Motion Control of Unmanned Aerial Vehicle

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***Abstract*—The abstract goes here.**

***Index Terms*—IEEEtran, journal, LaTeX, paper, template.**

I. Introduction

UNMANNED aerial vehicles, also known as UAVs, are becoming nowadays more and more popular because they are small, cheap to produce, have low operating and maintenance cost, have great maneuverability, can perform steady flight operations and are able to enter high-risk areas without having to compromise human safety. Most applications that involve UAVs have been used in open areas without any obstacles and with a human in control of the UAV. But in recent years people have come up with more modern applications of UAVs that will need UAVs to fly autonomously in densely populated areas, with a lot of other autonomous UAVs around, e.g. Amazon Prime Air delivery system, AltiGator drones services for inspection and data adquisition, or multi-UAVs used to deploy an aerial communications network. This places high demands on UAVs obstacle avoidance capabilities for both moving and static obstacles.

There are many different manufacturers and a vast amount of different UAV models, all with different motors, weights, sensors and lift-to-weight ratio. To make a standard autonomous flight applicable to all these kinds of UAVs, a simple and easy-to-implement multi-UAV mathematical model, that will still be able to avoid obstacles with as few sensors as possible, is needed.

This project aims to study and develop a mathematical model of a quadrotor UAV and the available sensors in it. From the trajectory and pose tracking a state feedback controller will be designed. In order to facilitate the multi-UAV navigation, potential fields or an A* algorithm will be used to make several quads fly to their goals while maintaining collision avoidance with respect to other quads and obstacles. To check the validity of the models, a simulated test environment in MatLab filled with a random reasonable amount of static obstacles and autonomous UAVs will be used.

II. Quadcopter modelling

A. Overview

The UAV is a rigid body quadcopter, with a cross-shaped body and four electrical propellers. Front and rear rotors rotate in a clockwise direction, while right and left rotors rotate in a counterclockwise direction, this is illustrated in Fig.1. Its motion has 6 degrees of freedom but there are only 4 propellers, therefore the system is underactuated.

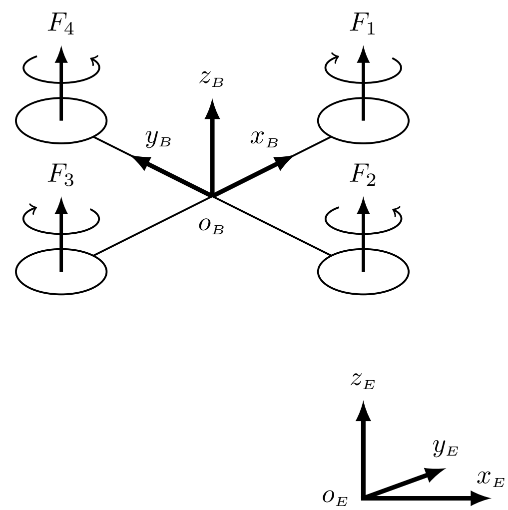

Fig. 1. Quadrotor with propellers and the two reference frames

Fig. 2. Euler angles with respect to Earth frame.

B. Notation

C. Kinematics

In order to describe its motion, a kinematic model for the UAV was developed.

Two right-hand reference frames are defined: the Earth frame and the body frame, as can be seen in Fig.1.

The Earth frame is static, with the x_E axis pointing towards the North, the y_E axis pointing towards the West, and z_E pointing upwards w.r.t. the Earth. The body frame is attached to the UAV, with the x_B axis pointing towards the quadrotor's front, the y_B axis pointing towards the left, and the z_B axis pointing upwards. In this case, the axis origin o_B coincides with the quadrotor's center of mass.

The generalized position $\boldsymbol{\xi}$ contains the linear and angular position and is described in the Earth frame, as in (1). The linear position $\boldsymbol{x}^E$ of the UAV is the vector between the origin of the Earth frame o_E and the origin of the body frame o_B, and the Euler angles $\boldsymbol{\eta}^E$ are defined as stated in Fig.2.

$$\boldsymbol{\xi} = \begin{bmatrix} \boldsymbol{x}^E & \boldsymbol{\eta}^E \end{bmatrix}^T = \begin{bmatrix} x & y & z & \phi & \theta & \psi \end{bmatrix}^T \quad (1)$$

The generalized velocity $\boldsymbol{\nu}$ (2) contains the linear and angular velocity, and it is expressed in the body frame.

$$\boldsymbol{\nu} = \begin{bmatrix} \boldsymbol{v}^B & \boldsymbol{\omega}^B \end{bmatrix}^T = \begin{bmatrix} u & v & w & p & q & r \end{bmatrix}^T \quad (2)$$

Three rotation matrixes around each of the x, y, z axes can be defined according to (3, 4, 5) respectively.

$$\boldsymbol{R}_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) & \cos(\phi) \end{bmatrix} \quad (3)$$

$$\boldsymbol{R}_y(\theta) = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \quad (4)$$

$$\boldsymbol{R}_z(\psi) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (5)$$

The complete rotation matrix $\boldsymbol{R}_\Theta$, that expresses the rotation from the body frame to the Earth frame, can be obtained by multiplying these three matrixes, as in (6).

$$\boldsymbol{R}_\Theta(\phi, \theta, \psi) = \boldsymbol{R}_x(\phi)\boldsymbol{R}_y(\theta)\boldsymbol{R}_z(\psi) \quad (6)$$

The transfer matrix $\boldsymbol{T}_\Theta$ that allows to change between the angular velocity in the body frame $\boldsymbol{\omega}^B$ and the Euler rates in the Earth frame $\dot{\boldsymbol{\eta}}^E$ can be determined and is as shown in (7).

$$\boldsymbol{T}_\Theta(\phi, \theta) = \begin{bmatrix} 1 & \sin(\phi)\cdot\tan(\theta) & \cos(\phi)\cdot\tan(\theta) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi)/\cos(\theta) & \cos(\phi)/\cos(\theta) \end{bmatrix} \quad (7)$$

A generalized matrix $\boldsymbol{J}_\Theta$ can be built joining the rotation and the transfer matrix (6, 7), as shown in (8).

$$\boldsymbol{J}_\Theta(\phi, \theta, \psi) = \begin{bmatrix} \boldsymbol{R}_\Theta & \boldsymbol{0}_{3\times3} \\ \boldsymbol{0}_{3\times3} & \boldsymbol{T}_\Theta \end{bmatrix} \quad (8)$$

Where the notation $\boldsymbol{0}_{3\times3}$ means a matrix filled with zeros with a 3×3 dimension.

In order to relate the derivate of the generalized position in the Earth frame with the generalized velocity on the body frame, the transfer matrix (7) can be used, and that is the final model of the quadrotor's kinematics.

$$\dot{\boldsymbol{\xi}} = \boldsymbol{J}_\Theta\ \boldsymbol{\nu} \quad (9)$$

D. Dynamics

The dynamic model for the UAV relates the acceleration of the vehicle with the forces and torques acting on the quadrotor. The Newton-Euler formulation allows to express the variables in the body frame, as in equations (10) and (11), as clearly stated by Bresciani in [1].

$$\boldsymbol{F}^B = m(\dot{\boldsymbol{v}}^B + \boldsymbol{\omega}^B \times \boldsymbol{v}^B) \quad (10)$$

$$\boldsymbol{\tau}^B = \boldsymbol{I}\ \dot{\boldsymbol{\omega}}^B + \boldsymbol{\omega}^B \times (\boldsymbol{I}\ \boldsymbol{\omega}^B) \quad (11)$$

III. Measuring the UAV state

The core of this project focuses on how to make aerial vehicles fly autonomously from an initial position to a goal. Therefore, apart from the main algorithm that makes this possible, it is really important that the vehicle can acquire precise information about its condition and its surroundings. Sensors do not only have to provide information about the state of the UAV as to close the loop for the controller, but also about the objects the vehicle may encounter throughput its path, as to make the navigation safe and prevent and block possible crashes.

A. Inertial Measurement Unit

This module is in charge of measuring almost all the variables related to the movement of the vehicle. Usually inside this module a 3-axis accelerometer, a 3-axis gyroscope and a 3-axis magnetometer can be found. The most affordable ones are those that cointain simply this three sensors integrated in the same circuit board by the manufacturer. The most expensive and precise IMUs integrate specially designed sensors and sometimes include a GPS, a RS232 transceiver and a processor, that runs a real-time Kalman filter in order to provide the most accurate data directly to the CPU.

1) Triple axis accelerometer: This sensor measures proper acceleration along the three axes on the body frame if the accelerometer's axes match these. It can measure dynamic acceleration as a result of the motion of the drone. As shown in (12), the rotation matrix is used to change from acceleration provided by the IMU to the acceleration in the Earth frame [2].

$$\boldsymbol{a}_{\mathrm{IMU}} = \boldsymbol{R}_\Theta^T\ (\ddot{\boldsymbol{x}}^E - g\vec{\boldsymbol{z}}) \quad (12)$$

2) Triple axis gyroscope: This device can measure angular rates in its three axes. Therefore, it gives the angular velocity of the body frame relative to the Earth frame, expressed in the body frame (13).

$$\boldsymbol{\omega}_{\mathrm{IMU}} = \boldsymbol{\omega}^B \quad (13)$$

3) Triple axis magnetometer: This kind of sensors are able to measure the ambient magnetic field. Ideally this corresponds to the Earth's magnetic field, therefore the orientation of the vehicle can be measured (14).

$$\boldsymbol{m}_{\mathrm{IMU}} = \boldsymbol{R}_\Theta^T\ \boldsymbol{m}_{\mathrm{Earth}} \quad (14)$$

Where $\boldsymbol{m}_{\mathrm{Earth}}$ corresponds to the Earth's magnetic field expressed in the Earth frame. This measure can be accurate if the bias caused by the local magnetic disturbance $\boldsymbol{b}_{\mathrm{m}}$ is taken into account (15) and the sensor is places as far as possible from the elements that may cause this disturbance onboard the UAV, such as the wires that power the rotors [2].

$$\boldsymbol{m}_{\mathrm{IMU}} = \boldsymbol{R}_\Theta^T\ \boldsymbol{m}_{\mathrm{Earth}} + \boldsymbol{b}_{\mathrm{m}} \quad (15)$$

In all the specified sensors, bias and noise are also present. Gyroscopes are usually robust against this noise. But one placed in an UAV, accelerometers are affected by the vibration, and need filtering for its masurements to be considered reliable.

B. GPS receiver

This device is basically a receiver that makes use of the satellite-based GPS system to calculate the vehicle's geographical position (longitude and latitude). Casual and inexpensive

GPS devices have some meters of accuracy, therefore suplementary information from other sensors is needed in order to estimate the position of the vehicle as accurately as possible.

C. Infrared sensors

D. Ultrasound sensors

IV. Quadcopter control

The controller used in this project is an LQ-controller. The controller is used to check that our mathematical model of the UAV is controllable and observable. An LQR utilizes a cost function and minimises said cost function. the reason behind using a LQR is that it usually has smaller errors than a normal PID-controller [SOURCE] and since the main goal of this project is to avoid colliding any of our UAVs it would be reasonable to focus on getting an error as small as possible.

V. Quadcopter navigation

VI. Simulation

VII. Results

It's bloody impossible to do this.

VIII. Discussion, conclusion and Future development

Appendix A
Dynamics or something that's heavy on the maths goes here

Appendix one text goes here.

Appendix B
Something else but similar here, matlab code?

Appendix two text goes here.

Acknowledgment

Praising of Christos goes here! The authors would like to thank... TEST

References

[1] T. Bresciani, "Modelling, identification and control of a quadrotor helicopter," 2008, student Paper.

[2] R. Mahony, V. Kumar, and P. Corke, "Multirotor aerial vehicles: Modeling, estimation, and control of quadrotor," *IEEE Robotics Automation Magazine*, vol. 19, no. 3, pp. 20–32, Sept 2012.