Lemma A.3. *Under Hypotheses 1 and 2, for every $\kappa \geq 2$ and $\delta > 0$ we have that for every*

$$\gamma > d - 2 + \alpha,$$

it holds

$$\mathbb{E} \sup_{t \in [0,T]} |z_\delta(t)|_H^\kappa \leq c_{\kappa,\gamma}(T)\, \delta^{-\frac{\gamma\kappa}{2}}, \qquad \delta \in (0,1). \tag{A.8}$$

Finally, by using again a factorization argument, for every $s > 0$ and $\kappa > \frac{2}{s} \vee 1$ we have

$$|z_\delta(t)|^\kappa_{H^{-s}(D)} \leq c \left(\int_0^T \sigma^{-\left(\frac{s}{2}-1\right)\frac{\kappa}{\kappa-1}}\, d\sigma \right)^{\kappa-1} \int_0^t |z_{\delta,s}(\sigma)|^\kappa_{H^{-\rho}(D)}\, d\sigma \leq c(T) \int_0^t |z_{\delta,s}(\sigma)|^\kappa_{H^{-s}(D)}\, d\sigma.$$

Therefore, due to (A.4) we can conclude that the following result is true.

Lemma A.4. *Under Hypotheses 1 and 2, for every $s > 0$, $\delta \in (0,1)$ and $\kappa \geq 1$we have that*

$$\mathbb{E} \sup_{t \in [0,T]} |z_\delta(t)|^\kappa_{H^{-s}(D)} \leq c_\rho(T) \begin{cases} \log \delta^{-1}, & \textit{if } d = 2, \\ \\ \delta^{-(d-2)}, & \textit{if } d \geq 3. \end{cases} . \tag{A.9}$$

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