



 $\begin{array}{c} \text{Attitude} \\ \text{Synch. in } \mathcal{S}^2 \end{array}$ 

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Introduction

D:-----

Function

Control

Doculto

Simulation

Summan

# Family of Controllers for Attitude Synchronization in $S^2$

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### Summary

 $\begin{array}{c} \text{Attitude} \\ \text{Synch. in } \mathcal{S}^2 \end{array}$ 

Pereira & Dimarogonas

Introduction

Distance

COILLIOI Law

Results

Simulation

Summa

#### Attitude Synchronization in $\mathcal{S}^2$

- Torque control (constrained torque)
- ② Distributed control laws
- Only local information
- Multiple equilibria



### Objective

 $\begin{array}{c} \text{Attitude} \\ \text{Synch. in } \mathcal{S}^2 \end{array}$ 

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Introduction

.....

Problem

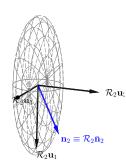
Function

Control

D 1.

Simulation Summary

 $\mathcal{R}_2\mathbf{u}_2$ 





(a)



## Objective

 $\begin{array}{c} \text{Attitude} \\ \text{Synch. in } \mathcal{S}^2 \end{array}$ 

Pereira & Dimarogonas

Introducti

Problem

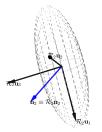
Distance

Control

Б 1.

Simulation

Summa





#### Objective

Given fixed  $\bar{\mathbf{n}}_1, \bar{\mathbf{n}}_2 \in \mathcal{S}^2$ , denote  $\mathbf{n}_1 = \mathcal{R}_1 \bar{\mathbf{n}}_1$  and  $\mathbf{n}_2 = \mathcal{R}_2 \bar{\mathbf{n}}_2$ .

Goal:  $\lim_{t\to\infty} \mathbf{n}_1(t) = \lim_{t\to\infty} \mathbf{n}_2(t)$ 



### Problem Statement

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Dimarogonas

Problem

#### Definition

(Incomplete) synchronization: when  $\mathbf{n}_1 = \cdots = \mathbf{n}_N$ 

#### Problem Statement

Given  $\{\bar{\mathbf{n}}_i\}_{i=1,\cdots,N}$ ,

$$\dot{\mathcal{R}}_i = \mathcal{R}_i \mathcal{S}(oldsymbol{\omega}_i),$$

$$\frac{d}{dt}\left(\mathcal{R}_i J_i \boldsymbol{\omega}_i\right) = \mathcal{R}_i \mathbf{T}_i,$$

design distributed torques  $\{T_i\}_{i=1,\dots,N}$  that guarantee that synchronization is asymptotically reached.



W. Song, X. Hu, et al. Distributed Control for Intrinsic Reduced Attitude Formation with Ring Inter-Agent Graph, CDC 2015



### Distance function in $S^2$

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Distance **Functions** 

#### Distance function $d(\mathbf{n}_1, \mathbf{n}_2)$

If

• 
$$d(\mathbf{n}_1, \mathbf{n}_2) > 0$$
,  $\mathbf{n}_1 \neq \mathbf{n}_2$ 

• 
$$d(\mathbf{n}_1, \mathbf{n}_2) = 0$$
,  $\mathbf{n}_1 = \mathbf{n}_2$ 

$$\bullet \ \mathcal{S}(\mathbf{n}_1) \frac{\partial d(\mathbf{n}_1, \mathbf{n}_2)}{\partial \mathbf{n}_1} = -\mathcal{S}(\mathbf{n}_2) \frac{\partial d(\mathbf{n}_1, \mathbf{n}_2)}{\partial \mathbf{n}_2},$$

Then

$$\dot{d}(\mathbf{n}_1, \mathbf{n}_2) = \underbrace{\begin{bmatrix} \boldsymbol{\omega}_1 \\ \boldsymbol{\omega}_2 \end{bmatrix}}^T \underbrace{\begin{bmatrix} \mathcal{R}_1^T & \mathbf{0} \\ \mathbf{0} & \mathcal{R}_2^T \end{bmatrix}}_{\boldsymbol{\mathcal{R}}^T} \underbrace{\left( \begin{bmatrix} 1 \\ -1 \end{bmatrix} \otimes \mathbf{I} \right)}_{B \otimes \mathbf{I}} \underbrace{\mathcal{S}(\mathbf{n}_1) \frac{\partial d(\mathbf{n}_1, \mathbf{n}_2)}{\partial \mathbf{n}_1}}_{\mathbf{e}}$$



### Distance function in $\mathcal{S}^2$

Attitude Synch. in  $S^2$ 

Pereira & Dimarogonas

Introduction

Distance Functions

Function

Results

Simulation

Summa

### Distance function $d(\mathbf{n}_1, \mathbf{n}_2)$

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• 
$$d(\mathbf{n}_1, \mathbf{n}_2) > 0$$
,  $\mathbf{n}_1 \neq \mathbf{n}_2$ 

• 
$$d(\mathbf{n}_1, \mathbf{n}_2) = 0$$
,  $\mathbf{n}_1 = \mathbf{n}_1$ 

• 
$$S(\mathbf{n}_1) \frac{\partial d(\mathbf{n}_1, \mathbf{n}_2)}{\partial \mathbf{n}_1} = -S(\mathbf{n}_2) \frac{\partial d(\mathbf{n}_1, \mathbf{n}_2)}{\partial \mathbf{n}_2},$$

Then

$$\sum_{k=1}^{k=M} \dot{d}_k(\mathbf{n}_{k_1},\mathbf{n}_{k_2}) = oldsymbol{\omega}^T oldsymbol{\mathcal{R}}^T \left( B \otimes \mathbf{I} 
ight) \mathbf{e}$$

$$(\mathbf{n}_1)$$
  $d_1(\mathbf{n}_1,\mathbf{n}_2)$   $(\mathbf{n}_2)$   $d_2(\mathbf{n}_2,\mathbf{n}_3)$   $(\mathbf{n}_3)$ 



### Distance function in $\mathcal{S}^2$

Attitude Synch. in  $S^2$ 

Pereira & Dimarogonas

Introduction

Problem

Distance Functions

00111101

Results

Simulation

Summa

#### Distance function $d(\mathbf{n}_1, \mathbf{n}_2)$

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• 
$$d(\mathbf{n}_1, \mathbf{n}_2) > 0$$
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,  $\mathbf{n}_1 = \mathbf{n}_2$ 

$$\bullet \ \mathcal{S}(\mathbf{n}_1) \frac{\partial d(\mathbf{n}_1, \mathbf{n}_2)}{\partial \mathbf{n}_1} = -\mathcal{S}(\mathbf{n}_2) \frac{\partial d(\mathbf{n}_1, \mathbf{n}_2)}{\partial \mathbf{n}_2},$$

Then, solution of the PDE is

$$d(\mathbf{n}_1, \mathbf{n}_2) = f(1 - \mathbf{n}_1^T \mathbf{n}_2),$$

where  $f:[0,2]\mapsto \mathbb{R}_{>0}$  and f(0)=0.



### Distance function in $\mathcal{S}^2$

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Introduction

....

#### Distance Functions

Function

Control I

Danula

Simulations

Summa

#### Distance function $d(\mathbf{n}_1, \mathbf{n}_2)$ :

$$d(\mathbf{n}_1, \mathbf{n}_2) = f(1 - \mathbf{n}_1^T \mathbf{n}_2).$$

Edge 
$$k$$
 error  $(\mathbf{e} = \begin{bmatrix} \mathbf{e}_1^T & \cdots & \mathbf{e}_M^T \end{bmatrix}^T)$ 

$$\bullet \ \mathbf{e}_k = \mathcal{S}(\mathbf{n}_1) \frac{\partial d_k(\mathbf{n}_1, \mathbf{n}_2)}{\partial \mathbf{n}_1} = -f_k' (1 - \mathbf{n}_1^T \mathbf{n}_2) \mathcal{S}(\mathbf{n}_1) \mathbf{n}_2.$$

$$\bullet \mathbf{e}_k = \mathbf{0} \Rightarrow \mathbf{n}_1 || \mathbf{n}_2$$



### Proposed control law

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Introductio

Distance

Functions

Results

Simulation

Summa

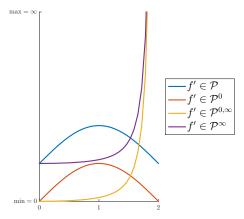


Figure : Classes of  $f':[0,2]\mapsto \mathbb{R}_{\geq 0}$   $\left(d^{\max}=\max_{s\in[0,2]}f(s)\right)$ 



### Proposed control law

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Introduction

\_\_\_\_

Distance Function

Control Law

Results

Simulations

Summa

### Proposed control law

$$\mathbf{T}_i = -\boldsymbol{\sigma}(\boldsymbol{\omega}_i) - \sum_{j \in \mathcal{N}_i} f'_{ij} (1 - \bar{\mathbf{n}}_i^T \mathcal{R}_i^T \mathbf{n}_j) \mathcal{S}(\bar{\mathbf{n}}_i) \mathcal{R}_i^T \mathbf{n}_j,$$

$$\mathbf{T} = -\boldsymbol{\sigma}(\boldsymbol{\omega}) - \boldsymbol{\mathcal{R}}^T(B \otimes \mathbf{I})\mathbf{e},$$

#### Proposed control law

$$\mathbf{T}_i = -\boldsymbol{\sigma}(\boldsymbol{\omega}_i) - \sum_{j \in \mathcal{N}_i} f'_{ij} (1 - \bar{\mathbf{n}}_i^T \mathcal{R}_i^T \mathbf{n}_j) \mathcal{S}(\bar{\mathbf{n}}_i) \mathcal{R}_i^T \mathbf{n}_j,$$

$$\mathbf{T} = -\boldsymbol{\sigma}(\boldsymbol{\omega}) - \boldsymbol{\mathcal{R}}^T(B \otimes \mathbf{I})\mathbf{e},$$

#### Lyapunov function



### Proposed control law

 $\begin{array}{c} \text{Attitude} \\ \text{Synch. in } \mathcal{S}^2 \end{array}$ 

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Introduction

....

Distance

Functions

Control Law

Simulation

#### Analysis

$$\dot{V}(\omega) \to 0 \Rightarrow \omega \to 0 \Rightarrow \dot{\omega} \to 0 \Rightarrow \mathrm{T} \to 0 \Rightarrow (B \otimes \mathbf{I})\mathbf{e} \to \mathbf{0}$$

#### Analysis

- $\bullet \ \, \text{Tree graph: } e \to 0$
- In general:  $\mathbf{e} \to \mathcal{N}(B \otimes \mathbf{I})$



## Proposed control law: Constrained Torque

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Introduction

Distance Function:

Control Law

Simulations

### Proposed control law

#### Lyapunov function & Analysis: $\bar{\mathbf{n}}_i$ principal axis

$$\dot{V}(\omega) = -\sum_{l=1}^{l=N} \omega_l^T \sigma(\omega_l)^{m{\sigma}} (\Pi(ar{\mathbf{n}}_l)\omega_l)$$

$$\dot{V} \to 0 \Rightarrow \Pi(\bar{\mathbf{n}}_i)\omega_i \to 0 \Rightarrow \Pi(\bar{\mathbf{n}}_i)\dot{\omega}_i \to 0 \Rightarrow$$
  
 $\Rightarrow \mathbf{T}_i \to (\bar{\mathbf{n}}_i^T\omega_i)^2 \mathbf{S}(\bar{\mathbf{n}}_i) J_i \bar{\mathbf{n}}_i \Rightarrow (B \otimes \mathbf{I})\mathbf{e} \to \mathbf{0}$ 



### Result: Tree Graphs

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$$d_{\min} := \min_{k \in \{1,\cdots,M\}} (d_k^{\max})$$

Introduction

Distance

Function

Control I

Results

Simulations

Summ

#### Result (Tree Graph)

If  $H(0) \leq d_{\min}$  and if, for all  $k \in \{1, \cdots, M\}$ ,

- $f'_k \in \bar{\mathcal{P}}$
- $d_k(\mathbf{n}_{k_1}, \mathbf{n}_{k_2})|_{t=0} \le \frac{1}{M} (d_{\min} H(0))$

then synchronization is asymptotically reached.

#### Result

In tree graph, if  $f'_k \in \mathcal{P}^{\infty} \cup \mathcal{P}^{0,\infty}$ , then synchronization is asymptotically reached for almost all initial conditions.



### Result

Attitude Synch. in  $S^2$ 

Introduction

Distance

Function

Control

Results

Simulation

Summa

# $d^{\star} := \min_{k} f_{k} \left( \min_{k} f_{k}^{-1} \left( \min_{k} f_{k} \left( \frac{\pi}{2} \frac{1}{N-1} \right) \right) \right)$

#### Result

If  $H(0) \leq d^*$  and, for all  $k \in \{1, \dots, M\}$ ,

- $f'_k \in \bar{\mathcal{P}}$
- $d_k(\mathbf{n}_{k_1}, \mathbf{n}_{k_2})|_{t=0} \leq \frac{1}{M} (d^* H(0))$

then synchronization is asymptotically reached for almost all initial conditions.

#### Remark:

$$f_k\left(\frac{\pi}{2}\frac{1}{N-1}\right) \le d_k^{\max} \Rightarrow d^* \le d_{\min}$$



# Equilibria solutions: $\mathbf{e} \in \mathcal{N}(B \otimes \mathbf{I})$

 $\begin{array}{c} \text{Attitude} \\ \text{Synch. in } \mathcal{S}^2 \end{array}$ 

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Introduction

Distance

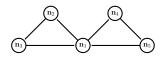
Functions

Control La

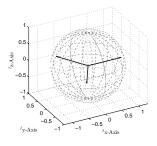
#### Results

Simulations

Summa



(a) No shared edges between cycles



(b) Planar unit vectors for each cycle



## Equilibria solutions: $\mathbf{e} \in \mathcal{N}(B \otimes \mathbf{I})$

 $\begin{array}{c} \text{Attitude} \\ \text{Synch. in } \mathcal{S}^2 \end{array}$ 

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Introduction

D:-----

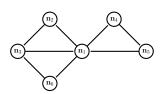
Functions

Control La

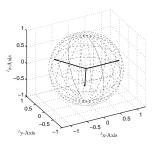
Results

Simulations

Summar



(c) Cycles that share only one edge



(d) Planar unit vectors



### Simulations

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Introductio

\_\_\_\_

Distance

Functions

Control La

Results

Simulations

Summary

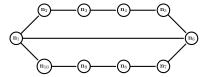


Figure : N=10, M=10



### Simulations

 $\begin{array}{c} {\sf Attitude} \\ {\sf Synch. in} \ \mathcal{S}^2 \end{array}$ 

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Introduction

5 11

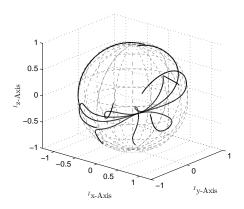
Distance

Function

Results

Simulations

Summary





### Summary

 $\begin{array}{c} \text{Attitude} \\ \text{Synch. in } \mathcal{S}^2 \end{array}$ 

Pereira & Dimarogonas

Introduction

Distance

Functions

Control Lav

Simulations

Summary

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- Torque control (constrained torque)
- ② Distributed control laws
- Only local information
- Multiple equilibria
- $\odot$  Distance functions in  $\mathcal{S}^2$



 $\begin{array}{c} \text{Attitude} \\ \text{Synch. in } \mathcal{S}^2 \end{array}$ 

Pereira & Dimarogonas

Introduction

Problem

Distance

Cambuall

Reculte

Simulations

Summary





Thank you! Questions?