Lyapunov-based Generic Controller Design for Thrust-Propelled Underactuated Systems

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Motivation

Thrust-propelled systems

1. Thrust along a body direction
2. Torque on body direction

Motivation:

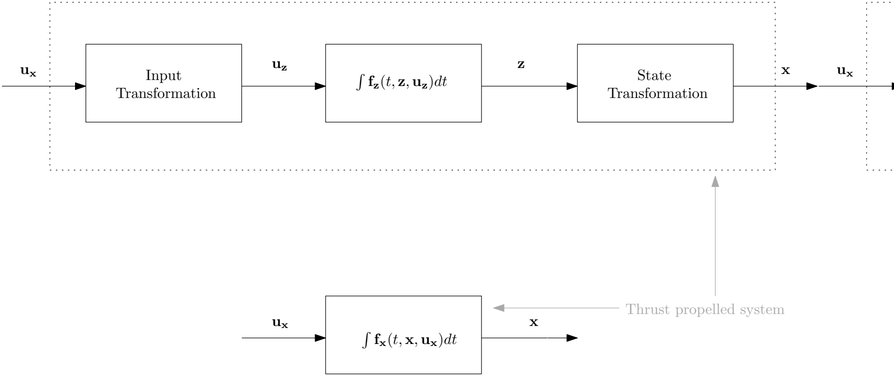
- Common controller: quadrotor, load-lifting

(a) Load lifting by uav

(b) DLR 7-jointed robot arm

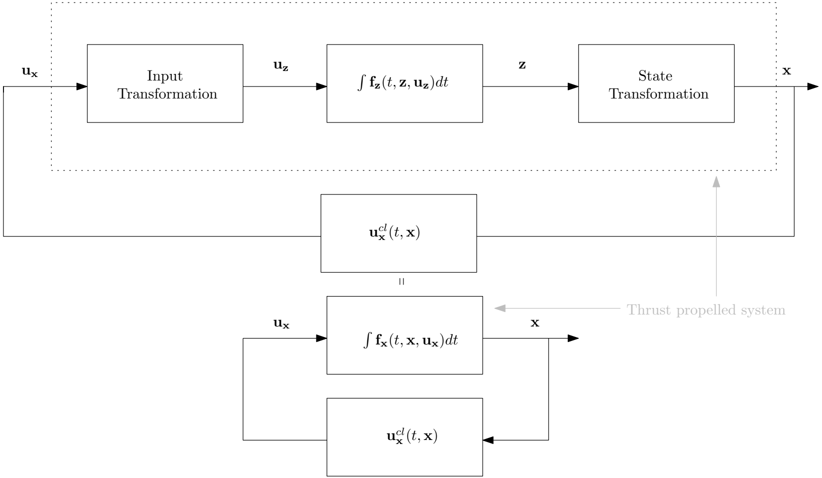
Summary

- Thrust-propelled vector field $\mathbf{f}_{\mathbf{x}}(t, \mathbf{x}, \mathbf{u}_{\mathbf{x}})$
- Transform vector field $\mathbf{f}_{\mathbf{z}}(t, \mathbf{z}, \mathbf{u}_{\mathbf{z}})$ to $\mathbf{f}_{\mathbf{x}}(t, \mathbf{x}, \mathbf{u}_{\mathbf{x}})$



Summary

- Design a controller $\mathbf{u}_{\mathbf{x}}^{cl}(t,\mathbf{x})$ for $\mathbf{f}_{\mathbf{x}}(t,\mathbf{x},\mathbf{u}_{\mathbf{x}})$



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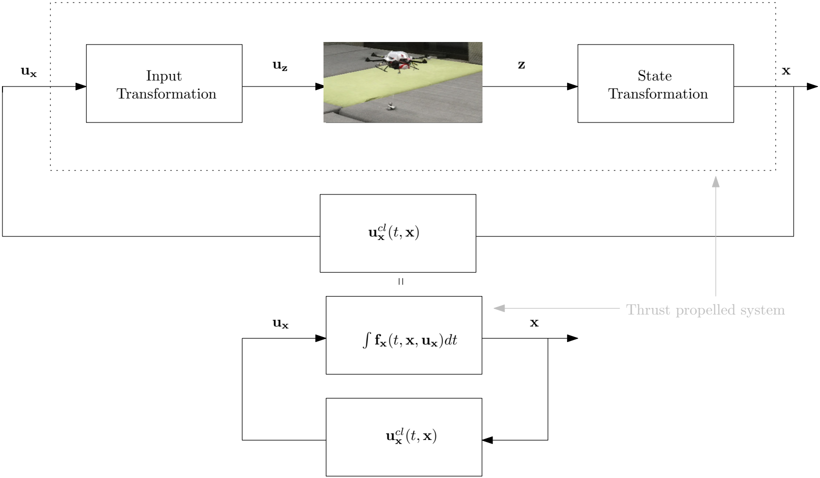

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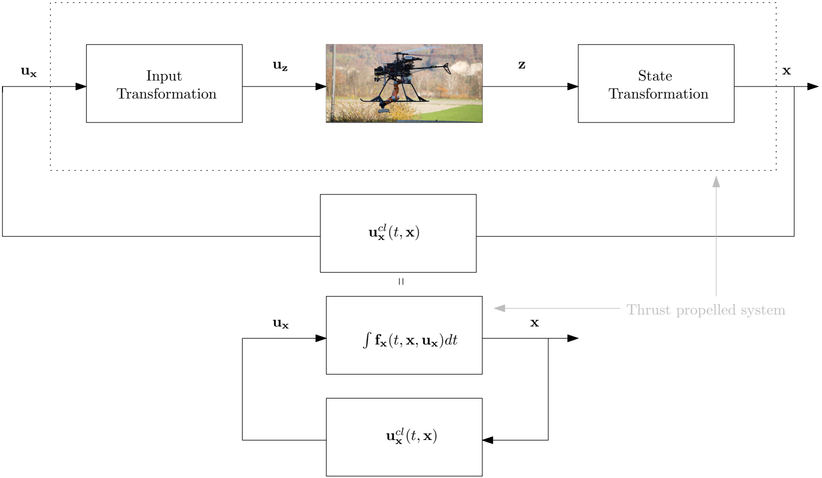
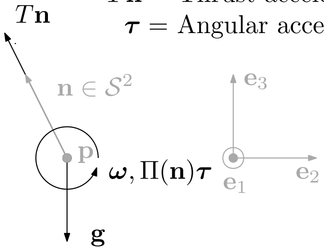


Thrust-propelled system

Vector field:

$$\dot{\mathbf{p}} = \mathbf{v}$$
$$\dot{\mathbf{v}} = T\mathbf{n} - \mathbf{g}(t)$$
$$\dot{\mathbf{n}} = \mathcal{S}(\boldsymbol{\omega})\mathbf{n}$$
$$\dot{\boldsymbol{\omega}} = \Pi(\mathbf{n})\boldsymbol{\tau}$$

$\mathbf{p}$ = position
$\mathbf{g}$ = gravity
$T\mathbf{n}$ = Thrust acceleration
$\boldsymbol{\tau}$ = Angular acceleration



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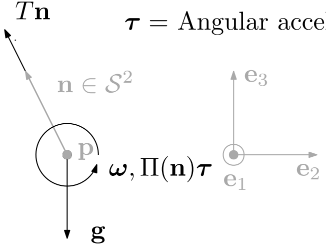

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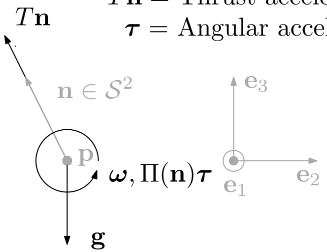


System model

Vector field:

$$\begin{aligned}\dot{\mathbf{p}} &= \mathbf{v}\\ \dot{\mathbf{v}} &= T\mathbf{n} - \mathbf{g}(t)\\ \dot{\mathbf{n}} &= \mathcal{S}(\boldsymbol{\omega})\mathbf{n}\\ \dot{\boldsymbol{\omega}} &= \Pi(\mathbf{n})\boldsymbol{\tau}\end{aligned}$$

State:

$$\begin{aligned}\mathbf{x} &= (\mathbf{p}, \mathbf{v}, \mathbf{n}, \boldsymbol{\omega})\\ \bar{\mathbf{x}} &= (\mathbf{p}, \mathbf{v}, \mathbf{n})\\ \boldsymbol{\xi} &= (\mathbf{p}, \mathbf{v})\end{aligned}$$

Input:

$$\mathbf{u} = (T, \boldsymbol{\tau})$$

Exogeneous input:

$$\mathbf{g} : [0, +\infty) \mapsto \mathbb{R}^3$$

System model

$$\dot{\mathbf{x}}(t) = \mathbf{f}(t, \mathbf{x}(t), \mathbf{u}(t))$$

$$\mathbf{f}(t, \mathbf{x}, \mathbf{u}) = \begin{bmatrix} \mathbf{v} \\ T\mathbf{n} - \mathbf{g}(t) \\ \mathcal{S}(\boldsymbol{\omega})\mathbf{n} \\ \Pi(\mathbf{n})\boldsymbol{\tau} \end{bmatrix}$$

State Space:

$$\mathbf{x} = (\mathbf{p}, \mathbf{v}, \mathbf{n}, \boldsymbol{\omega})$$
$$\Omega_{\mathbf{x}} = \{\mathbf{x} \in \mathbb{R}^{12} : \mathbf{n} \in \mathcal{S}^2, \boldsymbol{\omega}^T\mathbf{n} = 0\}$$
$$\mathbf{f}(\cdot, \mathbf{x}, \cdot) \in T_{\mathbf{x}}\Omega_{\mathbf{x}}$$

Objective

Control Objective

Design a control law

$$\mathbf{u}_{\mathbf{x}}^{cl} = (T^{cl}, \boldsymbol{\tau}^{cl}) : \mathbb{R}_{\geq 0} \times \Omega_{\mathbf{x}} \mapsto \mathbb{R}^4,$$

such that

$$\lim_{t \to \infty} \mathbf{p}(t) = \mathbf{0},$$

along any trajectory of $\dot{\mathbf{x}}(t) = \mathbf{f}(t, \mathbf{x}(t), \mathbf{u}_{\mathbf{x}}^{cl}(t, \mathbf{x}(t)))$.

Control Design Summary

$$\mathbf{f}(t,\mathbf{x},\mathbf{u}) = \begin{bmatrix} \mathbf{v} \\ T\mathbf{n} - \mathbf{g}(t) \\ \mathcal{S}(\boldsymbol{\omega})\mathbf{n} \\ \Pi(\mathbf{n})\boldsymbol{\tau} \end{bmatrix}$$

Steps

1. Position control: $\boldsymbol{\xi} = (\mathbf{p}, \mathbf{v})$
2. Kinematic attitude control: $\bar{\mathbf{x}} = (\boldsymbol{\xi}, \mathbf{n})$
3. Dynamic Attitude control: $\mathbf{x} = (\bar{\mathbf{x}}, \boldsymbol{\omega})$

1st Step

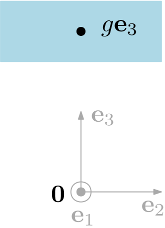
$$\mathbf{f}(t,\mathbf{x},\mathbf{u}) = \begin{bmatrix} \mathbf{f}_\xi(t,\bar{\mathbf{x}},T \qquad\qquad) \\ \mathbf{f}_n(\mathbf{n},\boldsymbol{\omega}) \\ \mathbf{f}_\omega(\mathbf{n},\boldsymbol{\tau}) \end{bmatrix}$$

$$= \begin{bmatrix} \begin{bmatrix} \mathbf{v} \\ T \qquad\qquad \mathbf{n} - \mathbf{g}(t) \end{bmatrix} \\ \mathcal{S}(\boldsymbol{\omega})\mathbf{n} \\ \Pi(\mathbf{n})\boldsymbol{\tau} \end{bmatrix}$$

1st Step

$$
\begin{aligned}
\mathbf{f}(t,\mathbf{x},\mathbf{u}) &= \begin{bmatrix} \mathbf{f}_\xi(t,\bar{\mathbf{x}},T^{cl}(t,\bar{\mathbf{x}})) \\ \mathrm{f}_n(\mathbf{n},\boldsymbol{\omega}) \\ \mathrm{f}_\omega(\mathbf{n},\boldsymbol{\tau}) \end{bmatrix} \\
&= \begin{bmatrix} \begin{bmatrix} \mathbf{v} \\ T^{cl}(t,\bar{\mathbf{x}})\mathbf{n} - \mathbf{g}(t) \end{bmatrix} \\ \mathcal{S}(\boldsymbol{\omega})\mathbf{n} \\ \Pi(\mathbf{n})\boldsymbol{\tau} \end{bmatrix}
\end{aligned}
$$

Constraints on time-varying gravity



Constraints on time-varying gravity

- $\mathbf{g} \in \mathcal{C}^2(\mathbb{R}_{\geq 0}, \mathcal{B}(g\mathbf{e}_3, \mathbf{r}))$
- $\sup_{t \geq 0} \|\mathbf{g}^{(i)}(t)\| < \infty, i \in \{0, 1, 2\}$

Example: $\mathbf{g}(t) = g\mathbf{e}_3 + r\omega^2 \begin{bmatrix} \cos(\omega t) \\ \sin(\omega t) \\ 0 \end{bmatrix}$

First Step

Double Integrator Controller

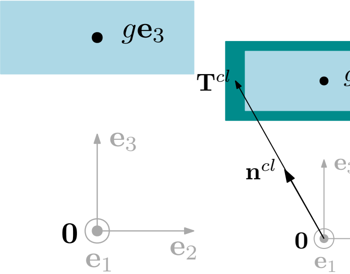
- $\ddot{\mathbf{p}} = \mathbf{u}_{di}(\mathbf{p}, \dot{\mathbf{p}})$
- $\mathbf{u}_{di} \in \mathcal{C}^2(\mathbb{R}^6, \{\mathbf{u} \in \mathbb{R}^3 : \|\mathbf{u}\| < g - r\})$
- $V_{di} \in \mathcal{C}^2(\mathbb{R}^6, \mathbb{R}^3)$

Full Thrust Actuation

- $\mathbf{T}^{cl}(t, \boldsymbol{\xi}) = \mathbf{g}(t) + \mathbf{u}_{di}(\boldsymbol{\xi})$

Desired Attitude

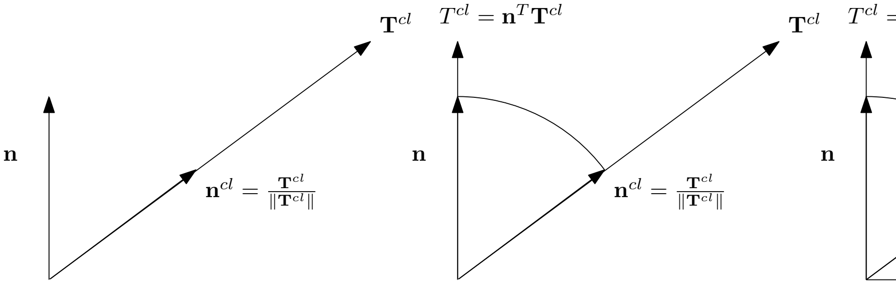
- $\mathbf{n}^{cl}(t, \boldsymbol{\xi}) = \frac{\mathbf{T}^{cl}(t,\boldsymbol{\xi})}{\|\mathbf{T}^{cl}(t,\boldsymbol{\xi})\|}$



First Step

Control Law for Thrust

- $T^{cl}(t, \bar{\mathbf{x}}) = \mathbf{n}^T \mathbf{T}^{cl}(t, \boldsymbol{\xi})$



End of First Step

Uncontrolled vector field

$$\mathbf{f}_\xi(t, \bar{\mathbf{x}}, \mathbf{n}, T) = \begin{bmatrix} \mathbf{v} \\ T\mathbf{n} - \mathbf{g}(t) \end{bmatrix}$$

Control Law for Thrust

- $T^{cl}(t, \bar{\mathbf{x}}) = \mathbf{n}^T \mathbf{T}^{cl}(t, \boldsymbol{\xi})$

Controlled Law for Thrust

$$\mathbf{f}_{\boldsymbol{\xi}}(t, \bar{\mathbf{x}}, T^{cl}(t, \bar{\mathbf{x}})) = \begin{bmatrix} \mathbf{v} \\ \mathbf{u}_{di}(\boldsymbol{\xi}) \end{bmatrix} + \underbrace{\begin{bmatrix} \mathbf{0} \\ \Pi(\mathbf{n}) T^{cl}(t, \bar{\mathbf{x}}) \end{bmatrix}}_{\text{error}}$$

2nd Step

$$
\mathbf{f}(t,\mathbf{x},\mathbf{u}) = \begin{bmatrix} \mathbf{f}_{\bar{\mathbf{x}}}(t,\bar{\mathbf{x}},\boldsymbol{\omega},T \qquad) \\ \mathbf{f}_{\omega}(\mathbf{n},\boldsymbol{\tau}) \end{bmatrix}
$$

$$
= \begin{bmatrix} \begin{bmatrix} \mathbf{f}_{\xi}(t,\bar{\mathbf{x}},T \qquad) \\ \mathbf{f}_{n}(\mathbf{n},\boldsymbol{\omega}) \end{bmatrix} \\ \mathbf{f}_{\omega}(\mathbf{n},\boldsymbol{\tau}) \end{bmatrix}
$$

2nd Step

$$\mathbf{f}(t,\mathbf{x},\mathbf{u}) = \begin{bmatrix} \mathbf{f}_{\bar{\mathbf{x}}}(t,\bar{\mathbf{x}},\boldsymbol{\omega},T^{cl}(t,\bar{\mathbf{x}})) \\ \mathbf{f}_{\omega}(\mathbf{n},\boldsymbol{\tau}) \end{bmatrix}$$

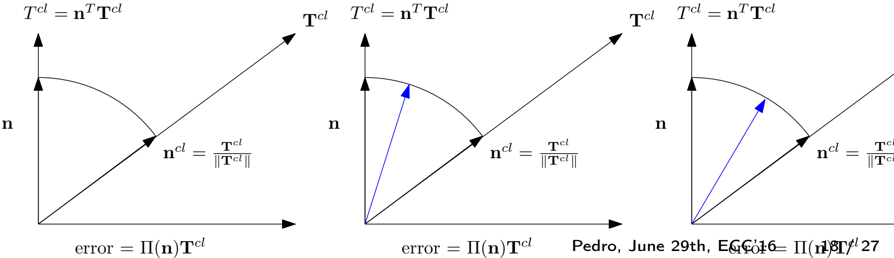
$$= \begin{bmatrix} \begin{bmatrix} \mathbf{f}_{\xi}(t,\bar{\mathbf{x}},T^{cl}(t,\bar{\mathbf{x}})) \\ \mathbf{f}_{n}(\mathbf{n},\boldsymbol{\omega}) \end{bmatrix} \\ \mathbf{f}_{\omega}(\mathbf{n},\boldsymbol{\tau}) \end{bmatrix}$$

Second Step

$$\mathbf{f}_{\bar{\mathbf{x}}}(t, \bar{\mathbf{x}}, \boldsymbol{\omega}, T^{cl}(t, \bar{\mathbf{x}})) = \begin{bmatrix} \mathbf{v} \\ \mathbf{u}_{di}(\boldsymbol{\xi}) \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \Pi(\mathbf{n}) T^{cl}(t, \bar{\mathbf{x}})\text{error} \\ \mathbf{f}_n(\mathbf{n}, \boldsymbol{\omega}) \end{bmatrix}$$

Kinematic attitude control

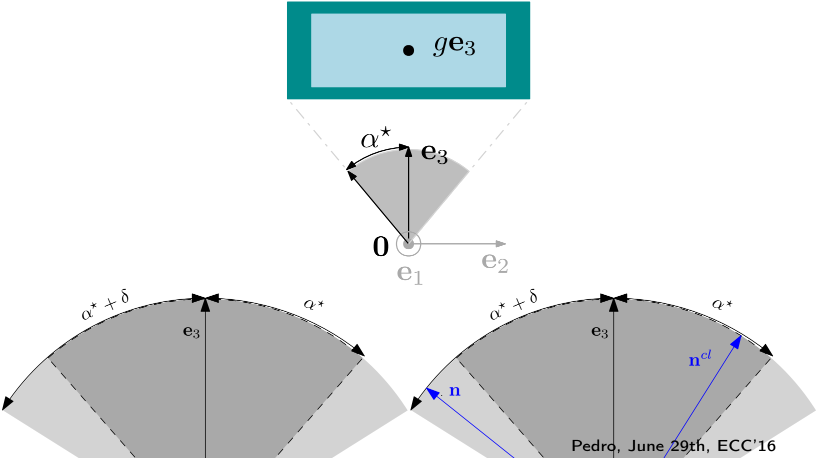
- Design angular velocity
- $\lim_{t\to\infty}(\mathbf{n}(t) - \mathbf{n}^{cl}(t, \boldsymbol{\xi}(t))) = \mathbf{0}$



Second Step

Kinematic Atittude control

- $\zeta(t, \bar{\mathbf{x}}) = 1 - \mathbf{n}^T \mathbf{n}^{cl}(t, \boldsymbol{\xi}) (= 1 - \cos(\theta))$
- $V_\theta \in \mathcal{C}^1([0, \epsilon), \mathbb{R}_{\geq 0})$ (control size of ζ)



$$\mathbf{f}_{\bar{\mathbf{x}}}(t, \bar{\mathbf{x}}, \boldsymbol{\omega}, T^{cl}(t, \bar{\mathbf{x}})) = \begin{bmatrix} \mathbf{v} \\ \mathbf{u}_{di}(\boldsymbol{\xi}) \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \Pi(\mathbf{n}) T^{cl}(t, \bar{\mathbf{x}}) \\ \mathbf{f}_n(\mathbf{n}, \boldsymbol{\omega}) \end{bmatrix}$$

Lyapunov function

$$\begin{aligned} V_{\bar{\mathbf{x}}}(\bar{\mathbf{x}}) =& V_{di}(\boldsymbol{\xi}) + V_{\theta}(\zeta(t, \bar{\mathbf{x}})) \\ W_{\bar{\mathbf{x}}}(\bar{\mathbf{x}}) =& -\frac{\partial V_{\bar{\mathbf{x}}}(\bar{\mathbf{x}})}{\partial \bar{\mathbf{x}}} \mathbf{f}_{\bar{\mathbf{x}}}(t, \bar{\mathbf{x}}, \boldsymbol{\omega}, T^{cl}(t, \bar{\mathbf{x}})) \\ =& W_{di}(\boldsymbol{\xi}) + V_{\theta}'(\mathcal{S}(\mathbf{n})\mathbf{n}^{cl})^T \left(\boldsymbol{\omega} - \boldsymbol{\omega}^{\mathbf{n}^{cl}} + \frac{\|\mathbf{T}^{cl}\|}{V_{\theta}'} \mathcal{S}(\mathbf{n}) \frac{\partial V_{di}}{\partial \boldsymbol{\xi}} \right) \end{aligned}$$

Control Objective

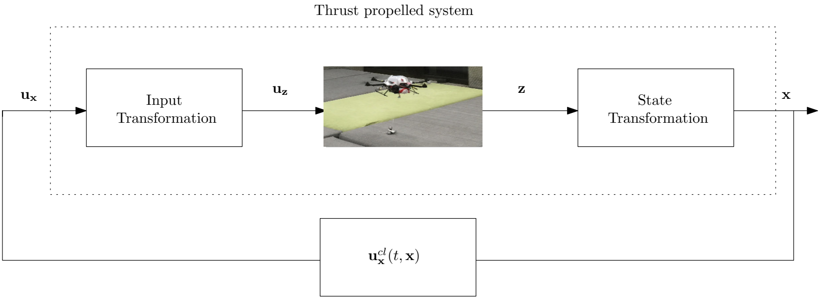
Design a control law

$$\mathbf{u}_{\mathbf{x}}^{cl} = (T^{cl}, \boldsymbol{\tau}^{cl}) : \mathbb{R}_{\geq 0} \times \Omega_{\mathbf{x}} \mapsto \mathbb{R}^4,$$

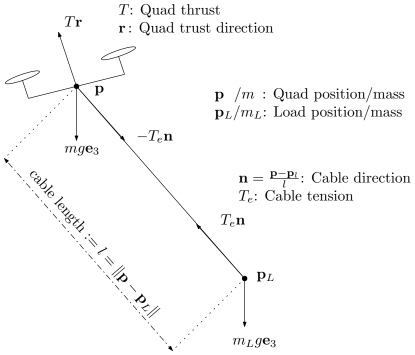
such that $\lim_{t\to\infty} \mathbf{p}(t) = \mathbf{0}$, along any trajectory of $\dot{\mathbf{x}}(t) = \mathbf{f}(t, \mathbf{x}(t), \mathbf{u}_{\mathbf{x}}^{cl}(t, \mathbf{x}(t)))$.

$$\frac{\partial V_{\mathbf{x}}(\mathbf{x})}{\partial \mathbf{x}} \mathbf{f}(t, \mathbf{x}, \mathbf{u}_{\mathbf{x}}^{cl}(t, \mathbf{x})) \leq -w(\mathbf{p})$$

Simulation



Pereira, Herzog Dimarogonas. Slung Load Transportation with a Single Aerial Vehicle and Disturbance Removal. MED'16

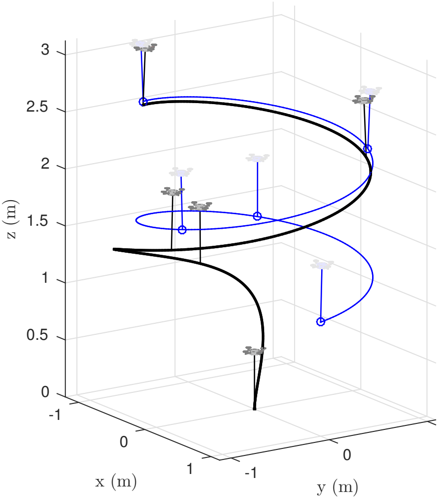


Thrust propelled state:

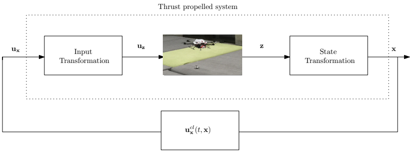
- $\mathbf{p}$: position tracking error
- $\mathbf{v}$: position tracking error
- $\mathbf{n}$: cable unit vector
- $\boldsymbol{\omega}$: cable angular velocity

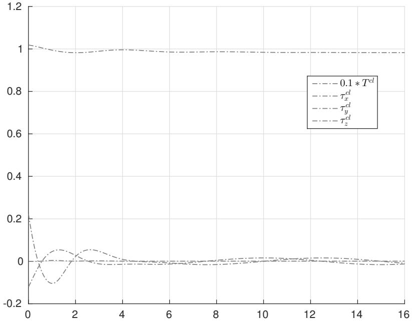
Pereira, Herzog Dimarogonas. Slung Load Transportation with a Single Aerial Vehicle and Disturbance Removal. MED'16

- Desired motion: load to describe a helix



- Input from controller





Conclusions

Summary

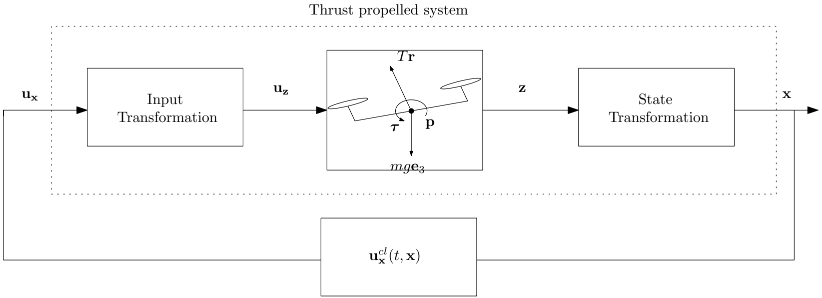
- Controller for thrust propelled system
- Applicable to different systems

Future Work

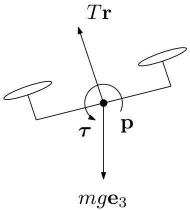
- Experimentally validate controller
- Study its robustness against disturbances
  - model uncertainty, input bias, ...

Thank you! Questions?

Simulation



Thrust propelled system
$\mathbf{u_x}$
Input
Transformation
$\mathbf{u_z}$
$T\mathbf{r}$
$\boldsymbol{\tau}$
$\mathbf{p}$
$mg\mathbf{e}_3$
$\mathbf{z}$
State
Transformation
$\mathbf{x}$
$\mathbf{u}_\mathbf{x}^{cl}(t,\mathbf{x})$



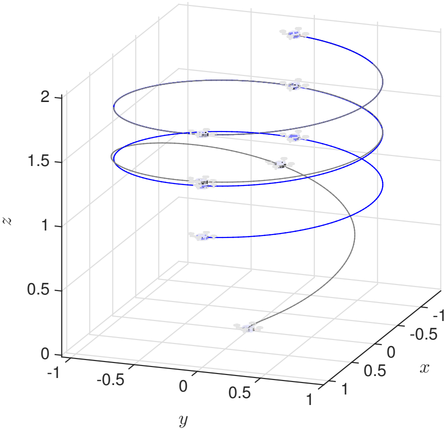
$\mathbf{p}/m$: Position/mass
T: Thrust
$\mathbf{r}$: Thust direction
$\boldsymbol{\tau}$: Torque

Thrust propelled state:

- $\mathbf{p}$: position tracking error
- $\mathbf{v}$: velocity tracking error
- $\mathbf{n}$: unit vector orthogonal to propellers plane
- $\boldsymbol{\omega}$: angular velocity

Simulation

- Desired motion: load to describe a helix



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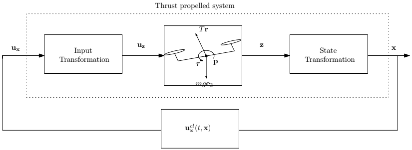

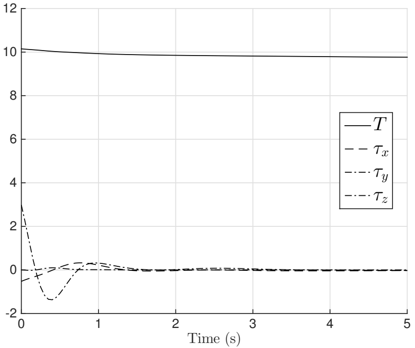

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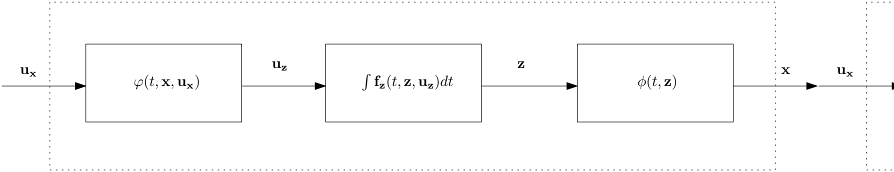

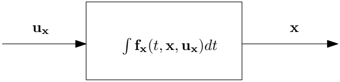

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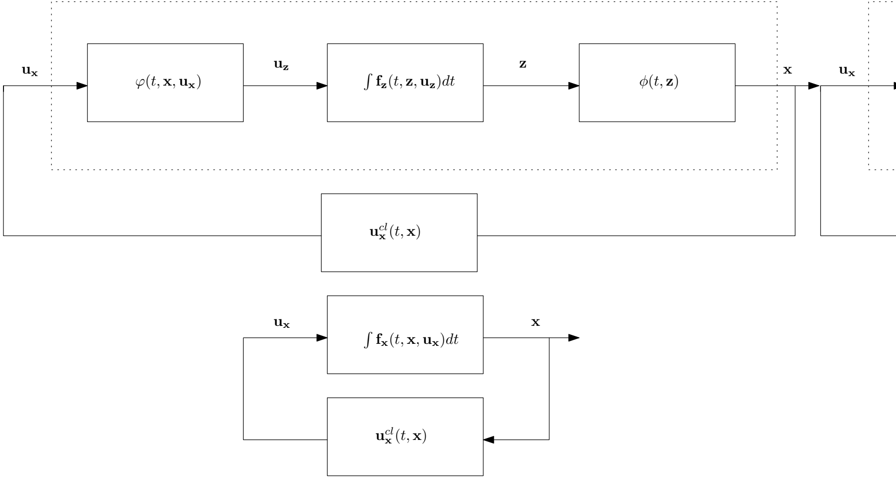

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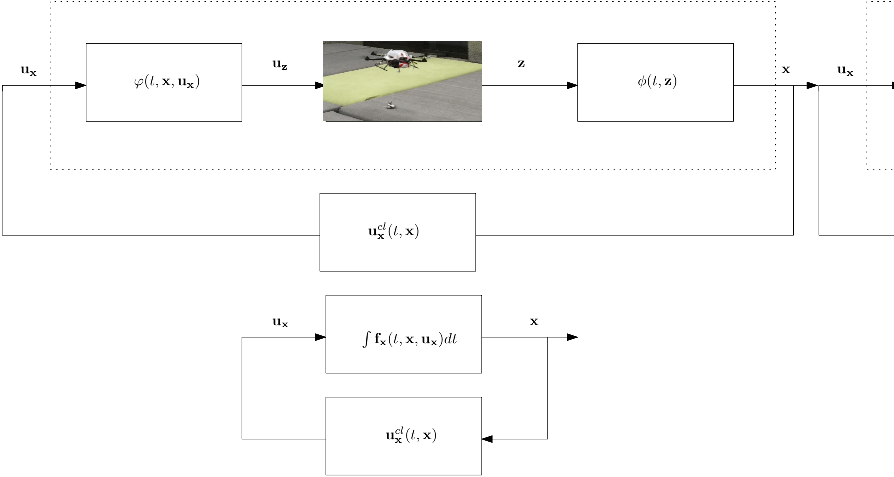

Summary

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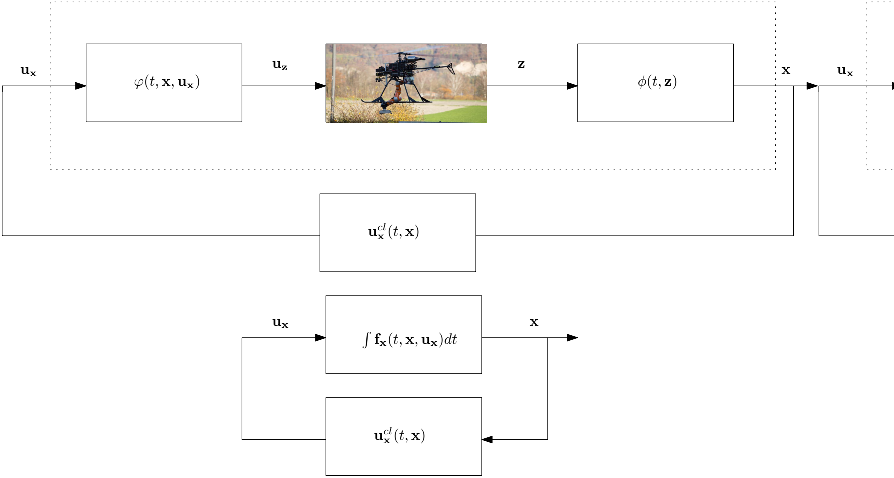

Second Step

$$\mathbf{f}_{\bar{\mathbf{x}}}(t, \bar{\mathbf{x}}, \boldsymbol{\omega}, T^{cl}(t, \bar{\mathbf{x}})) = \begin{bmatrix} \mathbf{v} \\ \mathbf{u}_{di}(\boldsymbol{\xi}) \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \Pi(\mathbf{n}) T^{cl}(t, \bar{\mathbf{x}}) \\ \mathbf{f}_n(\mathbf{n}, \boldsymbol{\omega}) \end{bmatrix}$$

Lyapunov function

$$\begin{aligned} V_{\bar{\mathbf{x}}}(\bar{\mathbf{x}}) =& V_{di}(\boldsymbol{\xi}) + V_\theta(\zeta(t, \bar{\mathbf{x}})) \\ W_{\bar{\mathbf{x}}}(\bar{\mathbf{x}}) =& -\frac{\partial V_{\bar{\mathbf{x}}}(\bar{\mathbf{x}})}{\partial \mathbf{x}} \mathbf{f}_{\bar{\mathbf{x}}}(t, \bar{\mathbf{x}}, \boldsymbol{\omega}, T^{cl}(t, \bar{\mathbf{x}})) \\ =& \underbrace{W_{di}(\boldsymbol{\xi}) - \frac{\partial V_{di}}{\partial \mathbf{v}} \Pi(\mathbf{n}) \mathbf{T}^{cl}}_{\frac{\partial V_{di}}{\partial \boldsymbol{\xi}} \mathbf{f}_{\boldsymbol{\xi}}} + \underbrace{V_\theta'(\mathcal{S}(\mathbf{n})\mathbf{n}^{cl})^T(\boldsymbol{\omega} - \boldsymbol{\omega}^{\mathbf{n}^{cl}})}_{\frac{\partial V_\theta}{\partial \bar{\mathbf{x}}} \mathbf{f}_{\bar{\mathbf{x}}}} \end{aligned}$$