Control of Single and Multiple Thrust Propelled Systems

Pedro Miguel Ótão Pereira

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October 19th

Control of TPS

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- Presentation at AEROWORKS Autumn School
- Thursday, October 19th, 2017
- 16:15-17:00
- PhD student at KTH, under the supervision of Dimos Dimarogonas

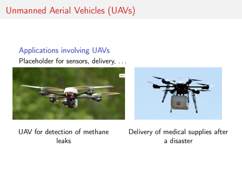
Unmanned Aerial Vehicles (UAVs)

Applications involving UAVs

Placeholder for sensors, delivery, . . .

UAV for detection of methane leaks

Delivery of medical supplies after a disaster



- Automate inspection, monitoring and delivery
- Relief support in natural (or non-natural) disasters

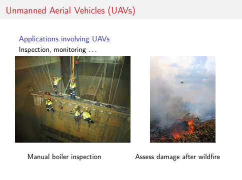
Unmanned Aerial Vehicles (UAVs)

Applications involving UAVs

Inspection, monitoring ...

Manual boiler inspection

Assess damage after wildfire



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Unmanned Aerial Vehicles (UAVs)

Rotorcraft (vs fixed wing aircraft)

- Ability to hover
- Ability to take-off and land vertically

Helicopter vs Multirotors

- Helicopter: mechanically complex
- Multicopters/Multirotors: control of differential rotation (mechanically simpler)

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Helicopter: mechanically complex
Multicopters/Multirotors: control of differential rotation (mechanically simpler)

- Brief taxonomy
- Rotorcraft: rotary wing aircraft
- Multirotors: high maneuverability, low maintenance costs, low mechanical complexity and their ability to hover and to take-off and land vertically;
- Airplanes cannot hover or VTOL (the advantage is the large forward speed)
- Single-rotor (Helicopter): mechanically complex (swash plate is mechanically complex); Multirotors: control of differential rotation

AEROWORKS: European project

- Develop a team of aerial robotic workers

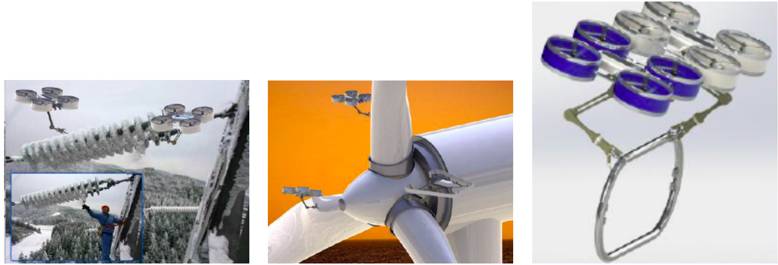
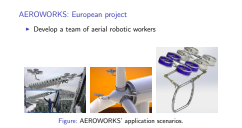


Figure: AEROWORKS' application scenarios.



- Five (six) academic partners: Lulea University of Technology – LTU, Swiss Federal Institute of Technology – ETHZ, Royal Institute of Technology – KTH, University of Twente – UT, University of Edinburgh – UEDIN, University of Patras – UPAT
- One (two) aerial robotics innovation SMEs: Ascending Technologies – ASC, Skybotix – SBX
- Two service robotics specialists: Alstom Inspection Robotics - AIR, and Skelleftea Kraft – SKL
- Removing humans from dangerous environments by performing aerial automated inspection.
- An automated fleet of aerial vehicles performing inspection and repair tasks on a wind mill.
- Cooperation between aerial vehicle when transporting a corona ring from an electrical pole.

Tethering vs Manipulator

Motivation

- Transportation of payloads in inaccessible locations
- Mechanically simplicity *vs* maneuverability
- Different useful payload capacity

Figure: Fukushima Nuclear Plant

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- Robotic arm provides extra degrees of freedom

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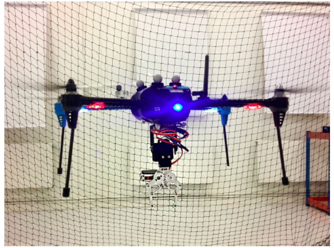

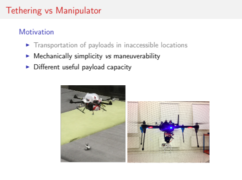

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Overview

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Motivation

Modeling of system of rigid bodies

Control of Thrust-propelled Systems

Conditions for local stability

Summary

Modeling: Point mass

- State

$$\begin{bmatrix} p \\ v \end{bmatrix} = \begin{bmatrix} \text{position} \in \mathbb{R}^3 \text{ w.r.t. I.F.} \\ \text{velocity} \in \mathbb{R}^3 \text{ w.r.t. I.F.} \end{bmatrix}$$

- Equations of motion

$$\begin{bmatrix} \dot{p} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} v \\ \frac{1}{m} F_{net} \end{bmatrix} = \begin{bmatrix} \text{kinematics} \\ \text{dynamics} \end{bmatrix}$$

- F_{net} is the net force applied on point mass
- m is the mass of the point mass

Modeling: Point mass
State

- Point mass kinematics and dynamics
- Net force = change of linear momentum
- I.F. = Inertial Frame

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Modeling: Rigid Body

- State

$$\begin{bmatrix} p \\ R \\ v \\ \omega \end{bmatrix} = \begin{bmatrix} \text{linear position} \in \mathbb{R}^3 \text{ w.r.t. I.F.} \\ \text{angular position} \in \mathbb{SO}(3) \text{ w.r.t. I.F.} \\ \text{linear velocity} \in \mathbb{R}^3 \text{ w.r.t. I.F.} \\ \text{angular velocity} \in \mathbb{R}^3 \text{ w.r.t. B.F.} \end{bmatrix}$$

- Equations of motion

$$\begin{bmatrix} \dot{p} \\ \dot{R} \end{bmatrix} = \begin{bmatrix} v \\ RS(\omega) \end{bmatrix} = \text{kinematics}$$

$$\begin{bmatrix} \dot{v} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} \frac{1}{m} F_{net} \\ J^{-1}\left(-S(\omega)J\omega + \tau_{net}\right) \end{bmatrix} \Big|_{W_{net}=(F_{net},\tau_{net})} = \text{dynamics}$$

- W_{net} is the net wrench applied on rigid body
- m/J is the mass/moment-of-inertia of the rigid body

- Rigid body kinematics and dynamics
- Net wrench (net force, and net torque)
- Newton's-Euler's equations of motion
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Modeling: System of Rigid Bodies and/or Point masses

- example: 1 point mass + 1 rigid body
- Internal forces pair: $(+F_{internal}, -F_{internal})$
- Net forces on point-mass

$$F_{net,1} = \underbrace{m_1 g e_3}_{\text{weight}} + \underbrace{F_{input,1}}_{\text{input force}} + \underbrace{(+F_{internal})}_{\text{internal force}}$$

- Net wrench on rigid-body

$$W_{net,2} = \begin{bmatrix} F_{net,2} \\ \tau_{net,2} \end{bmatrix}$$

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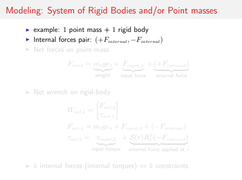
$$\tau_{net,2} = \underbrace{\tau_{input,2}}_{\text{input torque}} + \underbrace{S(r)R_2^T(-F_{internal})}_{\text{internal force applied at } r}$$

- k internal forces (internal torques) $\Leftrightarrow$ k constraints



- Weight does not apply any torque on rigid-body (since point of application coincides with center-of-mass; this is only true if gravity field is assumed uniform – reasonable assumption for *small* rigid bodies)
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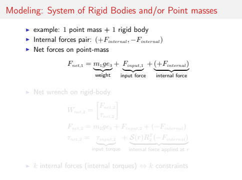

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Modeling: System of Rigid Bodies and/or Point masses

- example: 1 point mass + 1 rigid body
- Internal forces pair: $(+F_{internal}, -F_{internal})$
- 3 (holonomic) constraints

$$\{(p_1, p_2, R_2) \in \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{SO}(3) : C(p_1, p_2, R_2) = 0_3\} =: \mathbb{X}_{kinematic}$$

- Internal forces $F_{internal}$ are such that

$$\frac{d^2}{dt^2} C(p_1, p_2, R_2) = 0_3$$

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- Suppose in our example (1 point mass + 1 rigid body) we have 3 internal forces
- holonomic constraints: only depend on kinematic variables (i.e., don't depend on twists, or accelerations, ...)
- $\mathbb{X}_{kinematic}$: domain which all the kinematic variables belong to

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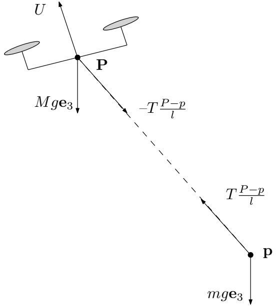
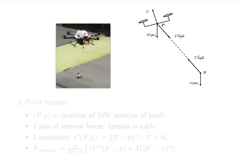
Example: Slung-load



2 Point-masses

- $(P, p) =$ (position of UAV, position of load)
- 1 pair of internal forces: tension in cable
- 1 constraint: $C(P, p) := \|P - p\|^2 - l^2 = 0_1$
- $F_{internal} = \frac{m}{M+m}\frac{1}{l}(U^T(P-p) + M\|V - v\|^2)$



- Massless cable
- Cable imposes 1 holonomic constraint
- $U \in \mathbb{R}^3$ is taken as an input force on UAV

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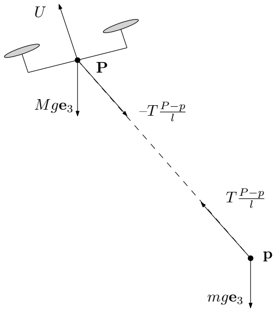

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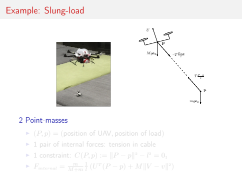

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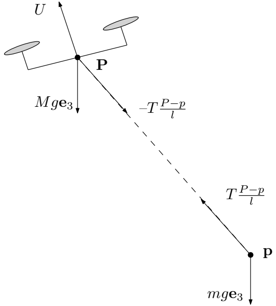

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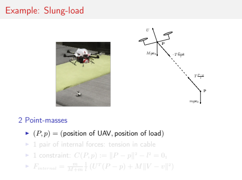

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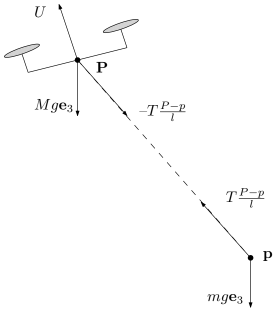

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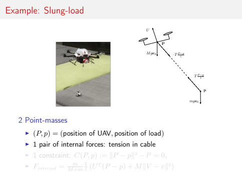

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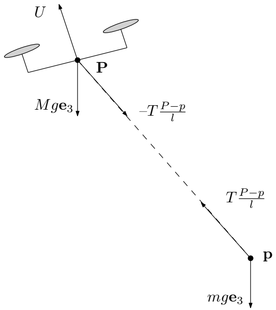

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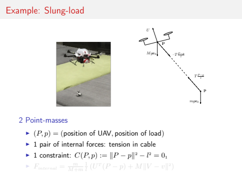

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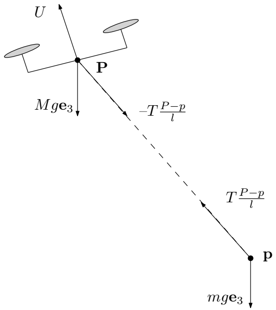

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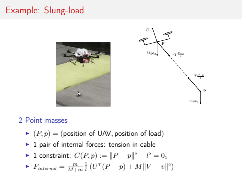
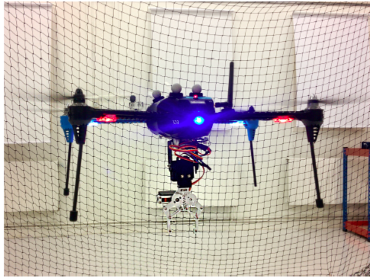
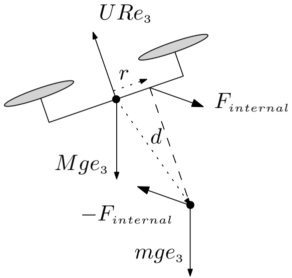


- Massless cable
- Cable imposes 1 holonomic constraint
- $U \in \mathbb{R}^3$ is taken as an input force on UAV

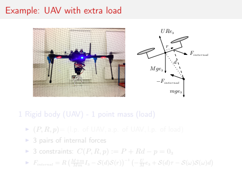
Example: UAV with extra load





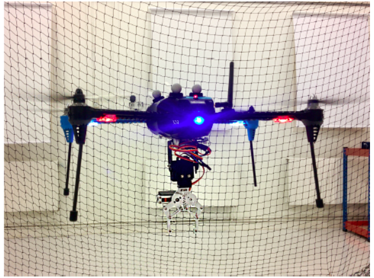
1 Rigid body (UAV) - 1 point mass (load)

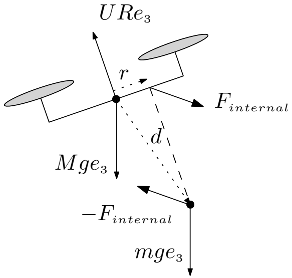
- (P, R, p)= (l.p. of UAV, a.p. of UAV, l.p. of load)
- 3 pairs of internal forces
- 3 constraints: $C(P, R, p) := P + Rd - p = 0_3$
- $F_{internal} = R\left(\frac{M+m}{Mm} I_3 - \mathcal{S}(d)\mathcal{S}(r)\right)^{-1}\left(-\frac{U}{M}e_3 + \mathcal{S}(d)\tau - \mathcal{S}(\omega)\mathcal{S}(\omega)d\right)$



- l.p. = linear position; a.p. = angular position
- For brevity, I took $J = I_3$: in mathematica the internal forces for generic moment of inertia are found.

Example: UAV with extra load





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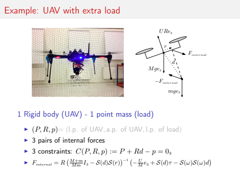

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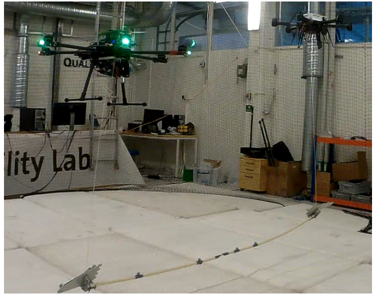
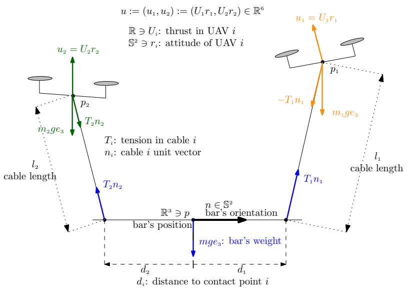
Example: 2 UAVs + 1 bar

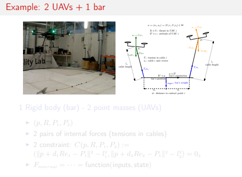




1 Rigid body (bar) - 2 point masses (UAVs)

- (p, R, P_1, P_2)
- 2 pairs of internal forces (tensions in cables)
- 2 constraint: $C(p, R, P_1, P_2) := (\|p + d_1 R e_1 - P_1\|^2 - l_1^2, \|p + d_2 R e_2 - P_1\|^2 - l_2^2) = 0_2$
- $F_{internal} = \cdots =$ function(inputs, state)

Pedro



- Pereira, P. and Dimarogonas, D.V. *Collaborative Transportation of a Bar by Two Aerial Vehicles with Attitude Inner Loop and Experimental Validation* CDC, 2017 (to appear).

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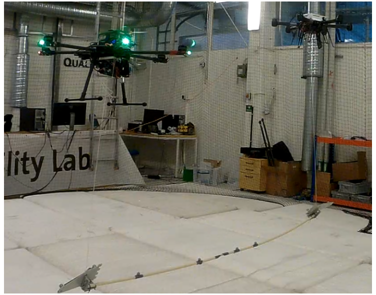

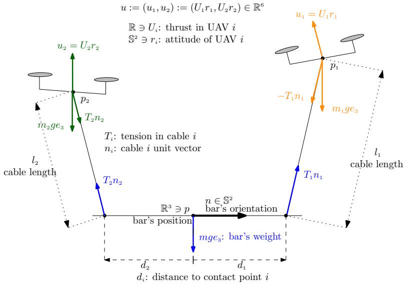

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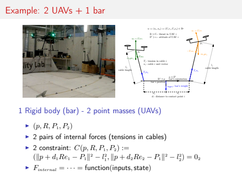

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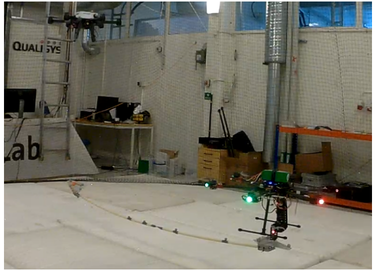
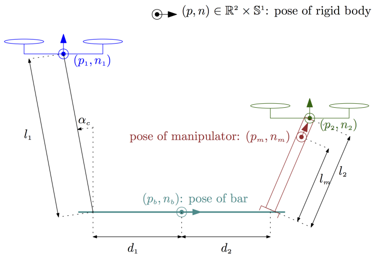
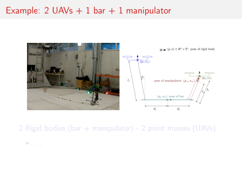
Example: 2 UAVs + 1 bar + 1 manipulator





2 Rigid bodies (bar + manipulator) - 2 point masses (UAVs)

- ...



- Roque, P., Pereira, P. and Dimarogonas, D.V. *Heterogeneous Collaborative Pose Stabilization of a Bar Via Aerial Manipulation and Tethering* ICRA, 2018 (submitted).

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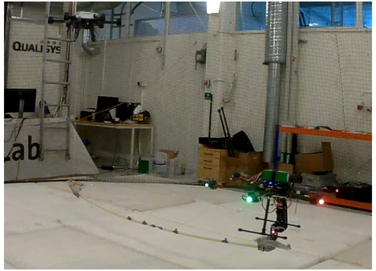

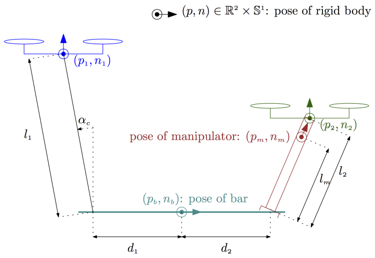

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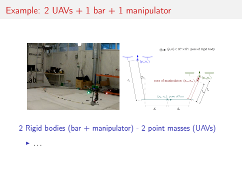

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Final remarks

- Model necessary for design/analysis
- Parameterizations: smaller set of coordinates

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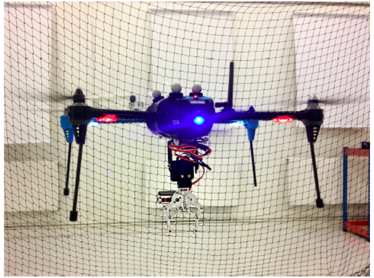
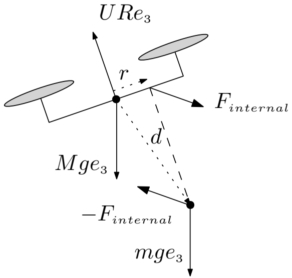
- Model necessary for design/analysis
- Parameterizations: smaller set of coordinates

- Other parameterizations: Euler-angles – see Professor Tzes' talk
- Other parameterizations: unit-quaternion – see Emil Fresk's talk
- I chose the $\mathbb{SO}(3)$ group to be parametrized by 3 by 3 matrices (orthonormal matrices)

Example: UAV with extra load





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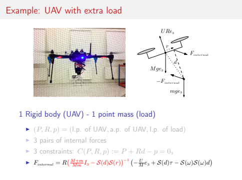


- Complexity of internal forces
- If we have k internal forces we have to invert a $k \times k$ matrix
- In this example we have to invert a 3×3 matrix
- See Professor Tzes' talk on modeling complexity

Overview

Overview

Motivation

Modeling of system of rigid bodies

Control of Thrust-propelled Systems

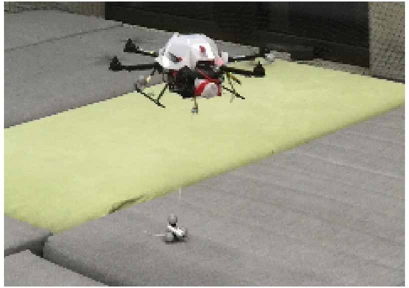
Conditions for local stability

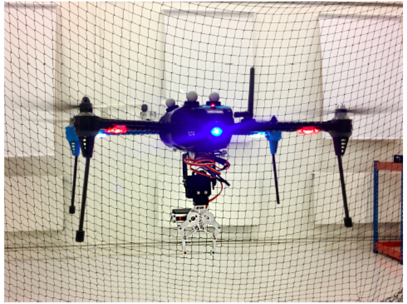
Summary

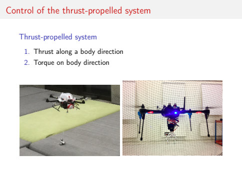
Control of the thrust-propelled system

Thrust-propelled system

1. Thrust along a body direction
2. Torque on body direction

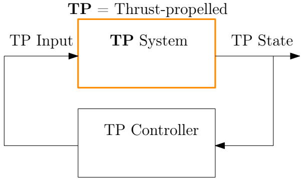




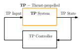


- We wish to design a controller that can be used for different systems
- These systems have something in common, which is what we explore in the control design
- Thrust-propelled systems, for example, trajectory tracking by quadrotor, trajectory tracking by load tethered to UAV, trajectory tracking with manipulator coupled with UAV

Control of the thrust-propelled system

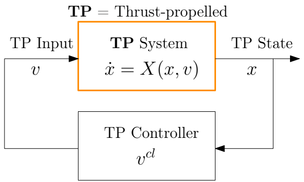


Control of the thrust-propelled system

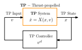


- Design state and input transformations
- Change original vector field, by finding maps that change the input and the state
- Transform physical vector field into vector field of thrust propelled sytem

Control of the thrust-propelled system



Control of the thrust-propelled system



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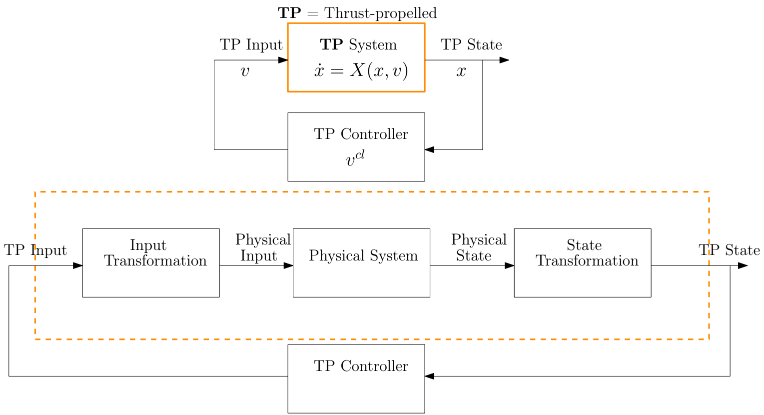

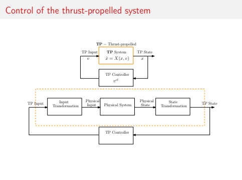

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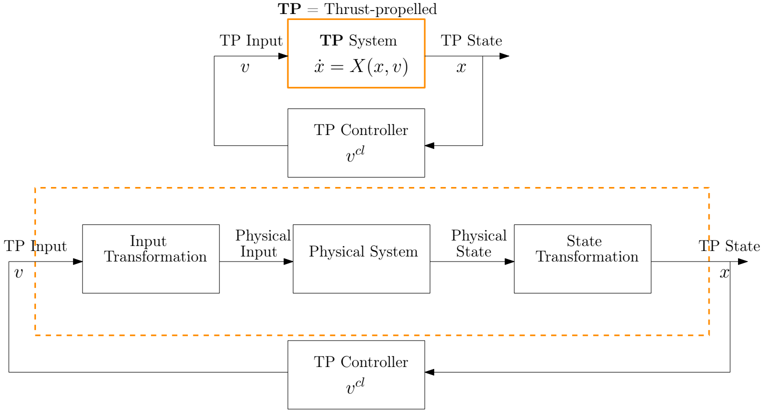

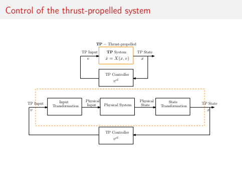

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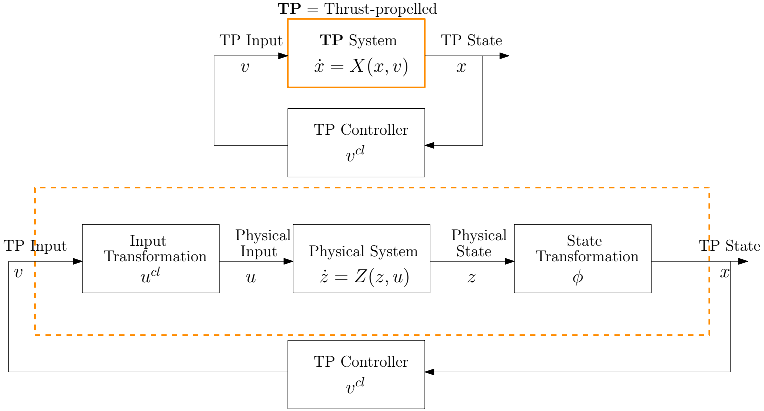

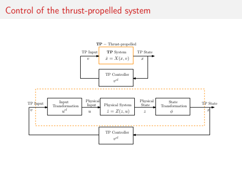

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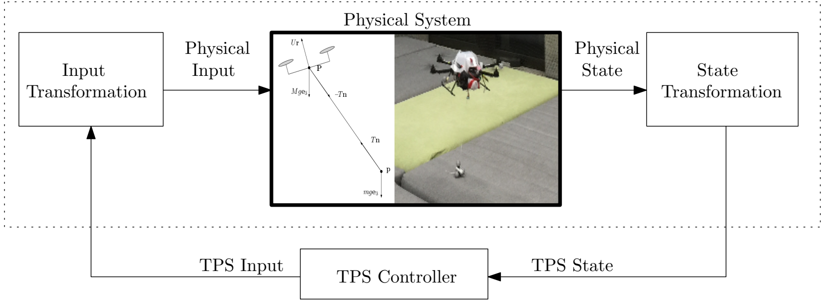
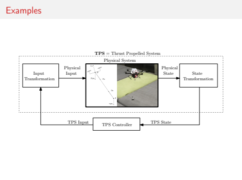
Examples

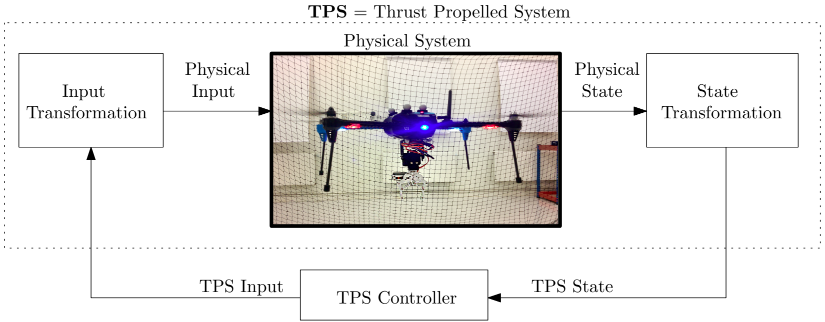
TPS = Thrust Propelled System

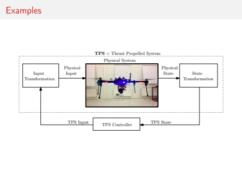




- Slung-load as a thrust-propelled system
- UAV with robotic arm as two decoupled thrust-propelled systems
- Bar tethered to two UAVs as an extended thrust-propelled system

Examples





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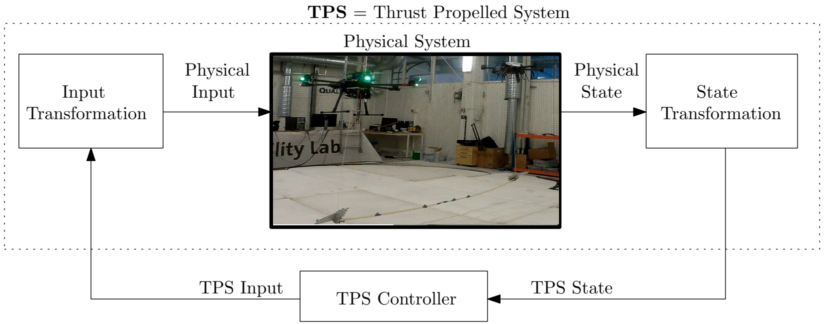

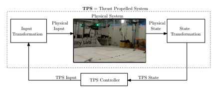

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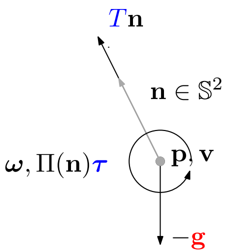
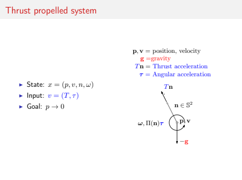
Thrust propelled system

- State: $x = (p, v, n, \omega)$
- Input: $v = (T, \tau)$
- Goal: $p \to 0$

$\mathbf{p}, \mathbf{v}$ = position, velocity
$\mathbf{g}$ =gravity
$T\mathbf{n}$ = Thrust acceleration
$\boldsymbol{\tau}$ = Angular acceleration





- State is composed of position, velocity, unit vector and angular velocity of unit vector
- Input: thrust and torque
- Time-varying gravity
- Pereira, P. and Dimarogonas, D.V. *Lyapunov-based Generic Controller Design for Thrust-Propelled Underactuated Systems.* IEEE European Control Conference, pp. 594–599, 2016.

Thrust-propelled system

Vector field: $\dot{x} = X(x, v)$

$$\dot{p} = v$$
$$\dot{v} = Tn - g$$
$$\dot{n} = \mathcal{S}(\omega)n$$
$$\dot{\omega} = \Pi(n)\tau$$

$\mathbf{p}, \mathbf{v}$ = position, velocity
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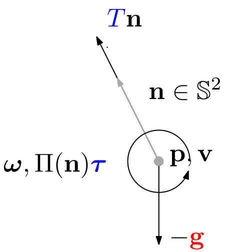

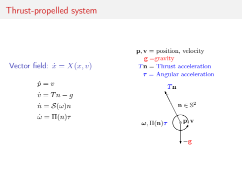

- Notation in this slide can be confusing: v (in blue) used as input to system; v (in black) used as velocity of the system.
- Unit vector along which thrust can be provided
- Torque: torque along unit vector makes unit vector spin around itself, and that is irrelevant, this is why we only use the torque orthogonal to the unit vector
- $\mathcal{S}$: skew operator; Π orthogonal projection operator

Thrust-propelled system

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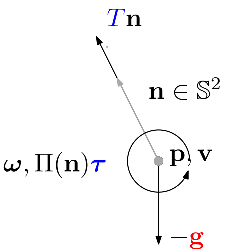

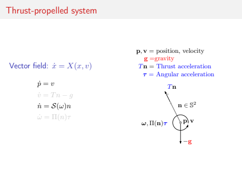

- Two kinematic equations

Thrust-propelled system

Vector field

$$\dot{p} = v$$
$$\dot{v} = Tn - g$$
$$\dot{n} = S(\omega)n$$
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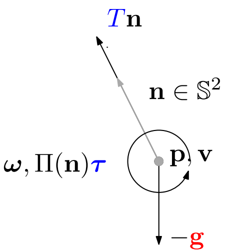
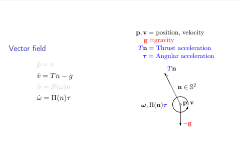
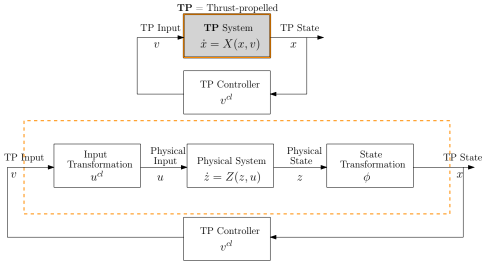




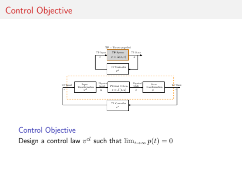
- Two dynamic equations

Control Objective



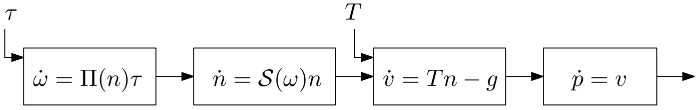
Control Objective

Design a control law v^{cl} such that $\lim_{t\to\infty} p(t) = 0$



- Stabilizing controller
- Steer position of TPS to origin (in physical systems, this will correspond to steering a position tracking error to zero)
- Pereira, P. and Dimarogonas, D.V. *Lyapunov-based Generic Controller Design for Thrust-Propelled Underactuated Systems.* IEEE European Control Conference, pp. 594–599, 2016.

Control Design Summary



$$\begin{bmatrix} \dot{p} \\ \dot{v} \\ \dot{n} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} v \\ Tn - g \\ \mathcal{S}(\omega)n \\ \Pi(n)\tau \end{bmatrix}$$

Steps

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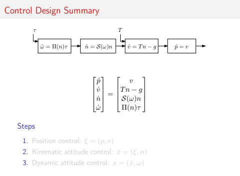
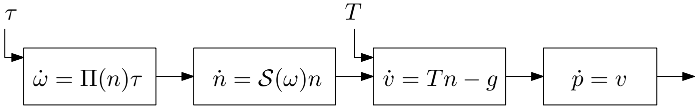


- Notice cascaded structure
- First step, we make system behave like a stable second order system
- However, an error is left, and in the next step we steer this error to zero
- Same logic in third step

Control Design Summary



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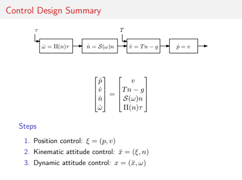

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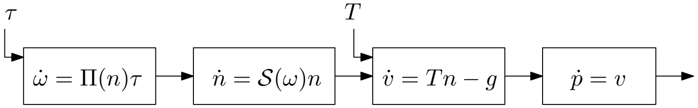

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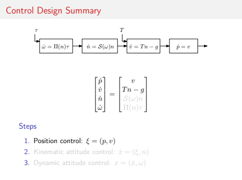

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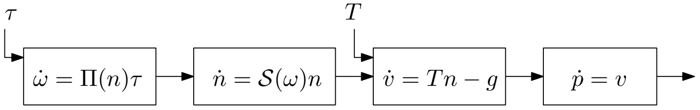

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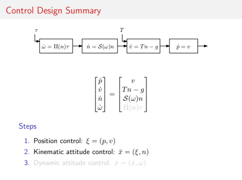

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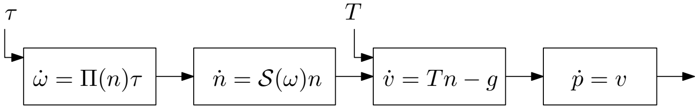

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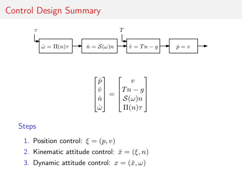

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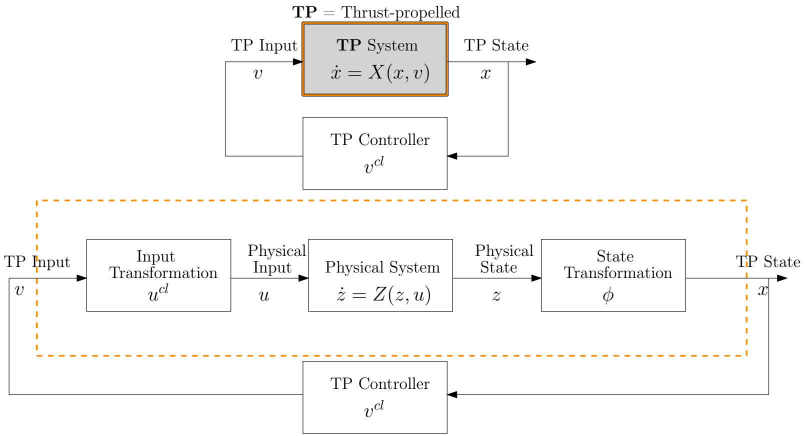
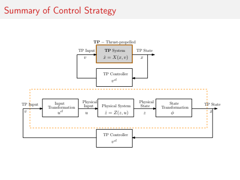
Summary of Control Strategy





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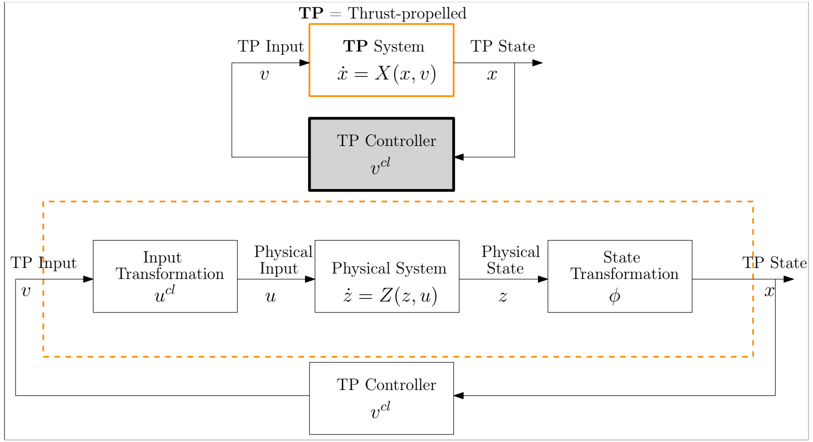

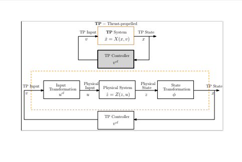

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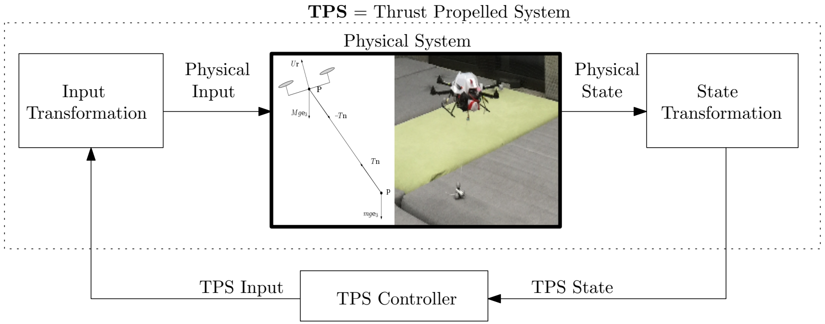
Example: Slung-load



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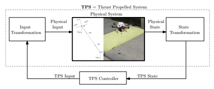
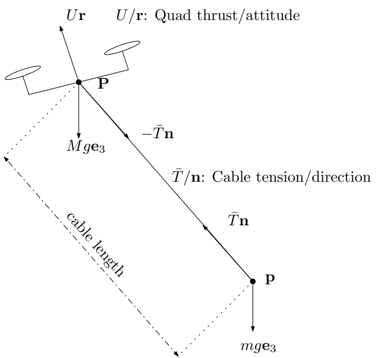
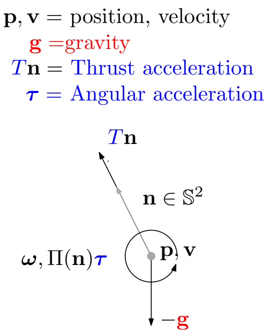


- Pereira, P. and Herzog, M. and Dimarogonas, D. V. *Slung Load Transportation with Single Aerial Vehicle and Disturbance Removal.* IEEE Mediterranean Conference on Control and Automation, pp. 671–676, 2016.

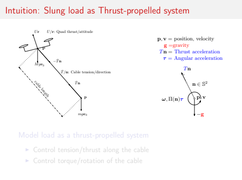
Intuition: Slung load as Thrust-propelled system





Model load as a thrust-propelled system

- Control tension/thrust along the cable
- Control torque/rotation of the cable



- thrust-propelled system: one where we can provide force along body axis, and where we can rotate that body axis
- load and cable form a thrust propelled system
- control uav actuation along cable direction, to control cable tension (thrust)
- control uav actuation orthogonal to cable to control cable rotation (torque)

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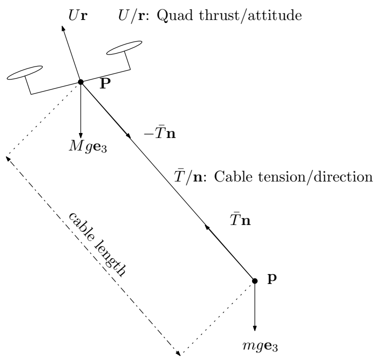
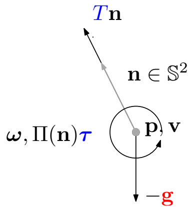


$\mathbf{p}, \mathbf{v}$ = position, velocity
$\mathbf{g}$ = gravity
$T\mathbf{n}$ = Thrust acceleration
$\boldsymbol{\tau}$ = Angular acceleration



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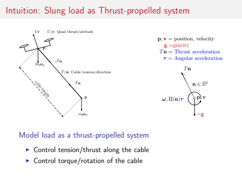

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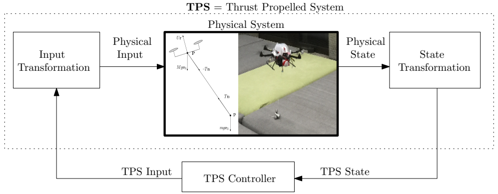
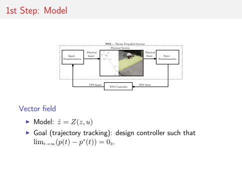
1st Step: Model



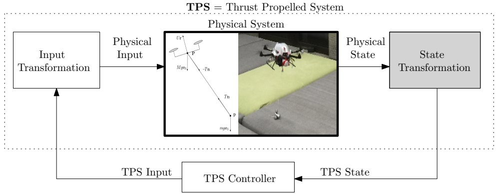
Vector field

- Model: $\dot{z} = Z(z, u)$
- Goal (trajectory tracking): design controller such that $\lim_{t\to\infty}(p(t) - p^\star(t)) = 0_3$.



- Vector field is composed of the velocities and accelerations of the load and uav
- Hybrid system (in general): vector field changes according to state and input (there may be discontinuities in the state as well)
- We must guarantee that with the control law we design that the tension remains positive
- Tracking objective: we want load to track a desired trajectory
- Trajectory needs to be sufficiently smooth ($\mathcal{C}^4$)
- There is an extra constraint on the trajectories we can track (related to the fact that the tension must remain positive)

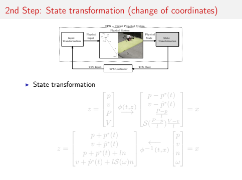
2nd Step: State transformation (change of coordinates)



- State transformation

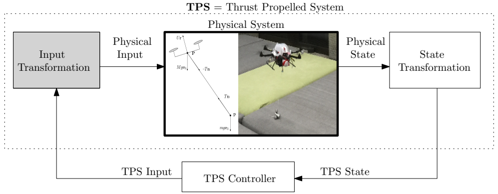
$$z = \begin{bmatrix} p \\ v \\ P \\ V \end{bmatrix} \xrightarrow{\phi(t,z)} \begin{bmatrix} p - p^\star(t) \\ v - \dot{p}^\star(t) \\ \frac{P-p}{l} \\ S(\frac{P-p}{l})\frac{V-v}{l} \end{bmatrix} = x$$

$$z = \begin{bmatrix} p + p^\star(t) \\ v + \dot{p}^\star(t) \\ p + p^\star(t) + ln \\ v + \dot{p}^\star(t) + lS(\omega)n \end{bmatrix} \xleftarrow{\phi^{-1}(t,x)} \begin{bmatrix} p \\ v \\ n \\ \omega \end{bmatrix} = x$$



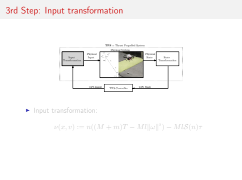
- State transformation = coordinate change
- Design this map using intution I provided before
- Map (for state transformation) must be invertible
- Thrust-propelled system: $\mathbf{p} \to \mathbf{0}$
- Slung load system: $\mathbf{p} - \mathbf{p}^\star(t) \to \mathbf{0}$

3rd Step: Input transformation



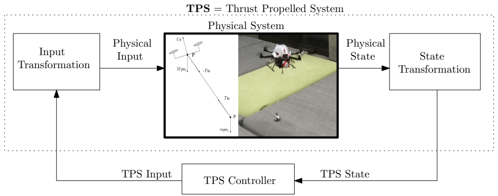
- Input transformation:

$$\nu(x, v) := n((M+m)T - Ml\|\omega\|^2) - MlS(n)\tau$$

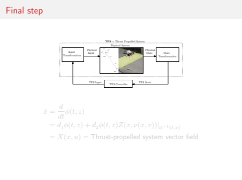


- Input transformation (from $v = (T, \tau)$ to physical input; recall that $v = (T, \tau)$ is the input to the thrust-propelled system)
- Design this map using intution I provided before

Final step

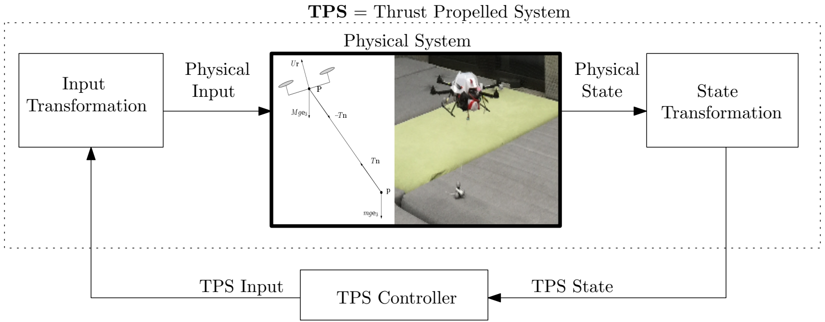


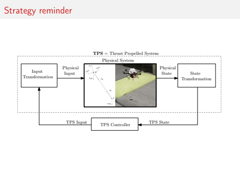
$$
\begin{aligned}
\dot{x} &= \frac{d}{dt}\phi(t,z) \\
&= d_1\phi(t,z) + d_2\phi(t,z)Z(z,\nu(x,v))|_{\phi^{-1}(t,x)} \\
&= X(x,u) = \text{Thrust-propelled system vector field}
\end{aligned}
$$



- Composing the three components: input transformation + vector field + state transformation
- d_1 = derivative w.r.t. 1st argument; d_2 = derivative w.r.t. 2nd argument
- We obtain the TPS vector field, thus we can use any controller for a TPS system for trajectory tracking of a load tethered to a UAV

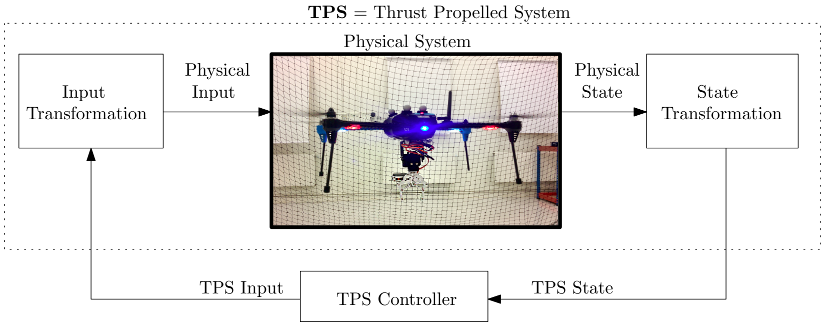
Strategy reminder





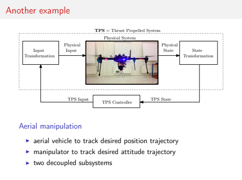
- Reminder of control scheme

Another example



Aerial manipulation

- aerial vehicle to track desired position trajectory
- manipulator to track desired attitude trajectory
- two decoupled subsystems



- Control of a system composed of a rigid manipulator and an aerial vehicle
- Disturbance removal: consider a constant thrust input disturbance (we design a disturbance estimator, and penalizes the position tracking error and the manipulation tracking error differently)
- Aerial vehicle: IRIS+; rigid manipulator: aluminum arm with a 1DOF revolute joint and a clamp gripper
- Pereira, P. and Zanella, R. and Dimarogonas, D.V. *Decoupled Design of Controllers for Aerial Manipulation with Quadrotors.* IEEE/RSJ International Conference on Intelligent Robots and Systems, pp. 4849–4855, 2016.

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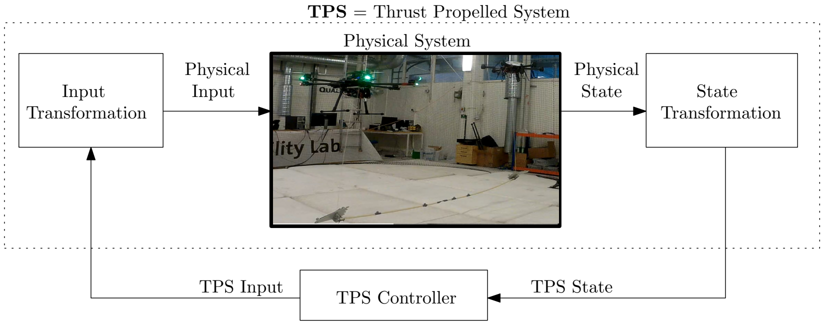
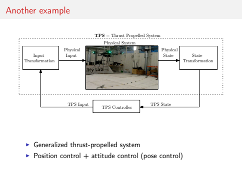


- Generalized thrust-propelled system
- Position control + attitude control (pose control)



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Disturbance removal

- Standard solutions for affine disturbances
- Input disturbance for input affine systems ($\dot{z} = f(z) + g(z)u$)
- Disturbance estimator and (dynamic) control law
- Helicopters dynamics are not input affine
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Overview

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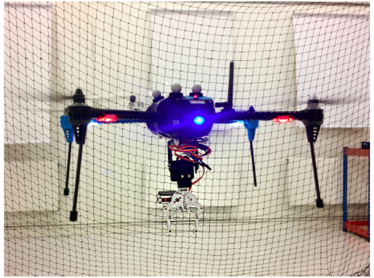
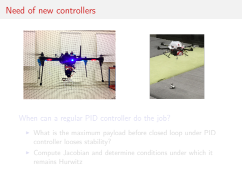
Motivation
Modeling of system of rigid bodies
Control of Thrust-propelled Systems
Conditions for local stability
Summary

Need of new controllers



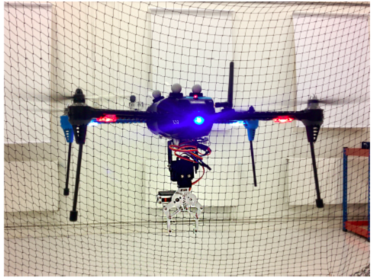
When can a regular PID controller do the job?

- What is the maximum payload before closed loop under PID controller looses stability?
- Compute Jacobian and determine conditions under which it remains Hurwitz



- Linearization around equilibrium point
- Determine conditions on gains or other parameters using Routh-Hurwitz criterion such that Jacobian is Hurwitz
- example condition: if proportinal gain is bigger than . . . , then Jacobian is Hurwitz
- example condition: if extra load is no further than . . . away from CoM, then Jacobian is Hurwitz

Need of new controllers



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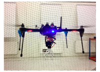

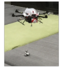

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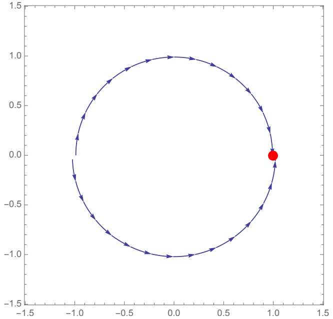
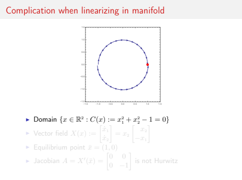
Complication when linearizing in manifold



- Domain $\{x \in \mathbb{R}^2 : C(x) := x_1^2 + x_2^2 - 1 = 0\}$
- Vector field $X(x) := \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = x_2 \begin{bmatrix} x_2 \\ -x_1 \end{bmatrix}$
- Equilibrium point $\bar{x} = (1, 0)$
- Jacobian $A = X'(\bar{x}) = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$ is not Hurwitz



- Recall that physical system (system of rigid bodies and point masses) is constrained in some set (defined, in our case, by holonomic constraints)
- Let us illustrate the problem with a constraint that defines a circle in $\mathbb{R}^2$.

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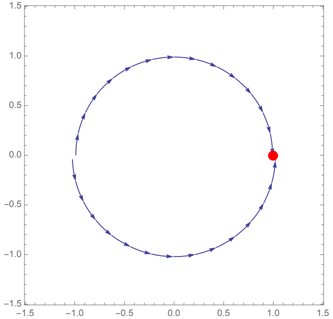

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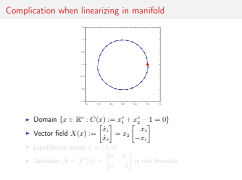

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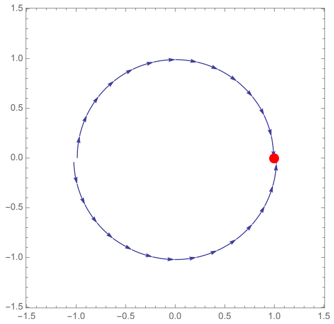

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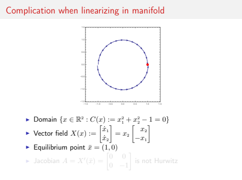

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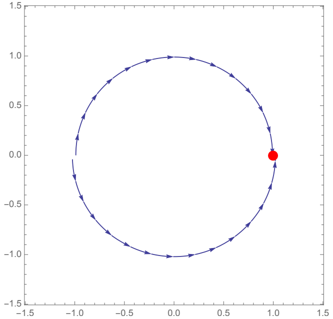

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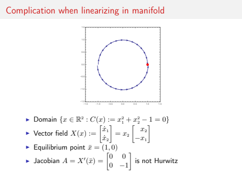

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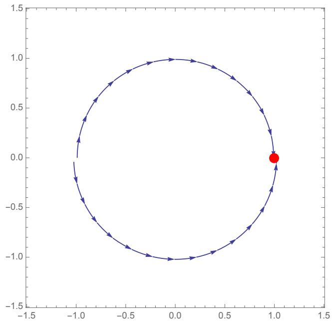
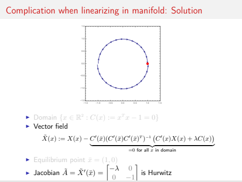
Complication when linearizing in manifold: Solution



- Domain $\{x \in \mathbb{R}^2 : C(x) := x^T x - 1 = 0\}$
- Vector field

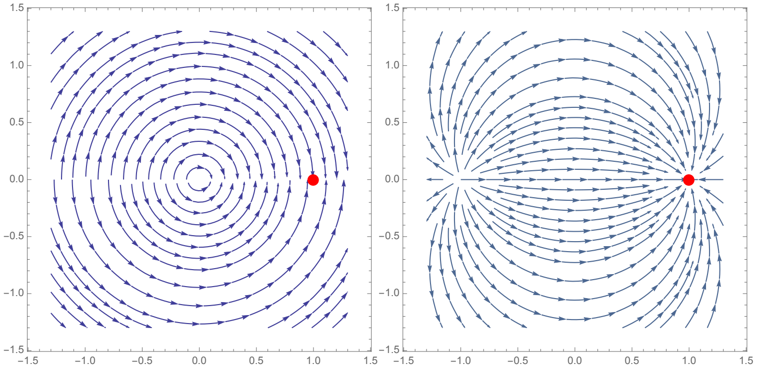
$$\tilde{X}(x) := X(x) - \underbrace{C'(\bar{x})(C'(\bar{x})C'(\bar{x})^T)^{-1}\left(C'(x)X(x) + \lambda C(x)\right)}_{=0 \text{ for all } x \text{ in domain}}$$

- Equilibrium point $\bar{x} = (1, 0)$
- Jacobian $\tilde{A} = \tilde{X}'(\bar{x}) = \begin{bmatrix} -\lambda & 0 \\ 0 & -1 \end{bmatrix}$ is Hurwitz

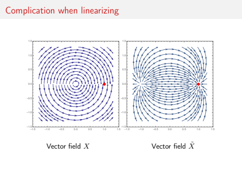


- λ is a positive number
- This method is an artifact/gimmick that we use, so that we can invoke results from linearization theorems
- See also: Palis and de Melo. *Geometric theory of dynamical systems*, 1982 (Chapter 2).

Complication when linearizing

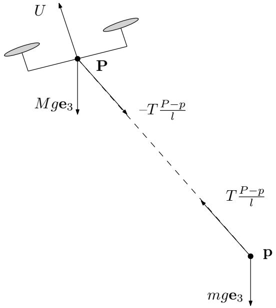


Vector field X Vector field $\tilde{X}$

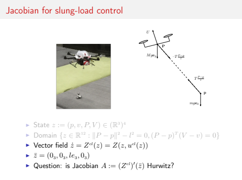


- On the circle, vector fiels are exactly the same
- Outside the circle, they are not
- Another option: parametrize circle with angle, and study stability of equilibrium with the vector field in the "angle coordinate" (however, more complicated sets, require more complicated parametrizations)

Jacobian for slung-load control

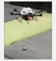


- State $z := (p, v, P, V) \in (\mathbb{R}^3)^4$
- Domain $\{z \in \mathbb{R}^{12} : \|P-p\|^2 - l^2 = 0, (P-p)^T(V-v) = 0\}$
- Vector field $\dot{z} = Z^{cl}(z) = Z(z, u^{cl}(z))$
- $\bar{z} = (0_3, 0_3, le_3, 0_3)$
- Question: is Jacobian $A := (Z^{cl})'(\bar{z})$ Hurwitz?



- Equilibrium $\bar{z}$
- u^{cl}: control law – in this case a PID control law based on position and velocity of UAV
- Jacobian is unstructured (find similarity transformation that exposes its structure)
- $(Z^{cl})' =$ “$\frac{dZ^{cl}}{dz}$”
- P. O. Pereira and D. V. Dimarogonas, *Stability of load lifting by a quadrotor under attitude control delay*, 2017 IEEE International Conference on Robotics and Automation (ICRA), Singapore, 2017, pp. 3287-3292.

Jacobian for slung-load control



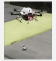
$$PAP^{-1} = \begin{bmatrix} A_z & 0 & 0 & \star \\ 0 & A_x & 0 & \star \\ 0 & 0 & A_y & \star \\ 0 & 0 & 0 & -\lambda I_2 \end{bmatrix}$$

$$\frac{d}{dt}\begin{bmatrix} z \\ \dot{z} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\frac{M}{M+m}k_{p,z} & -\frac{M}{M+m}k_{d,z} \end{bmatrix}}_{=:A_z}\begin{bmatrix} z \\ \dot{z} \end{bmatrix}$$

$$\frac{d}{dt}\begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \\ \dddot{x} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{g}{l}k_{p,x} & -\frac{g}{l}k_{d,x} & -k_{p,x} - \frac{g}{l}\frac{M+m}{M} & -k_{d,x} \end{bmatrix}}_{=:A_x}\begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \\ \dddot{x} \end{bmatrix}$$

- Jacobian is similar to a block diagonal matrix (thus, if each block diagonal matrix is Hurwitz, then the Jacobian is Hurwitz)
- z is associated to the z motion of the load (which behaves as a second order system – this is the case for a PD control law)
- x is associated to the x motion of the load (which behaves as a fourth order system)
- By the Routh-Hurwitz's criterion, both A_z and A_x are Hurwitz
- Lesson: PD control law does the job (local result though)

Jacobian for slung-load control



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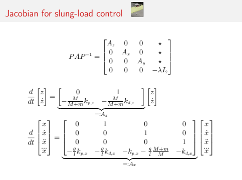

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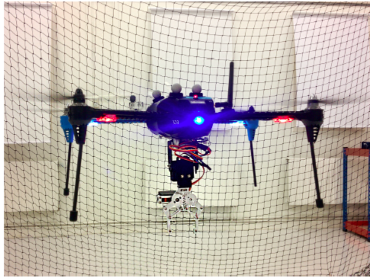
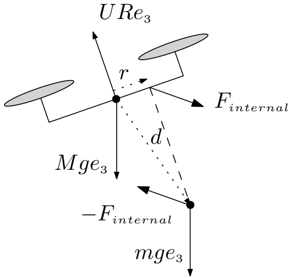
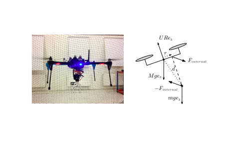
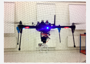
Jacobian for UAV with off-balanced load







- Same idea applied to a different system
- Does a *standard* PID control law does the job when the UAV is rigidly attached to some load (fixed in the body frame of the UAV)
- $d = (d_1, d_2, d_3)$: position of load w.r.t. UAV and expressed in the UAV body frame
- $r = (r_1, r_2, r_3)$: where internal force is applied – $r = (0, 0, 0)$ if arm is exactly placed on UAV CoM
- For simplicity, in the linearization, we assume that $r = (0, 0, 0)$.

Jacobian for slung-load control 

$$PAP^{-1} = \begin{bmatrix} A_z & \star_{d_1} & \star_{d_2} & 0 & \star \\ 0 & A_x & 0 & \star_{d_2} & \star \\ 0 & 0 & A_y & \star_{d_1} & \star \\ 0 & 0 & 0 & A_\psi & \star \\ 0 & 0 & 0 & 0 & -\lambda I_{12} \end{bmatrix}$$

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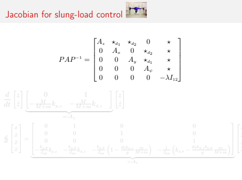

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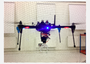
- We can find precise conditions on d_3 such that A_z is Hurwitz

Jacobian for slung-load control 

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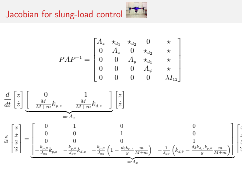
Pedro



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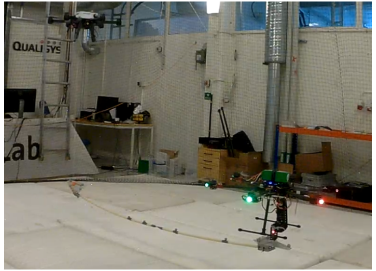
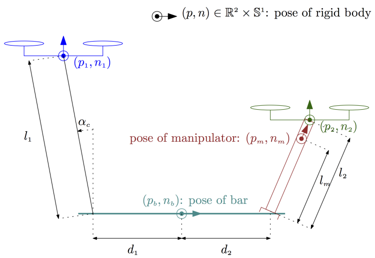
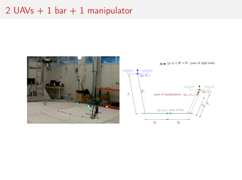
2 UAVs + 1 bar + 1 manipulator

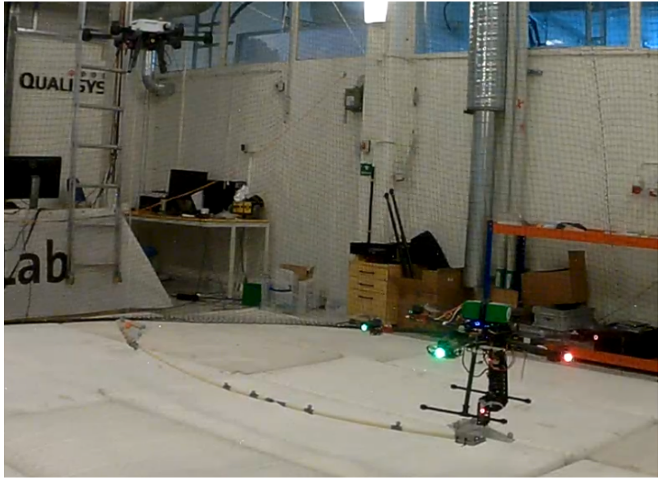






- Similar analysis
- Roque, P., Pereira, P. and Dimarogonas, D.V. *Heterogeneous Collaborative Pose Stabilization of a Bar Via Aerial Manipulation and Tethering* ICRA, 2018 (submitted).

Experiment



https://youtu.be/NADR9_VffBk

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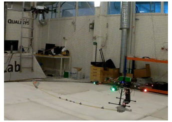

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- See video at https://youtu.be/NADR9_VffBk

Summary & Final remarks

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- Modeling
- Control of thrust propelled systems
- Conditions for local stability

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