長庚大學期中、期末考試答案用紙

學年度 第 學期 考 系 姓名 學號 科目

[3]

1. 不超過 5%

$P(X=10) = \binom{100}{10}(0.05)^{10}(0.95)^{90}$

$= 1.6715 \times 10^{-2}$

2. A buyer would suspect the claim isn't correct because consuming a correct claim probability of having 10 detective item in sample is 1.6715×10^{-2} and event would occur only 1.6715 % of time

[4]

$$b(x;n,p) = \binom{n}{x} p^x \cdot q^{n-x} \xrightarrow[n \cdot p \to \mu]{n \to \infty,\ p \to 0} p(x;\mu) = \frac{\mu^x}{x!} e^{-\mu}$$

$\Downarrow$

$$\frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} = \frac{n(n-1)(n-2)\cdots(n-x+1)}{x!} \left(\frac{\mu}{n}\right)^x \left(1-\frac{\mu}{n}\right)^{n-x}$$

$$= \left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{x-1}{n}\right)\frac{1}{x!}\mu^x\left[\left(1-\frac{\mu}{n}\right)^{-\frac{n}{\lambda}}\right]^{-x}\left(1-\frac{\mu}{n}\right)^{-x}$$

$$n \to \infty \Rightarrow \left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{x-1}{n}\right) \to 1$$

$$\left(1-\frac{\mu}{n}\right)^{-\frac{n}{\lambda}} \to e$$

$$\left(1-\frac{\mu}{n}\right)^{-x} \to 1$$

so $b(x;n,p) \to \frac{\mu^x e^{-\mu}}{x!}$

$$\sum_{x=0}^{\infty} p(x;\mu) = e^{-\mu}\sum_{x=0}^{\infty}\frac{\mu^{\infty}}{x!} = e^{-\mu}e^{\mu} = 1$$

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[1]

1\. $f(x) = \sum_{x=0}^{10} (x; 10, \frac{1}{10})$

```
import scipy.stats as st
prob = st.binom.pmf(k=[0,1,2,3,4,5,6,7,8,9,10], n=10, p=.1)
prob
```

x=0 1 2 3 4 5 6 7 8 9 10

array [0.3487, 0.3874, 0.1937, 0.0574 0.0112 0.0015 0.0001 0 0 0 0]

2\. $E(x) = np = 10 \times \frac{1}{10} = 1$

3\. Std [X] : $\sigma^2 = npq = 10 \times \frac{1}{10} \times \frac{9}{10} = 0.9$

$\sigma = \frac{3\sqrt{10}}{10}$

4\.

$f(y) = \frac{C^{10}_{k} C^{90}_{10-k}}{C^{100}_{10}} \rightarrow$

$f_y(0) = 0.3305$

$f_y(1) = 0.408$

$f_y(2) = 0.2015$

$f_y(3) = 0.0518$

$f_y(4) = 0.0076$

$f_y(5) = 0.0006$

$f_y(6) \doteq 0$

$f_y(7) \doteq 0$

$f_y(8) \doteq 0$

$f_y(9) \doteq 0$

$f_y(10) \doteq 0$

5\. 期望值與標準差與放不放回無關

$E[Y] + std[Y] \doteq 1 + 0.8487 = 1.8487$

[2]

1\. $f(w) = P(W; 100) = \frac{e^{-100} \times (100)^W}{W!}$

2\. $E(W) = 100 \quad std[W] = \sqrt{100} = 10$

$E(W) + Std[W] = 100 + 10 = 110$

3\.

$\sum_{W=80}^{120} P(W; 100)$

4\.

$P(W > 120) = 0.9819$

5\. 接受，因為結果符合且現實中時常發生不代表每一天都發生