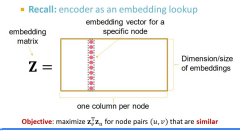
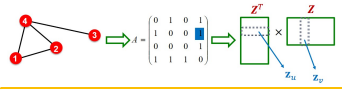
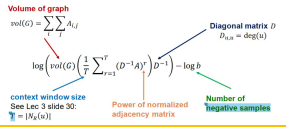
Matrix Factorization and Node Embeddings

Embeddings & Matrix Factorization



Connection to Matrix Factorization

- Simplest node similarity
  - $\mathbf{z}_v^{\mathrm{T}}\mathbf{z}_u = A_{u,v}$
  - 
  - 0 if no connection, 1 otherwise
- Matrix Factorization
  - $\min_{\mathbf{Z}} \| \mathrm{A} - \boldsymbol{Z}^T \boldsymbol{Z} \|_2$
  - objective
    - node2vec uses softmax not L2 norm
  - d << n
    - embedding dimension d (number of rows in)
  - **Inner product** decoder with node similarity defined by **edge connectivity** is equivalent to **matrix factorization** of A
  - treat A as the graph transformation, then it is a unified framework
- Random Walk-based Similarity
  - 
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limitations

Limitations of **node embeddings** via **matrix factorization** & **random walks**

1. no embedding for nodes **not in the training set**
   - needs to recompute all node embeddings
2. Cannot capture **structural similarity**
   - solution — anonymous walk
3. Cannot utilize node, edge and graph features
   - solution — GNN

Summary

- PageRank
  - Measures importance of nodes
  - efficiently computed by power iteration of adjacency matrix
- Personalized PageRank (PPR)
  - Measures importance of nodes — w.r.t a set of nodes or a particular node
  - efficiently computed by random walk
- Node embeddings based on **random walks** — expressed as **matrix factorization**
- Viewing graphs as matrices — plays a key role