

# Motion Control of Unmanned Aerial Vehicle

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**Abstract**—The abstract goes here.

**Index Terms**—IEEEtran, journal, L<sup>A</sup>T<sub>E</sub>X, paper, template.

## I. INTRODUCTION

UNMANNED aerial vehicles, also known as UAVs, are becoming nowadays more and more popular because they are small, cheap to produce, have low operating and maintenance cost, have great maneuverability, can perform steady flight operations and are able to enter high-risk areas without having to compromise human safety. Most applications that involve UAVs have been used in open areas without any obstacles and with a human in control of the UAV. But in recent years people have come up with more modern applications of UAVs that will need UAVs to fly autonomously in densely populated areas, with a lot of other autonomous UAVs around, e.g. Amazon Prime Air delivery system, AltitudeGator drones services for inspection and data adquisition, or multi-UAVs used to deploy an aerial communications network. This places high demands on UAVs obstacle avoidance capabilities for both moving and static obstacles.

There are many different manufacturers and a vast amount of different UAV models, all with different motors, weights, sensors and lift-to-weight ratio. To make a standard autonomous flight applicable to all these kinds of UAVs, a simple and easy-to-implement multi-UAV mathematical model, that will still be able to avoid obstacles with as few sensors as possible, is needed.

This project aims to study and develop a mathematical model of a quadrotor UAV and the available sensors in it. From the trajectory and pose tracking a state feedback controller will be designed. In order to facilitate the multi-UAV navigation, potential fields or an A\* algorithm will be used to make several quads fly to their goals while maintaining collision avoidance with respect to other quads and obstacles. To check the validity of the models, a simulated test environment in MatLab filled with a random reasonable amount of static obstacles and autonomous UAVs will be used.

## II. QUADCOPTER MODELLING

### A. Overview

The UAV is a rigid body quadcopter, with a cross-shaped body and four electrical propellers. Front and rear rotors rotate in a clockwise direction, while right and left rotors do it in a counterclockwise direction, can be seen in Fig.1. Its motion has 6 degrees of freedom and there are only 4 propellers, therefore the system is underactuated.

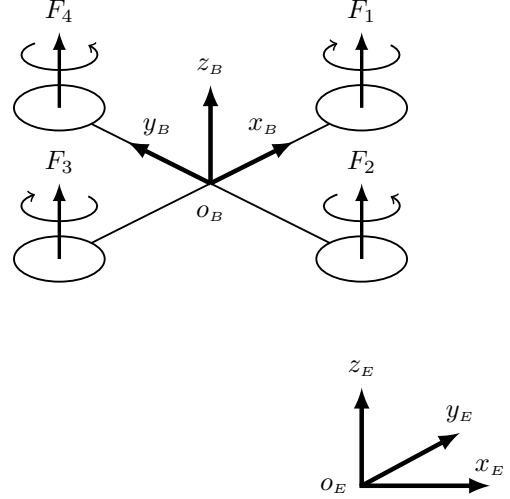


Fig. 1. Quadrotor with propellers and the two reference frames

Fig. 2. Euler angles with respect to Earth frame.

### B. Notation

### C. Kinematics

To describe the motion of the UAV without the forces and torques acting on it, a kinematic model for it will be developed.

Two right-hand reference frames need to be defined: the Earth frame and the body frame, as can be seen in Fig.1.

The Earth frame is static, with the  $x_E$  axis pointing towards the North, the  $y_E$  axis pointing towards the West, and  $z_E$  pointing upwards w.r.t. the Earth. The body frame is attached to the UAV, with the  $x_B$  axis pointing towards the quadrotor's front, the  $y_B$  axis pointing towards the left, and the  $z_B$  axis pointing upwards. In this case, the axis origin  $o_B$  coincides with the quadrotor's structure center.

The generalized position  $\xi$  contains the linear and angular position and it is described in the Earth frame, as in (1). The linear position  $\mathbf{x}^E$  of the UAV is the vector between the origin of the Earth frame  $o_E$  and the origin of the body frame  $o_B$ , and the Euler angles  $\boldsymbol{\eta}^E$  are defined as stated in Fig.2.

$$\xi = [\mathbf{x}^E \quad \boldsymbol{\eta}^E]^T = [x \quad y \quad z \quad \phi \quad \theta \quad \psi]^T \quad (1)$$

The generalized velocity  $\boldsymbol{\nu}$  (2) contains the linear and angular velocity, and it is expressed in the body frame.

$$\boldsymbol{\nu} = [\mathbf{v}^B \quad \boldsymbol{\omega}^B]^T = [u \quad v \quad w \quad p \quad q \quad r]^T \quad (2)$$

Three rotation matrixes around each of the  $x, y, z$  axes can be defined according to (3, 4, 5) respectively.

$$\mathbf{R}_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) & \cos(\phi) \end{bmatrix} \quad (3)$$

$$\mathbf{R}_y(\theta) = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \quad (4)$$

$$\mathbf{R}_z(\psi) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (5)$$

The complete rotation matrix  $\mathbf{R}_\Theta$ , that expresses the rotation from the body frame to the Earth frame, can be obtained by multiplying these three matrixes, as in (6).

$$\mathbf{R}_\Theta(\phi, \theta, \psi) = \mathbf{R}_x(\phi)\mathbf{R}_y(\theta)\mathbf{R}_z(\psi) \quad (6)$$

The transfer matrix  $\mathbf{T}_\Theta$  that allows to change between the angular velocity in the body frame  $\boldsymbol{\omega}^B$  and the Euler rates in the Earth frame  $\dot{\boldsymbol{\eta}}^E$  can be determined and is as shown in (7).

$$\mathbf{T}_\Theta(\phi, \theta) = \begin{bmatrix} 1 & \sin(\phi) \cdot \tan(\theta) & \cos(\phi) \cdot \tan(\theta) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi)/\cos(\theta) & \cos(\phi)/\cos(\theta) \end{bmatrix} \quad (7)$$

A generalized matrix  $\mathbf{J}_\Theta$  can be built joining the rotation and the transfer matrix (6, 7), as shown in (8).

$$\mathbf{J}_\Theta(\phi, \theta, \psi) = \begin{bmatrix} \mathbf{R}_\Theta & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{T}_\Theta \end{bmatrix} \quad (8)$$

Where the notation  $\mathbf{0}_{3 \times 3}$  means a matrix filled with zeros with a  $3 \times 3$  dimension.

In order to relate the derivate of the generalized position in the Earth frame with the generalized velocity on the body frame, the transfer matrix (7) can be used, and that is the final model of the quadrotor's kinematics.

$$\dot{\boldsymbol{\xi}} = \mathbf{J}_\Theta \boldsymbol{\nu} \quad (9)$$

#### D. Dynamics

The dynamic model for the UAV will relate the acceleration of the vehicle with the force and torques provided by the propellers. The Newton-Euler formulation allows to express the variables in the body frame, as in equations (10) and (11), as clearly stated by Bresciani in [1].

$$\mathbf{F}^B = m(\dot{\mathbf{v}}^B + \boldsymbol{\omega}^B \times \mathbf{v}^B) \quad (10)$$

$$\mathbf{T}_\Theta \boldsymbol{\tau}^B = \mathbf{I} \dot{\boldsymbol{\omega}}^B + \boldsymbol{\omega}^B \times (\mathbf{I} \boldsymbol{\omega}^B) \quad (11)$$

### III. SENSORS

### IV. QUADCOPTER CONTROL

### V. CONCLUSION

It's bloody impossible to do this.

### APPENDIX A

DYNAMICS OR SOMETHING THAT'S HEAVY ON THE MATHS  
GOES HERE

Appendix one text goes here.

### APPENDIX B

SOMETHING ELSE BUT SIMILAR HERE, MATLAB CODE?

Appendix two text goes here.

### ACKNOWLEDGMENT

Praising of Christos goes here! The authors would like to thank...

### REFERENCES

- [1] T. Bresciani, "Modelling, identification and control of a quadrotor helicopter," 2008, student Paper.