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Cooperative Assembling Using Multiple Robotic Manipulators

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Abstract: The scenario of cooperative assembling of multiple components in different locations is taken into consideration in this paper. It consists of two phases, including cooperative grasping and transporting respectively. Towards this end, a general nonlinear 3-DOF (degree of freedom) Euler-Lagrange model of the robotic manipulator is firstly established. A nonlinear distributed control law using a two-way-ring network topology is then proposed for each manipulator to cooperatively move from initial location to the desired location. Notably, we simply omit the control of end-effector of each manipulator and only focus on the cooperative control of multiple manipulators' trajectories. In the end, an assembling example of three components by three robotic manipulators is provided to illustrate the effectiveness of the proposed controller.

Key Words: Cooperative control, Robotic manipulators, Assembling and grasping

1 Introduction

Cooperative control of multiple robotic manipulators has attracted much attention due to its important role in the assembly automation, flexible manufacturing system and other potential applications. Compared with single robotic manipulator, multiple robotic manipulators has more superior maneuverability and manipulability in many tasks, which a single robotic manipulator cannot easily or even possibly undertake. In this paper, we focus on the general Euler-Lagrange model of robotic manipulator and the cooperative control law of the multiple robotic manipulators.

Various works on control methods of manipulators systems have been extensively studied based on the Euler-Lagrange dynamic model [1]. An attitude-tracking problem for a single manipulator was solved in [2] using sliding mode control method. In [3], an adaptive hybrid control of multiple robotic manipulators grasping a common object and moving along a predefined path was studied by designing the controller individually without information interaction. Regarding cooperative manipulation of a single object using multiple robotic arms, [4] proposed a nonlinear state feedback control law that linearizes and decouples the robotic system with respect to the objects motion, constraint force, and internal force. The authors in [5] presented general mathematical models of multiple rigid robotic manipulators performing cooperative task on a single dynamical object of which the motion was constrained by a dynamical environment. The aforementioned papers focused on either the control method of single manipulator or the control of multiple manipulators without information exchange. However, cooperative assembling problem requires multiple robotic manipulators to cooperatively transport different components to the predefined position and finalize the assembling. Therefore, to achieve the cooperative control of multiple manipulators and further better performance of the assembling, the communication among the manipulators is imperative.

Compared with traditional centralized coordination approaches, distributed cooperative control possesses more benefits, such as greater efficiency and higher robustness. Some researchers paid attention to the cooperative control of multiple robotic manipulators system with a network communication topology. Chung [6] proposed a synchronization method of Euler-Lagrange system that can achieve tracking desired trajectory with a specific topology network. A distributed leaderless consensus algorithm for multiple Euler-Lagrange systems can be found in [7], where it is assumed that the information is exchanged on a bidirectionally connected communication graph. In [8], a synchronized tracking problem for multiple Euler-Lagrange systems was studied when only a subset of the agents have access to the leader. A synchronized control approach with neuro-agents was proposed in [9] for multiple robotic manipulators based on the leader-follower network communication topology. The authors in [10] investigated the finite-time cooperative-tracking problem for a class of networked Euler-Lagrange systems with a leader-follower structure. [11] and [12] presented a robust cooperative control problem using NI systems theory with potential application in the control of flexible structures. It can be observed that a number of research has been conducted on the synchronized tracking problem of Euler-Lagrange systems. However, in practical situations, a complicated task of multiple manipulators such as assembling requires that each agent in the system reaches different attitude to form a desired assembling angle. Hence, it requires that the attitudes of the multiple manipulators should converge to the different desired attitudes.

In this paper, multiple 3-DOF robotic manipulators cooperative assembling problem including grasping and transporting of multiple components is taken into consideration. A general 3-DOF model of robotic manipulator is firstly established and then a contraction-based nonlinear cooperative control law for assembling of multiple components is proposed based on the result in [6]. The main contribution of this paper are three folds:

- the nonlinear Euler-Lagrange model of robotic manipulators is generalized and also adaptive for the case of load addition.
- the proposed algorithm is fully distributed dependent on communication topology and will thus yield a simulta-

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neous property in the assembling process.

- the problem we target is practical with engineering intuition and has solid theoretic basis.

The rest of this paper is organized as follows. Section 2 describes the general nonlinear Euler-Lagrange model of robotic manipulators with load addition. The analysis of assembling task, the cooperative control law design as well as the stability proof is given in Section 3. An assembling simulation of three components by three robotic manipulators is presented to illustrate the effectiveness of the proposed controller in the Section 4. Finally, conclusions are drawn in Section 5 and future directions are also pointed out.

2 Modeling of Robotic Manipulators

In this section, the general nonlinear model of three-link robotic manipulators with load addition was presented by Euler-Lagrange equations. It is commonly assumed that each link can be modelled as thin rods and their mass is uniformly distributed.

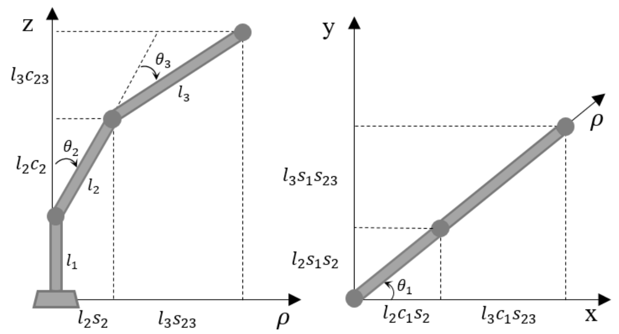


Fig. 1: Side view and top view of a single manipulator

Fig. 1 schematically shows the three-link manipulator under consideration, where l_j is the length of link $j\,(1 \le j \le 3)$, s_j, c_j, s_{jk}, c_{jk} denotes $\sin(\theta_j)$, $\cos(\theta_j)$, $\sin(\theta_j+\theta_k)$ and $\cos(\theta_j+\theta_k)$, axis-ρ is the projection of link 2 and link 3 in the x-y plane. Link 1 is attached to a fixed base on the ground, which can provide a torque τ_1 with an actuator to drive link 1 move alone the direction of the angle θ_1. Joint 1 connecting link 1 to link 2, can provide a torque τ_2 with an actuator to drive link 2 move alone the direction of the angle θ_2. Joint 2 connecting link 2 to link 3, can provide a torque τ_3 with an actuator to make link 3 move alone the direction of the angle θ_3. An end-effector, attached on the joint 3, is used for grasp the component. It is assumed that there is no offset between the end-effector and joint 3.

In our assembling task, we assumes that all robotic manipulators are three-link manipulators shown above. The equation of motion for ith robotic manipulator with load can be derived by exploiting the Euler-Lagrange equations.

$$L_i = \frac{1}{2}\dot{q}_i^T M_i(q_i)\dot{q}_i - V_i,\ \frac{d}{dt}\frac{\partial L_i(q_i,\dot{q}_i)}{\partial \dot{q}_i} - \frac{\partial L_i(q_i,\dot{q}_i)}{\partial q_i} = \tau_i, \tag{1}$$

where $i\,(1 \le i \le p)$ is the index of robotic manipulators and p is the total number of the manipulators. The generalized coordinate $q_i = [\theta_{1,i}, \theta_{2,i}, \theta_{3,i}]^T$ is the rotational degree of ith manipulator and the generalized force $\tau_i = [\tau_{1,i}, \tau_{2,i}, \tau_{3,i}]^T$ is the input torque acting on the ith manipulator, L_i is the Lagrange function, M_i is the inertia matrix and V_i is the potential energy function. Equation (1) can be represented as

$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i, \tag{2}$$

where $g_i(q_i) = \partial V_i/q_i$, $C_i(q_i,\dot{q}_i)$ is the vector of coriolis and centrifugal torques. We define $C_i(q_i,\dot{q}_i)$ such that $\dot{M}_i - 2C_i$ is skew-symmetric.

We only present $M_i(q_i)$, $C_i(q_i,\dot{q}_i)$, $g_i(q_i)$ as follows and omit the index i for convenience. For details of derivation, please refer [1].

$$M_i(q_i) = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}, \tag{3}$$

where

$$\begin{aligned} m_{11} =& \frac{1}{2}m_1r^2 + \frac{1}{3}m_2l_2^2s_2^2 + m_3\left(l_2^2s_2^2 + l_2l_3s_2s_{23}\right. \\ &\left.+\frac{1}{3}l_3^2s_{23}^2\right) + m_4(l_2s_2 + l_3s_{23})^2, \end{aligned} \tag{4}$$

$$\begin{aligned} m_{22} =& \frac{1}{3}m_2l_2^2 + m_3\left(l_2^2 + l_2l_3c_3 + \frac{1}{3}l_3^2\right) \\ &+ m_4\left(l_2^2 + 2l_2l_3c_3 + l_3^2\right), \end{aligned} \tag{5}$$

$$m_{23} = m_{32} = \frac{1}{2}m_3\left(l_2l_3c_3 + \frac{2}{3}l_3^2\right) + m_4\left(l_2l_3c_3 + l_3^2\right), \tag{6}$$

$$m_{33} = \frac{1}{3}m_3l_3^2 + m_4l_3^2, \tag{7}$$

$$m_{12} = m_{13} = m_{21} = m_{31} = 0. \tag{8}$$

$$C_i(q_i,\dot{q}_i) = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}, \tag{9}$$

where

$$\begin{aligned} c_{11} =& \frac{1}{3}m_2l_2^2s_2c_2 + m_3l_2^2s_2c_2 + \frac{1}{2}m_3l_2l_3\left(c_2s_{23} + s_2c_{23}\right) \\ &+ \frac{1}{3}m_3l_3^2s_{23}c_{23} + m_3\left(\frac{1}{2}l_2l_3s_2c_{2+3} + \frac{1}{3}l_3^2s_{23}c_{23}\right)\dot{q}_3 \\ &+ m_4\left(l_2s_2 + l_3s_{23}\right)\left(l_2c_2 + l_3c_{23}\right)\dot{q}_2 \\ &+ m_4\left(l_2s_2 + l_3s_{23}\right)l_3c_{23}\dot{q}_3, \end{aligned} \tag{10}$$

$$\begin{aligned} c_{12} = -c_{21} =& \left(\frac{1}{3}m_2l_2^2s_2c_2 + m_3l_2^2s_2c_2 + \frac{1}{3}m_3l_3^2s_{23}c_{23}\right. \\ &\left.+ \frac{1}{2}m_3l_2l_3\left(c_2s_{23} + s_2c_{23}\right)\right)\dot{q}_1 \\ &+ m_4\left(l_2s_2 + l_3s_{23}\right)\left(l_2c_2 + l_3c_{23}\right)\dot{q}_1, \end{aligned} \tag{11}$$

$$\begin{aligned} c_{13} = -c_{31} =& m_3\left(\frac{1}{2}l_2l_3s_2c_{23} + \frac{1}{3}l_3^2s_{23}c_{23}\right)\dot{q}_1 \\ &+ m_4l_3c_{23}\left(l_2s_2 + l_3s_{23}\right)\dot{q}_1, \end{aligned} \tag{12}$$

$$c_{22} = -\frac{1}{2}m_3 l_2 l_3 s_3 \dot{q}_3 - m_4 l_2 l_3 s_3 \dot{q}_3, \tag{13}$$

$$\begin{aligned} c_{23} = &-\frac{1}{2}m_3 l_2 l_3 s_3 \dot{q}_2 - \frac{1}{2}m_3 l_2 l_3 s_3 \dot{q}_3 \\ &- m_4 l_2 l_3 s_3 \dot{q}_3 - m_4 l_2 l_3 s_3 \dot{q}_2, \end{aligned} \tag{14}$$

$$c_{32} = \frac{1}{2}m_3 l_2 l_3 s_3 \dot{q}_2 + m_4 l_2 l_3 s_3 \dot{q}_2, \tag{15}$$

$$c_{33} = 0. \tag{16}$$

$$g_i(q_i) = \begin{bmatrix} 0 \\ g\left(\frac{1}{2}m_2 l_2 s_2 + m_3\left(l_2 s_2 + \frac{1}{2}l_3 s_{23}\right)\right) \\ +g m_4 \left(l_2 s_2 + l_3 s_{23}\right) \\ \frac{1}{2} g m_3 l_3 s_{23} + g m_4 l_3 s_{23} \end{bmatrix}, \tag{17}$$

where m_j $(1 \leq j \leq 3)$ is the mass of link j, m_4 is the mass of the load, r is the radius of link 1, g is the constant of gravity.

Remark 1. When the robotic manipulator captures the component, we assume that the component is firmly grasped by the end-effector. In other words, there is no relative motion between the end-effector and the component so that the component can be viewed as an additional mass attached at the end of link 3. When the robotic manipulator is moving without load, m_4 is zero.

3 Main Results

In this section, we describe the assembling task and give the proposed cooperative control law based on the dynamic model as shown in the last section.

3.1 Problem Statement

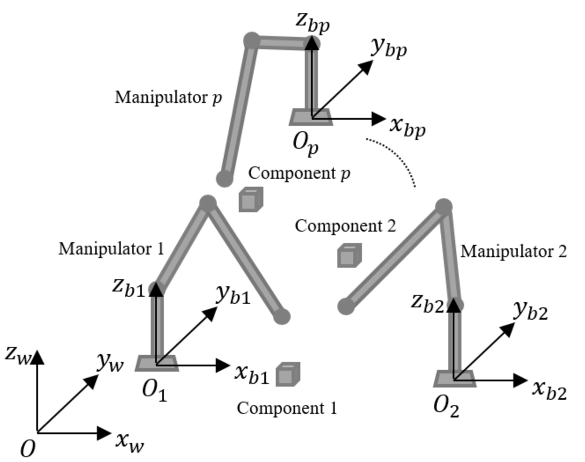


Fig. 2: Multiple robotic manipulators and components

Consider p cooperative robotic manipulators and p components located on the ground, as shown in Fig. 2. The problem of cooperative assembling is to cooperatively: 1) grasp the appointed component by the end-effector from arbitrary initial location (grasping phase) and 2) transport the components to the same predefined position to finalize the assembling (transporting phase). The mathematical interpretation can be seen as: 1) $\left[x_i^{(ee)}, y_i^{(ee)}, z_i^{(ee)}\right]^T \rightarrow \left[x_i^{(g)}, y_i^{(g)}, z_i^{(g)}\right]^T, \forall i \in p$ and 2) $\left[x_i^{(ee)}, y_i^{(ee)}, z_i^{(ee)}\right]^T \rightarrow \left[x^{(t)}, y^{(t)}, z^{(t)}\right]^T, \forall i \in p$. $\left[x_i^{(ee)}, y_i^{(ee)}, z_i^{(ee)}\right]^T$ is the position of the ith end-effector, $\left[x_i^{(g)}, y_i^{(g)}, z_i^{(g)}\right]^T$ is the initial location of the ith component in the grasping phase and $\left[x^{(t)}, y^{(t)}, z^{(t)}\right]^T$ is the predefined position to finalize the assembling in the transporting phase.

The world frame $Ox_w y_w z_w$ is defined by axes x_w, y_w, and z_w with z_w pointing upward. The body frame $O_i x_{bi} y_{bi} z_{bi}$ is attached to the base point of the ith robotic manipulators with x_{bi}, y_{bi}, z_{bi} coinciding with the preferred forward direction. Therefore, the position of the end-effector in the world frame and the attitude of the corresponding manipulator in the body frame satisfy the equation as follows:

$$\begin{bmatrix} x_i^{(ee)} \\ y_i^{(ee)} \\ z_i^{(ee)} \end{bmatrix} = \begin{bmatrix} x_i^{(b)} \\ y_i^{(b)} \\ z_i^{(b)} \end{bmatrix} + \begin{bmatrix} \left(l_{2,i} s_{2,i} + l_{3,i} s_{23,i}\right) c_{1,i} \\ \left(l_{2,i} s_{2,i} + l_{3,i} s_{23,i}\right) s_{1,i} \\ l_{1,i} + l_{2,i} c_{2,i} + l_{3,i} c_{23,i} \end{bmatrix}, \tag{18}$$

where $\left[x_i^{(b)}, y_i^{(b)}, z_i^{(b)}\right]^T$ is the coordinate of the base of the manipulator in the world frame.

3.2 Control Law Design

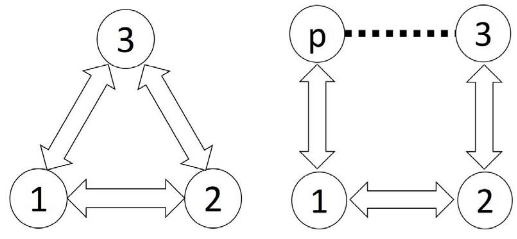
According to the above definitions, the task can be divided into two phases: grasping phase and transporting phase. First of all, the desired attitude of the manipulator could be solved by equation (18) with the initial location of the component and the coordinate of the base of the manipulator. Then multiple manipulators cooperatively move to the first desired position and capture the corresponding component from the initial attitude. The detail of grasping process is out of the scope of this paper and thus omitted. We assume that the component is captured when the distance between the end-effector and the component is small enough. After all of the end-effectors grasp the corresponding components steadily, all manipulators transport the components to the desired attitudes solved by the predefined assembling position of the components and the coordinate of the base of the manipulators. The assembling task will be accomplished when all end-effectors move to the same terminal position.

Given the desired attitude of each manipulator by calculation, the following control law is proposed for the ith robotic manipulator in a network comprised of p manipulators $(p \geq 3)$.

$$\begin{aligned} \tau_i = &M(q_i)\ddot{q}_{i,r} + C(q_i, \dot{q}_i)\dot{q}_{i,r} + g(q_i) - K_1(\dot{q}_i - \dot{q}_{i,r}) \\ &+ K_2(\dot{q}_{i-1} - \dot{q}_{i-1,r}) + K_2(\dot{q}_{i+1} - \dot{q}_{i+1,r}), \end{aligned} \tag{19}$$

where $\dot{q}_{i,r} = \dot{q}_{d,i} - \Lambda(q_i - q_{d,i})$, Λ is a positive diagonal matrix, $q_{d,i}$ is the desired attitude of the ith manipulator, a positive-definite matrix $K_1 \in \mathcal{R}^{3\times3}$ is a feedback gain for the ith manipulator, and another positive-definite matrix $K_2 \in \mathcal{R}^{3\times3}$ is a coupling gain with the adjacent members ($i-1$ and $i+1$).

Remark 2. From the control law (19), the ith robotic manipulator only can receive information from the adjacent manipulators ($i-1$ and $i+1$). Note that the last (pth) manipulator is



(a) (b)

Fig. 3: A two-way-ring network

connected with the first manipulator to form a ring network. In this paper, a two-way ring network structure as shown in Fig. 3 is adopted. This topology is a connected undirected graph where is a path from every nodes to every other nodes. Graph theory is used to model communication among the multi agent systems, which can be mathematically expressed by $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. $\mathcal{V} = \{v_1, v_2, \ldots, v_n\}$ is a nonempty finite set of n nodes and an edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is used to model the communications links among nodes. $\mathcal{A} = [a_{ij}] \in R^{n\times n}$ is the adjacency matrix, where $a_{ii} = 0$ and $\forall i, j$ with $i \neq j$, $a_{ij} = 1$ if $(v_i, v_j) \in \mathcal{E}$ and 0 otherwise. $\mathcal{L} = [l_{ij}] \in R^{n\times n}$ is the Laplacian matrix, where $l_{ii} = \sum_{j\neq i} a_{ij}$ and $l_{ij} = -a_{ij}$ for all $i \neq j$. The Laplacian matrix of Fig. 3(b) is given as follows.

$$\mathcal{L} = \begin{bmatrix} 2 & -1 & 0 & \cdots & 0 & -1 \\ -1 & 2 & -1 & 0 & \cdots & 0 \\ 0 & -1 & 2 & -1 & & \vdots \\ \vdots & & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & -1 & 2 & -1 \\ -1 & 0 & \cdots & 0 & -1 & 2 \end{bmatrix}_{p\times p} \tag{20}$$

Next, we are ready to present the main result of this paper.

Theorem 1. *If $K_1 - 2K_2$ is positive definite, then the cooperative assembling using multiple robotic manipulators is exponentially achieved using the proposed control law (19), i.e. $q_i \to q_{d,i}$, $\forall i \in p \geq 3$ from any initial condition (or in the grasping phase, $\left[x_i^{(ee)}, y_i^{(ee)}, z_i^{(ee)}\right]^T \to \left[x_i^{(g)}, y_i^{(g)}, z_i^{(g)}\right]^T$, $\forall i \in p$ and in the transporting phase $\left[x_i^{(ee)}, y_i^{(ee)}, z_i^{(ee)}\right]^T \to \left[x^{(t)}, y^{(t)}, z^{(t)}\right]^T$, $\forall i \in p$).*

Proof. The closed-loop system after apply in the control law (19) to system (2) can be written as follows:

$$M(q_i)\dot{s}_i + C(q_i, \dot{q}_i)s_i + K_1 s_i - K_2 s_{i-1} - K_2 s_{i-1} = 0, \tag{21}$$

where s_i denotes the composite variable $s_i = \dot{q}_i - \dot{q}_{d,i} + \Lambda(q_i - q_{d,i})$.

Let us define a $3p \times 3p$ block square matrix as follows.

$$\left[L^p_{K_1,-K_2}\right] = \begin{bmatrix} K_1 & -K_2 & 0 & \cdots & -K_2 \\ -K_2 & K_1 & -K_2 & \cdots & 0 \\ \vdots & \ddots & \ddots & & \vdots \\ 0 & & -K_2 & K_1 & -K_2 \\ -K_2 & \cdots & 0 & -K_2 & K_1 \end{bmatrix}_{3p\times 3p} \tag{22}$$

We can rewrite the closed-loop system in equation (21) in the following block matrix form

$$[M]\dot{x} + [C]x + \left[L^p_{K_1,-K_2}\right]x = 0, \tag{23}$$

where $[M] = \operatorname{diag}(M(q_1), ..., M(q_p))$, $[C] = \operatorname{diag}(C(q_1, \dot{q}_1), ..., C(q_p, \dot{q}_p))$, diag denotes a block diagonal matrix and $x = \left[s_1^T, ..., s_p^T\right]^T$. We construct the following virtual system by replacing x by y.

$$[M]\dot{y} + [C]y + \left[L^p_{K_1,-K_2}\right]y = 0 \tag{24}$$

This virtual system has two particular solutions: $x = \left[s_1^T, ..., s_p^T\right]^T$ and 0. Using the skew-symmetric property of $\left[\dot{M}\right] - 2[C]$, we have

$$\begin{aligned} \frac{d}{dt}\left(\delta y^T [M] \delta y\right) &= 2\delta y^T [M] \delta\dot{y} + \delta y^T \left[\dot{M}\right]\delta y \\ &= -2\delta y^T \left([C]\delta y + \left[L^p_{K_1,-K_2}\right]\delta y\right) + \delta y^T\left[\dot{M}\right]\delta y \\ &= -2\delta y^T \left[L^p_{K_1,-K_2}\right]\delta y. \end{aligned} \tag{25}$$

According to the contraction theory in [13], when $\left[L^p_{K_1,-K_2}\right]$ is positive, the system will contract leading $\delta y \to 0$. Then all solutions of y converge to a single trajectory globally and exponentially fast, resulting in globally exponential convergence of $s_i \to 0$, $\forall i \in p$. Due to the definition of $s_i = \dot{q}_i - \dot{q}_{d,i} + \Lambda(q_i - q_{d,i})$, it is easily obtained that $q_i \to q_{d,i}$, $\forall i \in p$. Given $K_1 > 0$, $K_2 > 0$, it can be shown that a sufficient condition for the positive-definiteness of $\left[L^p_{K_1,-K_2}\right]$ is $K_1 - 2K_2 > 0$.

We choose $q_{d,i} = q_{d,i}^{(g)} = \left[\theta_{d1,i}^{(g)}, \theta_{d2,i}^{(g)}, \theta_{d3,i}^{(g)}\right]^T$ in the grasping phase and $q_{d,i} = q_{d,i}^{(t)} = \left[\theta_{d1,i}^{(t)}, \theta_{d2,i}^{(t)}, \theta_{d3,i}^{(t)}\right]^T$ in the transporting phase, which satisfy the equation as follows:

$$\begin{bmatrix} x_i^{(g)} \\ y_i^{(g)} \\ z_i^{(g)} \end{bmatrix} = \begin{bmatrix} x_i^{(b)} \\ y_i^{(b)} \\ z_i^{(b)} \end{bmatrix} + \begin{bmatrix} \left(l_{2,i}s_{d2,i}^{(g)} + l_3 s_{d2d3,i}^{(g)}\right)c_{d1,i}^{(g)} \\ \left(l_{2,i}s_{d2,i}^{(g)} + l_3 s_{d2d3,i}^{(g)}\right)s_{d1,i}^{(g)} \\ l_{1,i} + l_{2,i}c_{d2,i}^{(g)} + l_3 c_{d2d3,i}^{(g)} \end{bmatrix}, \tag{26}$$

$$\begin{bmatrix} x^{(t)} \\ y^{(t)} \\ z^{(t)} \end{bmatrix} = \begin{bmatrix} x_i^{(b)} \\ y_i^{(b)} \\ z_i^{(b)} \end{bmatrix} + \begin{bmatrix} \left(l_{2,i}s_{d2,i}^{(t)} + l_3 s_{d2d3,i}^{(t)}\right)c_{d1,i}^{(t)} \\ \left(l_{2,i}s_{d2,i}^{(t)} + l_3 s_{d2d3,i}^{(t)}\right)s_{d1,i}^{(t)} \\ l_{1,i} + l_{2,i}c_{d2,i}^{(t)} + l_3 c_{d2d3,i}^{(t)} \end{bmatrix}. \tag{27}$$

Therefore, in the grasping phase, when $q_i \to q_{d,i}^{(g)}$, $\forall i \in p$, $\left[x_i^{(ee)}, y_i^{(ee)}, z_i^{(ee)}\right]^T \to \left[x_i^{(g)}, y_i^{(g)}, z_i^{(g)}\right]^T$, $\forall i \in p$; in the transporting phase, when $q_i \to q_{d,i}^{(t)}$, $\forall i \in p$, $\left[x_i^{(ee)}, y_i^{(ee)}, z_i^{(ee)}\right]^T \to \left[x^{(t)}, y^{(t)}, z^{(t)}\right]^T$, $\forall i \in p$, which completes the proof. ☐

Remark 3. The desired attitude of each manipulator $q_{d,i}^{(g)}$ and $q_{d,i}^{(t)}$ can be solved by equation (26) and equation (27) with the desired position of the end-effector and the coordinate of the base of each manipulator. Note that $\dot{q}_{d,i}$ is zero because

$q_{d,i}$ is constant. In the cooperative grasping phase, the desired position of the end-effector is the position of the each component. In the transporting phase, the desired position of the end-effector is the terminal position of assembling. Obviously, when the desired position of the end-effector and the coordinate of the base of the manipulator are given, there are more than one solution of the attitude of the manipulator. Then, the domain of the three angles is given as $\theta_1 \in [0, 2\pi)$, $\theta_2 \in [-\pi/2, \pi/2]$, $\theta_3 \in [0, \pi)$ to further narrow down the feasible solution of $q_{d,i}^{(g)}$ and $q_{d,i}^{(t)}$.

Remark 4. $\left[L^p_{K_1,-K_2}\right]$ can be viewed as the modified Laplacian of the network, which indicates the connectivity with adjacent manipulator as well as the strength of the coupling by K_2. $\left[L^p_{K_1,-K_2}\right]$ can be time-varying due to time-varying K_1 and K_2 and the sufficient condition for $q_i \to q_{d,i}$, $\forall i \in p$ is that $K_1 - 2K_2 > 0$ is uniformly positive definite [6].

4 Simulation Results

In order to demonstrate the performance of the proposed controller, an assembling example of three components by three identical robotic manipulators is provided in this section. The proposed control law is also appropriate for the heterogeneous case.

Three manipulators are located at the three vertices of an equilateral triangle on the ground and three components located at the three edges of the triangle. The coordinate of the base of the three robotic manipulators are $\left[x_1^{(b)}, y_1^{(b)}, z_1^{(b)}\right]^T = [2,2,0]^T m$, $\left[x_2^{(b)}, y_2^{(b)}, z_2^{(b)}\right]^T = [6,2,0]^T m$ and $\left[x_3^{(b)}, y_3^{(b)}, z_3^{(b)}\right]^T = \left[4, 2{+}2\sqrt{3}, 0\right]^T m$. The initial attitude of the three robotic manipulators are $q_1 = [0, 0, -0.35]^T rad$, $q_2 = [1.05, -0.52, 0.87]^T rad$ and $q_3 = [-1.05, 0.35, 0.35]^T rad$. The initial location of the three components is $\left[x_1^{(g)}, y_1^{(g)}, z_1^{(g)}\right]^T = [4.5, 2, 0]^T m$, $\left[x_2^{(g)}, y_2^{(g)}, z_2^{(g)}\right]^T = \left[4.5, 2{+}1.5 * \sqrt{3}, 0\right]^T m$, $\left[x_3^{(g)}, y_3^{(g)}, z_3^{(g)}\right]^T = \left[3, 2{+}\sqrt{3}, 0\right]^T m$. The terminal position of components to accomplish the assembling are both $\left[x^{(t)}, y^{(t)}, z^{(t)}\right]^T = \left[4, 2{+}2\sqrt{3}/3, 2\right]^T m$. The physical parameters of each robotic manipulator as well as the components are given in Table 1.

Table 1: Physical Parameter

$[l_1, l_2, l_3]$ lengths of link j	[1,1,2]m
r radius of the link 1	0.03m
$[m_1, m_2, m_3]$ mass of link j	[5,10,10]kg
$m_{4,1}$ mass of component 1	15kg
$m_{4,2}$ mass of component 2	10kg
$m_{4,3}$ mass of component 3	20kg

According to the equation (26) and equation (27), the desired attitude of three manipulators $\left[q_{d,1}^{(g)}, q_{d,2}^{(g)}, q_{d,3}^{(g)}\right] = \begin{bmatrix} 0 & 2.09 & -2.09 \\ 1.12 & 1.23 & 1.05 \\ 1.66 & 1.32 & 1.96 \end{bmatrix} rad$ and $\left[q_{d,1}^{(t)}, q_{d,2}^{(t)}, q_{d,3}^{(t)}\right] = \begin{bmatrix} 0.52 & 2.62 & -1.57 \\ 0.27 & 0.27 & 0.27 \\ 1.78 & 1.78 & 1.78 \end{bmatrix} rad$ can be easily obtained.

The simulation results are presented in Fig. 4 to Fig. 9. For the control gains in the equation (19), we utilize $K_1 = 300I$, $K_2 = 20I$, $\Lambda = 10I$ and I is the identity matrix. According to theorem 1, $K_1 - 2K_2$ is positive definite. Fig. 4 and Fig. 7 show the tracking errors of the three manipulators in the grasping phase and in the transporting phase, respectively. Fig. 5 and Fig. 8 show the torque of the three manipulators in the grasping phase and in the transporting phase, respectively. It can be seen that each manipulator converges to the desired position exponentially fast from initial conditions. Fig. 6 and Fig. 9 present several moments in the grasping phase and the transporting phase, respectively. The dashed lines represent the initial attitude of each manipulator, the dotted lines represents the attitude of each manipulator at $t = 1s$, and the solid lines represents the attitude of each manipulator at $t = 5s$. Video clips for this cooperative assembling simulation are available online at `http://v.youku.com/v_show/id_XMTQ1Mjc2ODEzNg==.html?from=y1.7-1.2`.

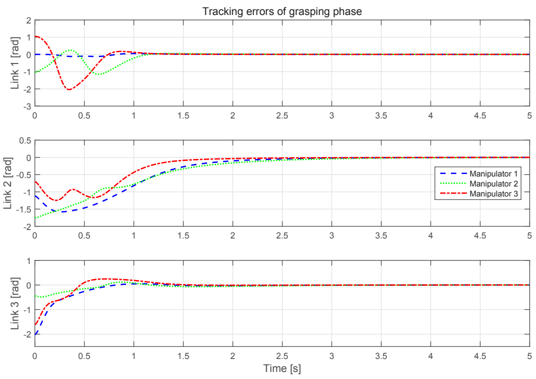


Fig. 4: Tracking errors of grasping phase

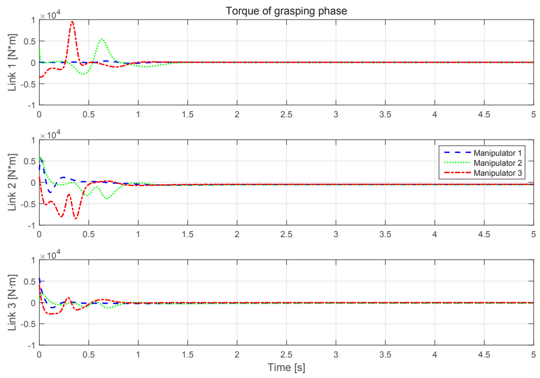


Fig. 5: Torque of grasping phase

5 Conclusions and Future Directions

In this paper, a general nonlinear 3-DOF Euler-Lagrange model of the robotic manipulator with load is firstly established. The cooperative assembling problem of multi-

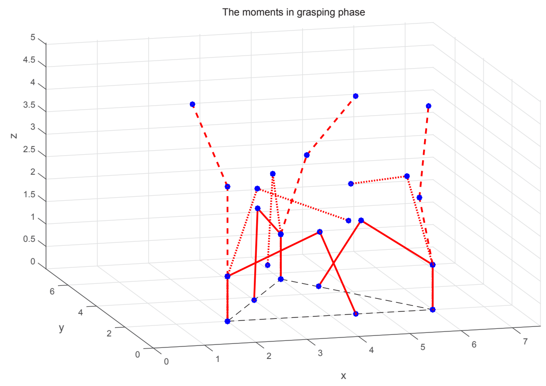


Fig. 6: The moments in grasping phase

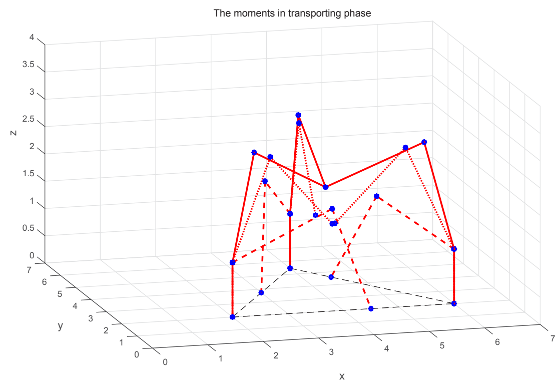


Fig. 9: The moments in transporting phase

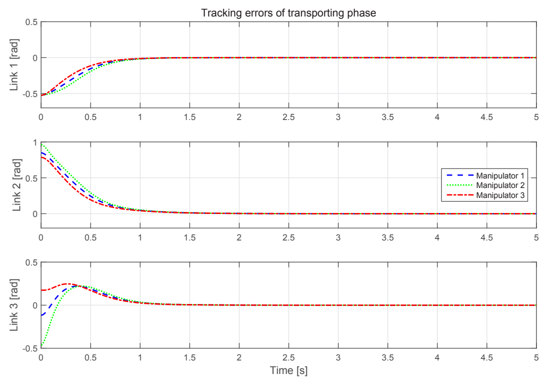


Fig. 7: Tracking errors of transporting phase

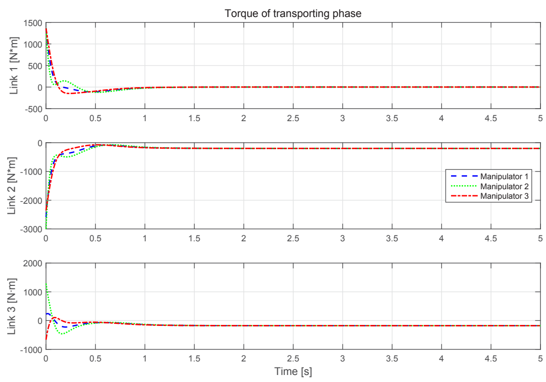


Fig. 8: Torque of transporting phase

ple components in different location is divided into grasping phase and transporting phase. The calculation method of the manipulators' desired attitude in different phase is given as well. In this context, a nonlinear distributed cooperative control law using a two-way-ring network topology is then proposed to solve the cooperative assembling problem.

Future research directions include the cooperative assembling problem of multiple robotic manipulators mounted in the UAV (Unmanned-Aerial-Vehicle) platform and collision avoidance among multiple robotic manipulators.

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