Slung Load Transportation with a Single Aerial Vehicle and Disturbance Removal

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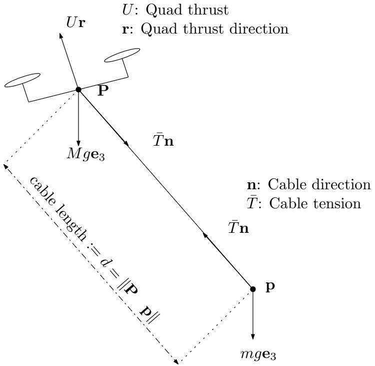
Motivation

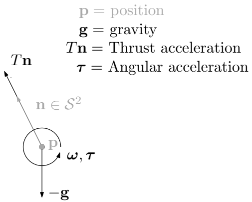
- Transportation of payloads in inaccessible locations
- Mechanically simple

Figure: Disaster environment

Summary

1. Model load as a thrust-propelled system
   - Control tension/thrust along the cable
   - Control torque/rotation of the cable
2. Disturbance removal for compensating model uncertainties

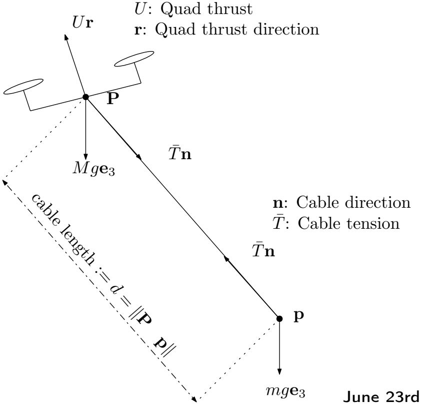




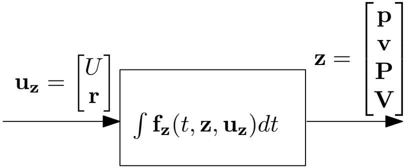
Modeling

Modeling

- State: $\mathbf{z} = [\mathbf{p}^T \, \mathbf{v}^T \, \mathbf{P}^T \, \mathbf{V}^T]^T \in \Omega_{\mathbf{z}}$
- State Set: $\Omega_{\mathbf{z}} \subset \mathbb{R}^{12}$ (cable of fixed length)
- Input $\mathbf{u}_{\mathbf{z}} = [U \, \mathbf{r}^T]^T \in \mathbb{R}_{\geq 0} \times \mathcal{S}^2$



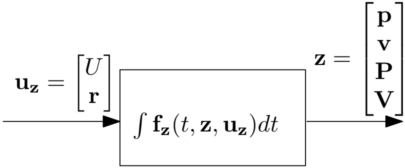
Modeling



Modeling: Vector field

- $\dot{\mathbf{z}} = \mathbf{f_z}(\mathbf{z}, \mathbf{u_z})$
- $\mathbf{f_z}(\mathbf{z}, \mathbf{u_z}) = \begin{bmatrix} \mathbf{v} \\ \frac{\bar{T}(\mathbf{z}, \mathbf{u_z} + b\mathbf{e}_1)}{m} \bar{\mathbf{n}}(\mathbf{z}) - g\mathbf{e}_3 \\ \mathbf{V} \\ \frac{U+b}{M}\mathbf{r} - \frac{\bar{T}(\mathbf{z}, \mathbf{u_z} + b\mathbf{e}_1)}{M} \bar{\mathbf{n}}(\mathbf{z}) - g\mathbf{e}_3 \end{bmatrix}$
- $\bar{\mathbf{n}}(\mathbf{z})$ is cable unit vector
- $\bar{T}(\mathbf{z}, \mathbf{u_z})$ is tension on the cable

Modeling



Modeling: Vector field

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- b is input disturbance

Objective

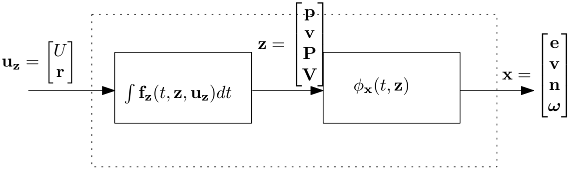
Problem

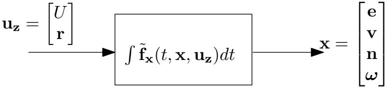
Given a desired trajectory $\mathbf{p}^\star \in \mathcal{C}^4(\mathbb{R}_{\geq 0}, \mathbb{R}^3)$, *design* $U : \mathbb{R}_{\geq 0} \mapsto \mathbb{R}$ *and* $\mathbf{r} : \mathbb{R}_{\geq 0} \mapsto \mathcal{S}^2$ *such that* $\lim_{t\to\infty}(\mathbf{p}(t) - \mathbf{p}^\star(t)) = \mathbf{0}$.

Change of Coordinates

- New state: $\mathbf{x} := [\mathbf{e}^T\ \boldsymbol{\upsilon}^T\ \mathbf{n}^T\ \boldsymbol{\omega}^T]^T \in \Omega_{\mathbf{x}}$
- State set: $\Omega_{\mathbf{x}} = \{\mathbf{x} \in \mathbb{R}^3 \times \mathbb{R}^3 \times \mathcal{S}^2 \times \mathbb{R}^3 : \mathbf{n}^T\boldsymbol{\omega} = 0\}$
- For each $t \geq 0$, diffeomorphism $\phi_{\mathbf{x}}(t, \cdot) : \mathbf{z} \in \Omega_{\mathbf{z}} \mapsto \mathbf{x} \in \Omega_{\mathbf{x}}$
- $\phi_{\mathbf{x}}(t, \mathbf{z}) = \begin{bmatrix} \mathbf{p} - \mathbf{p}^\star(t) \\ \mathbf{v} - \dot{\mathbf{p}}^\star(t) \\ \bar{\mathbf{n}}(\mathbf{z}) \\ \bar{\boldsymbol{\omega}}(\mathbf{z}) \end{bmatrix} = \mathbf{x} = \begin{bmatrix} \mathbf{e} \\ \boldsymbol{\upsilon} \\ \mathbf{n} \\ \boldsymbol{\omega} \end{bmatrix}$
- New vector field: $\dot{\mathbf{x}} = \tilde{\mathbf{f}}_{\mathbf{x}}(t, \mathbf{x}, \mathbf{u}_{\mathbf{z}})$

Change of Coordinates



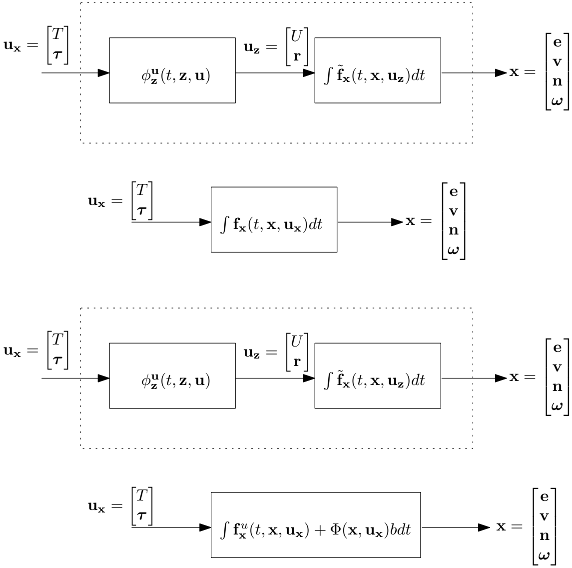


Change of Coordinates

- New input: $\mathbf{u}_\mathbf{x} = (T, \boldsymbol{\tau})$
- Actual input: $\mathbf{u}_\mathbf{z} = \phi^\mathbf{u}_\mathbf{z}(\mathbf{x}, \mathbf{u}_\mathbf{x})$
- New vector field: $\dot{\mathbf{x}} = \mathbf{f}_\mathbf{x}(t, \mathbf{x}, \mathbf{u}_\mathbf{x})$

$$
\begin{aligned}
\mathbf{f}_\mathbf{x}(t, \mathbf{x}, \mathbf{u}_\mathbf{x}) &:= \begin{bmatrix} \boldsymbol{v} \\ T\mathbf{n} - \mathbf{g}(t) \\ \mathcal{S}(\boldsymbol{\omega})\mathbf{n} \\ \mathcal{S}(\mathbf{n})\boldsymbol{\tau} \end{bmatrix} + \Phi(\mathbf{x}, \mathbf{u}_\mathbf{x})b \\
&:= \mathbf{f}^u_\mathbf{x}(t, \mathbf{x}, \mathbf{u}_\mathbf{x}) + \Phi(\mathbf{x}, \mathbf{u}_\mathbf{x})b
\end{aligned}
$$

Change of Coordinates



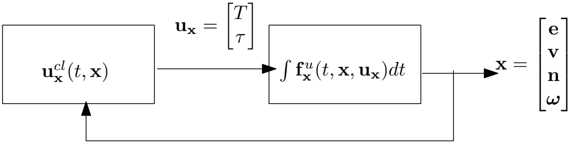
Thrust propelled vector field

$$\mathbf{f}^{u}_{\mathbf{x}}(t, \mathbf{x}, \mathbf{u}_{\mathbf{x}}) = \begin{bmatrix} \boldsymbol{v} \\ T\mathbf{n} - \mathbf{g}(t) \\ \mathcal{S}(\boldsymbol{\omega})\mathbf{n} \\ \mathcal{S}(\mathbf{n})\boldsymbol{\tau} \end{bmatrix}$$

- Control law: $\mathbf{u}^{cl}_{\mathbf{x}}(t, \mathbf{z}) = (T^{cl}(t, \mathbf{z}), \boldsymbol{\tau}^{cl}(t, \mathbf{z}))$
- Lyapunov function: $V(t, \mathbf{x}) : \mathbb{R}_{\geq 0} \times \Omega_{\mathbf{x}} \mapsto \mathbb{R}_{\geq 0}$
- $V(t, \mathbf{x})$ is smooth; gradient used to remove the disturbance
- $\dot{V}(t, \mathbf{x}(t)) \leq 0$

Pereira, Dimarogonas. Lyapunov-based Generic Controller Design for Thrust-Propelled Underactuated Systems. ECC 2016

Controller for Thrust propelled system



Pereira, Dimarogonas. Lyapunov-based Generic Controller Design for Thrust-Propelled Underactuated Systems. ECC 2016

Result

Theorem

Consider a trajectory $\mathbf{p}^\star \in \mathcal{C}^4(\mathbb{R}_{\geq 0}, \mathbb{R}^3)$, *the control law* $\mathbf{u}_{\mathbf{z}}^{cl} : \mathbb{R}_{\geq 0} \times \Omega_{\mathbf{z}} \mapsto \mathbb{R} \times \mathcal{S}^2$

$$\mathbf{u}_{\mathbf{z}}^{cl}(t, \mathbf{z}) = \phi_{\mathbf{z}}^{\mathbf{u}}(\mathbf{x}, \mathbf{u}_{\mathbf{x}}^{cl}(t, \mathbf{x}))|_{\mathbf{x}=\phi_{\mathbf{x}}(t,\mathbf{z})}$$

and a solution of

$$\dot{\mathbf{z}} = \mathbf{f}_{\mathbf{z}}(\mathbf{z}, \mathbf{u}_{\mathbf{z}}^{cl}(t, \mathbf{z})).$$

Then $\lim_{t\to\infty}(\mathbf{p}(t) - \mathbf{p}^\star(t)) = \mathbf{0}$, *and the tension in the cable is always positive, i.e.* $\inf_{t\geq 0} \bar{T}(\mathbf{z}(t), \mathbf{u}_{\mathbf{z}}^{cl}(t, \mathbf{z}(t)))$.

Disturbance Removal

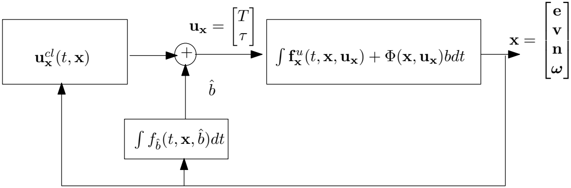
- Disturbance estimate: $\hat{b}$, where $\dot{\hat{b}} = f_b(t, \tilde{\mathbf{x}})$
- We design the vector field $f_b(t, \tilde{\mathbf{x}})$
- Augmented state: $\tilde{\mathbf{x}} = [\mathbf{x}^T\, \hat{b}]^T$
- New vector field $\dot{\tilde{\mathbf{x}}} = \mathbf{f}_{\tilde{\mathbf{x}}}(t, \tilde{\mathbf{x}}, \mathbf{u}_{\mathbf{x}})$

$$\mathbf{f}_{\tilde{\mathbf{x}}}(t, \tilde{\mathbf{x}}, \mathbf{u}_{\mathbf{x}}^{cl}(t, \mathbf{x}) - \hat{b}\mathbf{e}_1) = \begin{bmatrix} \mathbf{f}_{\mathbf{x}}^{u}(t, \mathbf{x}, \mathbf{u}_{\mathbf{x}}^{cl}) \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \Phi(\mathbf{x}, \mathbf{u}_{\mathbf{x}}^{cl})(b - \hat{b}) \\ f_b(t, \tilde{\mathbf{x}}) \end{bmatrix}.$$

$$f_b(t, \tilde{\mathbf{x}}) = \mathsf{Proj}\left(\Phi^T(\mathbf{x}, \mathbf{u}_{\mathbf{x}}^{cl})\frac{\partial V_{\mathbf{x}}(t, \mathbf{x})}{\partial \mathbf{x}}, \hat{b}\right)$$

Cai, Queiroz, Dawson. A sufficiently smooth projection operator. TAC 2006

Disturbance Removal



Feasible Trajectories

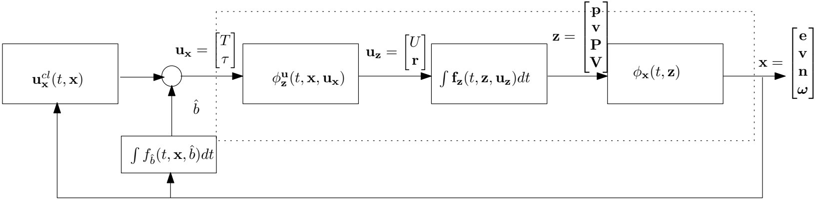
Definition

$\mathbf{p}^{\star} \in \mathcal{C}^4(\mathbb{R}_{\geq 0}, \mathbb{R}^3)$ *is a feasible trajectory if i)* $\sup_{t\geq 0} \|\mathbf{p}^{\star(i)}(t)\| < \infty$ *for* $i \in \{2, 3, 4\}$, *ii)* $\sup_{t\geq 0} \mathbf{e}_3^T \mathbf{p}^{\star(2)}(t) > -g$, *and iii)*

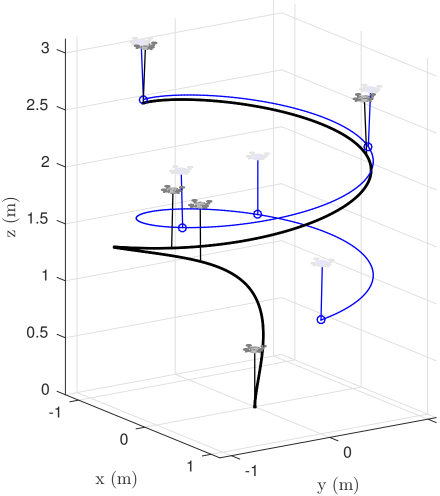
$$\inf_{t\geq 0} \frac{M}{M+m} \frac{d\|\mathcal{S}(g\mathbf{e}_3 + \mathbf{p}^{\star(2)}(t))\mathbf{p}^{\star(3)}(t)\|^2}{\|g\mathbf{e}_3 + \mathbf{p}^{\star(2)}(t)\|^5} < 1.$$

- Desired uav attitude $\mathbf{r}$ is well defined for feasible trajectory

Complete Diagram

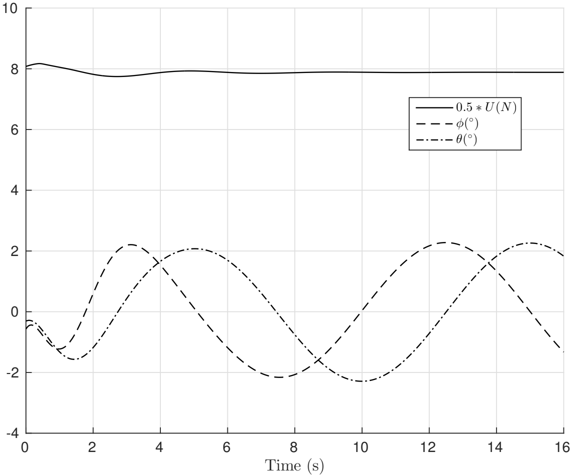


- Desired motion: load to describe a helix
- Input disturbance corresponding to $\approx 14\%$ of load's weight

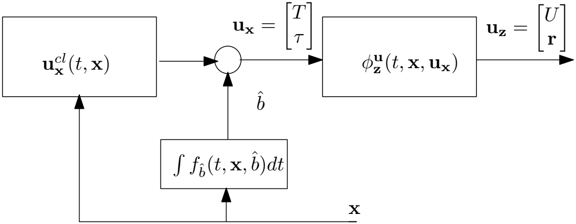


Simulations

- Inputs time evolution: Thrust and attitude of uav

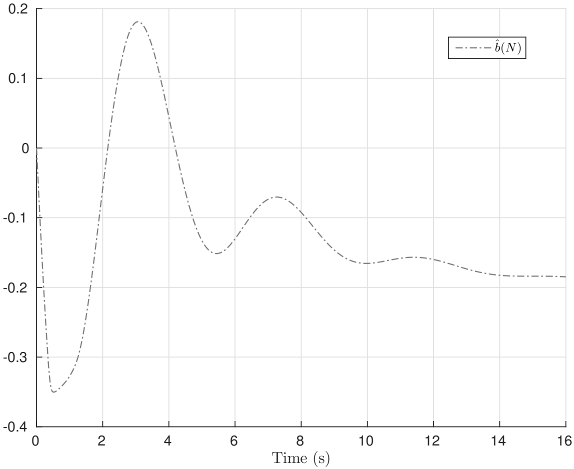


Simulations



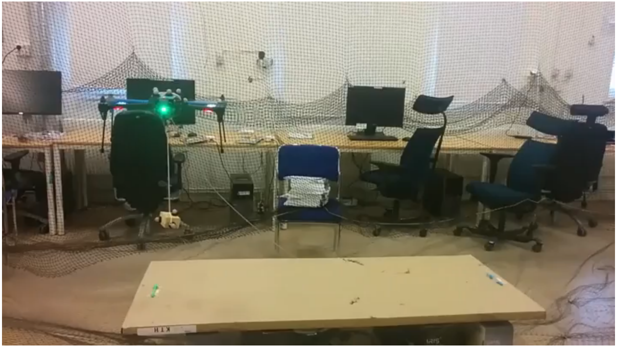
Simulations

- Disturbance estimate time evolution



Experiment

- Hover over green/blue pen



Future work

- Extension to multiple uav's
- Remove other types of disturbances
- Study robustness against model uncertainty

Thank you! Questions?