BANA4095: Final Optimization Project

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1 Introduction

UC is finally joining the Big 10! That is super exciting, but also gives rise to some logistical challenges for the football team. We (Bagriy, Posmik, Mehta, and Markel) examine the novel transportation challenges for the football team, having to travel around the country to play football. We propose an optimization model that minimizes for travel cost by (i) designing an optimal travel path and (ii) choosing the mode of transportation between Plane and Bus.

2 Proposed Model

2.1 Assumptions

This task has to be simplified in order to be feasible within the scope of this project. The cost is expressed per person travelling, regardless of whether they are a player, coach, or support staff. The distance between two universities does not change depending on mode of transportation as the minor distance changes that result from airport travel are viewed as irrelevant. The cost function for Bus is not spread out over multiple people as that is supposed to account for transportation of gear.

All cost and time data are assumed to be true within the scope of this project. For future work, they may be modified within the frame of this optimization task.

For simplicity, we assume that each game will be held in one week intervals. This yields a total of n-1 playing periods, since one university would not play itself.

2.2 Notation and Variables

We define the following notation and variables:

1. Let $U = \{1, 2, 3, ..., n\}$ denote the set of n universities in the Big 10 conference, including the University of Cincinnati. Moreover, let the distance between university i and j be denoted by d_{ij}. For simplicity, we define the travel distance between university i and j as the distance d_{ij}. The distance d_{ij} is obtained by pulling the distance between two universities i and j from Google Maps. By definition, $d_{ii} = d_{jj} = 0$. Note that $i, j \in U$.

2. Let $A = \{(i, j), i, j \in U, i \neq j\}$ denote the travel arcs from i to j.

3. There is two modes of transportation, Bus (b) and Plane (p). We index the modes of transportation, where X is the set containing b and p: $X = \{b, p\}$. There is two cost functions that correspond to both modes of transportation.

4. c_{ij}: The cost of traveling from university i to j for one person. Depending on mode of transport $X = b, p$, we define $c_{ij,X=p}$ as the cost of traveling by plane. Analogously, we define $c_{ij,X=b}$ as the cost of traveling by bus.

   Generally, we define the cost of travel as

$$c_{ij} = \begin{cases} c_{ij,X=b} = (f_i) * \frac{d_{ij}}{7} & if X = 0 \\ c_{ij,X=p} = 0.3 * d_{ij} + 150 & if X = 1 \end{cases}$$

where f_i is the cost of gas per gallon at university i. We assume that the buses yield a gas efficiency of 7 miles per gallon. Furthermore, 0.3 is the rate per mile that Delta has quoted for UC's team. The $150 constant processing fee remains constant. Both cost functions are represented in their respective cost matrices.

5. The time from traveling from university i to j is defined as t_{ij}.

2.3 Decision Variables

$$Gb_{ij} = \begin{cases} 1 & ifteamtravelslink[i,j]bybus \\ 0 & ifteamdoesnottravellink[i,j]bybus \end{cases}$$

$$Gp_{ij} = \begin{cases} 1 & ifteamtravelslinki,jbyplane \\ 0 & ifteamdoesnottravellinki,jbyplane \end{cases}$$

γ_i is a dummy variables for all nodes in U.

2.4 Objective Function

$$Min \sum_{i,j \in U} Gb_{i,j} * c_{ij,X=b} + Gp_{i,j} * c_{ij,X=p}$$

where $Gb_{i,j}$ and $Gp_{i,j}$ expresses whether the team travels path i,j by bus or plane respectively. The cost functions correspond to whether the team chooses bus or plane for link i,j.

2.5 Constraints

$\forall$ i $\in$ U

$$\sum_{i,j \in U} Gb_{i,j} + Gp_{i,j} = 1 \tag{1}$$

$\forall$ i $\in$ U

$$\sum_{i,j \in U} Gb_{i,j} + Gp_{i,j} = 1 \tag{2}$$

$\forall$ (i,j) $\in U | j \neq 0$

$$\gamma_j \geq \gamma_i + 1 - M(1 - Gb_{i,j} + Gp_{i,j}) \tag{3}$$

The first constraint requires that each university is arrived at from exactly one other university, and the second constraint requires that from each university there is a departure to exactly one other university. The third constraint enforces that there is only a single tour covering all universities, and not two or more disjointed tours that only collectively cover all universities. These three constraints are known as the Miller-Tucker-Zemlin (MTZ) subtour elimination constraints. Moreover,

$$Gb_{i,j} = \{0,1\}; Gp_{i,j} = \{0,1\}; X_{i,j} = \{b,p\} \tag{4}$$

$$\gamma_i \geq 0; \gamma_0 = 1 \tag{5}$$

where constraint (4) refers to the decision elements being elements of a finite set consisting of 0 and 1, more clearly, that they can indicate either yes or no. The mode of transportation can is a binary decision between plane and bus. Constraint (5) specifies that a starting node exists and that all subsequent travel nodes exist in non-negative space.

3 Findings

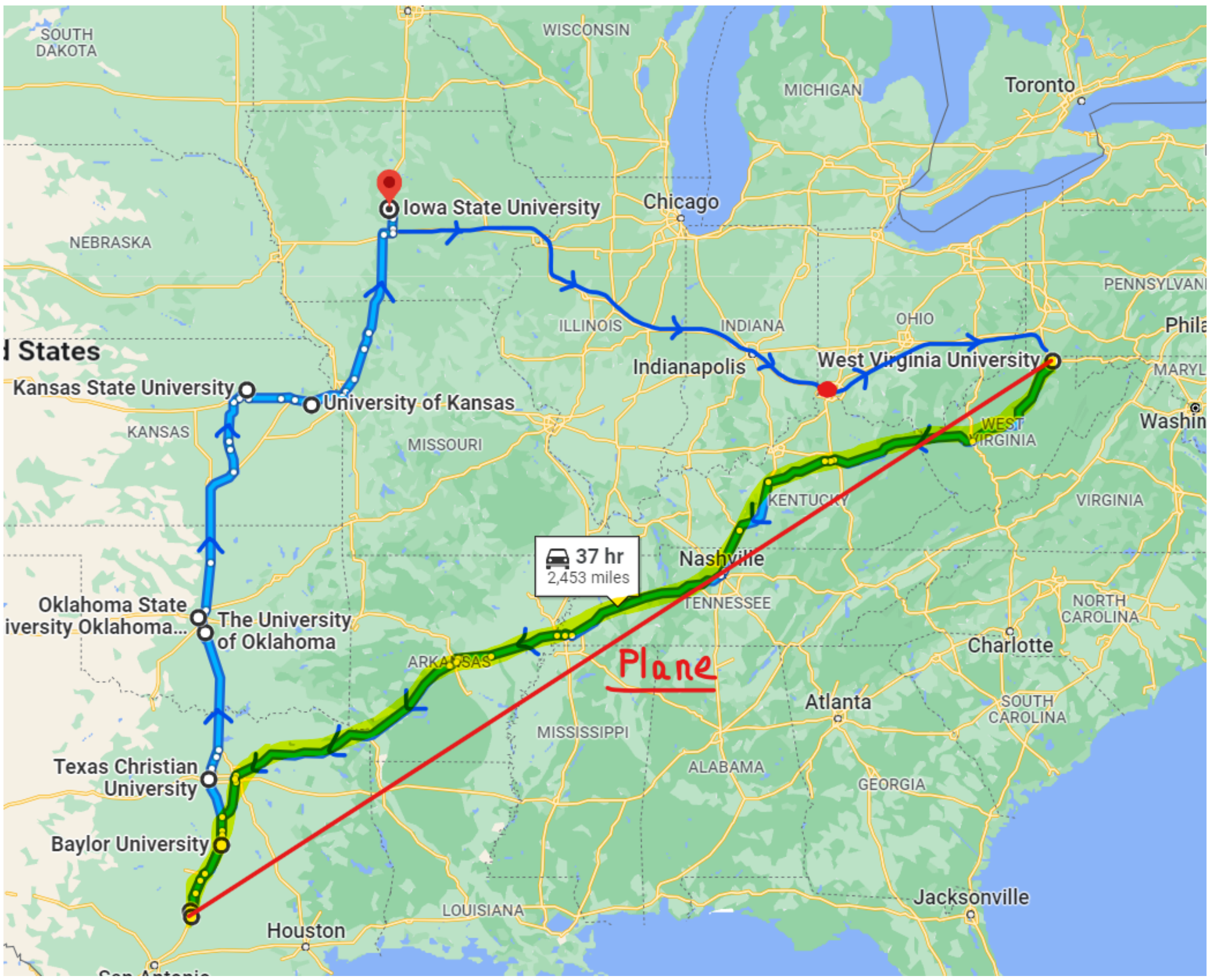
All code is attached in the corresponding ipynb file. Applying the optimization model to the data, we find that we can minimize transportation cost by adopting the following route with the respective modes of transportation:

1. Trip 1: Node 0 to Node 10 (Cost: $145; Mode: Bus)
2. Trip 2: Node 10 to Node 8 (Cost: $569; Mode: Plane)
3. Trip 3: Node 8 to Node 9 (Cost: $5; Mode: Bus)
4. Trip 4: Node 9 to Node 1 (Cost: $45; Mode: Bus)
5. Trip 5: Node 1 to Node 7 (Cost: $42; Mode: Bus)
6. Trip 6: Node 7 to Node 5 (Cost: $77; Mode: Bus)
7. Trip 7: Node 5 to Node 6 (Cost: $40; Mode: Bus)
8. Trip 8: Node 6 to Node 4 (Cost: $118; Mode: Bus)
9. Trip 9: Node 4 to Node 3 (Cost: $44; Mode: Bus)
10. Trip 10: Node 3 to Node 2 (Cost: $132; Mode: Bus)
11. Trip 11: Node 2 to Node 0 (Cost: $315; Mode: Bus)

where

- Node 0: University of Cincinnati
- Node 1: Baylor University
- Node 2: Iowa State University
- Node 3: University of Kansas
- Node 4: Kansas State University
- Node 5: University of Oklahoma
- Node 6: Oklahoma State University
- Node 7: Texas Christian University
- Node 8: University of Texas at Austin
- Node 9: Texas Tech University
- Node 10: West Virgina University

Observe how the plane is only used once. This is indicative of the high fixed costs of a plane ticket which makes the bus option cheaper 10/11 times. This yields a total minimum travel cost of $1,538 per person traveling.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