

is greater than  $K_1 n(1-a) |\log n(1-a)|$ , where  $K_1$  is a constant independent of  $a$  and  $n$ .

We now consider the Blaschke product

$$B(z) = \prod \frac{a_k^{n_k} - z^{n_k}}{1 - a_k^{n_k} z^{n_k}}.$$

The product converges if  $\sum n_k(1-a_k) < \infty$ , in particular, if

$$n_k(1-a_k) = 1/k(\log k)^{3/2} \quad (k = 2, 3, \dots).$$

If moreover the sequence  $\{n_k\}$  increases fast enough, we obtain disjoint intervals  $r_k < r < a_k$  such that

$$\begin{aligned} \int_{r_k}^{a_k} \int_0^{2\pi} |B'(re^{i\theta})| r d\theta dr &> K_2 n_k(1-a_k) |\log n_k(1-a_k)| \\ &> K_2/k(\log k)^{1/2}, \end{aligned}$$

and Mergeljan's theorem is proved.

From our construction, we see immediately that *there exists a continuous function f satisfying condition (1)*. Indeed, it is sufficient to choose finite Blaschke products  $B_m$  such that, for each of certain disjoint concentric annuli  $A_m$ ,

$$\iint_{A_m} |B'_m| dS - \sum_{j \neq m} \iint_{A_j} |B'_j| dS > m^3,$$

and to take  $f(z) = \sum m^{-2} B_m(z)$ .

Our second example is based on the function

$$g(z) = \exp\left(-a \frac{1+z^n}{1-z^n}\right).$$

Since the maximum and minimum modulus of  $g(z)$  on the circle  $|z^n| = \rho$  are

$$\exp\left(-a \frac{1-\rho}{1+\rho}\right) \text{ and } \exp\left(-a \frac{1+\rho}{1-\rho}\right),$$

the function  $g$  maps the circle  $C_r$  onto a curve of length greater than

$$2n \left\{ \exp\left(-a \frac{1-r^n}{1+r^n}\right) - \exp\left(-a \frac{1+r^n}{1-r^n}\right) \right\}.$$

To estimate the integral of this lower bound over the interval  $0 < r < 1$ ,