Image Formation

Computer Vision: AI3619

Image Formation and Optics

Image: Projection of 3D scene onto 2D plane.
We need to understand Geometric and Radiometric relations between the scene and its image.

Topics:

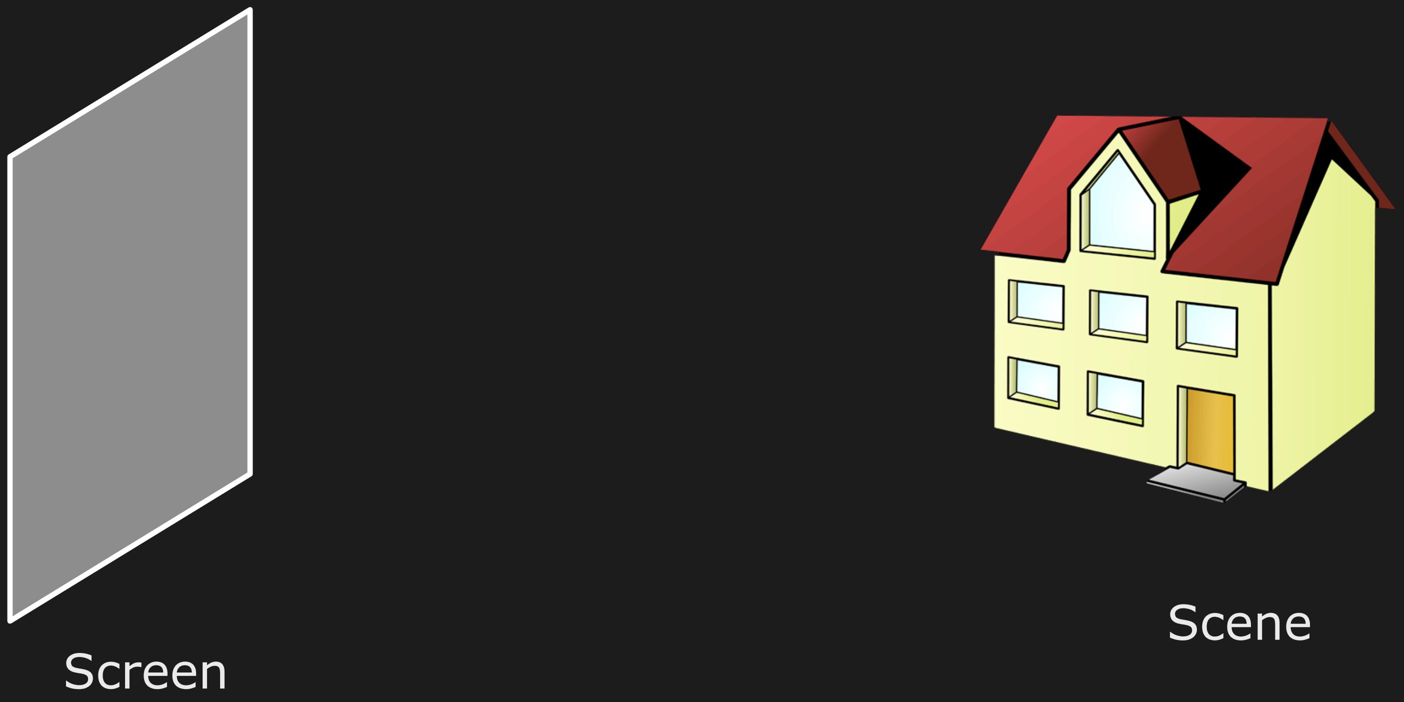
(1) Pinhole and Perspective Projection

(2) Vanishing Point

(3) Image formation using Lenses

(4) Lens Related Issues

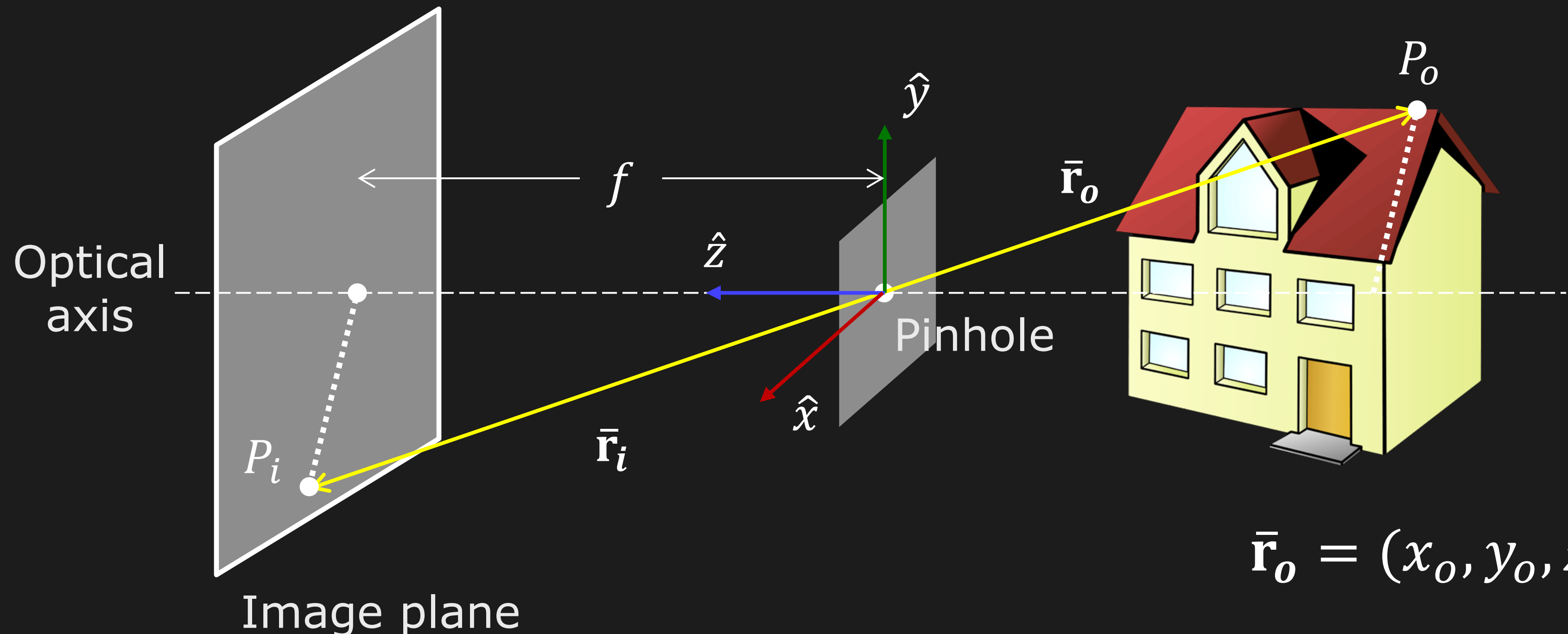
Image Formation



Is an image being formed on the screen?

Yes! But not a "clear" one.

Perspective Imaging with Pinhole



$$\bar{\mathbf{r}}_o = (x_o, y_o, z_o)$$

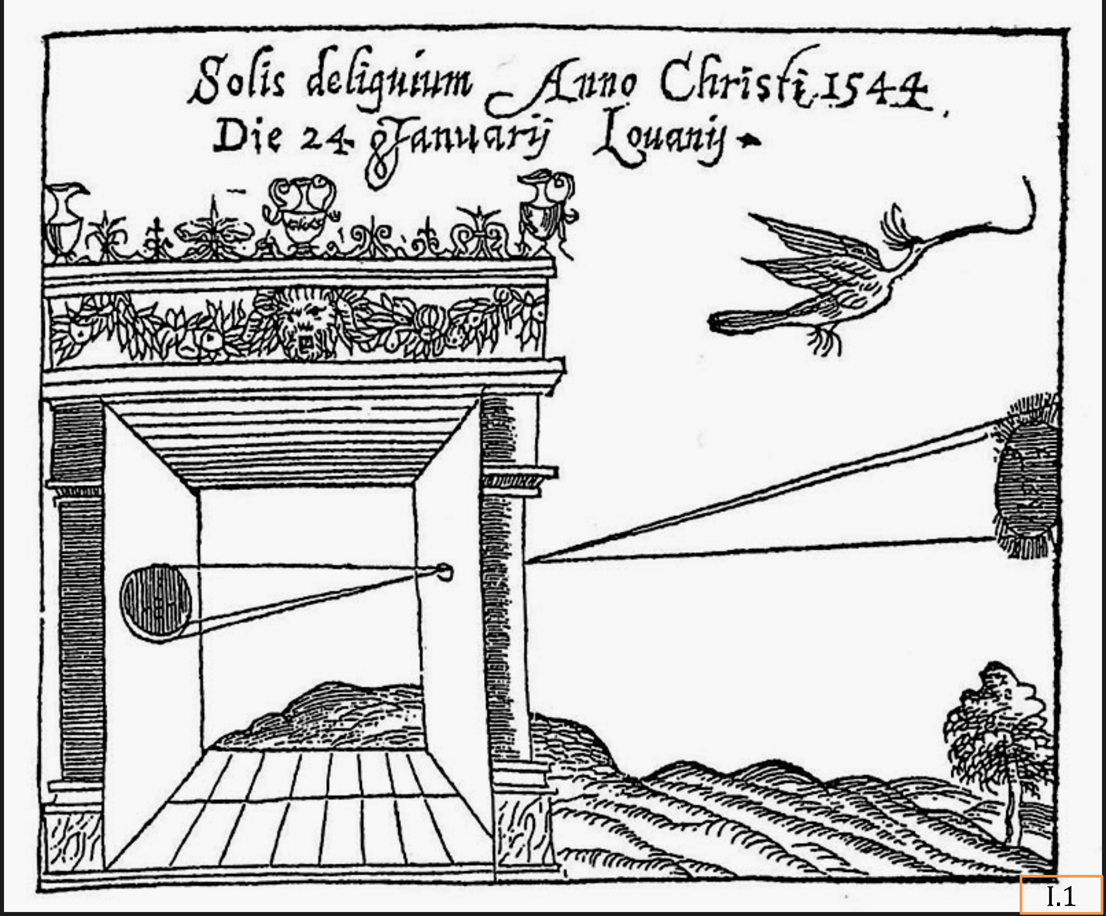
$$\bar{\mathbf{r}}_i = (x_i, y_i, f)$$

Using similar triangles:

$$\boxed{\frac{\bar{\mathbf{r}}_i}{f} = \frac{\bar{\mathbf{r}}_o}{z_o}} \Rightarrow \boxed{\frac{x_i}{f} = \frac{x_o}{z_o}, \quad \frac{y_i}{f} = \frac{y_o}{z_o}}$$

f: Effective Focal Length

Camera Obscura



"Dark Chamber"

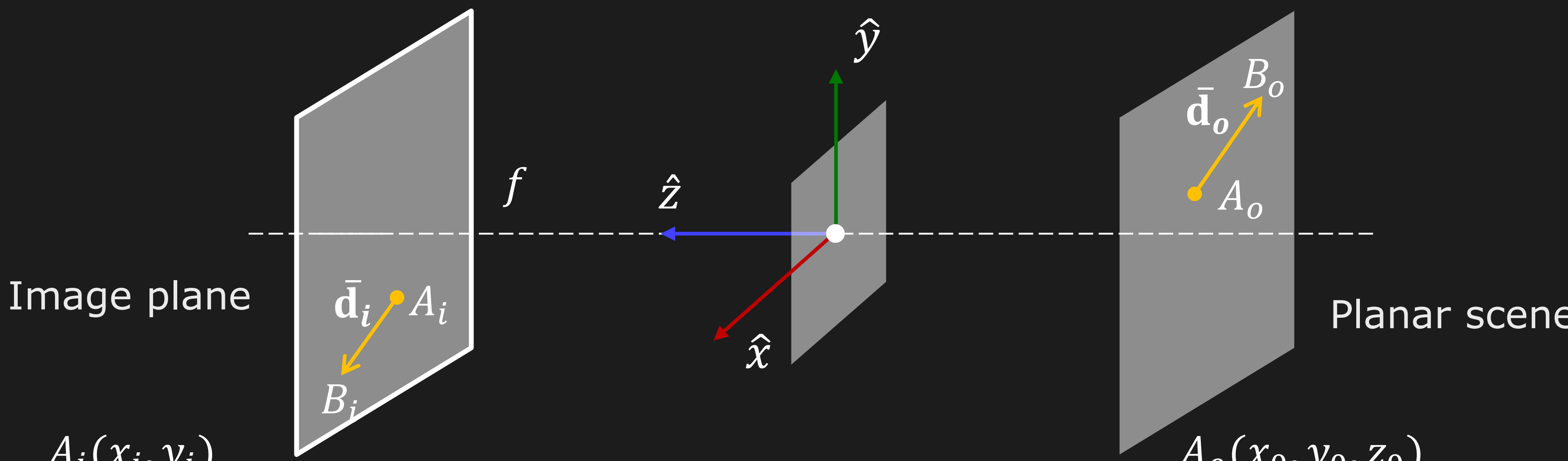
[Frisius 1545]

Pinhole Eye of *Nautilus pompilius*

I.2

I.3

Image Magnification



$A_i(x_i, y_i)$

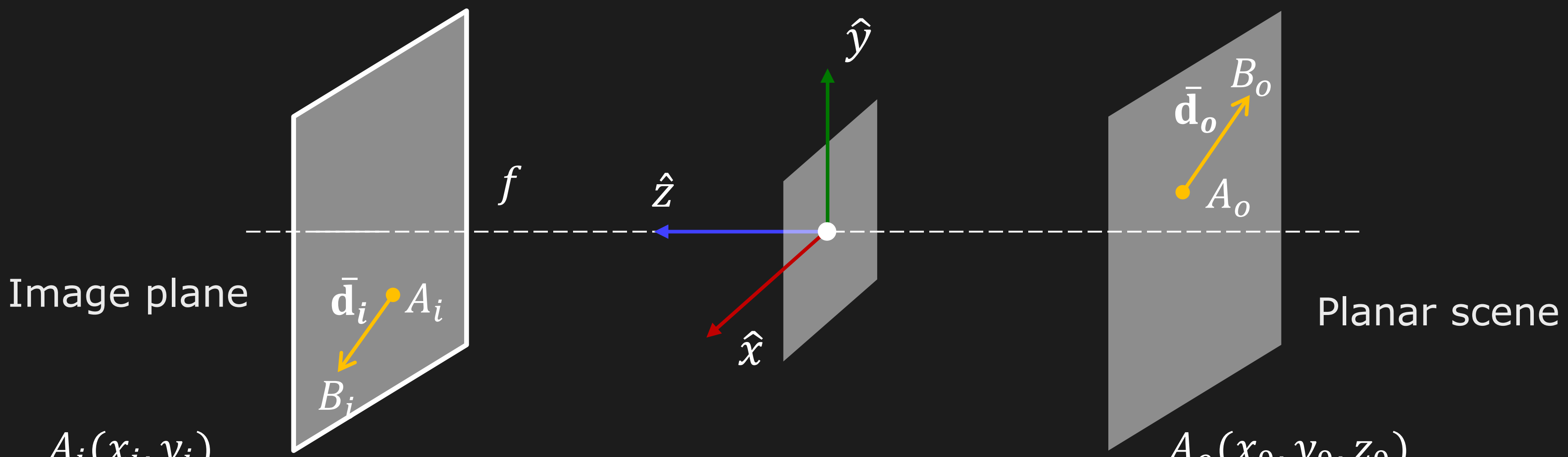
$B_i(x_i + \delta x_i, y_i + \delta y_i)$

$A_o(x_0, y_0, z_0)$

$B_o(x_0 + \delta x_o, y_0 + \delta y_o, z_0)$

Magnification: $$|m| = \frac{\|\bar{\mathbf{d}}_i\|}{\|\bar{\mathbf{d}}_o\|} = \sqrt{\delta x_i^{\,2} + \delta y_i^{\,2}} \Big/ \sqrt{\delta x_o^{\,2} + \delta y_o^{\,2}}$$

Image Magnification



$A_i(x_i, y_i)$

$B_i(x_i + \delta x_i, y_i + \delta y_i)$

$A_o(x_0, y_0, z_0)$

$B_o(x_0 + \delta x_o, y_0 + \delta y_o, z_0)$

From Perspective Projection:

$$\frac{x_i}{f} = \frac{x_o}{z_o} \quad \text{and} \quad \frac{y_i}{f} = \frac{y_o}{z_o} \quad \text{-----} \quad \text{(A)}$$

$$\frac{x_i + \delta x_i}{f} = \frac{x_o + \delta x_o}{z_o} \quad \text{and} \quad \frac{y_i + \delta y_i}{f} = \frac{y_o + \delta y_o}{z_o} \quad \text{-----} \quad \text{(B)}$$

Image Magnification

From (A) and (B) we get:

$$\frac{\delta x_i}{f} = \frac{\delta x_o}{z_o} \quad \text{and} \quad \frac{\delta y_i}{f} = \frac{\delta y_o}{z_o}$$

Magnification:

$$|m| = \frac{\|\bar{\mathbf{d}}_i\|}{\|\bar{\mathbf{d}}_o\|} = \sqrt{\delta x_i{}^2 + \delta y_i{}^2} \Big/ \sqrt{\delta x_o{}^2 + \delta y_o{}^2} = \left|\frac{f}{z_o}\right|$$

$$\boxed{m = \frac{f}{z_o}}$$

Image Magnification

$$m = \frac{f}{z_o}$$

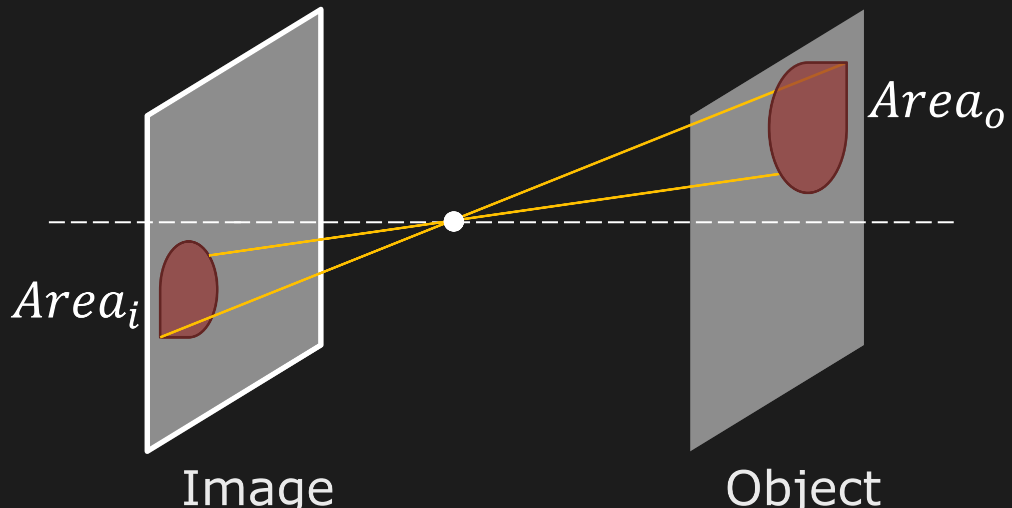
Image size inversely proportional to depth

Image Magnification

Notes:

- m can be assumed to be constant if the range of scene depth Δz is much smaller than the average scene depth $\tilde{z}$

- $$\frac{Area_i}{Area_o} = m^2$$



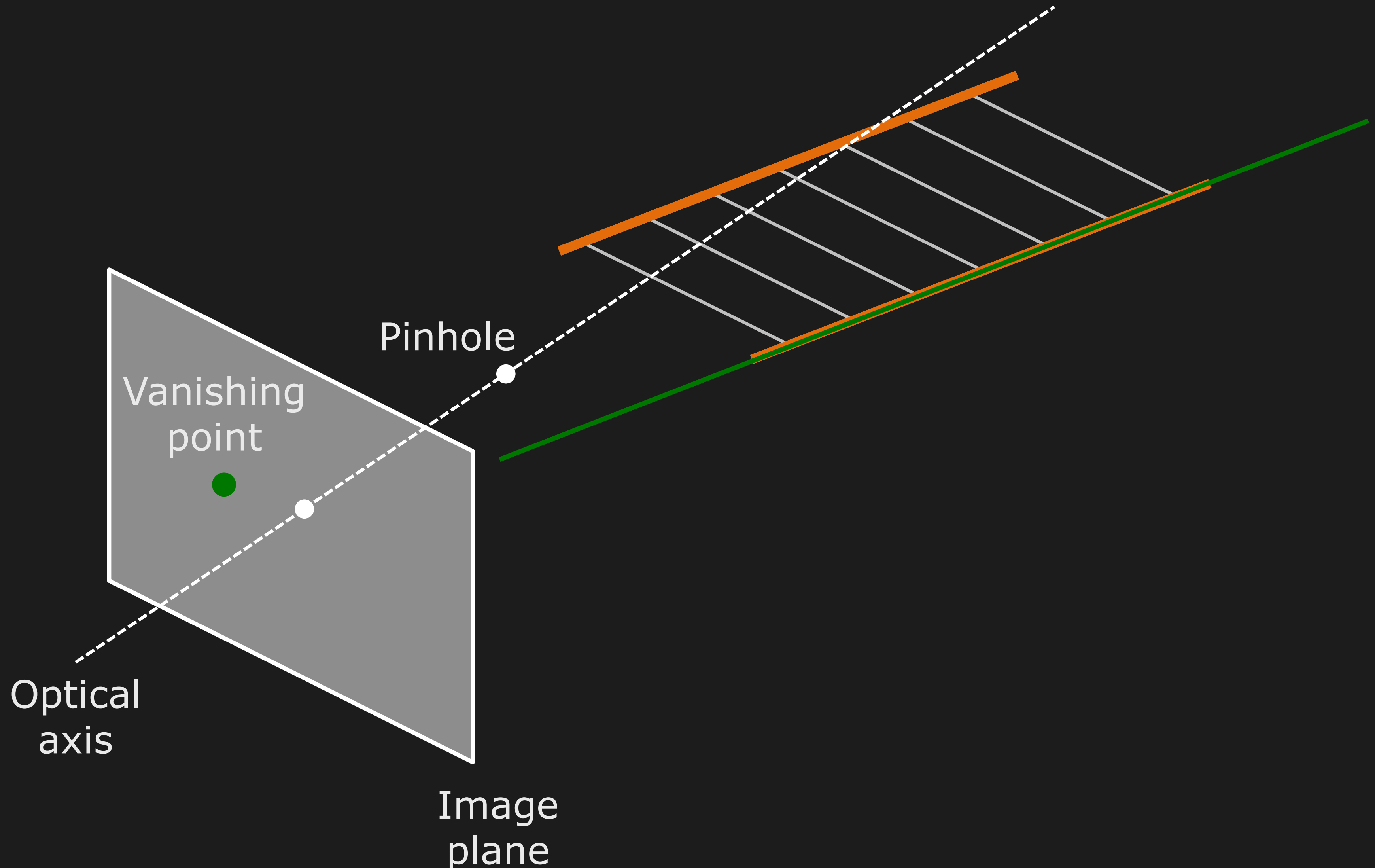
Vanishing Point

Parallel straight lines converge at a single image point

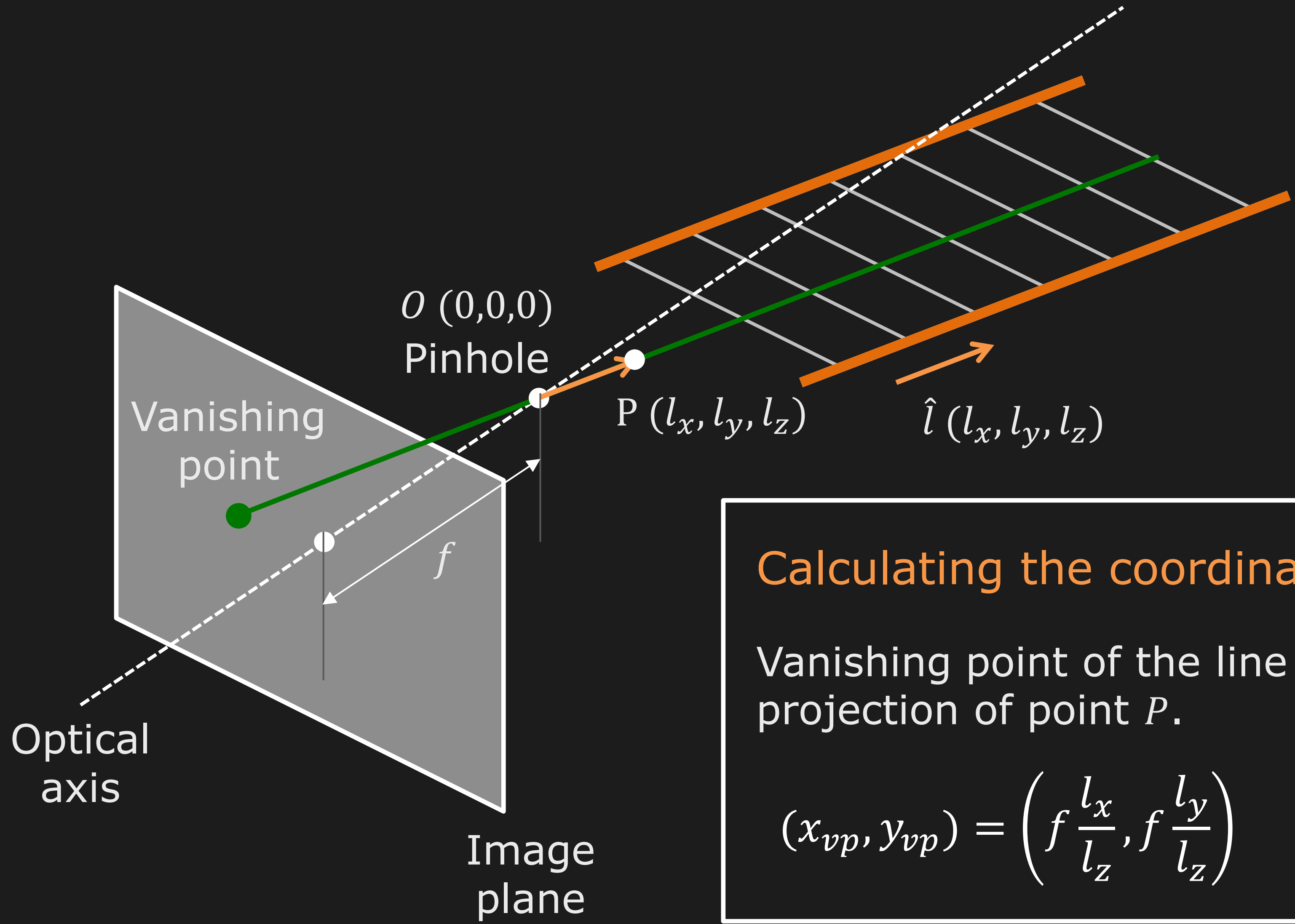
Vanishing Point

Location of Vanishing Point depends on the orientation of parallel straight lines.

Finding the Vanishing Point



Finding Vanishing Point

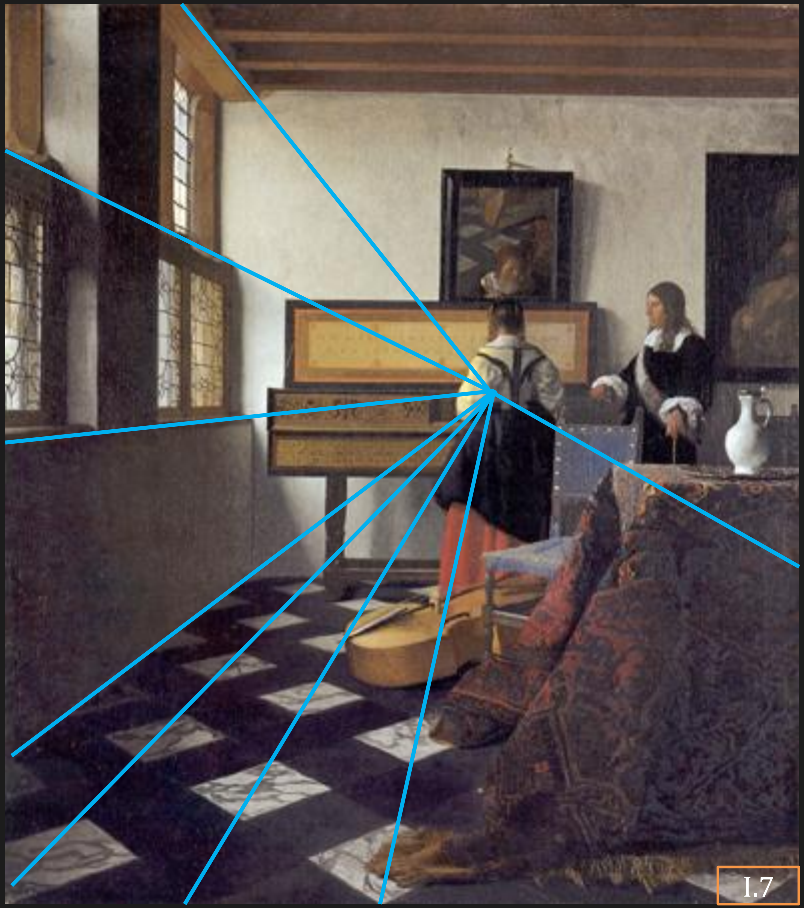


Calculating the coordinates:

Vanishing point of the line is the projection of point P.

$$(x_{vp}, y_{vp}) = \left(f\frac{l_x}{l_z}, f\frac{l_y}{l_z}\right)$$

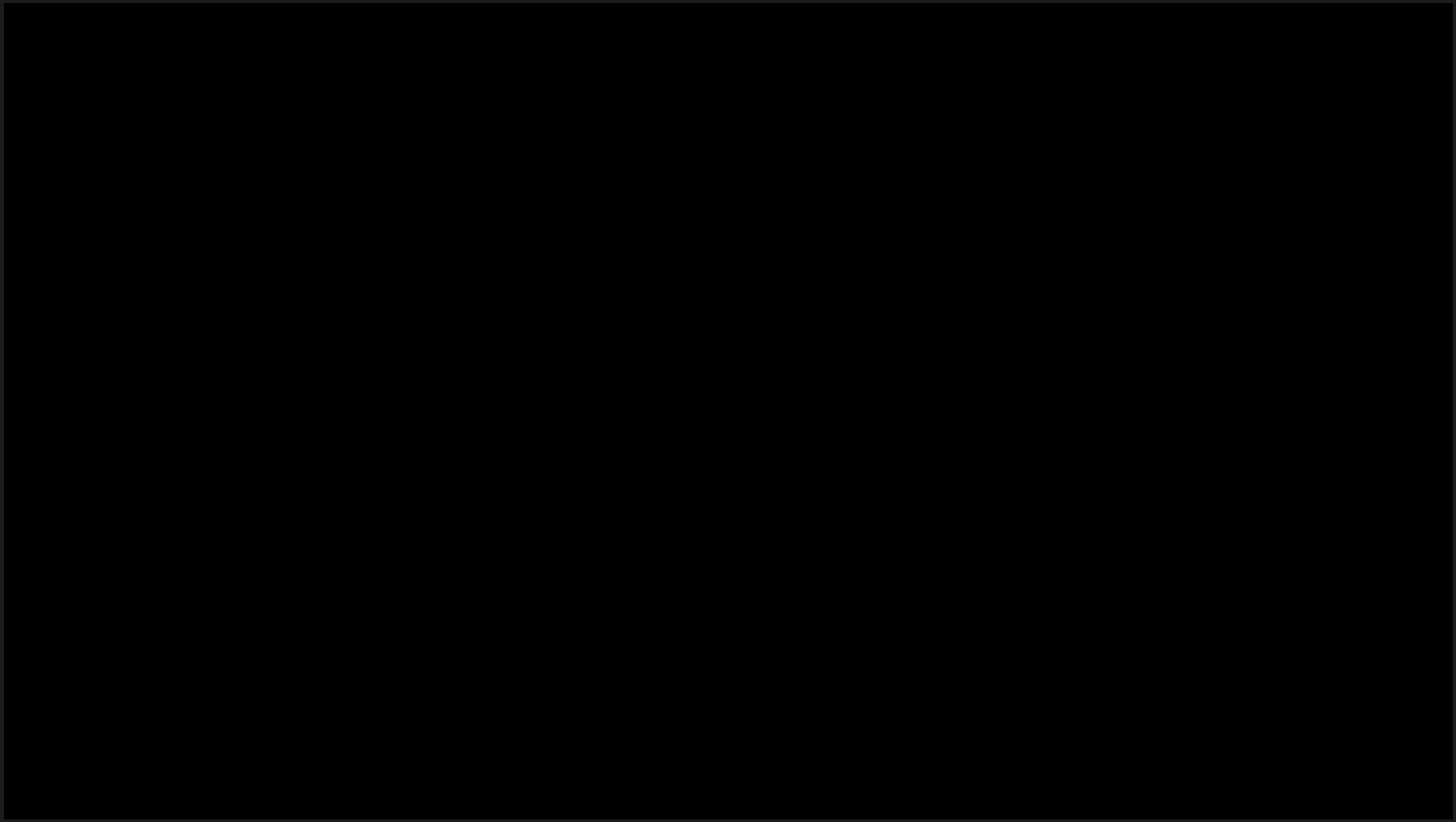
Use of Vanishing Point in Art



The Music Lesson, Johannes Vermeer, c. 1662-1664

Use of Vanishing Point in Art

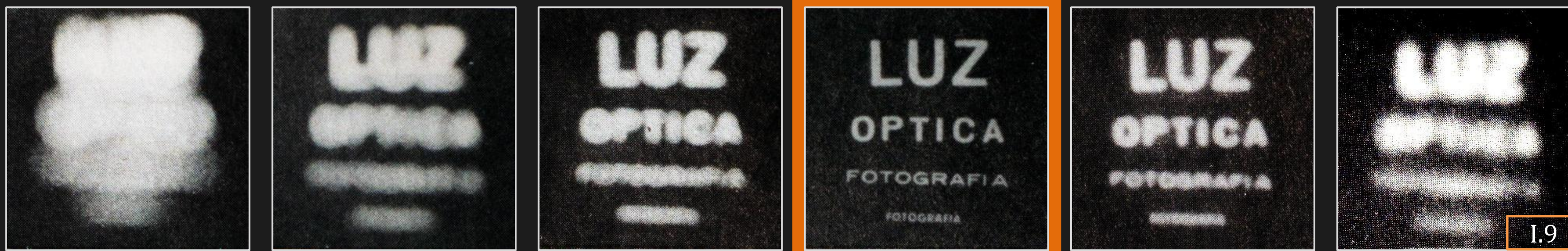
Use of Vanishing Point in Art



Use of Vanishing Point in Sport

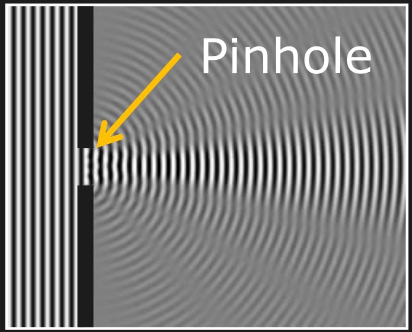
Use of Vanishing Point in Sport

What is the Ideal Pinhole Size?



2 mm | 1 mm | 0.6 mm | 0.35 mm | 0.15 mm | 0.07 mm

The pinhole must be tiny,
but if it's too tiny it will cause diffraction.



Diffraction

Ideal pinhole diameter: $\boxed{d \approx 2\sqrt{f\lambda}}$

f: effective focal length
λ: wavelength

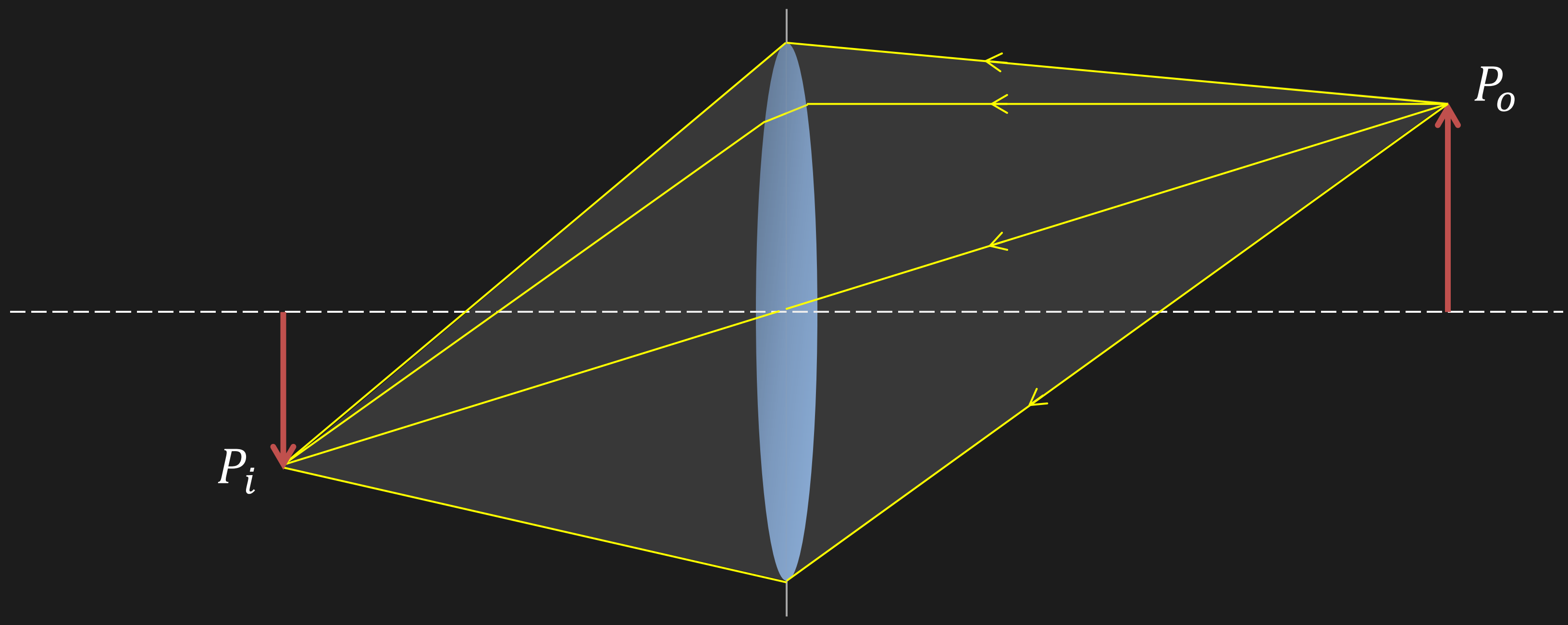
What about Exposure Time?

Pinholes pass less light and hence require long exposures to capture bright images.

$f = 73\ mm,\ d = 0.2\ mm,$
Exposure, $T = 12\ s$

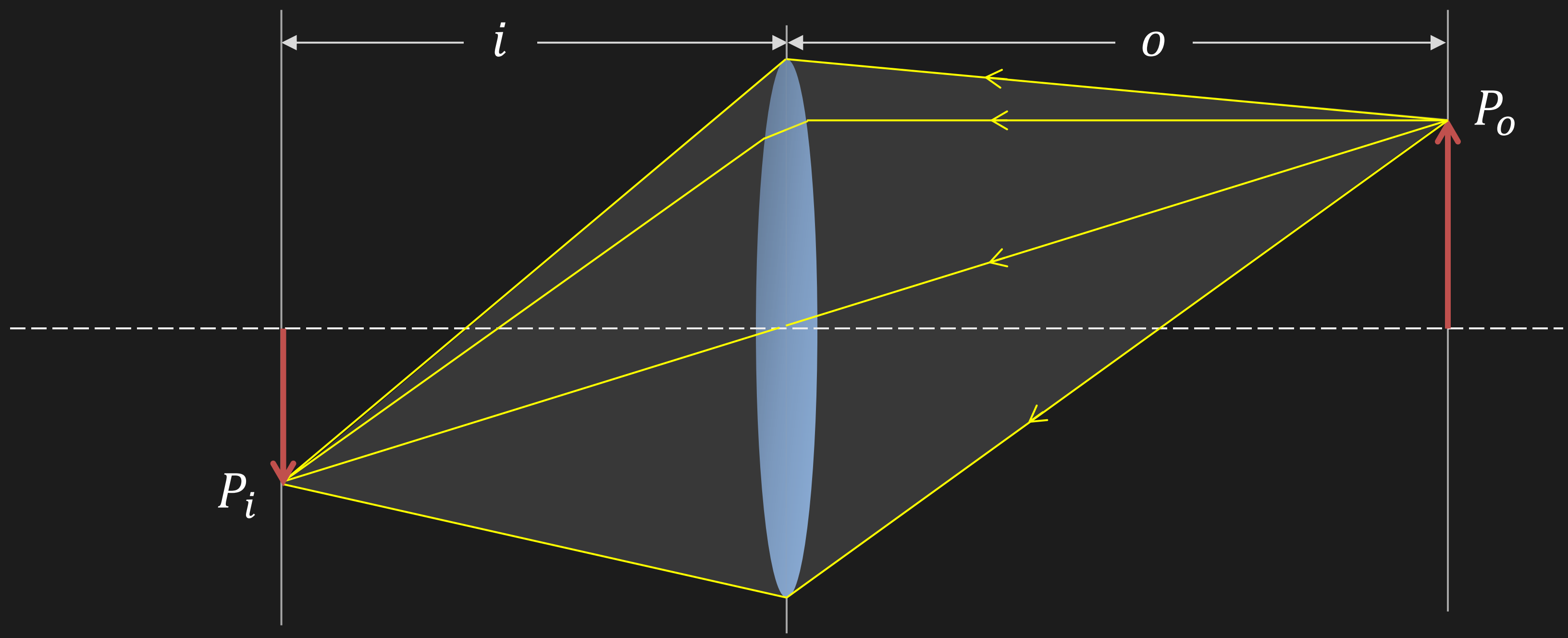
Lenses

Same projection as pinhole, but gather more light!



Focal length (f) determines the lens' bending power

Gaussian Lens (Thin Lens) Law



f: focal length

i: image distance

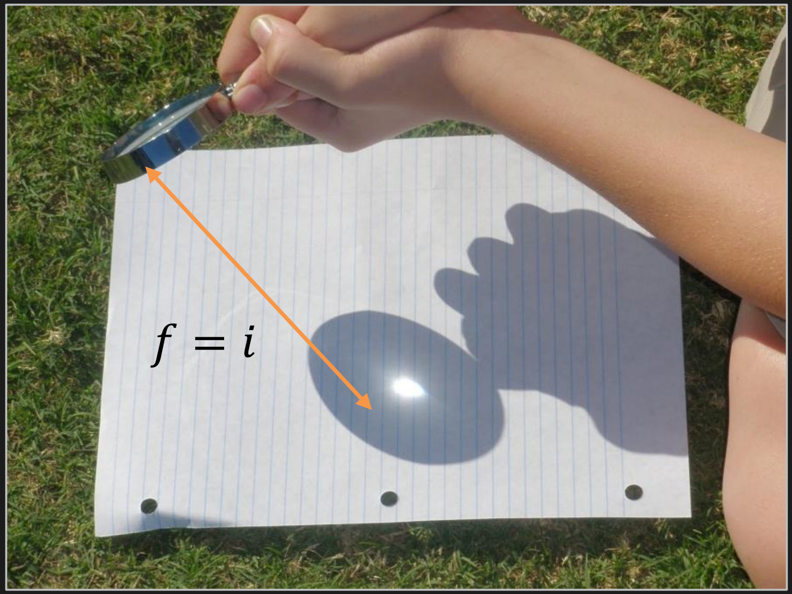
o: object distance

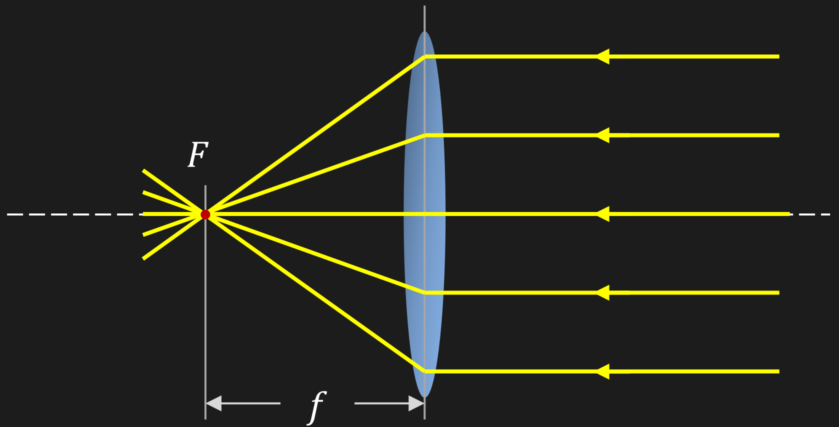
$$\boxed{\frac{1}{i} + \frac{1}{o} = \frac{1}{f}}$$

Example: If $f = 50mm$ & $o = 300mm$, then image distance $i = 60mm$

How to Find the Focal Length?

$$\frac{1}{i} + \frac{1}{o} = \frac{1}{f} \quad \Rightarrow \quad \text{If } o = \infty \text{, then } f = i$$





Focal length: Distance at which incoming rays that are parallel to the optical axis converge.

Relation Between Lens and Pinhole

$$\frac{1}{i}+\frac{1}{o}=\frac{1}{f} \quad \Rightarrow$$

Usually $o \gg i$, then $f \approx i$
Point A is approaching F, which is similar to a pinhole system

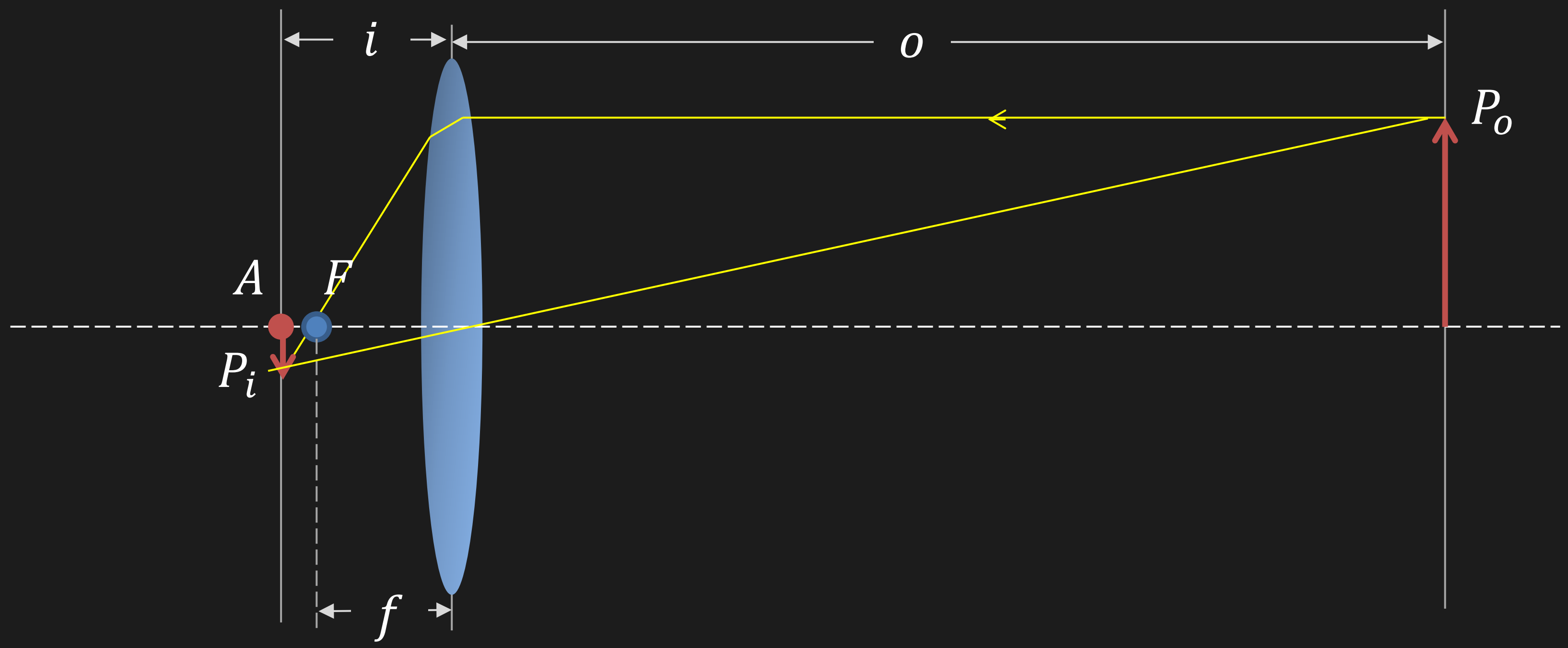
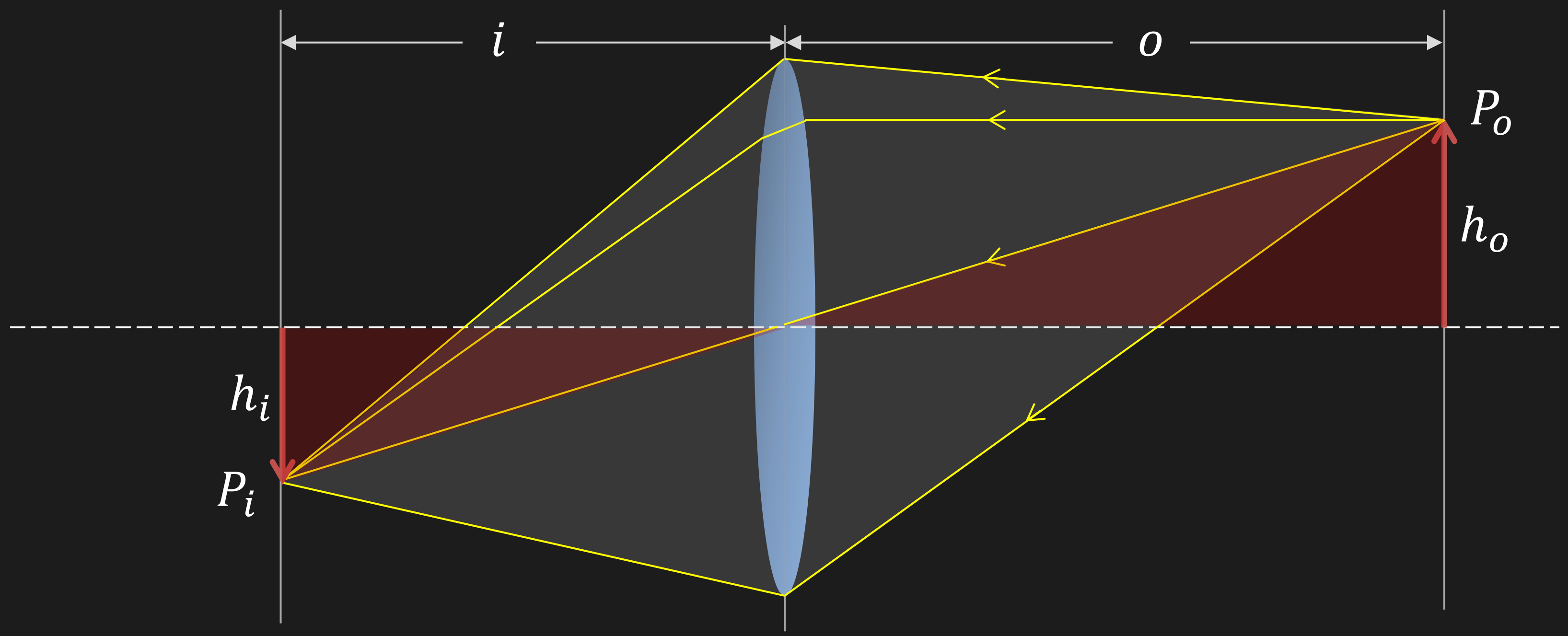
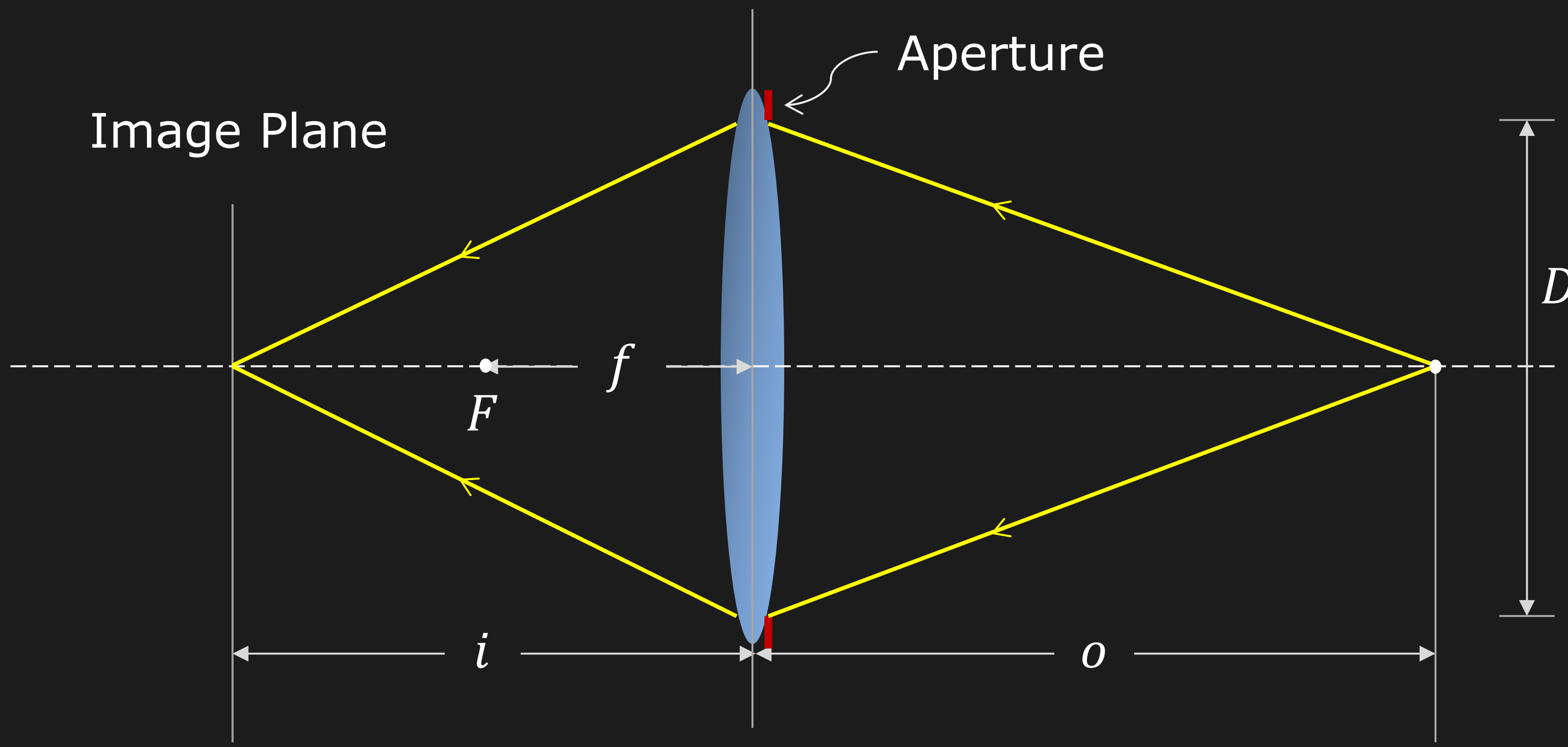


Image Magnification

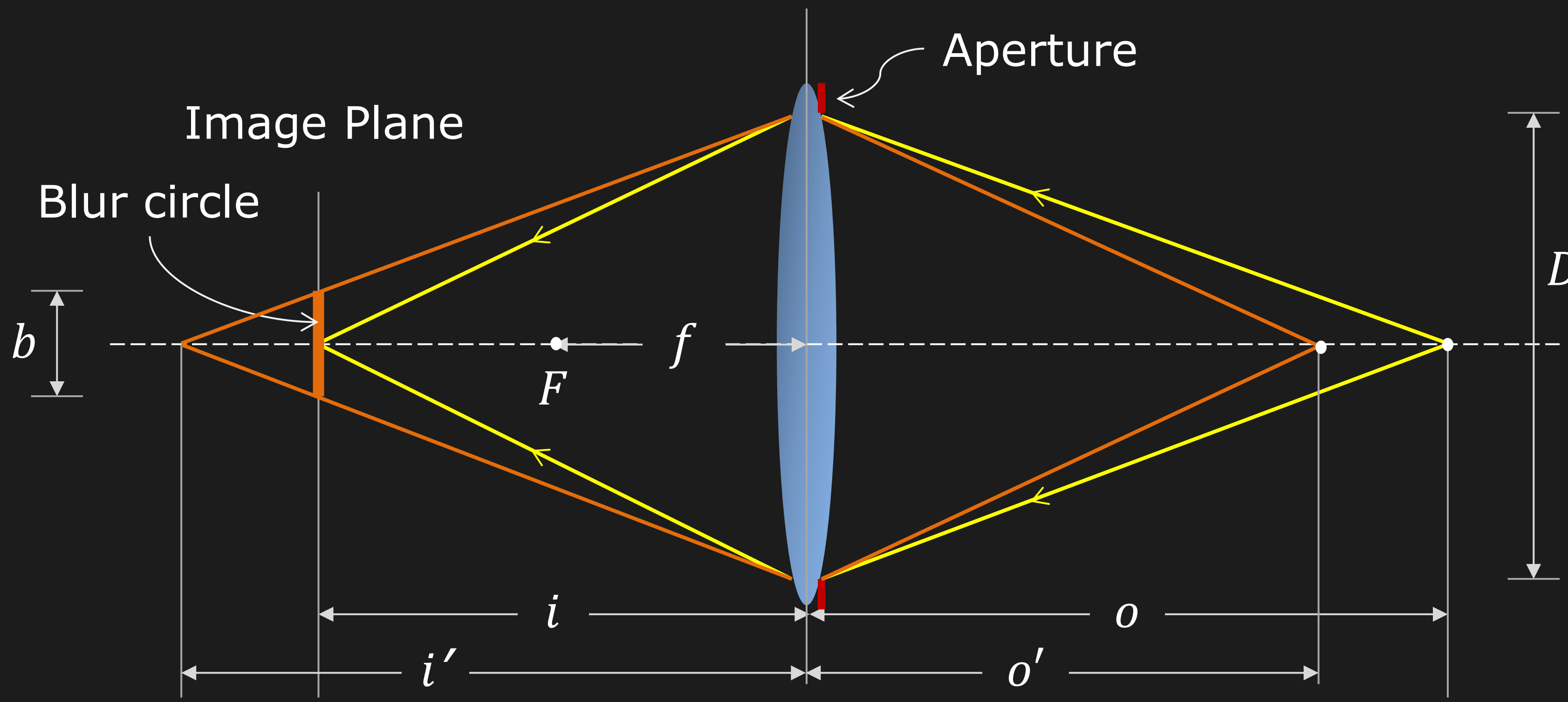


Magnification: $\boxed{m = \frac{h_i}{h_o} = \frac{i}{o}}$

Blur Circle (Defocus)



Blur Circle (Defocus)



From similar triangles:

$$\frac{b}{D} = \frac{|i' - i|}{i'}$$

Blur circle diameter:

$$\boxed{b = \frac{D}{i'}|i' - i|}$$

$$\boxed{b \propto D}$$

Blur Circle (Defocus)

Focused Point

$$\frac{1}{i} + \frac{1}{o} = \frac{1}{f}$$

$$i = \frac{of}{o - f}$$

Defocused Point

$$\frac{1}{i'} + \frac{1}{o'} = \frac{1}{f}$$

(Gaussian Lens Law)

$$i' = \frac{o'f}{o' - f}$$

$$i' - i = \frac{f}{(o' - f)} \cdot \frac{f}{(o - f)} \cdot (o - o')$$

$$\boxed{b = D \left| \frac{f(o - o')}{o'(o - f)} \right|}$$

Focusing

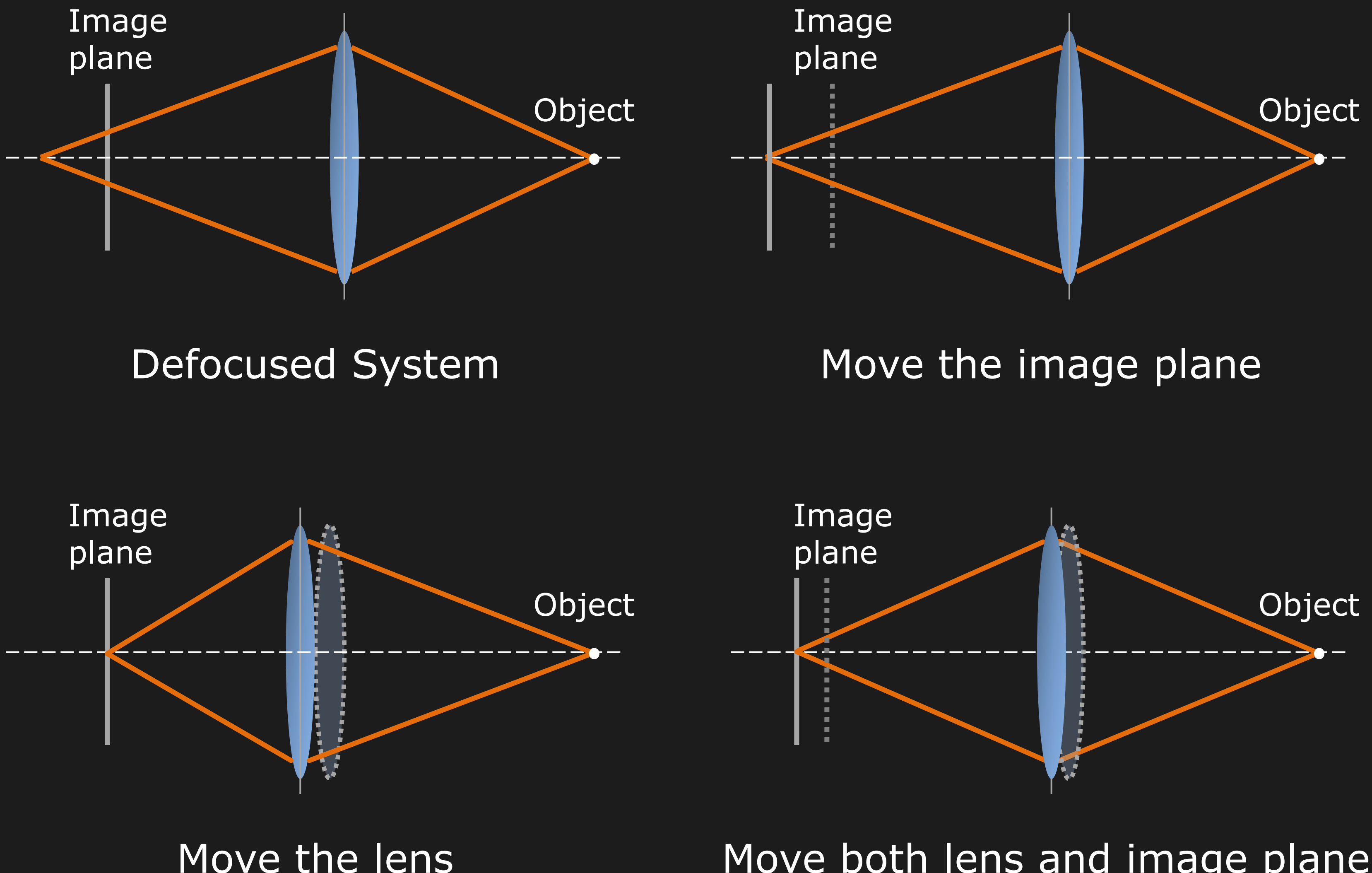
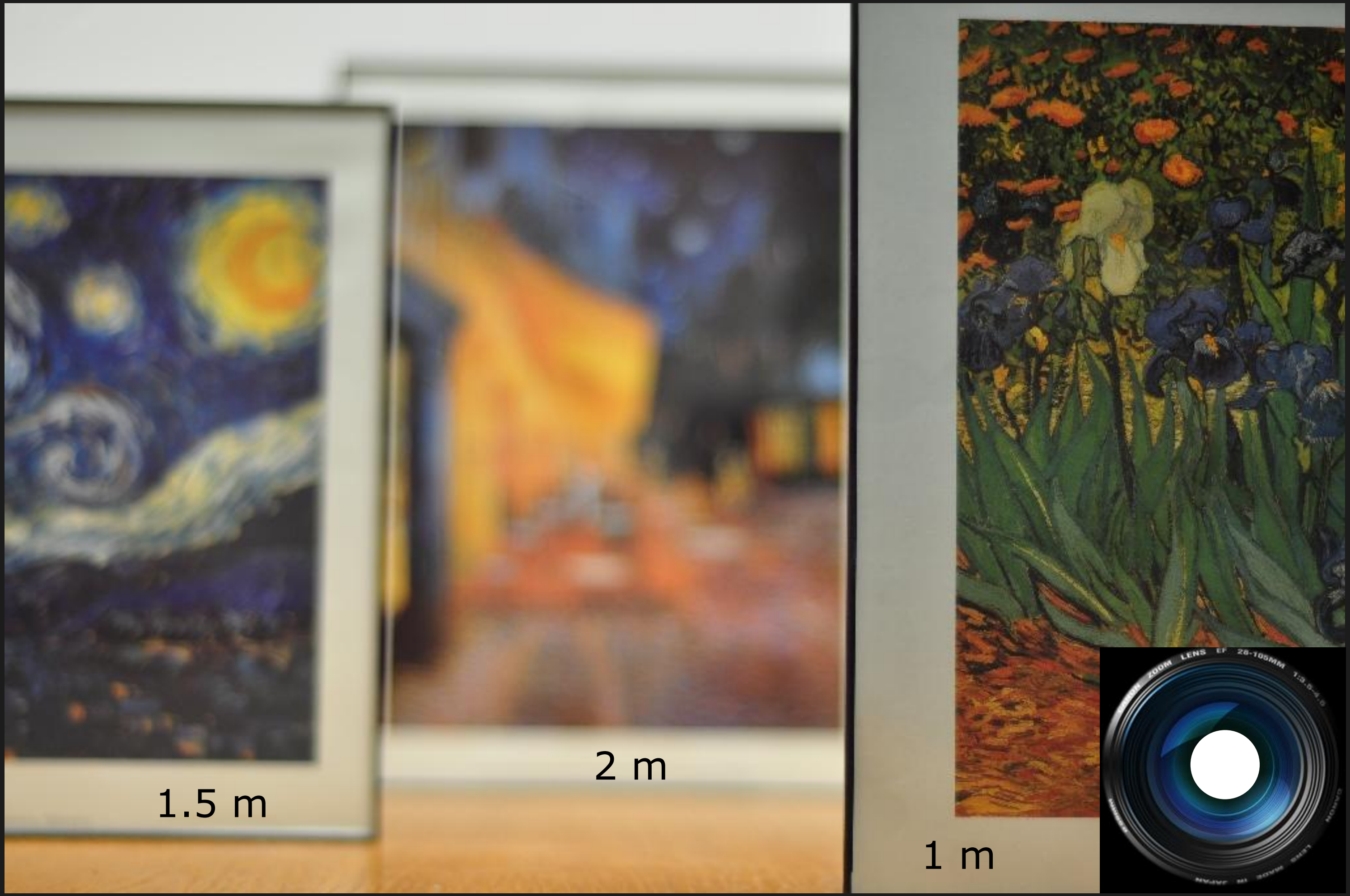


Image
plane
Object
Defocused System
Image
plane
Object
Move the image plane
Image
plane
Object
Move the lens
Image
plane
Object
Move both lens and image plane

Depth of Field (DoF)

Range of object distances $(o - o')$ over which the image is “sufficiently well” focused. i.e., Range $(o - o')$ for which blur b is less than pixel size.

Aperture Size: DOF vs. Brightness



Focal Length 50 mm, Focus = 1 m, Aperture D = 25 mm

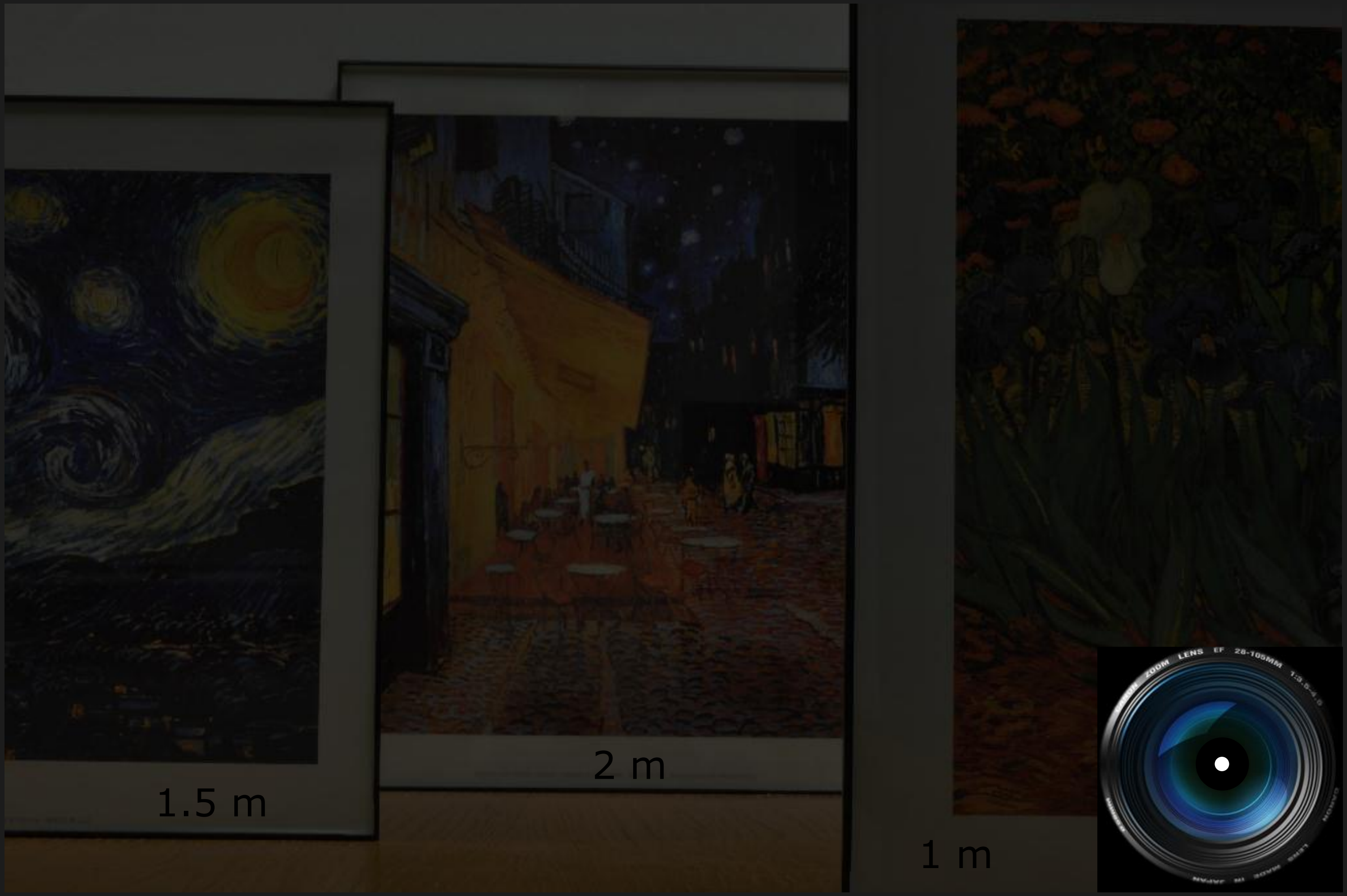
Aperture Size: DOF vs. Brightness

Focal Length 50 mm, Focus = 1 m, Aperture D = 12.5 mm

Aperture Size: DOF vs. Brightness

Focal Length 50 mm, Focus = 1 m, Aperture D = 6.25 mm

Aperture Size: DOF vs. Brightness



Focal Length 50 mm, Focus = 1 m, Aperture D = 3.125 mm

Aperture Size: DOF vs. Brightness

Large Aperture

- Bright Image or Small Exposure Time
- Shallow Depth of Field

Small Aperture

- Dark Image or Long Exposure Time
- Large Depth of Field

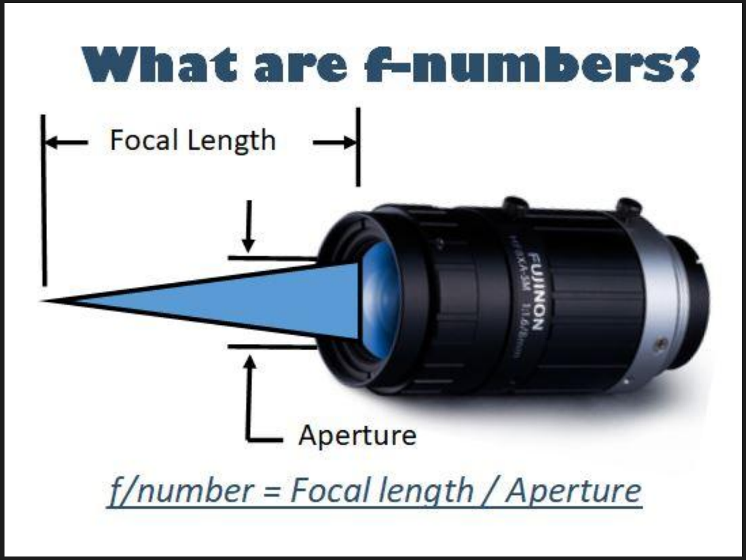
f-number

DOF is a function of both focus distance and aperture diameter d

f-number is a common number associated with cameras, which is usually denoted *f/#*

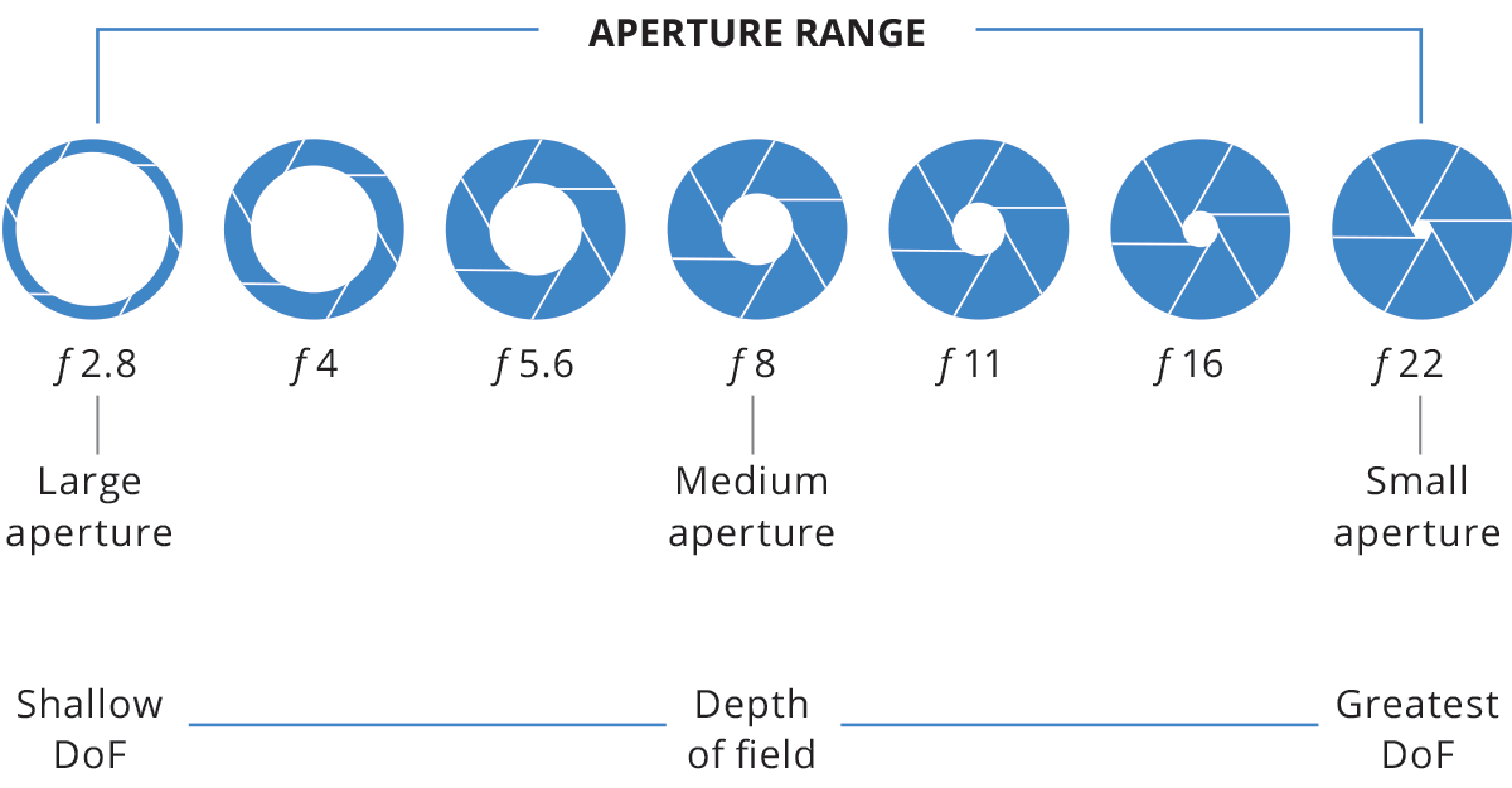
$f/\# = N = \frac{f}{d}$

f: focal length
d: aperture diameter



This is usually written as f/#, where # is the actual number *N* (e.g., *f*/1.4, *f*/2, *f*/2.8, …, *f*/22). We interpret these numbers by noticing that dividing the focal length by the f-number gives the aperture diameter *d*.

f-number


APERTURE RANGE
ƒ 2.8
ƒ 4
ƒ 5.6
ƒ 8
ƒ 11
ƒ 16
ƒ 22
Large aperture
Medium aperture
Small aperture
Shallow DoF
Depth of field
Greatest DoF

f-number


CANON ZOOM LENS EF-S 18-135mm 1:3.5-5.6 IS STM
φ67mm

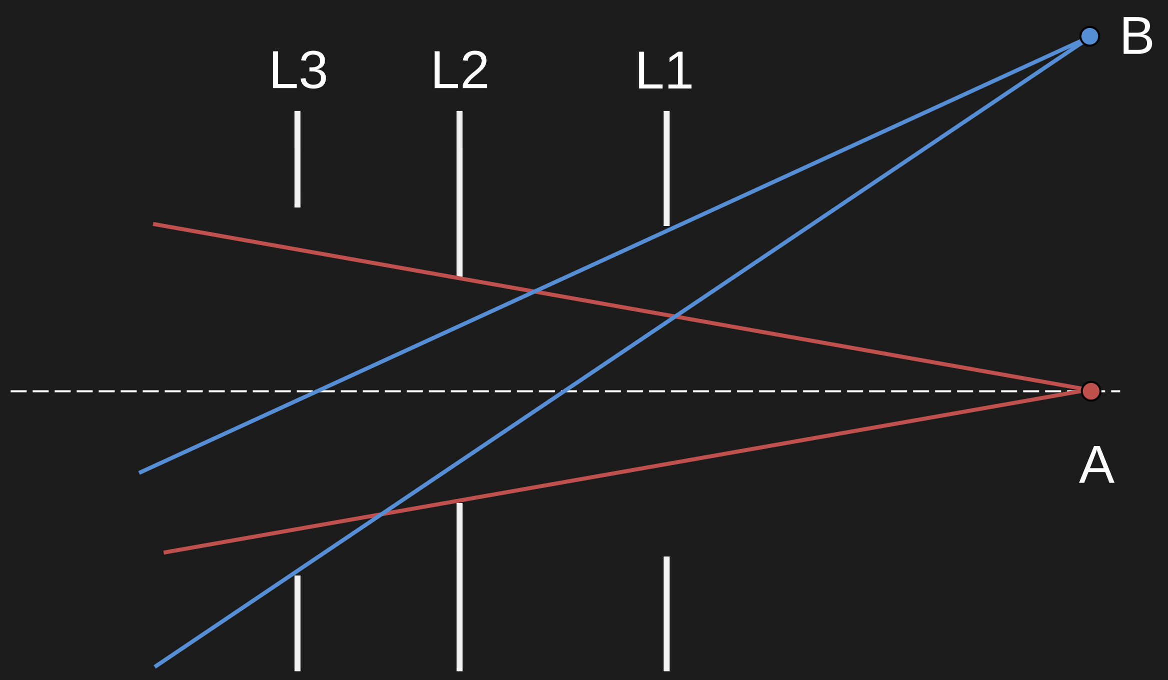

LENS MADE IN JAPAN
Ø77 mm
17-55 mm 1:2.8
ZOOM LENS

Vignetting

Brightness fall-off (Vignetting) in image of a White Wall

Brightness fall-off (Vignetting) in image of a Natural Scene

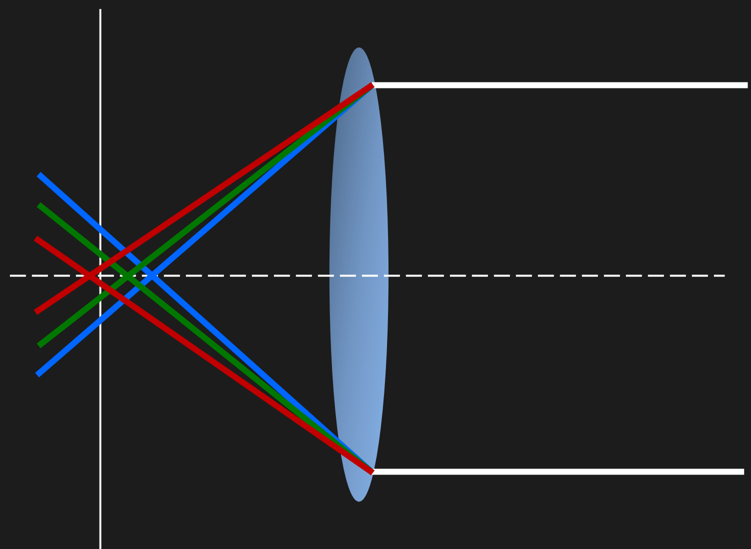
Vignetting



More light passes through L3 from point A than point B.

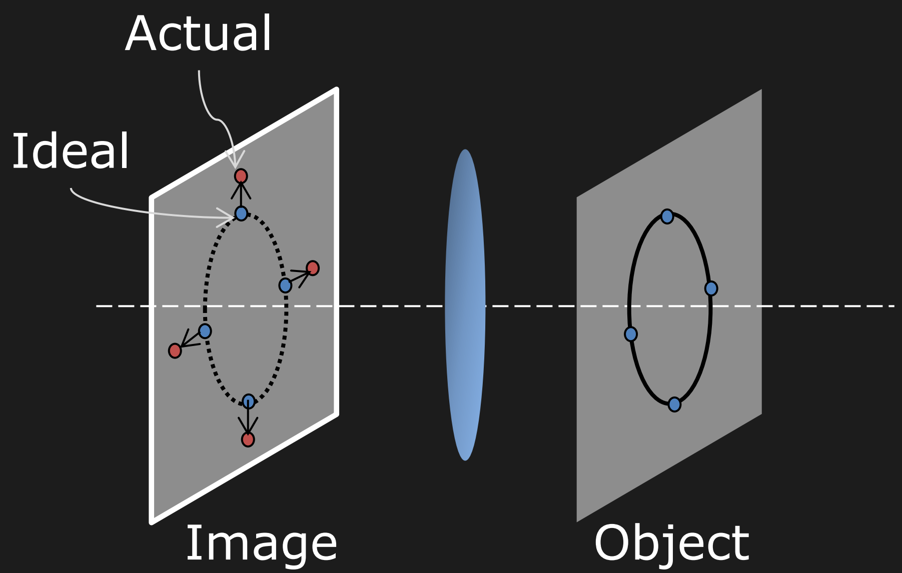
Results in a smooth fall-off of brightness from A to B.

Chromatic Aberration

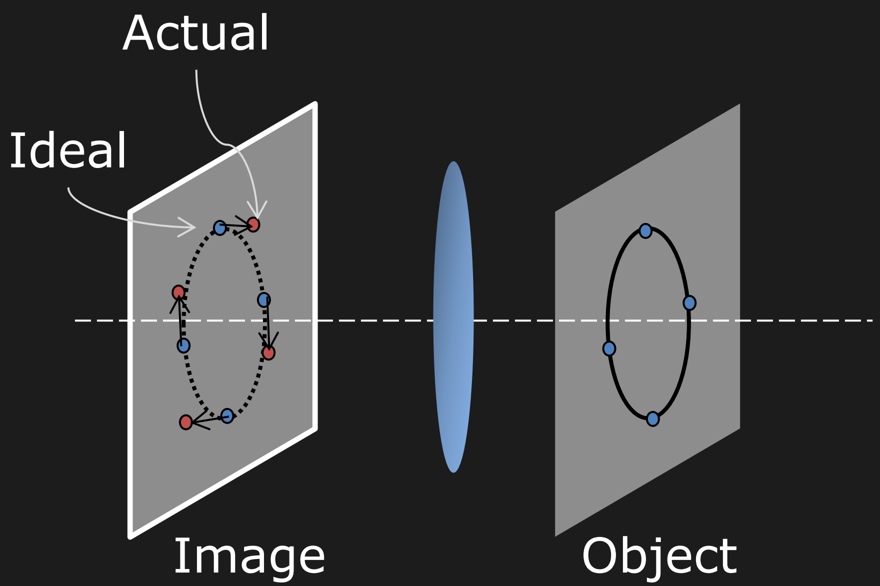


Refractive index (and hence focal length) of lens is different for different wavelengths.

Geometric Distortion



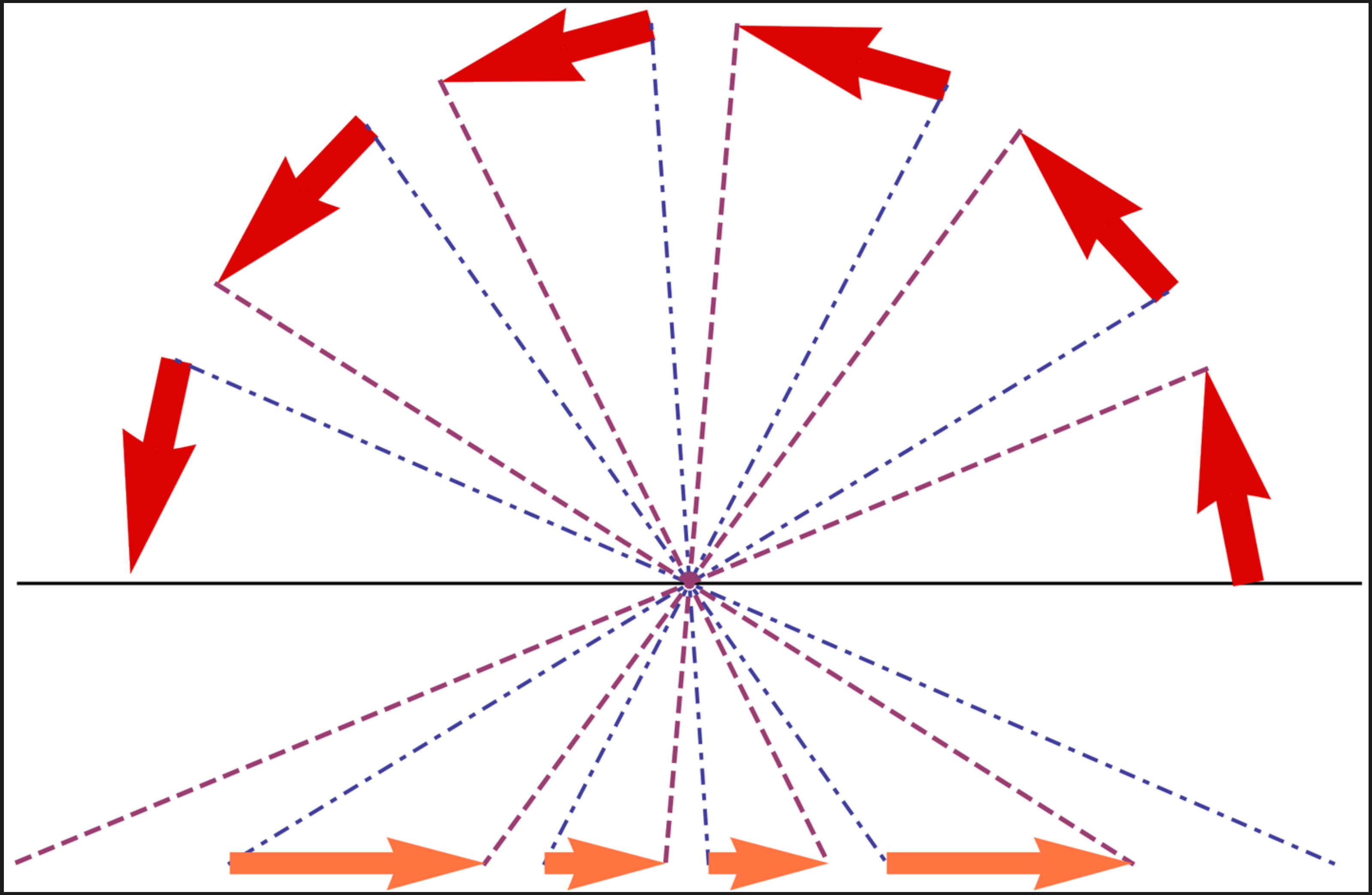
Radial distortion



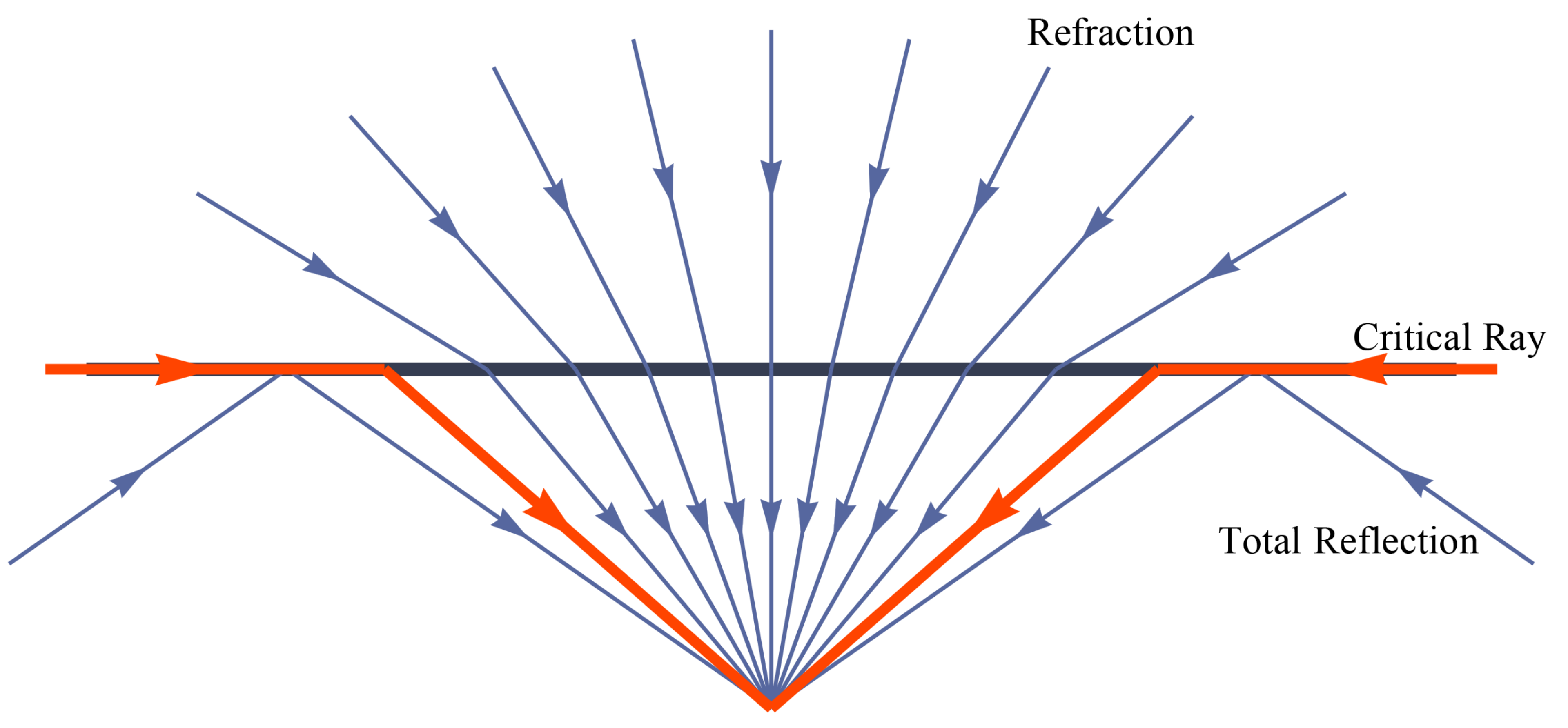
Tangential distortion

Due to lens imperfections

When Geometric Distortion is Useful?

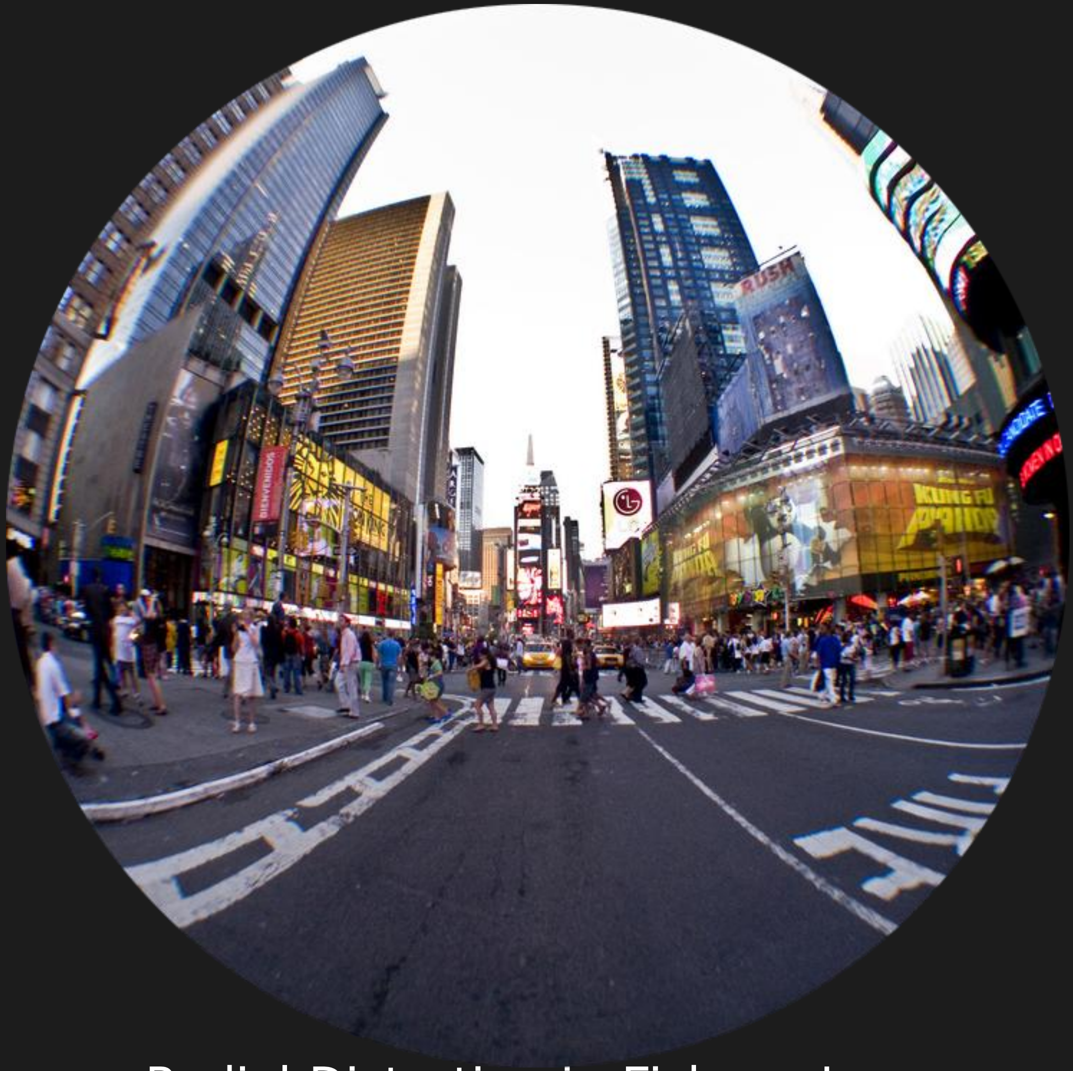


When Geometric Distortion is Useful?



When Geometric Distortion is Useful

Fisheye Lens



Radial Distortion in Fisheye Lens

Geometric Distortion Correction

Radial (Barrel) distortion

Undistorted image

The Digital Camera

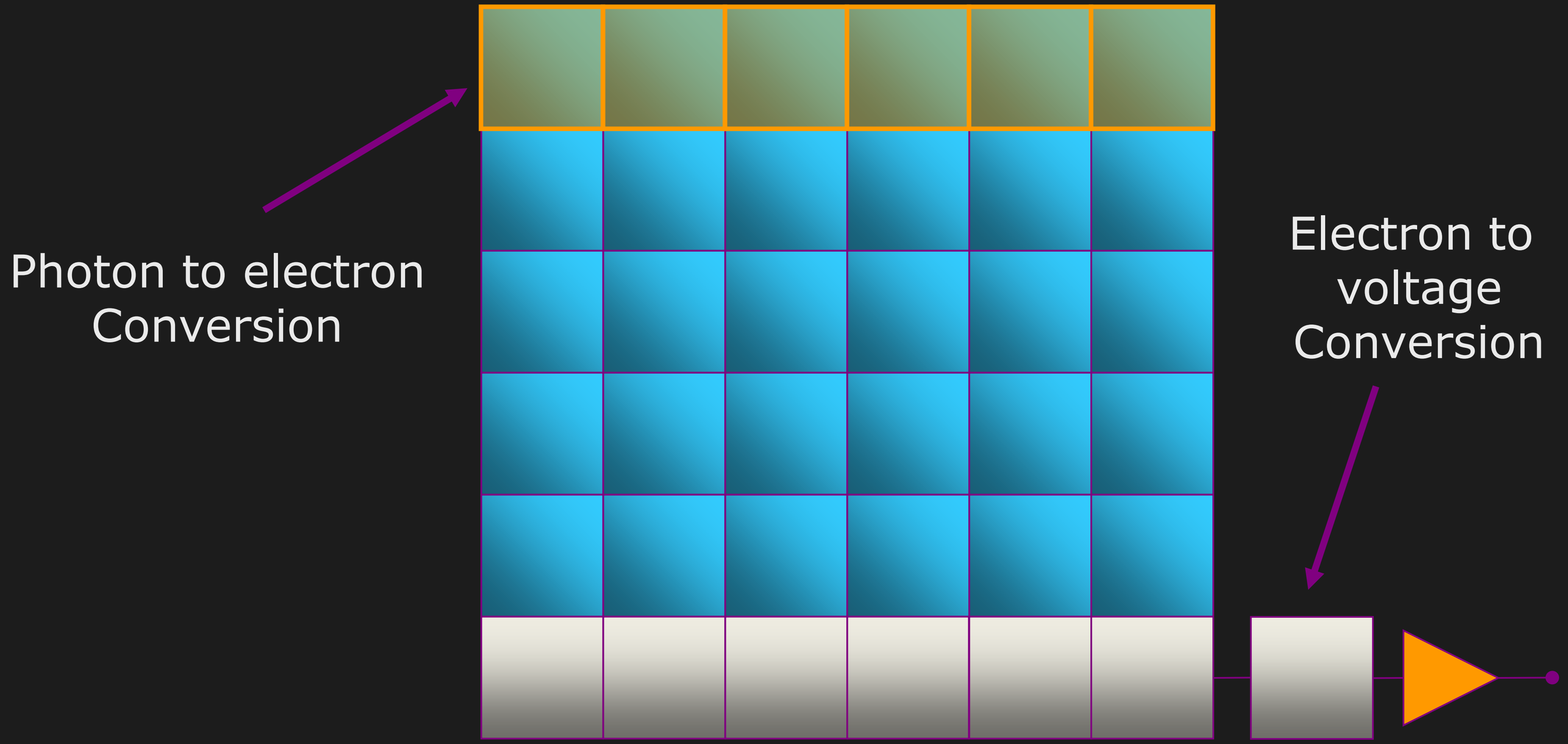
Two main kinds of sensors

Charge-Coupled Device (CCD): photons are accumulated in each active *well* during exposure and all charges are transferred from well-to-well until they are deposited at the amplifiers, and this is then passed to the Analog-to-Digital Converter (ADC).

Complimentary Metal Oxide on Silicon (CMOS)

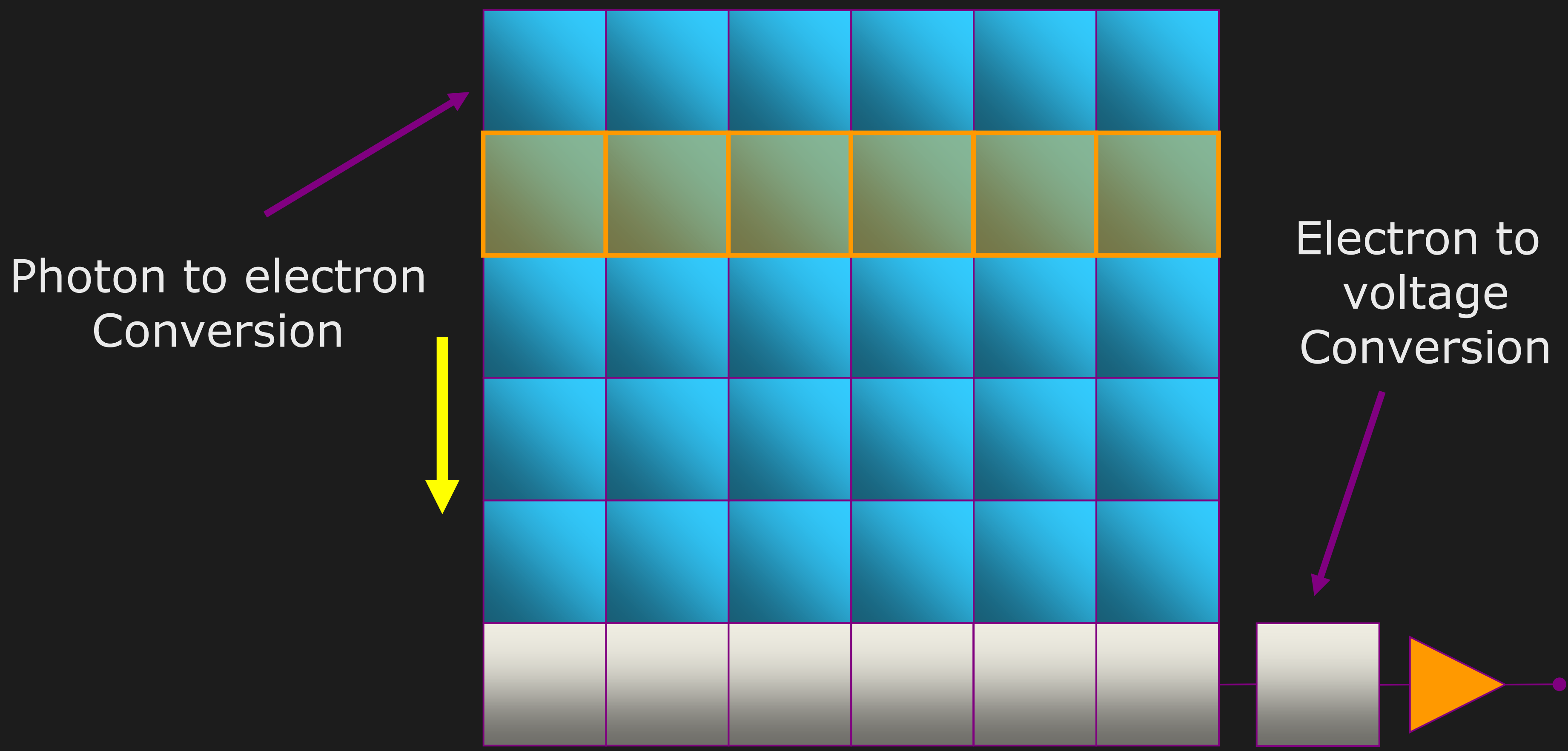
Popular Types of Image Detectors

Charge Coupled Device (CCD)



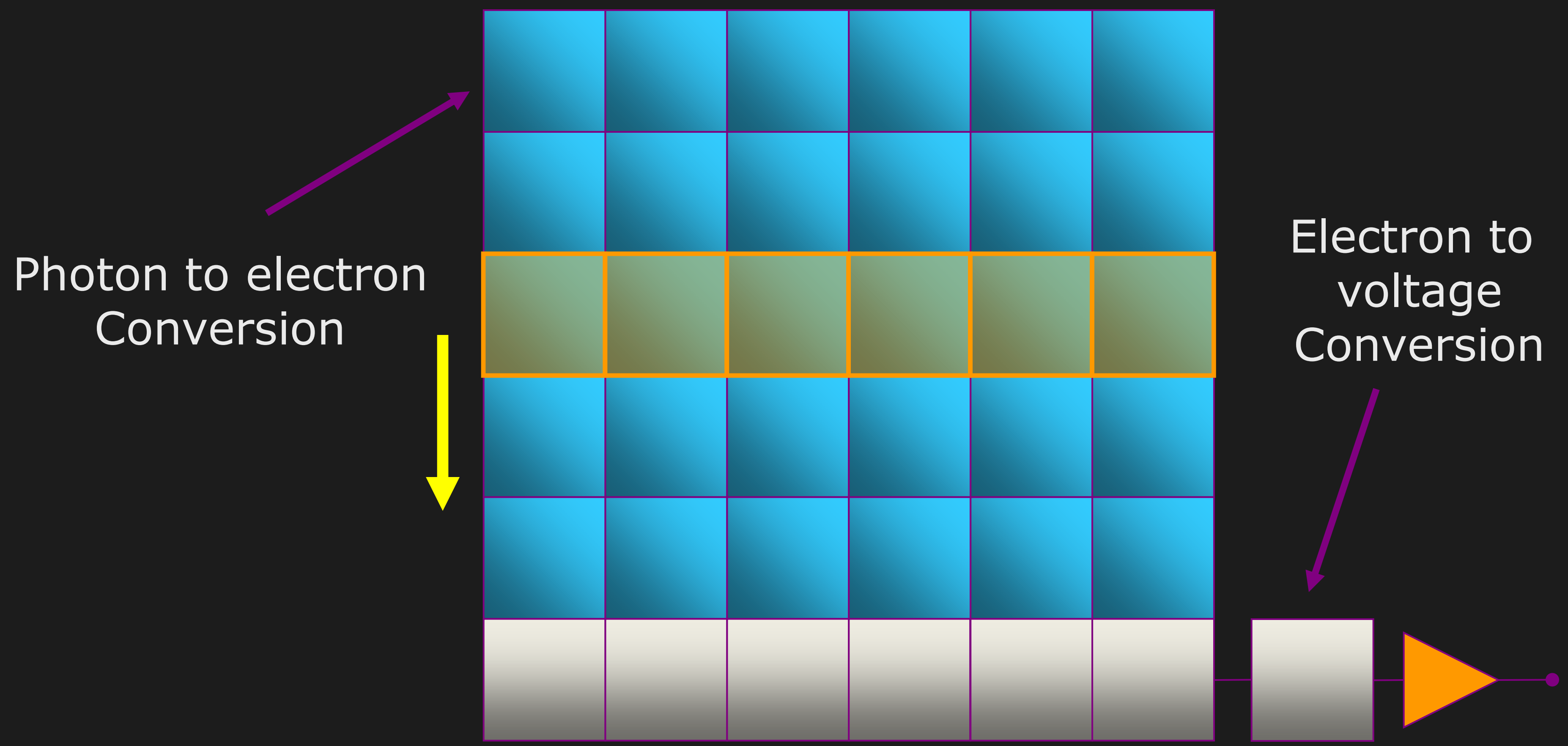
Popular Types of Image Detectors

Charge Coupled Device (CCD)



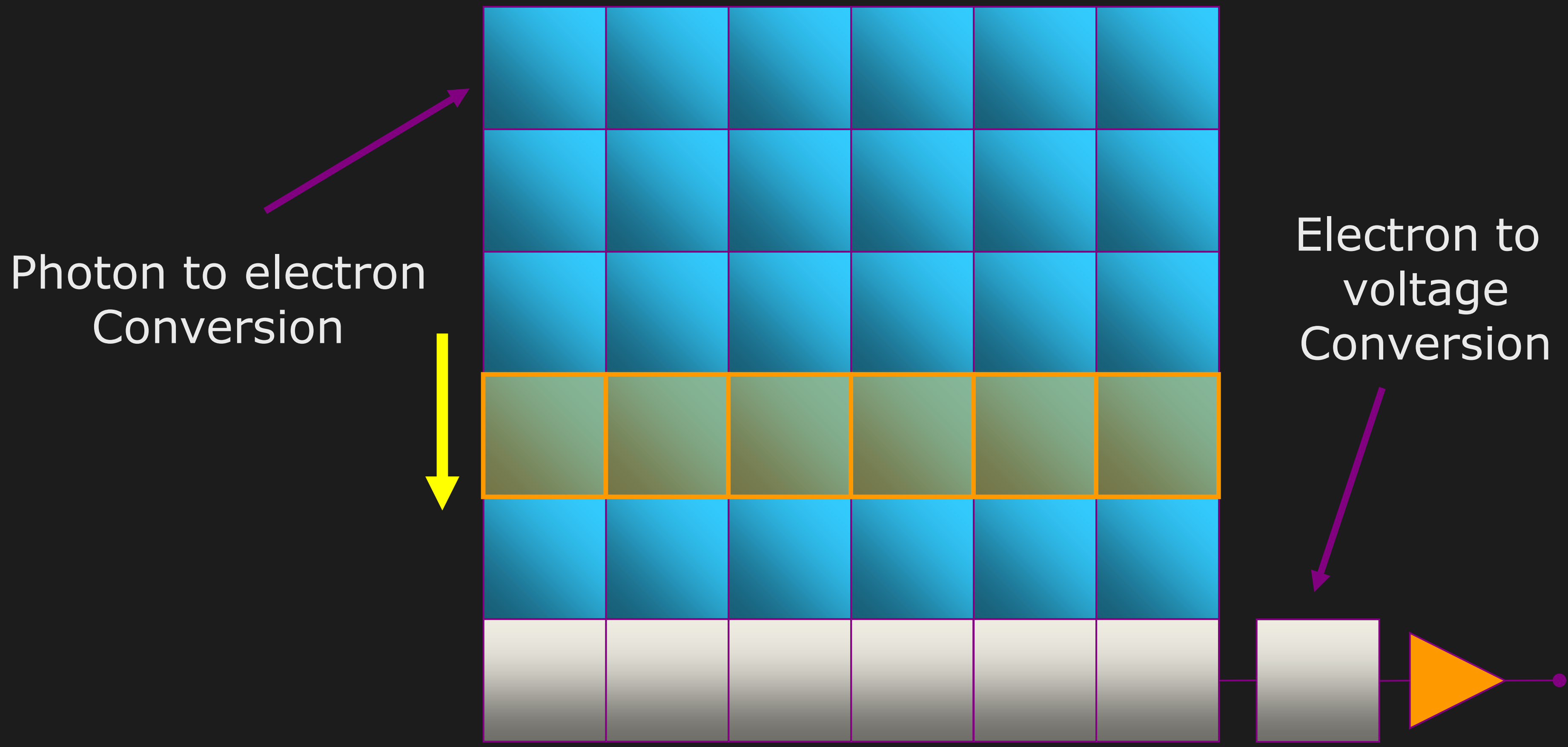
Popular Types of Image Detectors

Charge Coupled Device (CCD)



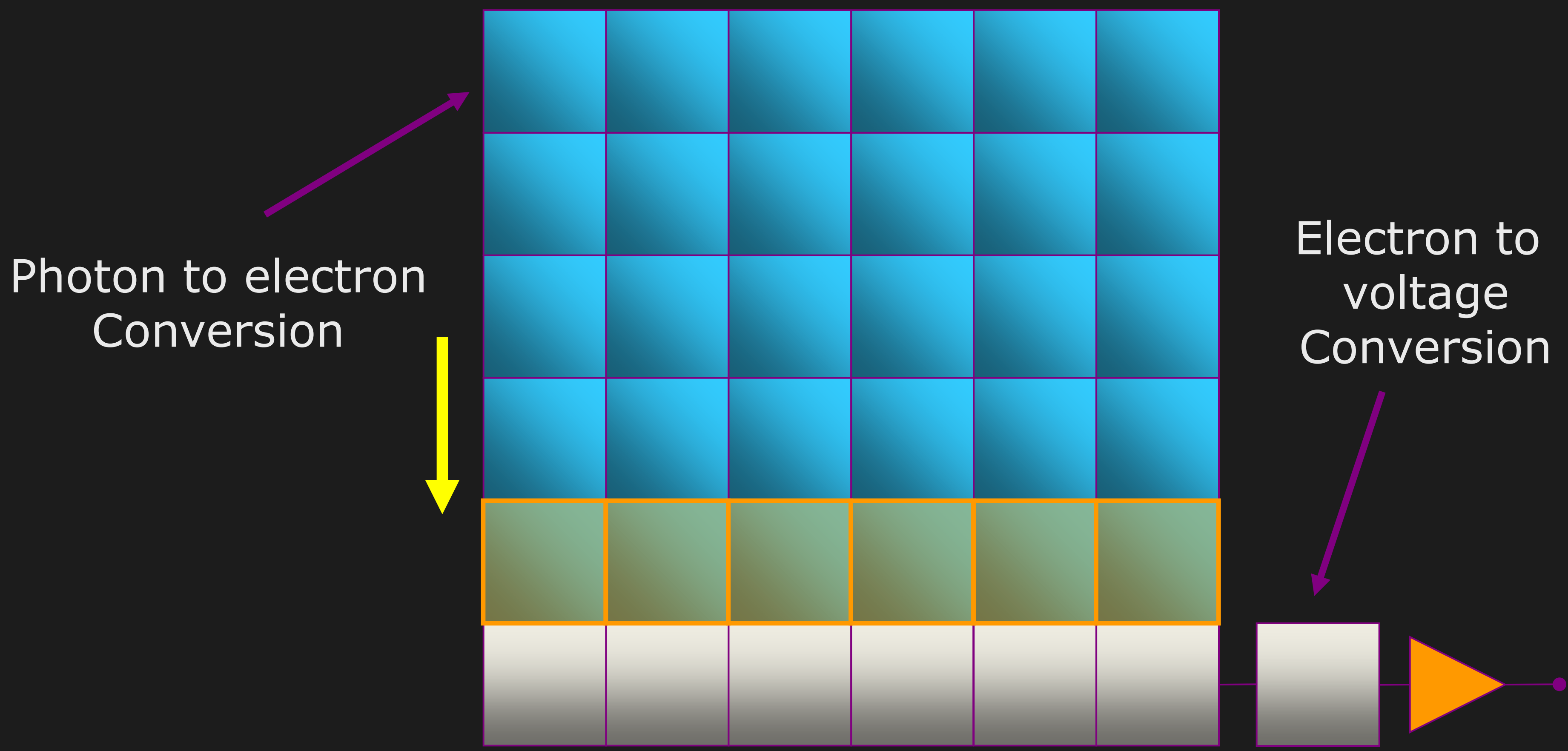
Popular Types of Image Detectors

Charge Coupled Device (CCD)



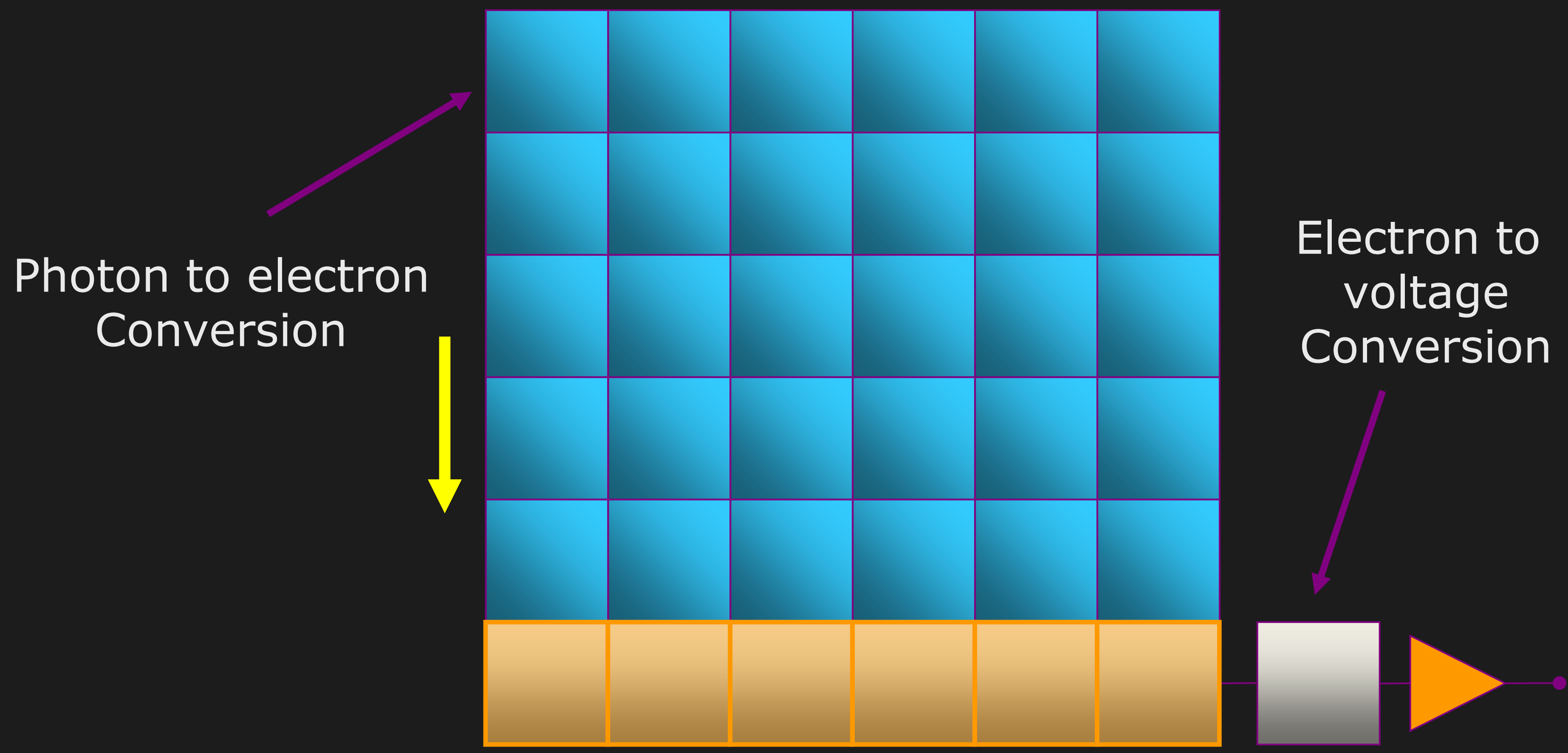
Popular Types of Image Detectors

Charge Coupled Device (CCD)



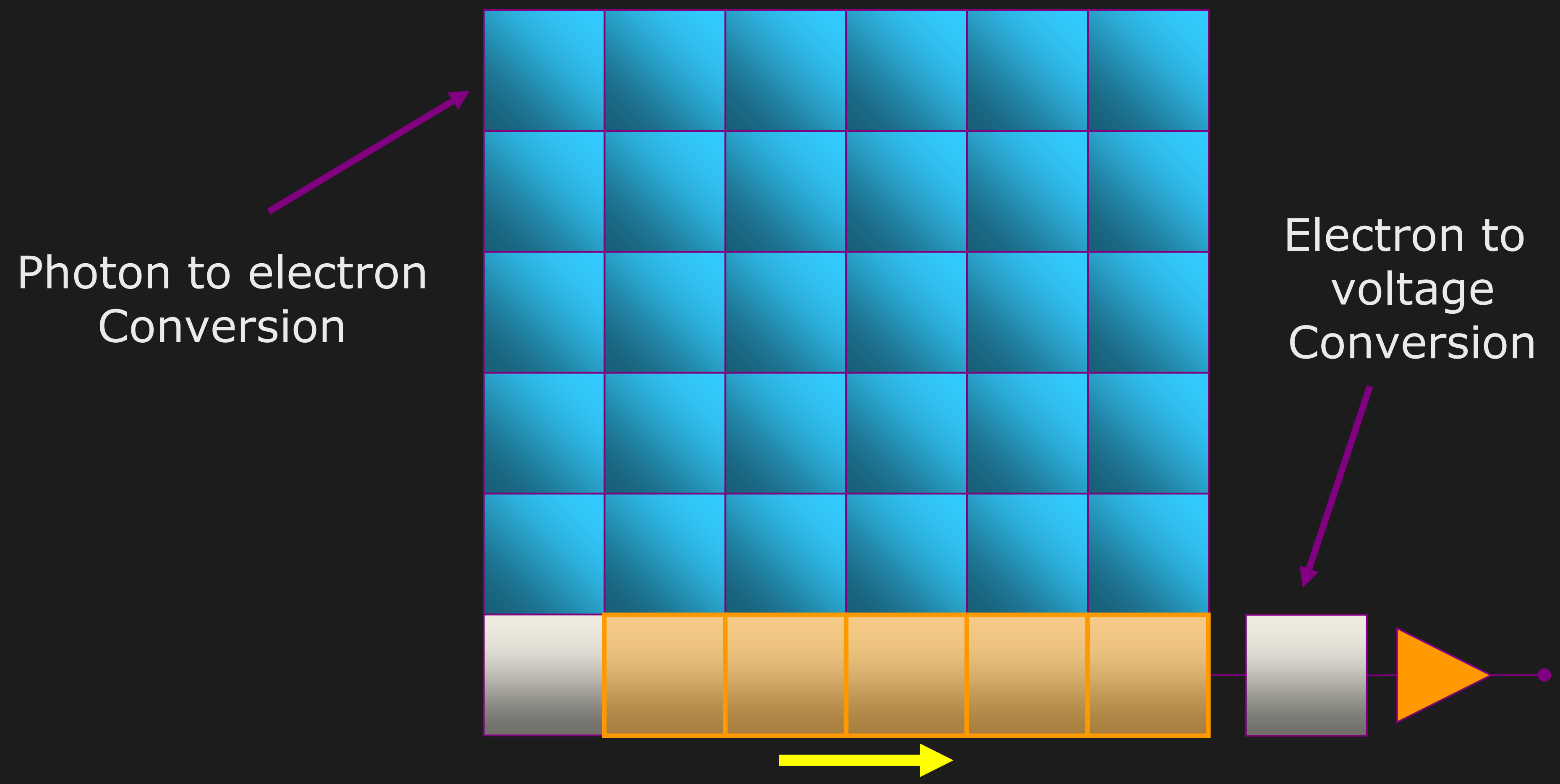
Popular Types of Image Detectors

Charge Coupled Device (CCD)



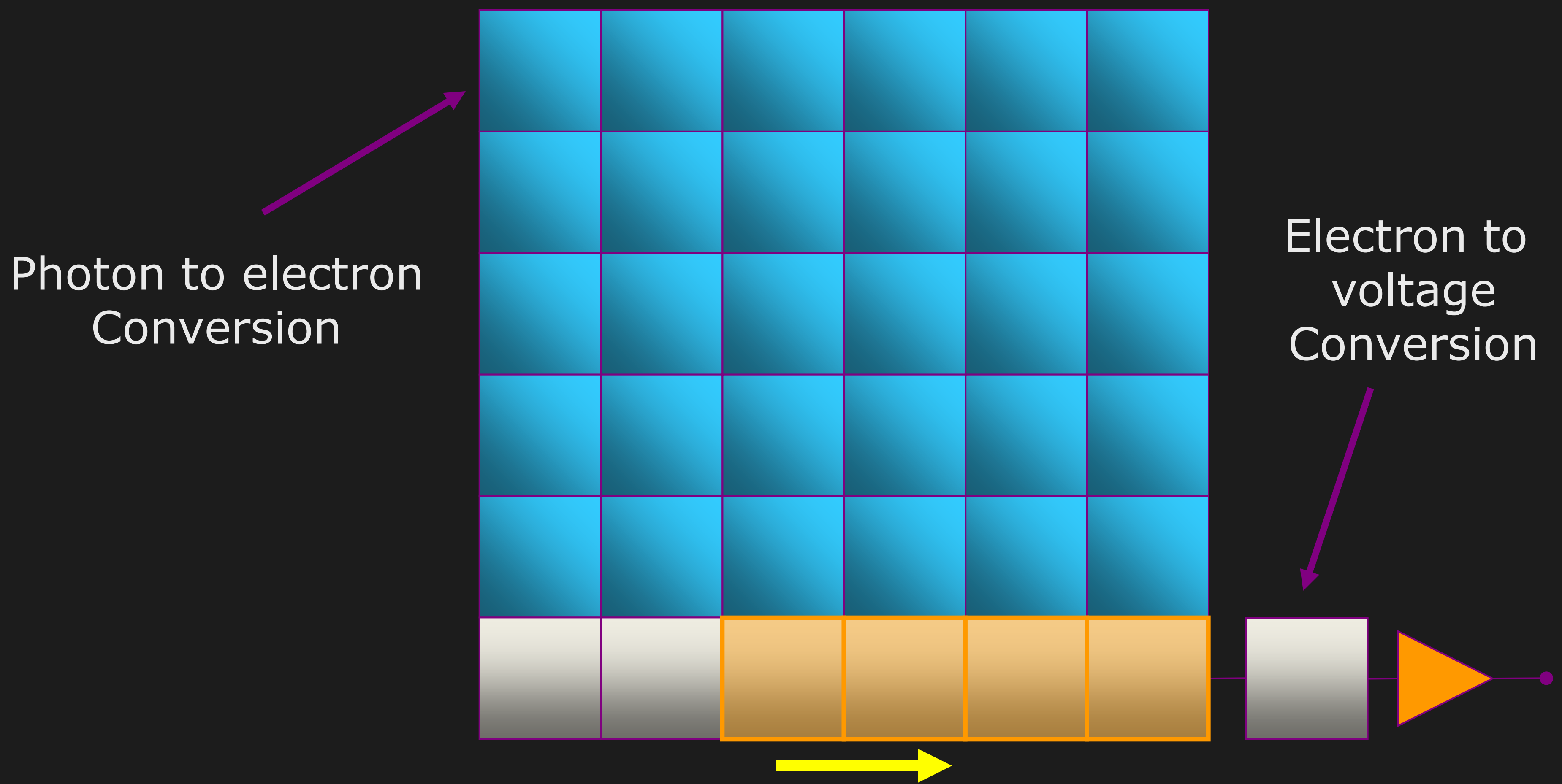
Popular Types of Image Detectors

Charge Coupled Device (CCD)



Popular Types of Image Detectors

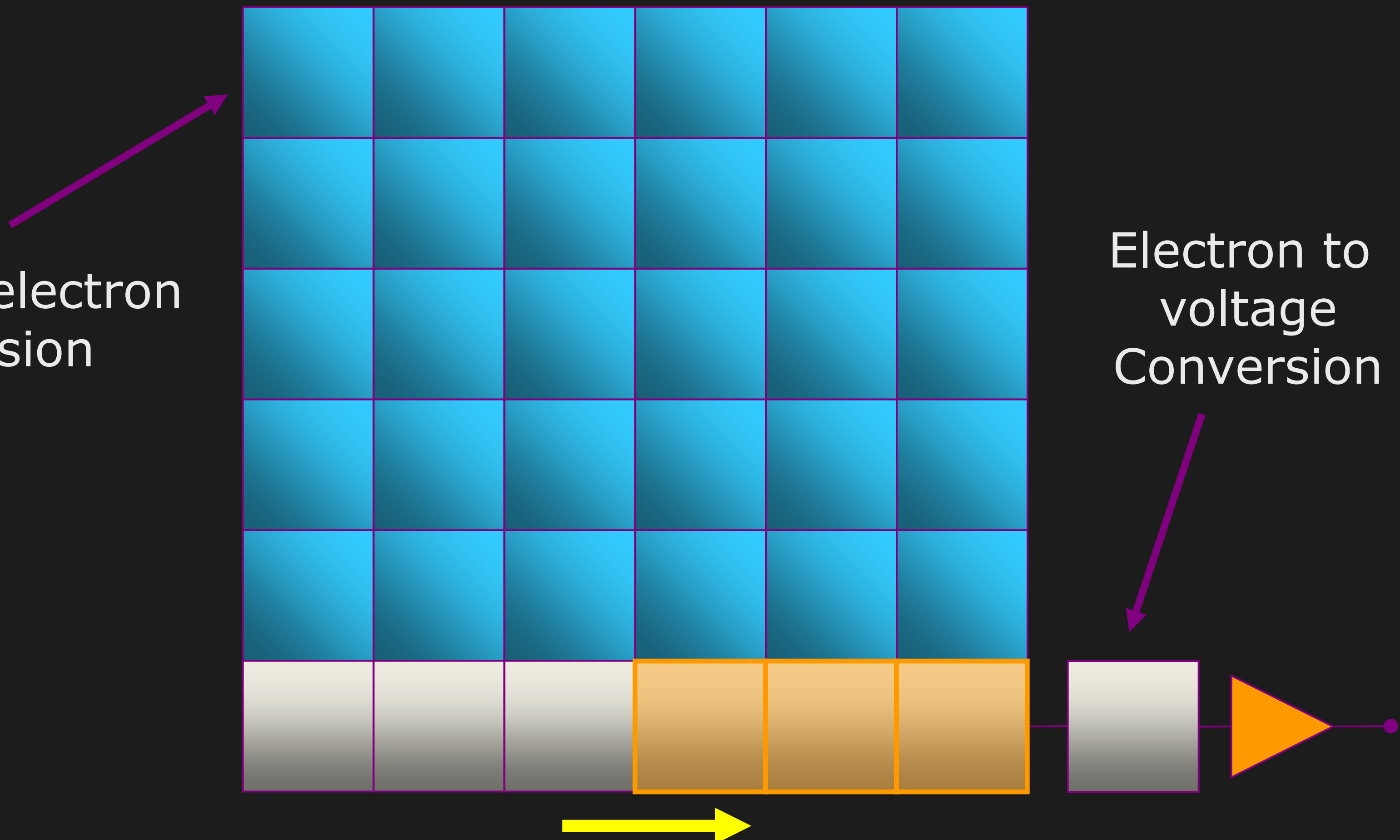
Charge Coupled Device (CCD)



Popular Types of Image Detectors

Charge Coupled Device (CCD)

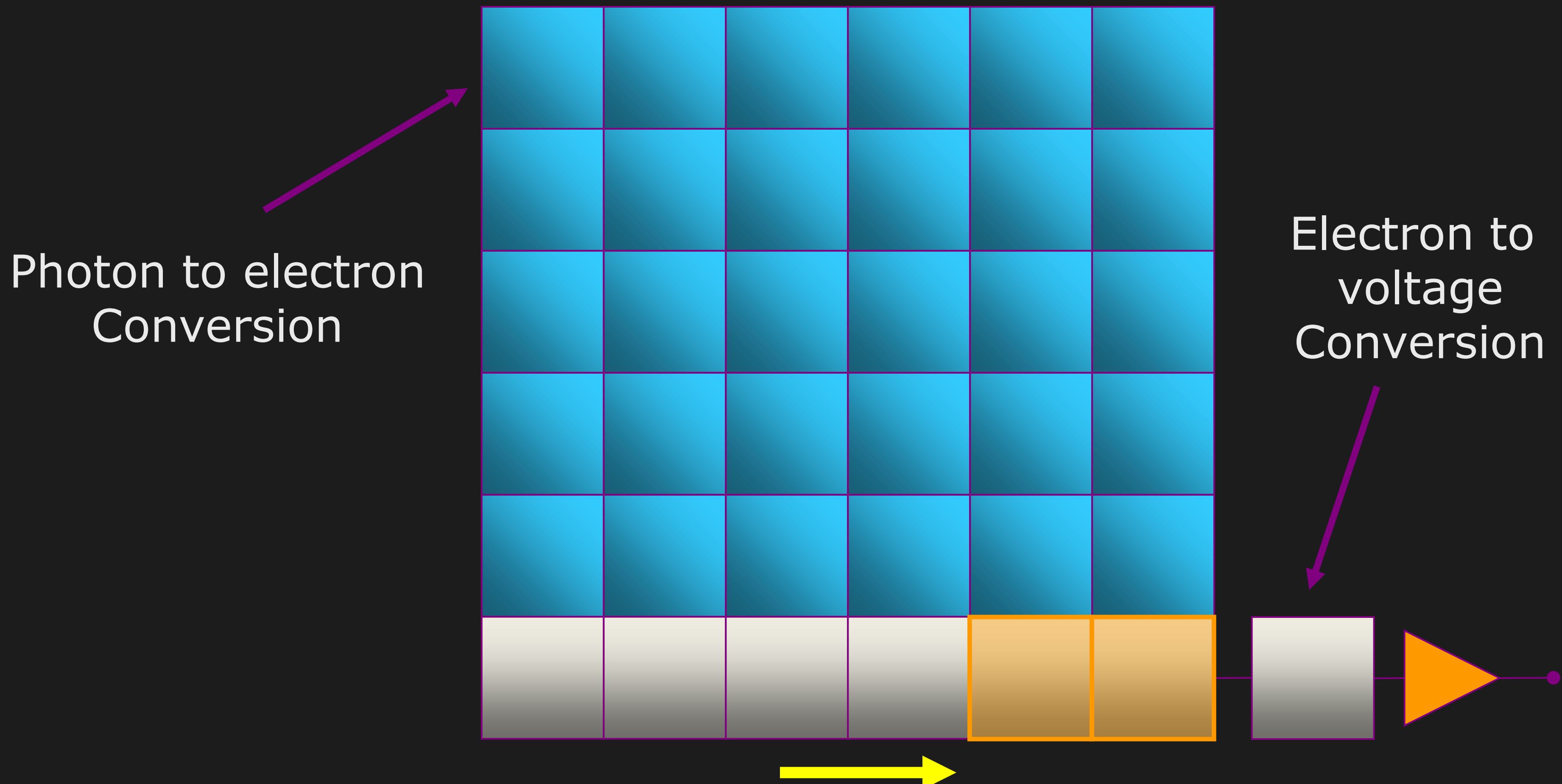
Photon to electron Conversion



Electron to voltage Conversion

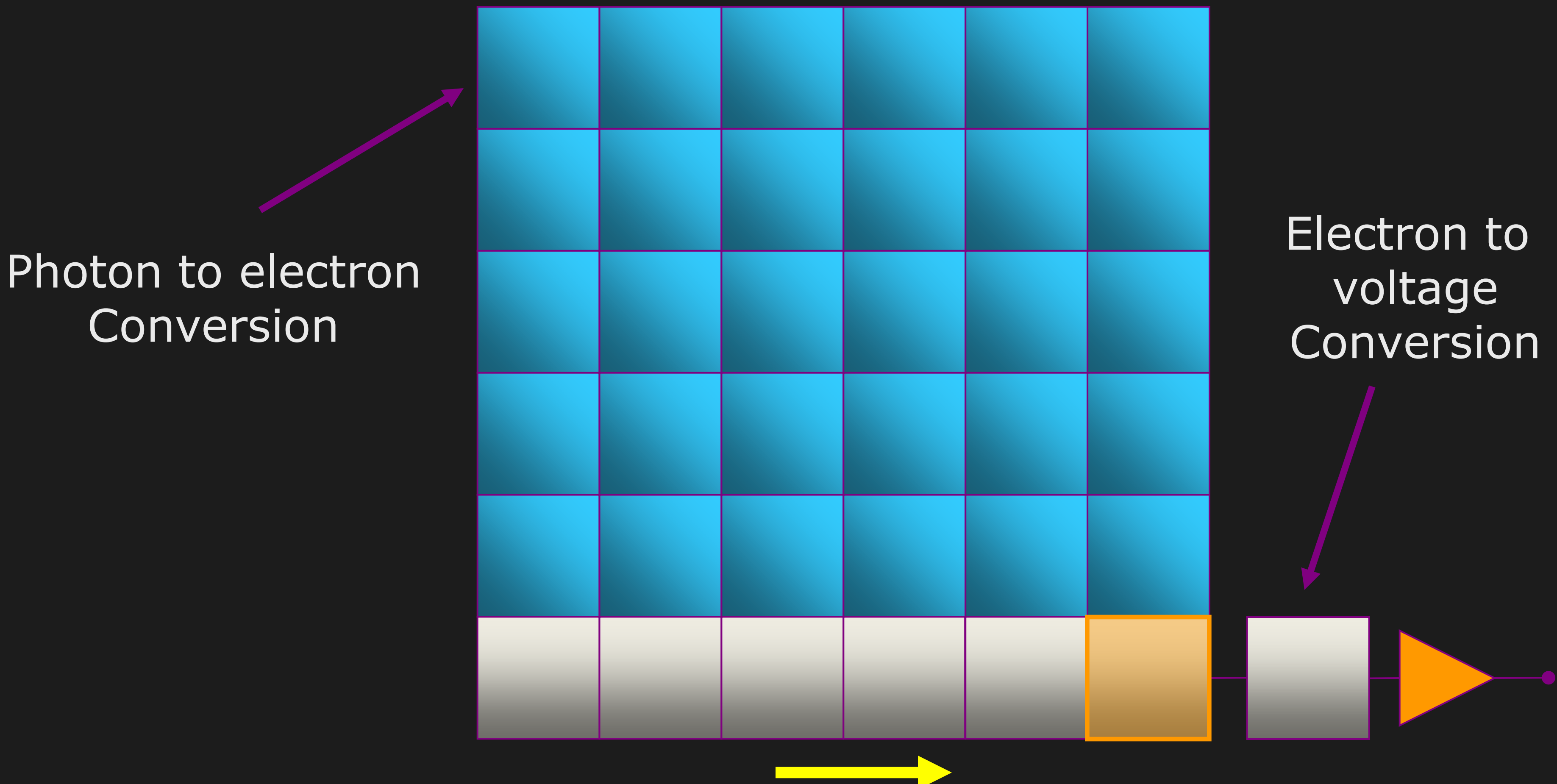
Popular Types of Image Detectors

Charge Coupled Device (CCD)



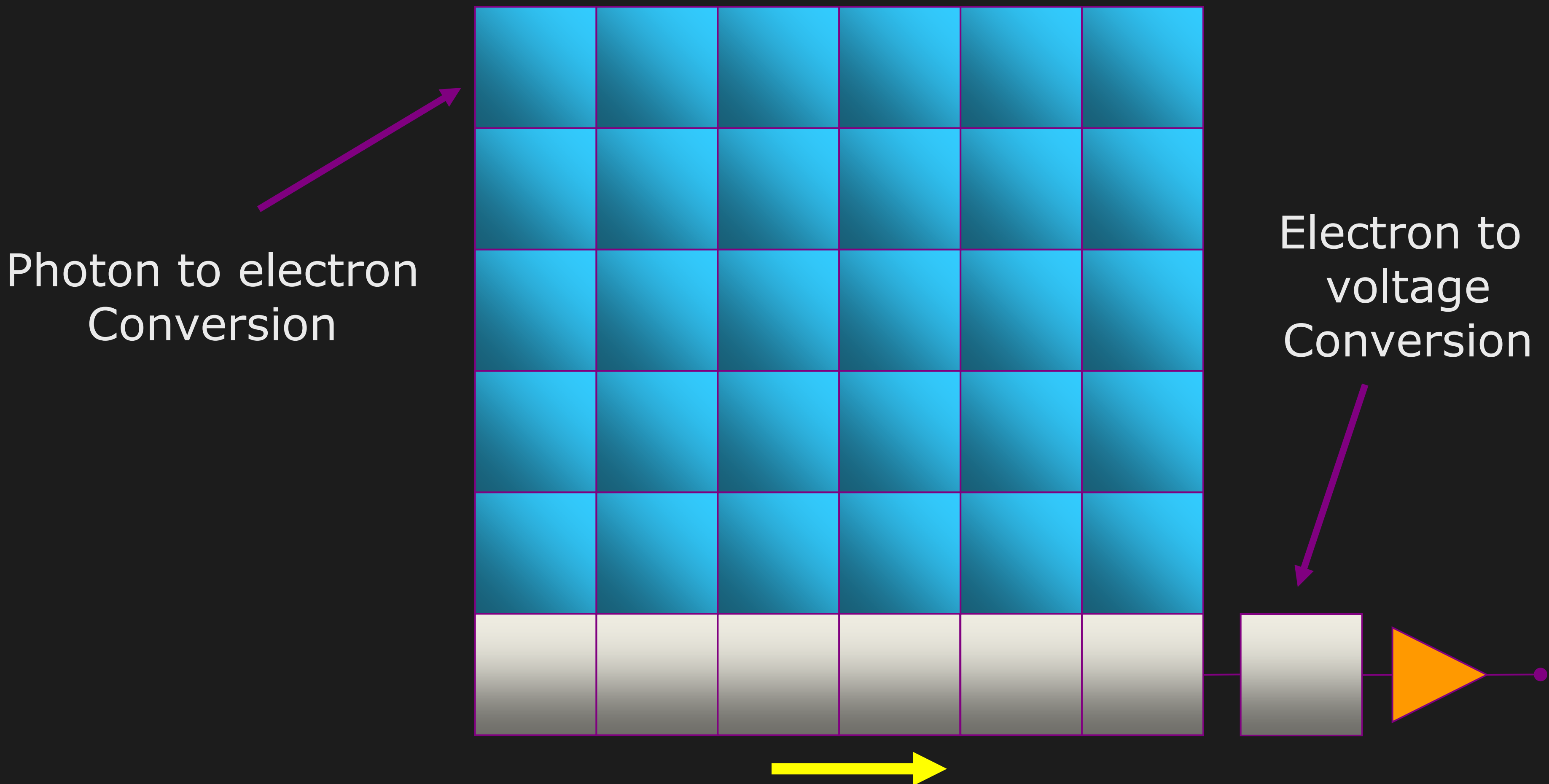
Popular Types of Image Detectors

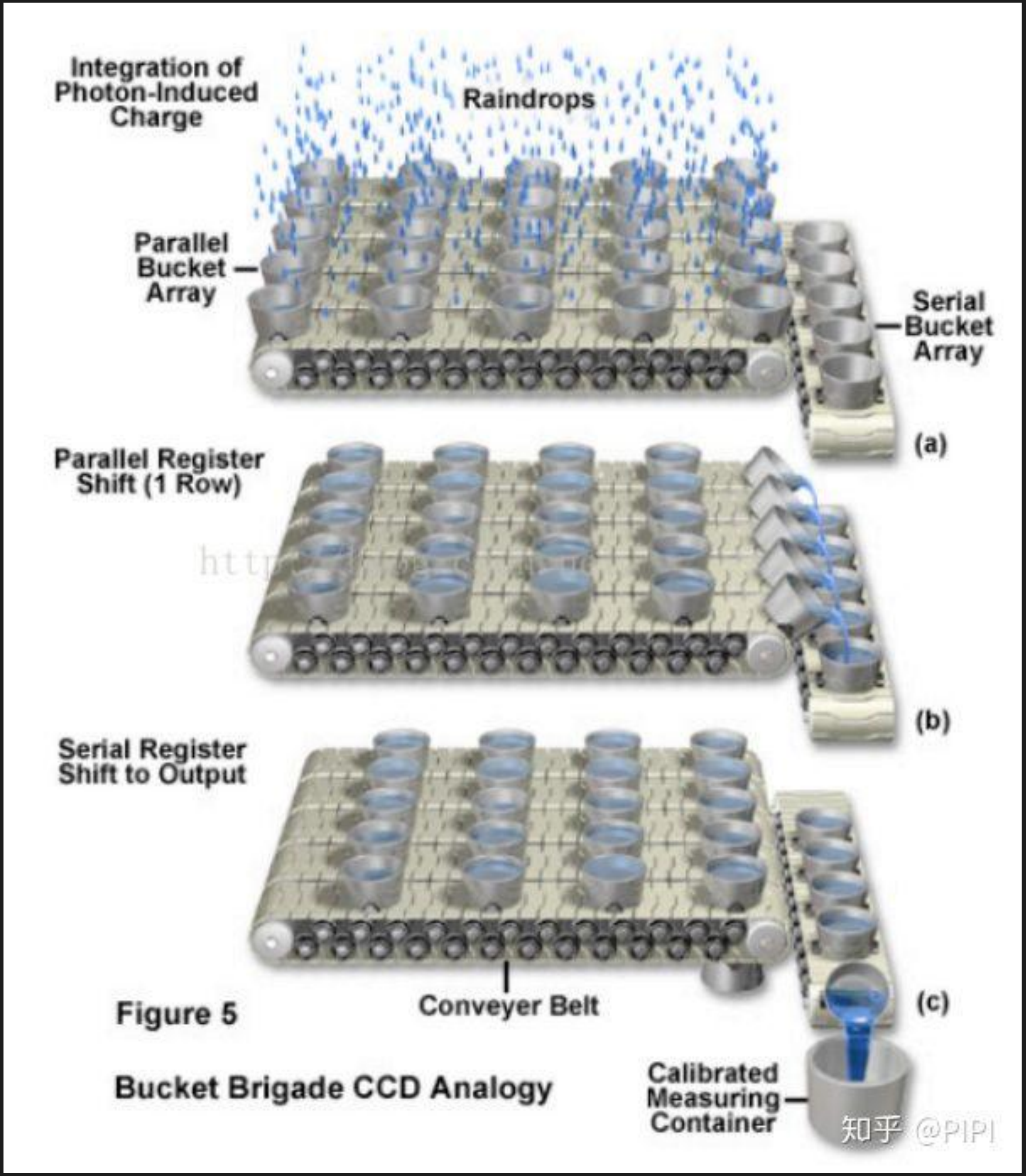
Charge Coupled Device (CCD)



Popular Types of Image Detectors

Charge Coupled Device (CCD)




Integration of Photon-Induced Charge
Raindrops
Parallel Bucket Array
Serial Bucket Array
(a)
Parallel Register Shift (1 Row)
(b)
Serial Register Shift to Output
Conveyer Belt
(c)
Figure 5
Bucket Brigade CCD Analogy
Calibrated Measuring Container
知乎 @PIPI

The Digital Camera

Two main kinds of sensors

Charge-Coupled Device (CCD): photons are accumulated in each active well during exposure and all charges are transferred from well-to-well until they are deposited at the amplifiers, and this is then passed to the Analog-to-Digital Converter (ADC).

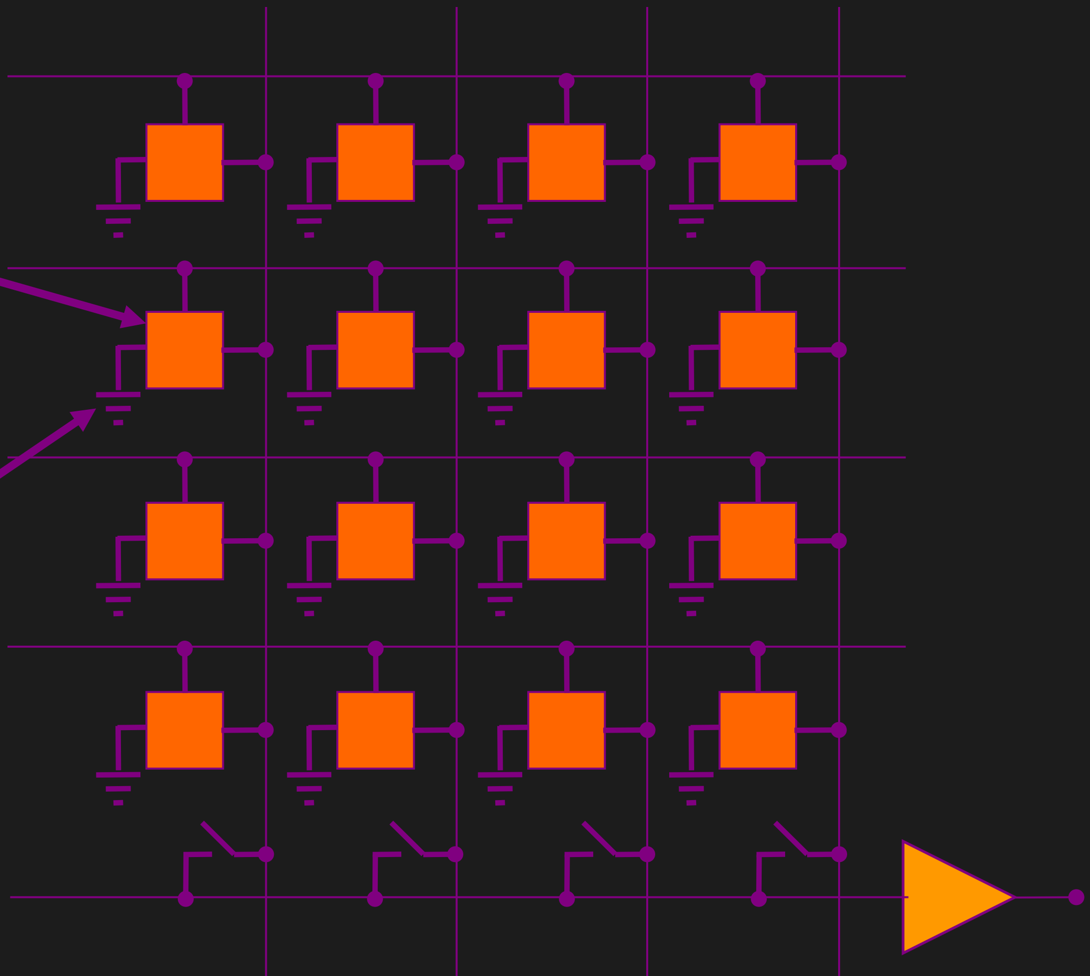
Complimentary Metal Oxide on Silicon (CMOS): photons hit the sensor directly and affect the conductivity of the photodetector, which can be selectively gated to control exposure duration and locally amplified before being read out.

Popular Types of Image Detectors

Complimentary Metal-Oxide Semiconductor (CMOS)

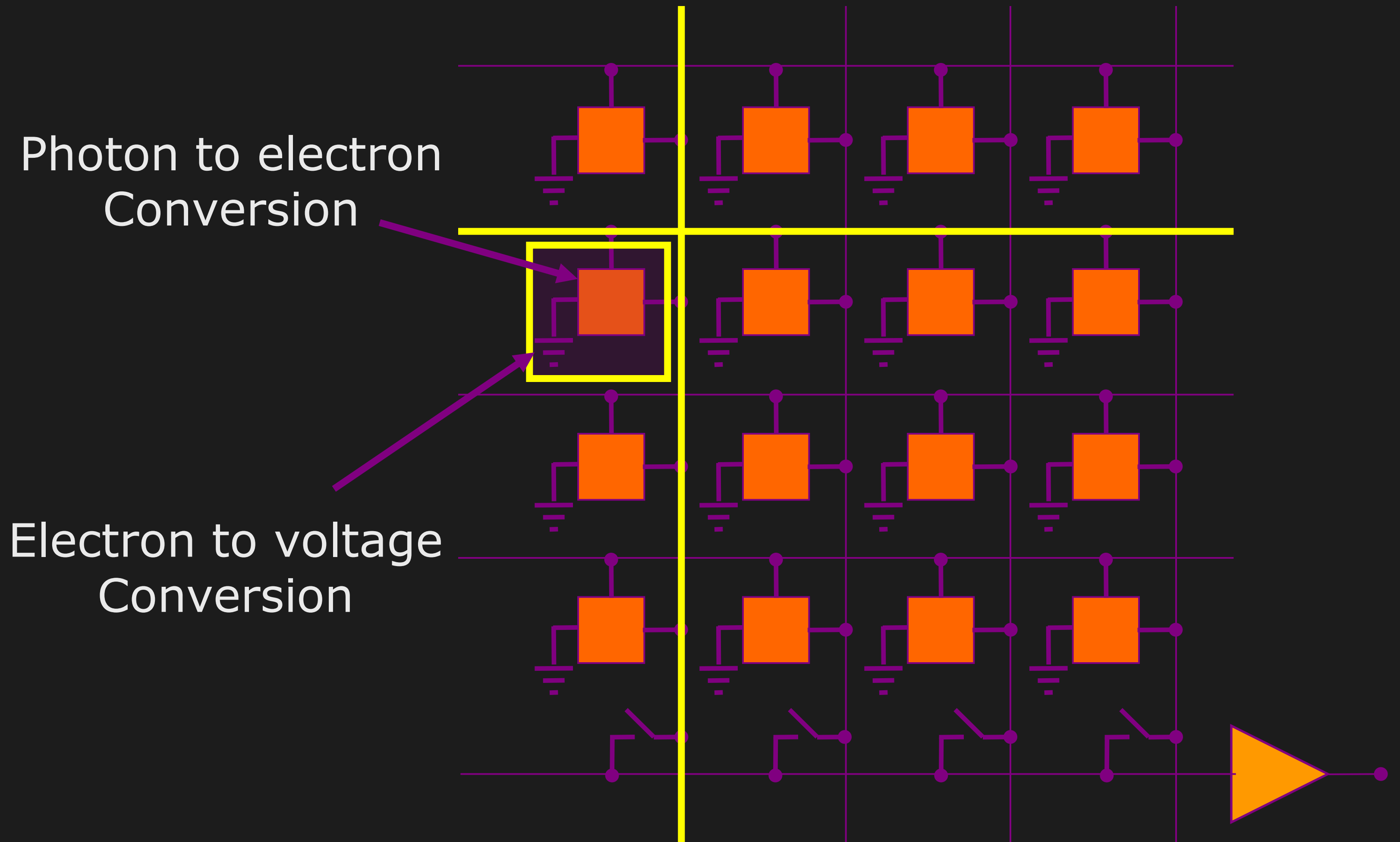
Photon to electron Conversion

Electron to voltage Conversion



Popular Types of Image Detectors

Complimentary Metal-Oxide Semiconductor (CMOS)

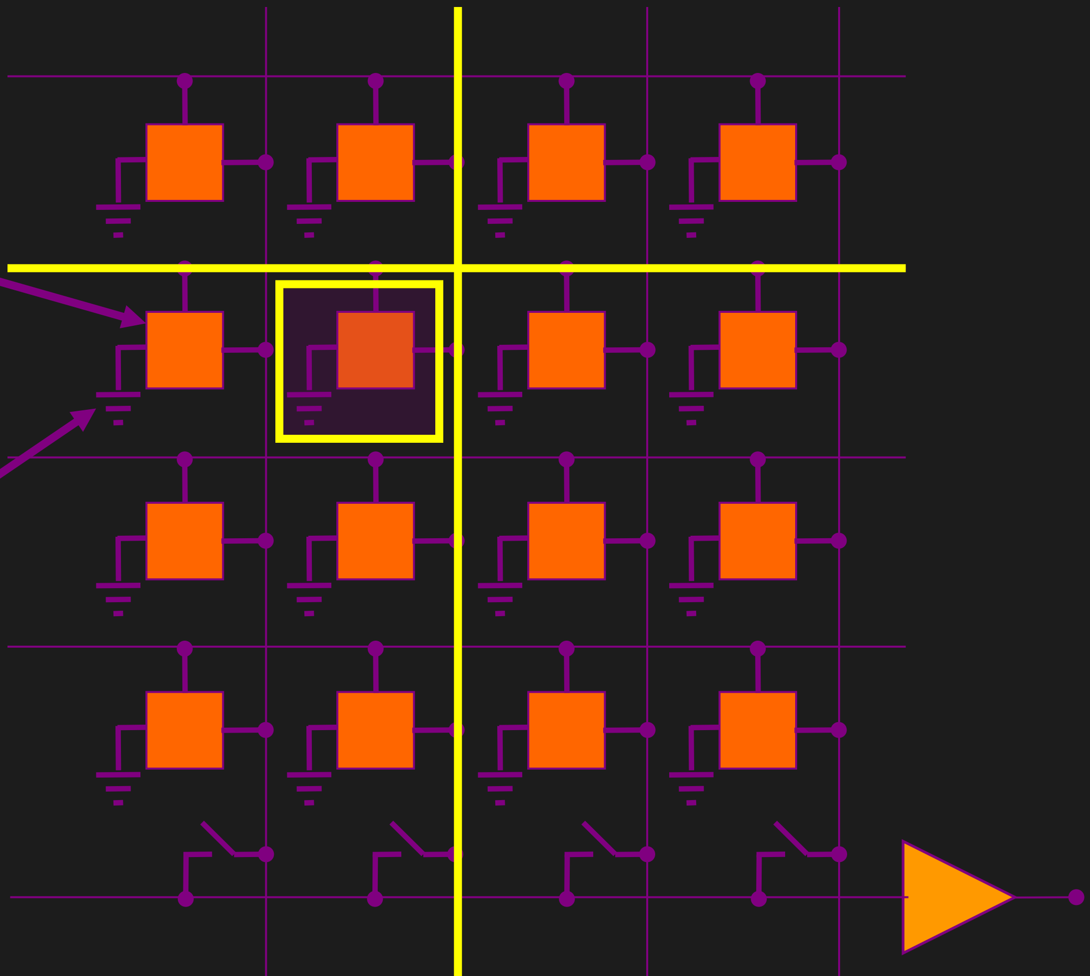


Popular Types of Image Detectors

Complimentary Metal-Oxide Semiconductor (CMOS)

Photon to electron
Conversion

Electron to voltage
Conversion

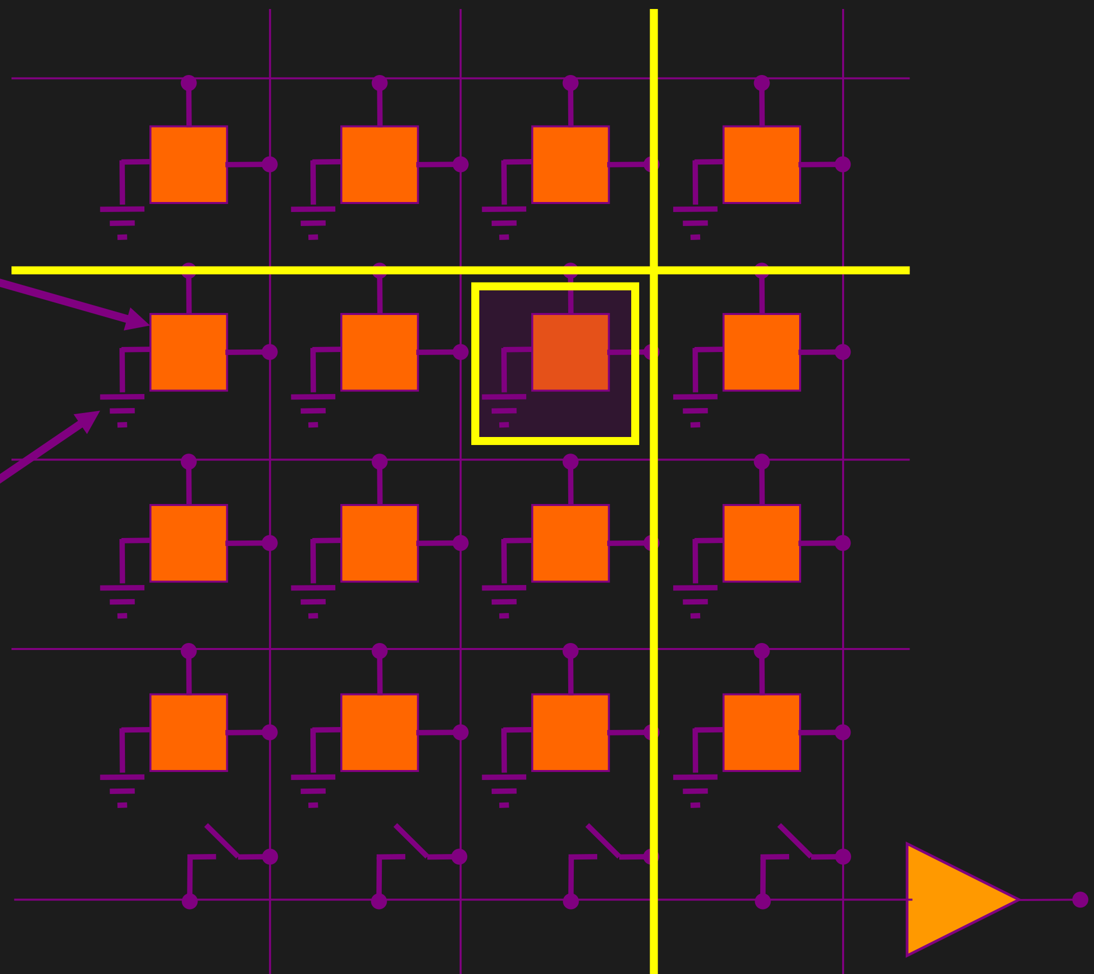


Popular Types of Image Detectors

Complimentary Metal-Oxide Semiconductor (CMOS)

Photon to electron Conversion

Electron to voltage Conversion

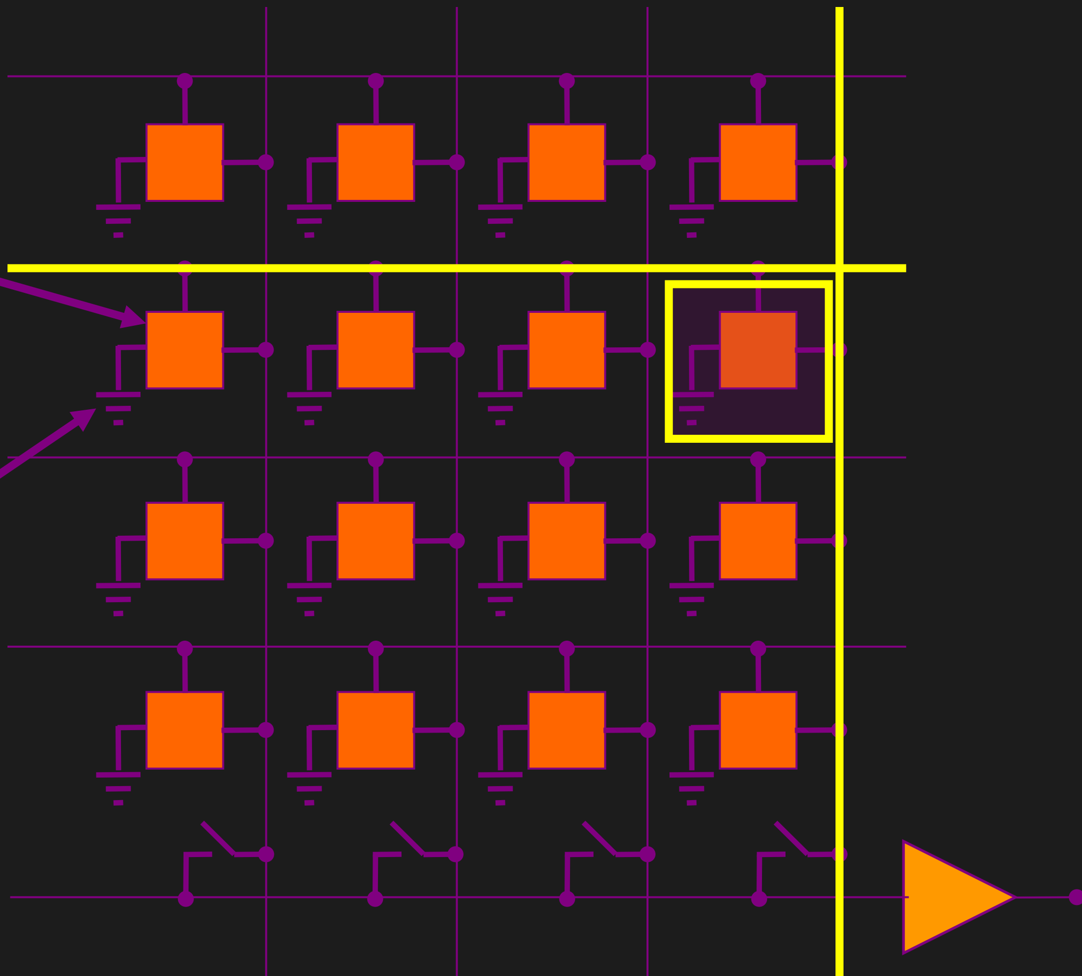


Popular Types of Image Detectors

Complimentary Metal-Oxide Semiconductor (CMOS)

Photon to electron Conversion

Electron to voltage Conversion



Comparison: CCD vs. CMOS

	CCD	CMOS
Signal Output	Separate circuit to convert photons to voltage	Convert photons to voltage within each pixel
Noise	Low	Moderate
Dynamic Range	High	Moderate
Uniformity	High	Low to Moderate
Windowing	Limited	Extensive
Power Consumption	Moderate	Low

What is "Color"?

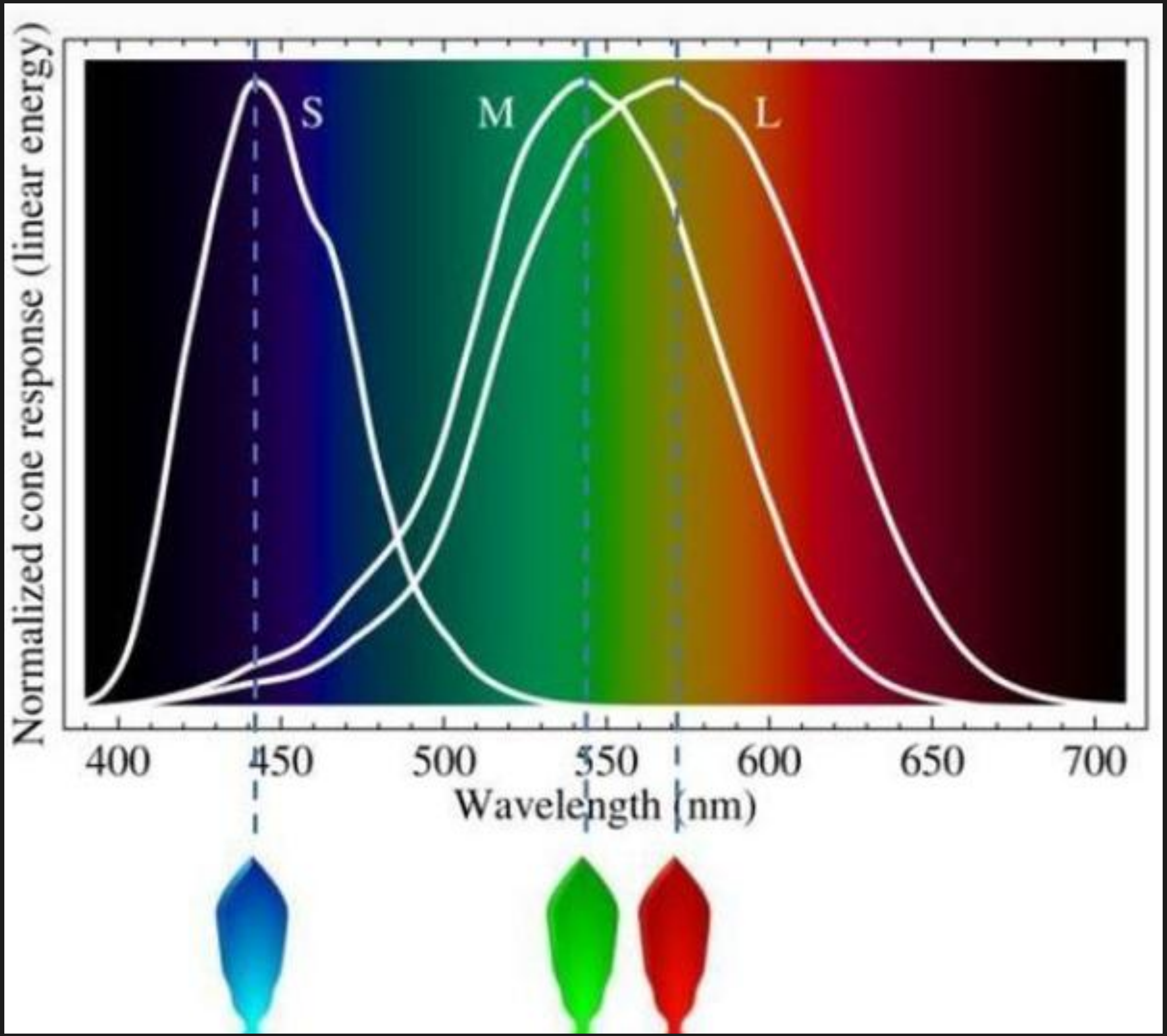
Human Response to different wavelengths

Visible light:

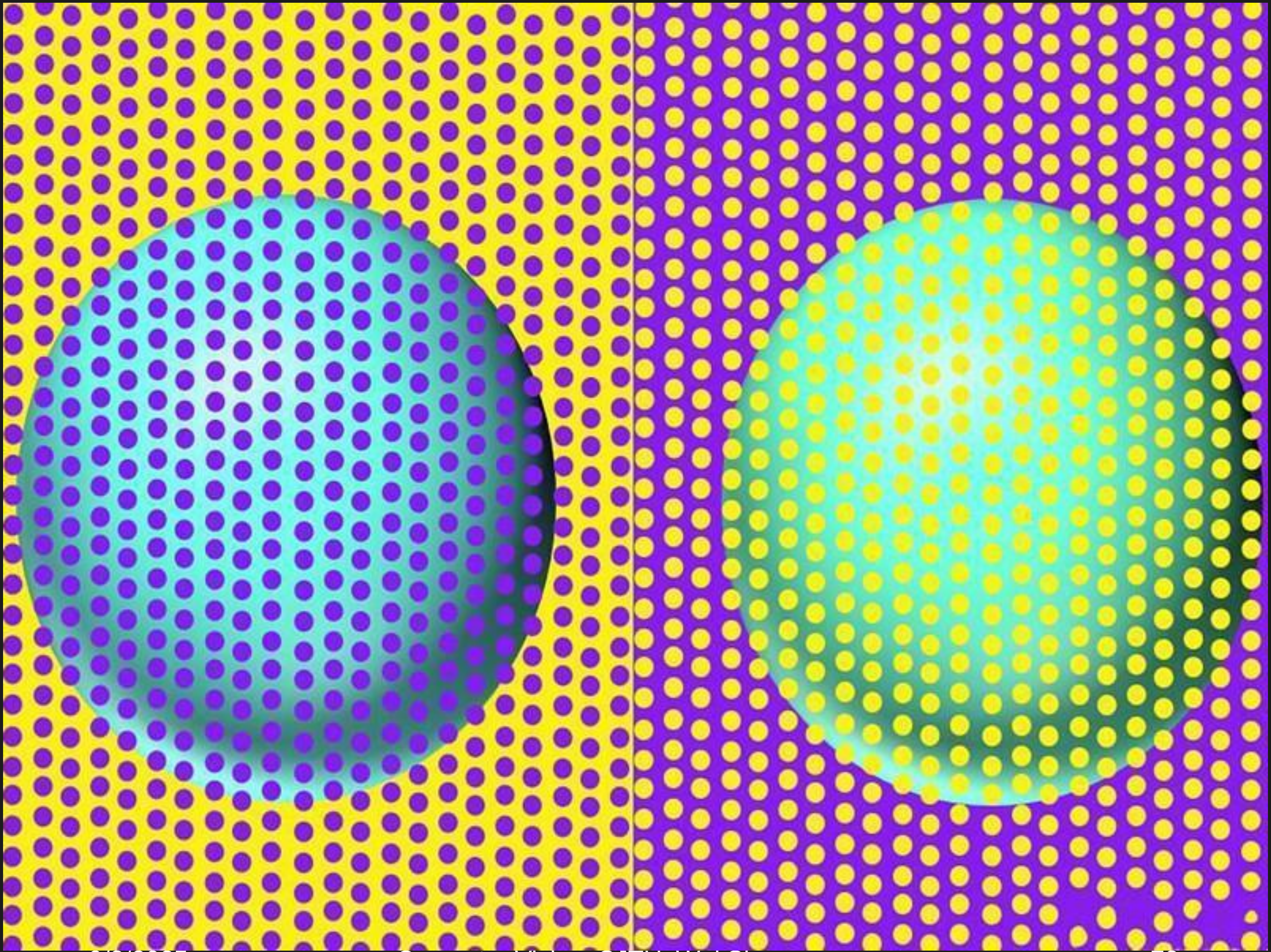
Do We recover Spectral Distribution $p(\lambda)$?

Sensors in the human eye: Rods & Cones
Neurochemical Sensors (3 types)

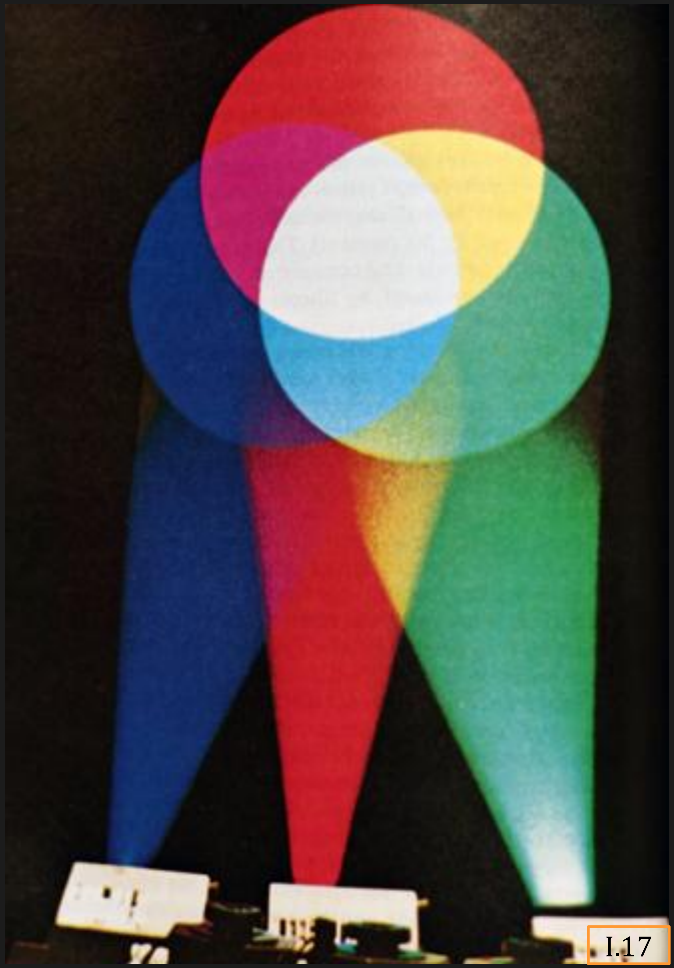
Cone Cell



Normalized cone response (linear energy)
S
M
L
400
450
500
550
600
650
700
Wavelength (nm)



The Mixing of Colors



Young's Experiment on Color Mixture

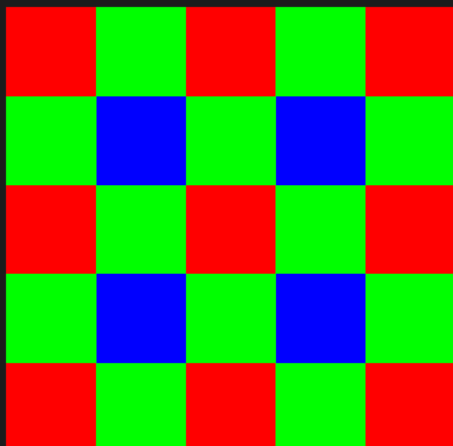
Human Sensation of nearly all colors can be produced using 3 wavelengths!

$$(\lambda_r,\ \lambda_g,\ \lambda_b) = (650, 530, 410)nm$$

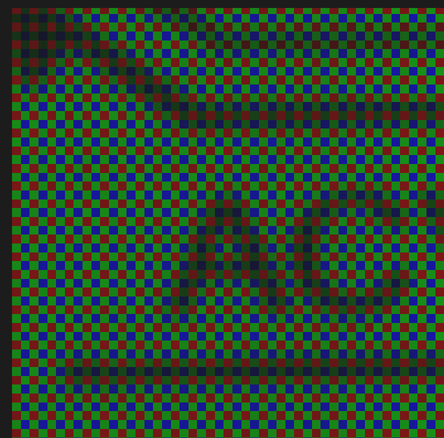
Hence, cameras and displays often use 3 filters:

(red, green, blue)

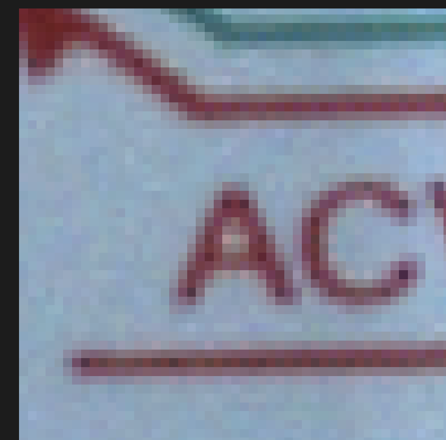
Sensing Color Using Color Mosaic



Bayer Pattern
(Color Filter Mosaic)



Raw Image



Interpolated Image

Color Filled in by Interpolation (Demosaicing)

High Dynamic Range: Multiple Exposures

Assume Camera Response $f(\cdot)$ is Linear

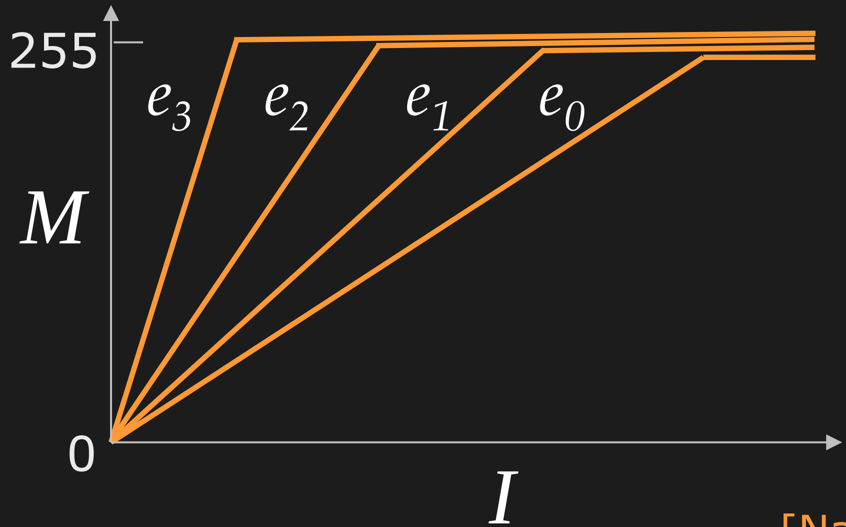
with e_0 e_1 e_2 e_3

$$M_0 = \min(e_0 \cdot \mathrm{I}, 255)$$

$$M_1 = \min(e_1 \cdot \mathrm{I}, 255)$$

$$M_2 = \min(e_2 \cdot \mathrm{I}, 255)$$

$$M_3 = \min(e_3 \cdot \mathrm{I}, 255)$$

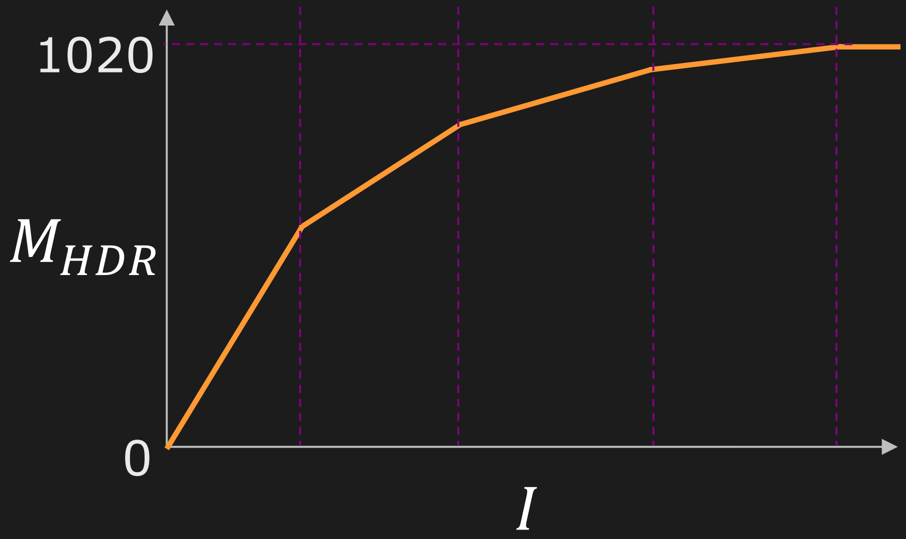


[Nayar 2000]

High Dynamic Range: Multiple Exposures

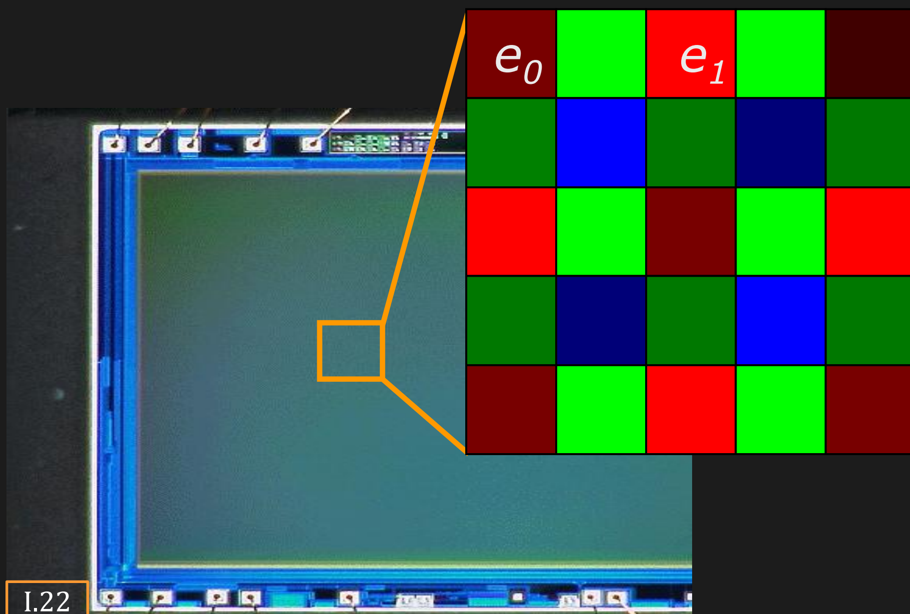
Aggregate Image: $M_{HDR} = M_0 + M_1 + M_2 + M_3$

Camera Response $f(\cdot)$ for Aggregate Image:

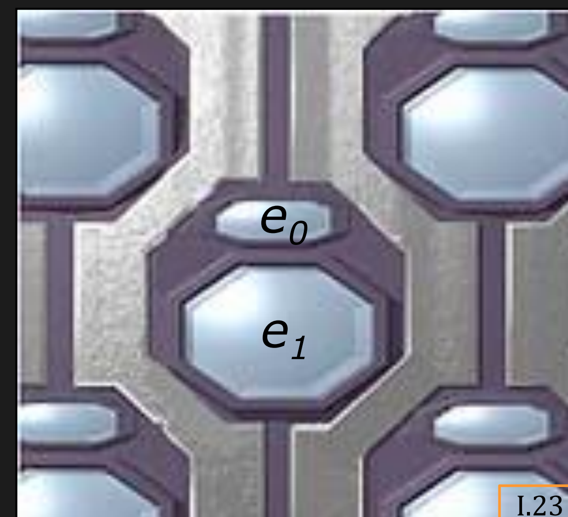


[Nayar 2000]

High Dynamic Range: Single Shot



Assorted Pixels:
Spatially Varying Color & Exposure



SuperCCD SR, FujiFilm:
Pixels with Subpixels

References: Textbooks

Robot Vision (Chapter 2 - Recommended Reading)
Horn, B. K. P., MIT Press

Computer Vision: A Modern Approach (Chapter 1)
Forsyth, D and Ponce, J., Prentice Hall

A Guided Tour of Computer Vision (Chapter 2, Pg:31-49)
Nalwa, V., Addison-Wesley Pub

Animal Eyes
Land, M. and Nilsson, D., Oxford University Press

Medical Physiology, Vol. I (Eye Physiology)
Mountcastle, V. B., C. V. Mosby Company

Eye and Brain (Human Vision)
Gregory, R., Princeton University Press

References: Papers

[Aizenberg 2001] J. Aizenberg, A. Tkachenko, S. Weiner, L. Addadi and G. Hendler. “Calcitic microlenses as part of the photoreceptor system in brittlestars.” Nature, 2001.

[Clarkson 2006] E. Clarkson, R. Levi-Setti, G. Horváth. ”The eyes of trilobites: The oldest preserved visual system”. Arthropod structure and development, 2006.

[Descartes 1637] R. Descartes. “La Dioptrique”. 1637.

[Frisius 1545] Gemma-Frisius. “De Radio Astronomica Et Geometrico”. 1545.

[Nillson 1994] D-E. Nilsson and S. Pelger. “A pessimistic estimate of the time required for an eye to evolve”. Proc of Royal Society, 1994.