

Spatial Data and Queries

- ❑ Spatial Data
- ❑ Spatial Relationships
- ❑ Spatial Queries
- ❑ Issues in Query Processing
- ❑ The R-tree
- ❑ Spatial Query Processing
 - Spatial Selections
 - Nearest Neighbor Queries
 - Spatial Joins

Spatial Data and Queries



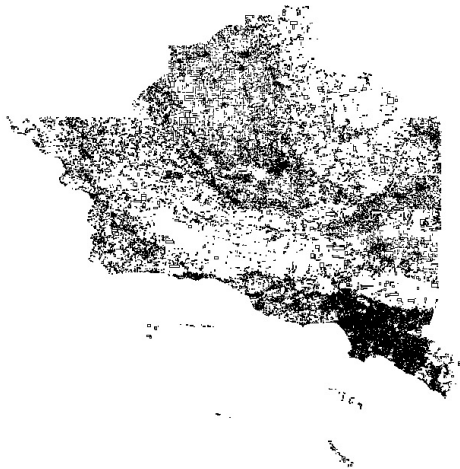
Spatial Data

- ❑ Spatial objects are a special case of multidimensional objects
 - ❑ Just **two** dimensions
 - ❑ Both dimensions are **interval-scaled**
 - ❑ Natural mapping of dimensions to real maps
- ❑ Spatial objects are characterized by
 - ❑ location
 - ❑ geometry (only for non-points)
- ❑ **Multidimensional objects are points**

Spatial Data Management



- ❑ **Spatial Database Systems** manage large collections of 2D/3D objects
- ❑ A **spatial object** (at least) one spatial attribute that describes its location and/or geometry
- ❑ A **spatial dataset** is an organized collection of spatial objects of the same entity (e.g. rivers, cities, road segments)



Road segments from an area in CA

ID	Name	Type	Polyline
1	Boulevard	avenue	(10023,1094), (9034,1567), (9020,1610)
2	Leeds	highway	(4240,5910), (4129,6012), (3813,6129), (3602,6129)
...

A spatial dataset (relation)

Spatial Data



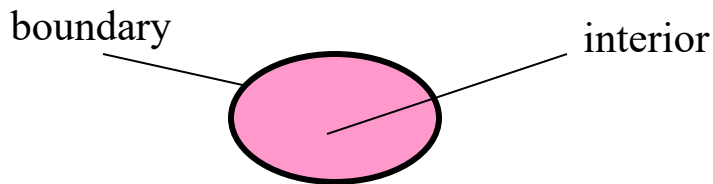
- ❑ Spatial data can be found in many applications
 - Geographic Information Systems
 - Segmented images (e.g., objects in X-rays)
 - Components of CAD constructs or VLSI circuits
 - Stars on the sky
 - ...
- ❑ Spatial database systems are used by
 - Users of mobile devices (find the nearest restaurant)
 - Geographers, astrologers, life scientists, army commanders, etc.

Spatial Relationships

- ❑ A **spatial relationship** or **spatial predicate** associates two objects according to their relative location and extent in space
 - Example: “My house is **close to** Central Park”
- ❑ Sometimes also called “spatial relation”.
- ❑ Spatial relationships are classified to
 - topological relationships (for objects with geometries)
 - distance relationships (mostly for point objects)
 - directional relationships (mostly for point objects)

Topological Relationships

- Each object is characterized by the space it occupies in the universe.
 - a (finite or infinite) set of elementary points (pixels)
- Each object has a boundary and an interior
 - **boundary**: the set of pixels the object occupies, that are adjacent to at least one pixel not occupied by the object
 - **interior**: the set of pixels occupied by the object, which are not part of its boundary



- Note: in some representation models, some objects may not have interior
 - e.g., points, line segments, etc.

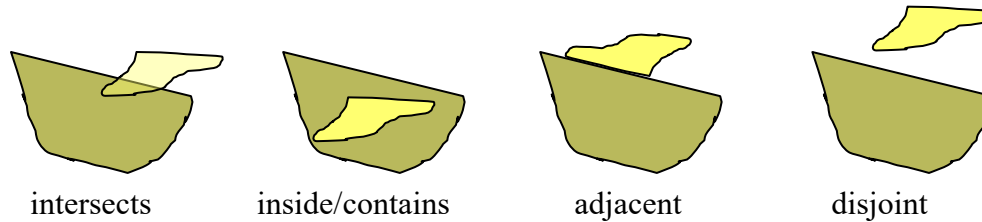
Topological Relationships

- A **topological relationship** between two objects is defined by a set of (set-based) relationships between their boundaries and interiors
 - E.g., o_1 is inside o_2 if $\text{interior}(o_1) \subset \text{interior}(o_2)$
- intersects (or overlap) means any of equals, inside, contains, adjacent
- intersects $\Leftrightarrow \neg \text{disjoint}$

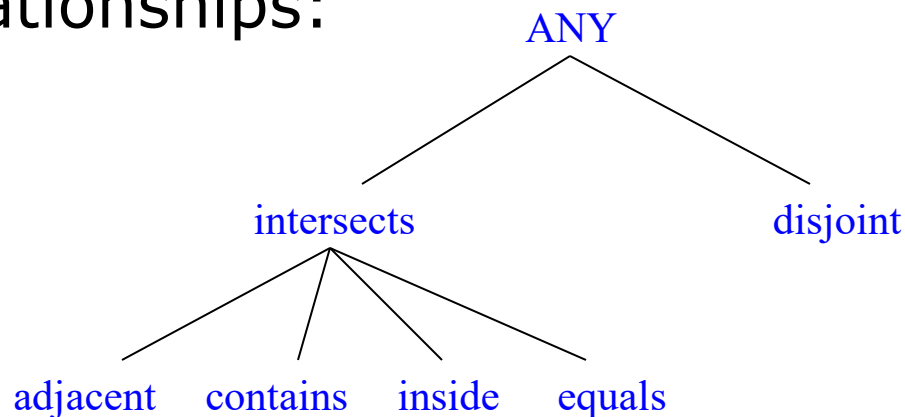
Topological relationship	equivalent boundary/interior relationships
$\text{disjoint}(o_1, o_2)$	$(\text{interior}(o_1) \cap \text{interior}(o_2) = \emptyset) \wedge (\text{boundary}(o_1) \cap \text{boundary}(o_2) = \emptyset)$
$\text{intersects}(o_1, o_2)$ (or $\text{overlaps}(o_1, o_2)$)	$(\text{interior}(o_1) \cap \text{interior}(o_2) \neq \emptyset) \vee (\text{boundary}(o_1) \cap \text{boundary}(o_2) \neq \emptyset)$
$\text{equals}(o_1, o_2)$	$(\text{interior}(o_1) = \text{interior}(o_2)) \wedge (\text{boundary}(o_1) = \text{boundary}(o_2))$
$\text{inside}(o_1, o_2)$	$\text{interior}(o_1) \subset \text{interior}(o_2)$
$\text{contains}(o_1, o_2)$	$\text{interior}(o_2) \subset \text{interior}(o_1)$
$\text{adjacent}(o_1, o_2)$	$(\text{interior}(o_1) \cap \text{interior}(o_2) = \emptyset) \wedge (\text{boundary}(o_1) \cap \text{boundary}(o_2) \neq \emptyset)$

Topological Relationships

□ Examples of topological relationships

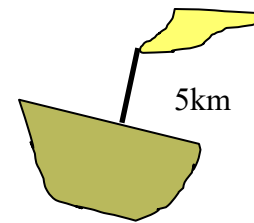


□ In fact, there is a **hierarchy** of topological relationships:



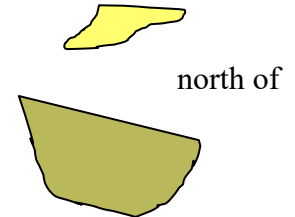
Distance Relationships

- Distance relationships associate two objects based on their **geometric distance** (typically, the minimum **Euclidean distance**)
- Distance is usually **abstracted** (i.e., discretized) into the human mind.
 - Example (distances in a city)
 - 0-100m: **near**
 - 100m-1km: **reachable**
 - 1km-10km: **far**
 - >10km: **very far**
- Distances are typically measured and then mapped to some abstract distance class (e.g., near, far)



Directional Relationships

- Directional relationships associate two objects based on their relative orientation according to a global reference system
- Example: My house is **north of** the river
 - relationship can also be a number:
 - e.g. house 96 degrees relative to river
- Examples of directional relationships:
 - north, south, east, west, northeast, etc.
 - left, right, above, below, front, behind, etc.
- Topological, distance, and directional relationships can be combined :
 - My house is **disjoint** with the park, **100 meters north of** it



Spatial Queries

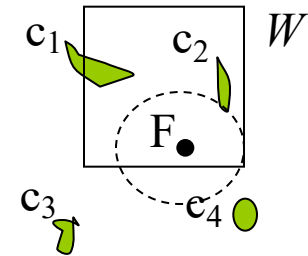
- ❑ Applied on one (or more) spatial datasets
- ❑ Retrieve objects (or combinations of objects) satisfying some spatial relationships
 - between them or
 - with a reference query object
- ❑ Examples:
 - **Nearest neighbor query:** What is the nearest city to my current location?
 - **Spatial join:** Find all pairs of hotels and restaurants within 100m distance from each other

Spatial Queries

- **Range query** (spatial selection, window query)

e.g. find all cities that *intersect* window W

Answer set: $\{c_1, c_2\}$



- **Nearest neighbor query**

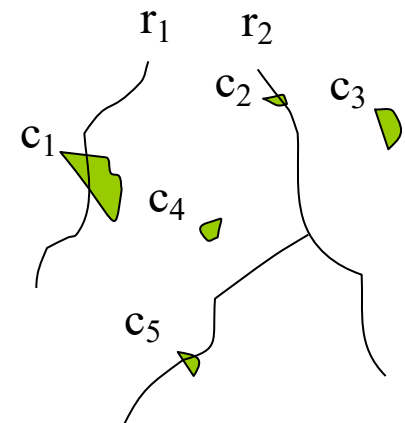
e.g. find the city closest to the forest F

Answer: c_2

- **Spatial join**

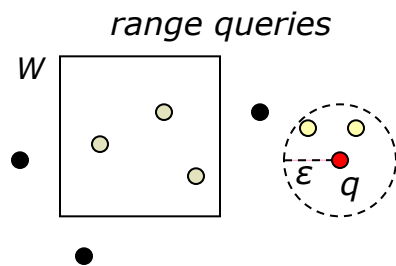
e.g. find all pairs of cities and rivers that intersect

Answer set: $\{(r_1, c_1), (r_2, c_2), (r_2, c_5)\}$

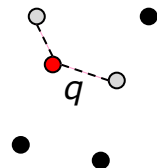


Spatial Queries for points

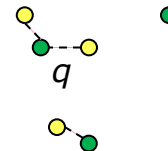
- For point data, the typical queries are
 - Range queries where the range can be:
 - A rectangular window
 - A circular range around a reference point
 - Nearest neighbor queries
 - Find k-nearest points
 - Spatial distance joins
 - Find pairs that are near each other
 - Find the closest pairs



2-nearest neighbor query



ε-distance join ----ε



Spatial Data Management



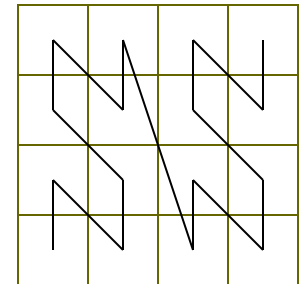
Issues in Spatial Data Management

- Data dimensionality

- There is **no total ordering** of objects in the multidimensional space that preserves proximity

- Example: space-filling curves

- Two nearby points could be far in curve order
 - Neighboring values in curve could be far in space



z-curve

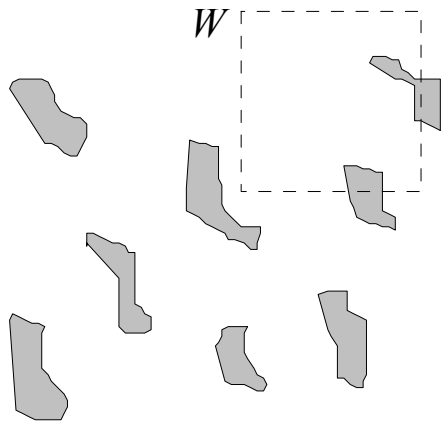
- Geometries of non-point objects

- adds to the complexity of partitioning the objects for indexing purposes

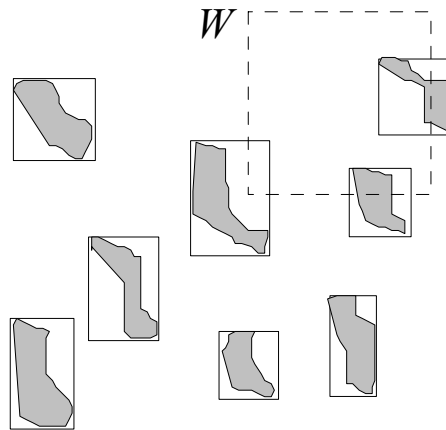
- Hence, traditional indexing and search techniques for 1D data are not applicable

Two-step Spatial Query Processing

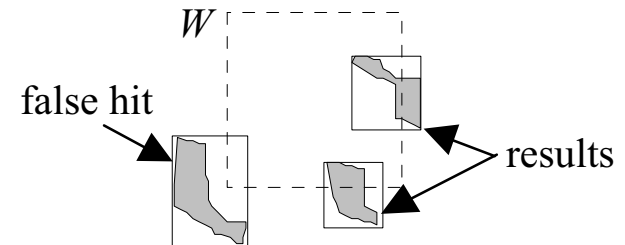
- ❑ Complex geometries of spatial objects are approximated by their minimum bounding rectangles (MBR)
- ❑ A spatial query is then processed in two steps:
 - ❑ **Filter step:** The MBR is tested against the query predicate
 - ❑ **Refinement step:** The exact geometry of objects that pass the filter step is tested for qualification



(a) objects and a query

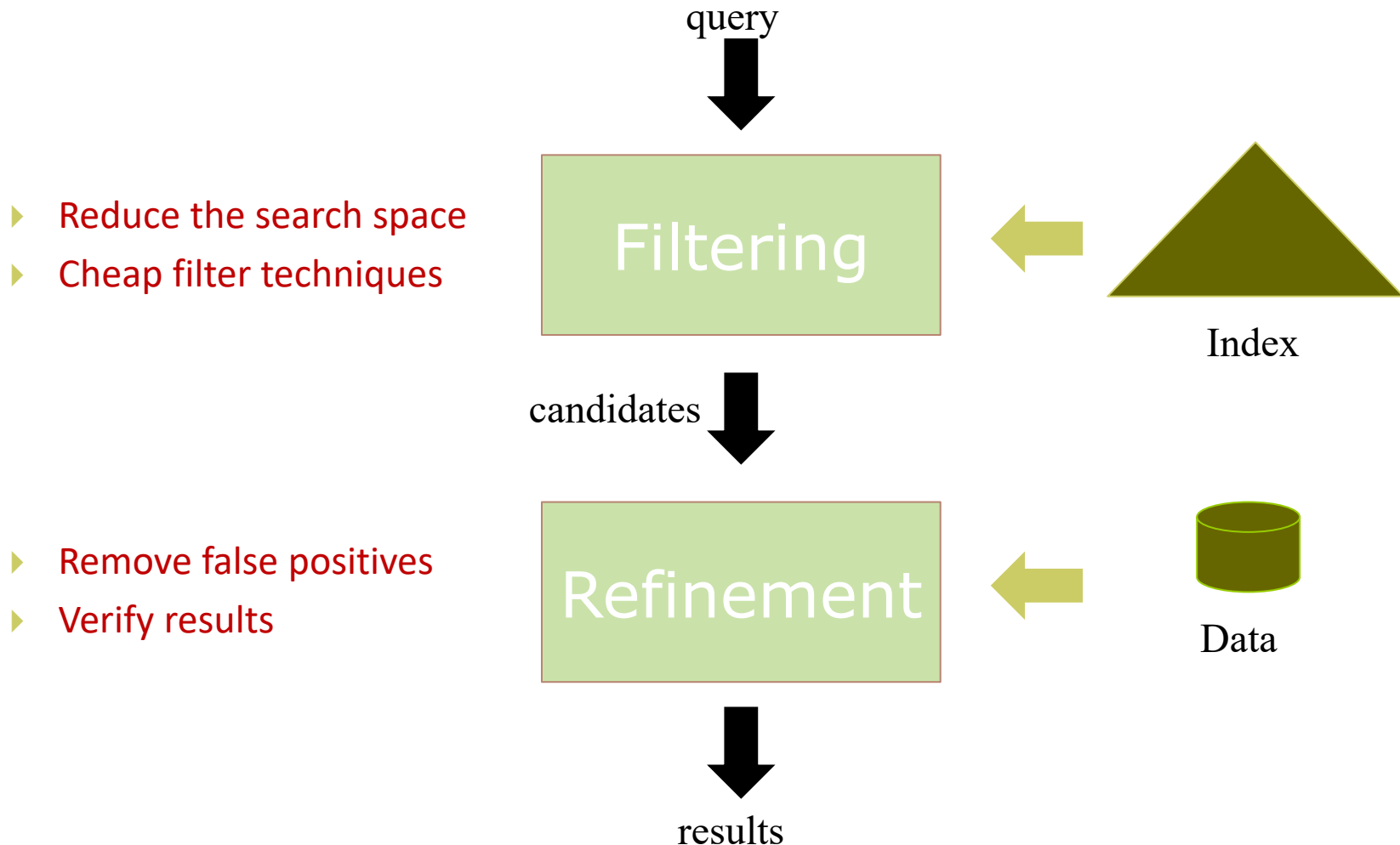


(b) object MBRs



(c) candidates and results ¹⁷

The Filter-and-Refine Paradigm



Indexing techniques for points

- ❑ Spatial point objects in applications:
 - ❑ GPS locations
 - ❑ Points of Interest on maps
 - ❑ Scientific applications (e.g., astronomy)
- ❑ Indexes for points
 - ❑ space-filling curves
 - ❑ grid
 - ❑ k-d-tree
 - ❑ quadtree
- ❑ All of them can be generalized for multi-dimensional points

Z-order curve (Morton curve)

- ❑ Reading:
 - ❑ https://en.wikipedia.org/wiki/Z-order_curve
- ❑ Maps multi-dimensional points to 1D values
 - ❑ Space is discretized, coordinates are converted to integers
 - ❑ Applies **bit-interleaving** to the binary representations of coordinates
- ❑ A multi-dimensional range query is equivalent to finding the points in a set of 1D ranges on the Z-order curve
- ❑ See also: Hilbert space-filling curve

Z-order curve (Morton curve)

	x: 0		1		2		3		4		5		6		7	
	000		001		010		011		100		101		110		111	
y: 0	000000		000001		000100		000101		010000		010001		010100		010101	
1	000010		000011		000110		000111		010010		010011		010110		010111	
2	001000		001001		001100		001101		011000		011001		011100		011101	
3	001010		001011		001110		001111		011010		011011		011110		011111	
4	100000		100001		100100		100101		110000		110001		110100		110101	
5	100010		100011		100110		100111		110010		110011		110110		110111	
6	101000		101001		101100		101101		111000		111001		111100		111101	
7	101010		101011		101110		101111		111010		111011		111110		111111	

Example: $(x,y) = (3,6)$

$x = 0b011$ $y = 0b110$

 $z = 0b101101 = 45$

Example: $z = 14$

$z = 0b001110$

 $x = 0b010$ $y = 0b011$

Z-order curve (Morton curve)

	x: 0 000	1 001	2 010	3 011	4 100	5 101	6 110	7 111
y: 0 000	000000	000001	000100	000101	010000	010001	010100	010101
1 001	000010	000011	000110	000111	010010	010011	010110	010111
2 010	001000	001001	001100	001101	011000	011001	011100	011101
3 011	001010	001011	001110	001111	011010	011011	011110	011111
4 100	100000	100001	100100	100101	110000	110001	110100	110101
5 101	100010	100011	100110	100111	110010	110011	110110	110111
6 110	101000	101001	101100	101101	111000	111001	111100	111101
7 111	101010	101011	101110	101111	111010	111011	111110	111111

Range query

x in [4,5]

y in [2,5]

Can be converted to 1D ranges:

[0b011000, 0b011011]

[0b110000, 0b111011]

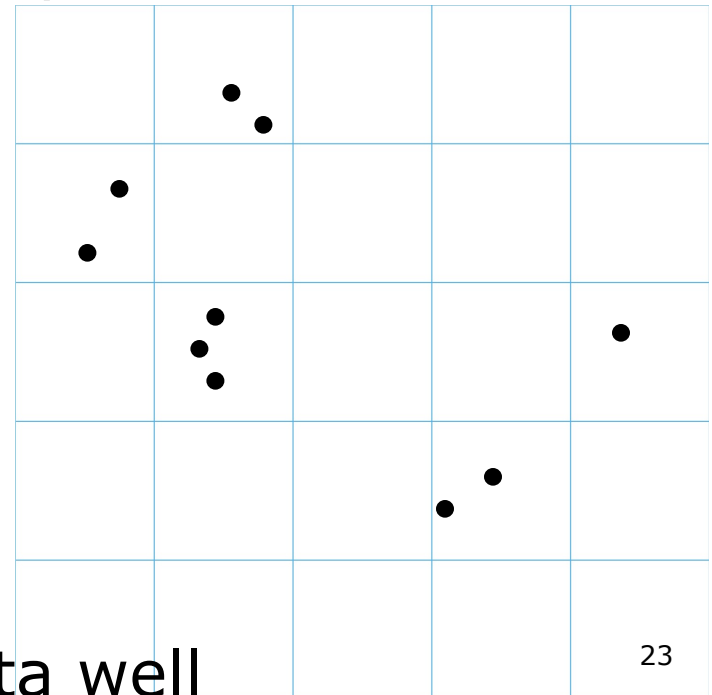
Evaluate query ranges if z-order values of points are indexed by a B⁺-tree

Alternative approach using a BST:

Tropf, H.; Herzog, H.
(1981), "Multidimensional
Range Search in Dynamically
Balanced Trees" Angewandte
Informatik, 2: 71–77

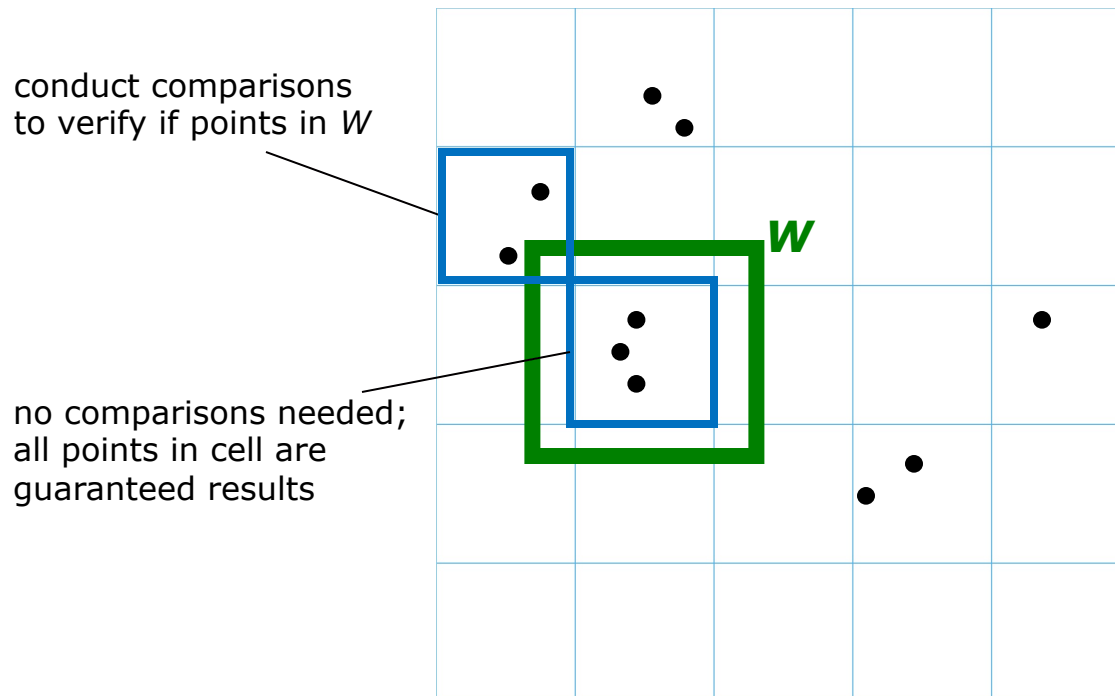
Grid Indexing

- ❑ Simple and suitable for highly dynamic data
- ❑ Space is divided by a **uniform grid** to disjoint cells
- ❑ Each grid cell defines a **space partition**
- ❑ Each point is assigned to one partition
 - ❑ Cell that contains the point is found in $O(1)$ using algebraic operations
- ❑ Supports fast insert/delete and search in main memory
 - ❑ Not as good for disk
- ❑ Range queries, NN queries and joins also fast
- ❑ Does not support skewed data well



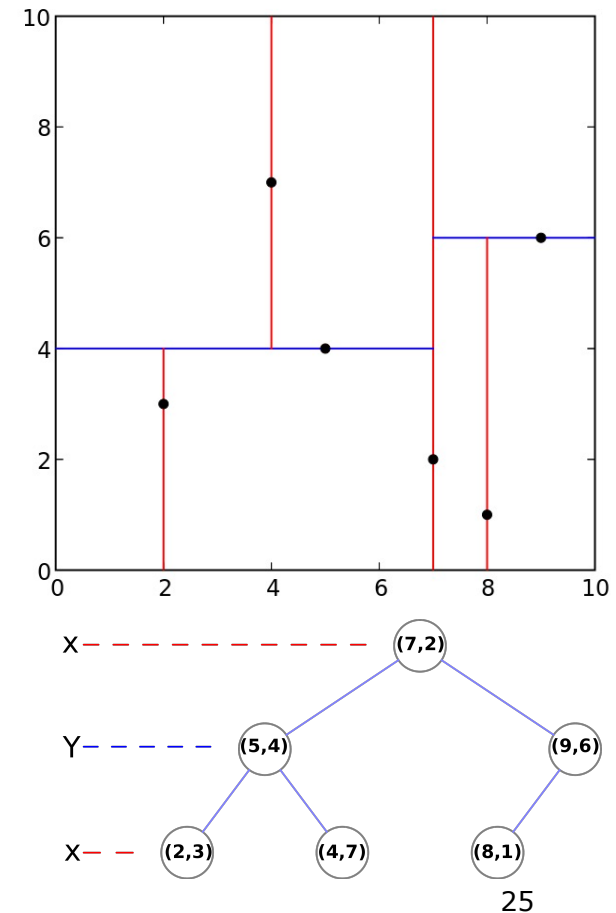
Grid Indexing – range queries

- Find cells that intersect W in $O(1)$ using range extent at each dimension at each dimension
- Empty cells are disregarded
- Cells totally covered by W need no comparisons



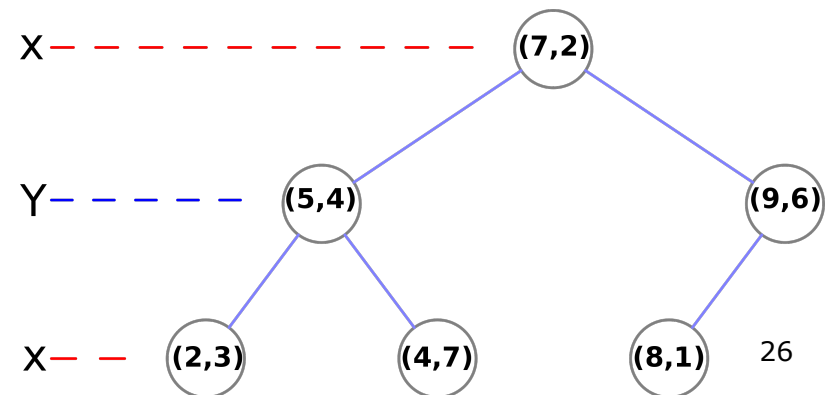
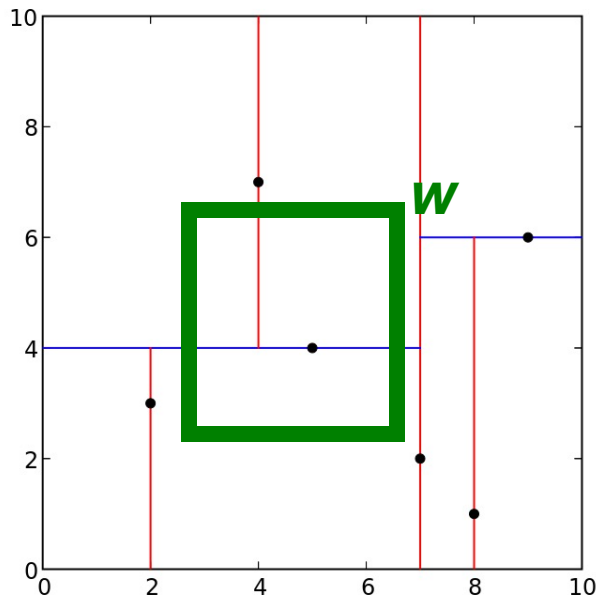
k-d tree

- ❑ Reading: https://en.wikipedia.org/wiki/K-d_tree
- ❑ A **binary tree** for points in a k-dimensional space
- ❑ Root uses one point (median) to divide the space into two subspaces using one dimension
 - ❑ sort a sample to find median
- ❑ Each node uses a point to divide its subspace in two parts
- ❑ Dimensions alternate per level



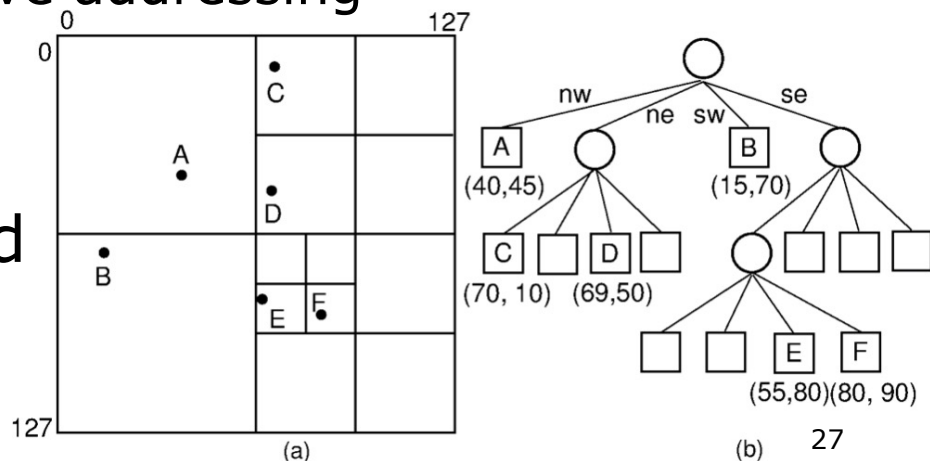
k-d tree – range queries

- Case 1: W entirely before node-point in splitting dimension
 - recursive call for left son
- Case 2: W entirely after node-point in splitting dimension
 - recursive call for right son
- Case 3: W covers node-point in splitting dimension
 - report node if in W ; recursive call for left and right son



Point-region (PR) quadtree

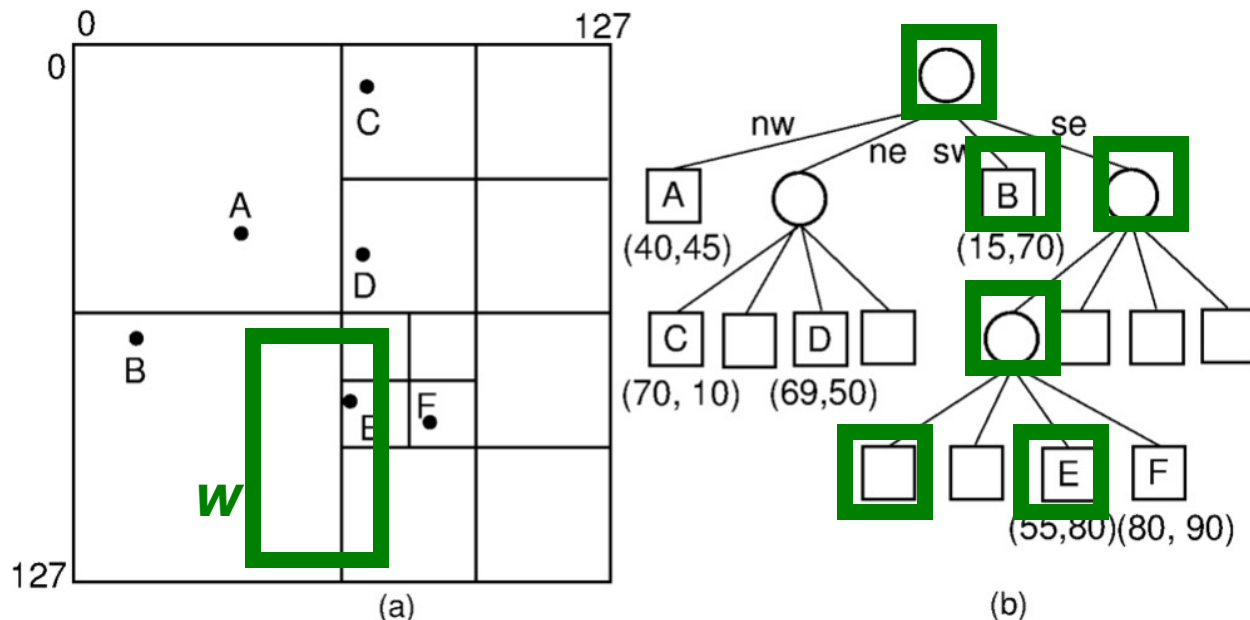
- ❑ Reading: <https://en.wikipedia.org/wiki/Quadtree>
- ❑ Each non-leaf divides its area to four quadrants
- ❑ Each leaf has points (up to a max capacity)
 - ❑ Leaves that exceed the max capacity are split
- ❑ Various tree representations
 - ❑ Traditional: use pointers to link nodes
 - ❑ Implicit: use z-order curve addressing
- ❑ Quadtree may not be balanced
- ❑ Quadtrees are also used for approximating regions or images



PR-quadtree, max capacity = 1

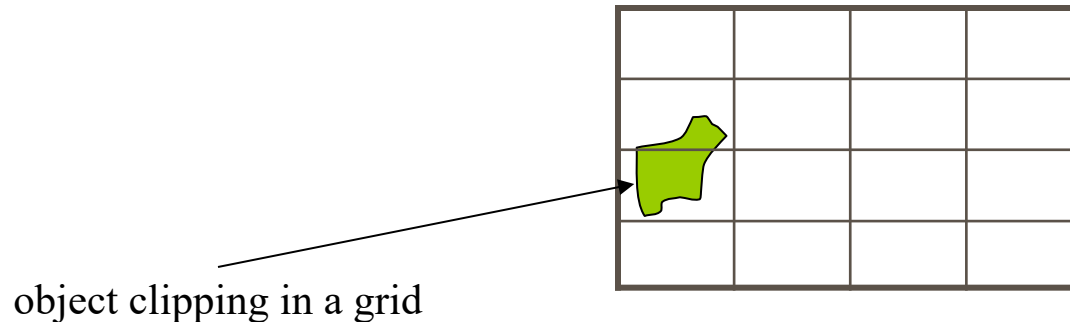
Point-region (PR) quadtree - queries

- Visit nodes that overlap with query range recursively
- Can use bit-interleaving and prefixes of query bounds to guide search at non-leaf nodes
 - That is, prefixes of z-order codes of query range



Spatial Access Methods

- ❑ **Point access methods** are not effective for extended objects
 - They divide the space into **disjoint partitions**
 - Objects may need to be **clipped** into several parts which leads to data redundancy and affects performance negatively



Spatial Access Methods

- ▣ Object clipping can be avoided if we allow the regions of object groups to overlap

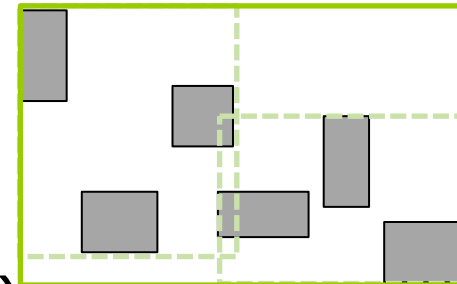
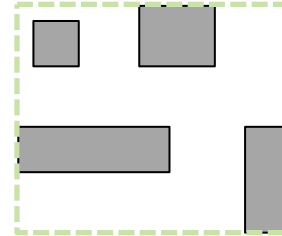


The R-tree

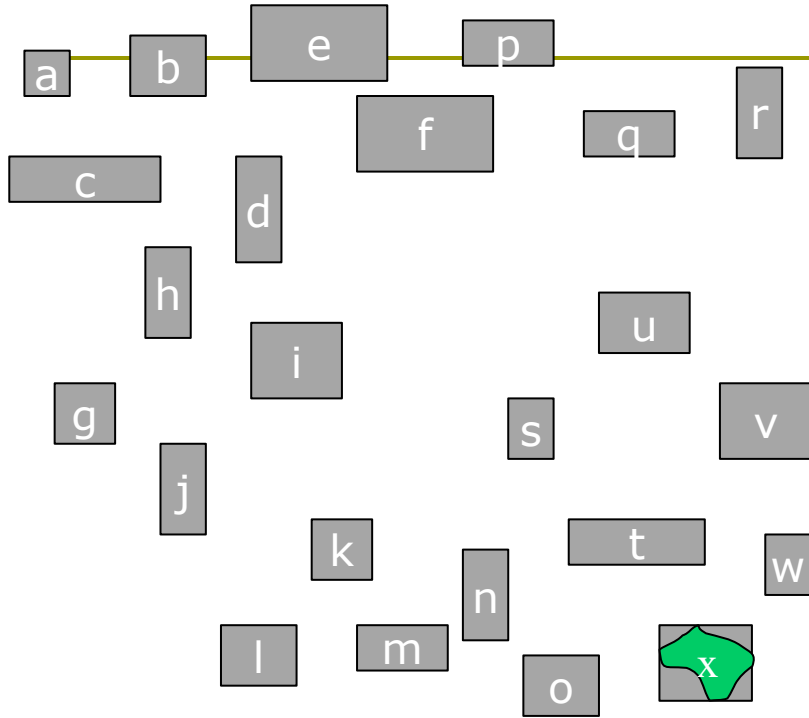
- ❑ Groups **object MBRs** to disk (or memory) blocks hierarchically
- ❑ Each group of objects (a block) is a leaf of the tree
- ❑ The MBRs of the leaf nodes are grouped to form nodes at the next level
- ❑ Grouping is recursively applied at each level until a single group (the root) is formed

The R-tree

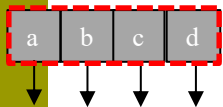
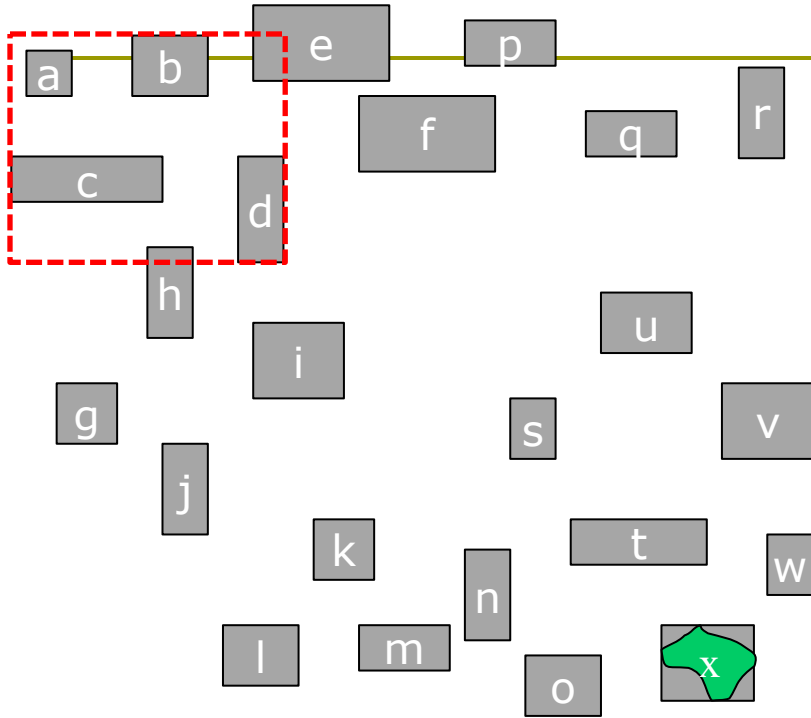
- ❑ Leaf node entries: $\langle \text{MBR}, \text{object-id} \rangle$
- ❑ Non-leaf node entries: $\langle \text{MBR}, \text{ptr} \rangle$
- ❑ The MBR of a non-leaf node entry is the MBR of all entries in the node pointed by it
- ❑ Parameters (except root):
 - M (max no of entries per node)
 - m (min no of entries per node)
 - $m \leq M/2$
 - usually $m=0.4M$
- ❑ Root has at least two children
- ❑ All leaves in same level (balanced tree)
- ❑ 1 node \rightarrow 1 disk block



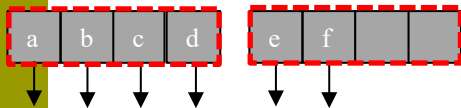
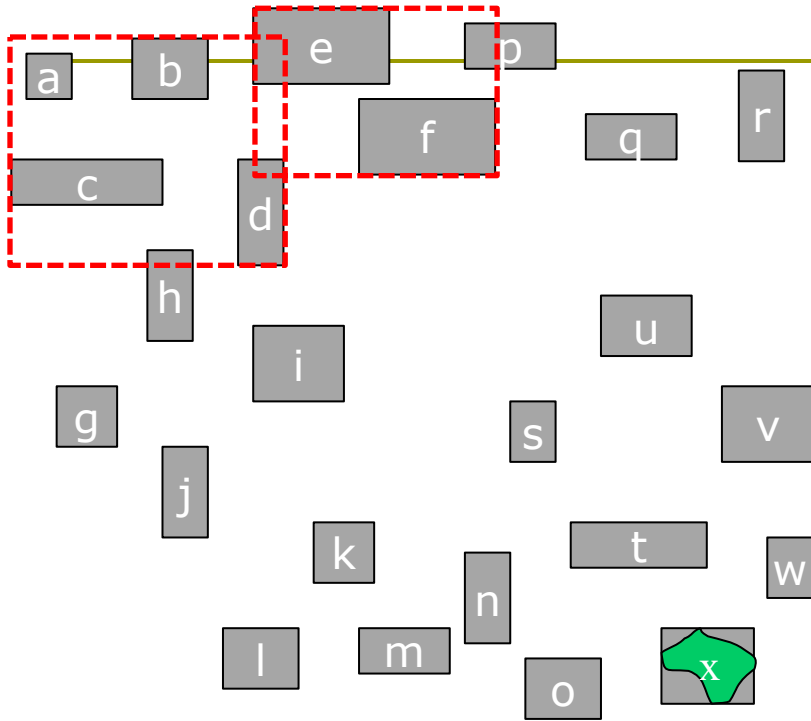
The R-tree - example



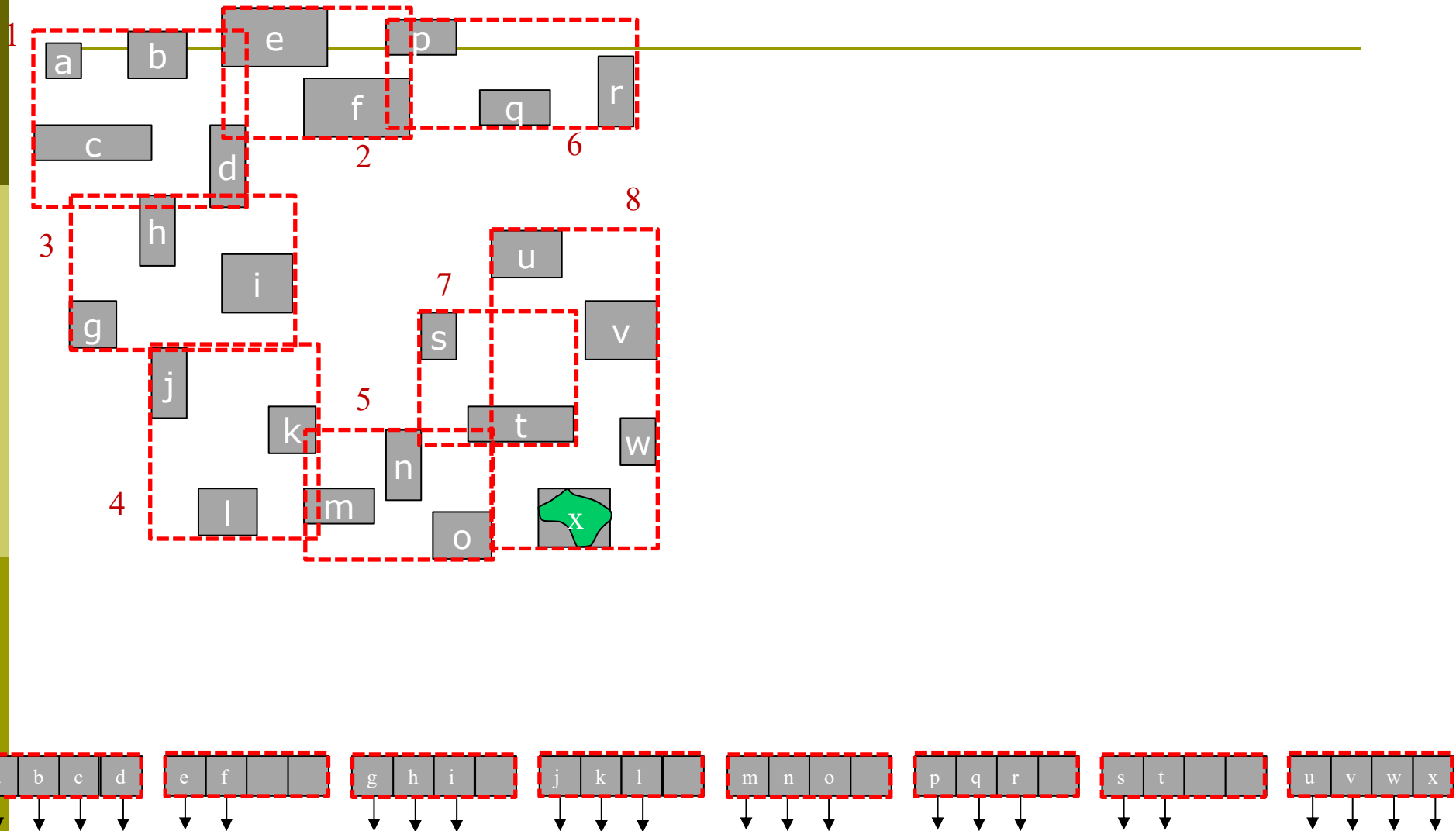
The R-tree - example



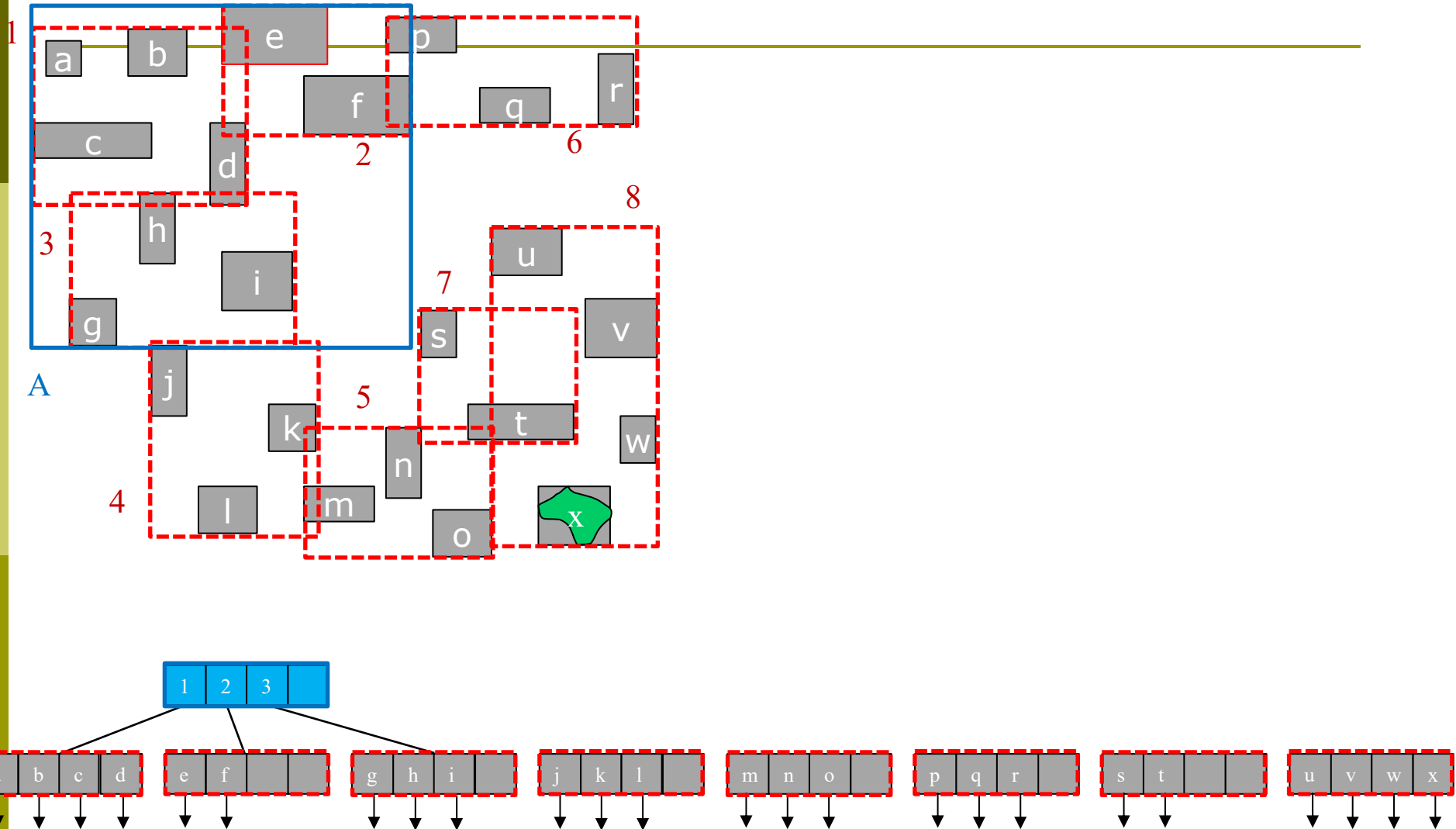
The R-tree - example



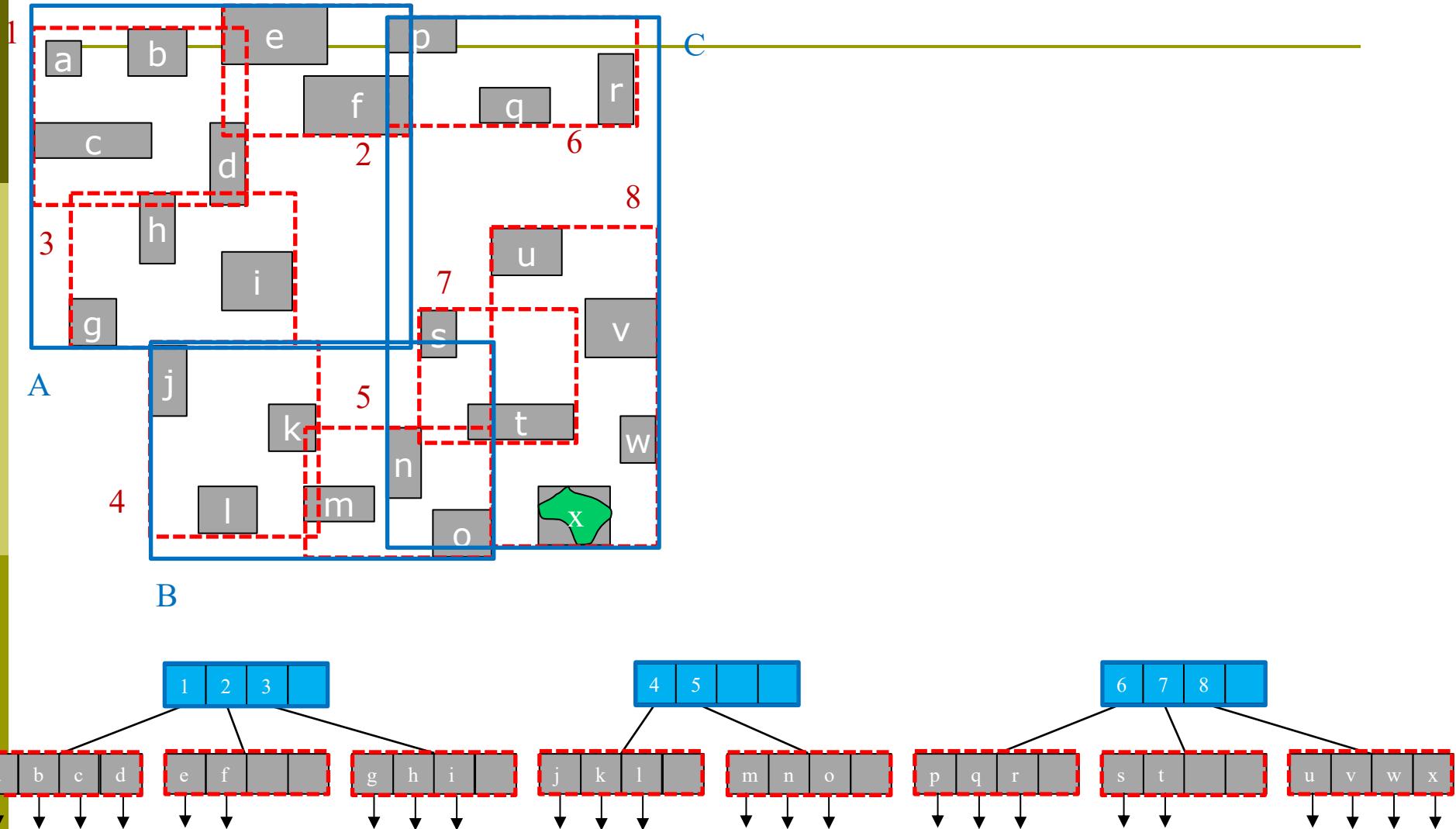
The R-tree - example



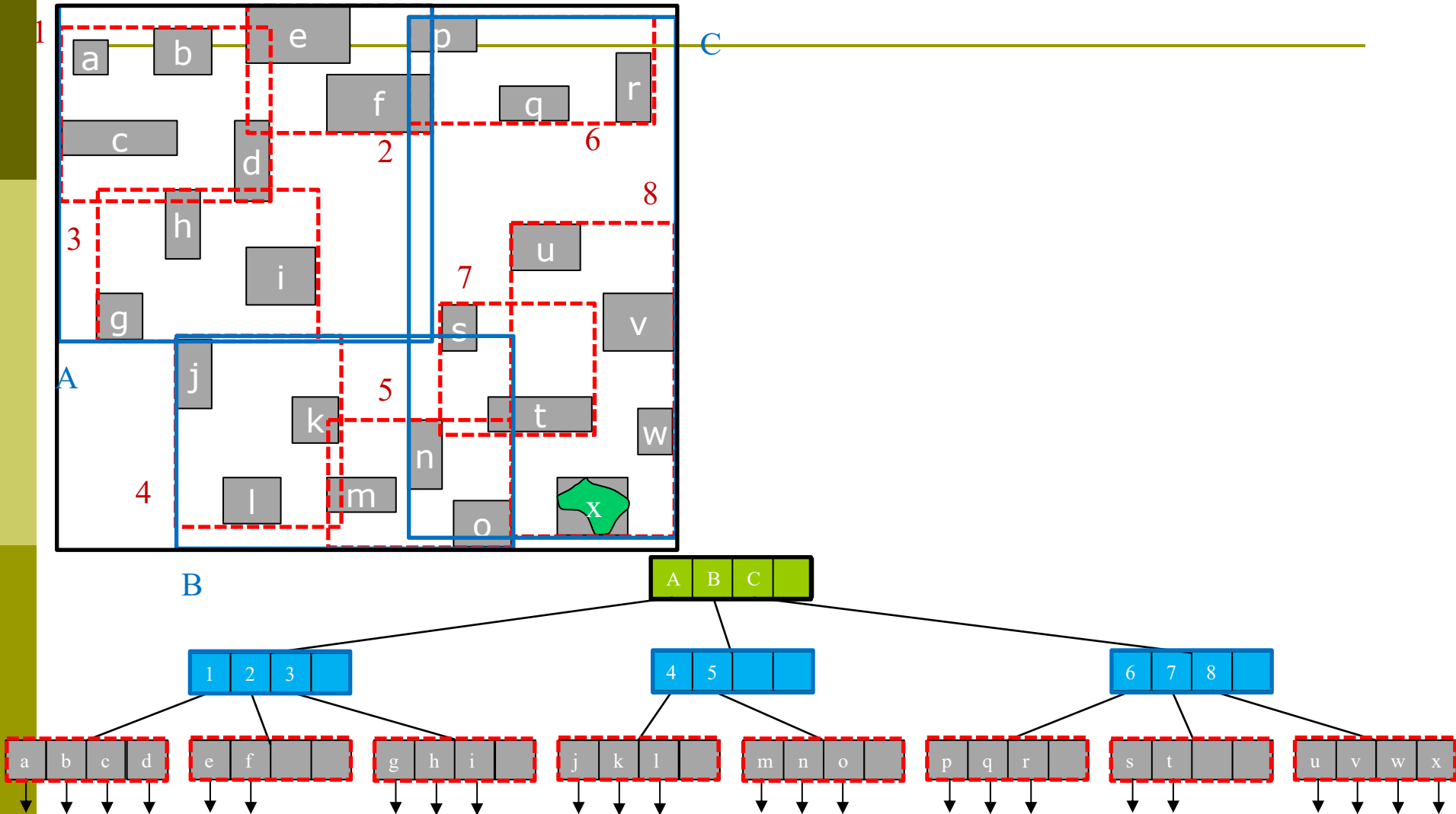
The R-tree - example



The R-tree - example



The R-tree - example



Spatial Query Evaluation



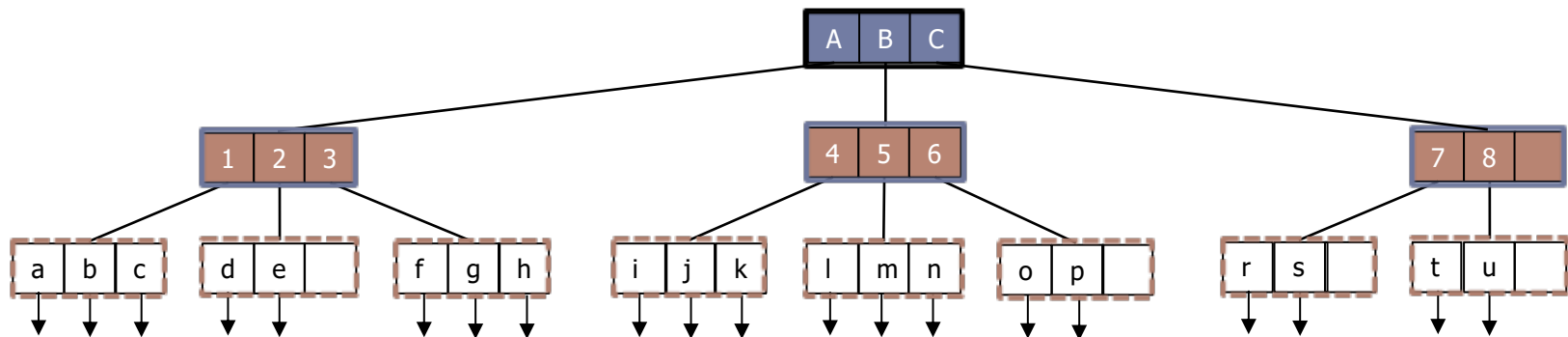
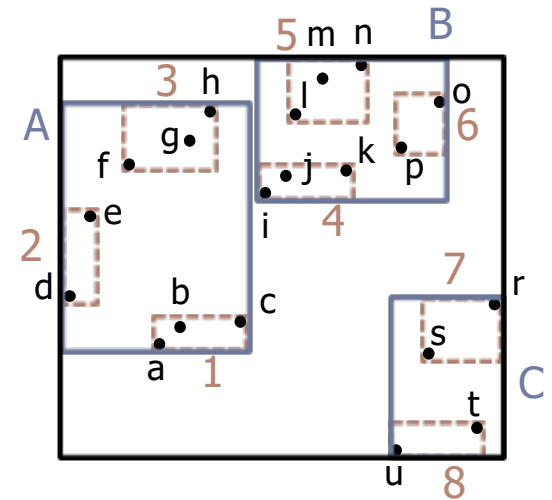
Range searching using an R-tree

- Range_query(query W , R-tree node n):
 - If n is not a leaf node
 - For each index entry e in n such that e .MBR intersects W
 - visit node n' pointed by $e.ptr$
 - Range_query(W , n')
 - If n is a leaf
 - For each index entry e in n such that e .MBR intersects W
 - visit object o pointed by $e.object-id$
 - test range query against exact geometry of o ; if o intersects W , report o
- May follow multiple paths during search
- Different search predicates are used for different relationships with W .
 - What if we want to find all objects inside W ?

Spatial range query

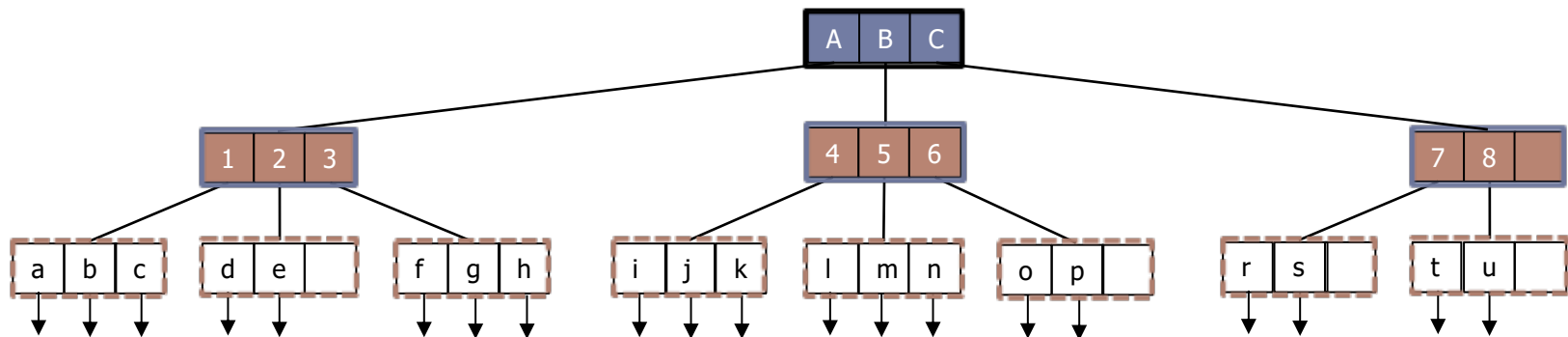
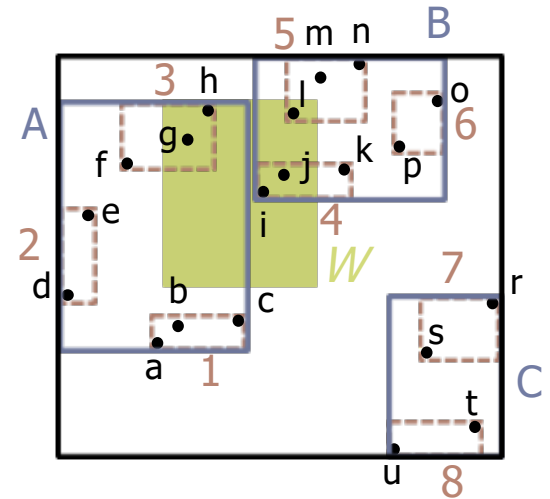
□ Traverse tree

- Top-down
- Visit nodes whose MBRs qualify predicate



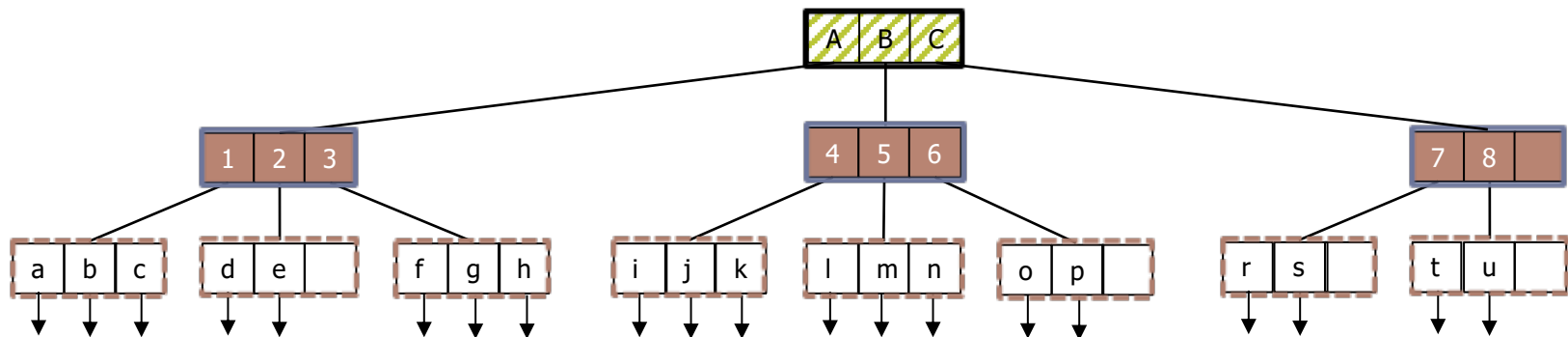
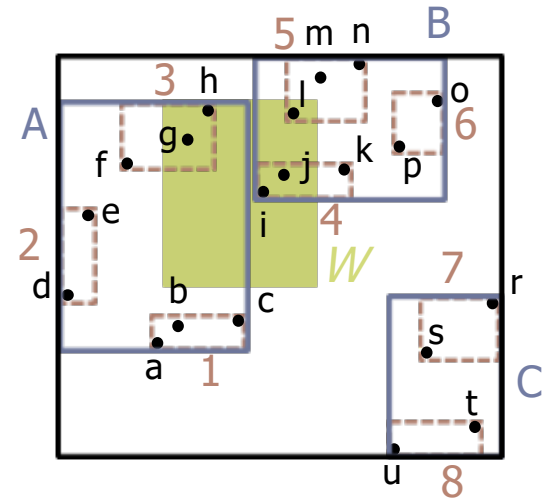
Spatial range query

- **Traverse tree**
 - **Top-down**
 - **Visit** nodes whose **MBRs** **qualify predicate**
- E.g., find points **inside** *W*



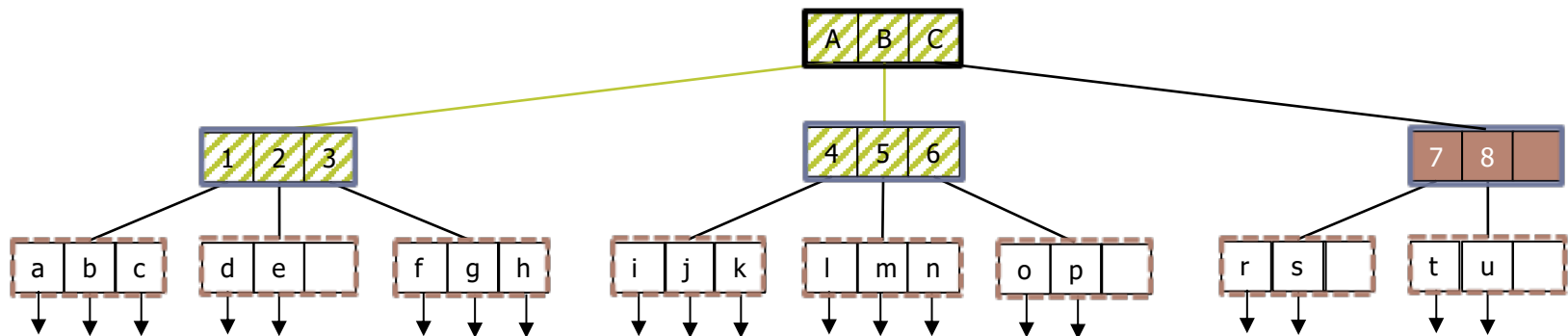
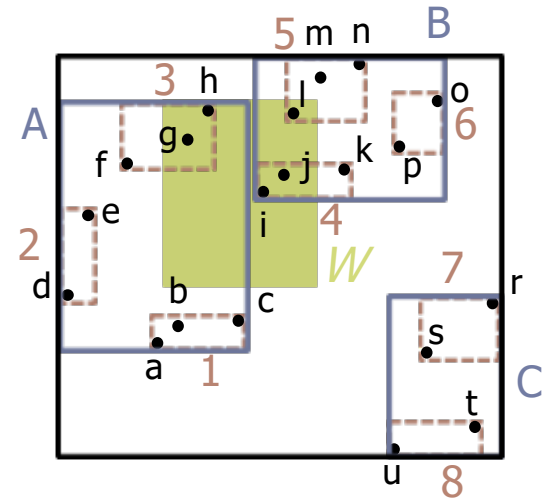
Spatial range query

- **Traverse tree**
 - **Top-down**
 - **Visit** nodes whose **MBRs** **qualify predicate**
- E.g., find points **inside** *W*



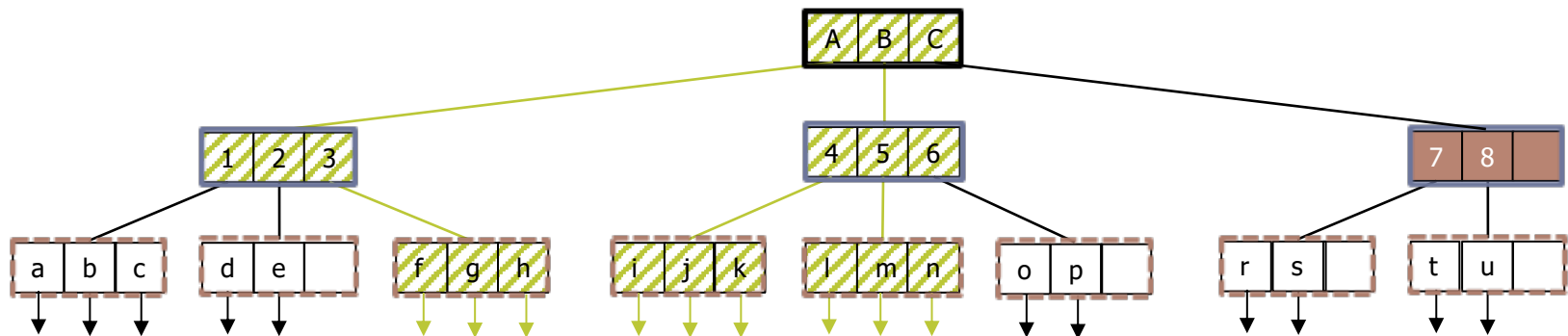
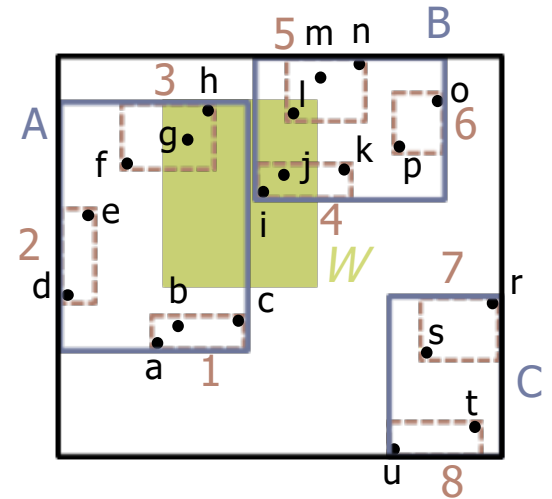
Spatial range query

- **Traverse tree**
 - **Top-down**
 - **Visit** nodes whose **MBRs** **qualify predicate**
- E.g., find points **inside** *W*



Spatial range query

- **Traverse tree**
 - **Top-down**
 - **Visit** nodes whose **MBRs** **qualify predicate**
- E.g., find points **inside** *W*



R-tree Construction



Construction of the R-tree

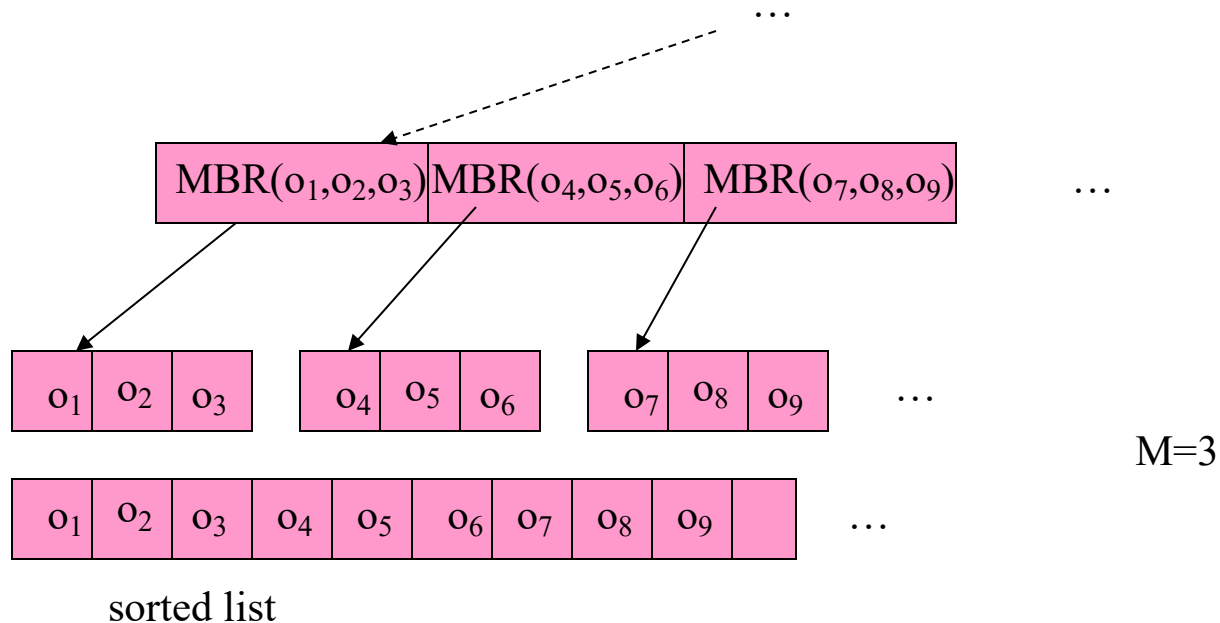
- ❑ Dynamically constructed/maintained
- ❑ Insertions/deletions interleave with search operations
- ❑ Insertion similar to B⁺-tree
 - However special optimization algorithms have to be designed for
 - ❑ choosing the path where a new MBR is inserted
 - ❑ splitting overflowed nodes
- ❑ Underflows in deletions are handled by
 - deleting the underflowed leaf node
 - re-inserting the remaining entries

Bulk-loading R-trees

- ❑ Given a **static set S** of rectangles, build an R-tree that indexes S.
- ❑ Method 1: iteratively insert rectangles into an initially empty tree
 - tree reorganization is slow
 - tree nodes are not as full as possible: more space occupied for the tree
- ❑ Method 2: **bulk-load** the rectangles into the tree using some fast (sort or hash-based) process
 - R-tree is built fast
 - good space utilization

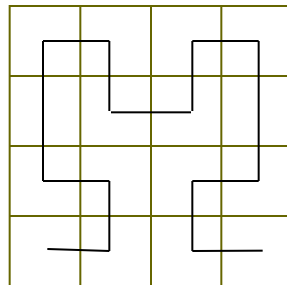
Bulk-loading R-trees

- Method 1: Sort using only one axis
 - sort rectangles using the x-coordinate of their center
 - pack M consecutive rectangles in leaf nodes
 - build tree bottom-up



Bulk-loading R-trees

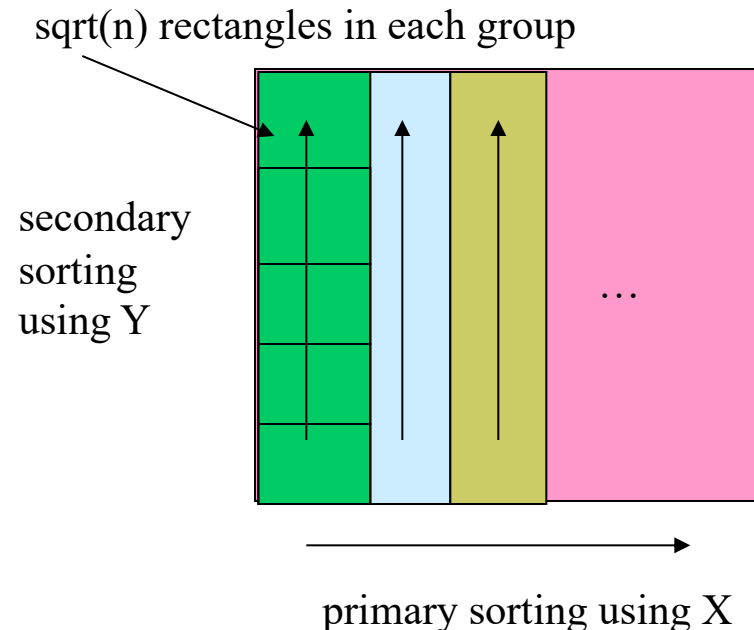
- ❑ Method 1 results in leaf nodes that have long stripes as MBRs
- ❑ Method 2: use a space-filling curve to order the rectangles
 - much better structure, but still the nodes have large overlap



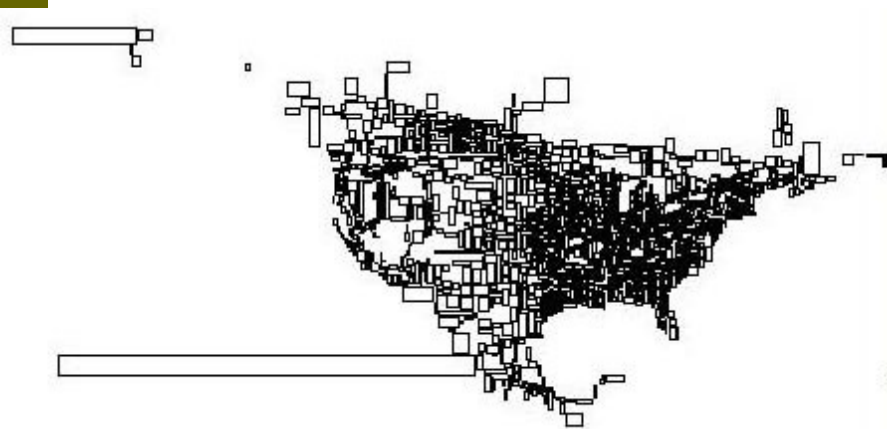
Hilbert-curve

Bulk-loading R-trees

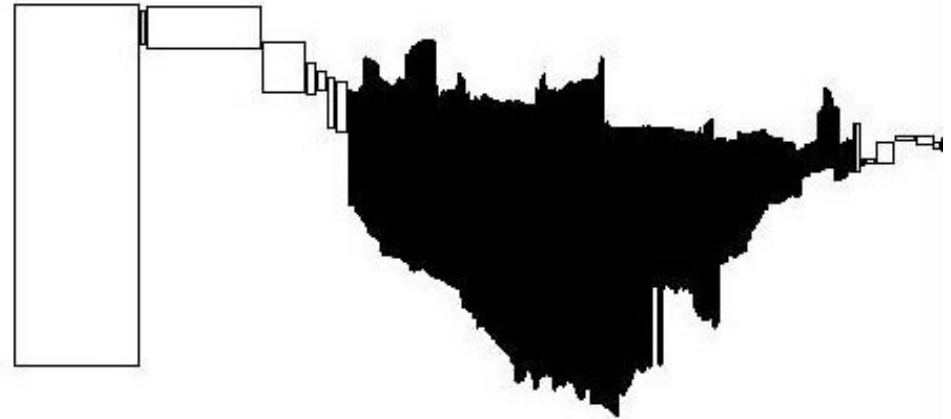
- Method 3: Sort using one axis first and then groups of \sqrt{n} rectangles using the other axis
- Usually the best structure compared to bulk-loading methods



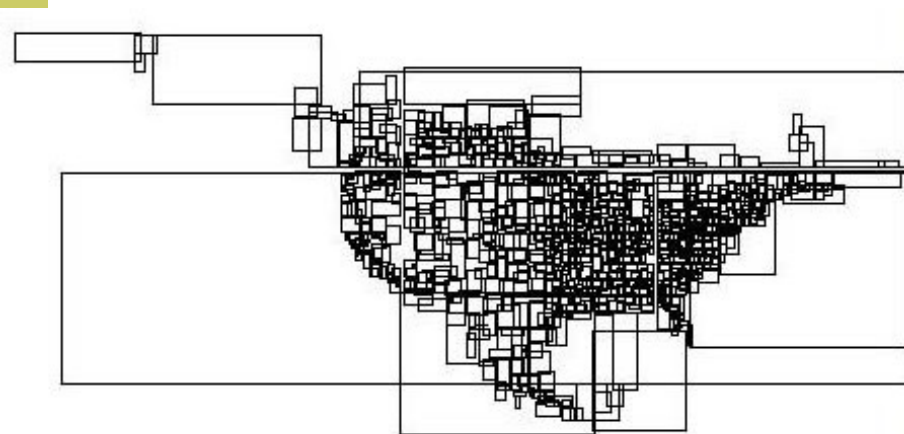
R-tree leaf nodes by different construction methods



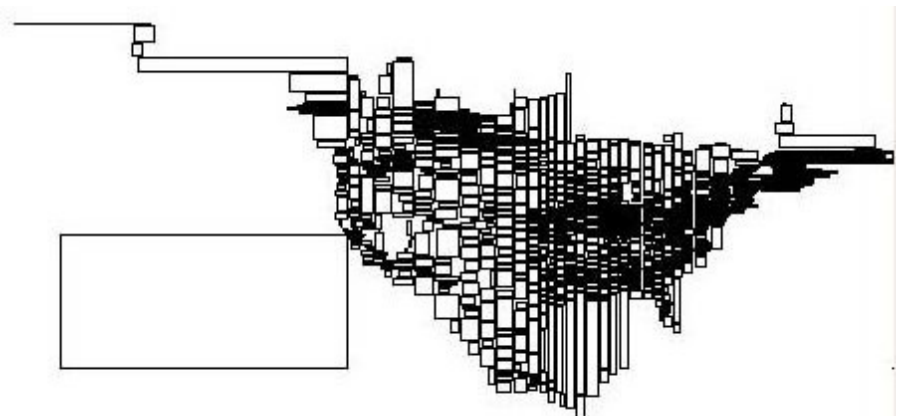
(a) R*-tree insertion



(b) x -sorting



(c) Hilbert sorting



(d) Sort-tile recursive

Nearest Neighbor Queries



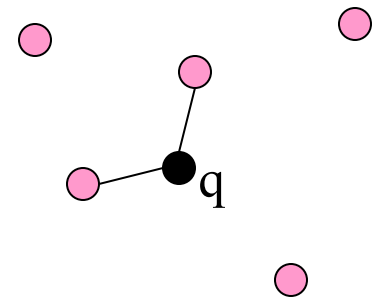
Nearest neighbor search

□ Basic problem:

- Given a spatial relation R and a query object q , find the nearest neighbor of q in R
- Formally:
 - $NN(q,R) = o \in R: \text{dist}(q,o) \leq \text{dist}(q,o'), \forall o' \in R$

□ Note:

- We can have more than one NN (with equal minimum distance)
- Break ties arbitrarily



Nearest neighbor search

□ Generalized problem:

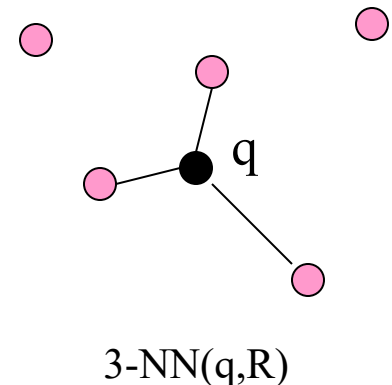
- Given a spatial relation R , a query object q , and a number $k < |R|$, find the k -nearest neighbors of q in R
- Formally:
 - $NN(q, k, R) = S \subset R : |S| = k, \text{dist}(q, o) \leq \text{dist}(q, o'), \forall o \in S \forall o' \in R - S$

□ Note:

- We can have more than one k -NN sets (with multiple possible equidistant furthest points in them)

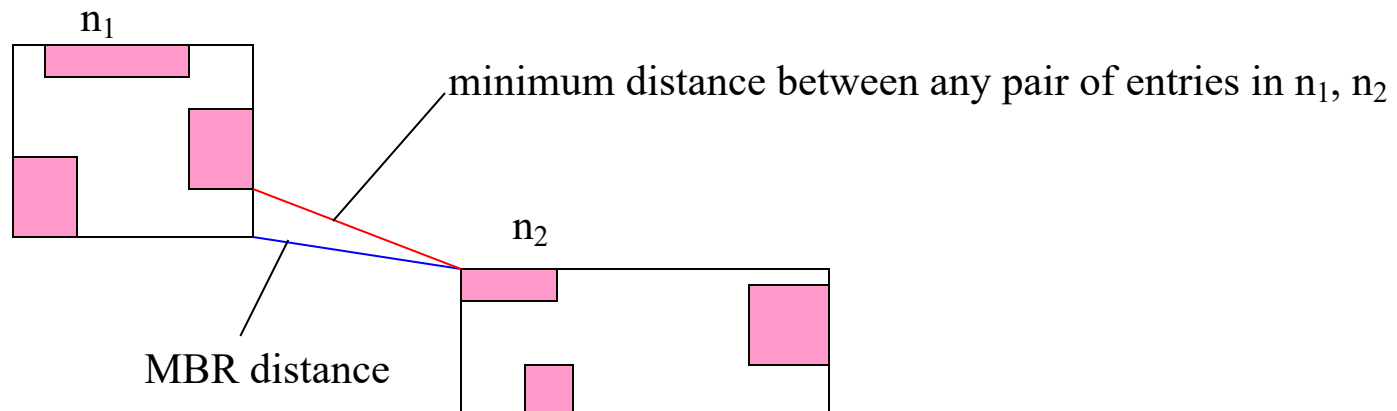
□ Simplification

- NN (and k -NN) operations return **any** NN (and k -NN sets)
- We usually focus on point-sets



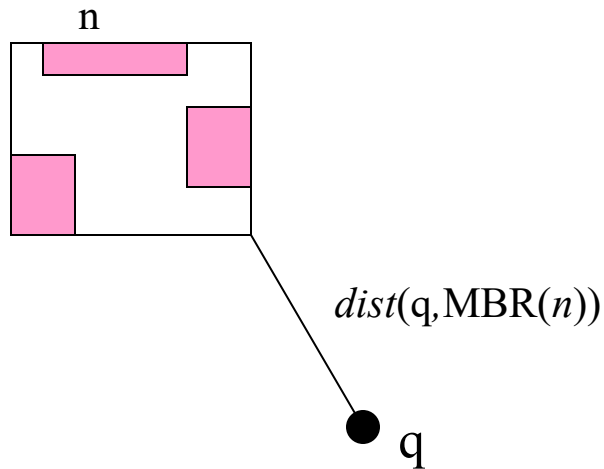
Distance measures and MBRs

- Distances between R-tree node MBRs lower-bound the distances between the entries in them
 - $\text{dist}(\text{MBR}(n_i), \text{MBR}(n_j)) \leq \text{dist}(e_i.\text{MBR}, e_j.\text{MBR}),$
 $\forall e_i \in n_i, e_j \in n_j$



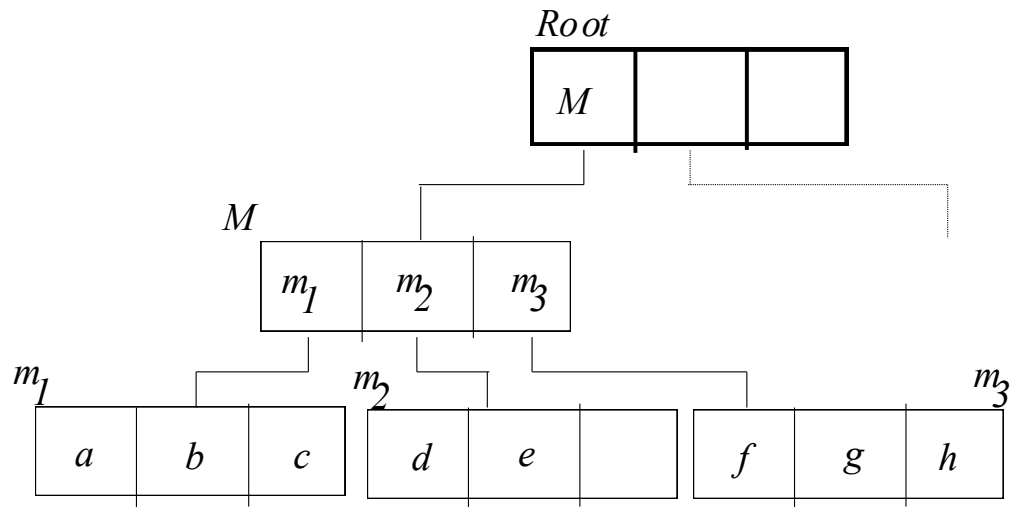
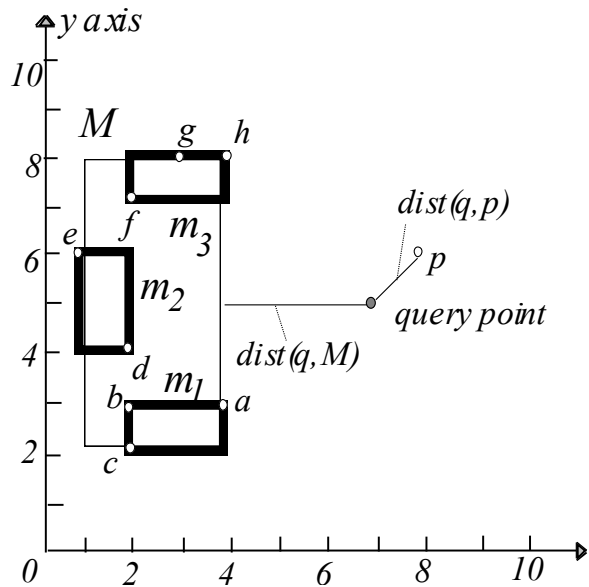
Distance measures and MBRs

- The distance between a query object q and an R-tree node MBR lower-bounds the distances between q and the objects indexed under this node
 - $dist(q, MBR(n)) \leq dist(q, o) \forall o$ indexed under n



Using MBR distances to guide/prune search in an R-tree

- ❑ Problem: find the NN of q
- ❑ Do we need to look for it in node M if we know $\text{dist}(q, p)$?



Depth-first NN search using an R-tree

- ❑ Start from the root and visit the first entry.
- ❑ Continue recursively, until a leaf node n_l is visited.
- ❑ Find the NN of q in n_l .
- ❑ Continue visiting other nodes after backtracking as long there are nodes closer to q than the current NN.
 - ❑ Do not visit a node, if the nearest distance between q and the node's MBR exceeds the current $\text{dist}(q, o_{\text{NN}})$

Depth-first NN search using an R-tree

▣ Recursive function (for data points only)

DFNN(query point q , node n , point o_{NN})

Initially: n is tree's root

Initially:
- $o_{NN} = \text{null}$
- $\text{dist}(q, o_{NN}) = \infty$

if n is a leaf node then

for each entry e in n do

if $\text{dist}(q, e) < \text{dist}(q, o_{NN})$ then

$o_{NN} = e$ // found closest point than current NN

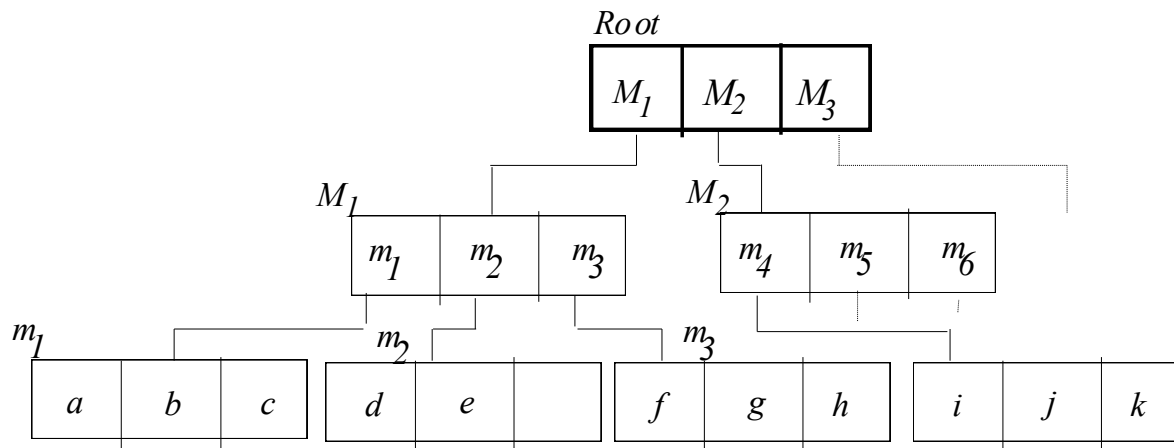
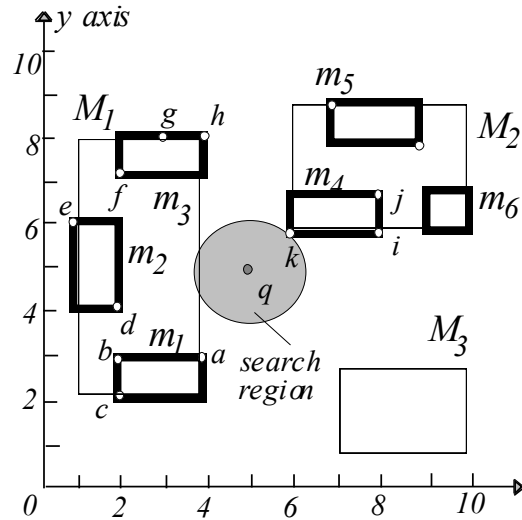
else

for each entry e in n do

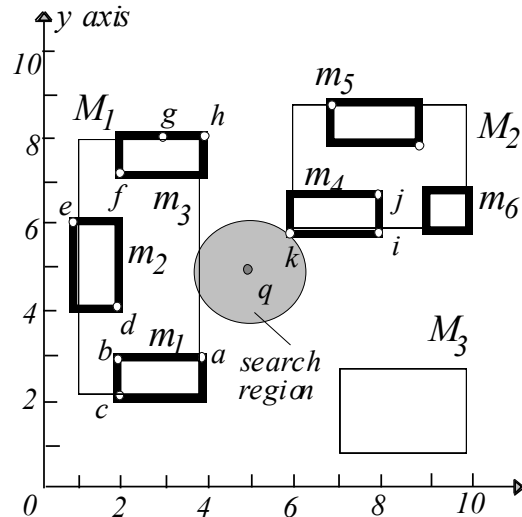
if $\text{dist}(q, e.MBR) < \text{dist}(q, o_{NN})$ then

DFNN(q , $e.ptr$, o_{NN}) // recursive call for node pointed by e

Depth-first NN search using an R-tree



Depth-first NN search using an R-tree



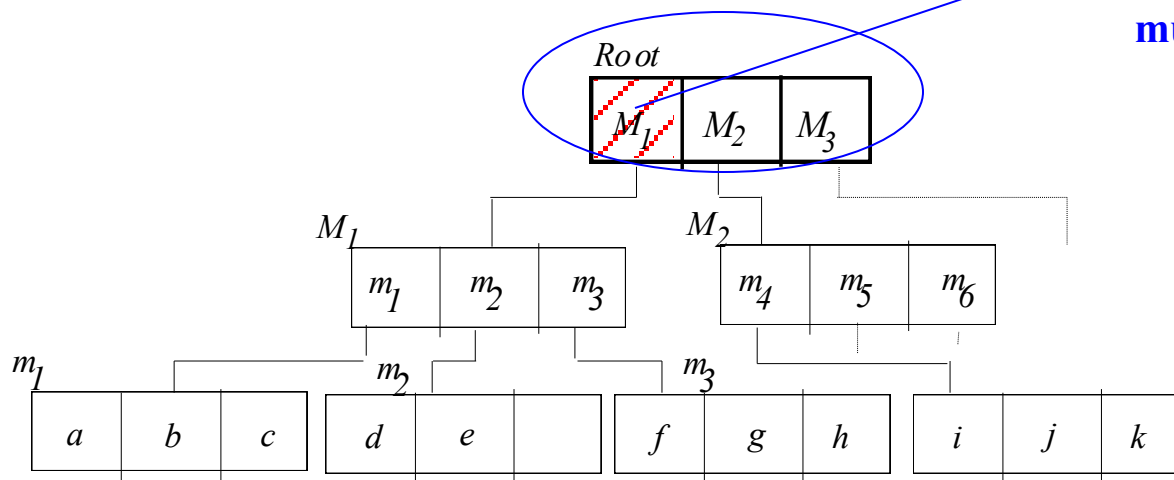
$o_{NN} = \text{NULL}$

$\text{dist}(q, o_{NN}) = \infty$

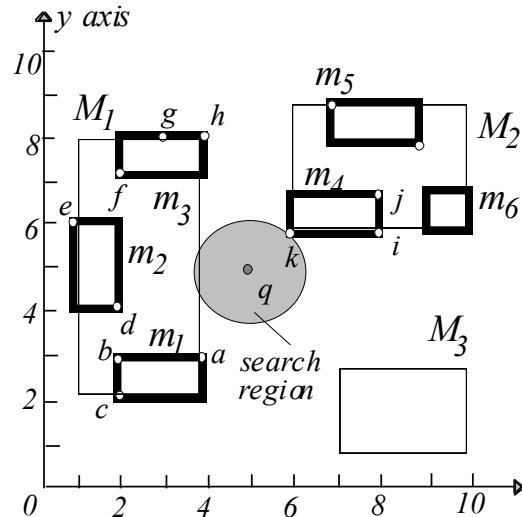
1. visit root

$\text{dist}(q, M_1) < \text{dist}(q, o_{NN})$

must visit node M_1



Depth-first NN search using an R-tree



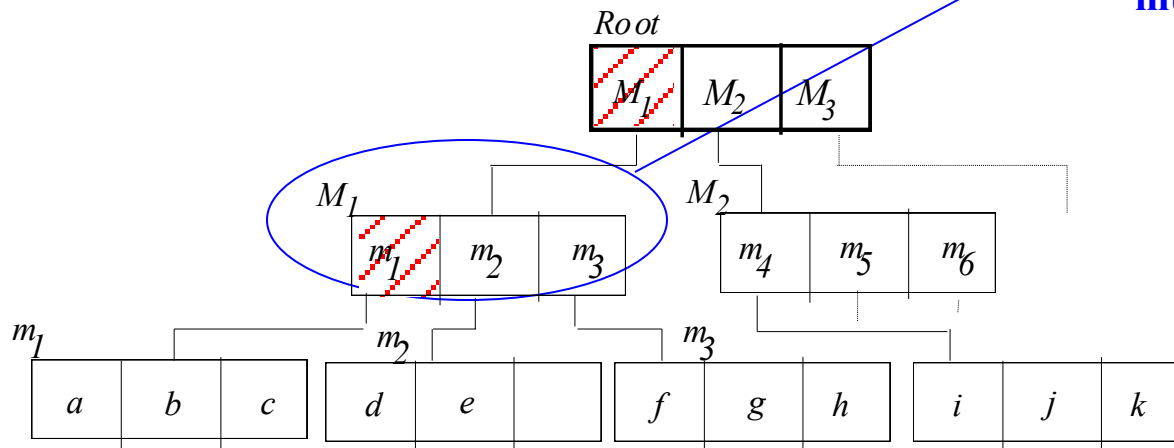
$o_{NN} = \text{NULL}$

$\text{dist}(q, o_{NN}) = \infty$

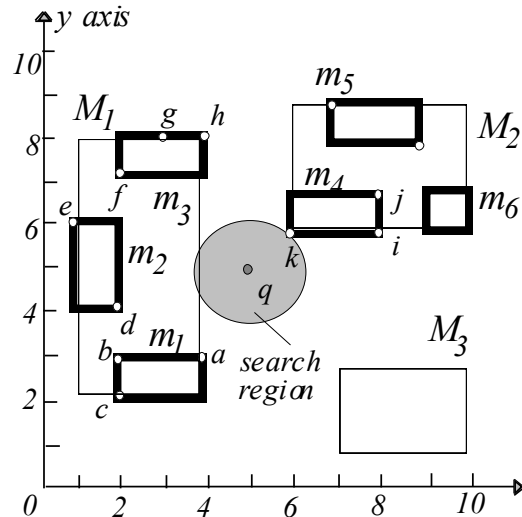
2. visit M_1

$\text{dist}(q, m_1) < \text{dist}(q, o_{NN})$

must visit node m_1



Depth-first NN search using an R-tree



$o_{NN} = \text{NULL}$

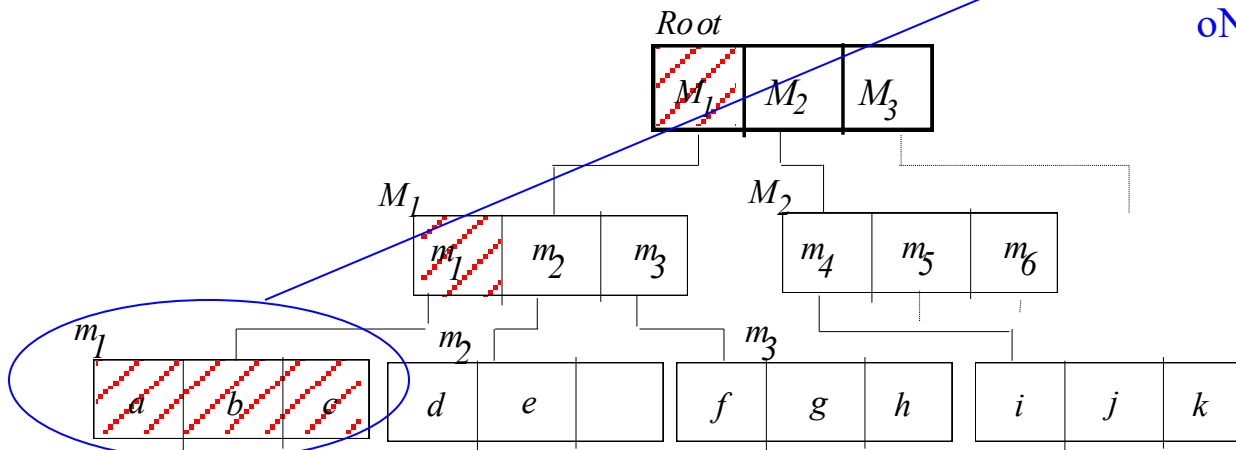
$\text{dist}(q, o_{NN}) = \infty$

3. visit m_1

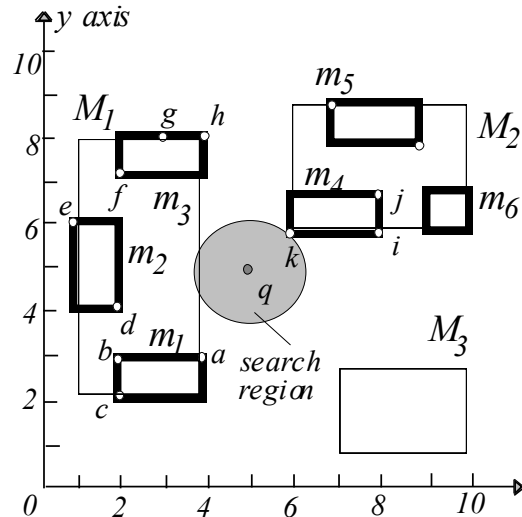
check a, b, c

found new NN:

$o_{NN} = a, \text{dist}(q, o_{NN}) = \sqrt{5}$



Depth-first NN search using an R-tree



$$o_{NN} = a$$

$$\text{dist}(q, o_{NN}) = \sqrt{5}$$

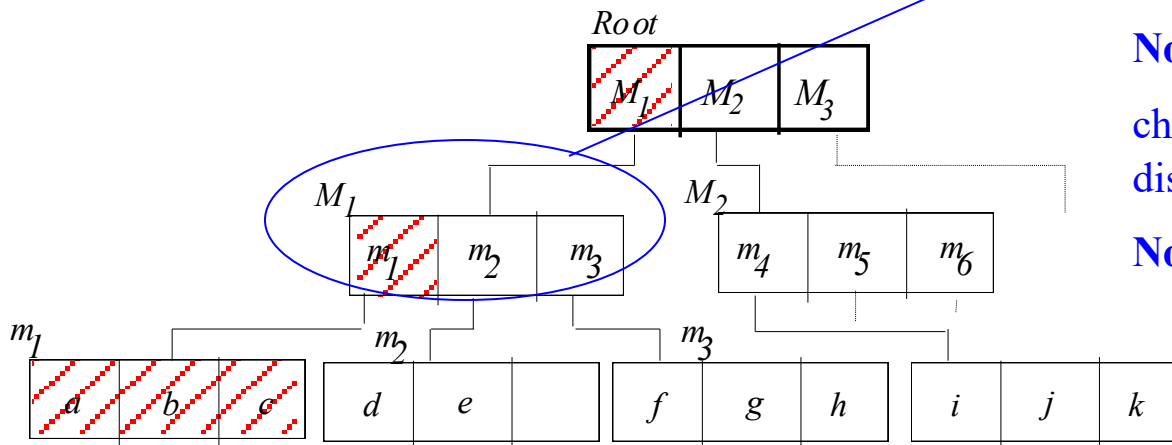
4. backtrack to M_1

check m_2
 $\text{dist}(q, m_2) = 3 \geq \sqrt{5}$:

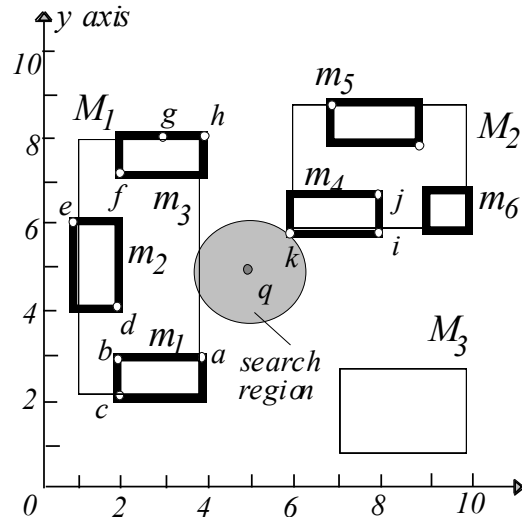
No need to visit node m_2

check m_3
 $\text{dist}(q, m_3) = \sqrt{5} \geq \sqrt{5}$:

No need to visit node m_3



Depth-first NN search using an R-tree



$$o_{NN} = a$$

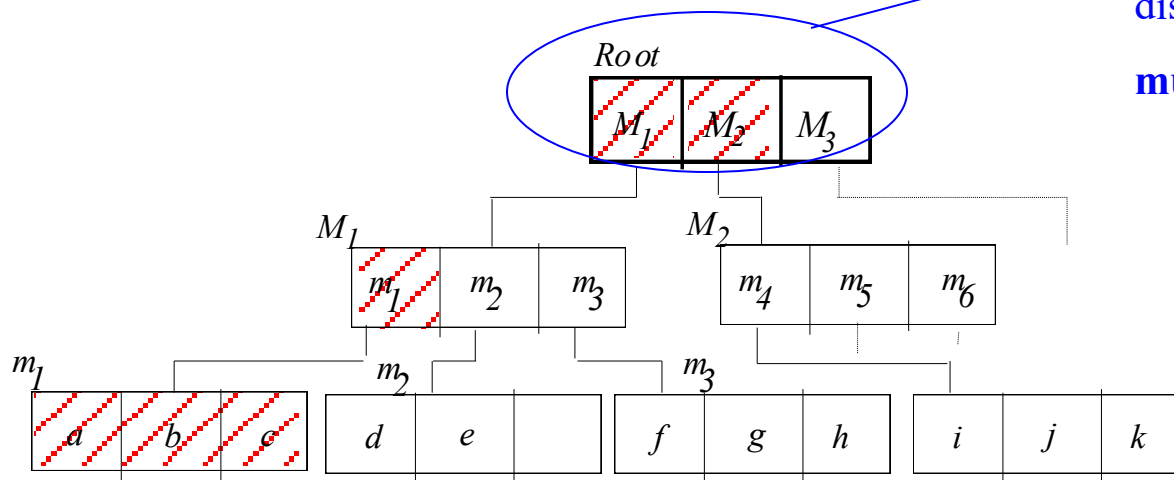
$$\text{dist}(q, o_{NN}) = \sqrt{5}$$

5. backtrack to root

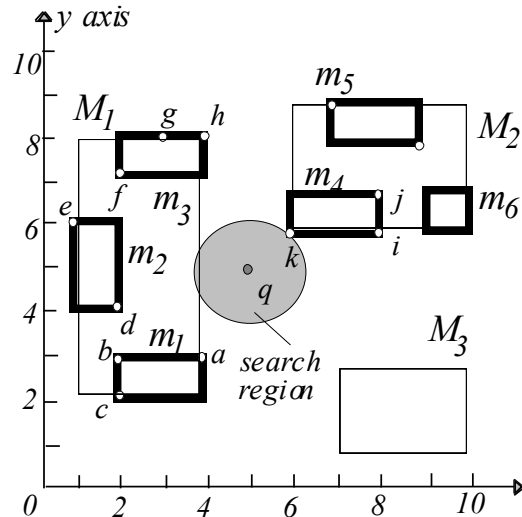
check M_2

$$\text{dist}(q, M_2) = \sqrt{2} < \sqrt{5}:$$

must visit node M_2



Depth-first NN search using an R-tree



$$o_{NN} = a$$

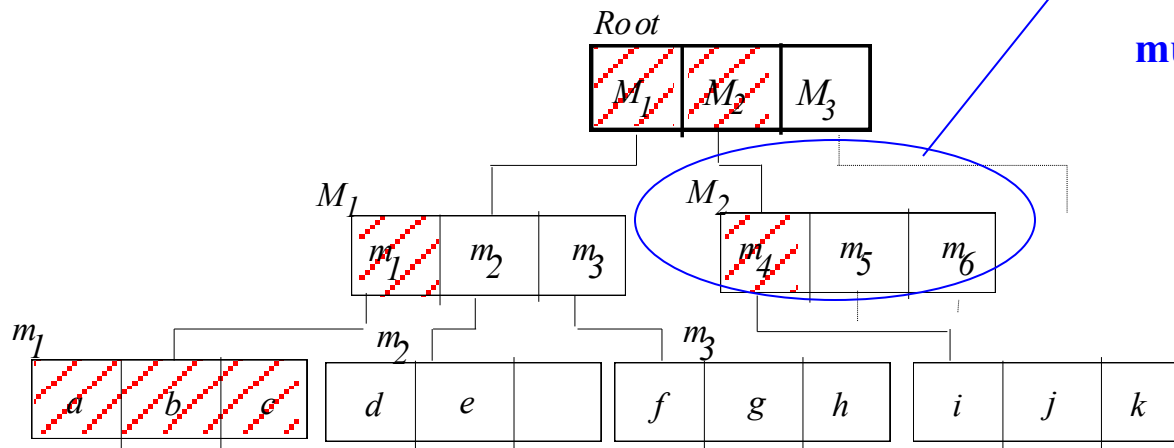
$$\text{dist}(q, o_{NN}) = \sqrt{5}$$

6. visit M_2

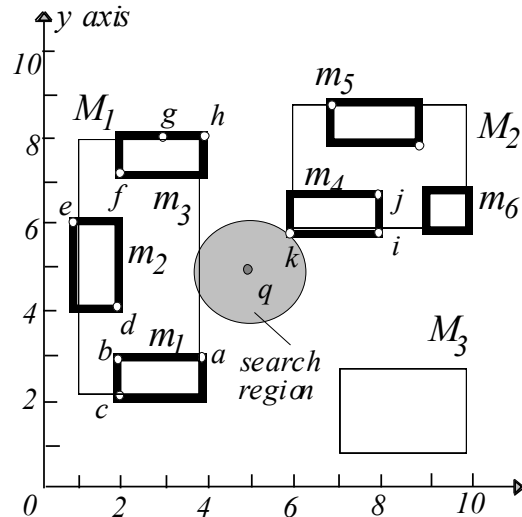
check m_4

$$\text{dist}(q, m_4) = \sqrt{2} < \sqrt{5}:$$

must visit node m_4



Depth-first NN search using an R-tree



$$o_{NN} = a$$

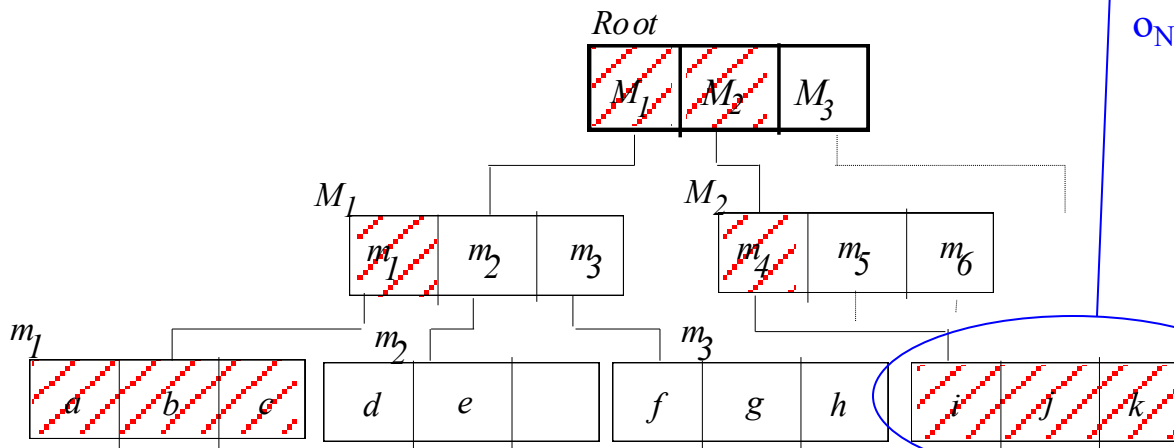
$$\text{dist}(q, o_{NN}) = \sqrt{5}$$

7. visit m_4

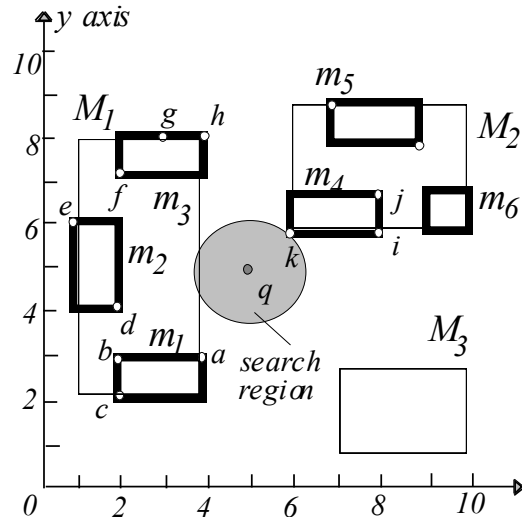
check i,j,k

found new NN:

$$o_{NN} = k, \text{dist}(q, o_{NN}) = \sqrt{2}$$



Depth-first NN search using an R-tree



$$o_{NN} = k$$

$$\text{dist}(q, o_{NN}) = \sqrt{2}$$

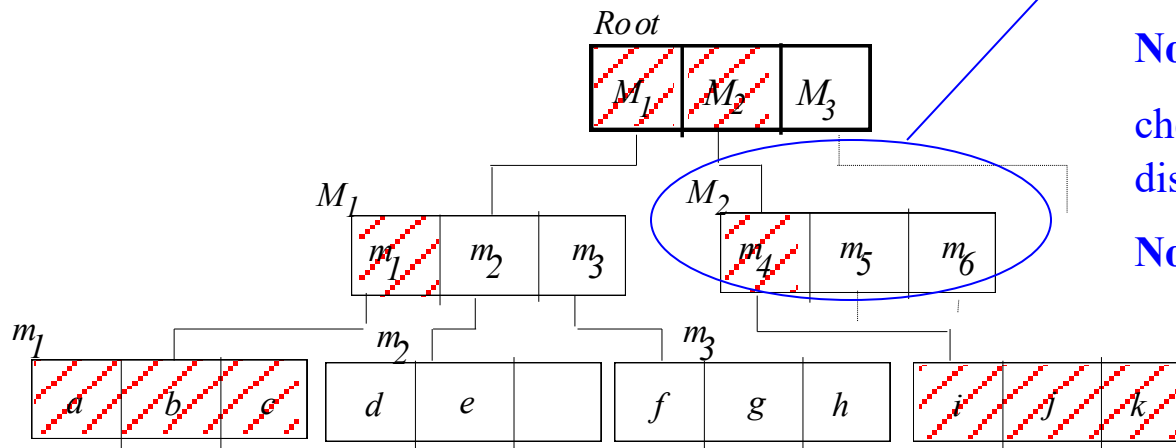
8. backtrack to M_2

check m_5
 $\text{dist}(q, m_5) \geq \sqrt{2}$:

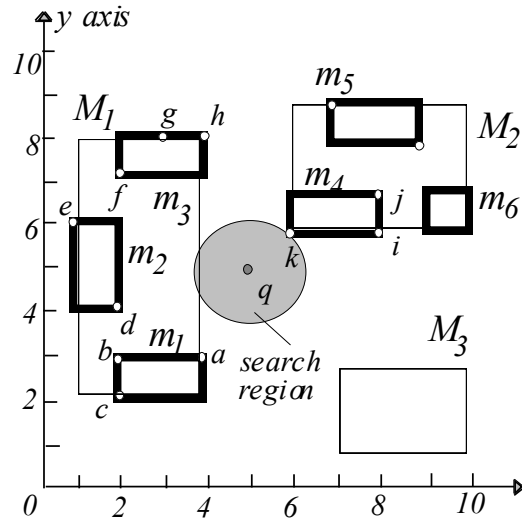
No need to visit node m_5

check m_6
 $\text{dist}(q, m_6) \geq \sqrt{2}$:

No need to visit node m_6



Depth-first NN search using an R-tree



$$o_{NN} = k$$

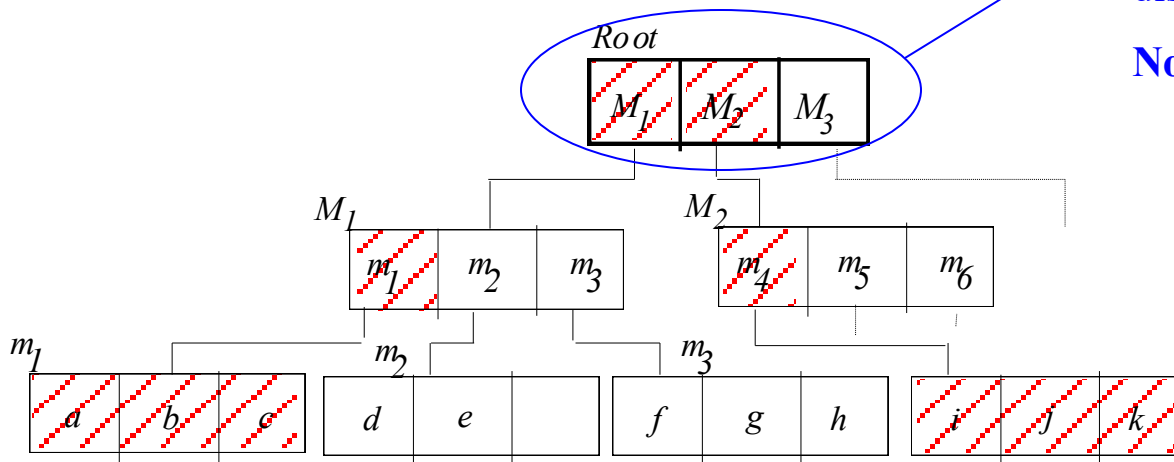
$$\text{dist}(q, o_{NN}) = \sqrt{2}$$

9. backtrack to root

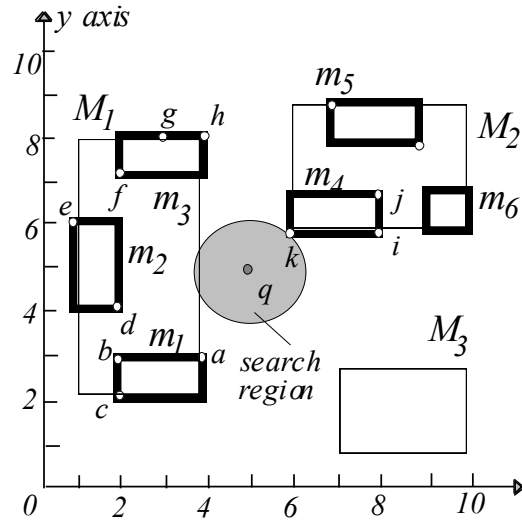
check M_3

$$\text{dist}(q, M_3) \geq \sqrt{2}:$$

No need to visit node M_3



Depth-first NN search using an R-tree



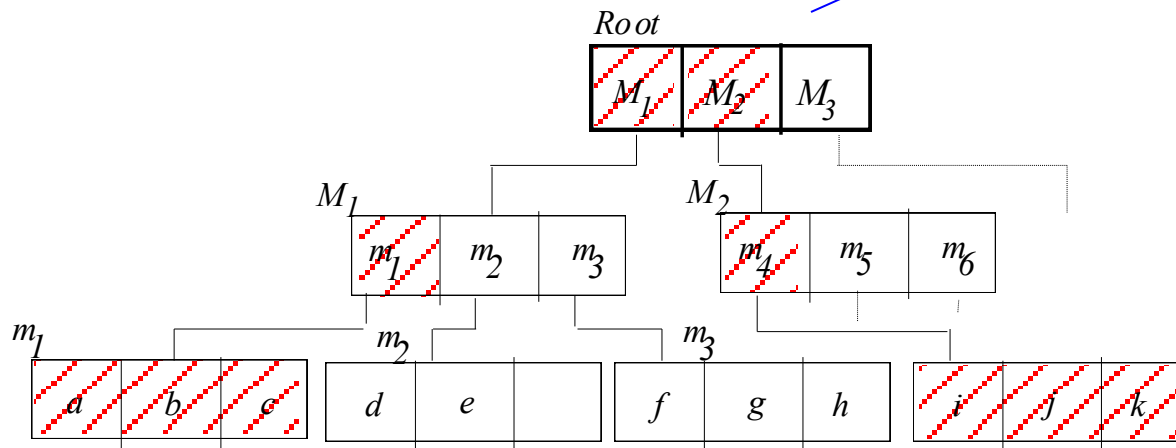
$$o_{NN} = k$$

$$\text{dist}(q, o_{NN}) = \sqrt{2}$$

10. backtrack from root

Algorithm terminates

$o_{NN} = k$ with $\text{dist}(q, o_{NN}) = \sqrt{2}$ found



Notes on Depth-first NN search

- ❑ Large space can be pruned by avoiding visiting R-tree nodes and their sub-trees
- ❑ Can order the entries of a node in increasing distance from q to maximize potential for a good NN found fast
- ❑ Can be easily adapted for k-NN search (how?)
- ❑ Requires at most one tree path to be currently in memory – good for small memories / caching
 - Characteristic of all depth-first search algorithms
 - Recall that the range search algorithm is also DF
- ❑ However, does not visit the least possible number of nodes
- ❑ Also, not incremental – more on this later...

Best-first NN search

- ❑ A more efficient algorithm
 - ❑ Performs fewer comparisons
 - ❑ Visits the smallest possible number of R-tree nodes, for a given query q
- ❑ Uses a priority queue to organize seen entries and prioritize the next node to be visited
- ❑ Can be used for **k**-NN search and **incremental** NN search

Best-first NN search

- ❑ Observation about DF-search:
 - The closest entry to q in the **current** node is “opened” and control is passed to the node pointed by it
 - However the entries in that node may not be the closest entries to q from those seen so far
- ❑ Idea of BF search:
 - Put all entries in a priority queue and always “open” the closest one, independently of the node that contains it
 - Thus the **best** (i.e., closest) entry is always visited first

Best-first NN search using an R-tree

▣ For data points only

BFNN(query point q , R-tree R)

add all entries of $R.root$ into a min-heap Q ——— Heap's key:
 $dist(q, e.MBR)$

while Q is not empty

$e = Q.top$; remove e from Q

 if e is a leaf entry // point

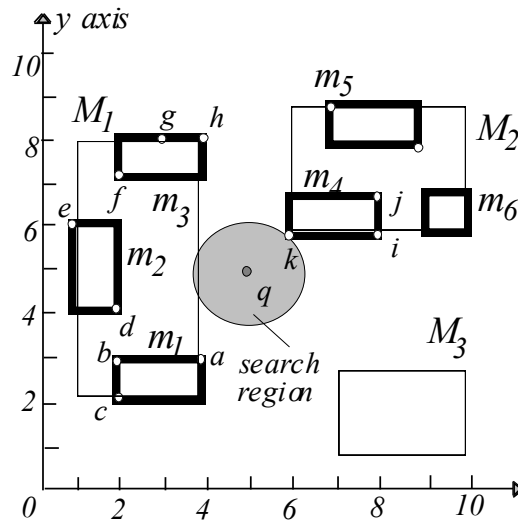
 return e // first removed data point is guaranteed to be the NN

$n =$ node of R pointed by e

 for each entry e in n do

$Q.enheap(e)$

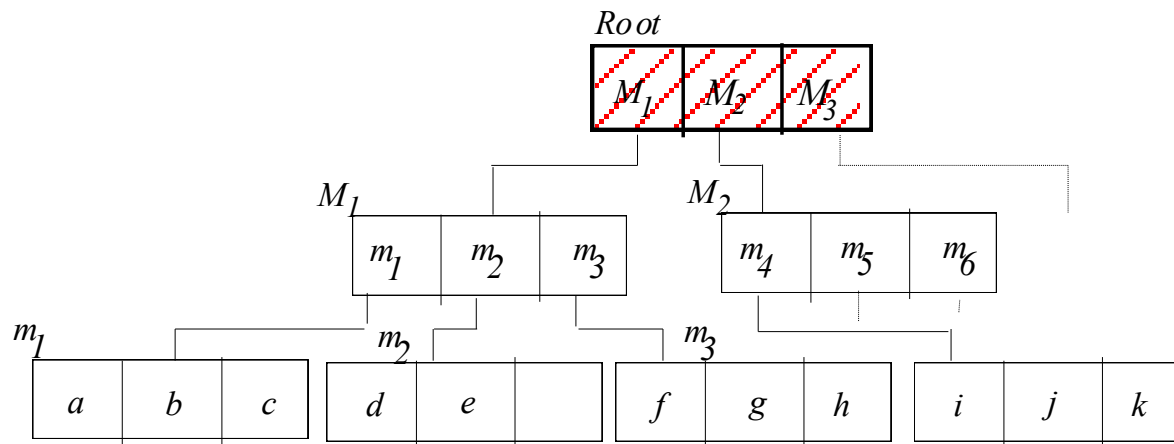
Best-first NN search



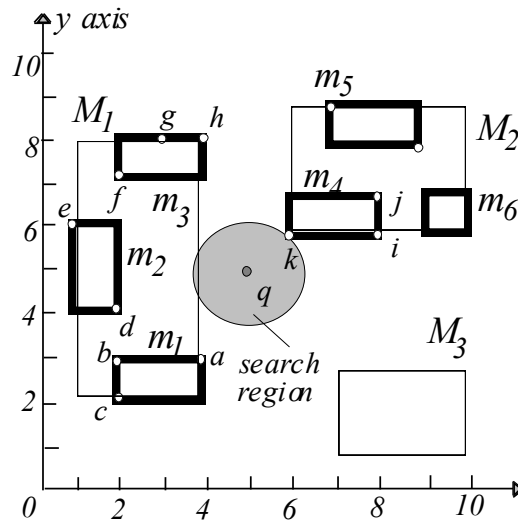
Step 1: put all entries of root on heap Q

$$Q = M_1(1), M_2(\sqrt{2}), M_3(\sqrt{8})$$

distance from q



Best-first NN search

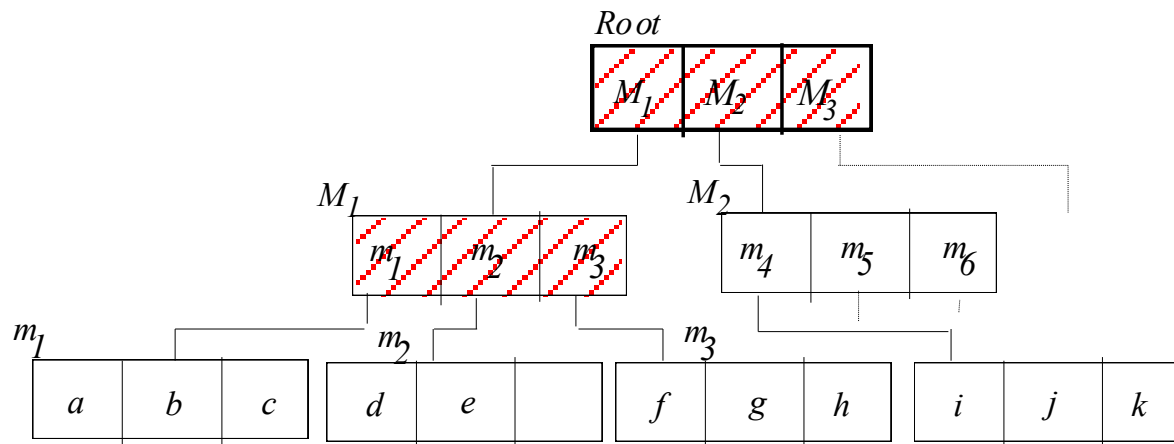


Step 2: get closest entry (top element of Q):

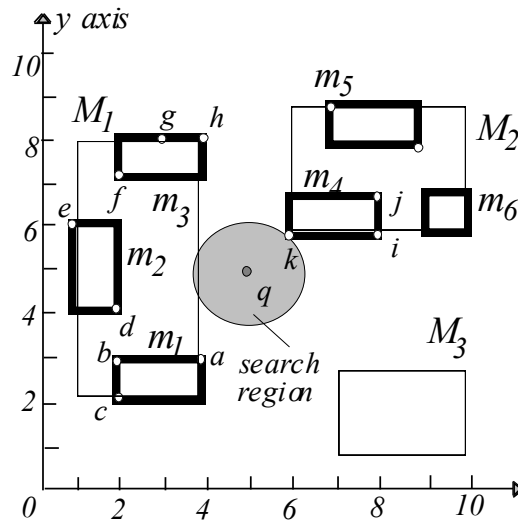
$M_1(1)$. Visit node M_1 . Put all entries of

visited node on heap Q

$Q = M_2(\sqrt{2}), m_1(\sqrt{5}), m_3(\sqrt{5}), M_3(\sqrt{8}), m_2(3)$



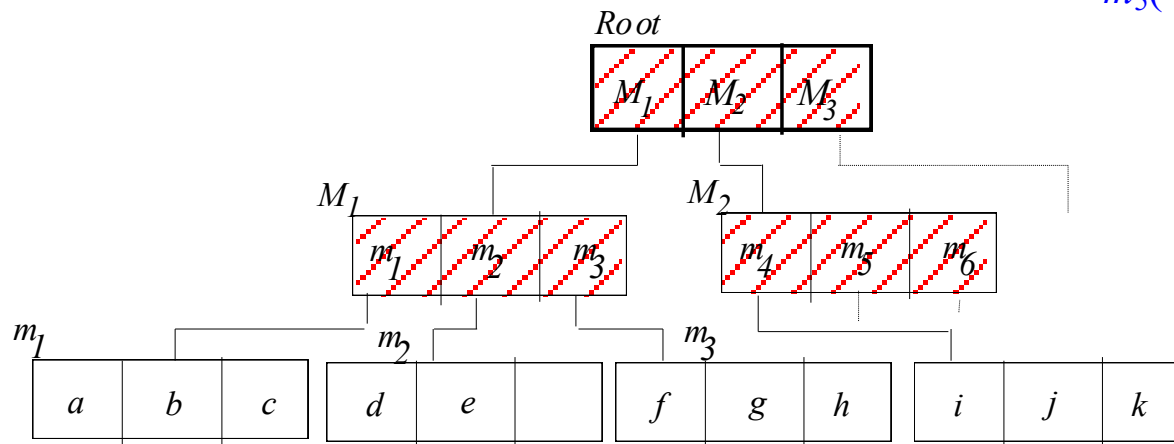
Best-first NN search



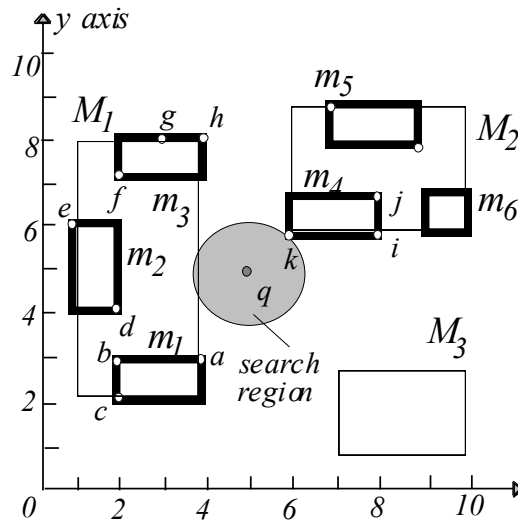
Step 3: get closest entry (top element of Q):

$M_2(\sqrt{2})$. Visit node M_2 . Put all entries of visited node on heap Q

$Q = m_4(\sqrt{2}), m_1(\sqrt{5}), m_3(\sqrt{5}), M_3(\sqrt{8}), m_2(3), m_5(\sqrt{13}), m_6(\sqrt{17})$



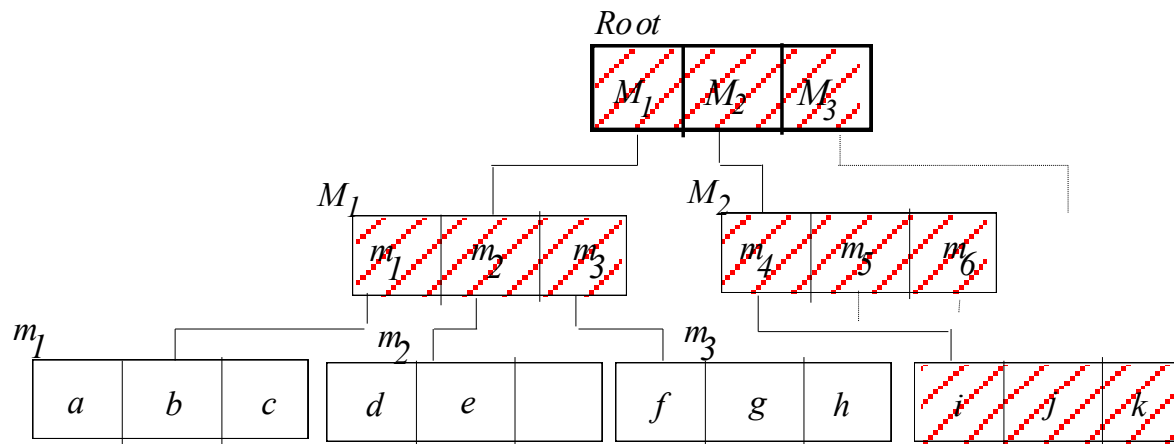
Best-first NN search



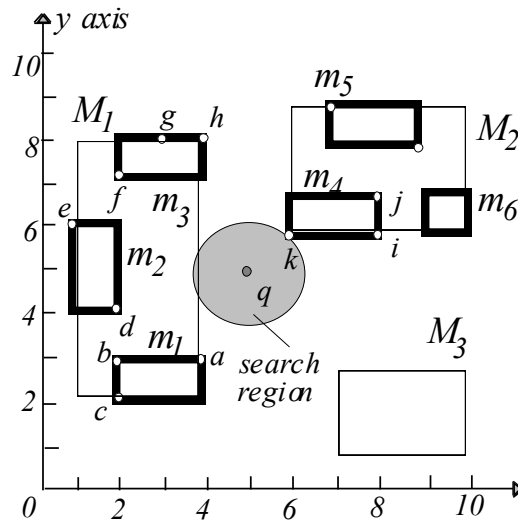
Step 4: get closest entry (top element of Q):

$m_4(\sqrt{2})$. Visit node m_4 . Put all entries of visited node on heap Q

$Q = k(\sqrt{2}), m_1(\sqrt{5}), m_3(\sqrt{5}), M_3(\sqrt{8}), m_2(3), i(\sqrt{10}), j(\sqrt{13}), m_5(\sqrt{13}), m_6(\sqrt{17})$



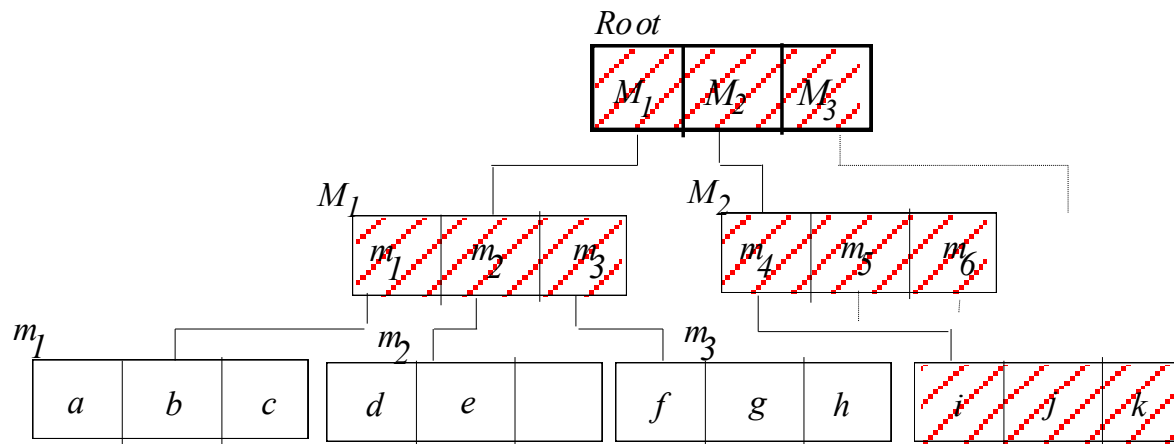
Best-first NN search



Step 5: get closest entry (top element of Q):

$k(\sqrt{2})$. Since k is a data point, search stops and k is returned as the NN of q

$Q = m_1(\sqrt{5}), m_3(\sqrt{5}), M_3(\sqrt{8}), m_2(3),$
 $i(\sqrt{10}), j(\sqrt{13}), m_5(\sqrt{13}), m_6(\sqrt{17})$



Notes on Best-first NN search

- ❑ In the previous example, we have visited fewer nodes compared to DF-NN algorithm
 - Only nodes whose MBR intersects the disk centered at q with radius the real NN distance are visited (see if you can prove this)
- ❑ The algorithm can be used for incremental NN search
 - After having found the NN, we can continue, until the next data point is de-heaped (the next NN) and so on without having to start the algorithm from the beginning
- ❑ The algorithm can be used for k-NN search
 - Continue the algorithm until k data points are de-heaped
- ❑ Drawback of best-first NN algorithm:
 - The heap can grow very large
 - In the worst case, all R-tree entries are en-heaped before the NN is found

Why incremental NN search?

- Example 1: find the nearest large city ($>10,000$ residents) to my current position
 - Solution 1:
 - find all large cities
 - apply NN search on the result
 - could be slow if many such cities
 - also R-tree may not be available for large cities only
 - Solution 2:
 - incrementally find NN and check if the large city requirement is satisfied; if not get the next NN
- Example 2: find the nearest hotel; see if you like it; if not get the next one; see if you like it; ...
- Also: similarity search in multimedia databases

Spatial Intersection Joins

□ Input:

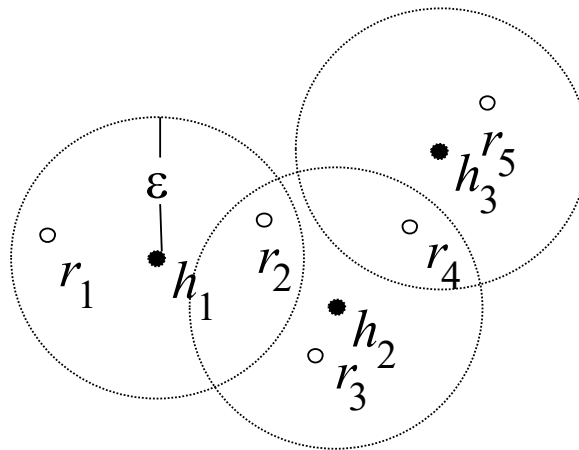
- two spatial relations R, S (e.g., $R=\text{cities}$, $S=\text{rivers}$)
- a spatial relationship θ (e.g., $\theta=\text{intersects}$)

□ Output:

- $\{(r,s): r \in R, s \in S, r \theta s \text{ is true}\}$
- Example: find all pairs of cities and rivers that intersect

Spatial Joins based on distances

- Distance join: Find pairs of hotels, restaurants close to each other (with distance smaller than 100m)
- Closest pairs: Find the closest pair of hotels, restaurants
- All-NN join: For each hotel find the nearest restaurant
- Iceberg distance join: Find hotels close to at least 10 restaurants



Spatial Join Algorithms

- ❑ Take advantage of existing indexes as much as possible
 - R-tree join
- ❑ Inspired from relational join algorithms
 - Spatial hash join
- ❑ Details are out of scope

Summary

- ❑ Spatial Data are ubiquitous
- ❑ Two main types of spatial data
 - ❑ Points
 - ❑ Extended objects
- ❑ Queries based on spatial relationships
 - ❑ topological, distance, directional
- ❑ Main query types
 - ❑ range selection, nearest neighbor search, spatial joins
- ❑ Indexes for points and/or extended objects
 - ❑ R-tree is the dominant index
- ❑ Spatial query algorithms for range and NN queries
 - ❑ Also applicable for multidimensional points