

Step 1: Find the derivative $i'(x)$

$$\text{Given: } i(x) = x^2 \cdot \sqrt{x^2 + 1}$$

Using the product rule: $i'(x) = 2x \cdot \sqrt{x^2 + 1} + x^2 \cdot (1/2)(x^2 + 1)^{-1/2} \cdot 2x$

$$i'(x) = 2x\sqrt{x^2 + 1} + x^3/\sqrt{x^2 + 1}$$

$$i'(x) = (2x(x^2 + 1) + x^3)/\sqrt{x^2 + 1}$$

$$i'(x) = (2x^3 + 2x + x^3)/\sqrt{x^2 + 1}$$

$$i'(x) = (3x^3 + 2x)/\sqrt{x^2 + 1}$$

Step 2: Integrate $i'(x)$ using substitution

$$\int (3x^3 + 2x)/\sqrt{x^2 + 1} dx$$

Let $u = x^2 + 1$, then $du = 2x dx$, so $x dx = du/2$

$$\text{Also, } x^2 = u - 1$$

$$\text{Substituting: } \int (3(u-1) + 2)/(2\sqrt{u}) du = \int (3u - 1)/(2\sqrt{u}) du$$

$$= (1/2) \int (3u^{1/2} - u^{-1/2}) du$$

$$= (1/2)[3 \cdot (2/3)u^{3/2} - 2u^{1/2}] + C$$

$$= (1/2)[2u^{3/2} - 2u^{1/2}] + C$$

$$= u^{3/2} - u^{1/2} + C$$

$$= (x^2 + 1)^{3/2} - (x^2 + 1)^{1/2} + C$$

Step 3: Simplify to match original form

$$= \sqrt{x^2 + 1}[(x^2 + 1) - 1] + C$$

$$= \sqrt{x^2 + 1} \cdot x^2 + C$$

$$= x^2\sqrt{x^2 + 1} + C$$

Step 4: Verification

This matches $i(x) = x^2\sqrt{x^2 + 1}$ when $C = 0$, confirming our integration is correct.