Implementation of unsteady flamelet progress model in OpenFOAM

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Abstract

This manual shows the implementation of a common turbulent combustion model, naming unsteady flamelet progress model with OpenFOAM.

1. Governing equations

$$\frac{\partial \bar{\rho}}{\partial t}+\frac{\partial}{\partial x_j}\left(\rho u_j\right)=0$$

$$\frac{\partial\left(\bar{\rho} u_i\right)}{\partial t}+\frac{\partial\left(\rho u_i u_j\right)}{\partial x_j}=-\frac{\partial \bar{p}}{\partial x_i}+2 \bar{\mu} \frac{\bar{\partial} S_{ij}}{\partial x_j}+\frac{\partial \tau_{t,ij}}{\partial x_j}$$

$$\frac{\partial \bar{\rho} Z}{\partial t}+\frac{\partial}{\partial x_j}\left(\bar{\rho} u_j Z\right)=\frac{\partial}{\partial x_j}\left[\bar{\rho}\left(\alpha_Z+\alpha_t\right) \frac{\partial Z}{\partial x_j}\right]$$

$$\frac{\partial\left(\bar{\rho} C\right)}{\partial t}+\frac{\partial}{\partial x_j}\left[\bar{\rho} u_j C\right]=\frac{\partial}{\partial x_j}\left[\bar{\rho}\left(\alpha_C+\alpha_t\right) \frac{\partial C}{\partial x_j}\right]+\rho \dot{\omega}_C$$

2. Model for each term

(1) $\bar{\mu}$, α_Z , α_C T , $\bar{\rho}$, $\dot{\omega}_C$: interpolated from the tabulated flamelet table.

(2) For momentum equation:

$$S_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$$

$$\tau_{t,ij}=2 \mu_t S_{ij}-\frac{1}{3} \bar{\rho} q^2 \delta_{ij} ,$$

$$\mu_t=C_\mu \bar{\rho} \Delta^2\left|S\right|^2,\left|S\right|=\sqrt{S_{ij} S_{ij}}$$

$$\rho q^2=C_k \bar{\rho} \Delta^2\left|S\right|^2$$

(3) For scalar terms:

$$\bar{\rho} \alpha_t=C_\alpha \bar{\rho} \Delta^2\left|S\right|$$

(4) For variance terms,

$$\bar{\rho} \phi''^2=C_\phi \bar{\rho} \Delta^2\left|\nabla \phi\right|^2$$

that is

$$\bar{\rho} Z''^2=C_\phi \bar{\rho} \Delta^2\left|\nabla Z\right|^2$$

with ϕ replaced by quantity Z during calculation in the model of C_ϕ. We should notice the limitation that $Z''^2 \leq Z\left(1-Z\right)$.

(5) Dynamic procedure:

C_μ, C_k, C_α, C_ϕ are obtained via dynamic procedure:

$$C_\mu = \frac{\langle L_{ij} M_{ij} \rangle}{2\langle M_{kl} M_{kl} \rangle}, \quad L_{ij} = -\bar{\rho} u_i u_j + \bar{\rho} u_i u_j, \quad M_{ij} = \bar{\rho}\Delta^2 |S| S_{ij} - \bar{\rho}\Delta^2 |S| S_{ij}$$

$$C_{\alpha,k} = \frac{\langle L_i M_i \rangle}{\langle M_j M_j \rangle}, \quad L_i = -\bar{\rho} u_i \phi_k + \bar{\rho} u_i \phi_k, \quad M_i = \bar{\rho}\Delta^2 |S| \phi_{k,i} - \bar{\rho}\Delta^2 |S| \phi_{k,i}$$

$$C_k = \frac{\langle LM \rangle}{\langle M^2 \rangle}, \quad L = -\bar{\rho} u_k u_k + \bar{\rho} u_k u_k, \quad M = \bar{\rho}\Delta^2 |S|^2 - \bar{\rho}\Delta^2 |S|^2$$

$$C_\phi = \frac{\langle LM \rangle}{\langle M^2 \rangle}, \quad L = -\bar{\rho}\phi\phi + \bar{\rho}\phi\phi, \quad M = \bar{\rho}\Delta^2 |\nabla S|^2 - \bar{\rho}\Delta^2 |\nabla \phi|^2$$

In dynamic procedure, there're two filters, the size of them is Δ and Δ. Correspondingly, filtered quantity on these two scales are represented as $\bar{\phi}$ and $\bar{\phi}$.

Table 1 Filtered quantity at two filters

	1 (Δ)	2 (Δ=2Δ)
Direct filtered	$\bar{\phi}$	$\bar{\phi} = \left(\bar{\phi}\right)$
Favre filtered	$\phi = \overline{\rho\phi} / \bar{\rho}$	$\phi = \overline{\rho\phi} / \bar{\rho}$

3. Tabulation
4. Solution procedure