0.1 Homework 2

The influence of back-lash in a control system will be discussed in this homework. Consider the drive line in a crane, where torque is generated by an electric motor, transformed through a gearbox, and finally used to lift a load. A block diagram describing the control of the angular position (aka, angular displacement) of the lifting axis θ_{out} is shown below to the left. Here the first block represents the P-controller ($G_{e\to V} = K > 0$), the second the dynamics in the motor ($G_{V\to\omega_{in}} = \frac{1}{Ts+1}$ with $T > 0$), and the third the back-lash between the gear teeth in the gearbox (there is a dynamic relation between the angular velocity of the driving gear and the angular displacement of the driven gear). The backlash model gives a dynamic relation between the angular

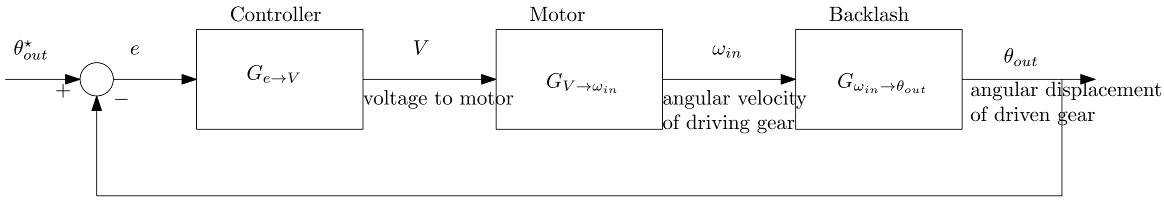

Figure 1: Block diagram

displacement of the driving gear (aka θ_{in}) and the angular displacement of the driven gear (aka θ_{out}). Another possibility is to model the backlash as the relation between the corresponding angular velocities ω_{in} and ω_{out}. In this modeling approach,

$$\dot{\theta}_{out} = \omega_{out}(\theta_{out}, \theta_{in}, \omega_{in}) = \begin{cases} \omega_{in} & \text{if contact}(\theta_{out}, \theta_{in}, \omega_{in}) \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where "in contact" corresponds to that $|\theta_{\text{in}} - \theta_{\text{out}}| \geq \Delta$ and $\dot{\theta}_{\text{in}}\,(\theta_{\text{in}} - \theta_{\text{out}}) > 0$. You may take $\Delta = 1$ degree, which means the driving gear can move 1 degree, while the drived gear stays at rest (experiment with other values for Δ if necessary).

1. [1p] Consider the back-lash model (1) described as a block, Assume

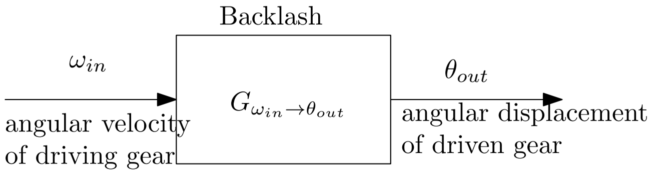

Figure 2: Backlash model

$$\dot{\theta}_{\text{in}}(t) = \begin{cases} 1, & t \in [0, 1] \\ -1, & t \in [1, 2] \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

is the input to BL (in open-loop). Sketch θ_{in}, θ_{out}, $\dot{\theta}_{\text{in}}$, and $\dot{\theta}_{\text{out}}$ for $\theta_{\text{in}}(0) = 0$ and $\theta_{\text{out}}(0) = -\Delta$ in a diagram.

2. [1p] Show that the gain of BL is equal to $\gamma(G_{\omega_{in}\to\theta_{out}}) = 1$. Show that BL is passive. Motivate why BL can be bounded by a sector $[k_1, k_2] = [0, 1]$.[1]

[1] You may argue that BL as defined here is not a memoryless nonlinearity, which is required for the application of the Circle Criterion in the lecture notes.It is, however, possible to circumvent this problem. If you want to know how, check Theorem (126) on page 361 in Vidyasagar (1993).

3. [1p] Consider the back-lash model BL in a feedback loop, Here d_{in} and d_{out} represent disturbances. Assume

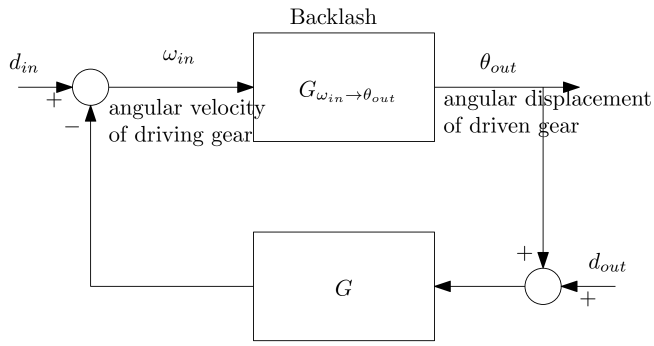

Figure 3: Backlash in closed loop

that $G(s)$ is an arbitrary transfer function (that is, not necessarily the one given by the crane control problem discussed previously).

a. Given $\gamma(G_{\omega_{in}\to\theta_{out}})$ derived in Exercise **2**, which constraints are imposed on $G(s)$ by the Small Gain Theorem in order to have a BIBO stable closed-loop system (from $(d_{\text{in}}, d_{\text{out}})$ to $(\dot\theta_{\text{in}}, \dot\theta_{\text{out}}) = (\omega_{\text{in}}, \omega_{\text{out}})$)?

b. Which constraints are imposed on $G(s)$ by the Passivity Theorem in order to have a BIBO stable closed-loop system?

c. Which constraints are imposed on $G(s)$ by the Circle Criterion in order to have a BIBO stable closed-loop system?

4. [1p] For the crane control system the transfer function in Exercise **3** is equal to $(G_{V\to\omega_{in}}(s) = \frac{K}{1+sT})$. For the driving gear angular displacement, the transfer function is

$$G_{V\to\theta_{in}}(s) = \frac{K}{s(1+sT)}.$$

Motivate why BIBO stability cannot be concluded from the Small Gain Theorem or the Passivity Theorem. Let $T = 1$ and determine for which $K > 0$ the Circle Criterion leads to BIBO stability.

5. [1p] Simulate the crane control system in Simulink. Download the Simulink model `hw2.mdl` and the short macro `macro.m` from the course homepage to your current directory and start Matlab. Open the Simulink model by running

```
> hw2
```

and compare with the block diagrams in this homework. What disturbance is added in `hw2`? What Δ is chosen? Simulate the system with the controller $K = 0.25$ by running

```
> K=0.25; macro
```

Does it seem like the closed-loop system is BIBO stable from d_{in} to $(\dot\theta_{\text{in}}, \dot\theta_{\text{out}})$? Why cannot the closed-loop system be BIBO stable from d_{in} to $(\theta_{\text{in}}, \theta_{\text{out}})$? Try other controller gains (for instance, $K = 0.5$ and $K = 4$). Compare with your conclusions in Exercise **4**. Compare your results to the case when the back-lash is neglected.