Dayou Du
N13825075
dd2645@nyu.edu

Homework 4
Fundamental Algorithms

2017-10-12

Ex 2.12

a)

Algorithm 1 Ex 2.12a

procedure $DFS(v;;sum)$
  $sum \leftarrow sum + v.dat$
  $v.ans \leftarrow sum$
  for each child c of v **do**
    $DFS(c, sum)$
  end for
end procedure

At beginning we call $DFS(Root, 0)$
The handout did NOT give the answer to Ex2.12

b)

Algorithm 2 Ex 2.12b

procedure $DFS(v;;sum)$
  for each child c of v **do**
    $DFS(c, sum)$
  end for
  $sum \leftarrow sum + v.dat$
  $v.ans \leftarrow sum$
end procedure

At beginning we call $DFS(Root, 0)$
The handout did NOT give the answer to Ex2.12

c)

Algorithm 3 Ex 2.12c

procedure $DFS(v;;sum)$
  if $v.left \neq Nil$ **then**
    $DFS(v.left, sum)$
  end if
  $sum \leftarrow sum + v.dat$
  $v.ans \leftarrow sum$
  if $v.right \neq Nil$ **then**
    $DFS(v.right, sum)$
  end if
end procedure

At beginning we call $DFS(Root, 0)$
The handout did NOT give the answer to Ex2.12

d)

Algorithm 4 Ex 2.12d

procedure $Ex2.12d(Root)$
  $sum \leftarrow 0$
  $DFS1(Root, sum)$
  $DFS2(Root, sum)$
end procedure
procedure $DFS1(v;;sum)$
  $sum \leftarrow sum + v.dat$
  for each child c of v **do**
    $DFS1(c, sum)$
  end for
end procedure
procedure $DFS2(v;;sum)$
  $sum \leftarrow sum - v.dat$
  $v.ans \leftarrow sum$
  for each child c of v **do**
    $DFS2(c, sum)$
  end for
end procedure

The handout did NOT give the answer to Ex2.12

e)

Algorithm 5 Ex 2.12e

procedure $DFS(v, sum;;)$
  $v.ans \leftarrow sum$
  for each child c of v **do**
    $DFS(c, sum + v.dat)$
  end for
end procedure

At beginning we call $DFS(Root, 0)$. Note: Actually we are counting the sum-of-path from $Root$ to v
The handout did NOT give the answer to Ex2.12

f)

All the vertices in the path from root of T to v(root included, v excluded)
The handout did NOT give the answer to Ex2.12

Ex 3.17

$$\begin{cases} Name(1) = 1 \\ Name(2n) = 2Name(n) - 1 & ,n \geq 1 \\ Name(2n+1) = 2Name(n) + 1 & ,n \geq 1 \end{cases} \tag{1}$$

I got this question CORRECT.

Just for fun: we can write down $Name(n)$ without recurrence.

$$Name(n) = 1 + 2n - 2(n\&(-n)) \tag{2}$$

Here we assume that n is represented in standard binary form(like int in C/C++), and $\&$ means $BinaryAnd$

Ex 3.18

a)

Let

$$1024n_s + 2T(\frac{n_s}{2}) = 4n_s^2 \tag{3}$$

Since

$$1024n + 2T(\frac{n}{2}) > 4n^2, n < n_s \tag{4}$$

Thus we have

$$T(\frac{n_s}{2}) = 4(\frac{n_s}{2})^2 = n_s^2 \tag{5}$$

Use equation (5) in equation (3), we have

$$1024n_s + 2n_s^2 = 4n_s^2 \tag{6}$$

Solve equation (6) We have $n_s = 512$
I got this question CORRECT.

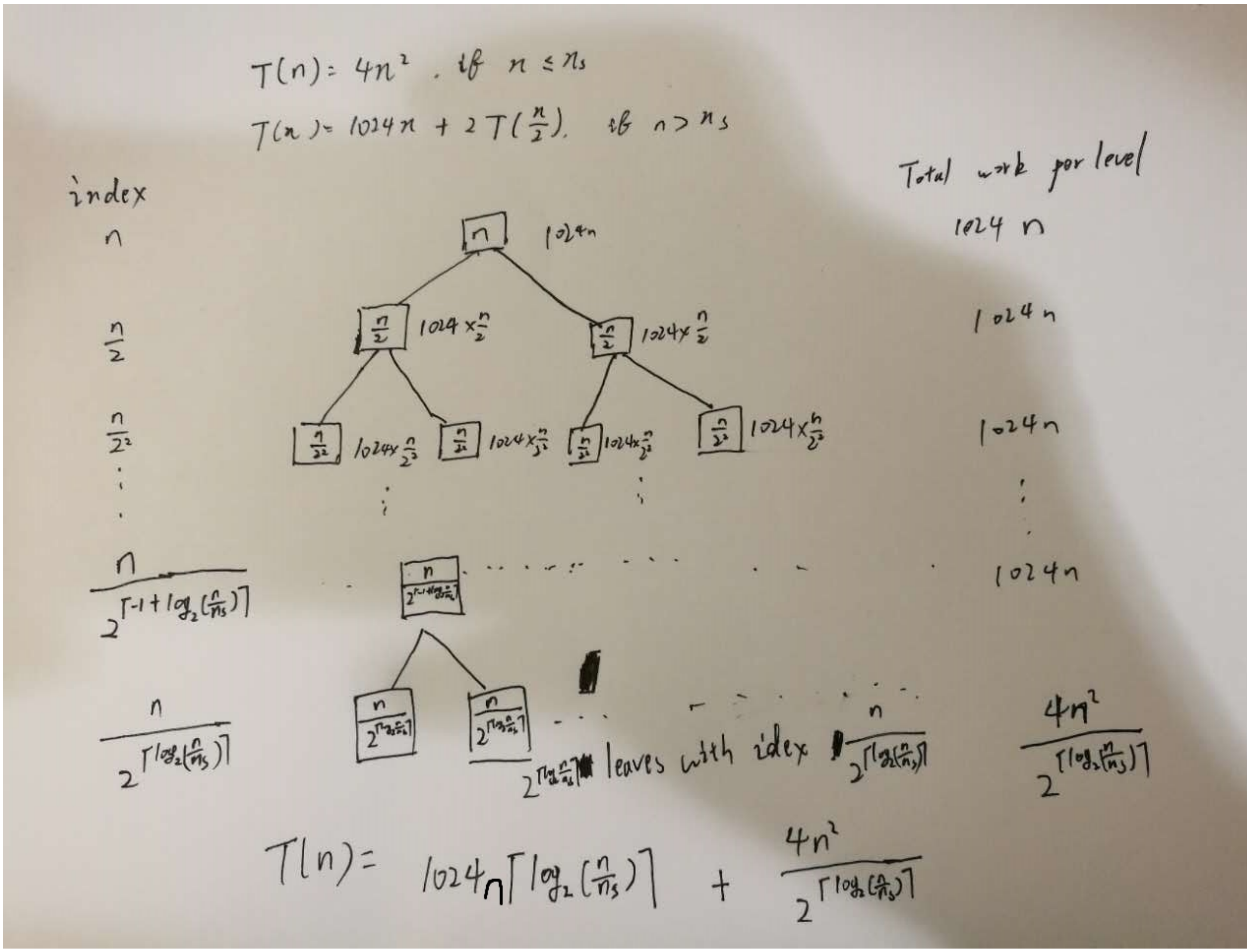
Actually we didn't prove that there indeed exist n_s and n_s is unique before we start calculating. We assumed that $T(n) = 4n^2 \ \ when \ \ n < n_s$ in order to get the value of $T(\frac{n}{2})$ and compute n_s, which actually missed something in our logic.
As the solution point out, we should compute the asymptotic growth for the two forms, which gives that the first form grows slower than the second one. then we can say that there must be at least one "cross point"between the two forms.

b)

Now we have n_s , thus we can rewrite our recurrence equation into a better form below:

$$\begin{cases} T(n) = 4n^2, & n \leq n_s \\ T(n) = 1024n + 2T(\frac{n}{2}), & n > n_s \end{cases} \tag{7}$$

Here we draw the recursion tree:



Which gives

$$T(n) = 1024n\lceil log_2(\frac{n}{n_s})\rceil + \frac{4n^2}{2^{\lceil log_2(\frac{n}{n_s})\rceil}}, n > n_s \tag{8}$$

I got this question CORRECT.
The formula above is more precise than the one on solution sheet(it doesn't assume that n is a power of 2)

Ex 3.19

a)

$$\begin{cases} L(0) = 1 \\ L(n) = 2L(n-1) + 1 \quad ,n \geq 1 \end{cases} \tag{9}$$

The "unit" that I used in equation is 1, but in question it ask for "length", which means each "unit" should be "2(inch)", thus we should change the constant 1s into 2s

b)

$$\begin{cases} V(0) = 1 \\ V(n) = 4V(n-1) + 3 \quad ,n \geq 1 \end{cases} \tag{10}$$

I got this question CORRECT.

c)

Solve $L(n)$:

$$\begin{cases} 2^0 L(n) = 2^1 L(n-1) + 2^0 \\ 2^1 L(n-1) = 2^2 L(n-2) + 2^1 \\ 2^2 L(n-2) = 2^3 L(n-3) + 2^2 \\ \ldots \\ 2^{n-1} L(1) = 2^n L(0) + 2^{n-1} \end{cases} \tag{11}$$

Sum up these n equations, we have

$$L(n) = 2^0 + 2^1 + .. + 2^{n-1} + 2^n L(0), n \geq 1 \tag{12}$$

$$L(n) = 2^{n+1} - 1, n \geq 1 \tag{13}$$

The formula above stands true for $n = 0$, thus

$$L(n) = 2^{n+1} - 1, n \geq 0 \tag{14}$$

Continue for the mistake I made in question a), the unit I use is "1", however the question ask us to use "2(inch)".Thus here $L(n)$ should be doubled and gives $L(n) = 2^{n+2} - 2, n \geq 0$

The calculation itself is CORRECT

Solve $V(n)$:

$$\begin{cases} 4^0 V(n) = 4^1 V(n-1) + 3 \times 4^0 \\ 4^1 V(n-1) = 4^2 V(n-2) + 3 \times 4^1 \\ 4^2 V(n-2) = 4^3 V(n-3) + 3 \times 4^2 \\ \ldots \\ 4^{n-1} V(1) = 4^n V(0) + 3 \times 4^{n-1} \end{cases} \tag{15}$$

Sum up these n equations, we have

$$V(n) = 3 \times (4^0 + 4^1 + .. + 4^{n-1}) + 4^n V(0), n \geq 1 \tag{16}$$

$$V(n) = 2 \times 4^n - 1, n \geq 1 \tag{17}$$

The formula above stands true for $n = 0$, thus

$$L(n) = 2 \times 4^n - 1, n \geq 0 \tag{18}$$

The calculation above is CORRECT

Ex 3.28

It's a little bit hard for me to draw a tree in Latex...thus I just write down the calculations for the recursion tree here.

a)

$$A(n) = n \times (\frac{2^0}{2^0} + \frac{2^1}{2^1} + ... + \frac{2^{-1+log_2 n}}{2^{-1+log_2 n}}) + 2^{log_2 n} \times 1, n > 1 \tag{19}$$

$$A(n) = nlog_2 n + n, n > 1 \tag{20}$$

The formula above stands true for A(1), thus we have

$$A(n) = nlog_2 n + n, n >= 1 \tag{21}$$

I got this question CORRECT.

b)

$$B(n) = n \times (\frac{3^0}{3^0} + \frac{3^1}{3^1} + ... + \frac{3^{-1+log_3 n}}{3^{-1+log_3 n}}) + 3^{log_3 n} \times 1, n > 1 \tag{22}$$

$$B(n) = nlog_3 n + n, n > 1 \tag{23}$$

The formula above stands true for B(1), thus we have

$$B(n) = nlog_3 n + n, n \geq 1 \tag{24}$$

I got this question CORRECT.

c)

$$C(n) = (4^0(\frac{n}{2^0})^2 + 4^1(\frac{n}{2^1})^2 + 4^2(\frac{n}{2^2})^2 + ... + 4^{-1+log_2 n}(\frac{n}{2^{-1+log_2 n}})^2) + 4^{log_2 n} \times 1, n > 1 \tag{25}$$

$$C(n) = n^2 log_2 n + n^2, n > 1 \tag{26}$$

The formula above stands true for C(1), thus we have

$$C(n) = n^2 log_2 n + n^2, n \geq 1 \tag{27}$$

I got this question CORRECT.

d)

$$D(n) = n^2 \times (\frac{9^0}{9^0} + \frac{9^1}{9^1} + ... + \frac{9^{-1+log_3 n}}{9^{-1+log_3 n}}) + 9^{log_3 n} \times 1, n > 1 \tag{28}$$

$$D(n) = n^2 log_3 n + n^2, n > 1 \tag{29}$$

The formula above stands true for D(1), thus we have

$$D(n) = n^2 log_3 n + n^2, n \geq 1 \tag{30}$$

I got this question CORRECT.

e)

$$E(n) = n^2 \times (\frac{r^0}{(r^2)^0} + \frac{r^1}{(r^2)^1} + ... + \frac{r^{-1+log_r n}}{(r^2)^{-1+log_r n}}) + r^{log_r n} \times 1, n > 1 \tag{31}$$

$$E(n) = n^2 \times \frac{1 - \frac{1}{n}}{1 - \frac{1}{r}} + n, n > 1 \tag{32}$$

The formula above stands true for E(1), thus we have

$$E(n) = \frac{n^2 - \frac{n}{r}}{1 - \frac{1}{r}}, n \geq 1 \tag{33}$$

I got this question CORRECT.

f)

$$F(n) = n^2 \times (\frac{2^0}{(2^2)^0} + \frac{2^1}{(2^2)^1} + ... + \frac{2^{-1+log_2 n}}{(2^2)^{-1+log_2 n}}) + 2^{log_2 n} \times 1, n > 1 \tag{34}$$

$$F(n) = n^2 \times (2 - \frac{2}{n}) + n, n > 1 \tag{35}$$

The formula above stands true for F(1), thus we have

$$F(n) = 2n^2 - n, n \geq 1 \tag{36}$$

I got this question CORRECT.

g)

$$G(n) = n \times (\frac{r^0}{s^0} + \frac{r^1}{s^1} + ... + \frac{r^{-1+log_s n}}{s^{-1+log_s n}}) + r^{log_s n} \times 1, n > 1 \tag{37}$$

$$G(n) = n \times \frac{1 - (\frac{r}{s})^{log_s n}}{1 - \frac{r}{s}} + r^{log_s n} = \frac{n - n^{log_s r}}{1 - \frac{r}{s}} + n^{log_s r}, n > 1 \tag{38}$$

The formula above stands true for G(1), thus we have

$$G(n) = \frac{n - \frac{r}{s} n^{log_s r}}{1 - \frac{r}{s}}, n \geq 1 \tag{39}$$

Since $r < s, log_s r < 1$, we have $G(n) = \Theta(n)$
I got this question CORRECT.

h)

$$H(n) = n \times (\frac{r^0}{s^0} + \frac{r^1}{s^1} + ... + \frac{r^{-1+log_s n}}{s^{-1+log_s n}}) + r^{log_s n} \times 1, n > 1 \tag{40}$$

$$H(n) = n \times \frac{1 - (\frac{r}{s})^{log_s n}}{1 - \frac{r}{s}} + r^{log_s n} = \frac{n^{log_s r} - n}{\frac{r}{s} - 1} + n^{log_s r}, n > 1 \tag{41}$$

The formula above stands true for H(1), thus we have

$$H(n) = \frac{\frac{r}{s} n^{log_s r} - n}{\frac{r}{s} - 1}, n \geq 1 \tag{42}$$

Since $r > s, log_s r > 1$, we have $H(n) = \Theta(n^{log_s r})$
I got this question CORRECT.

i)

$$I(n) = n^2 \times (\frac{r^0}{(s^2)^0} + \frac{r^1}{(s^2)^1} + ... + \frac{r^{-1+log_s n}}{(s^2)^{-1+log_s n}}) + r^{log_s n} \times 1, n > 1 \tag{43}$$

$$I(n) = \frac{n^2 - n^{log_s r}}{1 - \frac{r}{s^2}} + n^{log_s r}, n > 1 \tag{44}$$

The formula above stands true for I(1), thus we have

$$I(n) = \frac{n^2 - \frac{r}{s^2} n^{log_s r}}{1 - \frac{r}{s^2}}, n \geq 1 \tag{45}$$

Since $r < s^2, log_s r < 2$, we have $I(n) = \Theta(n^2)$
I got this question CORRECT.

j)

$$J(n) = n^2 \times (\frac{r^0}{(s^2)^0} + \frac{r^1}{(s^2)^1} + ... + \frac{r^{-1+log_s n}}{(s^2)^{-1+log_s n}}) + r^{log_s n} \times 1, n > 1 \tag{46}$$

$$J(n) = \frac{n^{log_s r} - n^2}{\frac{r}{s^2} - 1} + n^{log_s r}, n > 1 \tag{47}$$

The formula above stands true for J(1), thus we have

$$J(n) = \frac{\frac{r}{s^2} n^{log_s r} - n^2}{\frac{r}{s^2} - 1}, n \geq 1 \tag{48}$$

Since $r > s^2, log_s r > 2$, we have $J(n) = \Theta(n^{log_s r})$
I got this question CORRECT.

k)

Firstly we calculate the first several items.

$$81 = K(9) = 9^2 + 9K(3), K(3) = 0 \tag{49}$$

$$0 = K(3) = 3^2 + 9K(1), K(1) = -1 \tag{50}$$

Then we start to calculate $K(n), n > 1$

$$K(n) = n^2 \times (\frac{9^0}{9^0} + \frac{9^1}{9^1} + ... + \frac{9^{-1+log_3 n}}{9^{-1+log_3 n}}) + 9^{log_3 n} \times (-1), n > 1 \tag{51}$$

$$K(n) = n^2 log_3 n - n^2, n > 1 \tag{52}$$

The formula above stands true for K(1), thus we have

$$k(n) = n^2 log_3 n - n^2, n \geq 1 \tag{53}$$

I got this question CORRECT.

l)

$$L(n) = \sqrt{n} \times (\frac{3^0}{3^0} + \frac{3^1}{3^1} + ... + \frac{3^{-1+log_9 n}}{3^{-1+log_9 n}}) + 3^{log_9 n} \times 1, n > 1 \tag{54}$$

$$L(n) = \sqrt{n} log_9 n + \sqrt{n}, n > 1 \tag{55}$$

The formula above stands true for L(1), thus we have

$$L(n) = \sqrt{n} log_9 n + \sqrt{n}, n >= 1 \tag{56}$$

I got this question CORRECT.

General Recurrence Equation

Again it's hard to draw a tree in Latex... thus here I just write down the calculations of the equation tree:

Given that:

$$\begin{cases} T(n) = f(n), & n \leq 1 \\ T(n) = f(n) + \beta T(\delta n), & n > 1 \end{cases} \tag{57}$$

We have:

$$\begin{cases} T(n) = f(n), & n \leq 1 \\ T(n) = (\beta^0 f(\delta^0 n) + \beta^1 f(\delta^1 n) + ... + \beta^{\lceil -1+log_{\frac{1}{\delta}} n \rceil} f(n\delta^{\lceil -1+log_{\frac{1}{\delta}} n \rceil})) + \beta^{\lceil log_{\frac{1}{\delta}} n \rceil} f(n\delta^{\lceil log_{\frac{1}{\delta}} n \rceil}), & n > 1 \end{cases} \tag{58}$$

In a cleaner form gives:

$$\begin{cases} T(n) = f(n), & n \leq 1 \\ T(n) = \sum_{k=0}^{\lceil -log_\delta n \rceil} \beta^k f(n\delta^k), & n > 1 \end{cases} \tag{59}$$

I got this question CORRECT.

Note:
The leaf-level item that given on the question is $T(n) = f(n), n \leq 1$
However in the calculation of the solution is $T(n) = f(1), n \leq 1$
That's why there's a small different on the last item.