Control of Thrust Propelled Systems

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KTH Royal Institute of Technology

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Background and Biographical Information

Biographical Information

1. 2008-2013: MSc in Aerospace Engineering
   (IST Lisbon + TU Delft)
2. 2014: Worked as a researcher at DSOR
   (supervisor: Carlos Silvestre)
3. 2014-current: PhD at KTH (supervisor: Dimos Dimarogonas)
   - Control of single and multiple UAV for aerial manipulation
   - EU project: AEROWORKS

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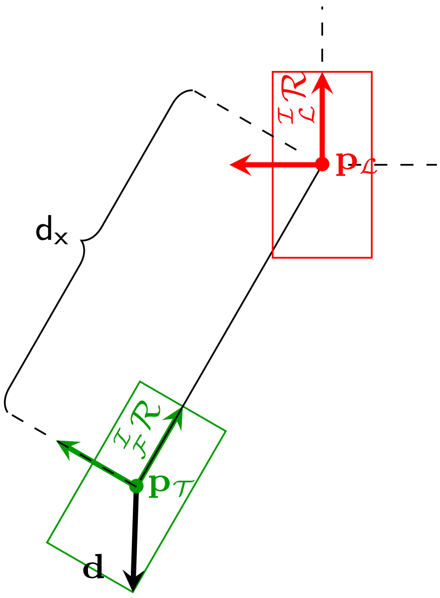
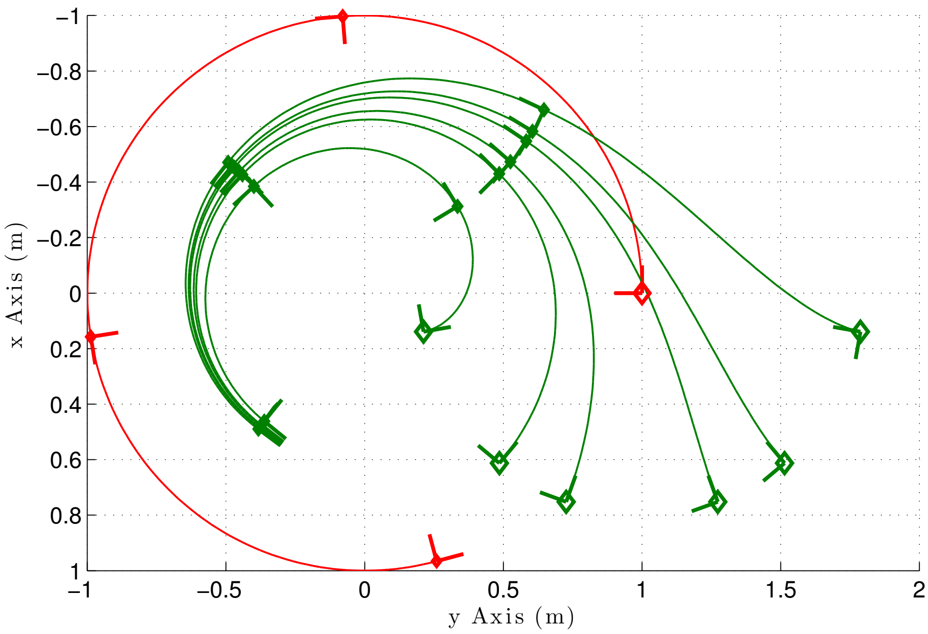
Leader following: Trailer like behavior



$\{\mathbf{p}_{\mathcal{T}}, {}^{\mathcal{I}}_{\mathcal{F}}\mathcal{R}\}$ = Trailer reference frame
$\{\mathbf{p}_{\mathcal{L}}, {}^{\mathcal{I}}_{\mathcal{L}}\mathcal{R}\}$ = Leader reference frame

Result of Trailer like behavior

Converge of all trailers to one nominal trailer



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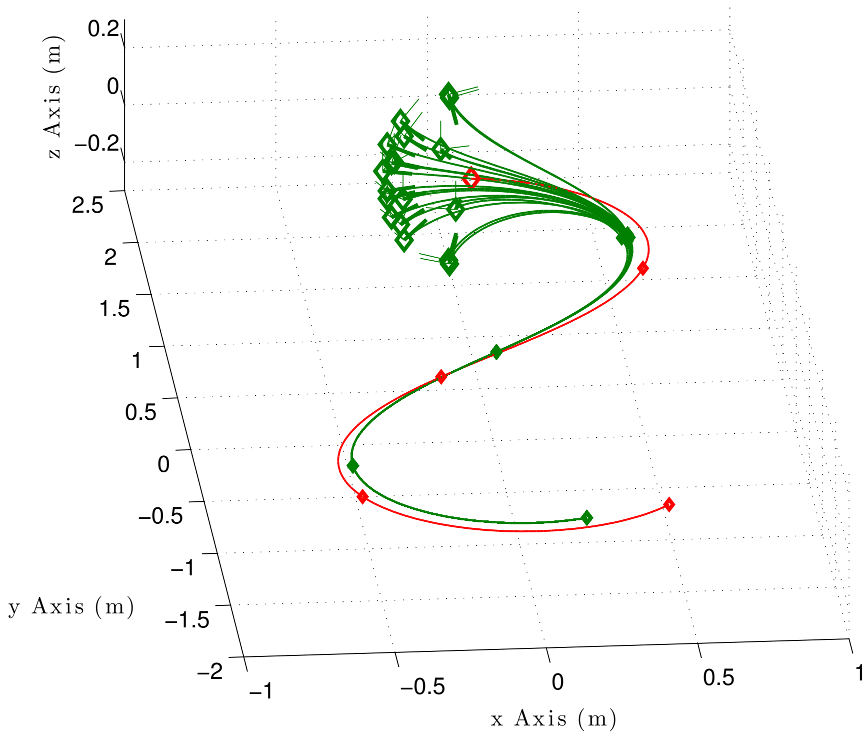
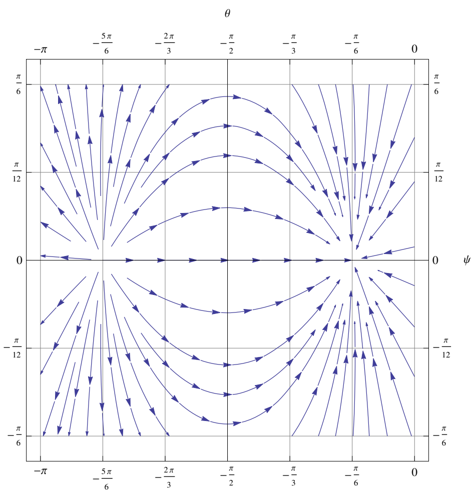
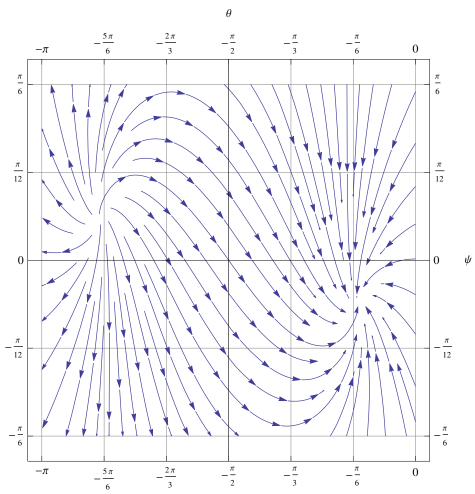


Angular Representation of $\mathbf{r}_1$

$$\begin{cases} \dot{\psi} = -\frac{\|\mathsf{v}_{\mathcal{L}}\|}{\mathsf{d_x}} \left(\frac{\sin(\psi)}{\cos(\theta)} + \kappa_{\mathcal{L}} \mathsf{d_x} + \tan(\theta)\cos(\psi)\tau_{\mathcal{L}} \mathsf{d_x} \right) \\ \dot{\theta} = -\frac{\|\mathsf{v}_{\mathcal{L}}\|}{\mathsf{d_x}} \left(\cos(\psi)\sin(\theta) - \sin(\psi)\mathsf{d_x}\tau_{\mathcal{L}} \right) \end{cases}$$



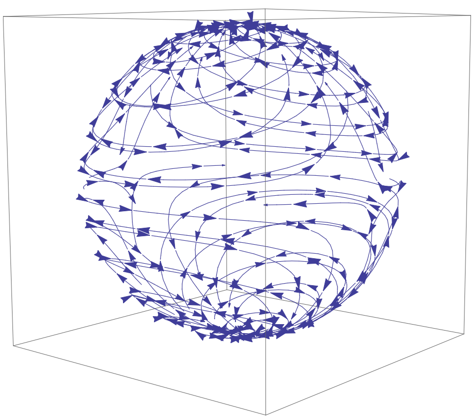
$\kappa_{\mathcal{L}} \mathsf{d_x} = 0.5, \tau_{\mathcal{L}} \mathsf{d_x} = 0$



$\kappa_{\mathcal{L}} \mathsf{d_x} = 0.5, \tau_{\mathcal{L}} \mathsf{d_x} = 0.5$

Representation of $\mathbf{r}_1$ on the sphere

$$\dot{\mathbf{r}}_1 = \frac{\|\mathsf{v}_{\mathcal{L}}\|}{\mathsf{d}_\mathsf{x}} \left(\Pi(\mathbf{r}_1)\mathbf{e}_1 + \mathcal{S}(\mathbf{r}_1) \begin{bmatrix} \tau_{\mathcal{L}} \\ 0 \\ \kappa_{\mathcal{L}} \end{bmatrix} \right)$$



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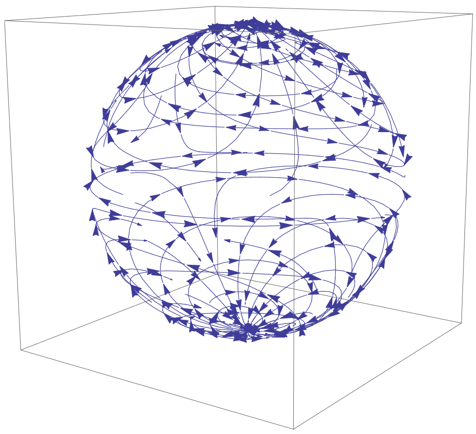

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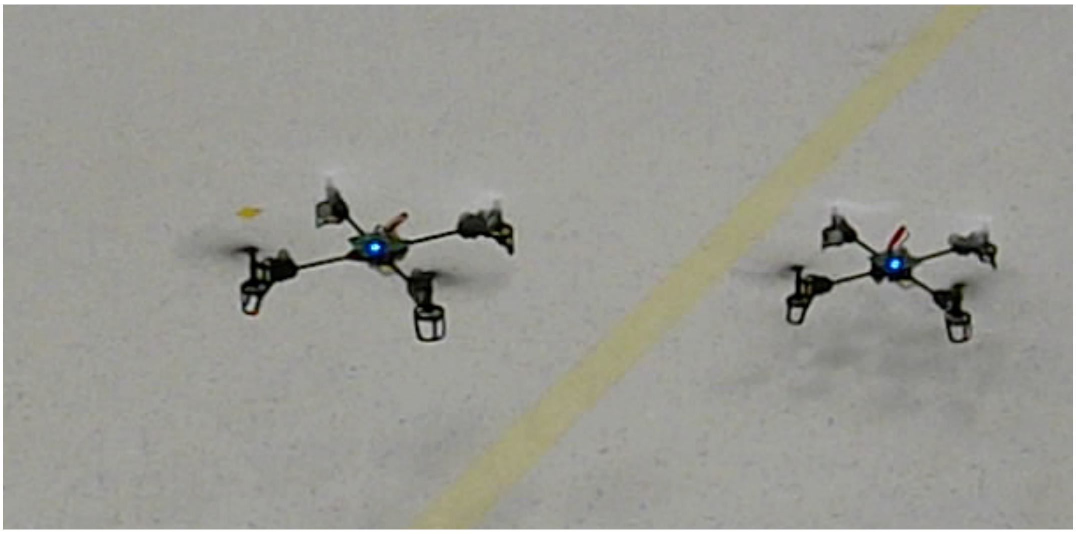
Video of Experiment

Leader describing Circular Paths with a non fixed normal



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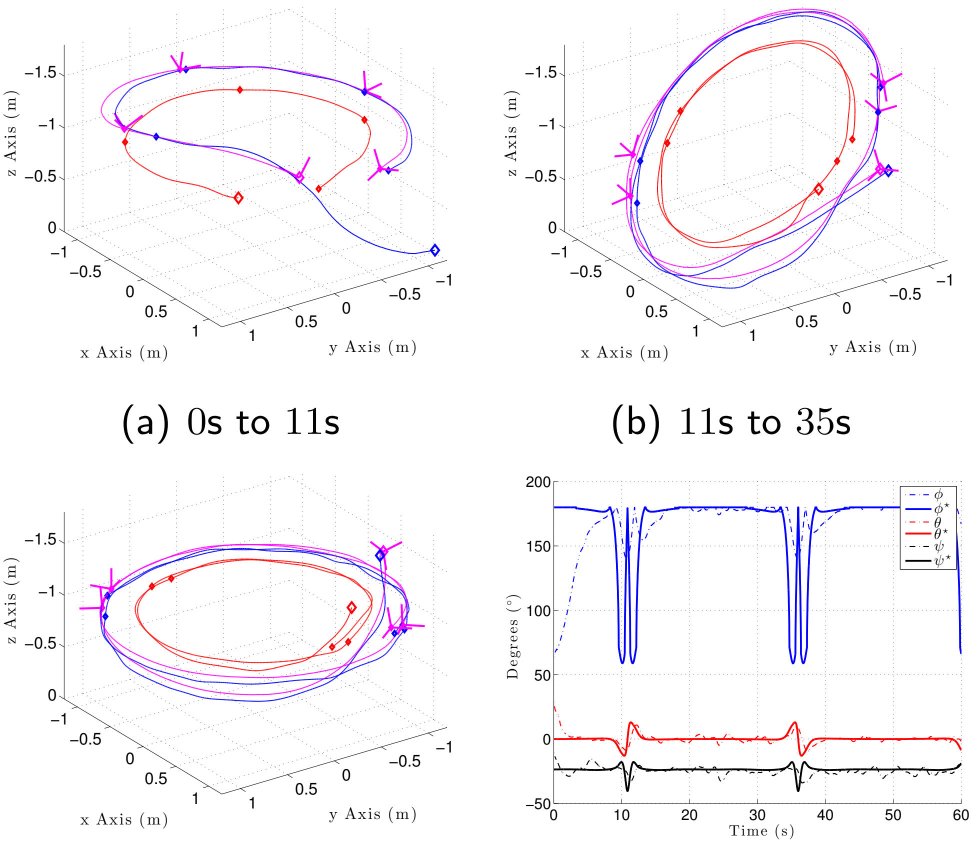


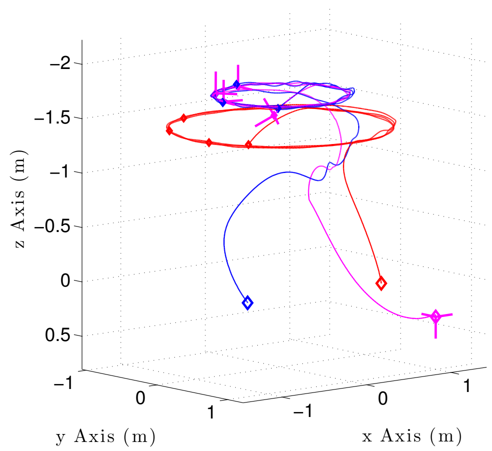
(a) 0s to 11s

(b) 11s to 35s

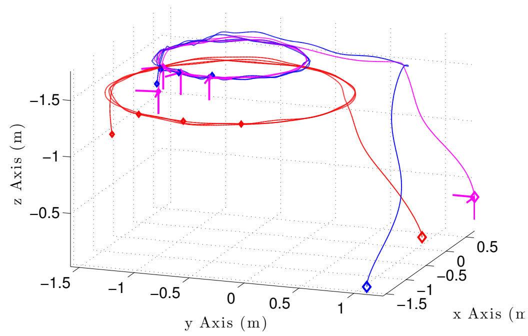
(c) 35s to 61s

(d) $\mathcal{R}$ and for $\mathcal{R}^\star$

Comparison with Planner from [1]



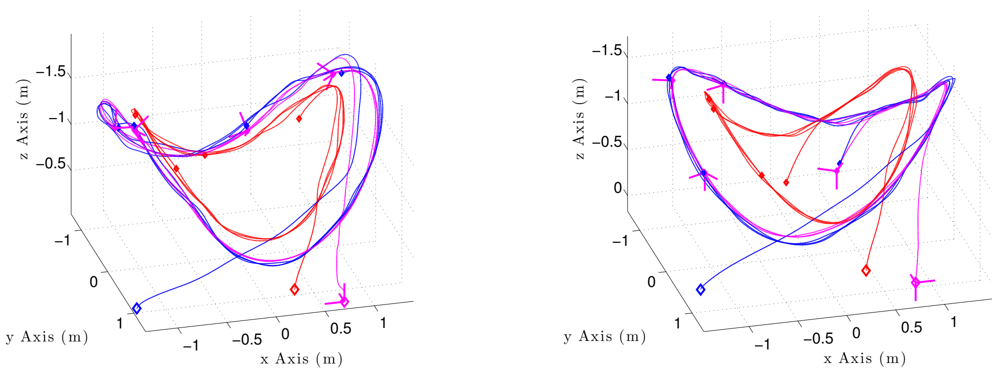
(a) 3D Planner



(b) Horizontal Planner +Vertical Offset

[1] Roldão. A novel leader-following strategy applied to formations of quadrotors. ECC 2013

Comparison with Planner from [1]



(a) 3D Planner

(b) Horizontal Planner +Vertical Offset

[1] Roldão. A novel leader-following strategy applied to formations of quadrotors. ECC 2013

EU Project: Collaboration between

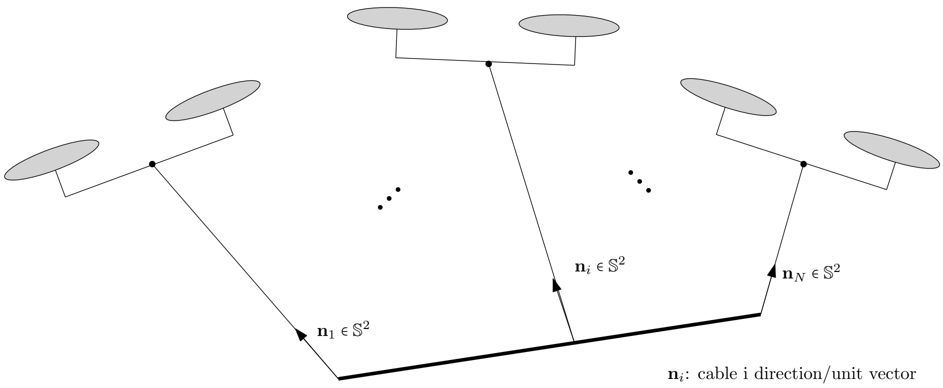
- LTU, SE: George Nikolakopoulos
- KTH, SE: Dimos Dimarogonas
- ETHZ, CH: (ASL) Roland Siegwart
- ETHZ, CH: (V4R) Margarita Chli
- UPatras, GR: Anthony Tzes
- UTwente, NL: Stefano Stramigioli
- ASC Technologies, DE
- Alstom, FR
- Skellefteå Kraft, SE

Figure : AEROWORKS' application scenarios.

Licentiate work

Licentiate (MSc < Licentiate < PhD)

1. Attitude synchronization of unit vectors and rotation matrices
2. Control of single and multiple UAV's for aerial manipulation



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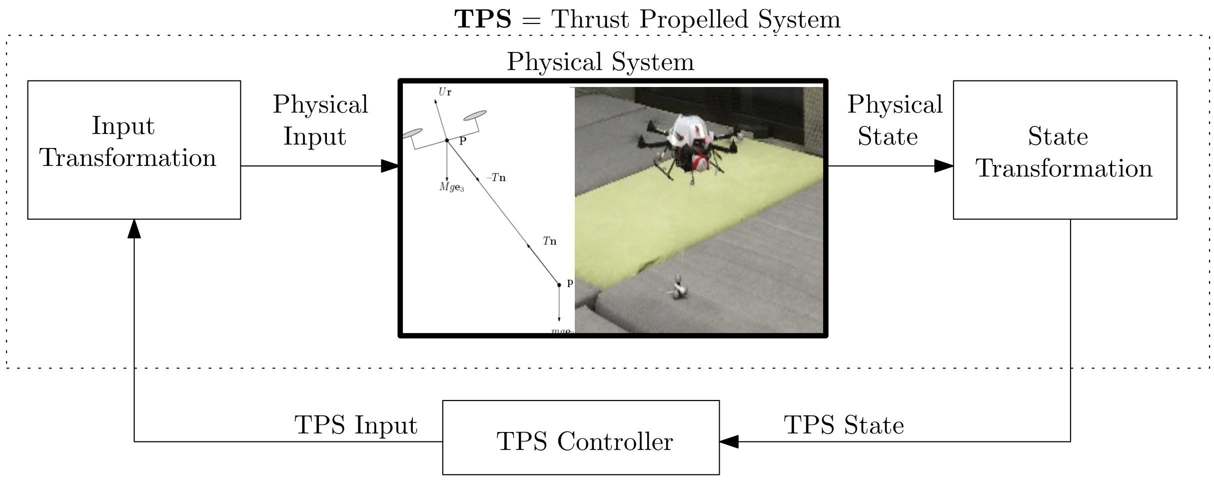


Control of the thrust-propelled system

Thrust-propelled system

1. Thrust along a body direction
2. Torque on body direction

(a) Load lifting by uav

(b) DLR 7-jointed robot arm

Summary of Control Strategy: Part 1

1. Controller $\mathbf{u}_x^{cl}$ for the thrust-propelled, with vector field $\mathbf{f}_x$

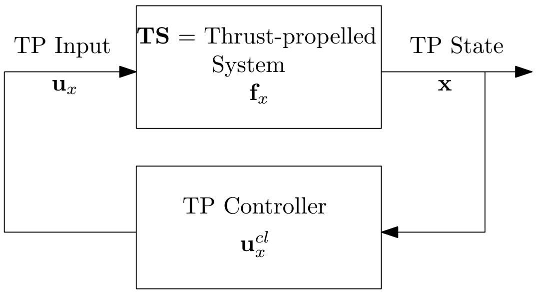
$$\dot{\mathbf{x}}(t) = \mathbf{f}_x(t, \mathbf{x}(t), \mathbf{u}_x(t)), \mathbf{x}(0) \in \Omega_x$$

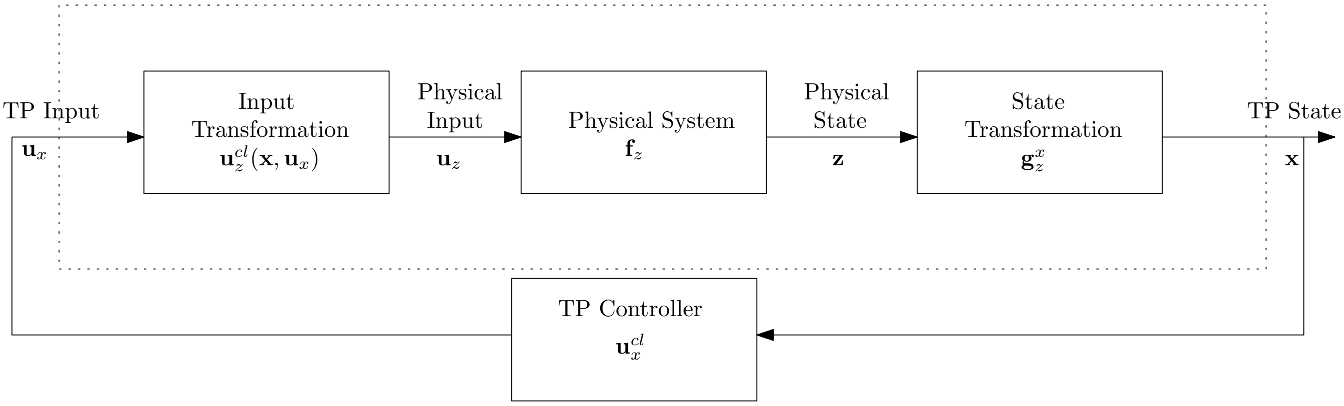
2. Given a physical system, with vector field $\mathbf{f}_z$

$$\dot{\mathbf{z}}(t) = \mathbf{f}_z(t, \mathbf{z}(t), \mathbf{u}_z(t)), \mathbf{z}(0) \in \Omega_z$$

   2.1 Transform $\mathbf{f}_z$ into $\mathbf{f}_x$
   2.2 Apply controller $\mathbf{u}_x^{cl}$ to physical system

Summary of Control Strategy: Part 2





System model

State:

$$\mathbf{x} = (\mathbf{p}, \mathbf{v}, \mathbf{n}, \boldsymbol{\omega})$$

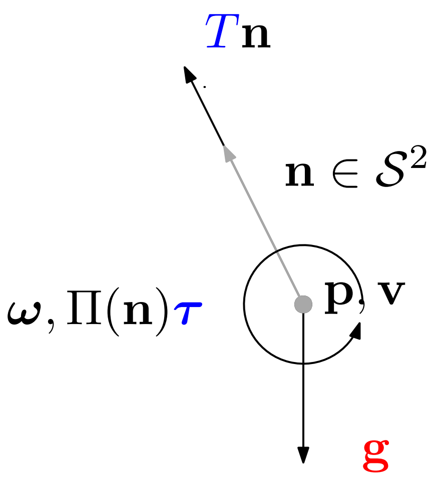
Input:

$$\mathbf{u}_x = (T, \boldsymbol{\tau})$$

Exogeneous input:

$$\mathbf{g} : [0, +\infty) \mapsto \mathbb{R}^3$$

$\mathbf{p}, \mathbf{v} =$ position, velocity
$\mathbf{g}$ =gravity
$T\mathbf{n} =$ Thrust acceleration
$\boldsymbol{\tau} =$ Angular acceleration



Thrust-propelled system

Vector field:

$$\dot{\mathbf{p}} = \mathbf{v}$$
$$\dot{\mathbf{v}} = T\mathbf{n} - \mathbf{g}(t)$$
$$\dot{\mathbf{n}} = \mathcal{S}(\boldsymbol{\omega})\mathbf{n}$$
$$\dot{\boldsymbol{\omega}} = \Pi(\mathbf{n})\boldsymbol{\tau}$$

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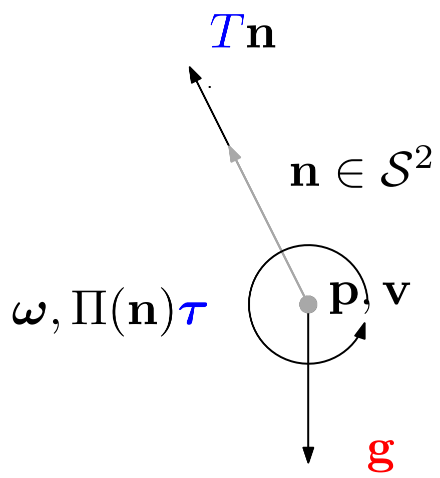

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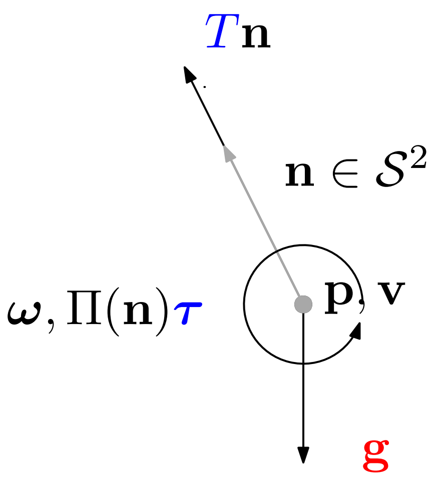

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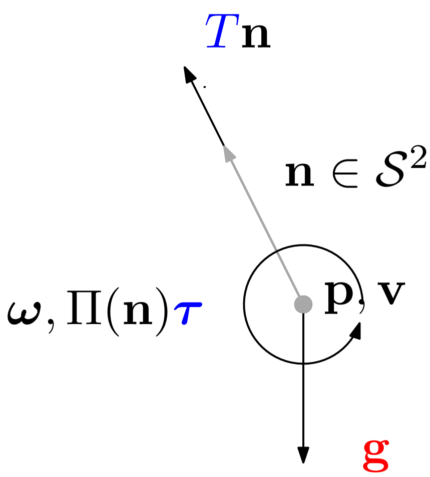

System model

$$\dot{\mathbf{x}}(t) = \mathbf{f}_x(t, \mathbf{x}(t), \mathbf{u}_x(t))$$

$$\mathbf{f}_x(t, \mathbf{x}, \mathbf{u}_x) = \begin{bmatrix} \mathbf{v} \\ T\mathbf{n} - \mathbf{g}(t) \\ \mathcal{S}(\boldsymbol{\omega})\mathbf{n} \\ \Pi(\mathbf{n})\boldsymbol{\tau} \end{bmatrix}$$

State Space:

$$\mathbf{x} = (\mathbf{p}, \mathbf{v}, \mathbf{n}, \boldsymbol{\omega}) \in \Omega_x$$
$$\Omega_x = \{\mathbf{x} \in \mathbb{R}^{12} : \mathbf{n} \in \mathbb{S}^2, \boldsymbol{\omega}^T\mathbf{n} = 0\}$$
$$\mathbf{f}_x(t, \mathbf{x}, \mathbf{u}_x) \in T_{\mathbf{x}}\Omega_x$$

Control Objective

Control Objective

Design a control law

$$\mathbf{u}_x^{cl} = (T^{cl}, \boldsymbol{\tau}^{cl}) : \mathbb{R}_{\geq 0} \times \Omega_x \mapsto \mathbb{R}^4,$$

such that

$$\lim_{t \to \infty} \mathbf{p}(t) = \mathbf{0},$$

along any trajectory of $\dot{\mathbf{x}}(t) = \mathbf{f}_x(t, \mathbf{x}(t), \mathbf{u}_x^{cl}(t, \mathbf{x}(t)))$.

Control Design Summary

$$\mathbf{f}_x(t, \mathbf{x}, \mathbf{u}_x) = \begin{bmatrix} \mathbf{v} \\ T\mathbf{n} - \mathbf{g}(t) \\ \mathcal{S}(\boldsymbol{\omega})\mathbf{n} \\ \Pi(\mathbf{n})\boldsymbol{\tau} \end{bmatrix}$$

Steps

1. Position control: $\boldsymbol{\xi} = (\mathbf{p}, \mathbf{v})$
2. Kinematic attitude control: $\bar{\mathbf{x}} = (\boldsymbol{\xi}, \mathbf{n})$
3. Dynamic attitude control: $\mathbf{x} = (\bar{\mathbf{x}}, \boldsymbol{\omega})$

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Control Objective Satisfaction

Control Objective Satisfaction

We find a control law

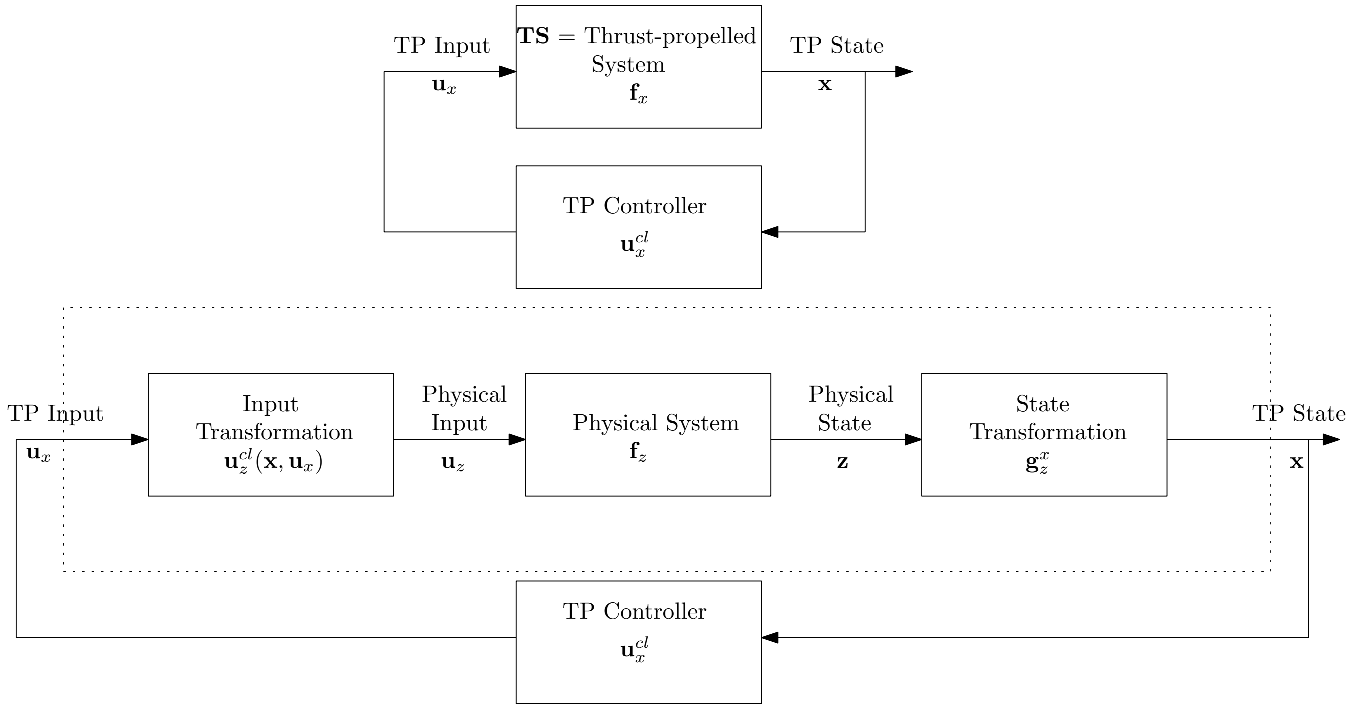
$$\mathbf{u}_x^{cl} = (T^{cl}, \boldsymbol{\tau}^{cl}) : \mathbb{R}_{\geq 0} \times \Omega_x \mapsto \mathbb{R}^4,$$

such that $\lim_{t \to \infty} \mathbf{p}(t) = \mathbf{0}$.

Proof

$$\begin{aligned}
\mathbf{f}_x^{cl}(t, \mathbf{x}) &:= \mathbf{f}_x(t, \mathbf{x}, \mathbf{u}_x^{cl}(t, \mathbf{x})) \\
\dot{V}_x(t, \mathbf{x}) &:= \partial_t V_x(t, \mathbf{x}) + \partial_{\mathbf{x}} V_x(t, \mathbf{x}) \mathbf{f}_x^{cl}(t, \mathbf{x}) \leq -w(\mathbf{p})
\end{aligned}$$

Summary of Control Strategy: Part 2



Slung Load Transportation

Motivation

- Transportation of payloads in inaccessible locations
- Mechanically simple (w.r.t. gripper/manipulator)

Figure : Disaster environment

Slung Load Transportation

Motivation

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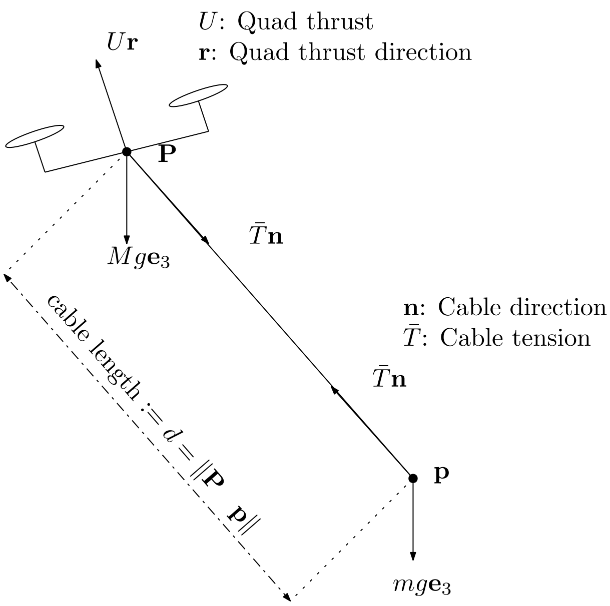
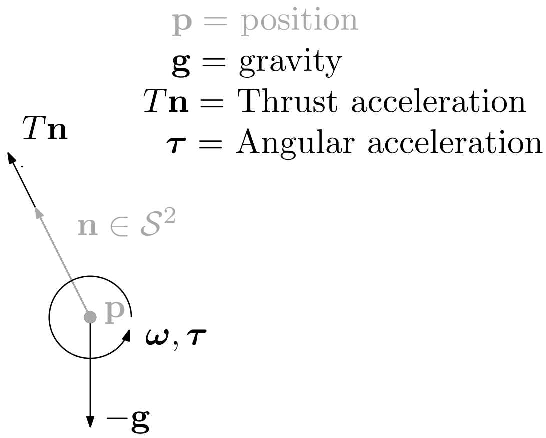
- Mechanically simple (w.r.t. gripper/manipulator)

(a) 2009 ICRA

(b) DLR 7-jointed robot arm

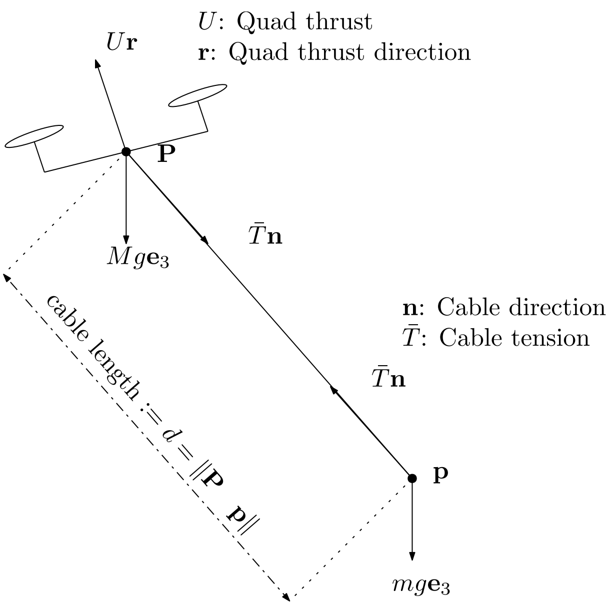
Summary

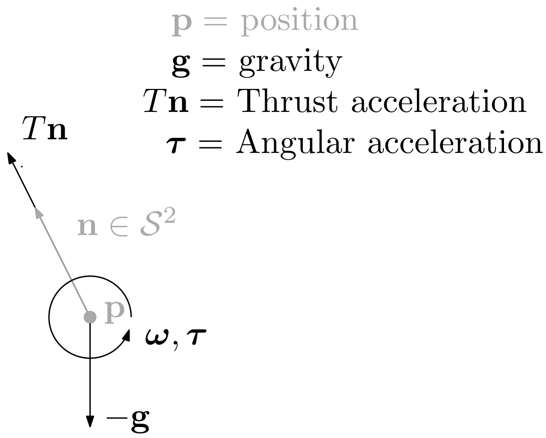




1. Model load as a thrust-propelled system
   - Control tension/thrust along the cable
   - Control torque/rotation of the cable
2. Disturbance removal for compensating model uncertainties

Summary





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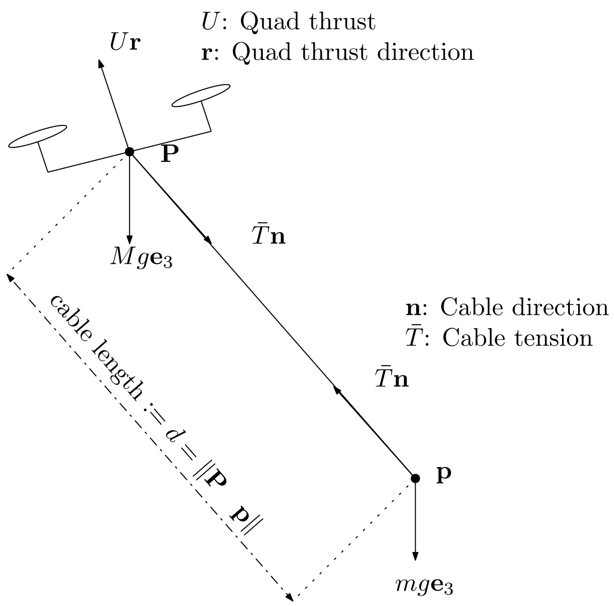

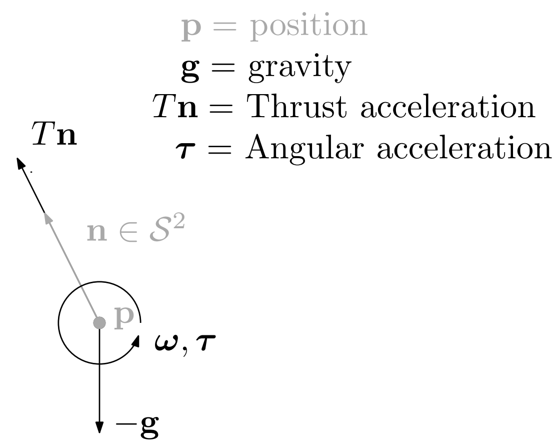

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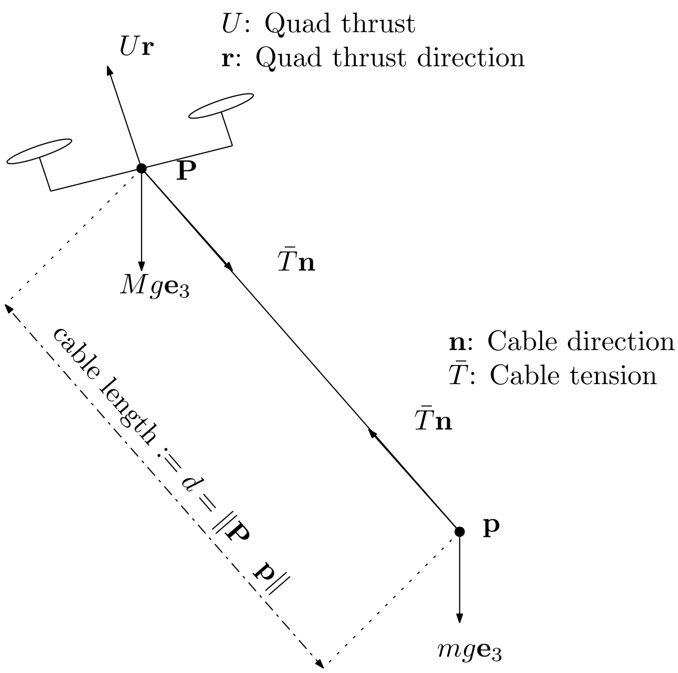
Modeling



- State:
  $\mathbf{z} = (\mathbf{p}, \mathbf{v}, \mathbf{P}, \mathbf{V}) \in \Omega_z$
- State Set: $\Omega_z \subset \mathbb{R}^{12}$
- Input
  $\mathbf{u}_z = (U, \mathbf{r}) \in \mathbb{R}_{\geq 0} \times \mathcal{S}^2$

Problem

Given a desired position trajectory $\mathbf{p}^\star \in \mathcal{C}^4(\mathbb{R}_{\geq 0}, \mathbb{R}^3)$*, design* $\mathbf{u}_z : \mathbb{R}_{\geq 0} \mapsto \mathbb{R}_{\geq 0} \times \mathcal{S}^2$ *such that* $\lim_{t \to \infty}(\mathbf{p}(t) - \mathbf{p}^\star(t)) = \mathbf{0}$*.*

Modeling

Vector field

- $\dot{\mathbf{z}} = \mathbf{f}_z(\mathbf{z}, \mathbf{u}_z)$
- $$\begin{bmatrix} \dot{\mathbf{p}} \\ \dot{\mathbf{v}} \\ \dot{\mathbf{P}} \\ \dot{\mathbf{V}} \end{bmatrix} = \mathbf{f}_z(\mathbf{z}, \mathbf{u}_z) := \begin{bmatrix} \mathbf{v} \\ \mathbf{a}(\mathbf{z}, \mathbf{u}_z) \\ \mathbf{V} \\ \mathbf{A}(\mathbf{z}, \mathbf{u}_z) \end{bmatrix} := \begin{bmatrix} \mathbf{v} \\ \frac{T(\mathbf{z}, \mathbf{u}_z)}{m}\bar{\mathbf{n}}(\mathbf{z}) - g\mathbf{e}_3 \\ \mathbf{V} \\ \frac{U}{M}\mathbf{r} - \frac{T(\mathbf{z}, \mathbf{u}_z)}{M}\bar{\mathbf{n}}(\mathbf{z}) - g\mathbf{e}_3 \end{bmatrix}$$
- $\bar{\mathbf{n}}(\mathbf{z})$ is cable unit vector
- $T(\mathbf{z}, \mathbf{u}_z)$ is tension on the cable
- Thrust input disturbance b: $\mathbf{f}_z(\mathbf{z}, \mathbf{u}_z + b\mathbf{e}_1)$

Modeling

Vector field

- $\dot{\mathbf{z}} = \mathbf{f}_z(\mathbf{z}, \mathbf{u}_z)$
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Modeling

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Change of Coordinates

- State transformation

$$\Omega_z \ni \mathbf{z} = \begin{bmatrix} \mathbf{p} \\ \mathbf{v} \\ \mathbf{P} \\ \mathbf{V} \end{bmatrix} \underset{g_x^z(t,\mathbf{x})}{\overset{g_z^x(t,\mathbf{z})}{\rightleftarrows}} \begin{bmatrix} \mathbf{p} - \mathbf{p}^\star(t) \\ \mathbf{v} - \dot{\mathbf{p}}^\star(t) \\ \bar{\mathbf{n}}(\mathbf{z}) \\ \bar{\boldsymbol{\omega}}(\mathbf{z}) \end{bmatrix} = \mathbf{x} \in \Omega_x$$

- Input transformation: construct $\mathbf{u}_z^{cl}$
- Since $\mathbf{x} = g_z^x(t, \mathbf{z})$

$$\mathbf{f}_x(t, \mathbf{x}, \mathbf{u}_x) = \left(\partial_t g_z^x(t, \mathbf{z}) + \partial_{\mathbf{z}} g_z^x(t, \mathbf{z}) \mathbf{f}_z(\mathbf{z}, \mathbf{u}_z)\right)\big|_{\substack{\mathbf{u}_z = \mathbf{u}_z^{cl}(\mathbf{z}, \mathbf{u}_x), \\ \mathbf{z} = g_x^z(t, \mathbf{x})}}$$

Change of Coordinates

- State transformation

$$\Omega_z \ni \mathbf{z} = \begin{bmatrix} \mathbf{p} \\ \mathbf{v} \\ \mathbf{P} \\ \mathbf{V} \end{bmatrix} \underset{g_x^z(t,\mathbf{x})}{\overset{g_z^x(t,\mathbf{z})}{\rightleftarrows}} \begin{bmatrix} \mathbf{p} - \mathbf{p}^\star(t) \\ \mathbf{v} - \dot{\mathbf{p}}^\star(t) \\ \bar{\mathbf{n}}(\mathbf{z}) \\ \bar{\boldsymbol{\omega}}(\mathbf{z}) \end{bmatrix} = \mathbf{x} \in \Omega_x$$

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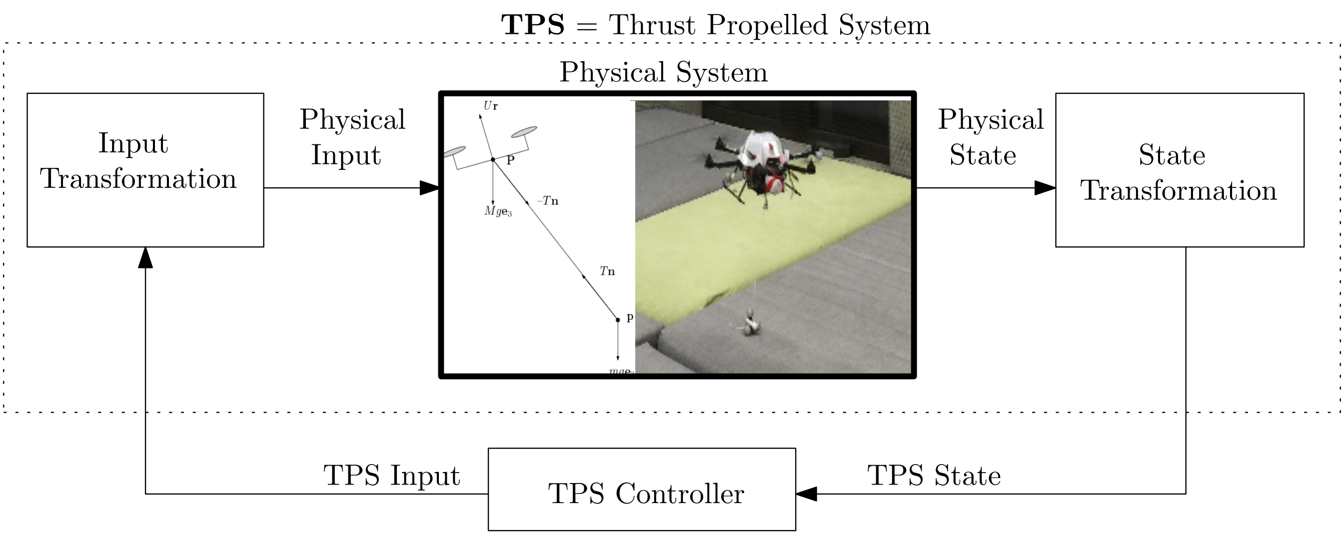
Reminder: Strategy



Constant input disturbance

- Physical Input: $\mathbf{u}_z = (T, \mathbf{r})$
- Constant input disturbance: $\mathbf{f}_z(\mathbf{z}, \mathbf{u}_z + b\mathbf{e}_1)$
- Disturbed transformed vector field

$$\mathbf{f}_z(\mathbf{z}, \mathbf{u}_z + b\mathbf{e}_1) \longleftrightarrow \mathbf{f}_x(t, \mathbf{x}, \mathbf{u}_x) + \Phi(\mathbf{x})b$$

- Disturbance estimator $\hat{b}$:

$$\dot{\hat{b}} = f_b(t, \tilde{\mathbf{x}}) = \text{Proj}\left(\Phi^T(\mathbf{x})\partial_{\mathbf{x}} V_x(t, \mathbf{x}), \hat{b}\right)$$

- Complete (dynamic) control law

$$\mathbf{u}_z^{cl}(\mathbf{x}, \mathbf{u}_x) - \hat{b}\mathbf{e}_1$$

Cai, Queiroz, Dawson. A sufficiently smooth projection operator. TAC 2006

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Cai, Queiroz, Dawson. A sufficiently smooth projection operator. TAC 2006

Feasible Trajectories

Definition

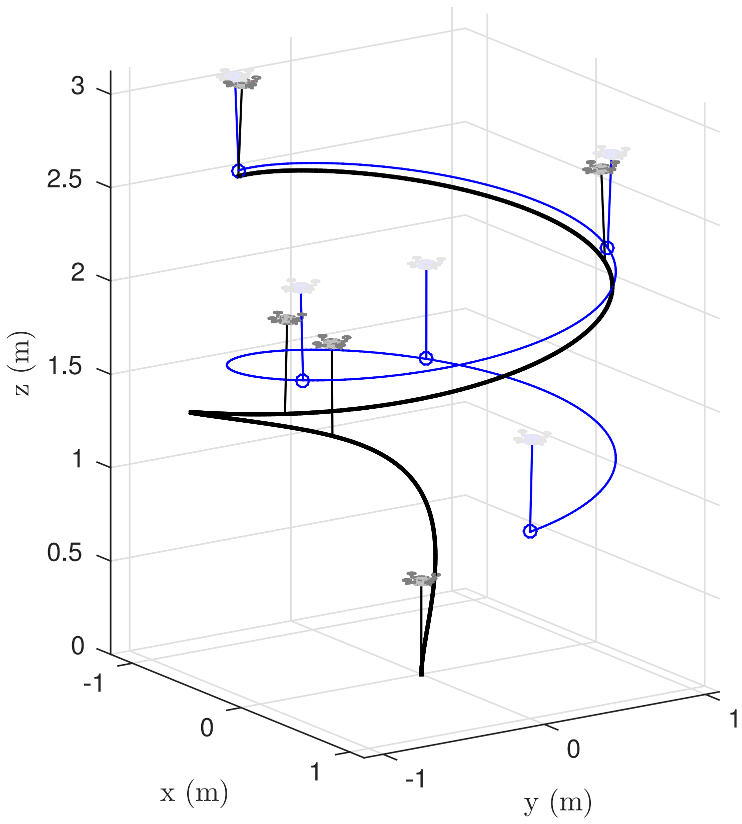
$\mathbf{p}^\star \in \mathcal{C}^4(\mathbb{R}_{\geq 0}, \mathbb{R}^3)$ *is a feasible trajectory if*

1. $\sup_{t\geq 0} \|\mathbf{p}^{\star(i)}(t)\| < \infty$ *for* $i \in \{2,3,4\}$
2. $\sup_{t\geq 0} \mathbf{e}_3^T \mathbf{p}^{\star(2)}(t) > -g$
3. $\inf\limits_{t\geq 0} \frac{M}{M+m} \frac{d\|\mathcal{S}(g\mathbf{e}_3+\mathbf{p}^{\star(2)}(t))\mathbf{p}^{\star(3)}(t)\|^2}{\|g\mathbf{e}_3+\mathbf{p}^{\star(2)}(t)\|^5} < 1.$

- Desired UAV attitude $\mathbf{r}$ is well defined for feasible trajectory
- Tension remains positive

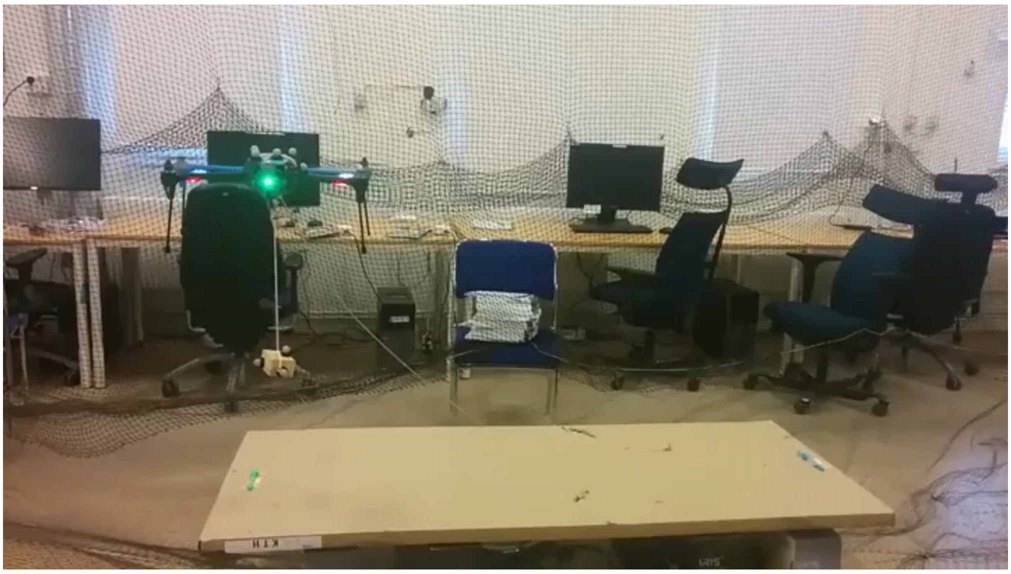
Simulations

- Desired motion: load to describe a helix
- Input disturbance corresponding to $\approx 14\%$ of load's weight



Experiment

- Hover over green/blue pen



How to tune gains: Load Lifting Controller

- *Simple* control law, used to provide initial guess for gains of *complicated* control law

Example

$$\mathbf{u}_z^{cl}(\mathbf{z}) = \begin{bmatrix} Mu(\mathbf{e}_1^T\mathbf{P}, \mathbf{e}_1^T\mathbf{V}) \\ Mu(\mathbf{e}_2^T\mathbf{P}, \mathbf{e}_2^T\mathbf{V}) \\ (M+m)(g + u(\mathbf{e}_3^T\mathbf{p}, \mathbf{e}_3^T\mathbf{v}) + \sigma(k_i \int \mathbf{e}_3^T\mathbf{p})) \end{bmatrix},$$

where (e.g. $\sigma = \frac{x}{\sqrt{1+x^2}}$)

$$u(p, v) = -k_p\sigma(p) - k_v\sigma(v).$$

Interesting results

Given:

$$\begin{aligned}\mathbf{z} &= (\mathbf{p}, \mathbf{v}, \mathbf{P}, \mathbf{V})\\ \dot{\mathbf{z}} &= \mathbf{f}_z^{cl}(\mathbf{z}) := \mathbf{f}_z(\mathbf{z}, \mathbf{u}_z^{cl}(\mathbf{z}))\end{aligned}$$

Facts:

Equilibrium

$$\mathbf{z}^\star = (\mathbf{p}^\star, \mathbf{v}^\star, \mathbf{P}^\star, \mathbf{V}^\star) = (\mathbf{0}, \mathbf{0}, L\mathbf{e}_3, \mathbf{0})$$

is asymptotically stable (for all positive gains).

Interesting results

Given:

$$\bar{\mathbf{z}} = (\mathbf{p}, \mathbf{v}, \mathbf{P}, \mathbf{V}, \mathbf{r})$$
$$\dot{\bar{\mathbf{z}}} = \mathbf{f}^{cl}_{\bar{z}}(\bar{\mathbf{z}}) := \left(\mathbf{f}_z(\mathbf{z}, (\mathbf{r}^T \mathbf{u}^{cl}_z(\mathbf{z}))\mathbf{r}), k_{\bar{\theta}}\Pi(\mathbf{r})\frac{\mathbf{u}^{cl}_z(\mathbf{z})}{\|\mathbf{u}^{cl}_z(\mathbf{z})\|})\right)$$

Facts:

Equilibrium

$$\bar{\mathbf{z}}^\star = (\mathbf{0}, \mathbf{0}, L\mathbf{e}_3, \mathbf{0}, \mathbf{e}_3)$$

is asymptotically stable iff

$$k_\theta > \frac{k_p}{k_v} = \frac{\omega_n^2}{2\xi\omega_n} = \frac{\omega_n}{2\xi}.$$

Interesting results

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Pedro

Interesting results

Given:

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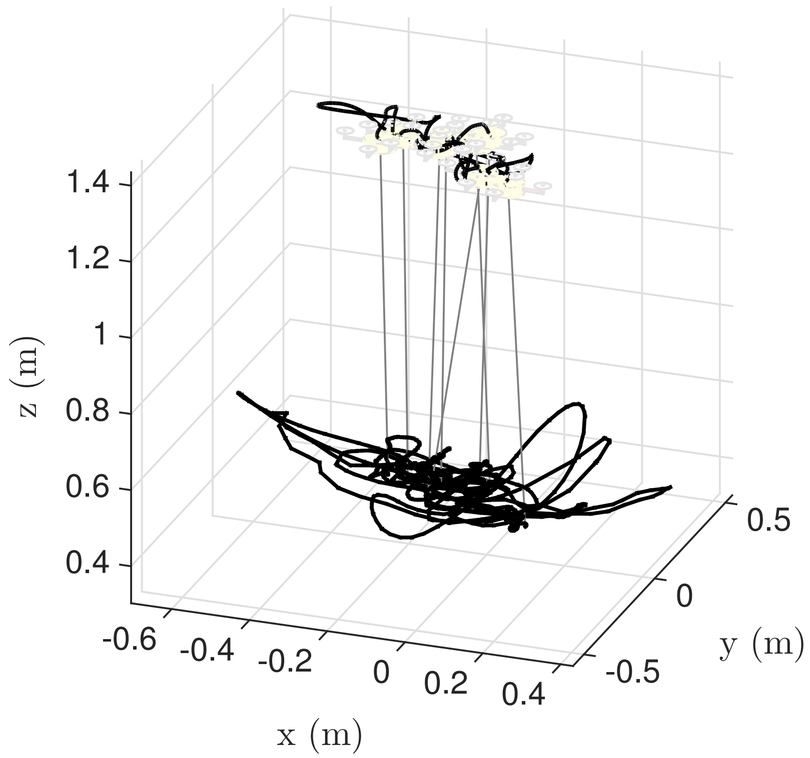
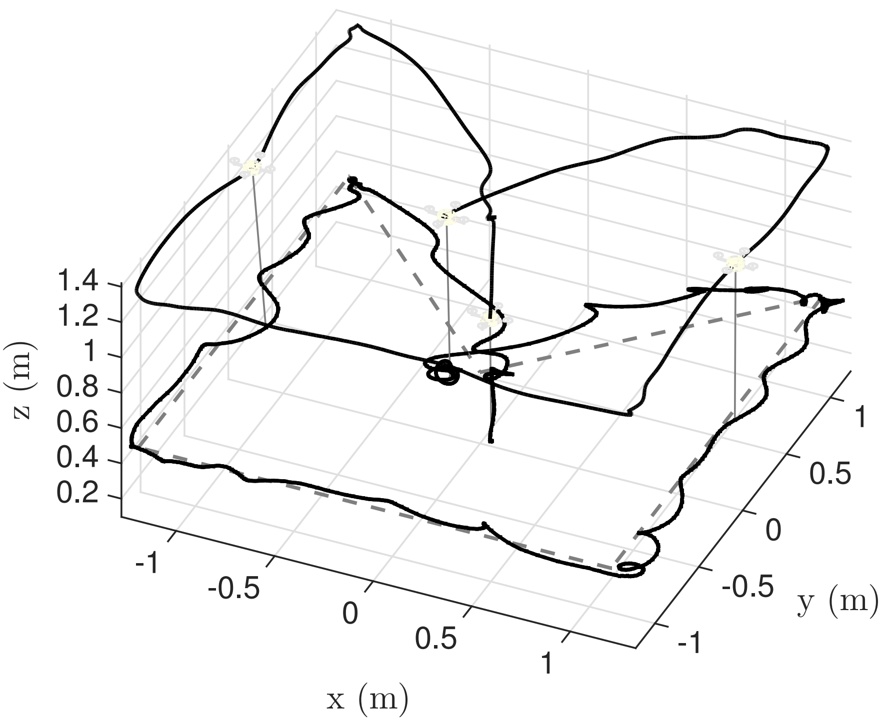
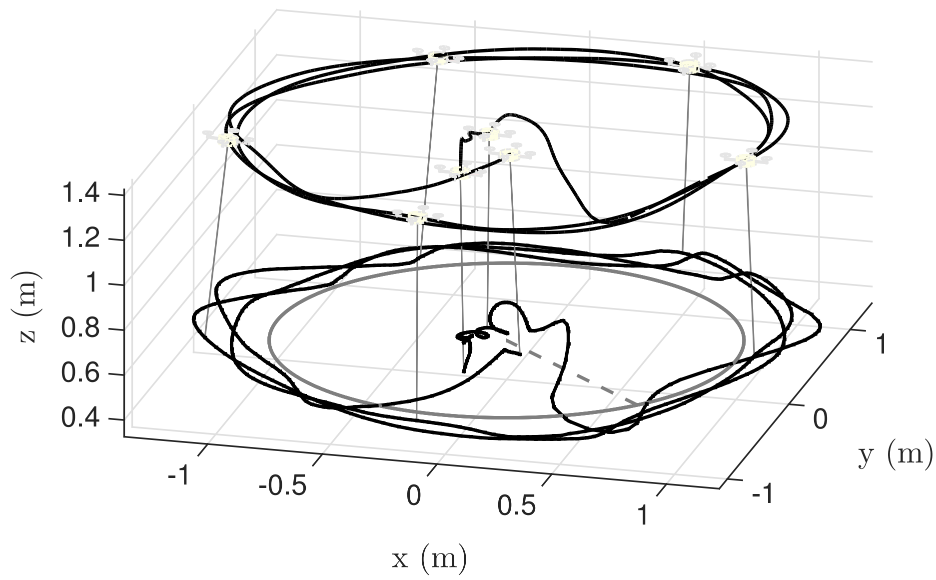
Experiment 1



Experiment 2

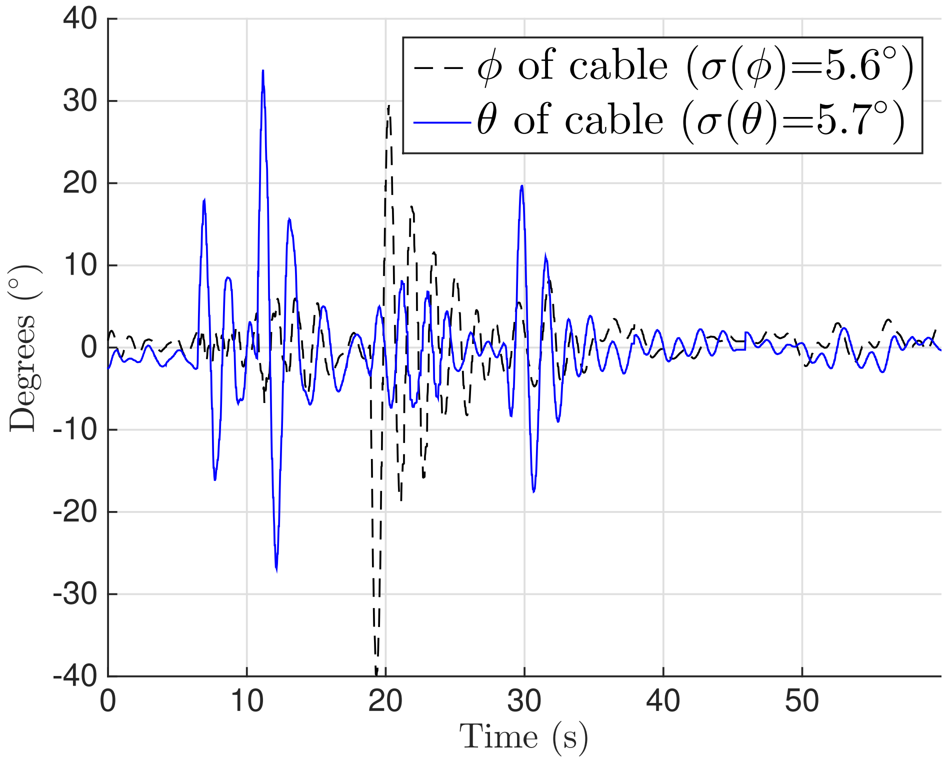


Experiment 3



Experiments' Videos

Experiment 1: Cable Angles



Aerial manipulation for interaction with the environment

Manipulator/gripper (vs cable)

- Mechanically complex
- More degrees of freedom

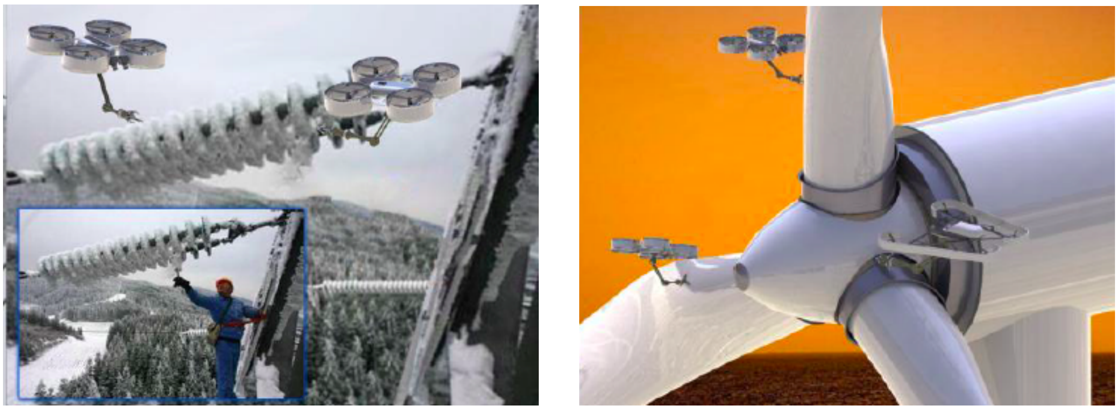
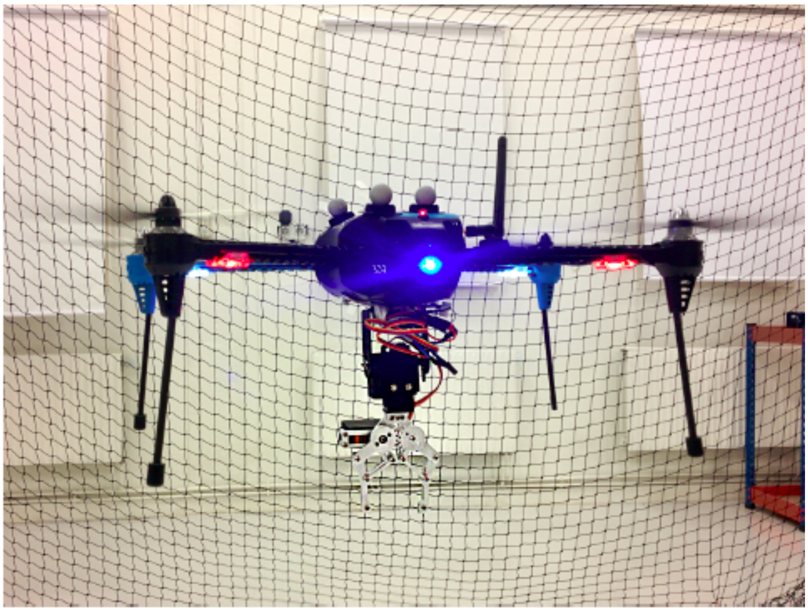


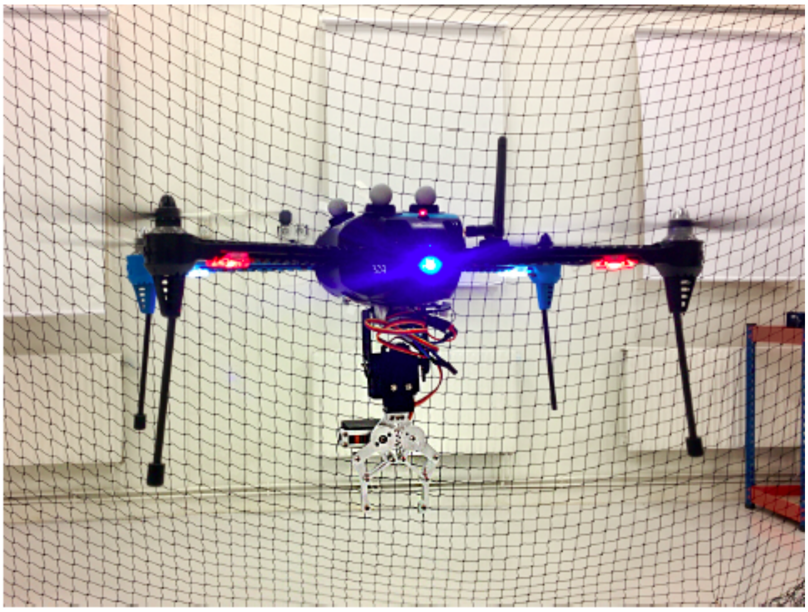
Figure : AEROWORKS's driving scenarios.

Control Problem



- System composed of aerial vehicle + manipulator w. gripper
- Objective:
  - Aerial vehicle to track desired position trajectory $(\mathbf{p}^{\star})$
  - Manipulator attitude to track desired attitude $(\mathbf{n}^{\star})$

Control Problem



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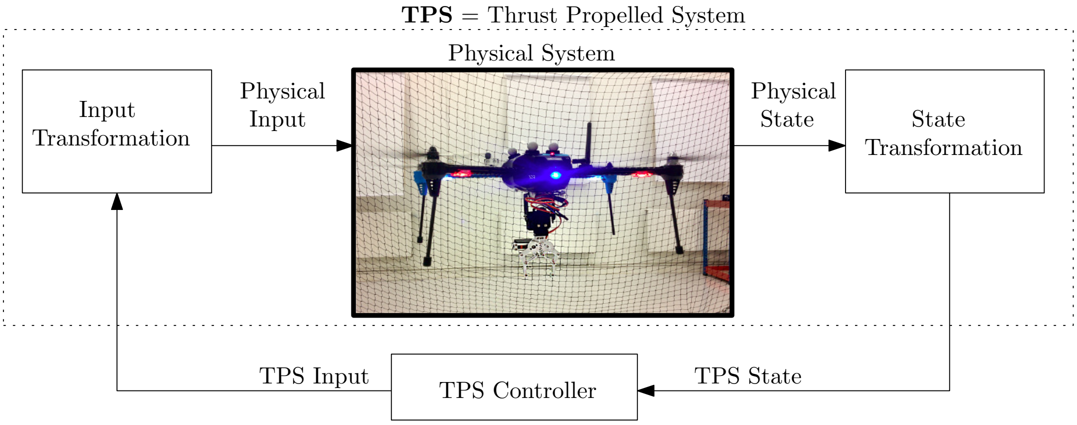
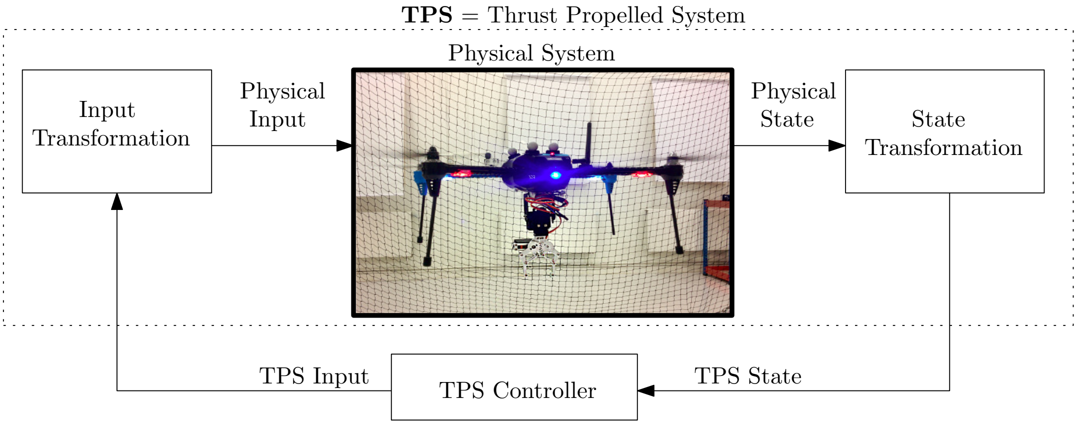
Solution to Control Problem



Solution

- Decomposed the system into two decoupled subsystems (thrust propelled systems)
- Leverage already available control laws
- Disturbance removal technique which penalizes position tracking error and attitude tracking error differently.

Solution to Control Problem



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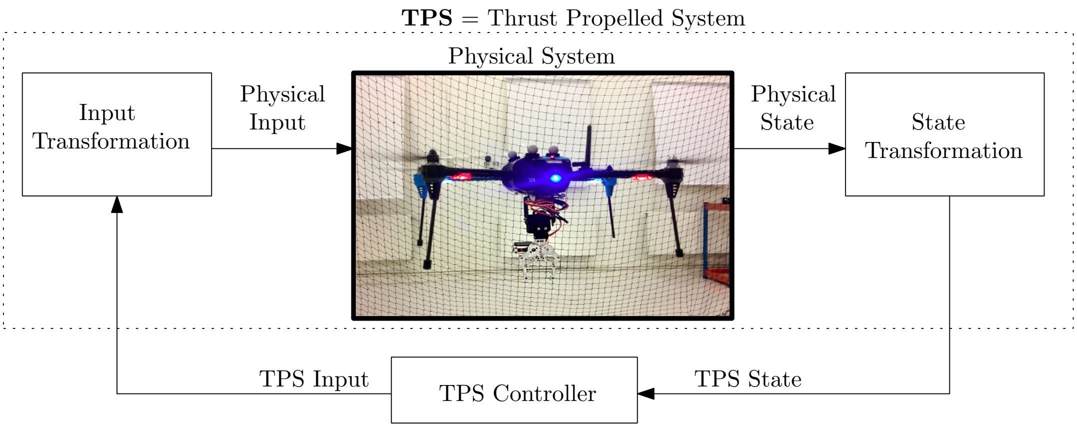

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Modeling

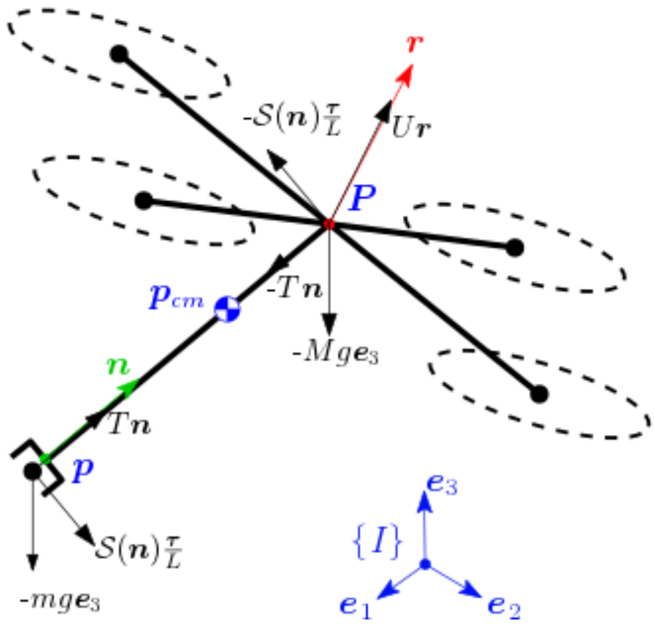
Modeling

- State:

$$\mathbf{z} = (\mathbf{p}, \mathbf{v}, \mathbf{P}, \mathbf{V}, \mathbf{r}) \in \Omega_z$$

- Physical input (thrust + angular velocity + torque):

$$\mathbf{u}_z = (T, \boldsymbol{\varpi}, \boldsymbol{\tau}) \in \mathbb{R} \times \mathbb{R}^3 \times \mathbb{R}^3$$



State set and its tangent space

System's State Set and Tangent Set

$$
\begin{aligned}
\Omega_z := \{(\mathbf{p},\mathbf{v},\mathbf{P},\mathbf{V},\mathbf{r}) \in (\mathbb{R}^3)^5 : & \ \mathbf{r}^T\mathbf{r} = 1, \\
& (\mathbf{P}-\mathbf{p})^T(\mathbf{P}-\mathbf{p}) = L, \} \\
& (\mathbf{V}-\mathbf{v})^T(\mathbf{P}-\mathbf{p}) = 0\} \\
T_{\mathbf{z}}\Omega_z := \{(\delta\mathbf{p},\delta\mathbf{v},\delta\mathbf{P},\delta\mathbf{V},\delta\mathbf{r}) \in (\mathbb{R}^3)^5 : & \ \mathbf{r}^T\delta\mathbf{r} = 0, \\
& (\delta\mathbf{P}-\delta\mathbf{p})^T(\mathbf{P}-\mathbf{p}) = 0, \\
& (\delta\mathbf{V}-\delta\mathbf{v})^T(\mathbf{P}-\mathbf{p}) + (\mathbf{V}-\mathbf{v})^T(\delta\mathbf{P}-\delta\mathbf{p}) = 0\}
\end{aligned}
$$

System Dynamics

(Open-loop) Vector field

$$\begin{bmatrix} \dot{\mathbf{p}} \\ \dot{\mathbf{v}} \\ \dot{\mathbf{P}} \\ \dot{\mathbf{V}} \\ \dot{\mathbf{r}} \end{bmatrix} = \mathbf{f}_z(\mathbf{z}, \mathbf{u}_z) := \begin{bmatrix} \mathbf{v} \\ \mathbf{a}(\mathbf{z}, \mathbf{u}_z) \\ \mathbf{V} \\ \mathbf{A}(\mathbf{z}, \mathbf{u}_z) \\ \mathcal{S}(\boldsymbol{\varpi})\mathbf{r} \end{bmatrix}$$

Vector fields is tangent to set Ω_z, thus Ω_z is positively invariant

$$\mathbf{f}_z(\mathbf{z}, \mathbf{u}_z) \in T_{\mathbf{z}}\Omega_z$$

System Dynamics

(Open-loop) Vector field

$$\begin{bmatrix} \dot{\mathbf{p}} \\ \dot{\mathbf{v}} \\ \dot{\mathbf{P}} \\ \dot{\mathbf{V}} \\ \dot{\mathbf{r}} \end{bmatrix} = \mathbf{f}_z(\mathbf{z}, \mathbf{u}_z) := \begin{bmatrix} \mathbf{v} \\ \mathbf{a}(\mathbf{z}, \mathbf{u}_z) \\ \mathbf{V} \\ \mathbf{A}(\mathbf{z}, \mathbf{u}_z) \\ \mathcal{S}(\boldsymbol{\varpi})\mathbf{r} \end{bmatrix}$$

Thrust input disturbance ($\mathbf{u}_z = (T, \boldsymbol{\varpi}, \boldsymbol{\tau})$ and $\mathbf{e}_1 \in \mathbb{R}^7$)

$$\mathbf{f}_z(\mathbf{z}, \mathbf{u}_z + b\mathbf{e}_1)$$

State and Input Transformation

Two decoupled subsystems ($\mathbf{x}_1$ and $\mathbf{x}_2$)

$$\Omega_z \ni \mathbf{z} = \begin{bmatrix} \mathbf{p} \\ \mathbf{v} \\ \mathbf{P} \\ \mathbf{V} \\ \mathbf{r} \end{bmatrix} \underset{g_x^z(t,\mathbf{x})}{\overset{g_z^x(t,\mathbf{z})}{\rightleftarrows}} \mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \in \Omega_{x_1} \\ \mathbf{x}_2 \in \Omega_{x_2} \end{bmatrix}$$

$$\mathbf{f}_z(\mathbf{z}, \mathbf{u}_z) \underset{g_x^z(t,\mathbf{x})}{\overset{g_z^x(t,\mathbf{z})}{\rightleftarrows}} \mathbf{f}_x(\mathbf{x}, \mathbf{u}_x) = \begin{bmatrix} \mathbf{f}_{x_1}(t, \mathbf{x}_1, \mathbf{u}_{x_1}) \\ \mathbf{f}_{x_2}(t, \mathbf{x}_2, \mathbf{u}_{x_2}) \end{bmatrix}$$

Decouple sub-systems

Center of Mass dynamics and Manipulator attitude dynamics are decoupled

$$\mathbf{z} \underset{g_x^z(t,\mathbf{x})}{\overset{g_z^x(t,\mathbf{z})}{\rightleftarrows}} \mathbf{x}_1 = \begin{bmatrix} \mathbf{p}_{cm}(\mathbf{z}) + \mathbf{p}_{cm}^{\star}(t) \\ \mathbf{v}_{cm}(\mathbf{z}) + \dot{\mathbf{p}}_{cm}^{\star}(t) \\ \mathbf{r} \end{bmatrix}$$

$$\mathbf{z} \underset{g_x^z(t,\mathbf{x})}{\overset{g_z^x(t,\mathbf{z})}{\rightleftarrows}} \mathbf{x}_2 \approx \begin{bmatrix} \bar{\mathbf{n}}(\mathbf{z}) - \mathbf{n}^{\star}(t) \\ \bar{\boldsymbol{\omega}}(\mathbf{z}) - \boldsymbol{\omega}^{\star}(t) \end{bmatrix}$$

$\mathbf{f}_{x_1}(t, \mathbf{x}_1, \mathbf{u}_{x_1})$: we find $\mathbf{u}_{x_1}^{cl}, V_{x_1}$ such that $\dot{V}_{x_1}(t, \mathbf{x}_{x_1}) \leq 0$

$\mathbf{f}_{x_1}(t, \mathbf{x}_2, \mathbf{u}_{x_2})$: we find $\mathbf{u}_{x_2}^{cl}, V_{x_2}$ such that $\dot{V}_{x_2}(t, \mathbf{x}_{x_2}) \leq 0$

Disturbance removal

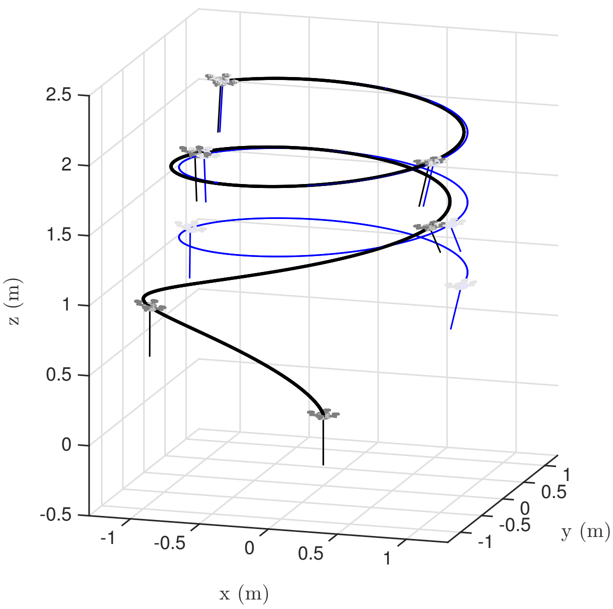
Remove effect of a constant thrust input disturbance $\mathbf{f}_z(\mathbf{z}, \mathbf{u}_z + b\mathbf{e}_1)$

- Model battery drainage as thrust input disturbance
- Disturbance estimate $\dot{\hat{b}} = f_{\hat{b}}(t, \tilde{\mathbf{x}})$

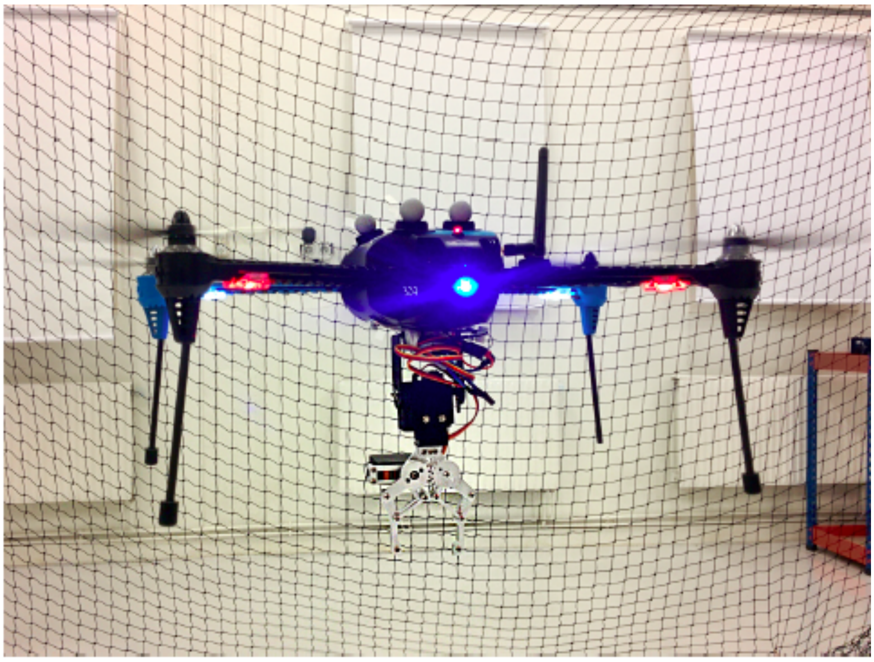
$$f_{\hat{b}}(t, \tilde{\mathbf{x}}) = \text{Proj}\left(\Phi^T(\mathbf{x})\left(\underbrace{k_{\hat{b}_1}\partial_{\mathbf{x}_1}V_{x_1}(t, \mathbf{x}_1)}_{\substack{\text{penalty position}\\ \text{tracking error}}} + \underbrace{k_{\hat{b}_2}\partial_{\mathbf{x}_2}V_{x_2}(t, \mathbf{x}_2)}_{\substack{\text{penalty attitude}\\ \text{tracking error}}}\right), \hat{b}\right),$$

Cai, Queiroz, Dawson. A sufficiently smooth projection operator. TAC 2006

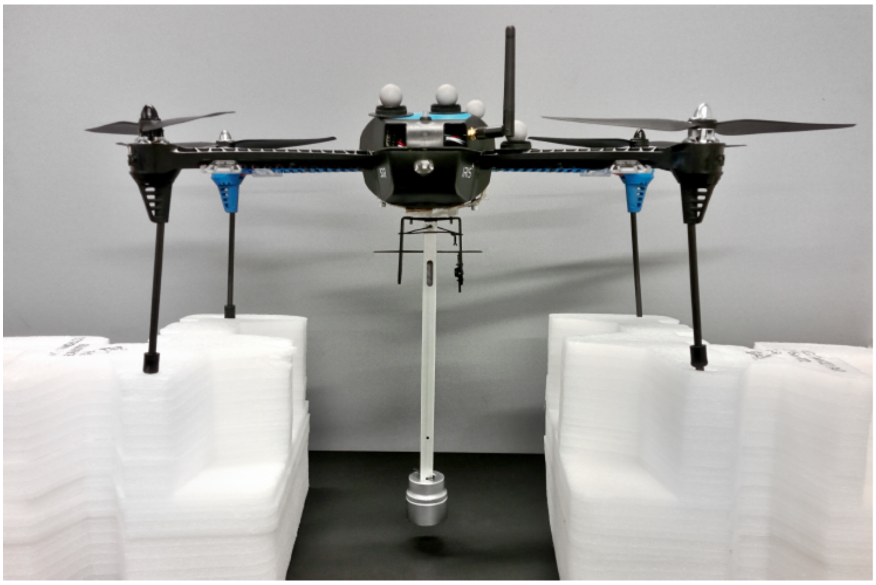
Simulations



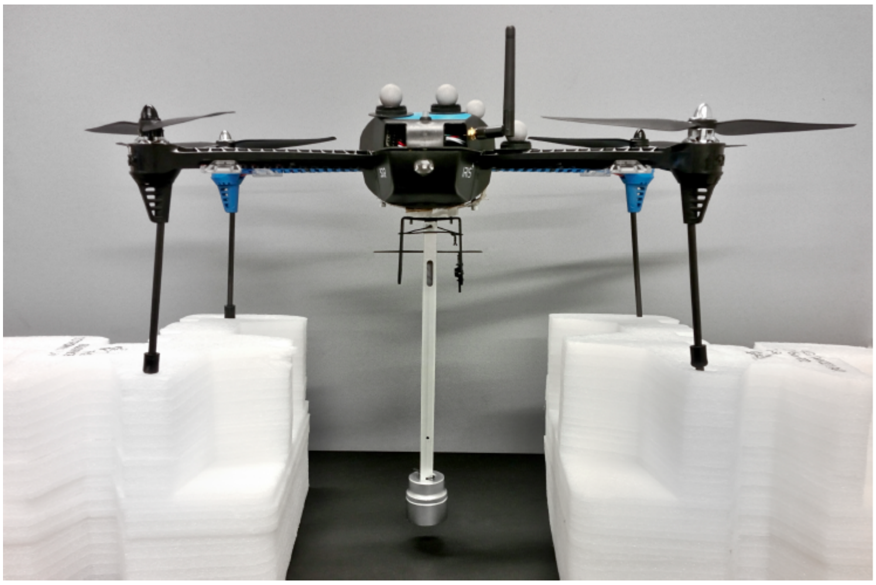
Experiments



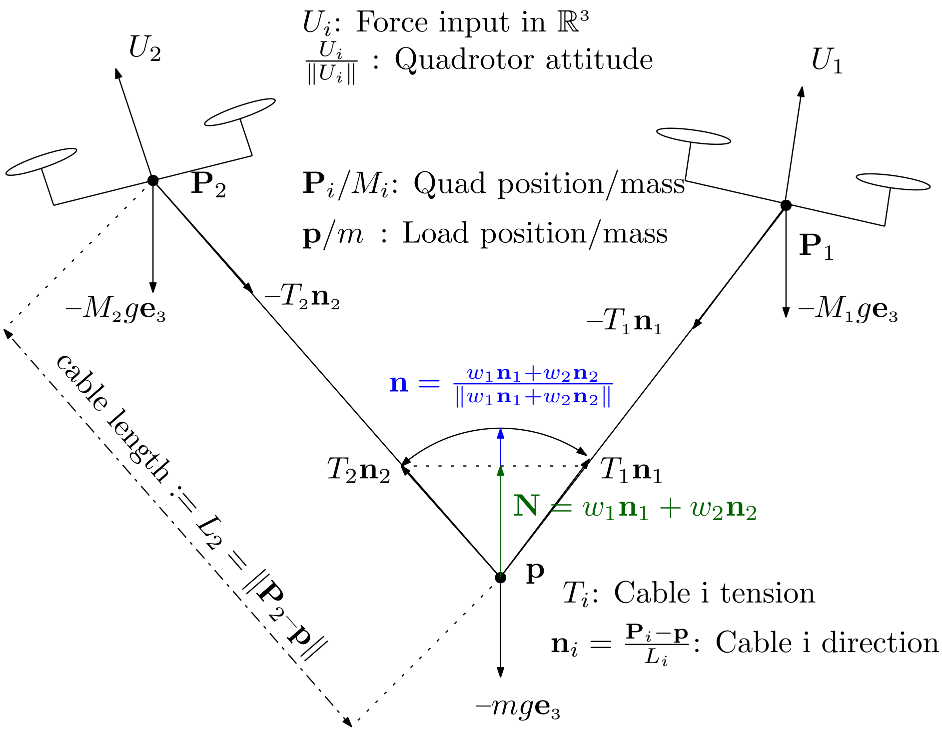
Experiments



Experiments

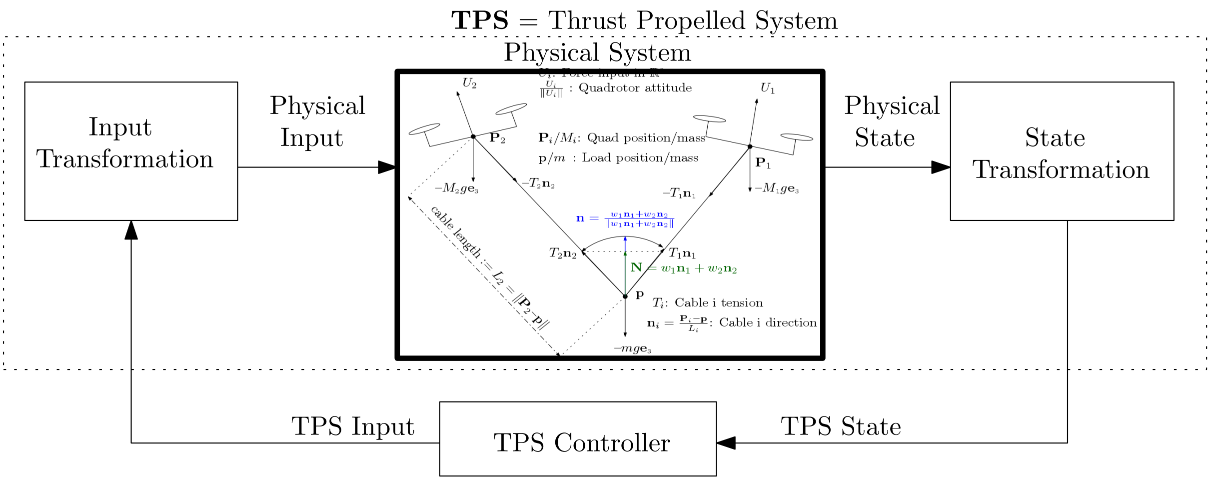


Collaborative load lifting



- Necessary when load is heavier than the individual UAVs' payload capacity
- Load's weight distribution can be chosen ($\frac{T_1}{T_2} = \frac{w_1}{w_2}$.)

Collaborative load lifting



Degrees of Freedom

- Angle between cables
- Rotation around vertical axis

Future and Current work

Future and Current work

- Experimental implementation of collaborative strategies
- Remove other types of disturbances
- Study robustness against model uncertainty

Thank you! Questions?