Asymmetric Collaborative Bar Stabilization Tethered to Two Heterogeneous Aerial Vehicles

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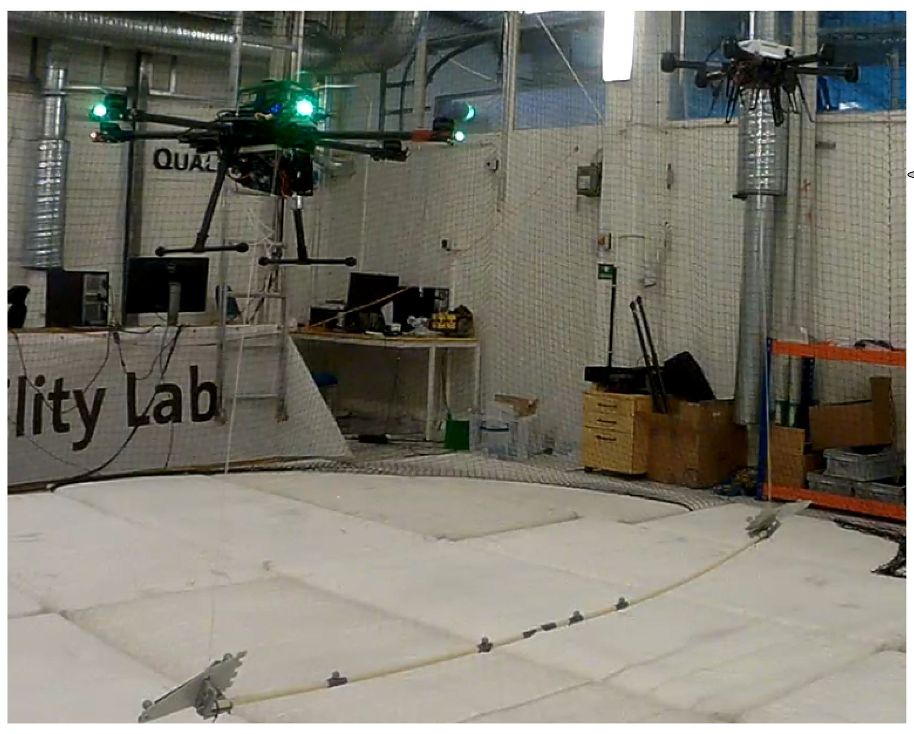
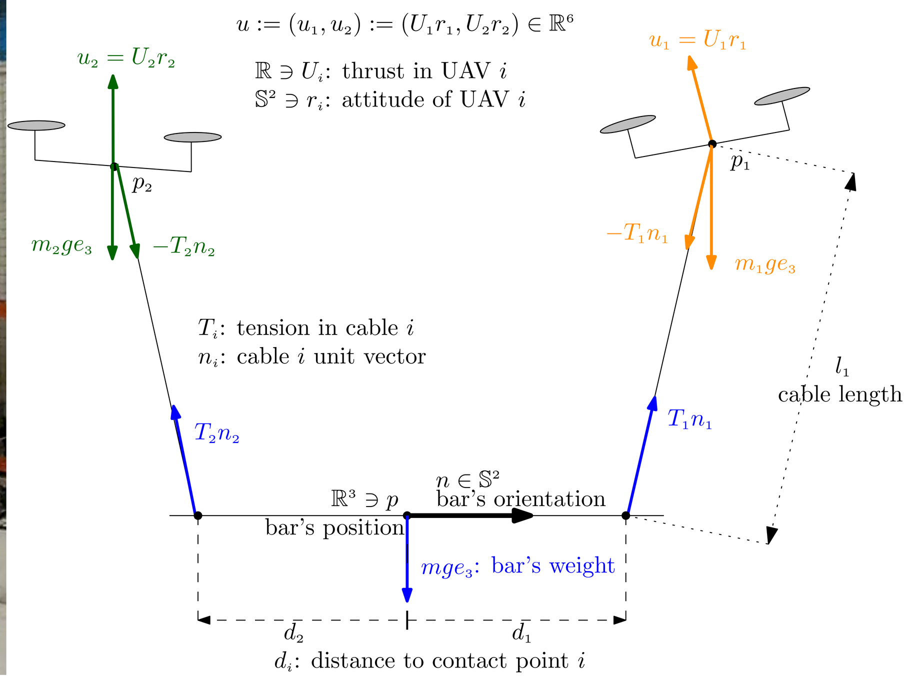
Motivation





Goal of collaborative transportation:

- Transportation of heavy cargos: individual UAVs' payload capacity exceeded
- Pose stabilization: requires two or more UAVs

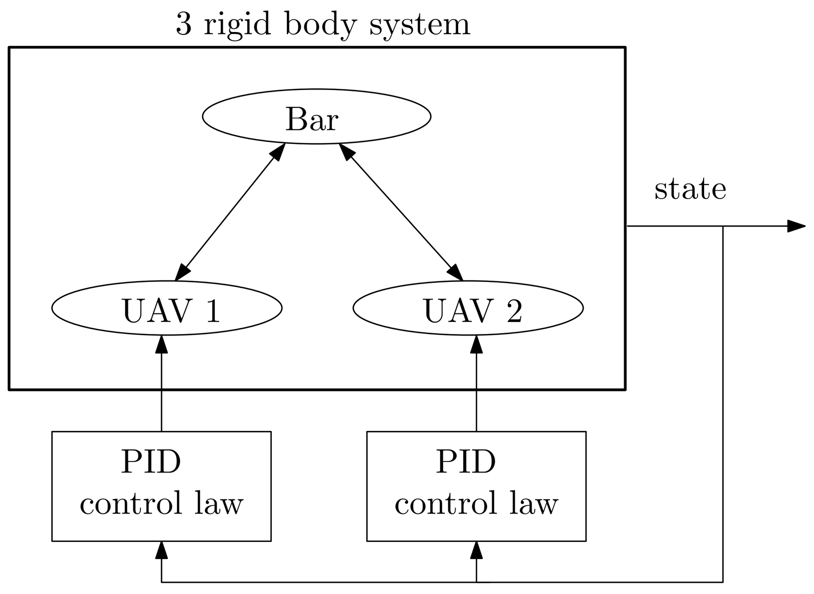
Tethered transportation:

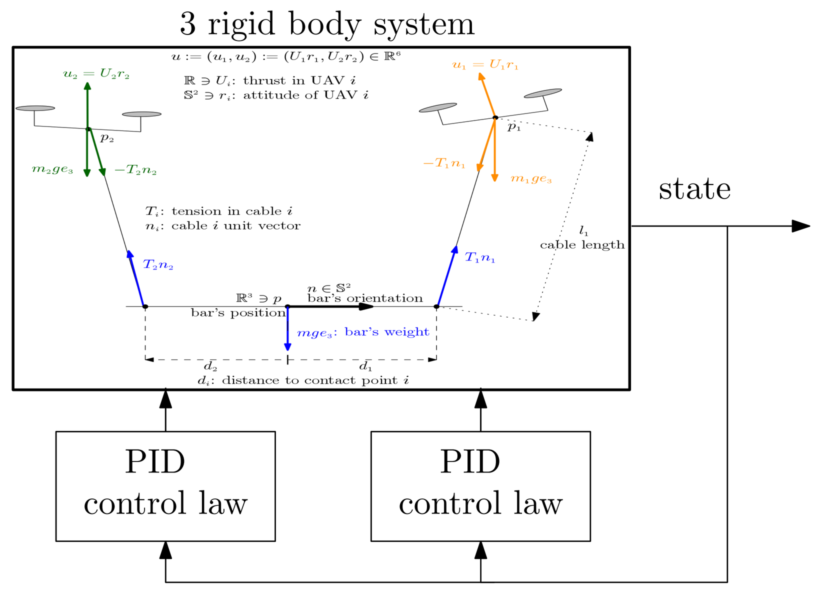
- Mechanically simple
- It does not consume useful payload capacity

Asymmetries:

- Non-identical UAVs: m_1, m_2
- Cables of different lengths: l_1, l_2
- Grasping points placed asymmetrically w.r.t. to bar's center-of-mass: d_1, d_2

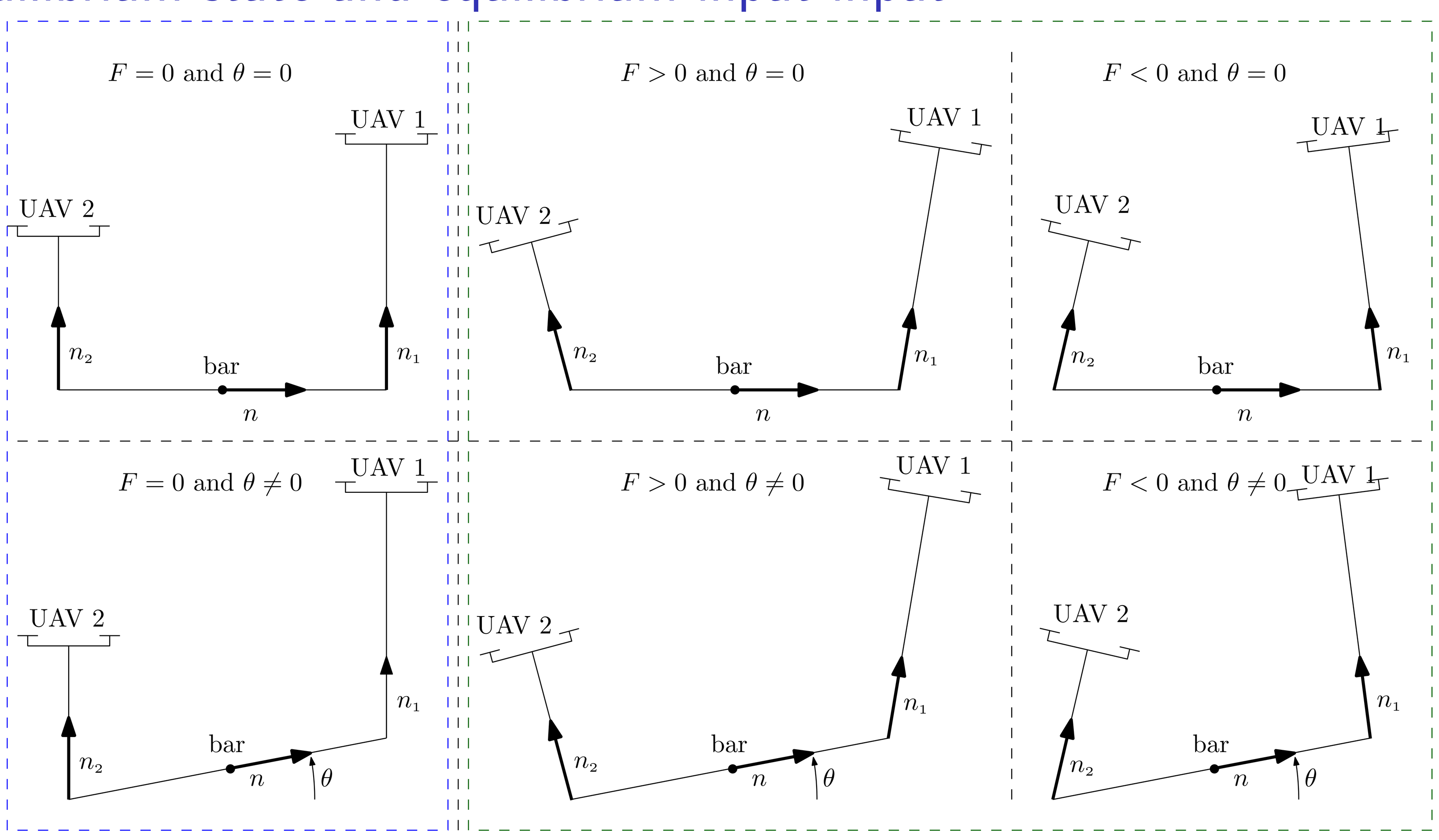
Problem and Strategy





- Modeling of system of three rigid bodies
  - Two geometric constraints imposed by each cable
  - Distance between UAV and grasping point on the bar is equal to cable length
  - Linearization around point in manifold requires special attention
- PID control laws on each UAV
- Relations between gains that guarantee stability

Equilibrium state and equilibrium input input



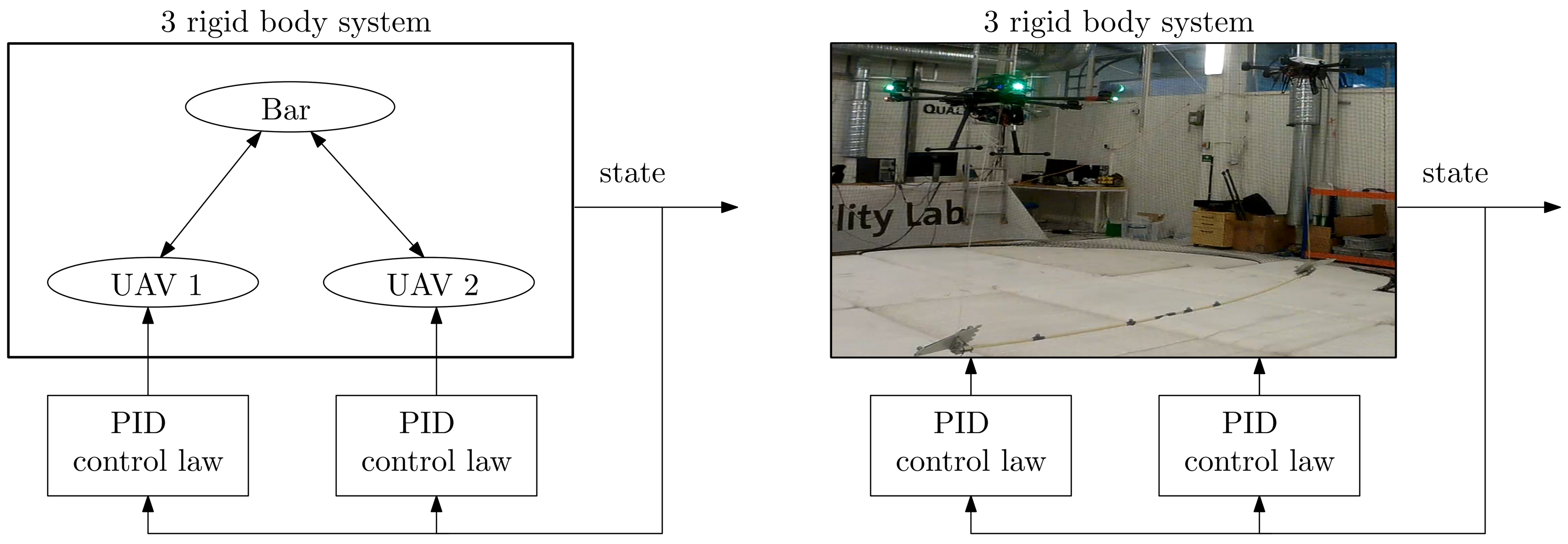
θ = pitch angle and F= normal force

- Equilibrium state:
  - Stabilize bar on the horizontal plane
  - Stabilize bar under no normal stress
- Equilibrium state requires non-zero equilibrium input along vertical direction

$$u_i^* = \underbrace{m_i g e_3}_{\text{UAV's weight}} + \underbrace{\frac{d_j}{d_1 + d_2} m g e_3}_{\text{portion of bar's weight}}$$

- Equilibrium input: requires exact model knowledge
- Other equilibria: requires separate stability analysis

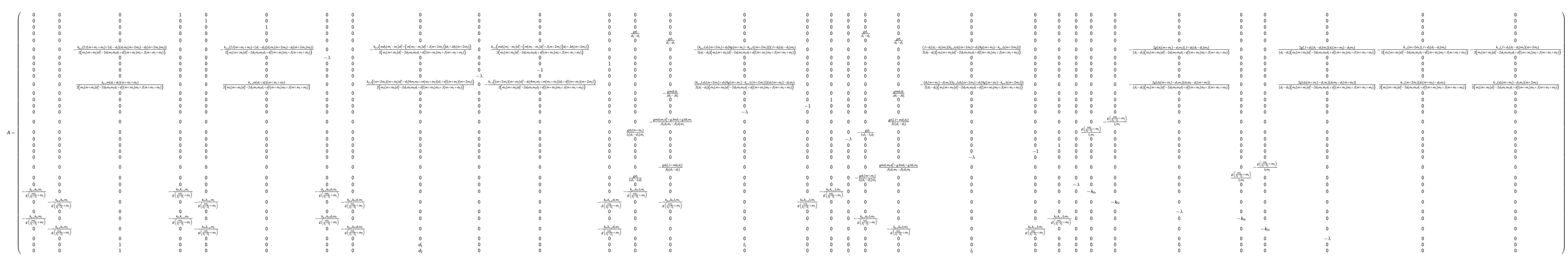
Control laws



$$u_i^{\text{PID}} = \begin{bmatrix} u^{\text{PD}}_{i,\text{longitudinal}} \\ u^{\text{PD}}_{i,\text{lateral}} \\ u_i^* + u^{\text{PID}}_{i,\text{vertical}} \end{bmatrix} = \begin{bmatrix} \text{PD} \\ \text{PD}_{\text{linear}} + \text{PD}_{\text{angular}} \\ \text{UAV's weight} + \text{Portion of bar's weight} + \text{PID} \end{bmatrix}$$

- Integral term along vertical direction compensates for model mismatch

Linearization



- Jacobian $A \in \mathbb{R}^{32\times32}$
- Find appropriate change of coordinates $P \in \mathbb{R}^{32\times32}$

Change of coordinates

Arbitrary gains

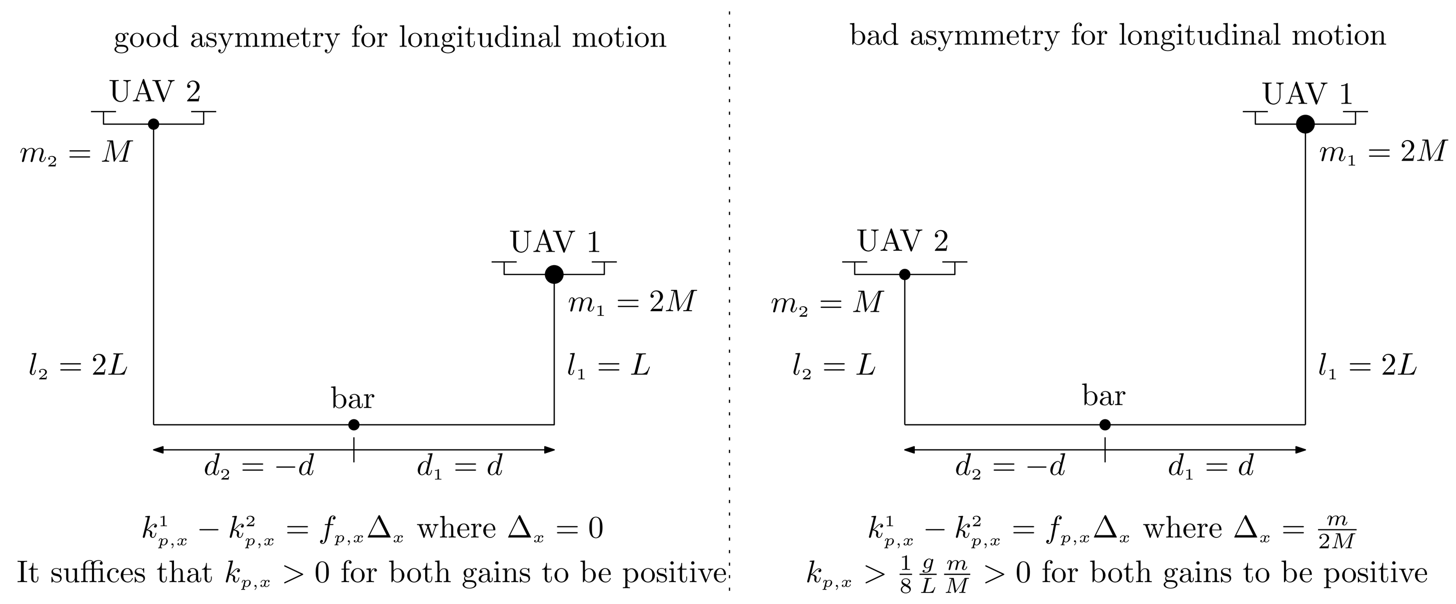
$$PAP^{-1} = \begin{bmatrix} \begin{matrix} A_z & \star \\ \star & A_\theta \end{matrix} & & & \\ & \begin{matrix} A_x & \star \\ \star & A_\delta \end{matrix} & & \star \\ & & \begin{matrix} A_y & \star \\ \star & A_\psi \end{matrix} & \\ & 0_{8\times32} & & -\lambda I_{8\times8} \end{bmatrix}$$

Choose gains such that

$$PAP^{-1} = \begin{bmatrix} \begin{matrix} A_z & 0 \\ \star & A_\theta \end{matrix} & & & \\ & \begin{matrix} A_x & 0 \\ \star & A_\delta \end{matrix} & & \star \\ & & \begin{matrix} A_y & 0 \\ \star & A_\psi \end{matrix} & \\ & 0_{8\times32} & & -\lambda I_{8\times8} \end{bmatrix}$$

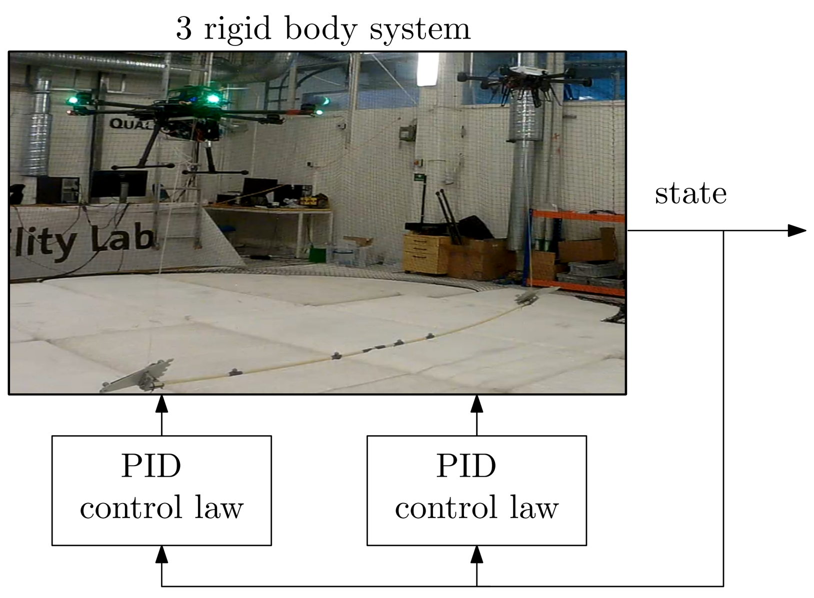
- Jacobian A is similar to PAP^{-1} (thus, they share the same eigenvalues)
- Three decoupled motions: vertical + longitudinal + lateral
- Each motion: linear component + angular component
- Choose gains that *cancel* asymmetries and render block triangular structure

Longitudinal motion



- Longitudinal proportional gains: $k^i_{p,x} = k_{p,x} + \frac{d_i l_i}{d_1 l_1 + d_2 l_2} f_{p,x}\Delta_x$
- Quantification of asymmetry: $\Delta_x = \frac{m(m_1 d_1 l_1 + m_2 d_2 l_2)}{m_1 m_2 (d_1 l_1 - d_2 l_2)}$
- Good/Bad asymmetry: gap $|\Delta_x|$ between gains is small/big

Main result



Link to video

Pose stabilization is asymptotically stable if

- Attitude inner loop is sufficiently fast
- Integral gain is sufficiently small