Binomial to Gamma

Binomial

$$S \sim \text{Binomial}(n, p) \tag{1}$$

Interpretation: $S \in \{0, \ldots, n\}$ is the number of successes in n repeated trials, each with probability p of success.

- If $n = 1$, it is called Bernoulli(p).

If we think of the n trials happening in a fixed period of time, and divide the period infinitesimally, so instead of n, p, we have a rate, λ, the expected number of successes, we get the Poisson.

Poisson

$$S \sim \text{Poisson}(\lambda) \tag{2}$$

Interpretation: $S \in \{1, \ldots, \infty\}$ is the number of successes in a fixed period, given constant rate λ.

If instead of being interested in the *number* of successes, we're interested in the *time between successes*, we get the Exponential distribution (or more generally, the Gamma distribution).

Exponential

$$S \sim \text{Exponential}(\lambda) \tag{3}$$

Interpretation: $S \in \mathbb{R} \geq 0$ is the time to wait before the first success in a Poisson process with rate λ.

- sometimes parametrized in terms of the mean $\frac{1}{\lambda}$ (units of time), rather than rate λ (units inverse time).
- if rate is not constant, but is proportional a power of time, see Weibull distribution (common for time-to-failure interpretation).

Gamma

$$S \sim \text{Gamma}(k, \lambda) \tag{4}$$

Interpretation: $S \in \mathbb{R} \geq 0$ is the time it takes to have k successes in a Poisson process with rate λ.

The 'shape' parameter k can be any positive real.

- if k is restricted to be an integer, then it is called Erlang(k, λ).
- if $k = 1$, it's Exponential(λ).

References

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