Propensity scores for continuous exposures

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The story so far

Propensity score weighting

1. Fit a propensity model predicting exposure `x`, `x + z` where `z` is all covariates
2. Calculate weights
3. Fit an outcome model estimating the effect of `x` on `y` weighted by the propensity score

Continous exposures

1. Use a model like `lm(x ~ z)` for the propensity score model
2. Scale weights to probability-like scale using `dnorm(true_value, fitted_value, estimated_sd)`
3. Apply the weights to the outcome model as normal!

Alternative: quantile binning

1. Bin the continuous exposure into quantiles and use categorical regression like a multinomial model to calculate probabilities.

2. Calculate the weights where the propensity score is the probability you fall into the quantile you actually fell into. Same as the binary ATE!

3. Same workflow for the outcome model

1. Fit a model for `exposure ~ confounders`

```r
model <- lm(
  exposure ~ confounder_1 + confounder_2,
  data = df
)
```

2. Calculate the weights with `dnorm()`

```r
model |>
  augment(data = df) |>
  mutate(denominator = dnorm(
    exposure,
    mean = .fitted,
    sd = mean(.sigma, na.rm = TRUE)
  ))
```

Does change in smoking intensity (`smkintensity82_71`) affect weight gain among lighter smokers?

```
nhefs_light_smokers <- nhefs_complete |>
  filter(smokeintensity <= 25)
```

1. Fit a model for `exposure ~ confounders`

```r
nhefs_denominator_model <- lm(
  smkintensity82_71 ~ sex + race + age + I(age^2) +
    education + smokeintensity + I(smokeintensity^2) +
    smokeyrs + I(smokeyrs^2) + exercise + active +
    wt71 + I(wt71^2),
  data = nhefs_light_smokers
)
```

2. Calculate the weights with `dnorm()`

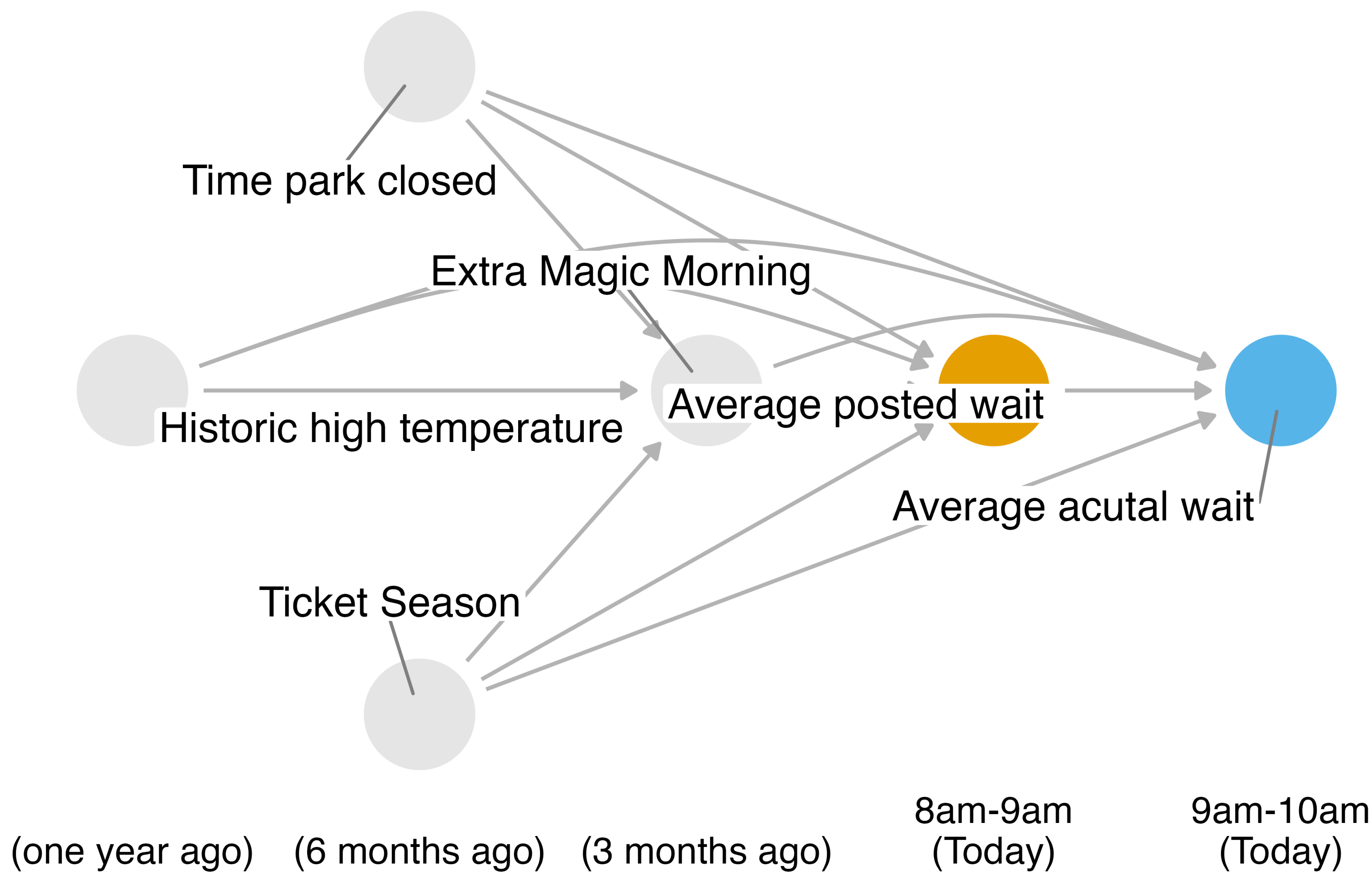
```
nhefs_denominators <- nhefs_denominator_model |>
  augment(data = nhefs_light_smokers) |>
  mutate(denominator = dnorm(
    smkintensity82_71,
    .fitted,
    mean(.sigma, na.rm = TRUE)
  )) |>
  select(id, denominator)
```

2. Calculate the weights with `dnorm()`

```
nhefs_denominators
```

```
# A tibble: 1,162 × 2
      id denominator
   <int>       <dbl>
 1     2      0.0265
 2     3      0.0275
 3     4      0.0314
 4     5      0.0371
 5     6      0.0262
 6     7      0.0364
 7     8      0.0381
 8     9      0.0386
 9    10      0.0129
10    13      0.0386
```

Do *posted* wait times at 8 am affect *actual* wait times at 9 am?



Your Turn 1

Fit a model using `lm()` with `avg_spostmin` as the outcome and the confounders identified in the DAG.

Use `augment()` to add model predictions to the data frame

In `dnorm()`, use `.fitted` as the mean and the mean of `.sigma` as the SD to calculate the propensity score for the denominator.

05:00

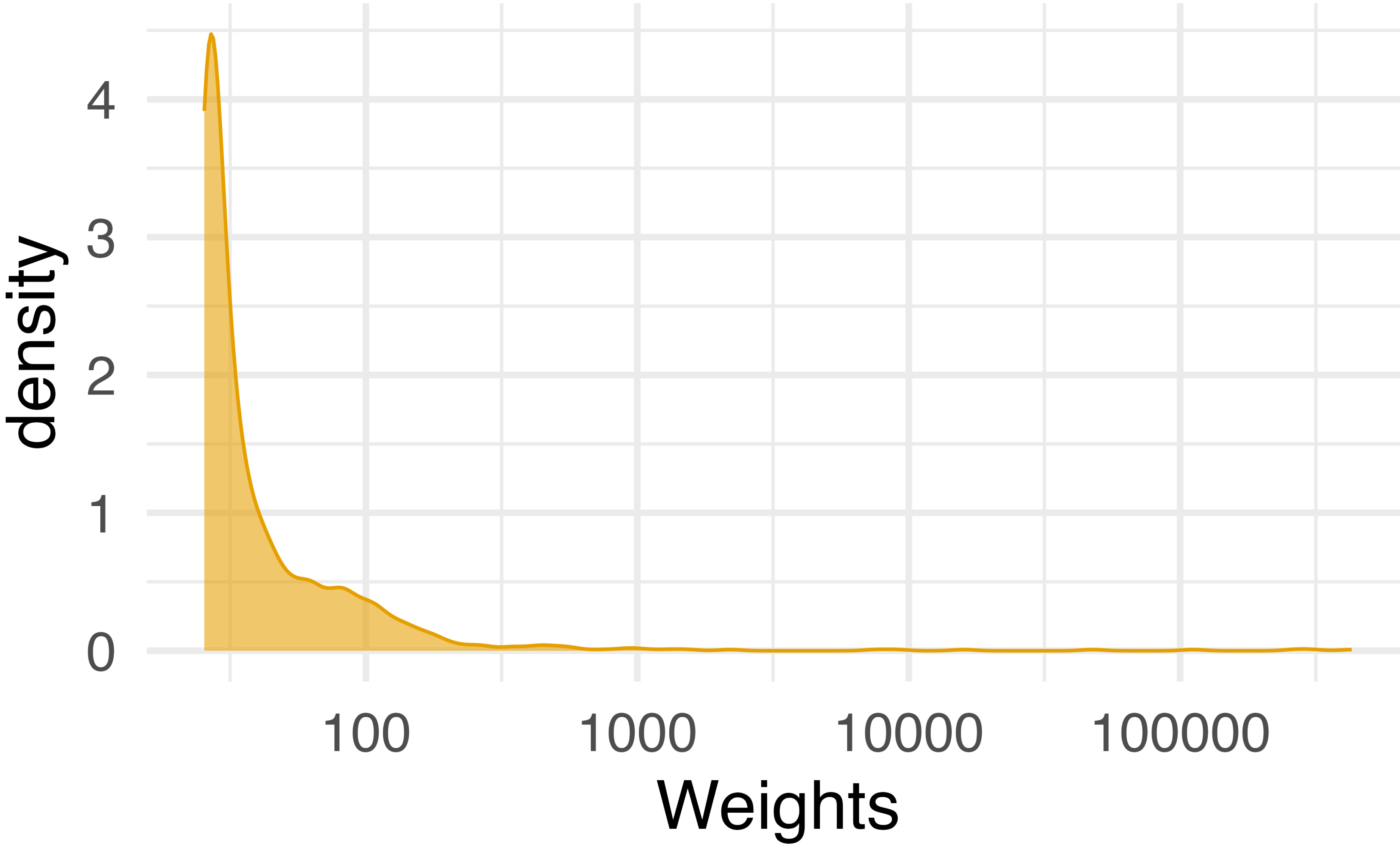
Your Turn 1

```r
denominator_model <- lm(
  avg_spostmin ~
    close + extra_magic_morning +
    weather_wdwhigh + wdw_ticket_season,
  data = wait_times
)
```

Your Turn 1

```r
denominators <- denominator_model |>
  augment(data = wait_times) |>
  mutate(
    denominator = dnorm(
      avg_spostmin, .fitted, mean(.sigma, na.rm = TRUE)
    )
  ) |>
  select(date, denominator)
```

Stabilizing extreme weights



Stabilizing extreme weights

1. Fit an intercept-only model (e.g. `lm(x ~ 1)`)
2. Calculate weights from this model
3. Divide these weights by the propensity score weights

1. Fit an intercept-only model

```
nhefs_numerator_model <- lm(
  smkintensity82_71 ~ 1,
  data = nhefs_light_smokers
)
```

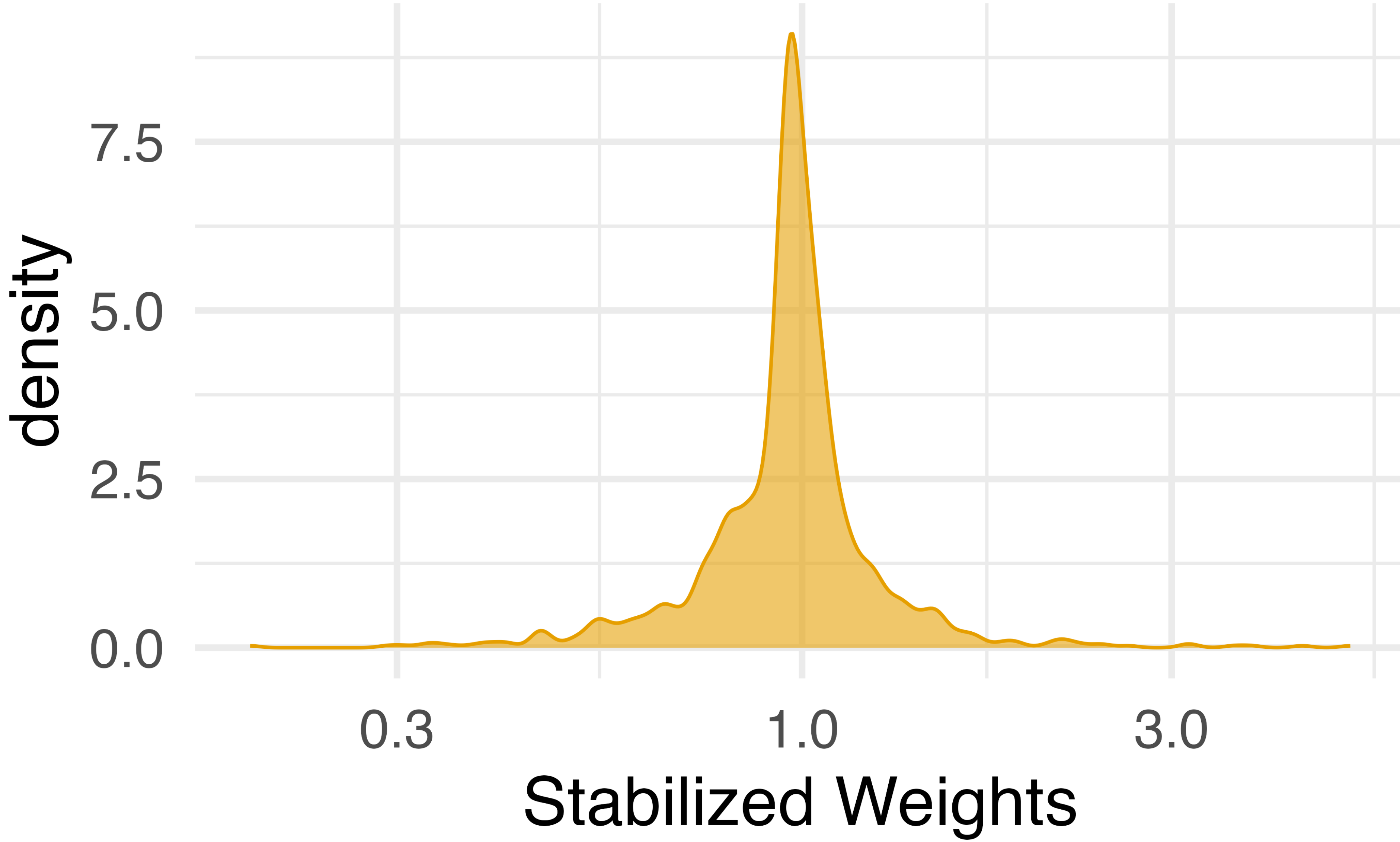
2. Calculate weights from this model

```r
nhefs_numerators <- nhefs_numerator_model |>
  augment(data = nhefs_light_smokers) |>
  mutate(numerator = dnorm(
    smkintensity82_71,
    mean = .fitted,
    sd = mean(.sigma, na.rm = TRUE))
  ) |>
  select(id, numerator)
```

3. Divide these weights by the propensity score weights

```r
nhefs_light_smokers <- nhefs_light_smokers |>
  left_join(nhefs_numerators, by = "id") |>
  left_join(nhefs_denominators, by = "id") |>
  mutate(swts = numerator / denominator)
```

Stabilizing extreme weights



Your Turn 2

Fit an intercept-only model of posted weight times to use as the numerator model

Calculate the numerator weights using `dnorm()` as above.

Finally, calculate the stabilized weights, `swts`, using the `numerator` and `denominator` weights

05:00

Your Turn 2

```r
numerator_model <- lm(
  avg_spostmin ~ 1,
  data = wait_times
)
```

Your Turn 2

```
numerators <- numerator_model |>
  augment(data = wait_times) |>
  mutate(
    numerator = dnorm(
      avg_spostmin, .fitted, mean(.sigma, na.rm = TRUE)
    )
  ) |>
  select(date, numerator)

wait_times_wts <- wait_times |>
  left_join(numerators, by = "date") |>
  left_join(denominators, by = "date") |>
  mutate(swts = numerator / denominator)
```

Fitting the outcome model

1. Use the stabilized weights in the outcome model. Nothing new here!

```r
lm(
  wt82_71 ~ smkintensity82_71,
  weights = swts,
  data = nhefs_light_smokers
) |>
  tidy() |>
  filter(term == "smkintensity82_71") |>
  mutate(estimate = estimate * -10)
```

```
# A tibble: 1 × 5
  term              estimate std.error statistic    p.value
  <chr>                <dbl>     <dbl>     <dbl>      <dbl>
1 smkintensity82_71    0.960    0.0210     -4.58 0.00000519
```

Your Turn 3

Estimate the relationship between posted wait times and actual wait times using the stabilized weights we just created.

03:00

Your Turn 3

```r
lm(
  avg_sactmin ~ avg_spostmin,
  weights = swts,
  data = wait_times_wts
) |>
  tidy() |>
  filter(term == "avg_spostmin") |>
  mutate(estimate = estimate * 10)
```

```
# A tibble: 1 × 5
  term         estimate std.error statistic p.value
  <chr>           <dbl>     <dbl>     <dbl>   <dbl>
1 avg_spostmin    -2.63    0.0807     -3.26 0.00162
```