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Regularization

K. Breininger, V. Christlein, Z. Yang, L. Rist, M. Nau, S. Jaganathan, C. Liu, N. Maul, L. Folle, M. Zinnen, K. Packhäuser
Pattern Recognition Lab, Friedrich-Alexander University of Erlangen-Nürnberg
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Tasks in this exercise

1. Optimization Constraints: Augmenting the loss function
2. Dropout **Layer**
3. Batch Normalization **Layer**
4. LeNet: Put everything together **(optional)**
5. RNN layer: Elman Unit
6. LSTM layer: Backpropagation at its best! **(optional)**

Optimization Constraints: Loss function augmentation

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- Since constraints generate part of the loss:

➔ Change Neural Network container class (and associated classes) to “channel” and gather **regularization loss** for **all layers**

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→ In each trainable layer, include **the gradient of norm** when calculating update

L_2 regularization

- Forward pass:

$$\tilde{L}(\mathbf{w}) = L(\mathbf{w}) + \lambda \|\mathbf{w}\|_2^2$$

- Backward pass:

$$\mathbf{w}^{(k+1)} = \underbrace{\left(1 - \eta\lambda\right)\mathbf{w}^{(k)}}_{\text{Shrinkage}} - \eta \frac{\partial L}{\partial \mathbf{w}^{(k)}}$$

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- Notice for matrices we compute here the Frobenius norm, not the Spectral norm.
- The influence of constraints is controlled via λ. Because `lambda` is a python keyword, you want to use e.g. `alpha` instead.

L_1 regularization

- Forward pass:

$$\tilde{L}(\mathbf{w}) = L(\mathbf{w}) + \lambda \|\mathbf{w}\|_1$$

- Backward pass:

$$\mathbf{w}^{(k+1)} = \underbrace{\mathbf{w}^{(k)} - \eta \lambda \operatorname{sign}\left(\mathbf{w}^{(k)}\right)}_{\text{Other shrinkage}} - \eta \frac{\partial L}{\partial \mathbf{w}^{(k)}}$$

Dropout

Method

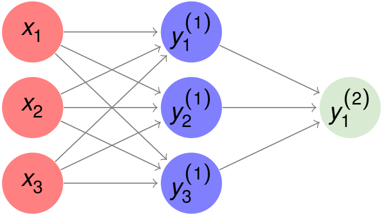

Figure: Dropout

- Implement this as a **fixed-function layer**

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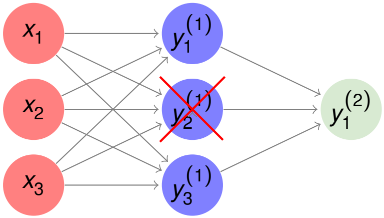

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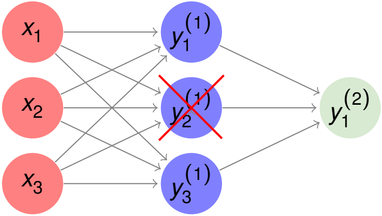

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➔ Change the behavior during training

- Multiply activations in forward-pass **only during training** by $\frac{1}{p}$
- Note: the backward pass has to be adapted as well!

Batch normalization

Forward pass

- Normalization as a new layer with 2 parameters, γ and β

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$\boldsymbol{\mu}_B$ and $\boldsymbol{\sigma}_B^2$ from **batch**

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- Notice that β is a **bias**

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- Therefore a **moving average** is common:

$$\tilde{\boldsymbol{\mu}}^{(k)} \approx \alpha \tilde{\boldsymbol{\mu}}^{(k-1)} + (1-\alpha)\boldsymbol{\mu}_B^{(k)}$$
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- Moving average **decay** α (e.g. 0.8)
- The exponent (k) and (k-1) are iteration-indices!

Backward pass

- Gradient **with respect to weights** is simply:

$$\frac{\partial L}{\partial \gamma} = \sum_{b=1}^{B} \frac{\partial L}{\partial \hat{\mathbf{Y}}_b} \tilde{\mathbf{X}}_b = \sum_{b=1}^{B} \mathbf{E}_b \tilde{\mathbf{X}}_b$$

- For the **bias** likewise we have:

$$\frac{\partial L}{\partial \beta} = \sum_{b=1}^{B} \frac{\partial L}{\partial \hat{\mathbf{Y}}_b} = \sum_{b=1}^{B} \mathbf{E}_b$$

Backward pass

The **gradient with respect to the input** is more complicated, but here it is:

$$\frac{\partial L}{\partial \tilde{\mathbf{X}}} = \frac{\partial L}{\partial \hat{\mathbf{Y}}} \odot \gamma$$

$$\frac{\partial L}{\partial \boldsymbol{\sigma}_B^2} = \sum_{b=1}^{B} \frac{\partial L}{\partial \tilde{\mathbf{X}}_b} \odot (\mathbf{X}_b - \boldsymbol{\mu}_B) \odot \frac{-1}{2} \left(\boldsymbol{\sigma}_B^2 + \epsilon\right)^{\frac{-3}{2}}$$

$$\frac{\partial L}{\partial \boldsymbol{\mu}_B} = \left(\sum_{b=1}^{B} \frac{\partial L}{\partial \tilde{\mathbf{X}}_b} \odot \frac{-1}{\sqrt{\boldsymbol{\sigma}_B^2 + \epsilon}}\right) + \underbrace{\frac{\partial L}{\partial \boldsymbol{\sigma}_B^2} \odot \frac{\sum_{b=1}^{B} -2(\mathbf{X}_b - \boldsymbol{\mu}_B)}{B}}_{0}$$

$$\frac{\partial L}{\partial \mathbf{X}} = \frac{\partial L}{\partial \tilde{\mathbf{X}}} \odot \frac{1}{\sqrt{\boldsymbol{\sigma}_B^2 + \epsilon}} + \frac{\partial L}{\partial \boldsymbol{\sigma}_B^2} \odot \frac{2(\mathbf{X} - \boldsymbol{\mu}_B)}{B} + \frac{\partial L}{\partial \boldsymbol{\mu}_B} \odot \frac{1}{B}$$

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- `compute_bn_gradients`

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- ... and do the **same** in the **backward pass**

LeNet (optional)

LeNet architecture

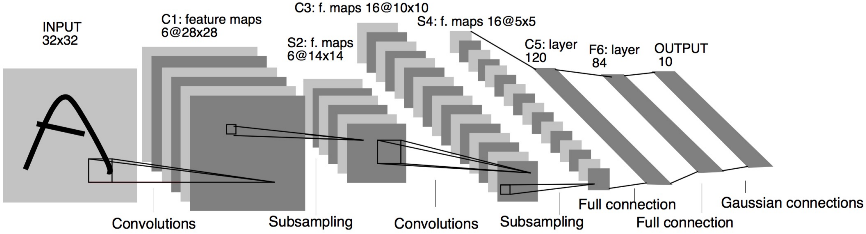

Figure: LeNet

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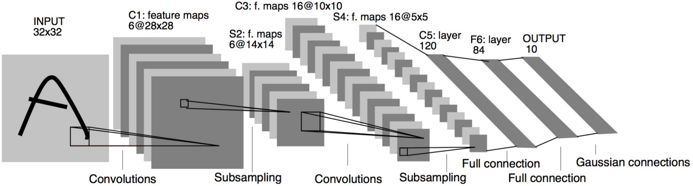
Modified LeNet architecture

Deviations

- Input is 28×28
- Our conv only supports “same” padding - so C3 has **larger activation maps**
- Input to **C5** is also **larger**
- We only implemented ReLUs, so **no** TanH
- We also use the implemented SoftMax **instead of** RBF units



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Thanks for listening.

Any questions?