

**Step 1: Find the derivative  $i'(x)$** 

Given:  $i(x) = x^2 \cdot \sqrt{x^2 + 1}$

Using the product rule:  $i'(x) = 2x \cdot \sqrt{x^2 + 1} + x^2 \cdot (1/2)(x^2 + 1)^{-1/2} \cdot 2x$

$$i'(x) = 2x\sqrt{x^2 + 1} + x^3/\sqrt{x^2 + 1}$$

$$i'(x) = (2x(x^2 + 1) + x^3)/\sqrt{x^2 + 1}$$

$$i'(x) = (2x^3 + 2x + x^3)/\sqrt{x^2 + 1}$$

$$i'(x) = (3x^3 + 2x)/\sqrt{x^2 + 1}$$

**Step 2: Integrate  $i'(x)$  using substitution**

$$\int (3x^3 + 2x)/\sqrt{x^2 + 1} \, dx$$

Let  $u = x^2 + 1$ , then  $du = 2x \, dx$ , so  $x \, dx = du/2$

Also,  $x^2 = u - 1$

Substituting:  $\int (3(u-1) + 2)/(2\sqrt{u}) \, du = \int (3u - 1)/(2\sqrt{u}) \, du$

$$= (1/2) \int (3u^{1/2} - u^{-1/2}) \, du$$

$$= (1/2) [3 \cdot (2/3)u^{3/2} - 2u^{1/2}] + C$$

$$= (1/2) [2u^{3/2} - 2u^{1/2}] + C$$

$$= u^{3/2} - u^{1/2} + C$$

$$= (x^2 + 1)^{3/2} - (x^2 + 1)^{1/2} + C$$

**Step 3: Simplify to match original form**

$$= \sqrt{x^2 + 1}[(x^2 + 1) - 1] + C$$

$$= \sqrt{x^2 + 1} \cdot x^2 + C$$

$$= x^2\sqrt{x^2 + 1} + C$$

**Step 4: Verification**

This matches  $i(x) = x^2\sqrt{x^2 + 1}$  when  $C = 0$ , confirming our integration is correct.