

Graph as Matrix: PageRank, Random Walks & Embeddings

Graph as Matrix

- Determine node importance via **random walk** (PageRank)
- Obtain node embeddings via **matrix factorization (MF)**
- View other **node embeddings** (e.g. Node2Vec) as MF

The Web as a Graph

- Example
 - links
 - navigational
 - transactional
- Question
 - how to recursively define importance (depends on others)

the focus

- Link Analysis approaches
 - PageRank
 - Personalized PageRank (PPR)
 - Random Walk with Restarts

PageRank (Google Algo)

- Links as Votes
 - in-links as votes
- The "Flow" Model
 - link importance
 - page importance / #out links
 - Matrix Formulation
 - Stochastic adjacency matrix M
 - Rank vector r
 - flow equations
 - $M * r$
 - turn equations to matrix multi
- connection to random walks
 - random web surfer
 - Models a random web surfer using the stochastic adjacency matrix
 - $p(t+1) = M * p(t) = p(t)$
 - stationary distribution
 - $r = M * r$
 - directed graphs
 - similar to **eigenvector** centrality formula (undirected graphs)
 - $\lambda c = A c$
 - c: eigenvector; λ : eigenvalue
 - matrix*vector**
 - $1 * r = M * r$
 - long range random walk $M(M(...M(M u)))$
- summary
 - 3 eqns
 - $r = M * r$
 - principle eigenvector of M
 - stationary distribution of a random walk
 - problem
 - random walks eqn
 - flow eqn
 - eigenvalue, eigenvector (linear algebra)
 - how to efficiently solve for r
 - Power iteration