

1. (30 points) 分析用 FTFS、FTBS、FTCS 和 BTCS 格式离散线性对流方程的稳定性.

$$\begin{cases} \frac{\partial u}{\partial t} + c \frac{\partial u}{\partial x} = 0, x \in \mathbb{R}, t > 0 \\ u(x, 0) = g(x), x \in \mathbb{R} \end{cases}$$

2. (30 points) 分析用 FTCS、BTCS 和 Richardson 格式离散扩散方程的稳定性.

$$\begin{cases} \frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial x^2}, x \in \mathbb{R}, t > 0 \\ u(x, 0) = g(x), x \in \mathbb{R} \end{cases}$$

3. (40 points) 求解对流扩散方程

$$\begin{cases} \frac{\partial u}{\partial t} + c \frac{\partial u}{\partial x} = \mu \frac{\partial^2 u}{\partial x^2}, x \in \mathbb{R}, t > 0 \\ u(x, 0) = g(x), x \in \mathbb{R} \end{cases}$$

的一种差分格式为

$$\frac{u_j^{n+1} - u_j^n}{\tau} + c \frac{u_{j+1}^n - u_{j-1}^n}{2h} = \mu \frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{h^2}$$

试分析其稳定性, 并给出稳定性条件.