偏微分方程数值解法　春季学期　2017
第一次作业 3.4-3.8　　姓名: _____

1. (*10 points*) 下列哪个算子是线性的?

1. $\mathcal{L}u = u_x + xu_y$
2. $\mathcal{L}u = u_x + uu_y$
3. $\mathcal{L}u = u_x + u_y^2$
4. $\mathcal{L}u = u_x + u_y + 1$
5. $\mathcal{L}u = \sqrt{1+x^2}(\cos y)u_x + u_{yxy} + [\arctan(x/y)]u$

2. (*20 points*) 对于下列方程, (1) 指出方程阶数; (2) 说明方程是线性的还是非线性的; (3) 说明方程是齐次的还是非齐次的; (4) 阐明做出以上结论的理由.

1. $u_t - u_{xx} + 1 = 0$
2. $u_t - u_x x + xu = 0$
3. $u_t - u_{xxt} + uu_x = 0$
4. $u_x\sqrt{1+u_x^2} + u_y\sqrt{1+u_y^2} = 0$
5. $u_t + u_{xxxx} + \sqrt{1+u} = 0$

3. (*30 points*) 对下列方程进行分类

1. $u_{xx} - 5u_{xy} = 0$
2. $4u_{xx} - 12u_{xy} + 9u_{yy} + u_y = 0$
3. $4u_{xx} + 6u_{xy} + 9u_{yy} = 0$
4. $yu_{xx} - 2u_{xy} + xu_{yy} = 0$

4. (*40 points*) 求解下列偏微分方程

1. $u_t + u_x = 0 \quad$ with $u(x,0) = -\sin \pi x$
2. $u_t + u_x = 0 \quad$ with $u(x,0) = \begin{cases} 1 & for \quad |x| < \frac{1}{3} \\ 0 & for \quad \frac{1}{3} < |x| \le 1 \end{cases}$
3. $u_x + 2xy^2 u_y = 0$