# Clitic Movement in Pregroup Grammar: A Cross-Linguistic Approach

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Abstract. The calculus of pregroups is a kind of type (or categorial) grammar introduced by Lambek [17] for the analysis and computation of natural languages; it has been applied to a wide range of languages from English and German, to French and Italian, and many others [11]. Like Lambek's Syntactic Calculus, pregroups are non-commutative structures, but the syntax of natural languages shows also the presence of cyclic patterns, in particular those exhibited by the phenomenon known in the literature as movement of clitic pronouns in different languages. In this paper we propose an extension of the calculus of pregroups including two cyclic rules and use them to formally analyze movement of clitic clusters in Persian, French, and Italian. In the final part of the paper, we discuss the relations of these rules to Yetter's and Abrusci's cyclic rules for Linear Logic.

**Keywords:** Type Grammars, Pregroup, Clitic Movement, Cyclic Rules.

#### 1 Introduction

Pregroups are mathematical structures introduced by Lambek [17]20,18 to allow suitable computational procedures for the analysis and generation of natural languages. The calculus of pregroups has been developed as an alternative to the Syntactic Calculus, well known as Lambek Calculus [16], used in theoretical and computational linguistics in the context of categorial grammars, see [23]24]22: while the last is an intuitionistic system, based on the operation of implication, the former is a classical system, based on the operation of multiplicative conjunction (for details and discussion see [5]6]10]20). The mathematical and logical properties of pregroups have been studied in [4]5]17, and they have been applied to the grammatical analysis of a large number of languages: English [19],

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French [2,21], Italian [7], Arabic [3], Polish [14,15], Persian [28] and many more, see [11].

The story of this paper is motivated by the following observations: in Persian the subject and object of a sentence occur in pre-verbal position (Persian is a SOV language), but they may attach themselves as clitic pronouns to the end of the verb and form a one-word sentence. By doing so, the word order changes from SOV to VSO. In Arabic we find an analogous pattern, but the word order remains VSO in both cases. A similar phenomenon can be found in some European languages, but in a less extreme format. For example, in Italian and French nouns or pronouns occurring as complements in post-verbal position, can take a clitic form and move to a pre-verbal position, still keeping their relative order of occurrence. To account for these kind of movements, Casadio and Lambek in [9] and Bargelli and Lambek in 2 introduce into the pregroup types for verbal inflectors and infinitives: verbs taking accusative and dative complements are assigned two different types of infinitive, extended infinitive of type j and short infinitive of type i, and somewhat lengthy types are introduced for pre-verbal clitic pronouns taking advantage of the iterated adjoints that are allowed by the pregroup grammar. In fact, pregroups are a non-conservative extension of noncommutative linear logic [1117], in which "left" and "right" iterated negations (equivalently "left" and "right" iterated adjoints) do not cance 2.

In this paper we propose a different approach that simplifies the above analyses for Italian and French and provides a new solution for Persian, offering a unified account. We show how one can explain these movements by adding two *clitic metarules* to the pregroup grammars of these languages. Metarules are postulates introduced into the dictionary of the grammar to simplify lexical assignments and make syntactic calculations quicker. In Lambek's words: "The types assigned to words are assumed to be stored permanently in our 'mental' dictionary. To prevent overloading the dictionary, we have employed *inflectors*, operations which transform a dictionary entry into the inflected form occurring in actual discourse and certain *metarules*, which assert that, if the dictionary assigns a certain type to a word, then this word may also have certain other types" [21, p. 105].

The effect of the two metarules is that the *clitic* type of each verb form is derivable from its original type. Interestingly, the same rule works for third person and count-noun clitics in French and pre-verbal clitic clusters in Italian, whereas the converse of the rule works for Persian. In Arabic and post-verbal Italian, in turn, we don't need these rules, since the clitic forms preserve the order of their non-clitic peers. To allow a clear understanding of the cross-linguistic generalization obtained by the application of the two clitic metarules, we confine

<sup>&</sup>lt;sup>1</sup> For example, in Italian the type  $(jo^{ll}i^{l})$  for the clitic pronoun "lo" (him, it) in accusative case, with the iterated adjoint  $o^{ll}$ , which cancels with the accusative argument of the infinitive type  $(io^{l})$ , assigned e.g. to the verb "vedo" (I see), giving an extended infinitive of type j, "lo vedo"  $(I him see): (jo^{ll}i^{l})(io^{l}) \rightarrow j$ .

<sup>&</sup>lt;sup>2</sup> See the discussion of the relations between Syntactic Calculus, non-commutative linear logic and pregroups in [10].

our analysis to the basic sentence structures involving subject, verbal predicate and different kinds of verbal complements.

Our clitic rules remind the *cyclic* rules of Yetter's cyclic linear logic [30]. These rules were later added to Lambek's calculus by Abrusci [1], proving that a non-commutative calculus with these rules is still a conservative extension of the Syntactic Calculus. Motivated by this fact, we studied the possibility to add cyclic rules as extra axioms to the calculus of pregroups or, alternatively, as extra rules to its sequent calculus. However this idea is still under investigation since, as discussed below in §6, the addition of cyclic rules to the pregroup calculus has the effect of making it collapse into an ordered group. We therefore decided to introduce the cyclic rules in the form of metarules applying just to the verb types. In this way the calculus, so enriched, is still a free multiplicative pregroup [20].

#### 2 Two Clitic Metarules

A pregroup P is a partially ordered monoid  $(P, ..., 1, \leq, ()^l, ()^r)$  in which P is a set of types, '.' is a non-commutative multiplicative operation, 1 is the unit of the monoid, and each element  $p \in P$  has both a left adjoint  $p^l$  and a right adjoint  $p^r$ , to the extent that the following rewritings hold:

$$p^l p \le 1 \le p p^l$$
 ,  $p p^r \le 1 \le p^r p$  .

The two inequalities on the left side of 1 are referred to as *contractions*, while the two at the right side as *expansions*. One can show that the adjoints are unique and contravariant, i.e.

$$p \le q \implies q^l \le p^l \text{ and } q^r \le p^r$$
,

the unit 1 and the multiplication (the dot '.' here omitted) are self dual 17.5:

$$1^{l} = 1 = 1^{r}$$
  $(pq)^{l} = q^{l}p^{l}$   $(pq)^{r} = q^{r}p^{r}$ .

To analyze a natural language, we define a set of basic types and assign them to the words of its dictionary; e.g. the types n for noun phrase,  $\pi$  for subject, o for object, o for indirect object, o for locative object, o for sentence, o for question. To frame the partial order on the set of basic types, convenient order postulates are introduced, allowing order relations to be derived by transitivity. In this paper we assume the partial orders (or order postulates):  $n \le \pi$ ,  $n \le o$ ,  $n \le \omega$ ,  $n \le \lambda$ . The free pregroup of the language is generated from these basic types and will include simple types such as  $n^l, n^r, \pi^l, \pi^r, o^l, o^r$ , and compound types such as

<sup>&</sup>lt;sup>3</sup> A partial order ' $\leq$ ' (here denoted by the arrow ' $\rightarrow$ ') is a binary relation which is reflexive:  $x \leq x$ , transitive:  $x \leq y$  and  $y \leq z$  implies  $x \leq z$ , and anti-symmetric:  $x \leq y$  and  $y \leq x$  implies x = y. We may read  $x \leq y$  as saying that everything of type x is also of type y.

<sup>&</sup>lt;sup>4</sup> The operation is the multiplicative conjunction of compact bilinear logic and its counterpart is the tensor product '⊗' of non-commutative linear logic, see [10,117].

 $(\pi^r s \, o^l)$ . For example, the declarative sentence 'I saw him.' is typed as follows (for  $\pi_1$  a first person pronoun,  $s_2$  a sentence in the past tense)

I saw him. 
$$\pi_1 \ (\pi_1^r s_2 o^l) \ o$$

A sentence is grammatical iff it *reduces* to the type s, and the derivation procedure:  $\pi_1$  ( $\pi_1^r s_2 o^l$ )  $o \to s$ , is depicted by the under-link diagrams.

A categorical semantics for pregroups in terms of compact bi-categories has been developed by Preller and Lambek [25]. It is based on the result that pregroups are skeletons of compact closed categories, that is, a category whose objects are elements of a pregroup and whose morphisms are the partial orders thereof. This category is a monoidal one, with the multiplication being the monoid multiplication of the pregoup. But moreover, each element has a left and right adjoint with regard to which this multiplication is closed, that is we have two units and co-units for the adjunctions, that correspond to the adjunction inequalities in a pregroup. On this basis, Preller and Sadrzadeh have recently elaborated a vector spaces semantics for pregroups developing a translation between the functional and vector semantics of pregroups and showing the equivalence of the two [26].

We extend the pregroup introducing the following metarules, for  $p, q \in P$ :

Clitic Rule (1): If  $p^rq$  is the *original* type of the verb, then so is  $q \overline{p}^l$ .

Clitic Rule (2): If  $qp^l$  is the *original* type of the verb, then so is  $\overline{p}^rq$ .

The over-lined types  $\overline{p}^l$ ,  $\overline{p}^r$  are introduced as a notational convenience to distinguish the clitic pronouns from the non-clitic stressed pronouns or arguments. For any clitic pronoun  $\overline{p}$ , we postulate the partial order  $\overline{p} \leq p$  to express the fact that a clitic pronoun is also a kind of pronoun. We assume that for all  $p, q \in P$ , we have  $\overline{pq} = \overline{p} \ \overline{q}$ . The mention of the word original in the rules is to forbid the application of the rule to the derived types of the verb. We want to be able to apply the rule only to the type of the verb in the full sentence without any clitic pronouns. If we do not assume this restriction and allow the rule to be applied to the type of the verb also when clitics occur, then non-grammatical sentences can be formed. We will give examples in the following sections. Note that these rules are converse to each other, not considering the use of the over-lined types.

### 3 Clitic Movement in Persian

The clitic clusters (pre-verbal vs. post-verbal) for the sentence *I saw him*, "man u-ra didam" in Persian, exhibit the following general pattern:

<sup>&</sup>lt;sup>5</sup> These diagrams are reminiscent of the planar poof nets of non-commutative linear logic, as shown in the calculus developed by Abrusci; see 115124.



The over-lined types  $\overline{\pi}, \overline{o}$ , stand for the clitic versions of the subject and object pronouns. The "am" at the end of "didam" in the non-clitic sentence above is a conjugation pronoun and liaises with the first person subject "man". Conjugation pronouns are also used as clitic pronouns for the subject. Typing the conjugation and the accord can be done by indexing the subject and the s in the verb with its person and require that they should cancel out only if they have the same index. Making the typing for conjugation pronouns explicit would be distracting for the purpose of this paper, so we shall ignore it and focus on the clitic pronouns. Then the conjugation pronoun "am" in the first sentence is included into the type of the verb, whereas the "am" in the second sentence, being a clitic pronoun, receives a type.

We postulate the first clitic rule as a metarule in the pregroup grammar of Persian. Applying this rule to the above example will allow us to obtain the clitic form of the verb from its original type. The original Persian verb has the type  $o^r \pi^r s = (\pi o)^r s$ , which is of the form  $p^r q$ ; after applying the clitic rule (1) we obtain  $s(\overline{\pi o})^l = s(\overline{\pi o})^l = s\overline{o}^l \overline{\pi}^l$ , i.e. the type of the verb with post-verbal clitics. The clitic rule can be seen as a re-write rule and the derivation can be depicted as one-linear as follows

$$o^r \pi^r s = (\pi o)^r s \quad \rightsquigarrow \quad s(\overline{\pi o})^l = s \overline{o}^l \overline{\pi}^l$$

To form these one-word sentences, one does not need to have pronouns as subject and object in the original sentence. They can as well be nouns or noun phrases, for example the sentence I saw Nadia, in Persian "man Nadia-ra didam", becomes "did-am-ash", being typed exactly as in the example above. This movement takes a more subtle form when the subject is third person singular, since in this case the subject clitic pronoun is the same as the last letter of the verb. But the types and the movement are exactly the same as in our previous examples.

One can form a yes-no question from any of the sentences above, by adding the question form "aya" to the beginning of the sentence. Since in Persian the word order of the question form is the same as that of the original sentence, clitic movement obeys the same rule, as follows

Did Hassan Nadia see? Did see he her? aya Hassan Nadia-ra did? aya di d ash? 
$$qs^l$$
  $\pi$   $o$   $(o^r\pi^rs) \rightarrow q$   $qs^l$   $(s\overline{o}^l\overline{\pi}^l)$   $\overline{\pi}$   $\overline{o}$   $o$ 

<sup>&</sup>lt;sup>6</sup> In the pregroup calculus, like in non-commutative linear logic, a form of De Morgan rule holds to the extent that  $(pq)^l = (q^l p^l)$  and  $(pq)^r = (q^r p^r)$ ; see ¶17.

#### 3.1 Partial Clitic Movement

One can also obtain partial clitic movement, like in *Hassan saw her*, in Persian "Hassan did-ash". This would make sense as an answer to the wh-question *Who saw Nadia*? that in Persian translates as "Che-kasi u-ra did?"

$$\begin{array}{ccc} \text{Hassan} & \text{saw} & \text{her.} \\ \text{Hassan} & \text{did} & \text{ash.} \\ \pi & \left(\pi^r s \ \overline{o}^l\right) & \overline{o} \ \rightarrow \ \text{s} \end{array}$$

This partial clitic type of the verb is derivable via the same clitic rule, but applied in a different manner. Here one takes  $p^r$  to be  $o^r$  and q to be  $\pi^r s$ , the application of the rule results in  $q\overline{p}^l$  equal to  $(\pi^r s) \overline{o}^l = (\pi^r s \overline{o}^l)$ , via associativity of '.'.

Another example of a partial clitic movement is the sentence "Nadia saw I" (*I saw Nadia*), in Persian "Nadia-ra did-am". This is more controversial than the previous example, and might not be considered as grammatical, being a phrase that starts neither with a subject nor with a verb. Perhaps it only makes sense as an immediate answer to the question *Whom did I see*?, in Persian "Man che-kasi-ra did-am?".

Nadia saw I.  
Nadia-ra did am.  

$$o (o^r s \overline{\pi}^l) \overline{\pi} \rightarrow s$$

We need to point out that this new type  $(o^r s \overline{\pi}^l)$  of the verb is not derivable from its original type  $(o^r \pi^r s)$ . This is in a sense a good new, since the sentence under consideration is not really grammatical; one should rather say "man Nadia-ra did-am". However, if one insists on a solution, here is a fix. Although we do not 100% agree with these partial clitic sentences in Persian, we are nonetheless going to present a solution, because it is also a fix for partial clitic movement in French, with the difference that there the sentences obtained from such movement are grammatical. To be able to derive this type of the verb, we need to generalize our clitic rule to the effect that it can be applicable to parts (and not necessarily to the whole) of the type string of the verb. Here is the definition of such a partial clitic rule:

For 
$$p, q \in P$$
, if  $p^r q$  is in the *original* type of the verb, then so is  $q \overline{p}^l$ .

This means that if a string of the type  $p^rq$  occurs somewhere inside the type of a verb, it can be replaced with a string of the type  $q \overline{p}^l$  and the resulting type is considered as another type for that verb.

One can now apply this rule to the  $\pi^r s$  part of the full type of the verb, that is  $(o^r \pi^r s)$ , and obtain  $(s \overline{\pi}^l)$ , leaving the  $o^r$  at the beginning untouched. In this way we obtain the type that we need, that is  $(o^r s \overline{\pi}^l)$ . It has to be pointed out that the partial clitic rule is harmless. The only other way one can apply it to the original type of the verb is by fixing s at the end and taking  $p^r = o^r$  and  $q = \pi^r$ , applying the rule will then result in  $(\pi^r \overline{o}^l s)$ , which cannot make any ungrammatical sentence grammatical. This is due to the occurrence of the left

adjoint  $\overline{o}^l$  for the clitic pronoun, which needs a double left adjoint  $\overline{o}^{ll}$  to cancel out, but we have assigned no grammatical role to types such as  $\overline{o}^{ll}$  in this paper.

Picking up on the bad habit of uttering half-grammatical sentences, one can use this type to form yes-no questions from the above question *Did I See Nadia*?", in Persian "aya Nadia-ra did-am?", etc.

Did Nadia see I? aya Nadia-ra did am? 
$$qs^l$$
  $o$   $(o^r s \overline{\pi}^l)$   $\overline{\pi} \rightarrow q$ 

The grammatical form of this question would be "aya man Nadia-ra did-am?", just like the grammatical form of the original sentence. Had we allowed for the application of the clitic rule to the derived types of the verb, rather than only to its original type, we would have been able to apply it to the type of the verb in the partial clitic sentence above for  $p^rq = o^r(s\overline{\pi}^l)$ , by taking  $p^r = o^r$  and  $q = s\overline{\pi}^l$  and obtaining  $q\overline{p}^l = s\overline{\pi}^l\overline{o}^l$ , which would have made the ungrammatical one-word sentence "did-ash-am" grammatical.

saw him I
\* did ash am
$$s \overline{\pi}^l \overline{o}^l = \overline{a} = \overline{\pi}$$

### 3.2 Locative Objects and Auxiliaries

One can introduce a locative object in the example above and utter *I saw him* in the library, in Persian "man u-ra dar-ketabkhaneh didam". There are no clitic pronouns for the locative object in Persian, so the *short-form* of this sentence is not one word any more. One can still move the pronouns and form shorter sentences, but in this case the locative object "dar-ketabkhaneh" stays before the verb and only the subject and object pronouns become post-verbal.

Similarly to the second example of partial clitic movement, this new type for the verb is not derivable from its original type by means of the clitic rule, since according to this rule, one can only move the types from the beginning or the end of the type string. There is no way to pull out the type s from the middle of  $(\lambda^r s \, \overline{o}^l \, \overline{\pi}^l)$  and move it to the end of the type string in  $(\lambda^r o^r \pi^r s)$ , while keeping the initial type  $\lambda^r$  untouched. If one insists on applying the rule, from  $(\lambda^r o^r \pi^r s)$  one obtains  $(s \overline{\lambda}^l \, \overline{o}^l \overline{\pi}^l)$ , which would only type check if there was a clitic pronoun for location in Persian, something similar to the y in French. But this is not the case in Persian. This may be again good news, because while there is no doubt that one word sentences like "did-am-ash" are grammatical, the two word ones "dar-ketabkhaneh did-am-ash" are controversial; in fact, it would be a hard job to find a Persian grammar book which claims that the latter is grammatical.

Its use is perhaps only justified as the short answer to the question Where did you see him?, in Persian "dar-koja did-i-ash?", which is itself a short form of the full question "To u-ra dar-koja didi?". If one does not agree with the above argument, one can use the partial version of the clitic rule, the in rule given above: we apply it only to the  $(o^r\pi^r s)$  part of the verb and obtain  $(s \, \overline{o}^l \, \overline{\pi}^l)$ , without touching the type  $\lambda^r$  at the beginning.

Persian sentences in perfect tenses are built by using auxiliary verbs and either the participle or the subjunctive form of the verb. A full analysis of the Persian tenses has been provided in [13]; we skip the details here and, in order to focus on clitic movement, we introduce the types p for the  $past\ participle$  and p for the p for the  $past\ participle$  and p for the  $past\ participle$  and p for the  $past\ participle$  and p for the p for

I him seen had seen had I him man u-ra dideh budam. dideh bud am ash. 
$$\pi$$
 o  $p$   $(p^r o^r \pi^r s) \to s$   $p$   $(p^r s \overline{o}^l \overline{\pi}^l) \overline{\pi} \overline{o} \to s$ 

The type of the verb of the second sentence is obtained from the verb type of the first one by applying the partial in rule: fix p and move the  $(o^r \pi^r) = (\pi o)^r$  from the left of s to its right, so it becomes  $(\overline{\pi o})^l = \overline{o}^l \overline{\pi}^l$ . As another example, consider the following sentence in present subjunctive form, involving the modal verb "mikhaham" (want) and the verb "bebinam"  $(I \ see)$  to express a wish. We treat the subjunctive form of the verb similarly to its infinitive. Like in French, this form needs an object but no external subject; it is the modal verb instead that takes the external subject and a subjunctive j to produce the sentence s.

I want him see want I see him man mikhaham u-ra bebinam. mikhah am bebinam ash. 
$$\pi$$
  $(\pi^r s j^l)$   $o$   $(o^r j) \rightarrow s$   $(s j^l \overline{\pi}^l)$   $\overline{\pi}$   $(j \overline{o}^l)$   $\overline{o} \rightarrow s$ 

Clitic movement takes an interesting form here: since the modal verb inputs the subject, clitic movement ends up into the form "mikhah-am":  $(\pi^r s j^l)$  becomes  $(s j^l \overline{\pi}^l)$ ; and since the subjunctive form of the verb inputs the object, objective clitic movement results in "bebinam-ash":  $(o^r j)$  becomes  $(j \overline{o}^l)$ .

### 3.3 An Insight into Cliticization in Arabic

We conclude this section by taking a brief look at Arabic. In general (not considering word order changes caused by putting into focus different parts of speech) Arabic is a VSO language. Consider the following transitive sentence

carries Hassan a book. yahmela Hassan ketaban. 
$$(so^l\pi^l)$$
  $\pi$   $o \to s$ 

where "yahmela" is the verb, "Hassan" is the subject, and "ketaban" is the object. In its short form, this sentence becomes just one word: "yahmel-a-ha", but the order remains the same VSO, see [27]3].

carries he it. yahmel a ha. 
$$(s \overline{o}^l \overline{\pi}^l) \overline{\pi} \overline{o} \rightarrow s$$

Here the vowel "a" stands for the clitic form of the subject pronoun and refers to "Hassan", "ha" is the clitic form of the object pronoun and refers to "ketaban". Since there is no movement, we need not use the clitic metarules. The clitic type of the verb is easily derivable from its original type by applying the postulate  $\overline{x} \leq x$ , the conventions about bars  $\overline{p} \overline{q} = \overline{p} \overline{q}$ , and by remembering that adjoints reverse the order  $x^l \leq \overline{x}^l$ . The derivation is as follows

$$so^l\pi^l = s(\pi o)^l \le s(\overline{\pi o})^l = s(\overline{\pi o})^l = s\overline{o}^l\overline{\pi}^l$$

### 4 Clitic Movement in French

In French, clitic clusters move in the opposite direction with respect to Persian. We will therefore invoke the second clitic metarule on the basis of which we derive the type of the clitic form of the verb from its original type. Let's start with a simple example, the sentence "Jean voit Marie" (Jean sees Marie) and its clitic form "Jean la voit". Following the Persian examples, we derive the type  $(\overline{o}^r\pi^rs)$  of the clitic form of the verb starting from its original type  $(\pi^rs\,o^l)$ : we take  $q=(\pi^rs)$ ,  $p^l=o^l$ , apply the clitic rule (2) and obtain  $(\pi^rs\,o^l)=(\pi^rs)\,o^l$   $\longrightarrow \overline{o}^r(\pi^rs)=(\overline{o}^r\pi^rs)$ . Here are the derivations for the two sentences

Jean voit Marie. Jean la voit. 
$$\pi \quad (\pi^r s \, o^l) \quad o \quad \to \mathbf{s} \qquad \qquad \pi \quad \overline{o} \quad (\overline{o}^r \pi^r s) \to \mathbf{s}$$

Consider a second simple example, this time with the locative object  $\lambda$  and its clitic pronoun  $\overline{\lambda}$ 

Again the clitic rule easily derives  $(\overline{\lambda}^r \pi^r s)$  from  $(\pi^r s \lambda^l)$ . Now consider the more complicated example "Jean donne une pomme à Marie." (Jean gives an apple to Marie), we type it as follows

Jean donne une pomme à Marie. 
$$\pi (\pi^r s w^l o^l)$$
  $o$   $w \to s$ 

While learning French at school, we always had difficulty in remembering the order of the clitic pronouns in this kind of sentences. Our clitic rule (2) offers a solution: according to it a verb of type  $(\pi^r s \, w^l \, o^l)$  can also be of type  $(\overline{w}^r \overline{o}^r \pi^r s)$ , taking  $q = (\pi^r s), p = (ow)^l$  and deriving  $(\pi^r s \, w^l \, o^l) = (\pi^r s)(w^l \, o^l) = (\pi^r s)(ow)^l$   $\rightarrow (\overline{ow})^r (\pi^r s) = (\overline{w}^r \overline{o}^r)(\pi^r s) = (\overline{w}^r \overline{o}^r \pi^r s)$ . This type for the clitic verb form allows one to obtain grammatical sentences like the following

Jean la lui donne.  

$$\pi \quad \overline{o} \quad \overline{w} \ (\overline{w}^r \overline{o}^r \pi^r s) \to s$$
,

but it will not allow the following incorrect order

Jean lui la donne 
$$\pi \overline{w} \overline{o} (\overline{w}^r \overline{o}^r \pi^r s)$$

We can also predict clitic movement within the sentence "Jean met une pomme sur la table" (Jean puts an apple on the table), assigning the type  $(\pi^r s \lambda^l o^l)$  to the original verb. Clitic rule (2) assures that this verb can also receive the type  $(\overline{\lambda}^r \overline{o}^r \pi^r s)$ , allowing the derivation of grammatical sentences like the following

but not of the following ungrammatical one

As a more complicated example, consider a sentence with a modal verb like "Jean peut donner une pomme à Marie" (Jean can give an apple to Marie), which we type as follows

Jean peut donner une pomme à Marie. 
$$\pi (\pi^r si^l) (iw^l o^l)$$
 o  $w \to s$ 

This time we apply the clitic rule to the infinitive of the verb  $(iw^lo^l)$  and obtain  $(\overline{w}^r\overline{o}^ri)$  for its clitic form. This types the following sentence

Jean peut la lui donner.  

$$\pi (\pi^r s i^l) \ \overline{o} \ \overline{w} (\overline{w}^r \overline{o}^r i) \to s$$
,

but not the following incorrect one

Jean peut lui la donner 
$$\pi$$
  $(\pi^r s i^l)$   $\overline{w}$   $\overline{o}$   $(\overline{w}^r \overline{o}^r i)$ 

The same technique of applying the rule to the type of the infinitive of the verb works for simpler sentences such as "Jean peut voire Marie".

In order to type clitic movement for sentences in the past tense like "Jean a donné une pomme à Marie" (Jean gave an apple to Marie), we apply the clitic rule to the verb phrase that is the combination of the auxiliary and the past-participle of the verb (see [2,9]). Having typed the sentence as follows

Jean a donné une pomme à Marie. 
$$\pi \quad (\pi^r s \, p^l) \; (p \, w^l o^l) \qquad o \qquad w \quad \to \; s$$

we need the type of "a donné", which is  $(\pi^r sp^l)(pw^l o^l) \to (\pi^r sw^l o^l)$ , after the contraction  $p^l p \leq 1$ ; then we take  $q = (\pi^r s)$ ,  $p^l = (ow)^l$ , and applying clitic rule (2) we obtain  $\overline{p}^r q = (\overline{ow})^r (\pi^r s) = (\overline{w}^r \overline{o}^r \pi^r s)$ . The clitic sentence is thus typed

Jean la lui a donnée.  

$$\pi \quad \overline{o} \quad \overline{w} \quad (\overline{w}^r \overline{o}^r \pi^r s) \rightarrow s$$

#### 4.1 Partial Clitic Movement

In the above, we have analyzed sentences where both objects are replaced with clitic pronouns. This need not always be the case. For instance our example sentence "Jean donne une pomme à Marie", can also turn into the sentence "Jean la donne à Marie" by partial clitic movement. The type  $(\overline{o}^r \pi^r s w^l)$  of the verb in the latter sentence is obtainable from the original type of the verb  $(\pi^r s w^l o^l)$  by applying clitic rule (2) as before, for  $q = (\pi^r s w^l)$  and  $p^l = o^l$ . However the same will not work for the other partial clitic sentence "Jean lui donne une pomme", where we need the type  $(\overline{w}^r \pi^r s o^l)$  for the verb. Similarly to the Persian case, to derive this type, we generalize clitic rule (2) as follows

For  $p, q \in P$ , if  $qp^l$  is in the *original* type of the verb then so is  $\overline{p}^r q$ .

With this version of the rule one can derive  $(\overline{w}^r \pi^r s o^l)$  from  $(\pi^r s w^l o^l)$ , by applying the rule to the  $(\pi^r s w^l)$  part of the type string. To do so, one takes  $q = (\pi^r s)$  and  $p^l = w^l$ ; after applying the in rule one obtains  $(\overline{w}^r \pi^r s)$ , to which one then glues the type  $o^l$  at the end of the type string.

A similar problem occurs with locative clitic movement in "Jean met une pomme sur la table". The partial clitic movement giving "Jean la met sur la table" works as before: the new type of the verb  $(\overline{o}^r \pi^r s \lambda^l)$  is derivable from its original type  $(\pi^r s \lambda^l o^l)$ . But for the other partial movement "Jean y met une pomme", one needs to use the [in] rule to obtain  $(\overline{\lambda}^r \pi^r s o^l)$ : we fix the  $o^l$  at the end of the type string, then apply the rule to the  $(\pi^r s \lambda^l)$  part deriving  $(\overline{\lambda}^r \pi^r s)$ .

Had we allowed the clitic rule to apply to non-original types of the verb, then we would have been able to apply the rule also to its derived types, for example those assigned to a sentence with a partial clitic movement. In the above examples these types would be  $(\overline{w}^r \pi^r s o^l)$  or  $(\overline{\lambda}^r \pi^r s o^l)$ . The application of the rule to these types would have derived the types  $(\overline{o}^r \overline{w}^r \pi^r s)$  and  $(\overline{o}^r \overline{\lambda}^r \pi^r s)$ , which would have respectively made the ungrammatical sentences "Jean lui la donne" and "Jean y la met" grammatical.

### 5 Clitic Movement in Italian

Clitic movement in Italian exhibits two general patterns: clitic pronouns can occur both in pre-verbal and post-verbal position, keeping the same relative order: locative/indirect object, direct object. Differently from Persian, subjects do not allow clitic counterparts and concerning the other verbal arguments, clitic

<sup>&</sup>lt;sup>7</sup> Two main patterns don't follow from the above rules for clitic movement in French: when the direct object is a mass noun, and when the indirect object is a first or second person pronoun. An example of the first is "Jean donne de la farine à Marie" with clitic form "Jean lui en donne", and of the second "Jean donne une pomme à nous", the clitic form of which is "Jean nous la donne". A number of papers show that these constructions can be obtained in the pregroup grammar of French, and similarly for the patterns occurring in Italian see e.g. [2]21]11].

unstressed elements can be attached both to the main verb or to auxiliaries and modal verbs. Therefore the set of types will include  $\overline{o}$  (direct object clitic),  $\overline{w}$  (indirect object clitic), and  $\overline{\lambda}$  (locative clitic). Sentences with one occurrence of a pre-verbal clitic can be obtained similarly to French, as shown in these examples corresponding to the French cases given above: "Gianni vede Maria" and its clitic form "Gianni la vede".

Gianni vede Maria. Gianni la vede. 
$$\pi \quad (\pi^r s \, o^l) \quad o \quad \to s \qquad \qquad \pi \quad \overline{o} \, (\overline{o}^r \pi^r s) \to s$$

To derive the clitic type of the verb we start with the original type  $(\pi^r s \, o^l)$ , take  $q = (\pi^r s), \, p^l = o^l$ , apply clitic rule (2) and obtain the type  $(\overline{o}^r \pi^r s)$ . The same process applies with a locative argument  $\lambda$  and the corresponding clitic pronoun  $\overline{\lambda}$ , where the clitic rule derives  $(\overline{\lambda}^r \pi^r s)$  from  $(\pi^r s \lambda^l)$ .

Gianni va a Roma. Gianni ci va. 
$$\pi \quad (\pi^r s \, \lambda^l) \quad \lambda \quad \to s \qquad \qquad \pi \quad \overline{\lambda} \; (\overline{\lambda}^r \pi^r s) \to s$$

When considering a verb with two arguments like in "Gianni da un libro a Maria" (Gianni gives a book to Maria), or "Gianni mette un libro sul tavolo" (Gianni puts a book on the table), we find that clitics pronouns occur in the opposite order with respect to French e.g. "dare" (to give) has the clitic form "Gianni glie lo da", and "mettere" (to put) the clitic form "Gianni ce lo mette". In [9] this problem was handled by assigning verbs of type  $(\pi^r s \, o^l w^l)$ , a second type in which the arguments order is reversed:  $(\pi^r s \, o^l w^l)$  we will follow the same line here and show that from the verb type  $(\pi^r s \, o^l w^l)$ , all the pre-verbal clitic patterns follow, while the post-verbal patterns follow from the type  $(\pi^r s \, w^l \, o^l)$ . The advantage of the present analysis is that we will make use only of the clitic metarules and of some order postulates. Applying clitic rule (2) to the new types we obtain the correct clitic verb forms to handle the cases of pre-verbal cliticization mentioned above:  $(\pi^r s \, o^l w^l) = (\pi^r s)(wo)^l \rightsquigarrow (\overline{wo})^r (\pi^r s) = (\overline{o}^r \, \overline{w}^r s)$ , and the same with  $\lambda$  in place of w.

Gianni glie lo da. Gianni ce lo mette. 
$$\pi \quad \overline{w} \quad \overline{o} \quad (\overline{o}^r \overline{w}^r \pi^r s) \to s \qquad \qquad \pi \quad \overline{\lambda} \quad \overline{o} \quad (\overline{o}^r \overline{\lambda}^r \pi^r s) \to s$$

The following diagram shows the general pattern of preverbal cliticization in Italian with a verb taking two arguments:

<sup>&</sup>lt;sup>8</sup> For the analysis of *clitic climbing* in Italian and French we refer to 22111.

<sup>&</sup>lt;sup>9</sup> In French the order of the accusative and dative/locative third person clitic pronouns is reversed with respect to Italian and other Romance languages such as Spanish or Portuguese; but the order is similar in the case of first or second person pronouns; see [21], pp. 77-79.

Similar types, that we can name *prototypes*, are admitted by other languages such as English where you find sentences like "John gives Mary a kiss" or "John took to the station the bag", and we can consider them as reminiscent of the basic SOV order holding in Latin or other Indo-European languages; see 31.

I (nom) you (dat) it (acc) say io te lo dico 
$$\pi$$
  $\overline{w}$   $\overline{o}$   $(\overline{o}^r \overline{w}^r \pi^r s)$ 

To get the post-verbal clitic forms we do not need apply the clitic metarules; in the case of transitive verbs such as "vedere" (to see), or motion verbs such as "andare" (to go), the post-verbal clitic forms are obtained from the original types (io<sup>l</sup>), (i $\lambda$ <sup>l</sup>) assuming the order conditions:  $\overline{o} \leq o$ ,  $\overline{\lambda} \leq \lambda$ , on the basis of which an argument of type  $\overline{o}$  is also of type o, and the same for  $\overline{\lambda}$  [17]9].

Gianni vuole veder.la . Gianni vuole andar.ci . 
$$\pi \quad (\pi^r s \, i^l) \quad (i \, o^l) \quad \overline{o} \, \to \mathbf{s} \qquad \qquad \pi \quad (\pi^r s \, i^l) \quad (i \, \lambda^l) \ \overline{\lambda} \, \to \mathbf{s}$$

Similarly the post-verbal clitic forms with two arguments follow from the types introduced above, by means of the same order conditions: e.g. assigning the type  $(\pi^r s \, o^l w^l)$  to the verb "dare"  $(to \, give)$  it applies to the clitic types  $\overline{o}, \, \overline{w}$ , under the order conditions  $\overline{o} \leq o, \, \overline{w} \leq w$ ; the same with the verb type  $(\pi^r s \, o^l \lambda^l)$  and the condition  $\overline{\lambda} \leq \lambda$ . The following diagram gives a general picture of the post-verbal construction with a verb taking two arguments, cf.  $\square$ .

I (nom) want say you (dat) it (acc) io voglio dir te lo 
$$\pi$$
 ( $\pi^r$  s  $j^r$ ) ( $i$   $o^l$   $w^l$ )  $\overline{w}$   $\overline{o}$ 

#### 5.1 Partial Clitic Movement

Also Italian allows partial clitic movement in sentences with two arguments predicates, like the verb "dare" (to give) considered above. These sentences are characterized by the presence of a full argument and a clitic pronoun that can occur in pre-verbal or in post-verbal position. The preverbal cases are like in the following examples where the clitic pronoun is the *indirect* object in the first case and the *direct* object in the second

Gianni gli da un libro. Gianni lo da a Maria. 
$$\pi \overline{w} (\overline{w}^r \pi^r s o^l) o \to s \qquad \pi \overline{o} (\overline{o}^r \pi^r s w^l) w \to s$$

To derive the first sentence we choose the type  $(\pi^r s \, o^l w^l)$  for the verb "dare" and take  $p^l = w^l$ ,  $q = (\pi^r s \, o^l)$ , then by applying clitic rule (2) we obtain  $p^r q = (\overline{w}^r)(\pi^r s \, o^l) = (\overline{w}^r \, \pi^r s \, o^l)$ . For the second case of pre-verbal cliticization we need to invoke the [in] rule given above for French, the content of which is: if a type string  $qp^l$  occurs somewhere inside the *original* type of a verb, it can be replaced by a type string  $\overline{p}^r q$  and the resulting type is considered as another type for that verb. In this case the string in question is  $(\pi^r s \, o^l)$  where q is  $(\pi^r s)$  and  $p^l$  is

 $o^l$ ; the resulting type string is  $(\overline{o}^l\pi^r s) w^l = (\overline{o}^l\pi^r s w^l)$  that allows the preverbal clitic to apply on the left side of the verb and the indirect argument to apply to the other side. Finally, post-verbal partial clitic movement is so exemplified

Gianni vuole dar.gli un libro. Gianni vuole dar.lo a Maria. 
$$\pi$$
  $(\pi^r s i^l) (io^l w^l) \overline{w}$   $o \to s$   $\pi$   $(\pi^r s i^l) (iw^l o^l) \overline{o} w \to s$ 

The first sentence follows directly from the verb type  $(io^l w^l)$  and the order condition  $\overline{w} \leq w$ : we interpret this result as meaning that a post-verbal clitic occurs in situ. The second sentence is more critical, since we cannot derive it from the same type. However, it make sense to derive it from the other type of the Italian two-arguments verb, that is  $(i \, w^l \, o^l)^{\square}$ . Observe that if we choose this type, we easily derive the type of "vuole dar", i.e.  $(\pi^r s \, w^l \, o^l)$  by the  $(i^l \, i)$  contraction, via the order postulate  $\overline{o} < d \overline{\square}$ .

### 5.2 Application of Cyclic Rules to Other Word Types

We wish to shortly address a number of distinctive properties of cliticization in Romance languages, although a detailed analysis of these problems goes far behind the length and goal of this paper. Of central interest are the cases of *clitic climbing*, where a clitic attaches to an auxiliary or a modal verb, or appears on the main verb, but semantically belongs to a subordinate clause, as in "Marie la lui laisse manger" (*Marie it to him allowed to eat*), where "Marie laisse la lui manger" is ungrammatical. As pointed out by one referee, there are differences between languages, e.g. climbing is obligatory in French, while is optional in Italian, depending on the triggers and on the possibility of applying alternatively post-verbal cliticization.

There are also non clausal structures, where cliticization applies to some phrase within the noun phrase that is the direct object, eg. "Marie en connaît la fin" i.e. "la fin du livre" (*Marie of-it knows the end - of the book*). These are cases of partitive constructions, with "en" in French, or "ne" in Italian.

All these cases can been successfully treated in the context of pregroup grammar (see e.g. [21]7]11]), invoking the long distance cancellations allowed by double adjoints, the dependency restrictions imposed on the partial order and, more recently, the featural architecture designed by the parallel computations of two or more pregroups [21]8]. In our context, these constructions can be accounted for by allowing clitic rule (2) to apply to extended infinitives involving auxiliaries, modals, and predicates taking e.g. nominal complements. As an example, consider the following application of the rule to the Italian string "dovere dare" ( $must\ give$ ) formed by the modal verb and the infinitive of the main verb (extended infinitive), where  $C_{11}$  is an operator (Inflector) introduced for computing verbal inflection like in

<sup>&</sup>lt;sup>11</sup> Post-verbal attachment appeared later in the development of modern Italian, see [31].

An alternative solution is to assign types  $(\pi^r s w^l \overline{o}^l)$ ,  $(\pi^r s o^l \overline{w}^l)$  to the contracted forms like dar, allowing composition just with clitic arguments  $\overline{o}$ ,  $\overline{w}$ , as in  $\underline{\Theta}$ .

Obviously restrictions are needed to avoid ungrammatical results due to repeated applications of the clitic metarules. For example, as suggested by the referee, when a clitic attaches to verbal auxiliaries like in "la lui a donnée", to allow that the clitic rule (2) apply to the outcome of the cancellation of the type strings for the auxiliary "a" and the participle "donnée", we need excluding it having been applied before. It is observed that allowing such applications of clitic metarules to extended infinitives is contradicted by cases where some expression intervenes between the auxiliary and the participle, like in "Marie les a tous vus", in French, or "Maria li ha tutti visti", in Italian, obtained from sentences like "Marie a vu tous les livres" (Mary has seen all the books). We think that these cases are rather marginal in the languages considered, due to discourse functions like emphasis, although they may have grammatical counterparts e.g. in English (Mary has seen them all). However, if you want to obtain them in the context of a free pregroup grammar, you can follow different strategies (some are discussed in  $\Pi$ ), and a promising one is that of taking advantage of the parallel computations allowed by a multiple pregroup grammar where e.g. one free pregroup takes care of syntactic composition, and a second pregroup will compute features relations (see 21,29,8).

## 6 Clitic Rules and Cyclic Pregroups

Following Lambek [16]17]18, we have formulated the clitic rules as metarules. At some risk of overgeneration, one is tempted to formulate these rules as axioms and add them to the pregroup calculus (or add their rule version to the sequent calculus of compact bilinear logic [6]). Its worth pointing out that the addition of cyclic axioms (or rules) is not equivalent to the reintroduction of the structural rule of Commutativity into the pregroup calculus (a logic without structural rules like the Syntactic Calculus) [13]. These axioms belong to the cyclic calculus studied by Abrusci [1] and introduced in the following way

$$\frac{\vdash \varGamma, \varDelta}{\vdash \varDelta^{+2}, \varGamma}(rr) \qquad \qquad \frac{\vdash \varGamma, \varDelta}{\vdash \varDelta, \varGamma^{-2}}(ll)$$

Via the standard translation from the Syntactic Calculus to pregroups [17]5] (positive formulae as right adjoints and negative formulae as left adjoints), the axiomatic version of these rules becomes

$$(1) qp \le pq^{ll} \qquad (2) qp \le p^{rr}q$$

An approach in this line is proposed by [12], where a free pregroup grammar is extended by a finite set of additional (commutative) inequations between types, leading to a class of mildly context-sensitive languages, allowing the analysis of crossed dependencies and extractions.

We can refer to (1) and (2) as *cyclic axioms*, in particular to the first one as the *left cyclic axiom* and to the second one as the *right cyclic axiom*. We can then re-formulate our clitic *metarules* as *clitic axioms* 

Persian 
$$p^r q \leq q \overline{p}^l$$
 French-Italian  $q p^l \leq \overline{p}^r q$ 

where the latter is derivable from the former, and prove the following results:

**Proposition 1.** The clitic axioms are derivable from the cyclic axioms.

**Proof.** The axiom for French and Italian is derivable form the right cyclic axiom as follows, take p to be  $p^l$  and observe that  $(p^l)^{rr} = p^r$ , then one obtains  $qp^l \le p^rq$ . Since  $\overline{p} \le p$ , and since adjoints are contravariant, we have  $p^r \le \overline{p}^r$ , thus  $p^rq \le \overline{p}^rq$ , and by transitivity of order we obtain  $qp^l \le \overline{p}^rq$ . The axiom for Persian is derivable from the left cyclic axiom as follows: take q to be  $p^r$  and p to be q. Now since  $(p^r)^{ll} = p^l$ , we obtain  $p^rq \le qp^l$ , and since  $\overline{p} \le p$ , by contravariance,  $p^l \le \overline{p}^l$ , thus  $qp^l \le q\overline{p}^l$ , and by transitivity of order  $p^rq \le q\overline{p}^l$ .

It is interesting that the rules for clitic movement correspond to logical rules of cyclicity. Accordingly, one may call French and Italian *right cyclic* languages and Persian a *left cyclic* language. The consequences of enriching a pregroup with these cyclic axioms (or rules) are however not so desirable.

**Proposition 2.** A pregroup P with either of the cyclic axioms is a partially ordered group.

**Proof.** Consider the left cyclic axiom; if one takes q=1, we obtain  $p^r \leq p^l$  for all  $p \in P$ , from which one obtains  $p^{ll} \leq p$ . Here take  $p=w^r$  for some  $w \in P$  and obtain  $w^l \leq w^r$ . Now since we have  $p^r \leq p^l$  for all p, we obtain  $w^r = w^l$ . A similar argument can be made for the right cyclic axiom.

Although, as proven by Abrusci and Lambek, cyclic bilinear logic is a conservative extension of bilinear logic (or non-commutative linear logic), this is not the case for cyclic compact bilinear logic and compact bilinear logic (the logical calculus of pregroups) [5]4]20]. The relations among these system are however of real interest to be studied both from the logical and, particularly, from the linguistic point of view.

We conclude observing that the present analysis is consistent with previous work on French [2] and Italian [9], where *iterated* adjoints are used to type clitic pronouns. We can prove in fact that iterated adjoints show up in our work too, since as observed by Lambek, the  $\overline{p}^r$  used in the metarule for French and Italian is nothing but  $(\overline{p}^l)^{rr}$ , and the  $\overline{p}^l$  used for Persian is nothing but  $(\overline{p}^r)^{ll}$ .

### 7 Conclusions

The paper presents a unified approach to clitic movement in Persian, French and Italian. Two general rules accounting for this movement have been introduced in the framework of pregroup grammar and applied to several examples, also

checking that they do not produce ungrammatical strings. A general pattern of clitic movement has then be defined for these languages: French and Italian result as *right-cyclic* languages, whereas Persian results as a *left cyclic* language. That is, the clitics in French and Italian move in the opposite direction with respect to those in Persian. Also the analysis of sentences with partial clitic movement has been successfully performed, again using a uniform pattern of partial applications of the rules.

We have given arguments for introducing these rules as *metarules* that can only apply to specific type strings, associated with the finite or infinitive forms of the verbs, that we call *original* types, and we have given reasons why one shouldn't add these rules as extra axioms to the pregroup or as extra rules to its calculus, since such additions would reduce the pregroup to a partially ordered group. We hope that the present analysis, focusing on a set of kernel examples in a cross-linguistic perspective, will be useful in stimulating further investigations in the field.

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