Floating Bar Model Derivation

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August 11, 2021

We treat the mantis shrimp striking appendage as a bar with center of mass position $\mathbf{r} = (x, y)$ and orientation angle θ, measured counterclockwise from the horizontal. We combine these together into the state vector $\mathbf{X}$:

$$\mathbf{X} = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} \tag{1}$$

We assume that at initial equilibrium (no muscles activated), the bar has position $\mathbf{r} = \mathbf{0}$ and angle θ_0. While θ_0 is technically redundant (changing the orientations of the actuators has the same effect), it helps us to visualize the system. Note that $\mathbf{X}$ is not *teeeeechnically* dimensionally consistent... we'll circle back to that in the readme. Throughout, we will follow the convention of organizing vectors with translational and rotational components into columns in the order (horizontal, vertical, rotational).

To calculate the magnitudes and directions of the actuator forces, we first need to know where and how the actuators are positioned. The following description is pictured in Fig 1. Attached to the bar are two muscles and two springs, with one muscle-spring pair on either side. Each actuator acts at its attachment point $\mathbf{P}_i$ (where $i \in \{ES, EM, FS, FM\}$ is a label indicating the side [extensor E/flexor F] and type [spring S/muscle M] of the actuator). The distance from the attachment point to the bar's center of mass is a fixed value $d_i = |\mathbf{r} - \mathbf{P}_i|$. Note that this means we can determine the attachment points of the actuators as a function of the state vector $\mathbf{X}$ by

$$\mathbf{P}_i = \mathbf{r} + d_i \hat{\mathbf{n}}(\theta) \tag{2}$$

where $\hat{\mathbf{n}}(\theta) = (\cos\theta, \sin\theta)^\top$ is a unit column vector making an angle θ with the horizontal. The other endpoint of each actuator is its *anchor point* $\mathbf{A}_i$. We determine this using the rest length of the actuator ℓ_i and the angle α_i^0 it makes with the horizontal at equilibrium: to find the anchor point, start at the center of mass, then "backtrack" along the bar to the attachment point, and then "backtrack" again a distance ℓ_i along the initial actuation angle α_i^0. Mathematically,

$$\mathbf{A}_i = \mathbf{r} - d_i \hat{\mathbf{n}}(\theta_0) - \ell_i \hat{\mathbf{n}}(\alpha_i^0) \tag{3}$$

Having calculated the locations of these points, we are now ready to determine the actuator forces and torques.

Since we assume the bar is loaded quasi-statically ($\dot{\mathbf{X}} = \mathbf{0}$), the actuator forces are dependent only on their lengths L_i. Specifically, they depend on the actuator extensions $L_i - \ell_i$. We determine the length and direction of the actuator using the actuation vector $\mathbf{a}_i = \mathbf{P}_i - \mathbf{A}_i$:

$$L_i = |\mathbf{a}_i| \tag{4}$$

$$\hat{\mathbf{a}}_i = \frac{\mathbf{a}_i}{L_i} \tag{5}$$

having determined the actuator extensions, we use the appropriate force-length relation to determine the actuator force vector:

$$\mathbf{F}_i = F_i(L_i - \ell_i)\hat{\mathbf{a}} \tag{6}$$

Finally, we can determine the torque of the actuator using the cross product:

$$\tau_i = |\ell_i \hat{\mathbf{n}}(\theta) \times \mathbf{F}_i| \tag{7}$$

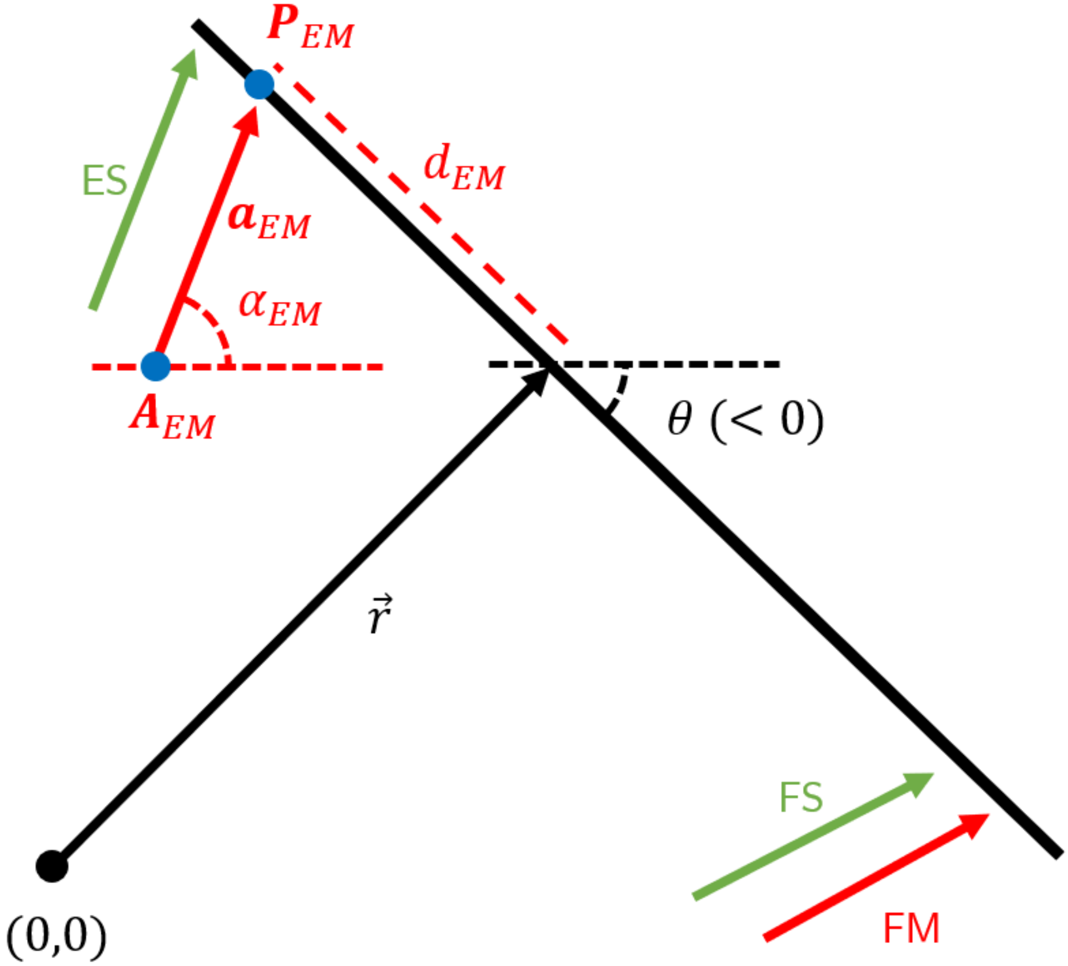

Figure 1: Model setup. The bar (diagonal black line) with state variables x, y, θ is shown, as well as the spring (green) and muscle (red) actuation forces, with points of contact. The force orientation angle α and actuation distance d are shown for the extensor muscle (EM). Note that in this figure $\theta < 0$, so the bar has a negative slope.