A Tale of Four Grammars

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Source: *Studia Logica: An International Journal for Symbolic Logic*, Vol. 71, No. 3, The Lambek Calculus in Logic and Linguistics (Aug., 2002), pp. 315-329
Published by: Springer
Stable URL: https://www.jstor.org/stable/20016439
Accessed: 25-09-2019 14:04 UTC

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Abstract. In this paper we consider the relations existing between four deductive systems that have been called "categorial grammars" and have relevant connections with linguistic investigations: the syntactic calculus, bilinear logic, compact bilinear logic and Curry's semantic calculus.

Keywords: categorial grammar, linear logic, pregroup, adjoint, *wh*-questions

1. Introduction

The grammars we are considering have been called "categorial", but we prefer to call them "type grammars", in order to avoid confusion with the categories of Eilenberg and Mac Lane, which will also enter our story. The idea of a type grammar is to assign a type to each word of the language to be studied, say English, so that the sentencehood of a string of words can be determined by a calculation on their types, hopefully without recourse to further language specific grammatical rules. It seems that this idea cannot be carried out except for a first approximation to the language. As shown by the history of categorial grammars, there are linguistic environments that represent a serious problem for this approach.[1] For example, consider the English sentences:

Girls he admired tended to love boys he dislikes.
The horse raced around the barn fell.

No type assignment we know will enable us to show that these are indeed well-formed sentences, unless we refer to grammatical rules not in the dictionary or unless we insert additional words with appropriate types, *e.g. whom* twice in the first sentence and *which was* in the second. For the time being we shall therefore look at a first approximation to English in which elliptical constructions such as the above are not admitted.

2. Four grammars

We shall look at four formal systems in which the calculations on types are to be carried out:

(1) the syntactic calculus,

[1] For detailed discussion we refer to Morrill (1994), Moortgat (1988), (1997).

Studia Logica **71**: 315–329, 2002.

(2) classical bilinear logic,

(3) compact bilinear logic,

(4) Curry's semantic calculus.

2.1. The syntactic calculus was developed by the second author (1958) and was based on earlier systems by Ajdukiewicz (1935) and Bar-Hillel (1953). From given basic types, it allows compound types to be built up by three operations:

$$A \otimes B \text{ (henceforth abbreviated as AB)}, \ A/B \text{ and } A\backslash B,$$

subject to the following axioms and rules of inference:

$$(AB)C \leftrightarrow A(BC),$$
$$AB \to C \ \text{ iff } \ A \to C/B \ \text{ iff } \ B \to A\backslash C,$$

where $\to$ is a deduction symbol satisfying the usual reflexive and transitive rules. Later, a constant 1 was also introduced satisfying:

$$A1 \leftrightarrow A \leftrightarrow 1A.$$

Note however that the syntactic calculus with 1 is not a conservative extension of that without 1. For example, $A/(B/B) \to A$ is provable in the former but not in the latter.

In the syntactic calculus one could prove a number of theorems, in particular:

$(A/B)\,B \to A$	*Ajdukiewicz's law*,
$B \to (A/B)\backslash A$	*type raising*,
$(A/B)(B/C) \to A/C$	*composition*,
$A/B \to (A/C)/(B/C)$	*Geach's law*,
$(C/B)/A \leftrightarrow C/(AB)$	*Curry's law*,

as well as their mirror images such as:

$$B\,(B\backslash A) \to A \quad \text{etc.}$$

We have also

$$(A\backslash B)/C \leftrightarrow A\backslash(B/C),$$

hence we often write $A\backslash B/C$ for either side, and the following rules of inference indicating what categorists call *functoriality*:

$$\frac{A \to B \quad C \to D}{AC \to BD} \qquad \frac{A \to B \quad C \to D}{A/D \to B/C}$$

and the mirror dual of the latter.

2.2. Under the influence of Girard (1987), Abrusci (1991) and Lambek (1993) had developed a non-commutative version of his linear logic, here called classical bilinear logic. Casadio (1997), (2001) realized that this too had a linguistic application. It can be obtained from the syntactic calculus by adjoining a constant symbol 0 satisfying:

$$0/(\mathrm{A}\backslash 0) \leftrightarrow \mathrm{A} \leftrightarrow (0/\mathrm{A})\backslash 0$$

In what follows, it will be convenient to abbreviate

$$\mathrm{A}\backslash 0 = \mathrm{A}^r, \quad 0/\mathrm{A} = \mathrm{A}^\ell.$$

One can show that

$$(\mathrm{B}^r\mathrm{A}^r)^\ell \leftrightarrow (\mathrm{B}^\ell\mathrm{A}^\ell)^r,$$

for which it is convenient to write $\mathrm{A} \oplus \mathrm{B}$, or simply A+B. We may think of $\oplus$ as the De Morgan dual of $\otimes$: it corresponds to what Girard calls "par".[2] Here are some theorems of classical bilinear logic, which had been anticipated by Grishin (1983):

$$\begin{gathered}
1^r \leftrightarrow 0 \leftrightarrow 1^\ell \\
\mathrm{A}+0 \leftrightarrow \mathrm{A} \leftrightarrow 0+\mathrm{A} \\
(\mathrm{A}+\mathrm{B})+\mathrm{C} \leftrightarrow \mathrm{A}+(\mathrm{B}+\mathrm{C}) \\
\mathrm{A}^\ell\mathrm{A} \to 0, \quad \mathrm{A}\mathrm{A}^r \to 0 \\
1 \to \mathrm{A}+\mathrm{A}^\ell, \quad 1 \to \mathrm{A}^r+\mathrm{A} \\
\mathrm{A}/\mathrm{B} \leftrightarrow \mathrm{A}+\mathrm{B}^\ell, \quad \mathrm{B}\backslash\mathrm{A} \leftrightarrow \mathrm{B}^r+\mathrm{A} \\
(\mathrm{A}+\mathrm{B})\,\mathrm{C} \to \mathrm{A}+\mathrm{BC}, \quad \mathrm{C}\,(\mathrm{B}+\mathrm{A}) \to \mathrm{CB}+\mathrm{A}
\end{gathered}$$

The last two are the *mixed associative* laws of Grishin.[3]

2.3. When hearing Casadio's exposition of her ideas, discussing linguistic applications of bilinear logic, Lambek observed that there was a simplification if one assumed that

$$\mathrm{A}+\mathrm{B} \leftrightarrow \mathrm{AB}, \qquad 0 \leftrightarrow 1.$$

The word "compact" had been used by Kelly (1972) and Barr (1979) to describe this situation in a categorical context, though then still restricted

[2] The duality relation between $\otimes$ and $\oplus$ plays a crucial role in non-commutative linear logic (or bilinear logic), as developed in Abrusci (1991).

[3] The name *weak distributivity* is also introduced for these laws by Cockett and Seely (1997b).

to the commutative case. Finally, it was realized that *compact* bilinear logic allowed a simpler description as follows:

$$(\mathrm{AB})\mathrm{C} \leftrightarrow \mathrm{A}(\mathrm{BC}), \qquad \mathrm{A}1 \leftrightarrow \mathrm{A} \leftrightarrow 1\mathrm{A}$$
$$\mathrm{A}\,\mathrm{A}^r \rightarrow 1 \rightarrow \mathrm{A}^r\mathrm{A}, \qquad \mathrm{A}^\ell\,\mathrm{A} \rightarrow 1 \rightarrow \mathrm{A}\,\mathrm{A}^\ell.$$

Models in which $\rightarrow$ stands for a partial order were called "pregroups"; they reduce to groups if the order is discrete, that is, is the equality relation, and to partially ordered groups in the so-called *cyclic* case when $\mathrm{A}^\ell \leftrightarrow \mathrm{A}^r$. If however the arrows are allowed to stand for morphisms in a category, one would also demand that the above occurrences of $\leftrightarrow$ represent isomorphisms and that the composite arrows

$$\mathrm{A} \rightarrow \mathrm{A}\,\mathrm{A}^\ell\mathrm{A} \rightarrow \mathrm{A}, \qquad \mathrm{A} \rightarrow \mathrm{A}\,\mathrm{A}^r\mathrm{A} \rightarrow \mathrm{A}$$

are identity arrows, making A^ℓ the *left adjoint* and A^r the *right adjoint* of A in a 2-category. It follows from the work of Abrusci [(1991), (1995), (1996)] that the passage from the syntactic calculus to classical bilinear logic is a conservative extension, but it would be nice to have a direct proof of this. On the other hand, the passage to compact bilinear logic is not conservative, as in the latter

$$\mathrm{A}\,(\mathrm{B}/\mathrm{C}) \;\leftrightarrow\; \mathrm{ABC}^\ell \;\leftrightarrow\; (\mathrm{AB})/\mathrm{C},$$

but in the former

$$(\mathrm{AB})/\mathrm{C} \;\not\rightarrow\; \mathrm{A}\,(\mathrm{B}/\mathrm{C}).$$

In view of the fact that this counter-example involves the tensor product (here denoted by juxtaposition), one may ask whether compact bilinear logic is a conservative extension of a pure division calculus, that is to say a variant of the syntactic calculus without the tensor product. We are indebted to Wojciech Buszkowski for pointing out that even this is not the case, since

$$\mathrm{B}/((\mathrm{A}/\mathrm{A})/\mathrm{A})/\mathrm{A} \rightarrow \mathrm{B}$$

is provable in compact bilinear logic, but not in the pure division calculus.

3. The syntactic calculus (I)

In order to compare the three type grammars, we shall adopt the same set of basic types, actually a partially ordered set or, in the categorical context, a graph, first used in connection with compact bilinear logic:

$$\pi_1,\ \pi_2,\ \pi_3$$

stand for *first*, *second* and *third* person pronouns *I*, *you*, *he*, *she* and *it*. In English it is possible to assign π_2 also to the three plural pronouns *we*, *you* (= *you all*), and *they*. We also write π when the person is irrelevant and postulate $\pi_k \to \pi$ ($k = 1, 2, 3$). Such a postulate may be viewed as an oriented edge of the graph of the basic types. The types:

$$\mathrm{s}_1, \mathrm{s}_2, \mathrm{s}$$

stand for statements in the *present*, the *past* tense and when the tense is not relevant, respectively. We postulate: $\mathrm{s}_i \to \mathrm{s}$ ($i = 1, 2$). The types:

$$\mathrm{q}_1, \mathrm{q}_2, \mathrm{q}$$

stand for *yes*-or-*no* questions in the present, the past tense and when the tense does not matter, respectively. We postulate: $\mathrm{q}_i \to \mathrm{q}$ ($i = 1, 2$). We also assign the type $\overline{\mathrm{q}}$ to all questions[4], including *wh*-questions, and postulate $\mathrm{q} \to \overline{\mathrm{q}}$.

The type i stands for the infinitive of intransitive verbs. The types:

$$\mathrm{p}_1, \mathrm{p}_2$$

stand for present and past participles.

The type o stands for objects such as *me*, *him*, *her*, *us*, and *them*. Note that *it* can have type π_3 or o and that *you* can have type π_2 or o.

We list some sample sentences, with types of the syntactic calculus attached, bearing in mind that proofs in the syntactic calculus will be required to show that they are statements or questions. First some statements:

$$\begin{array}{ll} \textit{you} \quad \textit{go}, & \textit{I} \quad \textit{went}, \\ \pi_2 \quad (\pi_2\backslash \mathrm{s}_1) & \pi_1 \quad (\pi\backslash \mathrm{s}_2) \\[1ex] \textit{he} \quad \textit{is} \quad \textit{going}, & \textit{we} \quad \textit{have} \quad \textit{gone}, \\ \pi_3 \quad (\pi_3\backslash \mathrm{s}_1/\mathrm{p}_1) \quad \mathrm{p}_1 & \pi_2 \quad (\pi_2\backslash \mathrm{s}_1/\mathrm{p}_2) \quad \mathrm{p}_2 \\[1ex] \textit{they} \quad \textit{had} \quad \textit{been} \quad \textit{going}, & \textit{you} \quad \textit{saw} \quad \textit{him}. \\ \pi_2 \quad (\pi\backslash \mathrm{s}_2/\mathrm{p}_2) \quad (\mathrm{p}_2/\mathrm{p}_1) \quad \mathrm{p}_1 & \pi_2 \quad (\pi\backslash \mathrm{s}_2/\mathrm{o}) \quad \mathrm{o} \end{array}$$

From these statements we form questions, *e.g.*:

$$\begin{array}{ll} \textit{Do} \quad \textit{you} \quad \textit{go}\,? & \textit{Did} \quad \textit{I} \quad \textit{go}\,? \\ ((\mathrm{q}_1/i)/\pi_2) \quad \pi_2 \quad i & ((\mathrm{q}_2/i)/\pi) \quad \pi_1 \quad i \\[1ex] \textit{Is} \quad \textit{he} \quad \textit{going}? & \textit{Have} \quad \textit{we} \quad \textit{gone}? \\ ((\mathrm{q}_1/\mathrm{p}_1)/\pi_3) \quad \pi_3 \quad \mathrm{p}_1 & ((\mathrm{q}_1/\mathrm{p}_2)/\pi_2) \quad \pi_2 \quad \mathrm{p}_2 \end{array}$$

[4] On analogy with the principles of X-bar theory, $\overline{\mathrm{q}}$ may be considered as the first level expansion of q.

Have	*they*	*been*	*going*?
$((q_1/p_2)/\pi_2)$	π_2	(p_2/p_1)	p_1

Did	*you*	*see*	*him*?
$((q_2/i)/\pi)$	π_2	(i/o)	o

Whom	*did*	*you*	*see* _ ?
$(\overline{q}/(q/o))$	$((q_2/i)/\pi)$	π_2	(i/o)

Note that in these questions the auxiliary verbs, accompanied by a rising intonation, have different types than they would have in declarative sentences. The dash indicates a Chomskian trace, here inserted for comparison only. For example, to prove that the last is a well-formed question, we invoke the axiom $\pi_2 \to \pi$, hence we have

$$((q_2/i)/\pi)\ \pi_2 \to ((q_2/i)/\pi)\ \pi \to q_2/i$$

by functoriality of $\otimes$ and Ajdukiewicz's rule, that

$$(q_2/i)\ (i/o) \to q_2/o$$

by composition, that

$$q_2/o \to q/o$$

by the axiom $q_2 \to q$ and functoriality of /, and finally that

$$(\overline{q}/(q/o))\ (q/o) \to \overline{q}$$

by Ajdukiewicz's rule. Of course, in reality, the hearer would process the information from left to right and begin by calculating the type of *whom did*, which has to be $(\overline{q}/(i/o))/\pi$, but which is a little more difficult to arrive at (see the *Appendix* below).

It is interesting that the above type for *whom* still works in more complex sentences, such as the following:

Whom	*did*	*you*	*say*	*you*	*saw* _ ?
$(\overline{q}/(q/o))$	$((q_2/i)/\pi)$	π_2	(i/s)	π_2	$(\pi_2\backslash s/o)$

Since the part of the question following *whom* is easily seen to have type $q_2/o \to q/o$, the entire question has type $\overline{q}$. However, there are limits to what used to be called *wh*-transformations. For instance the following are not well-formed questions, even though we can also calculate their types to be $\overline{q}$:

* *Whom did you see him and* _ ?
* *Whom did you see the man who loved* _ ?

One of us has attempted an explanation of this phenomenon, in the easier context of compact bilinear logic.

4. Classical bilinear logic (II)

To pass from the above examples to classical bilinear logic, following Casadio (1997) and (2001), all we have to do is to replace A/B by $A+B^{\ell}$ and B\A by $B^{r}+A$. For instance, we thus obtain the following type assignments:

$$\begin{array}{ll} \textit{goes} : \pi_3^r + s_1 \,, & \textit{went} : \pi^r + s_2, \\ \textit{is} : \pi_3^r + s_1 + p_1^{\ell} \,, & \textit{is} : q_1 + p_1^{\ell} + \pi_3^{\ell} \,. \end{array}$$

Let us look again at the question:

$$\begin{array}{ccccc} \textit{Whom} & \textit{did} & \textit{you} & \textit{see} & \_\,? \\ (\overline{q} + o^{\ell\ell}q^{\ell}) & (q_2 + i^{\ell} + \pi^{\ell}) & \pi_2 & (i + o^{\ell}) & \end{array}$$

But, this time, we shall analyze it from left to right. By the axiom $q_2 \to q$ and mixed associativity, *whom did* has type:

$$\begin{array}{rcl} (\overline{q} + o^{\ell\ell}q^{\ell})\ (q + i^{\ell} + \pi^{\ell}) & \to & \overline{q} + (o^{\ell\ell}\ q^{\ell}\ )(q + i^{\ell} + \pi^{\ell}) \\ & \to & \overline{q} + o^{\ell\ell}\ (q^{\ell}\ q + i^{\ell} + \pi^{\ell}) \\ & \to & \overline{q} + o^{\ell\ell}\ (0 + i^{\ell} + \pi^{\ell}) \\ & \to & \overline{q} + o^{\ell\ell}\ (i^{\ell} + \pi^{\ell}) \end{array}$$

hence, by the axiom $\pi_2 \to \pi$, *whom did you* has type:

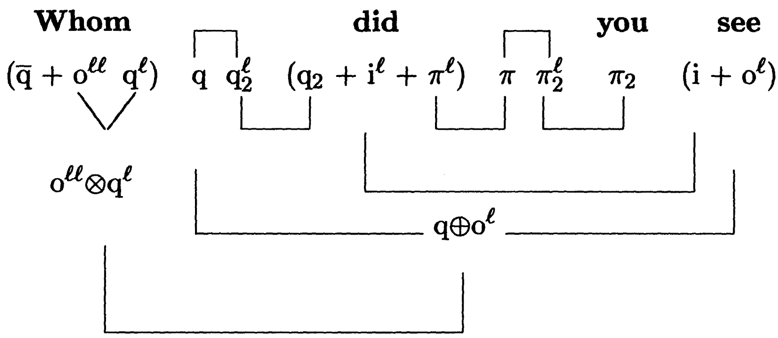
$$\begin{array}{rcl} (\overline{q} + o^{\ell\ell}\ (i^{\ell} + \pi^{\ell}))\ \pi & \to & \overline{q} + o^{\ell\ell}\ (i^{\ell} + \pi^{\ell})\ \pi \\ & \to & \overline{q} + o^{\ell\ell}\ (i^{\ell} + \pi^{\ell}\pi) \\ & \to & \overline{q} + o^{\ell\ell}\ (i^{\ell} + 0) \\ & \to & \overline{q} + o^{\ell\ell}\ i^{\ell} \end{array}$$

and the whole sentence has type:

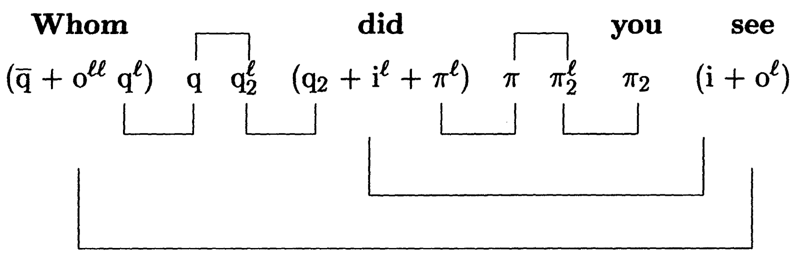
$$\begin{array}{rcl} (\overline{q} + o^{\ell\ell}\ i^{\ell})(i + o^{\ell}) & \to & \overline{q} + o^{\ell\ell}\ (i^{\ell}\ i + o^{\ell}) \\ & \to & \overline{q} + o^{\ell\ell}\ (0 + o^{\ell}) \\ & \to & \overline{q} + (o^{\ell\ell}\ o^{\ell}) \\ & \to & \overline{q} + 0 \\ & \to & \overline{q} \end{array}$$

In the above calculations we have taken advantage of the usual algebraic convention that products inside sums require no parentheses.

This derivation may also be represented by means of the proof nets of linear logic. The proof nets of non-commutative linear logic have the nice geometrical property of being planar graphs in which the order of the premises and of the conclusions is preserved.[5] We can *e.g.* introduce the following non-commutative proof net for the example above, where the axiom $q_2 \to q$ becomes $q + q_2^{\ell}$ and the axiom $\pi_2 \to \pi$ becomes $\pi + \pi_2^{\ell}$:



Through the elimination[6] of the major link connecting the types $o^{\ell\ell} \otimes q^{\ell}$ and $q \oplus o^{\ell}$, we obtain the proof net in normal form (*i.e.* with simple links only):



5. Compact bilinear logic (III)

Finally, let us look at the same example from the viewpoint of compact bilinear logic[7], where $A + B = A\,B$ and $0 = 1$:

[5] Abrusci, in (1991) and particularly in (1995), defines a correctness criterion for non-commutative proof nets and shows that the proof nets of (multiplicative) non-commutative linear logic may be converted into corresponding proof nets of Girard's (multiplicative) commutative linear logic. We assume that his criterion is satisfied here.

[6] This operation corresponds to cut-elimination in the sequent calculus for non-commutative linear logic that, as proved in Abrusci (1991), enjoys the cut-elimination property. Through repeated applications of this procedure, we obtain normalized non-commutative proof nets with simple cuts only.

[7] See Lambek (1999) and (2001).

$$
\begin{array}{cccc}
\textbf{Whom} & \textbf{did} & \textbf{you} & \textbf{see} \\
(\overline{\mathrm{q}}\ \mathrm{o}^{\ell\ell}\ \mathrm{q}^{\ell}) & (\mathrm{q}_2\ \mathrm{i}^{\ell}\ \pi^{\ell}) & \pi_2 & (\mathrm{i}\ \mathrm{o}^{\ell}) \\
\mathrm{q} & & \pi &
\end{array}
$$

The underlining linkage makes it clear which cancellations take place to reduce the composite types to $\overline{\mathrm{q}}$. It is but a degenerate form of the proof nets we have considered above and indicates that we calculate first $\mathrm{q}^{\ell}\mathrm{q}_2 \to \mathrm{q}^{\ell}\mathrm{q} \to 1$, then $\pi^{\ell}\pi_2 \to \pi^{\ell}\pi \to 1$, then $\mathrm{i}^{\ell}\mathrm{i} \to 1$ and finally $\mathrm{o}^{\ell\ell}\mathrm{o}^{\ell} \to 1$.

We note that, in this example, only the contractions $\mathrm{A}^{\ell}\mathrm{A} \to 1$ are involved, and not the expansions $1 \to \mathrm{A}\,\mathrm{A}^{\ell}$. This is explained by a metatheorem (Lambek 1999), which shows that, in compact bilinear logic, one may assume, without loss of generality, that the contractions $\mathrm{A}^{\ell}\mathrm{A} \to 1$ and $\mathrm{A}\,\mathrm{A}^{r} \to 1$ precede the expansions $1 \to \mathrm{A}\,\mathrm{A}^{\ell}$ and $1 \to \mathrm{A}^{r}\mathrm{A}$. It follows that, if one is interested in showing that a string of types reduces to a basic type, such as $\overline{\mathrm{q}}$, only contractions are needed.

We note similarly that, for the purpose of verifying sentencehood, type raising was not required for the syntactic calculus and the expansions $1 \to \mathrm{A}\,\mathrm{A}^{\ell}$, $1 \to \mathrm{A}^{r}\mathrm{A}$ play no role in classical bilinear logic, at least in the above examples. As far as we know, no corresponding metatheorem has been established there, although Buszkowski (1986) and Kandulski (1988, 1999) have come close. It would seem to follow from such a metatheorem that the grammar implicit in classical bilinear logic is context-free and one might then deduce the same result for the syntactic calculus, thus providing a new proof of the theorem of Pentus (1993, 1997).

A. K. Joshi has drawn our attention to an interesting paper by Z. Harris (1966), which anticipates the algorithm and even the diagrams implicit in the application of compact bilinear logic to natural languages. In attempting to specify "the simplest device sufficient for recognizing sentence structure", Harris also introduces what we have called "adjoints", in our notation A^{r} and A^{ℓ}, but only for basic types, so he does not consider $\mathrm{A}^{\ell\ell}$ or A^{rr}. He does not relate his calculus to any known algebraic or logical system nor does he refer to other papers on categorial grammar.

6. Curry's semantic calculus (IV)

We shall now turn to a fourth grammar, the semantic calculus of Haskell B. Curry (1961). Formally this is the same as the positive propositional calculus, which may be obtained from the syntactic calculus by introducing Gentzen's three structural rules: *interchange*, *contraction* and *weakening.* Gentzen had formulated these rules in terms of his sequent calculus, but they may also be expressed without reference to his sequents as follow[8]:

$\mathrm{a\ b} \to \mathrm{b\ a}$	*interchange*
$\mathrm{a} \to \mathrm{a\ a}$	*weakening*
$\mathrm{a\ b} \to \mathrm{a}\ , \quad \mathrm{b\ a} \to \mathrm{a}$	*contraction*

When Gentzen's structural rules are in force, one usually writes

$$\mathrm{a\ b} = \mathrm{a} \wedge \mathrm{b}, \quad \mathrm{a/b} = \mathrm{a} \Leftarrow \mathrm{b} = \mathrm{b} \Rightarrow \mathrm{a} = \mathrm{b} \backslash \mathrm{a},$$

or even, with a set-theoretical interpretation in mind,

$$\mathrm{a\ b} = \mathrm{a} \times \mathrm{b}, \quad \mathrm{a/b} = \mathrm{a}^{\mathrm{b}} = \mathrm{b} \backslash \mathrm{a}.$$

We may think of the passage from the syntactic to the semantic calculus as semantic interpretation. Once basic types have been interpreted, say π by the set $[\pi]$ of persons, o by the set [o] of entities and s_2 by the set $[\mathrm{s}_2]$ of past truth values, the interpretation of compound types follows. For example, the type $(\pi \backslash \mathrm{s}_2)/\mathrm{o}$ of *saw* is interpreted as $([\mathrm{s}_2]^{[\pi]})^{[o]}$, the set of all functions from [o] to the set of all functions from $[\pi]$ to $[\mathrm{s}_2]$. Curry's semantic calculus is closely related to Montague semantics, but to see that we have to pass from the calculus to its proof theory. Instead of just reading $\mathrm{A} \to \mathrm{B}$ to mean that B may be deduced from A, we should look at the actual deduction $\mathrm{f}\colon \mathrm{A} \to \mathrm{B}$ and consider the problem when two such deductions are equal. In this way the deductive systems discussed above are turned into categories:

(1) the syntactic calculus into a residuated (*i.e.* biclosed) monoidal category;

(2) classical bilinear logic into a non-commutative *-autonomous category;

(3) compact bilinear logic into a monoidal category in which each object (thought of as a bimodule) has both a *left* and a *right* adjoint;

(4) positive intuitionistic propositional logic (*i.e.* Curry's semantic calculus) into a cartesian closed category.

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[8] When expressed in this way, interchange can be derived from the other rules:

$$\mathrm{a\ b} \to \mathrm{a\ b\ a\ b} \to \mathrm{b\ a\ b} \to \mathrm{b\ a}.$$

This may explain why no one has studied a substructural logic which allows contraction and weakening, but not interchange. On the other hand, interchange is admitted in linear logic, BCK logic and relevance logic; the second also allows weakening, but not contraction, while the third allows contraction but not weakening.

To understand these categories we think of their objects as types and introduce variables and other terms of each type. We shall only discuss this briefly for the semantic calculus. If a and b are terms of types A and B respectively, there is a term (a, b) of type $\mathrm{A} \times \mathrm{B}$. If $\varphi(x)$ is a term of type C, x being a variable of type A, and f is a term of type C^A (think of it as a function $\mathrm{A} \to \mathrm{C}$), then $f`a$ is a term of type C satisfying

$$\lambda_{x \in A}\, \varphi(x)`a = \varphi(a), \qquad \lambda_{x \in A}\,(f`x) = f\,.$$

In this way one obtains the so-called Curry-Howard isomorphism between the proof theory of positive intuitionistic logic and the λ-calculus. In this way one also obtains Montague semantics which interprets the syntactic calculus (I) into the semantic calculus (IV).

For example, if the name *John* has type n in (I), then it denotes a person of semantic type [n] in (IV). If *John works* is a sentence of type s, then it denotes a truth-value of type [s]. Therefore, the word *walks* of type n\s is interpreted as the function $\lambda_{x \in [n]}(x \textit{ walks})$ which to any person $[a]$ of type [n] assigns the truth value of *a walks*.

If we now look at the sentence *somebody walks*, we cannot view *somebody* as a name, but we may still assign it type s/(n\s) which denotes a function from $[\mathrm{s}]^{[n]}$ to [s]. Since *somebody walks* has the logical form $\exists \mathrm{x} \in [\mathrm{n}](x \textit{ walks})$ and *somebody drinks* has the logical form $\exists \mathrm{x} \in [\mathrm{n}](x \textit{ drinks})$, *somebody* should be thought of as denoting the function $\lambda \mathrm{u} \in [\mathrm{s}]^{[s]^{[n]}} \exists \mathrm{x} \in [\mathrm{n}](\mathrm{u}`x)$ and this meaning should be entered into the dictionary. Of course, this is not the whole story of semantics; the dictionary should also tell us that the noun *uncle* has the primary meaning *parent's male sibling* and that *learn* means *get to know*.

Comparing the deductive systems (I), (II) and (III) from the viewpoint of easy computation, we find that calculations in (II) are easier than those in (I) and that calculations in (III) are easier than those in (II). On the other hand, comparing the same systems from the viewpoint of ease of interpretation, we find the reverse order. (I) has the intriguing property of leading immediately to the Curry-Montague semantics (IV) by introducing the three structural rules. Since (II) is a conservative extension of (I), it might seem likely that this semantic interpretation can be extended to (II).[9] However, (III) is not a conservative extension of (I), so it is less likely that this semantic interpretation can be made to work for (III). How can we reconcile these conflicting demands?

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[9] There is a difficulty: a cartesian closed category with a dualizing object is just a Boolean algebra, *i.e.* any two arrows with the same source and target are equal.

7. Appendix

We present a calculation of the type $(\bar{q}/(i/o))/\pi$ for *whom did*, using a cut-free Gentzen style argument (see Lambek 1958), where juxtaposition no longer stands for the tensor product, but for Gentzen's comma:

$$\dfrac{\bar{q} \to \bar{q} \qquad \dfrac{\dfrac{\dfrac{q_2 \to q \qquad i \to i}{(q_2/i)\ i \to q} \qquad o \to o}{(q_2/i)\ (i/o)\ o \to q}}{(q_2/i)\ (i/o) \to q/o}}{\dfrac{(\bar{q}/(q/o))\ (q_2/i)\ (i/o) \to \bar{q}}{\dfrac{(\bar{q}/(q/o))\ (q_2/i) \to \bar{q}/(i/o) \qquad \pi \to \pi}{\dfrac{(\bar{q}/(q/o))\ ((q_2/i)/\pi)\ \pi \to \bar{q}/(i/o)}{(\bar{q}/(q/o))\ ((q_2/i)/\pi) \to \bar{q}/(i/o)/\pi}}}}$$

This example should convince the reader that the original syntactic calculus does not realistically reflect the hearer's information processing and it motivates the transition to classical or compact bilinear logic.

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