid each year
- The principal is paid in a lump sum when the bond matures. It is similar to equity financing in that flotation costs are involved when bonds are issued.

### b. Term Loans

Term loans involve an equal repayment arrangement according to which the sum of interest payment and the principal payment is uniform. Interest payment decrease, while principal increase, over the life of the loan. Term loans are usually negotiated directly between the borrowing company and a financial institution, generally a commercial bank, an insurance company.

### Example: 15.2

Consider previous example, suppose SSI instead decided to raise the \$10 million by debt financing.

SSI could mortgage bond or secure a term loan.

a. Bond financing: The floatation cost is 1.8% of \$10 million issue. The company's investment banker have indicated that a 5 year bond issue with a face value of \$1000 can be sold at \$985 per share. The bond would require annual interest payments of 12%.

b. Term loan: A \$10 million bank loan can be secured at an annual interest rate of 11%. for five years it would require five equal annual installments

(i) How many \$1000 per value bonds would SSI have to sell to raise \$10 million?

(ii) what are the annual payment (int + principal) on the bond?

(iii) what are the annual payments (interest and principal) on the term loans.

→ Solution

(i) To net \$10 million, SSI would have to sell.

$$\text{floatation rate} = 1.8\% = 0.018$$

$$\text{share rate} = 1 - 0.018 = 0.982$$

then

\$10,000,000

0.982

$$= 10183299.39$$

$\approx 10183300$  worth of bonds and pay \$10183300 in floatation cost. since the \$1000 bond will be sold at 1.5% discount, the total no. of bonds to be sold would be.

$$= \frac{\$10,183,300}{\$985} = 10,338.37 \\ \approx 10,339$$

(b) for the bond financing, The annual interest is :

$$\$10,338,380 * 0.12 = \$1,240,606.$$

Only the interest is paid each period; thus, the principal amount owned remains unchanged.

(c) for the ~~term~~ term loan, annual payment are

$$\$10,000,000 (\text{AIP}, 11\%, 5)$$

$$\frac{\$10,000,000}{\$10,000,000 + \frac{0.11 * 1.11^5}{1.11^5 - 1}}$$

$$\$2705703.045$$

The principal and interest components of each annual payment are summarized

## \* Capital Structure:

The ratio of total debt to total capital, generally called the debt ratio or capital structure, represent the percentage of the total capital provided by borrowed funds. For example: a debt ratio of 0.4 indicates that 40% of the capital is borrowed and the remaining funds are provided from the company's equity (retained earning or stock offerings). This type of financing is called mixed financing.

## # Cost of Capital

We assumed that the firms under considerations were financed entirely with equity funds. In those cases, the cost of capital may be represented the firm's required return on equity.

### 1. Cost of Capital: Cost of Equity

It is the minimum rate of return a firm must offer shareholder to compensate for waiting their returns and bearing some risk.

The debt and preferred stock are easily determined but cost of equity is not easy to measure. In principle, the cost of equity capital involves opportunity cost. In fact, the firm's after tax cash flows belongs to the stockholder. Management may either pay out these earning in the form of dividends or retain the earnings, an opportunity cost is involved.

→ Cost of Retained Earnings ( $k_r$ )

$$k_r = \frac{D_1}{P_0} + g$$

where  $k_r$  = Cost of retained earnings

$D_1$  = first year dividend

$P_0$  = current stock price

$g$  = growth rate of dividend

→ Cost of Issuing New Common Stock ( $k_e$ )

$$k_e = \text{Cost of Common Stock} = \frac{D_1}{P_0(1-f_c)} + g$$

$D_1$  = first year dividend

$g$  = growth rate of dividend

$f_c$  = floatation cost as a percentage of the stock price.

→ Cost of equity

$$i_e = \left( \frac{C_r}{C_e} \right) * k_r + \left( \frac{C_c}{C_e} \right) * k_e + \left( \frac{C_p}{C_p} \right) * k_p$$

Where

Cost of Preferred stock ( $k_p$ )

$$k_p = \frac{D^*}{P^*(1-f_c)}$$

where  $D^*$  = Fixed annual dividend

$P^*$  = issuing pricing

### Example : Cost of Equity

Alpha corporation needs to raise \$10 million for plant modernization. Alpha's target capital structure calls for a debt ratio of 0.4, indicating that \$6 million has to be financed from equity.

- Alpha is planning to raise \$6 million from the following equity sources:

Source	Amount	fraction of total equity
Retained earnings	\$ 1 million	0.167
New common stock	4 million	0.666
Preferred stock	1 million	0.167

- Alpha's current common stock price is \$40, the market price that reflects the firm's future plant modernization. Alpha is planning to pay an annual cash dividend of \$5 at the end of first year, the annual cash dividend will grow at an annual rate of 8%. thereafter.
- Additional common stock can be sold at the same price of \$40, but there will be 12.4% floatation cost.
- Alpha can issue \$100 per preferred stock with 9% dividend (ie Alpha calculate dividend on the basis of per value, which is \$9 per share). The stock can be sold on the market for \$95, and Alpha ~~not~~ must pay floatation cost of 6% of market price.

Determine the cost of equity to finance the plant modernization

$$\text{and } Ce = Cr + Cc + Cp$$

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Page \_\_\_\_\_

### Solution

We will itemize the cost of each component of equity:

- Cost of retained earnings with

$$D_1 = \$5; g = 0.08(8\%); P_0 = \$40$$

$$K_Y = \frac{D_1}{P_0} + g = \frac{5}{40} + 0.08 = 20.5\%$$

- Cost of new stock with

$$D_1 = \$5, g = 0.08, F_c = 12.4\%, P_0 = \$40$$

$$K_e = \frac{\cancel{P_0}}{40(1-\cancel{F_c})} - \frac{D_1}{P_0(1+F_c)} + g = \frac{5}{40*(1+0.124)} + 0.08 \\ = 22.27\%.$$

- Cost of preferred stock

$$\text{with } D^* = \$9, P^* = \$95, F_c = 0.06$$

$$K_P = \frac{D^*}{P^*(1-F_c)} = \frac{9}{95(1-0.06)} = 10.08\%$$

→ Cost of equity:

$$\text{with } \frac{Cr}{Ce} = 0.167, \frac{Cc}{Ce} = 0.666, \frac{Cp}{Cp} = 0.162$$

$$ie = \frac{Cr}{Ce} * K_Y + \frac{Cc}{Ce} * K_e + \frac{Cp}{Ce} * K_P$$

$$= (0.167 * 0.205) + (0.666 * 0.2227) + (0.162 * 0.1008)$$

$$= 19.96\%$$

where

$C_E$  = amount of equity financed from retained earnings

$C_C$  = from issuing new stock

$C_P$  from preferred stock

$$C_E = C_E + C_C + C_P$$

$i_E$  = weighted-average cost of equity

# Alternative way of determining cost of equity.

Cost of equity:

$$i_E = r_f + \beta [r_m - r_f]$$

where

$r_f$  = risk free rate

$r_m$  = market rate of return

$\beta$  = firms beta

Example

- \* Alpha corporation needs to raise \$10 million for plant modernization. Alpha's target capital structure calls for a debt ratio of 0.4 indicating that \$6 million has to be financed from equity.
- Alpha is planning to raise \$6 million from market
- Alpha's  $\beta$  is 1.99
- The risk free rate is 6% & average market return is 13%.

Determine cost of equity to finance it.

$$\rightarrow \text{Given } r_m = 13\% = 0.13, r_f = 6\% = 0.06, \beta = 1.99$$

then

$$i_E = r_f + \beta [r_m - r_f]$$

$$= 0.06 + 1.99 [0.13 - 0.06]$$

$$= 19.93\%$$

Note: If Alpha invest finances the project entirely from its equity funds, the project must earn at least 19.93% return on investment

### \* Cost of debt:

To calculate the after-tax cost of debt ( $i_d$ ) we evaluate the expression

$$i_d = \left( \frac{c_s}{c_d} \right) k_s (1 - t_m) + \left( \frac{c_b}{c_d} \right) * k_b (1 - t_m)$$

where

$c_s$  = short-term loan

$c_b$  = bond-financing

$c_d = c_s + c_b$

$k_s$  = before tax interest rate on term loan

$k_b$  = before tax interest rate on bond

$t_m$  = firms marginal tax rate.

cost of debt ( $i_d$ ): the effective rate that company pays on its current debt.

### - Example

Suppose that Alpha has decided to finance the remaining \$4 million by securing a term loan and 20-year \$1000 par bonds under the following condition

Source	Amount	fraction	Interest rate	floatation cost
Term loan	\$1 million	0.333	12% per year	
Bonds	\$3 million	0.667	10% per year	6%

If the bond can be sold to net \$940 (after deducting the 6.1% floating cost) determine the cost of debt to raise \$4 million for the plant + modernization. Alpha's marginal tax rate is 38%. and it is expected to remain constant in the future.

→ Solution

$$tm = \text{firm's marginal tax rate} = 0.38$$

floatation cost ( $F_c$ ) =

$$\frac{Cs}{Cd} = 0.333, \quad k_s = 0.12$$

$$\frac{cb}{cd} = 0.667 \quad k_b = \cancel{\text{?}}$$

Here, we need to find the effective ~~tax rate~~<sup>cost</sup> of issuing the bond with a floatation cost of 6.1%. The before tax specific cost is found by solving the equivalence formula

$$\text{Present value} = \$940$$

$$PV = \$100 * (P/A, k_b, 20) + \$1000 * (P/F, k_b, 20)$$

$$\$940 = 100 * \left[ \frac{(1+k_b)^{20} - 1}{k_b * (1+k_b)^{20}} \right] + 1000 * \frac{1}{(1+k_b)^{20}}$$

Solving, we get

$$k_b = 10.74\%$$

so

$$\begin{aligned} id &= 0.333 * 0.12 * (1 - 0.38) + 0.667 * 0.1074 * (1 - 0.38) \\ &= 6.92\% \end{aligned}$$

- \* Calculating the cost of capital (Marginal cost of capital)
 

The cost of capital represents a composite index reflecting the cost of raising funds from different sources. The cost of capital is defined as:

$$K = \frac{i_d * c_d}{V} + \frac{i_e * c_e}{V}$$

where  $c_d$  = total debt capital

$c_e$  = total equity capital

$V = c_d + c_e$

$i_e$  = average equity interest rate per period

$i_d$  = After tax debt rate

$K$  = tax adjusted cost of capital

→ Example

$$c_d = \$4 \text{ million}, c_e = \$6 \text{ million}$$

$$V = c_d + c_e = 10 \text{ million}$$

$$i_d = 6.92\%$$

$$i_e = 19.93\%$$

Now

$$\begin{aligned} K &= \frac{0.692 * 4}{10} + \frac{0.1993 * 6}{10} \\ &= 14.73\% \end{aligned}$$

\* Method of describing project risks:

- Sensitivity Analysis
- Break-even Analysis
- Scenario Analysis

### ① Sensitivity Analysis

[done in old go copy Brinda's Assignment]

#### Example

If initial investment is 11,500, Annual revenue is 3,000, Time period: 6 years, MARR = 10%.

Perform sensitivity analysis over the range of  $\pm 40\%$  by the parameters

- i) Initial investment
- ii) Annual revenue
- iii) Useful life.

$$\begin{aligned}
 PW(10\%) &= -11,500 + 3,000 (P/A, 10\%, 6) + 1000 * (P/F, 10\%, 6) \\
 &= -11,500 + 3000 * \left[ \frac{(1+i)^N - 1}{i(1+i)^N} \right] + 1000 * \frac{1}{(1+i)^N} \\
 &= -11,500 + 3000 * 4.3552 + 1000 * 0.5644 \\
 &= 2130
 \end{aligned}$$

a. When initial investment varies by  $\pm 40\%$ ,  
PW would be

$$\begin{aligned}
 \text{At } I(+40\%) \Rightarrow PW &= -11,500 * (1.4) + 3000 * (P/A, 10\%, 6) \\
 &\quad + 1000 * (P/F, 10\%, 6) \\
 &= -2470
 \end{aligned}$$

$$\begin{aligned}
 \text{At } I(-40\%), PW &= -11,500 * (0.6) + 3000 * (P/A, 10\%, 6) \\
 &\quad + 1000 * (P/F, 10\%, 6) \\
 &= 6730
 \end{aligned}$$

b. When Annual revenues varies  $\pm 40\%$ , PW would be

$$\text{At AR} (+40\%) = \text{PW} = -11,500 + 3000 * (1.4) * (4.3552) \\ + 1000 * (0.5644) = 7356$$

$$\text{At AR} (-40\%) \Rightarrow \text{PW} = -11,500 + 3000 * 0.6 * 4.3552 \\ + 1000 * 0.5664 = -3096$$

c. when useful life varies  $\pm 40\%$ , the PW would be

$$\text{At } (N = +40\%) \quad N = 6 + 0.4 * 6 = 8.4$$

$$\text{PW} = -11,500 + 3000 * (P/A, 10\%, 8.4) + 1000 * (P/F, 10\%, 8.4) \\ =$$

$$\text{At } (N = -40\%) \quad N = 6 - 0.4 * 6 = 3.6$$

$$-11,500 + 3000 * (P/A, 10\%, 3.6) + 1000 * (P/F, 10\%, 3.6) \\ =$$

### Sensitivity Analysis table

PW(10%) with sensitivity of  $\pm 40\%$ .

Parameters	$-40\%$	$0\%$	$+40\%$
I	6730	2130	-2470
AR	-3096	2130	7356
N	-2077	2130	5476

## \* Break-even Analysis

BEP = Break even Point

$$- \text{BEP (unit)} = \frac{\text{fixed cost}}{\text{SPPU} - \text{VCPU}}$$

where

SPPU = Selling price per unit

VCPU = Variable cost per unit

BEP = Break even point

$$- \text{BEP (Amount)} = \text{BEP (unit)} * \text{S.P.P.U}$$

$$- \text{Total cost} = \text{fixed cost (FC)} + \text{total variable cost (TVC)}$$

$$- \text{TVC} = \text{unit demand} * \text{variable cost (V.C.)}$$

$$- \text{Profit} = \text{Revenue} - \text{Expenses}$$

At BEP, profit = 0

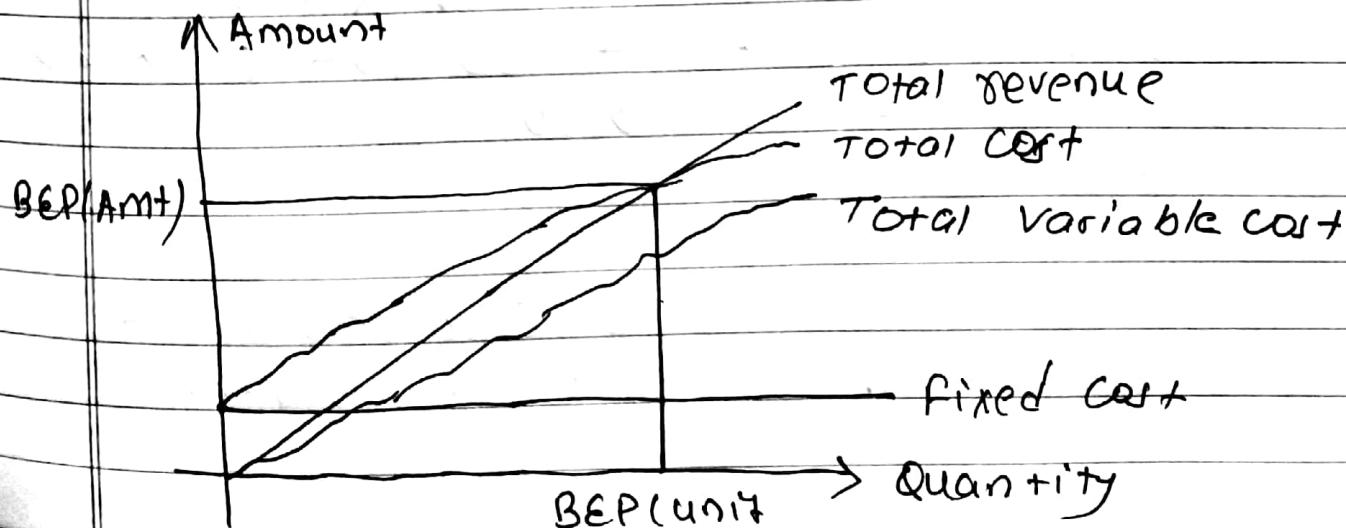
$$\text{Revenue} - \text{Expenses} = 0$$

$$\therefore \text{Revenue} = \text{expenses}$$

$$- \text{Net profit} = \text{Revenue} - \text{Expenses}$$

$$= (\text{S.P.P.U} * x) - (\text{FC} + x * \text{V.C.P.U})$$

where  $x$  = unit of demand



Example

Find BEP (volume/unit) and BEP (Amount) if  
fixed cost (FC) = Rs. 5,00,000

Variable cost per unit (V.C.P.U) = Rs. 20,000

Selling Price per unit (S.P.P.U) = Rs. 30,000

ALSO what would be the output (production  
unit) if company wishes profit of Rs. 5,00,000

→ Solution:

Given

$$FC = \text{Rs. } 5,00,000$$

$$VCPU = \text{Rs. } 20,000$$

$$SPPU = \text{Rs. } 30,000$$

$$\begin{aligned} \text{BEP (unit)} &= \frac{FC}{SPPU - VCPU} = \frac{5,00,000}{30000 - 20000} \\ &= 50 \text{ units} \end{aligned}$$

$$\begin{aligned} \text{BEP (Amount)} &= \text{BEP (unit)} * S.P.P.U \\ &= 50 * 30,000 \\ &= \text{Rs. } 1,50,000 \end{aligned}$$

For Profit 5,00,000

let  $X$  be a output (units)

Profit = Revenue - Expenses

Profit =  $X * S.P.P.U - (FC + X * V.C.P.U)$

$$5,00,000 = 30,000 * X - 5,00,000 - 20,000 * X$$

$$10,000 X = 10,00,000$$

$$X = 100 \text{ units}$$

Thus, for generating Rs 5,00,000 profit,  
we need to sell 100 units of product.