Type Grammar Revisited

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To Claudia Casadio, who persuaded me that there is something to categorial grammar after all.

Abstract. A *protogroup* is an ordered monoid in which each element a has both a left proto-inverse a^ℓ such that $a^\ell a \leq 1$ and a right proto-inverse a^r such that $aa^r \leq 1$. We explore the assignment of elements of a free protogroup to English words as an aid for checking which strings of words are well-formed sentences, though ultimately we may have to relax the requirement of freeness. By a *pregroup* we mean a protogroup which also satisfies $1 \leq aa^\ell$ and $1 \leq a^r a$, rendering a^ℓ a left adjoint and a^r a right adjoint of a. A pregroup is precisely a poset model of classical non-commutative linear logic in which the tensor product coincides with it dual. This last condition is crucial to our treatment of passives and Wh-questions, which exploits the fact that $a^{\ell\ell} \neq a$ in general. Free pregroups may be used to recognize the same sentences as free protogroups.

1 Protogroups

Traditional dimensional analysis assigns to each quantity of classical mechanics a type, usually called *dimension*, $L^a T^b M^c$, where a, b, and c are integers and L, T and M stand for *length, time* and *mass* respectively. (We shall ignore the later attempt to incorporate electro-magnetic quantities by allowing fractional exponents.) These types form a free Abelian group and the well-foundedness (though of course not the correctness) of a physical equation can be verified by checking that the two sides of the equation have the same type.

A seductive idea, which has occured to many people, is to apply a similar technique to natural languages such as English, using a free (non-commutative) group of what might be called syntactic types. Having assigned types to words, one would expect to be able to decide whether given strings of words are sentences, by checking that the product of their types is $\mathbf{s}$, the type of a sentence. This naïve attempt breaks down as soon as one realizes that every element of a group has a unique inverse, whereas syntax would seem to require a distinction between the left inverse a^ℓ and the right inverse a^r of a. Unfortunately, if $a^\ell a = 1 = aa^r$, it follows that

$$a^\ell = a^\ell 1 = a^\ell a a^r = 1 a^r = a^r.$$

Now, if we assign type $\mathbf{n}$ to *John*, presumably we should assign type $\mathbf{n}\mathbf{n}^\ell$ to *poor* so that *poor John* has type $\mathbf{n}\mathbf{n}^\ell\mathbf{n} = \mathbf{n}1 = \mathbf{n}$. But, if $\mathbf{n}^\ell = \mathbf{n}^r$, *poor* would have type 1, meaning that it could pop up anywhere in a sentence.

A. Lecomte, F. Lamarche, and G. Perrier (Eds.): LACL '97, LNCS 1582, pp. 1–27, 1999.

To get around this difficulty, we define a *protogroup* to be an ordered monoid, that is a semigroup with a unity element 1 and a partial order rendering multiplication order preserving, such that each element a has two "proto-inverses" a^ℓ and a^r satisfying $a^\ell a \leq 1$ and $aa^r \leq 1$ respectively. If the order is discrete, a protogroup is just a group.

For mathematical purposes, at first sight without linguistic significance, it is convenient to consider also the following equations:

$$(E) \qquad 1^r = 1, \quad a^{\ell r} = a, (ab)^r = b^r a^r; \quad 1^\ell = 1, \quad a^{r\ell} = a, \quad (ab)^\ell = b^\ell a^\ell,$$

as well as the implications:

$$(I) \qquad \text{if } a \leq b \text{ then } b^\ell \leq a^\ell \text{ and } b^r \leq a^r.$$

Note, however, that in general $a^{\ell\ell} \neq a$ and $a^{rr} \neq a$. While we do not incorporate these conditions into the definition of a protogroup, they will reappear in Section 6 as giving rise to "pregroups". We mention them now for motivating why the following construction describes a protogroup.

Given a poset (= partially ordered set) A, we construct the *free protogroup* $F(A)$ as follows. For the moment we shall write

$$\cdots a^{(-2)},\ a^{(-1)},\ a^{(0)},\ a^{(1)},\ a^{(2)}, \cdots$$

for

$$\cdots a^{\ell\ell},\ a^\ell,\ a,\ a^r,\ a^{rr}, \cdots$$

Then an element of $F(A)$ has the form

$$\alpha = a_1^{(n_1)} \cdots a_k^{(n_k)},$$

where the $a_i \in A$ and the $n_i \in \mathbb{Z}$.

Multiplication is defined by concatenation and proto-inverses are defined thus:

$$\begin{aligned} \alpha^r &= a_k^{(n_k+1)} \cdots a_1^{(n_1+1)}, \\ \alpha^\ell &= a_k^{(n_k-1)} \cdots a_1^{(n_1-1)}. \end{aligned}$$

The equations (E) ensure that proto-inverses have to be defined in this way.

Finally we define $\alpha \leq \beta$ to mean that there is a sequence

$$\alpha = \alpha_1, \alpha_2, \cdots, \alpha_m = \beta$$

of elements of $F(A)$, where α_i and α_{i+1} are related as follows:

Case 1. $$\alpha_i = \gamma a^{(n)} \delta, \gamma b^{(n)} \delta = \alpha_{i+1},$$

where n is even and $a \leq b$ in A or n is odd and $b \leq a$ in A, in view of (I).

Case 2. $$\alpha_i = \gamma a^{(n)} a^{(n+1)} \delta, \alpha_{i+1} = \gamma \delta.$$

In case 1 we shall call the step from α_i to α_{i+1} an *induced step*, in case 2 we call it a *contraction.*

A *generalized contraction* has the form

$$\gamma a^{(n)} b^{(n+1)} \delta \leq \gamma \delta,$$

where n is even and $a \leq b$ in A or n is odd and $b \leq a$ in A. This may be justified by interposing either $\gamma a^{(n)} a^{(n+1)} \delta$ or $\gamma b^{(n)} b^{(n+1)} \delta$ between the two sides of the inequality.

We claim that, without loss in generality, it may be assumed that no induced step immediately precedes a generalized contraction. There are essentially two cases:

Case A. Say $\delta = \delta' c^m \delta''$
and $\gamma a^{(n)} b^{(n+1)} \delta' c^{(m)} \delta'' \leq \gamma a^{(n)} b^{(n+1)} \delta' d^{(m)} \delta'' \leq \gamma \delta' d^{(m)} \delta''$,

then we may replace the intermediate term by $\gamma \delta' c^{(m)} \delta''$. (The case $\gamma = \gamma' c^{(m)} \gamma''$ is treated similarly.)

Case B. $$\gamma a^{(n)} c^{(n+1)} \delta \leq \gamma b^{(n)} c^{(n+1)} \delta \leq \gamma \delta,$$

where either n is even and $a \leq b \leq c$ or n is odd and $c \leq b \leq a$. Then we may delete the intermediate term altogether. (The case when the intermediate term is $\gamma a^{(n)} d^{(n+1)} \delta$ is treated similarly.)

We thus have the following:

Proposition 1. *If $\alpha \leq \beta$ in $F(A)$ then there exists a string γ such that $\alpha \leq \gamma$ by generalized contractions only and $\gamma \leq \beta$ by induced steps only.*

The following sketch of a small part of English grammar is very provisional. I have several times been forced to make revisions and there is no reason to suppose that further revision will not be needed. I thought it best not to hide my false starts, but to discuss the reasons for abandoning them.

2 Typing the English verb

2.1 Intransitive verbs

Originally, the English verb, like that in other European languages, had six persons, three singular and three plural. The old second person associated with the pronoun *thou* is now obsolete; even the Quaker *thee* takes the third person singular. Taking advantage of this fact, we shall assume from now on that there are three types of personal pronouns: π_1, π_2 and π_3. Provisionally, the second person will also do for other plurals. Thus

I has type π_1,
you, we, they have type π_2,
he, she, it, one have type π_3.

Sometimes the person is irrelevant, so we shall introduce the type π and postulate:

$$\pi_1 \leq \pi, \ \pi_2 \leq \pi, \ \pi_3 \leq \pi.$$

We shall also assume that there are two tenses, present and past, and we shall ignore the subjunctive. Accordingly we distinguish two types of declarative sentences (or statements): $\mathbf{s}_1$ in the present tense and $\mathbf{s}_2$ in the past tense. Sometimes the tense of a statement is irrelevant, then we just assign it type $\mathbf{s}$ and postulate:

$$\mathbf{s}_1 \leq \mathbf{s}, \quad \mathbf{s}_2 \leq \mathbf{s}.$$

Given an intransitive verb such as *go*, we consider three non-finite forms:

the infinitive *go* of type $\mathbf{i}$,
the present participle *going* of type $\mathbf{p}_1$,
the past participle *gone* of type $\mathbf{p}_2$.

There are also, in principle, six finite forms, of which only three are distinct, making up the 3×2 conjugation matrix

$$\begin{bmatrix} go & go & goes \\ went & went & went \end{bmatrix}.$$

(The analogous matrix in French has 6×7 entries.) The corresponding types are given by $\pi_k^r \mathbf{s}_i$, when $k = 1, 2$ or 3 and $i = 1$ or 2. For example, *he goes* has type

$$\pi_3(\pi_3^r \mathbf{s}_1) = (\pi_3 \pi_3^r)\mathbf{s}_1 \leq 1\mathbf{s}_1 1 = \mathbf{s}_1,$$

making it a statement in the present tense.

To emphasize a statement, one makes use of the auxiliary verb *do*, as in *he does go*, which leads us to assign the type $\pi_3^r \mathbf{s}_1 \mathbf{i}^\ell$ to *does* so that

$$\pi_3(\pi_3^r \mathbf{s}_1 \mathbf{i}^\ell)\mathbf{i} = (\pi_3 \pi_3^r)\mathbf{s}_1(\mathbf{i}^\ell \mathbf{i}) \leq 1\mathbf{s}_1 1 = \mathbf{s}_1.$$

The conjugation matrix for *do* is

$$\begin{bmatrix} do & do & does \\ did & did & did \end{bmatrix}$$

with types $\pi_k^r \mathbf{s}_i \mathbf{i}^\ell$, the subscript k ranging from 1 to 3, the subscript i from 1 to 2.

2.2 Inflectors

Suppose we want to type the adverb *quietly* as in *goes quietly*, *going quietly, gone quietly,* etc. Unless we want to assign different types to each of these occurrences of the adverb, we are led to say that it modifies the infinitive *go*, as in *I want to go quietly*, hence has type $\mathbf{i}^r \mathbf{i}$, and that somehow the forms *goes, going, gone* etc are obtained from the infinitive. Elsewhere I had suggested that these forms should be analyzed as

$$\mathrm{C}_{13}\ go,\ \mathrm{Part}\ go,\ \mathrm{Perf}\ go,$$

where C_{13}, Part and Perf are what I called "inflectors", standing for the present tense third person, the present participle and the past (= perfect) participle respectively.

We can incorporate this idea into the present context by typing

C_{ik} with $\pi_k^r \mathbf{s}_i \mathbf{i}^\ell$,
Part with $\mathbf{p}_1 \mathbf{i}^\ell$,
Perf with $\mathbf{p}_2 \mathbf{i}^\ell$.

This is consistent with the old types for *goes, going* and *gone*, since for instance

$$(\pi_3^r \mathbf{s}_1 \mathbf{i}^\ell)\mathbf{i} \leq \pi_3^r \mathbf{s}_1.$$

The old types are still valid, but the new types make explicit the relation between the verb form and the infinitive and they imply the old types.

2.3 Transitive verbs

A transitive verb like *see* requires an object, as in *I saw her.* We shall use $\mathbf{o}$ for the type of objects and assign type $\mathbf{o}$ to the accusative form of the pronouns: *me*; *you, us, them*; *him, her, it, one.* We are thus led to assign type $\pi_1^r \mathbf{s}_2 \mathbf{o}^\ell$ to *saw* in *I saw her.* The conjugation matrix of a transitive verb then has types $\pi_k^r \mathbf{s}_i \mathbf{o}^\ell$, the person of the object being irrelevant. There are also non-finite forms of types $\mathbf{i}\mathbf{o}^\ell$, $\mathbf{p}_1 \mathbf{o}^\ell$ and $\mathbf{p}_2 \mathbf{o}^\ell$. If we analyze *saw* as C_{21}*see*, we assign to it the type

$$(\pi_1^r \mathbf{s}_2 \mathbf{i}^\ell)(\mathbf{i}\mathbf{o}^\ell) \leq \pi_1^r \mathbf{s}_2 \mathbf{o}^\ell$$

as required.

2.4 Auxiliary verbs

Let us take a look at the modal verbs *may, can, will, shall* and *must.* It is convenient to say that the first four of them have past tenses *might, could, would* and *should.* In view of such sentences as *he may go*, *I might go*, we are tempted to assign to *may* the type $\pi_k^r \mathbf{s}_1 \mathbf{i}^\ell$ and to *might* the type $\pi_k^r \mathbf{s}_2 \mathbf{i}^\ell$, where k ranges from 1 to 3. But this assignment will have to be revised presently.

The word *have*, in addition to being a transitive verb, can also act as a perfect auxiliary, as in *I have gone*, where it must have type $\pi_1^r \mathbf{s}_1 \mathbf{p}_2^\ell$. Altogether, the conjugation matrix

$$\begin{bmatrix} have & have & has \\ had & had & had \end{bmatrix}$$

will have types $\pi_k^r \mathbf{s}_i \mathbf{p}_2^\ell$. Whereas the transitive verb *have* also has three non-finite forms (*have, having, had*), the perfect auxiliary only has the infinitive *have*. At first sight we are tempted to assign to it the type $\mathbf{i}\mathbf{p}_2^\ell$, but this would permit the non-sentence

$$^*\textit{he does have gone.}$$

Therefore we shall assign to the infinitive *have* the type $\mathbf{j}\mathbf{p}_2^\ell$ instead, which will rule this out. However, we do wish to admit

$$\textit{I may go,}\quad \textit{she may have gone,}$$

so we postulate $\mathbf{i} \leq \mathbf{j}$ and retype *may* with $\pi^r \mathbf{s}_1 \mathbf{j}^\ell$ and *might* with $\pi^r \mathbf{s}_2 \mathbf{j}^\ell$. Since $\pi_k \leq \pi$ and $\mathbf{i} \leq \mathbf{j}$, we thus obtain e.g.

$$\pi_1(\pi^r \mathbf{s}_1 \mathbf{j}^\ell)\mathbf{i} \leq (\pi\pi^r)\mathbf{s}_1(\mathbf{j}^\ell \mathbf{j}) \leq \mathbf{s}_1.$$

We may say that $\mathbf{i}$ is the type of infinitives of intransitive main verbs, while $\mathbf{j}$ is the type of all infinitival intransitive verb phrases. To account for the conjugation of the auxiliary verbs *have* and *be*, we should retype C_{ik} with $\pi_k^r \mathbf{s}_i \mathbf{j}^\ell$, Part with $\mathbf{p}_1 \mathbf{j}^\ell$ and Perf with $\mathbf{p}_2 \mathbf{j}^\ell$.

Forms of the verb *be* can be used as progressive auxiliaries or as passive auxiliaries, as in

$$\textit{I am going,}\quad \textit{he was seen} - ,$$

where the dash represents a Chomskian trace. (The notion of trace is not really necessary for our analysis; we only refer to it to facilitate comparison with generative-transformational grammars.) Here *am* should have type $\pi_1^r \mathbf{s}_1 \mathbf{p}_1^\ell$ and *was* should have type $\pi_3^r \mathbf{s}_2 \mathbf{o}^{\ell\ell} \mathbf{p}_2^\ell$, as is verified by the calculations:

$$\pi_1(\pi_1^r \mathbf{s}_1 \mathbf{p}_1^\ell)\mathbf{p}_1 \leq (\pi_1\pi_1^r)\mathbf{s}_1(\mathbf{p}_1^\ell \mathbf{p}_1) \leq \mathbf{s}_1,$$

$$\pi_3(\pi_3^r \mathbf{s}_2 \mathbf{o}^{\ell\ell} \mathbf{p}_2^\ell)(\mathbf{p}_2 \mathbf{o}^\ell) \leq (\pi_3\pi_3^r)\mathbf{s}_2(\mathbf{o}^{\ell\ell}(\mathbf{p}_2^\ell \mathbf{p}_2)\mathbf{o}^\ell) \leq \mathbf{s}_2.$$

Note that the double ℓ in $\mathbf{o}^{\ell\ell}$ signals the presence of a Chomskian trace.

Altogether, the conjugation matrix

$$\begin{bmatrix} \textit{am} & \textit{are} & \textit{is} \\ \textit{was} & \textit{were} & \textit{was} \end{bmatrix}$$

is assigned the types $\pi_k^r \mathbf{s}_i \mathbf{p}_1^\ell$ for the progressive auxiliary and $\pi_k^r \mathbf{s}_i \mathbf{o}^{\ell\ell} \mathbf{p}_2^\ell$ for the passive auxiliary.

The progressive auxiliary also has an infinitive *be* of type $\mathbf{j}\mathbf{p}_1^\ell$, as witnessed by *I may be going* of type

$$\pi_1(\pi \mathbf{s}_1 \mathbf{j}^\ell)(\mathbf{j}\mathbf{p}_1^\ell)\mathbf{p}_1 \leq \mathbf{s}_1,$$

and a past participle *been* of type $\mathbf{p}_2 \mathbf{p}_1^\ell$, as witnessed by *I have been going* of type

$$\pi_1(\pi_1^r \mathbf{s}_1 \mathbf{p}_2^\ell)(\mathbf{p}_2 \mathbf{p}_1^\ell)\mathbf{p}_1 \leq \mathbf{s}_1.$$

It does not have a present participle, in view of the non-sentence

$$^{*}I\ am\ being\ going.$$

The passive auxiliary has an infinitive *be* of type $\mathbf{j}\mathbf{o}^{\ell\ell}\mathbf{p}_2^{\ell}$, as witnessed by *I may be seen–* of type

$$\pi_1(\pi^r\mathbf{s}_1\mathbf{j}^{\ell})(\mathbf{j}\mathbf{o}^{\ell\ell}\mathbf{p}_2^{\ell})(\mathbf{p}_2\mathbf{o}^{\ell}) \leq \mathbf{s}_1.$$

(We ignore here the optional agent as in *seen by her.*) It also has a present participle *being* of type $\mathbf{p}_1\mathbf{o}^{\ell\ell}\mathbf{p}_2^{\ell}$, as witnessed by *I am being seen –* of type

$$\pi_1(\pi_1^r\mathbf{s}_1\mathbf{p}_1^{\ell})(\mathbf{p}_1\mathbf{o}^{\ell\ell}\mathbf{p}_2^{\ell})(\mathbf{p}_2\mathbf{o}^{\ell}) \leq \mathbf{s}_1,$$

and a past participle *been* of type $\mathbf{p}_2\mathbf{o}^{\ell\ell}\mathbf{p}_2^{\ell}$, as witnessed by *I have been seen* - of type

$$\pi_1(\pi_1^r\mathbf{s}_1\mathbf{p}_2^{\ell})(\mathbf{p}_2\mathbf{o}^{\ell\ell}\mathbf{p}_2^{\ell})(\mathbf{p}_2\mathbf{o}^{\ell}) \leq \mathbf{s}_1.$$

These type assignments also justify

I have been going, I may have been going, I have been seen –,

and even

I have been being seen –,

although many people might doubt the sentencehood of the latter. Our type assignments do not justify

**I do be going*, **I do be seen –*, **I have had gone.*

However, the passive can also be expressed with the help of *get* instead of *be*, and *I do get seen –* sounds alright and suggests the type $\mathbf{i}\mathbf{o}^{\ell\ell}\mathbf{p}_2^{\ell}$ for *get*.

2.5 Bitransitive verbs

A word should be said about doubly transitive verbs such as *give, teach* and *tell*, as in

I gave him books, she taught me English, she told me she is going.

Here *books* and *English* are direct objects of type $\mathbf{o}$, but *him* and *me* are indirect objects of type $\mathbf{o}'$, say. We are led to assign to the infinitives *give* and *teach* the type $\mathbf{i}\mathbf{o}^{\ell}\mathbf{o}'^{\ell}$ and to *tell* also the type $\mathbf{i}\mathbf{s}^{\ell}\mathbf{o}'^{\ell}$. To recognize that any expression of type $\mathbf{o}$ may serve as an indirect object we postulate $\mathbf{o} \leq \mathbf{o}'$.

Formerly the indirect object was in the dative case and the direct object in the accusative, but in modern English these two cases have coalesced. It is a peculiarity of English that the indirect object can become the subject in passive sentences:

he was given – books, I was taught – English, I was told – she is going.

To account for these passives, we must assign new types to the passive auxiliary *be*, as summarized by the following metarule (apologies to Gazdar):

Corresponding to any verb of type $\mathbf{i}x^\ell\mathbf{o}'^\ell$, the passive auxiliary *be* may have type $\mathbf{j}x^\ell\mathbf{o}'^{\ell\ell}x^{\ell\ell}\mathbf{p}_2^\ell$.

In our examples $x = \mathbf{o}$ or $\mathbf{s}$; but, taking $x = 1$, we recapture the earlier type of *be* in Section 2.4.

Semantically, the bitransitive verbs may all be viewed as indicating causation. Thus, the above sample sentences are roughly equivalent to:

I let him have books, she made me learn English, she let me know she is going.

They may all be rephrased with the help of the preposition *to*:

I gave books to him, she taught English to me, she told that she is going to me.

(For the complementizer *that* see Section 4.2 below.) These alternative statements may also be passivized as follows:

books were given – to him,
English was taught – to me,
that she is going was told – to me.

The above metarule may be amended to account for these alternative passives, but we shall refrain from doing so here.

3 Questions

3.1 Negation

Interrogative sentences, as I shall use the term here, are tied to the notion of negation: to ask whether a statement holds is to choose between it and its negation. The easiest way to introduce grammatical negation is to assign to the word *not* the type xx^ℓ, where $x = \mathbf{i}, \mathbf{j}, \mathbf{p}_1$ or $\mathbf{p}_2$. This will justify the following sentences:

he does not go, with *does* of type $\pi_3^r\mathbf{s}_1\mathbf{i}^\ell$,
he is not going, with *is* of type $\pi_3^r\mathbf{s}_1\mathbf{p}_1^\ell$,
I was not seen –, with *was* of type $\pi_1^r\mathbf{s}_2\mathbf{o}^{\ell\ell}\mathbf{p}_2^\ell$,
he has not gone, with *has* of type $\pi_3^r\mathbf{s}_1\mathbf{p}_2^\ell$,
he will not be going, with *will* of type $\pi^r\mathbf{s}_1\mathbf{j}^\ell$.

It will explain why the optional contracted forms

doesn't, isn't, wasn't, hasn't, won't

also have the types of

does, is, was, has, will

respectively. It does not explain why the spelling *cannot* is obligatory.

3.2 Interrogatives

We shall distinguish between interrogative sentences of types $\mathbf{q}_1$ in the present tense and $\mathbf{q}_2$ in the past tense. If the tense is irrelevant, we make use of type $\mathbf{q}$ and postulate $\mathbf{q}_1 \leq \mathbf{q}$ and $\mathbf{q}_2 \leq \mathbf{q}$. The quickest way to summarize the so-called interrogative transformation in English in the present context is with the help of the following metarule, which assigns new types to *does, is, was*, etc:

If the finite form V of an auxiliary verb has type $\pi_k^r \mathbf{s}_i x^\ell$, where $x = \mathbf{i}, \mathbf{j}, \mathbf{p}_1$ or $\mathbf{p}_2$, then V (with rising intonation) may be assigned the type $\mathbf{q}_i x^\ell \pi_k^\ell$.

For example, this metarule will account for the following questions:

does he go?	with *does* of type $\mathbf{q}_1 \mathbf{i}^\ell \pi_3^\ell$,
is he going?	with *is* of type $\mathbf{q}_1 \mathbf{p}_1^\ell \pi_3^\ell$,
was I seen –?	with *was* of type $\mathbf{q}_2 \mathbf{o}^{\ell\ell} \mathbf{p}_2^\ell \pi_1^\ell$,
has he gone?	with *has* of type $\mathbf{q}_1 \mathbf{p}_2^\ell \pi_3^\ell$,
will he be going?	with *will* of type $\mathbf{q}_1 \mathbf{j}^\ell \pi^\ell$.

It will also account for:

does he not go?
is he not going?

etc. However, to account for

doesn't he go?
isn't he going?

etc, we must allow the contracted forms *doesn't, isn't*, etc as values for V in the metarule. Indeed, in Section 6 below, we shall assign to them exactly the same types as to *does, is*, etc.

The types $\mathbf{q}_i$ may be useful in handling other inversions, though without rising intonation, for example:

Seldom does he sleep, never did he yawn.

Here we may assign type $\mathbf{s}_i \mathbf{q}^\ell$ to *seldom* and *never*, $i = 1$ or 2.

3.3 Wh-questions

We shall now look at questions such as *whom does he see –*?

I realize that many people, even some linguists, say *who* instead of *whom*, but I shall follow Inspector Morse in distinguishing between the two. (To account for the distinction between *whom* or *who* and *what* we would have to subdivide type π_3 into animate versus inanimate, something I hesitate to do in this preliminary sketch of English grammar.)

We shall take the view that a Wh-question is a request to complete a statement, present or past, and is itself without tense. We assign type $\mathbf{q}'$ to all questions, interrogative as well as Wh-questions, hence we must postulate $\mathbf{q} \leq \mathbf{q}'$.

Noting that *does he see* has type

$$(\mathbf{q}_1\mathbf{i}^\ell\pi_3^\ell)\pi_3(\mathbf{io}^\ell) \le \mathbf{q}_1\mathbf{o}^\ell,$$

we are led to assign type $\mathbf{q}'\mathbf{o}^{\ell\ell}\mathbf{q}_1^\ell$ to *whom*, so that *whom does he see* – ? has type

$$(\mathbf{q}'\mathbf{o}^{\ell\ell}\mathbf{q}_1^\ell)(\mathbf{q}_1\mathbf{o}^\ell) = \mathbf{q}'(\mathbf{o}^{\ell\ell}(\mathbf{q}_1^\ell\mathbf{q}_1)\mathbf{o}^\ell) \le \mathbf{q}'.$$

But *whom* also occurs in *whom did he see* –?, which suggests also type $\mathbf{q}'\mathbf{o}^{\ell\ell}\mathbf{q}_2^\ell$ for *whom*. These two type assignments may be combined into $\mathbf{q}'\mathbf{o}^{\ell\ell}\mathbf{q}^\ell$, if we recall that $\mathbf{q}_k \le \mathbf{q}$.

Surprisingly, the same type assignment will work for *whom did she say she saw* -? Here *she saw* has type $\mathbf{s}_2\mathbf{o}^\ell \le \mathbf{so}^\ell$ and *did she say* has type $\mathbf{q}_2\mathbf{s}^\ell \le \mathbf{qs}^\ell$, so we may calculate

$$(\mathbf{q}'\mathbf{o}^{\ell\ell}\mathbf{q}^\ell)(\mathbf{q}_2\mathbf{s}^\ell)(\mathbf{s}_2\mathbf{o}^\ell) \le \mathbf{q}'(\mathbf{o}^{\ell\ell}(\mathbf{q}^\ell\mathbf{q}_2)(\mathbf{s}^\ell\mathbf{s}_2)\mathbf{o}^\ell) \le \mathbf{q}'.$$

The situation is a little more complicated for the nominative form *who*. To account for *who is/was going*? we are led to assign to *who* the type $\mathbf{q}'\mathbf{s}^\ell\pi_3$. However, in *who did she say - is going*?, *did she say* has type $\mathbf{q}_2\mathbf{s}^\ell$, *is going* has type $\pi_3^r\mathbf{s}_1$, hence we require *who* to have type $\mathbf{q}'\mathbf{s}^\ell\pi_3\mathbf{s}^{\ell\ell}\mathbf{q}^\ell$ so that we can calculate

$$(\mathbf{q}'\mathbf{s}^\ell\pi_3\mathbf{s}^{\ell\ell}\mathbf{q}^\ell)(\mathbf{q}_2\mathbf{s}^\ell)(\pi_3^r\mathbf{s}) \le \mathbf{q}'.$$

Fortunately, this second type assignment is now stable, it will also account for

who did she say he believes – is going?

as it is easily calculated that

$$(\mathbf{q}'\mathbf{s}^\ell\pi_3\mathbf{s}^{\ell\ell}\mathbf{q}^\ell)(\mathbf{q}_2\mathbf{s}^\ell)(\mathbf{s}_1\mathbf{s}^\ell)(\pi_3^r\mathbf{s}) \le \mathbf{q}'.$$

We can thus handle unbounded dependencies. However, it is known that some constraints are necessary, as one does not wish to admit

**whom did she see John and* –?

and similar non-sentences. I still hope to find a reason why these are ruled out.

Unfortunately, the above non-sentence will receive type $\mathbf{q}'$ if *and* is here given type $\mathbf{o}^r\mathbf{oo}^\ell$. Perhaps it is relevant to observe that this non- sentence will have type $\mathbf{q}'\mathbf{o}^{\ell\ell}\mathbf{o}^\ell\mathbf{oo}^r\mathbf{oo}^\ell$ at the penultimate stage of its type calculation and that one must defer the contractions $\mathbf{o}^{\ell\ell}\mathbf{o}^\ell \le 1$ and $\mathbf{o}^\ell\mathbf{o} \le 1$ until after the contraction $\mathbf{oo}^r \le 1$ has been carried out.

Note that we cannot say **who am going*? and **who are going?*, so *who* does not have type $\mathbf{q}'\mathbf{s}^\ell\pi_1$ or $\mathbf{q}'\mathbf{s}^\ell\pi_2$. Concerning the confusion between *who* and *whom*, even Inspector Morse would not approve of the butler announcing:

**whom shall I say – is calling*?

3.4 Discontinuous dependencies

Unfortunately, we are not quite finished with the type assignment to *whom*. It remains to consider such questions as:

whom did you see – yesterday?
whom did you see – with Jane?
whom did you see – when you left?

We shall assume that the adverb *yesterday*, the propositional phrase *with Jane* and the subordinate clause *when you left* all have type $\mathbf{i}^r\mathbf{i}$ (as did *quietly* in Section 2.2). Then we must assign to *whom* the new type $\mathbf{q}'\mathbf{i}^\ell\mathbf{io}^{\ell\ell}\mathbf{q}^\ell$, so that *whom did you see* - before an adverbial phase has type

$$(\mathbf{q}'\mathbf{i}^\ell\mathbf{io}^{\ell\ell}\mathbf{q}^\ell)(\mathbf{q}_2\mathbf{i}^\ell\pi^\ell)\pi_2(\mathbf{io}^\ell)$$

$$\leq \mathbf{q}'\mathbf{i}^\ell\mathbf{io}^{\ell\ell}\mathbf{i}^\ell\mathbf{io}^\ell \leq \mathbf{q}'\mathbf{i}^\ell\mathbf{io}^{\ell\ell}\mathbf{o}^\ell \leq \mathbf{q}'\mathbf{i}^\ell\mathbf{i}.$$

In the absence of an adverbial phrase, we are justified in contracting $\mathbf{i}^\ell\mathbf{i} \leq 1$; in other words, the new type implies the old. But, if an adverbial phrase follows, this contraction should be delayed, to permit the following argument:

$$(\mathbf{q}'\mathbf{i}^\ell\mathbf{i})(\mathbf{i}^r\mathbf{i}) \leq \mathbf{q}'\mathbf{i}^\ell\mathbf{i} \leq \mathbf{q}'.$$

Yet another type assignment is required to handle

whom did you teach – English?

Here *whom* should have type $\mathbf{q}'x^\ell\mathbf{o}'^{\ell\ell}x^{\ell\ell}\mathbf{q}^\ell$, where $x = \mathbf{o}$. A similar type assignment will justify

whom did you ask – whether he is leaving?

and

whom did you ask – to leave?

where we should take $x = \overline{\mathbf{q}}$ and $x = \bar{\mathbf{i}}$ respectively, anticipating sections 4.2 and 4.4. However, there seems to be no reason for invoking an indirect object here, so *whom* should have type $\mathbf{q}'x^\ell\mathbf{o}^{\ell\ell}x^{\ell\ell}\mathbf{q}^\ell$. We recapture the original type of *whom* by taking $x = 1$.

A further complication arises when we allow both generalizations simultaneously, as in

whom did you teach – English yesterday?

for example. We shall ignore this here. Evidently, the longer the type of *whom* becomes, the less confidence we have in the present approach. Perhaps we should try to formulate an appropriate metarule instead.

4 Typing the noun phrase

4.1 Mass nouns versus count nouns.

We shall use $\mathbf{n}$ as the type of names and other singular noun phrases. We postulate

$$\mathbf{n} \leq \pi_3, \quad \mathbf{n} \leq \mathbf{o}$$

to ensure that names can occupy the subject or object position of a sentence. We will use $\hat{\mathbf{n}}$ for plural noun phrases, such as *many people*, and postulate $\hat{\mathbf{n}} \leq \pi_2$, $\hat{\mathbf{n}} \leq \mathbf{o}$.

Most noun phrases are made up from nouns preceded by determiners, such as articles, quantifiers or numerals. We distinguish three types of nouns:

mass nouns of type $\mathbf{m}$, e.g. *water, pork*;
count nouns of type $\mathbf{c}$, e.g. *apple, pig*;
plurals of type $\mathbf{p}$, e.g. *police, pigs.*

Typical determiners are *much* of type $\mathbf{n}\mathbf{m}^\ell$, *one* of type $\mathbf{n}\mathbf{c}^\ell$ and *many* of type $\mathbf{n}\mathbf{p}^\ell$, as in:

much pork, one pig, many pigs.

Many determiners have several types, in fact the article *the* has all three of the above. Mass nouns and plurals can have zero determiners, which we will express with the help of two postulates:

$$\mathbf{m} \leq \mathbf{n}, \quad \mathbf{p} \leq \hat{\mathbf{n}}.$$

But we don't have $\mathbf{c} \leq \mathbf{n}$, to the consternation of many Slavs!

The types $\mathbf{c}$ and $\mathbf{m}$ are not rigidly attached to the associated nouns. Some type shifting may occur accompanied by a corresponding shift in meaning. For example, while *beer* usually has type $\mathbf{m}$ and *man* almost always has type $\mathbf{c}$, one may order *two beers* in a tavern and a cannibal may *prefer man to pork.*

Sometimes names can also be used as count nouns, as in

six Nguyens were registered in my course.

This is not the case for other noun phrases and suggests that $\mathbf{n}$ should really be split into two types.

Most plurals arise from count nouns by adding a morpheme $+s$, e.g. *pigs* may be analyzed as *pig* $+s$, which seems to suggest the morpheme $+s$ has type $\mathbf{c}^r\mathbf{p}$. On the other hand, the best we can do with *men* is to analyze it as Plur *man*, the plural of *man*, which suggests the inflector Plur has type $\mathbf{p}\mathbf{c}^\ell$. In a production grammar we would postulate

$$\text{Plur } \textit{pig} \geq \textit{pig} + s \geq \textit{pigs},$$

but it seems difficult to incorporate such rules into a type grammar.

The contrast between mass nouns and count nouns reflects an early dispute in Greek philosophy, concerning substance versus discrete entitites, and in Greek mathematics, concerning measuring versus counting. The dispute in mathematics was resolved by Eudoxus, who essentially invented what we now call Dedekind cuts; the dispute in philosophy was resolved by Aristotle, who suggested that to measure water is to count *cups of water*. Linguistically, we could type the expression Plur *cup of water* as:

$$(\mathbf{p}\mathbf{c}^{\ell})\mathbf{c}(\mathbf{c}^{r}\mathbf{c}\mathbf{m}^{\ell})\mathbf{m} \leq \mathbf{p}.$$

4.2 Propositional noun phrases

Behaving much like noun phrases are indirect sentences, namely indirect statements of type $\overline{\mathbf{s}}$ and indirect questions of type $\overline{\mathbf{q}}$. (I suppose this use of the bar bears some correlation to that in the literature.) For example:

that he goes has type $\overline{\mathbf{s}}$,
whether he went has type $\overline{\mathbf{q}}$.

We are led to assign types $\overline{\mathbf{s}}\mathbf{s}^{\ell}$ and $\overline{\mathbf{q}}\mathbf{s}^{\ell}$ to the so-called complementizers *that* and *whether* respectively.

Indirect sentences can appear both in subject and in object position. To take care of the former, we postulate $\overline{\mathbf{s}} \leq \mathbf{n}$ and $\overline{\mathbf{q}} \leq \mathbf{n}$, so that $\overline{\mathbf{s}} \leq \pi_3$ and $\overline{\mathbf{q}} \leq \pi_3$. To take care of the latter, we assign types $\mathbf{i}\overline{\mathbf{s}}^{\ell}$ and $\mathbf{i}\overline{\mathbf{q}}^{\ell}$ to certain verbs such as *say* and *know*.
Thus we can account for

whether he went does not concern me

of type $\overline{\mathbf{q}}(\pi_3^{r}\mathbf{s}_1) \leq \mathbf{s}_1$ and

I did not say that he is going

of type

$$(\mathbf{s}_2\mathbf{i}^{\ell})(\mathbf{i}\overline{\mathbf{s}}^{\ell})\overline{\mathbf{s}} \leq \mathbf{s}_2.$$

Thus, we must allow *say* to have not only type $\mathbf{i}\mathbf{s}^{\ell}$ as before, but also type $\mathbf{i}\overline{\mathbf{s}}^{\ell}$ and $\mathbf{i}\overline{\mathbf{q}}^{\ell}$. Presumably, this multiplicity of type assignments calls for some unification. In object position, $\overline{\mathbf{s}}$ can usually be replaced by $\mathbf{s}$, but in subject position the complementizer *that* is obligatory. It remains to be explained why this complementizer cannot occur before a trace.

4.3 Indirect Wh-questions

There are also indirect Wh-questions which can be assigned type $\overline{\mathbf{q}}$, for example in:

I don't know whom he saw –

of type

$$(\mathbf{s}_1\overline{\mathbf{q}}^\ell)(\overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{s}^\ell)(\mathbf{s}_3\mathbf{o}^\ell) \leq \mathbf{s}_1,$$

provided we assign yet another type $\overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{s}^\ell$ to *whom* in this context. The same assignment will work in:

I don't know whom she says he saw –.

Of course, to account for indirect questions of type $\overline{\mathbf{q}}$ such as

whom he saw – yesterday, *whom you taught – English*

we require modified or additional types for *whom*, as we did for direct questions. The easiest way to accomplish this is with the help of a metarule:

if *whom* has type $\mathbf{q}' \cdots \mathbf{q}^\ell$ in a direct question, then it has type $\overline{\mathbf{q}} \cdots \mathbf{s}^\ell$ in an indirect one.

This metarule will yield types $\overline{\mathbf{q}}\mathbf{i}^\ell\mathbf{i}\mathbf{o}^{\ell\ell}\mathbf{s}^\ell$ and $\overline{\mathbf{q}}x^\ell\mathbf{o}'^{\ell\ell}x^{\ell\ell}\mathbf{s}^\ell$, with $x = \mathbf{o}, \overline{\mathbf{q}}$ or $\bar{\mathbf{i}}$, for *whom*.

As in direct questions, we seem to require two types for *who*, $\overline{\mathbf{q}}\mathbf{s}^\ell\pi_3$ and $\overline{\mathbf{q}}\mathbf{s}^\ell\pi_3\mathbf{s}^{\ell\ell}\mathbf{s}^\ell$, to account for *who is going* of type

$$(\overline{\mathbf{q}}\mathbf{s}^\ell\pi_3)(\pi_3^\ell\mathbf{s}_1) \leq \overline{\mathbf{q}}$$

and *who she said – is going* of type

$$(\overline{\mathbf{q}}\mathbf{s}^\ell\pi_3\mathbf{s}^{\ell\ell}\mathbf{s}^\ell)(\mathbf{s_2}\mathbf{s}^\ell)(\pi_3^r\mathbf{s}_1) \leq \overline{\mathbf{q}}.$$

Note that $\mathbf{s}^{\ell\ell}\mathbf{s}^\ell \leq 1$, so $\overline{\mathbf{q}}\mathbf{s}^\ell\pi_3\mathbf{s}^{\ell\ell}\mathbf{s}^\ell \leq \overline{\mathbf{q}}\mathbf{s}^\ell\pi_3$, hence the second type would do for both examples. However, it would be a mistake to contract $\mathbf{s}^{\ell\ell}\mathbf{s}^\ell$ to 1 when interpreting *who she said – is going.*

4.4 Infinitival noun phrases and gerunds

There are other expressions that should be assigned type $\overline{\mathbf{q}}$, e.g.

whether to go, whom to see –

Let us provisionally assign type $\bar{\mathbf{i}}$ to *to go*, hence $\bar{\mathbf{i}}\mathbf{o}^\ell$ to *to see*, then we are led to supply *to*, *whether* and *whom* in this context with types $\bar{\mathbf{i}}\mathbf{j}^\ell, \overline{\mathbf{q}}\bar{\mathbf{i}}^\ell$ and $\overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\bar{\mathbf{i}}^\ell$ respectively.

If we now look at

I may wish to go, you may want to go

we are led to say that *wish* and *want* have type $\mathbf{i}\bar{\mathbf{i}}^{\ell}$. However, we shall here refrain from typing the more elaborate sentences

I may wish for him to go, you may want her to go,

where *for* seems to be another kind of complementizer. These expressions correspond to the Latin "accusative with infinitive".

We shall also here refrain from typing gerunds such as

him seeing her, his seeing her,

which form some kind of noun phrases, the second construction being obligatory in subject position. But we will point out that participles of transitive verbs can also be used to form mass nouns, such as: *killing of pigs*. On the other hand, the similar construction *killer of pigs* must be treated as a count noun.

One is tempted to assign the types $\mathbf{o}\mathbf{i}^r\mathbf{c}$ and $\mathbf{o}\mathbf{i}^r\mathbf{m}$ to the morphemes $+ing$ and $+er$ respectively. However, it is not clear how to establish a systematic link between the inflector Part of type $\mathbf{p}_1\mathbf{i}^{\ell}$ and the morpheme $+ing$ for transitive verbs. In a production grammar one can state a rule:

$$V + ing \leq \text{ Part } V$$

for any infinitivial verb form V, but how can such a rule be incorporated into a type grammar?

4.5 Relative clauses

The words *who* and *whom* also appear in relative clauses, of which there are two kinds: the nonrestrictive ones which modify a name or noun phrase and the restrictive ones which modify a noun.

The nonrestrictive relative clauses should have type $\mathbf{n}^r\mathbf{n}$, so in *John, who left* we assign to *who* the type $\mathbf{n}^r\mathbf{n}\mathbf{s}^{\ell}\mathbf{n}$. Here we have exploited the facts that $\mathbf{n} \leq \pi_3$ and $\mathbf{s}_2 \leq \mathbf{s}$. On the other hand, in *John, whom I saw -*, we assign to *whom* the type $\mathbf{n}^r\mathbf{n}\mathbf{o}^{\ell\ell}\mathbf{s}^{\ell}$. Like the interrogative pronoun *whom*, this should be modified to account for such clauses as

whom I saw – yesterday, whom I saw – leave.

Incidentally, it is not a good idea to treat the relative and interrogative pronouns in common, since they have different inanimate versions *which* and *what* respectively.

The restrictive relative clause should have type $x^r x$, where $x = \mathbf{c}, \mathbf{m}$ or $\mathbf{p}$. The most economic way to handle the restrictive relative pronouns *who* and *whom* is to assign to them the types $x^r x\mathbf{s}^\ell\mathbf{n}$ and $x^r x\mathbf{o}^{\ell\ell}\mathbf{s}^\ell$ respectively. Both *who* and *whom*, when occurring restrictively, can be replaced by *that*, so this should be assigned the same types. However, the restrictive *whom* can be left out altogether, as in

the man I saw – , *people I know –*.

This causes a problem: should the type of *whom* be attached to a zero morpheme, advocated e.g. by Moortgat? At one time I thought of assigning a new type to the article *the* in *the man I saw* -, namely $\mathbf{n}\mathbf{o}^{\ell\ell}\mathbf{s}^\ell\mathbf{c}^\ell$, but this would not help with *people I know*-.

If we don't like naked types or zero morphemes, there is another way: we could adopt the rule

$$x\mathbf{s}\mathbf{o}^\ell \leq x.$$

In either case, we must abandon the attempt to push all the grammar into the dictionary. If the second remedy is adopted, as I prefer, this means that there are grammatical rules in addition to the type assignments, hence our protogroup is no longer free. Having accepted this, we can also explain *the man that I saw* – by the rule

$$x\bar{\mathbf{s}}\mathbf{o}^\ell \leq x,$$

rather than assign a new type to *that*.

4.6 Adjectives

Once we have decided to abolish freeness, we can also attack other problems, for example, how to handle adjectives. While adjectives modifying nouns could be assigned types xx^ℓ, with $x = \mathbf{c}, \mathbf{m}$ or $\mathbf{p}$, as in *the good old man*, this won't work for adjectives in predicative position, as in **the man is good old.* I think it is best to adopt a new basic type $\mathbf{a}$ for all adjectives and account for their attributive function by the rule

$$\mathbf{a}x \to x.$$

Their predicative function can be handled by assigning an appropriate type to the copula *be*. Thus, in *John may be old*, *be old* should have type $\mathbf{j}$, hence *be* the type $\mathbf{j}\mathbf{a}^\ell$. To account for *John is old*, we recognize *is* as the third person present tense of *be*, that is, we decompose it as C_{13}*be* of type

$$(\pi_3^r\mathbf{s}_1\mathbf{j}^\ell)(\mathbf{j}\mathbf{a}^\ell) \leq \pi_3^r\mathbf{s}_1\mathbf{a}^\ell.$$

Occasionally adjectives can also modify names, as in *poor John*, but not other noun phrases, thus supporting the remark near the end of Section 4.1 that $\mathbf{n}$ should be split into two types.

Among adjectives we should also count present participles of intransitive verbs and past participles of transitive verbs as in

the bird flying past , *the kettle watched by him*,
water boiling , *a woman scorned.*

Thus, we should adopt the rules

$$x\mathbf{p}_1 \leq x \quad , \quad x\mathbf{p}_2\mathbf{o}^\ell \leq x,$$

where $x = \mathbf{c}, \mathbf{m}$ or $\mathbf{p}$.

4.7 Historical digression

My early work on what I called the "syntactic calculus" was inspired by ideas from logic (type theory) and homological algebra. As it turned out, it had been partly anticipated by Ajdukiewicz and Bar-Hillel, and similar ideas were being pursued independently by Curry on what, in retrospect, should be called "semantic types", later to give rise to Montague semantics.

Ajdukiewicz, influenced by Husserl, had explored the rule $(c/b)b \leq c$, and Bar-Hillel distinguished this from the rule $a(a\backslash c) \leq c$ (in my notation). In my own work, I explored a more general setup:

$$ab \leq c \Leftrightarrow a \leq c/b \Leftrightarrow b \leq a\backslash c.$$

After assigning types $\mathbf{n}$, $\mathbf{n}\backslash\mathbf{s}$ and $\mathbf{s}/(\mathbf{n}\backslash\mathbf{s})$ to *John, goes* and *he* respectively, one could then prove not only that *John goes* has type $\mathbf{n}(\mathbf{n}\backslash\mathbf{s}) \leq \mathbf{s}$, and *he goes* has type $(\mathbf{s}/(\mathbf{n}\backslash\mathbf{s}))(\mathbf{n}\backslash\mathbf{s}) \leq \mathbf{s}$, but also that $\mathbf{n} \leq \mathbf{s}/(\mathbf{n}\backslash\mathbf{s})$, which may be interpreted as saying that every name also has the type of a pronoun.

Over time, I have vacillated between the associative and the non-associative syntactic calculus. Although the latter has recently gained popularity, e.g. in the work of Moortgat and Oehrle, who introduce special modality operators for licensing associativity, I shall here stick to the associative calculus. I have also vacillated between the syntactic calculus with 1 and that without 1, 1 being the type of the empty string. I may take this opportunity to point out that the latter is not a conservative extension of the former, since, in the presence of 1, we have

$$a = a1 \leq a(b/b),$$

but $a \leq a(b/b)$ does not hold in the original syntactic calculus, unless we give it a Gentzen style presentation allowing empty strings of types on the left of a sequent.

Poset models of the syntactic calculus are known as "residuated semigroups" or "residuated monoids", the latter if the unity element 1 of the semigroup is admitted. One may think of 1 as the type of the empty string. By an "AB-monoid" I shall understand a poset model of the original system of Ajdukiewicz as modified by Bar- Hillel, provided the type 1 is incorporated.

The protogroups we have studied here are special cases of AB-monoids, as in an AB- monoid one can define

$$a^r = a\backslash 1, \quad a^\ell = 1/a.$$

But also, conversely, in a protogroup one can define

$$c/b = cb^\ell, \quad a\backslash c = a^r c.$$

although these two definitions are not inverse to one another.

My early work on the syntactic calculus was largely ignored by the linguistic community and I myself became converted to the generative-transformational grammars pioneered by Chomsky. However, in recent years there has been a revival of interest in categorial grammars based on the syntactic calculus, largely initiated by van Benthem. Far reaching applications to natural languages were made by Casadio, Moortgat, Morrill and Oehrle among others. Much important work was done in resolving theoretical questions associated with the syntactic calculus viewed as a formal system, for example by Buszkowski, Došen, Kanazawa, Kandulski, Mikulás and Pentus. There were also a number of excellent doctoral dissertations written in the Netherlands, mostly directed by van Benthem. For detailed references see Moortgat [1997].

A new impetus was given to this revival of interest after the influential introduction of (classical commutative) linear logic into computer science by Girard, when it was realized that the syntactic calculus was just the multiplicative fragment of intuitionistic non-commutative linear logic, or "bilinear logic", as I prefer to call it. It was the brilliant insight by Claudia Casadio that classical bilinear logic could also be used in linguistics which prompted me to take another look at type grammars.

It so happens that the multiplicative fragment of classical bilinear logic had been studied by Grishin [1983] even before the advent of linear logic. Unfortunately, his article was published in Russian. I have called poset models of this logic "Grishin monoids". They are residuated monoids with a dualizing object 0 satisfying

$$0/(a\backslash 0) = a = (0/a)\backslash 0.$$

It will be convenient to write

$$a\backslash 0 = a^r, \quad 0/a = a^\ell,$$

so that $a^{r\ell} = a = a^{\ell r}$. It is easily shown that

$$(b^\ell a^\ell)^r = (b^r a^r)^\ell,$$

for which I have written $a + b$, although Girard in his commutative version has used an upside-down ampersand in place of $+$. We may think of $a + b$ as the De Morgan dual of $a \cdot b$.

We now have

$$aa^r \leq 0, \quad 1 \leq a^r + a; \quad a^\ell a \leq 0, \quad 1 \leq a + a^\ell,$$

and we may introduce

$$c/b = c + b^\ell, \quad a\backslash c = a^r + c.$$

To show the first of these, for example, one proceeds thus:

$$(c/b)b = (c + b^\ell)b \leq c + (b^\ell b) \leq c + 0 = c,$$

making use of one of what Grishin calls "mixed associative laws" (and what Cockett and Seely at one time called "weak distributive laws"), which are easily proved:

$$(a + b)c \leq a + (bc), \quad c(a + b) \leq (ca) + b.$$

Classical bilinear logic, or rather its multiplicative fragment, is known to be a conservative extension of the syntactic calculus, as follows e.g. from the work of Abrusci.

After listening to Claudia's exposition of her ideas, I felt that there were too many operations and that it might be reasonable to require that

$$a + b = a \cdot b, \quad 0 = 1,$$

rendering the Grishin monoid "compact". (This word, attributed to Max Kelly, had been used by Barr in a similar context, namely that of star-autonomous categories.)

One reason for proposing compactness has to do with Gentzen style deductions for bilinear logic. These have the form $\alpha \to \beta$, when α and β are strings of formulas, here types. According to Gentzen, juxtaposition on the left of an arrow should stand for the tensor product, here represented by a dot (usually omitted), a kind of non-commutative conjunction, and juxtaposition on the right should stand for its De Morgan dual, here denoted by $+$. However, we know that one way to describe the grammar of a natural language is with the help of rewrite rules, or productions, $\alpha \to \beta$, where juxtaposition on the two sides of the arrow (corresponding to our $\geq$) has exactly the same interpretation.

I should say a word about the proofnets proposed by Girard for checking proofs in linear logic geometrically. Many people have been enthusiastic about this method, even in linguistic applications. Although, following Descartes, I personally have held the opposing view that one should use algebra to explicate geometry and not vice versa, in the present context even I must admit the advantage of the geometric method. To verify that a string α of types ultimately contracts to the single type b it is necessary and sufficient that one can draw non-crossing linkages for all generalized contractions. I had planned to illustrate this method by a single example:

whom did Jane say she saw –?

$$(\mathbf{q}'\mathbf{o}^{\ell\ell}\mathbf{q}^\ell) \;\; (\mathbf{q}_2\mathbf{i}^\ell\pi_3^\ell) \;\; \mathbf{n} \;\; (\mathbf{i}\mathbf{s}^\ell) \;\; \pi_3 \;\; (\pi^\ell\mathbf{s}_2\mathbf{o}^\ell)$$

Unfortunately the typist has difficulty drawing the linkages; but the reader can easily supply them by linking corresponding left and right parentheses in the following:

$$\mathbf{q}'(\mathbf{o}^{\ell\ell}(\mathbf{q}^\ell\mathbf{q}_2)(\mathbf{i}^\ell(\pi_3^\ell\mathbf{n})\mathbf{i})(\mathbf{s}^\ell(\pi_3\pi^\ell)\mathbf{s}_2)\mathbf{o}^\ell).$$

5 Pregroups

From now on, I shall call a compact Grishin monoid a "pregroup". A *pregroup* is just an ordered monoid in which each element a has both a left adjoint a^ℓ and a right adjoint a^r such that

$$a^\ell a \leq 1 \leq aa^\ell, \quad aa^r \leq 1 \leq a^r a.$$

These adjoints can be defined in terms of residual quotients:

$$a^\ell = 1/a, \quad a^r = a\backslash 1.$$

Conversely, one may recover the residual quotients from the adjoints:

$$a/b = ab^\ell, \quad b\backslash a = b^r a.$$

Again, these two definitions are not inverse to one another, see the argument at the end of this section.

Pregroups are easily seen to be the same as protogroups which satisfy the equations (E) and the implications (I) of Section 1. For example, under these conditions we have

$$a^r a = a^r a^{\ell r} = (a^\ell a)^r \geq 1^r = 1.$$

If we stipulate $a^r = a^\ell$ (the so-called "cyclic" case), the pregroup becomes an ordered group. One advantage of pregroups over protogroups is this: the dual of a pregroup, replacing $\leq$ by $\geq$ and interchanging r and ℓ, is also a pregroup.

At the time of writing this, I know of only one example of a non-cyclic pregroup, aside from the free pregroups to be considered presently. This is the ordered monoid of unbounded order preserving mappings $\mathbb{Z} \to \mathbb{Z}$. To see that this is not cyclic, take $f(n) = 2n$ and calculate

$$f^r(n) = [n/2], \quad f^\ell(n) = [(n+1)/2].$$

where $[x]$ denotes the greatest integer $\leq x$.

The *free pregroup* $F(A)$ generated by a poset A is defined like the free protogroup of Section 1, only now there is an additional case in the definition of $\alpha \leq \beta$, to be called an *expansion*:

Case 3. $$\alpha_i = \gamma\delta, \alpha_{i+1} = \gamma a^{(n)} a^{(n-1)} \delta.$$

It is easily verified that $F(A)$ is a pregroup with the expected universal property.

A *generalized expansion* has the form

$$\gamma\delta \leq \gamma a^{(n)} b^{(n-1)} \delta,$$

where n is odd and $a \leq b$ in A or n is even and $b \leq a$ in A.

The following proposition will yield an effective procedure for deciding when $\alpha \leq \beta$, for elements α, β of $F(A)$. While the corresponding problem for free residuated monoids was solved with the help of a technique borrowed from logic, namely Gentzen's method of cut elimination, the present decision procedure (like that for protogroups) is borrowed from highschool algebra, where one simplifies each side of a potential equation as much as possible, expecting to arrive at a common answer.

Proposition 2. *If $\alpha \leq \beta$ in a free pregroup, then there exist strings α' and β' such that $\alpha \leq \alpha' \leq \beta' \leq \beta$, where $\alpha \leq \alpha'$ by generalized contractions only, $\alpha' \leq \beta'$ by induced steps only and $\beta' \leq \beta$ by generalized expansions only.*

Proof. Consider a sequence

$$\alpha \leq \alpha_1 \leq \alpha_2 \leq \cdots \leq \alpha_n = \beta$$

in $F(A)$, where each step is induced from A or is a generalized contraction or expansion. We claim that all generalized expansions can be postponed to the end, after which the proof of Proposition 1 takes over.

Indeed, suppose a generalized expansion, say

$$\gamma\delta \leq \gamma a^{(n+1)} b^{(n)} \delta,$$

with n odd and $a \leq b$ in A, immediately precedes an induced step, say

$$\gamma a^{(n+1)} b^{(n)} \delta \leq \gamma a^{(n+1)} c^{(n)} \delta,$$

where $b \leq c$ in A, then we may combine these two steps into a single generalized expansion with $a \leq c$. (Other subcases of this situation are left to the reader.)

Suppose a generalized expansion is immediately followed by a generalized contraction, then we have essentially two cases:

Case A. Say m odd, $a \leq b$, n even and $c \leq d$, so that

$$\gamma\lambda b^{(m)} a^{(m+1)} \delta \leq \gamma c^{(n)} d^{(n-1)} \lambda b^{(m)} a^{(m+1)} \delta \leq \gamma c^{(n)} d^{(n-1)} \lambda\delta.$$

This can be replaced by

$$\gamma\lambda b^{(m)} a^{(m+1)} \delta \leq \gamma\lambda\delta \leq \gamma c^{(n)} d^{(n-1)} \lambda\delta.$$

(Other subcases are treated similarly.)

Case B. Say n odd and $a \leq b \leq c$, then

$$\gamma a^{(n)} \delta \leq \gamma a^{(n)} b^{(n-1)} c^{(n)} \delta \leq \gamma c^{(n)} \delta.$$

This can be replaced by a single induced step. (Other subcases are treated similarly.)

Corollary 1. *If $\alpha \leq \beta$ and β has length 1, one can go from α to β by contractions and induced steps only.*

Can we use pregroups in place of protogroups when looking at English grammar? Whereas a pregroup may be viewed as a residuated monoid satisfying

$$1 \leq (a\backslash 1)a, \quad 1 \leq a(1/a),$$

these inequalities are not justified by the usual models of the syntactic calculus, e.g. the monoid of subsets of the free monoid generated by the English vocabulary. Fortunately, they play no role in checking the sentencehood of a string of words. For, in view of the above corollary, to verify $\alpha \leq \mathbf{s}$ requires contractions and induced steps only, just as though we were operating in a free protogroup instead of a free pregroup.

While pregroups may be more interesting mathematically than protogroups, what can they do for linguistics that protogroups can't? We shall look at a few potential applications.

The phrase *does not*, often contracted to *doesn't*, has type

$$(\pi_3^r \mathbf{s}_1 \mathbf{i}^\ell)(\mathbf{i}\mathbf{i}^\ell) \leq \pi_3^r \mathbf{s}_1 \mathbf{i}^\ell.$$

However, using the expansion $1 \leq \mathbf{i}\mathbf{i}^\ell$, we can reverse the inequality, hence obtain equality. Perhaps this observation lends some weight to the suggestion in Section 3.2 that *doesn't* should be regarded as a verb form.

We have analyzed *he goes* and *John goes* as $\pi_3(\pi_3^r \mathbf{s}_1)$ and $\mathbf{n}(\pi_3^r \mathbf{s}_1)$ respectively and justified $\mathbf{n}(\pi_3^r \mathbf{s}_1) \leq \mathbf{s}_1$ by postulating $\mathbf{n} \leq \pi_3$. An alternative approach, in the spirit of my older work, would be to give to *goes* the type $\mathbf{n}^r \mathbf{s}_1$ and to define π_3 as $\mathbf{s}_1 \mathbf{s}_1^\ell \mathbf{n}$. Then $\mathbf{n} \leq \pi_3$ could be proved as a consequence of $1 \leq \mathbf{s}_1 \mathbf{s}_1^\ell$ and need not be postulated. The only problem with this approach is that it won't account for *he went*, which would require that *he* has two types: not only $\mathbf{s}_1 \mathbf{s}_1^\ell \mathbf{n}$, but also $\mathbf{s}_2 \mathbf{s}_2^\ell \mathbf{n}$. Since π_3 cannot be defined to be equal to both of them, we would have to dispense with π_3 altogether.

This problem would not arise with *him*, which pronoun could be assigned the single type $\mathbf{n}\mathbf{i}^r \mathbf{i}$. In a similar fashion we can treat *it* and *so* in

you may be saying so, you may have said it.

I would assign to *so* the type $\mathbf{s}\mathbf{i}^r \mathbf{i}$ and to *it* the type $\overline{\mathbf{s}}\mathbf{i}^r \mathbf{i}$, assuming that *saying* and *said* here have types $(\mathbf{p}_1 \mathbf{i}^r)(\mathbf{i}\mathbf{s}^\ell)$ and $(\mathbf{p}_2 \mathbf{i}^r)(\mathbf{i}^r \overline{\mathbf{s}}^\ell)$ respectively.

Of course the extension of a free residuated monoid into a pregroup is not conservative, since $(1/a)\backslash 1 = a$ holds in the latter but not in the former. Unfortunately, the same objection applies to the extension of a free residuated semigroup, since after the extension we obtain the equation

$$a(b/c) = abc^\ell = (ab)/c,$$

which does not hold originally. This objection already applies to embedding a free *AB*-semigroup into a protogroup.

6 Concluding remarks

This article advocates a number of ideas, which are essentially independent of one another:

(1) to use classical bilinear logic in place of the syntactic calculus, as had been suggested by Claudia Casadio;
(2) to allow the set of basic types to be partially ordered;
(3) to require ordered models of the resulting calculus to be freely generated by the poset of basic types, thus ensuring that all grammatical information is found in the dictionary;
(4) to simplify the algebra by identifying multiplication with addition, its De Morgan dual, hence the unit 1 with the dualizing object 0 (since $1 = 1+0 = 1 \cdot 0 = 0$).

Of these ideas, I share Casadio's confidence in (1). (2) is quite useful, though not essential, as it can be circumvented by multiple type assignments. (3) proved useful as a starting point, but ultimately had to be abandoned when confronted with typed zero morphemes, as in

$$\textit{people} \quad \emptyset \quad \textit{I know} \ –$$

$$\mathbf{p} \quad (\mathbf{p}^r \mathbf{p} \mathbf{o}^{\ell\ell} \mathbf{s}^{\ell}) \pi_1 (\pi_1^r \mathbf{s}_1 \mathbf{o}^{\ell})$$

or with the equivalent postulate

$$\mathbf{p}\mathbf{s}\mathbf{o}^{\ell} \leq \mathbf{p}.$$

It would appear that a speaker of English can avoid such constructions, in this example by replacing the zero morpheme by *that*. Anyway, there is no problem with generating *people I know* -; all one has to do is to suppress the complementizer *that*. The difficulty arises with recognition, as illustrated by the well-known example:

the horse raced – past the barn fell,

where recognition depends on the postulate

$$\mathbf{c}\mathbf{p}_2\mathbf{o}^{\ell}\mathbf{i}^r\mathbf{i} \leq \mathbf{c}.$$

Finally, (4) causes serious problems when we want the grammar to account for semantics in the spirit of Curry and Montague. The syntactic calculus carries with it an implicit semantics: by introducing Gentzen's three structural rules, interchange, contraction and thinning, one essentially turns it into Curry's semantic calculus. On the level of proofs rather than provability, the syntactic calculus may be viewed as a biclosed monoidal category and the semantic calculus as a cartesian closed category. Since cartesian closed categories are known to be equivalent to lambda calculi, Montague semantics arises naturally.

Unfortunately, the compact bilinear logic advocated in (4) is not a conservative extension of the syntactic calculus, hence its relationship with semantics is less evident. Perhaps we should retreat and abandon compactness, thus returning to Casadio's original suggestion?

Indeed, many of the linguistic examples studied here can also be handled without compactness. All one has to do is to replace multiplication of types inside a word by addition, but to allow multiplication between words to stand.

Consider, for instance, the example:

I may be going.

$$\pi_1(\pi^r + \mathbf{s}_1 + \mathbf{j}^\ell)(\mathbf{j} + \mathbf{p}_1^\ell)\mathbf{p}_1$$

We calculate successive initial segments of the associated string of types, making use of the mixed associative laws:

$$\pi_1(\pi^r + \mathbf{s}_1 + \mathbf{j}^\ell) \leq (\pi_1\pi^r) + \mathbf{s}_1 + \mathbf{j}^\ell \leq 0 + \mathbf{s}_1 + \mathbf{j}^\ell = \mathbf{s}_1 + \mathbf{j}^\ell;$$
$$(\mathbf{s}_1 + \mathbf{j}^\ell)(\mathbf{j} + \mathbf{p}_1^\ell) \leq \mathbf{s}_1 + \mathbf{j}^\ell(\mathbf{j} + \mathbf{p}_1^\ell) \leq \mathbf{s}_1 + (\mathbf{j}^\ell\mathbf{j}) + \mathbf{p}_1^\ell \leq \mathbf{s}_1 + 0 + \mathbf{p}_1^\ell = \mathbf{s}_1 + \mathbf{p}_1^\ell;$$
$$(\mathbf{s}_1 + \mathbf{p}_1^\ell)\mathbf{p}_1 \leq \mathbf{s}_1 + (\mathbf{p}_1^\ell\mathbf{p}_1) \leq \mathbf{s}_1 + 0 = \mathbf{s}_1.$$

Similarly, looking at

did he see her?

$$(\mathbf{q}_2 + \mathbf{i}^\ell + \pi^\ell)\pi_3(\mathbf{i} + \mathbf{o}^\ell)\mathbf{o}$$

we calculate successive initial segments, making implicit use of mixed associativity:

$$\mathbf{q}_2 + \mathbf{i}^\ell, \quad \mathbf{q}_2 + \mathbf{o}^\ell, \quad \mathbf{q}_2.$$

A problem arises with examples involving double negation such as $\mathbf{o}^{\ell\ell}$, indicative of a Chomskian trace. Consider

I may be seen –

$$\pi_1(\pi^r + \mathbf{s}_1 + \mathbf{j}^\ell)(\mathbf{j} + \mathbf{o}^{\ell\ell} + \mathbf{p}_2^\ell)(\mathbf{p}_2 + \mathbf{o}^\ell)$$

The initial segments here are:

$$\mathbf{s}_1 + \mathbf{j}^\ell, \quad \mathbf{s}_1 + \mathbf{o}^{\ell\ell} + \mathbf{p}_2^\ell, \quad \mathbf{s}_1 + \mathbf{o}^{\ell\ell} + \mathbf{o}^\ell.$$

Unfortunately, the last does not contract to $\mathbf{s}_1$, unless $\mathbf{o}^{\ell\ell} + \mathbf{o}^\ell \leq 0$. This would however follow from $\mathbf{o}^{\ell\ell}\mathbf{o}^\ell \leq 1$ by compactness.

Similar, consider:

whom did you see –?

$$(\mathbf{q}' + \mathbf{o}^{\ell\ell} + \mathbf{q}^\ell)(\mathbf{q}_2 + \mathbf{i}^\ell + \pi^\ell)\pi_2(\mathbf{i} + \mathbf{o}^\ell)$$

The initial segments are:

$$\mathbf{q}' + \mathbf{o}^{\ell\ell} + \mathbf{i}^\ell + \pi^\ell, \quad \mathbf{q}' + \mathbf{o}^{\ell\ell} + \mathbf{i}^\ell, \quad \mathbf{q}' + \mathbf{o}^{\ell\ell} + \mathbf{o}^\ell.$$

Again, compactness is required to ensure that this contracts to $\mathbf{q}'$.

However, according to Claudia Casadio, there is no reason why the types inside a word should be linked only by addition. Indeed, the last two examples will work out if we assign to the passive auxiliary *be* the type $\mathbf{j} + \mathbf{o}^{\ell\ell}\mathbf{p}_2^\ell$ and to the question word *whom* the type $\mathbf{q}' + \mathbf{o}^{\ell\ell}\mathbf{q}^\ell$.

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