PREFACE (I) 22/06/19

Utilizar caracteres más grandes, como el resto del libro

0.1 Views of the nucleus

0.1.1 The liquid drop and the shell model

In the atom, the nucleus provides the Coulomb field in which negatively charged electrons ($-1 \times e$) move indenpendently of each other in single–particle orbitals. The filling of these orbitals explains Mendeleev's periodic table. Thus the valence of the chemical elements as well as the particular stability of the noble gases (He, N, Ar, Kr, Xe and Ra) associated with the closing of shells (Fig. 0.1.1). The dimension of the atom is measured in angstroms (Å=10^{-8}cm), and typical energies in eV, the electron mass being $m_e \approx 0.5$ MeV (MeV=10^6eV).

The atomic nucleus is made out of positively charged protons ($1 \times e$) and of (uncharged) neutrons, nucleons, of mass $\approx 10^3$ MeV ($m_p = 938.3$ MeV, $m_n = 939.6$ MeV). Nuclear dimensions are of the order of few fermis (fm= 10^{-13} cm). While the stability of the atom is provided by a source external to the electrons, namely the atomic nucleus, this system is self–bound as a result of the strong interaction of range $a_0 \approx 0.9$ fm and strength $v_0 \approx -100$ MeV acting among nucleons. ~~Carrying with the paral~~lel, While most of the atom is empty space, the density of the atomic nucleus is conspicuous ($\rho = 0.17$ nucleon/fm^3). The "closed packed" nature of this system implies a short mean free path as compared to nuclear dimensions. This can be estimated from classical kinetic theory $\lambda \approx (\rho\sigma)^{-1} \approx 1$ fm, where $\sigma \approx 2\pi a_0^2$ is the nucleon–nucleon cross section. It seems then natural to liken the atomic nucleus to a liquid drop (Bohr and Kalckar (1937)). This picture of the nucleus provided the framework to describe the basic features of the fission process (Meitner and Frisch (1939); Bohr and Wheeler (1939)).

The leptodermic properties of the atomic nucleus are closely connected with the semi–empirical mass formula (Weizsäcker (1935))

$$m(N,Z) = (Nm_n + Zm_p) - \frac{1}{c^2}B(N,Z), \tag{0.1.1}$$

the binding energy being

$$B(N,Z) = \left(b_{vol}A - b_{surf}A^{2/3} - \frac{1}{2}b_{sym}\frac{(N-Z)^2}{A} - \frac{3}{5}\frac{Z^2e^2}{R_C}\right). \tag{0.1.2}$$

The second term in (0.1.2) represents the surface energy, while

$$b_{surf} = 4\pi r_0^2 \gamma. \tag{0.1.3}$$

The nuclear radius is written as $R = r_0 A^{1/3}$, with $r_0 = 1.2$ fm, the surface tension energy being $\gamma \approx 0.95$ MeV/fm^2.

When, in a heavy–ion reaction, ~~the~~ a two nuclei come within the range of the nuclear forces, the trajectory of relative motion will be changed by the attraction which will act between the nuclear surfaces. This surface interaction is a fundamental quantity in all heavy ion reactions. Assuming two spherical nuclei at a relative distance $r_{aA} = R_a + R_A$, where R_a and R_A are the corresponding half–density

N. Bohr and F. Kalckar, On the transmutation of atomic nuclei by impact of material particles, Mat. Fys. Medd. Dans. Vid. Selsk 14 no. 10 (1937) radii, the force acting between the two surfaces is

$$\left(\frac{\partial U_{aA}^{N}}{\partial r}\right)_{r_{aA}} = 4\pi\gamma \frac{R_a R_A}{R_a + R_A} \tag{0.1.4}$$

This result allows for the calculation of the ion–ion (proximity) potential which, supplemented with a position dependent absorption, can be used to accurately describe heavy ion reactions (Broglia and Winther (2004) and refs. therein).

In such reactions, not only elastic processes are observed, but also anelastic ~~ones~~ processes in which one, or both of the nuclear surfaces is set into vibration (Fig. 0.1.2). The restoring force parameter associated with oscillations of multipolarity λ is

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$$C_\lambda = (\lambda - 1)(\lambda + 2)R^2\gamma - \frac{3}{2\pi}\frac{\lambda - 1}{2\lambda + 1}\frac{Z^2e^2}{R}, \tag{0.1.5}$$

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where the second term corresponds to the contribution of the Coulomb energy to C_λ. Assuming the flow associated with surface vibration to be irrotational, the associated inertia for small amplitude oscillations is,

$$D_\lambda = \frac{3}{4\pi}\frac{1}{\lambda}AMR^2, \tag{0.1.6}$$

the energy of the corresponding mode being

$$\hbar\omega_\lambda = \hbar\sqrt{\frac{C_\lambda}{D_\lambda}}. \tag{0.1.7}$$

The label λ stand for the angular momentum of the vibrational mode, μ being its third component (see Eq. (0.1.18)). Aside from λ, μ, surface vibrations can also be characterize by an integer n $(=1, 2, \ldots)$, an ordering number indicating increasing energy. For simplicity, a single common label α will be also used.

Experimental information associated with low–energy quadrupole vibrations, namely $\hbar\omega_2$ and the electromagnetic transition probabilities $B(E2)$, allow to determine C_2 and D_2. The resulting C_2 values exhibit variations by more than a factor of 10 both above and below the liquid–drop estimate. The observed values of D_2 are large as compared with the mass parameter for irrotational flow.

A picture apparently antithetic to that of the liquid drop, the shell model, emerged from the study of experimental data, plotting them against either the number of protons (atomic number), or the number of neutrons in the nuclei, rather than against the mass number. One of the main nuclear features which led to the development of the shell model was the study of the stability and abundance of nuclear species and the discovery of what are usually called magic numbers (Elsasser (1933); Mayer (1948); Haxel et al. (1949)). What makes a number magic is that a configuration of a magic number of neutrons, or of protons, is unusually stable whatever the associated number of other type of nucleons (Mayer (1949); Mayer and Teller (1949)).

The strong binding of a magic number of nucleons and weak binding for one more reminds, only relatively much weaker, the results displayed in Fig. 0.1.1 concerning the atomic stability of rare gases. In the nuclear case, at variance with the atomic case, the spin–orbit coupling play an important role, as can be seen from

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shell

the level scheme shown in Fig. 0.1.3, obtained by assuming that nucleons move independently of each other in an average potential of spherical symmetry.

A closed shell, or a filled level, has angular momentum zero. Thus, nuclei with one nucleon outside (missing from) closed, should have the spin and parity of the orbital associated with the odd nucleon (–hole), a prediction confirmed by the data (available at that time) throughout the mass table. Such a picture implies that the nucleon mean free path is large compared to nuclear dimensions.

The systematic studies of the binding energies leading to the shell model found also that formula (0.1.2) has to be supplemented to take into account the fact that nuclei with both odd number of protons and of neutrons are energetically unfavored compared with even–even ones (inset Fig. 0.1.1) by a quantity of the order of as a rule, $\delta \approx 33MeV/A^{3/4}$ called the pairing energy[1].

The low–lying state of closed shell nuclei can be interpreted as harmonic quadrupole or octupole collective vibrations (Fig. 0.1.4) described by the Hamiltonian *)

excited

$$H_{coll} = \sum_{\lambda\mu} \left(\frac{1}{2D_\lambda} |\Pi_{\lambda\mu}|^2 + \frac{C_\lambda}{2} |\alpha_{\lambda\mu}|^2 \right) \tag{0.1.8}$$

Following Dirac (1930) one can describe the oscillatory motion introducing boson creation (annihilation) operator $\Gamma^\dagger_{\lambda\mu}$ $(\Gamma_{\lambda\mu})$ obeying

$$\left[\Gamma_\alpha, \Gamma^\dagger_{\alpha'}\right] = \delta(\alpha, \alpha'), \tag{0.1.9}$$

$$\sqrt{\frac{\hbar\omega_\lambda}{2C_\lambda}}$$

leading to

ω_λ

$$\hat{\alpha}_{\lambda\mu} = \sqrt{\frac{\hbar\omega}{2C}} \left(\Gamma^\dagger_{\lambda\mu} + (-1)^\mu \Gamma_{\lambda-\mu} \right), \tag{0.1.10}$$

λ C_λ ~~20/6/19~~

and a similar expression for the conjugate momentum variable $\hat{\Pi}_{\lambda\mu}$, resulting in

ω_λ

$$\hat{H}_{coll} = \sum \hbar\omega_\lambda \left((-1)^\mu \Gamma^\dagger_{\lambda\mu} \Gamma_{\lambda-\mu} + 1/2 \right). \tag{0.1.11}$$

$|n_{\lambda\mu} = 1\rangle$

The frequency is $\omega_\lambda = (C_\lambda/D_\lambda)^{1/2}$, while $(\hbar\omega_\lambda/2C_\lambda)^{1/2}$ is the amplitude of the zero–point fluctuation of the vacuum state $|0\rangle_B$, $\Gamma^\dagger_{\lambda\mu}|0\rangle_B$ being the one–phonon state. To simplify the notation, in many cases one writes $|n_\alpha = 1\rangle$

The ground and low–lying states of nuclei with one nucleon outside closed shell can be described by the Hamiltonian

$$H_{sp} = \sum_\nu \epsilon_\nu a^\dagger_\nu a_\nu, \tag{0.1.12}$$

[1]Connecting with further developments associated with the BCS theory of superconductivity (Bardeen et al. (1957a,b)) and its extension to the atomic nucleus (Bohr et al. (1958)), the quantity δ is identified with the pairing gap Δ parametrized according to $\Delta = 12\text{MeV}/\sqrt{A}$ (Bohr and Mottelson (1969)). It is of notice that for typical superfluid nuclei like ^{120}Sn, the expression of δ leads to $\delta \approx 10$ MeV$/\sqrt{A}$. a numerical value which can be parametrized as

*) Classically $\Pi_{\lambda\mu} = D_\lambda \dot{\alpha}_{\lambda\mu}$

where $a^\dagger_\nu(a_\nu)$ is the single–particle creation (annihilation) operator,

$$|\nu\rangle = a^\dagger_\nu|0\rangle_F, \qquad (0.1.13)$$

being the single–particle state of quantum numbers $\nu(\equiv nljm)$ and energy ϵ_ν, $|0\rangle_F$ being the Fermion vacuum.*)

(a) -(a)

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Both the existence of drops of nuclear matter displaying collective surface vibrations, and of independent–particle motion in a self–confining mean field are emergent properties not contained in the particles forming the system, neither in the NN–force, but on the fact that these particles behave according to the rules of quantum mechanics, move in a confined volume and that there are many of them.

Generalized rigidity as measured by the inertia parameter D_λ, as well as surface tension closely connected to the restoring force C_λ, implies that acting on the system with an external time–dependent (nuclear and/or Coulomb) field, the system reacts as a whole. This behavior is to be found nowhere in the properties of the nucleons, nor in the nucleon–nucleon scattering phase shifts at the basis of Yukawa prediction of the existence of a π–meson as the carrier of the strong force acting among nucleons.

Similarly, the fact that nuclei probed through fields which change in one unit particle number (e.g. (d, p) and (p, d) reactions) react in term of independent particle motion, feeling the pushings and pullings of the other nucleons only when trying to leave the nucleus, is not apparent in the detailed properties of the NN–forces, not even in those carrying the quark–gluon input. Within this context, independent particle motion can be considered a *bona fide* emergent property.

atención !!

Collective surface vibrations and independent particle motion are examples of what are called elementary modes of excitation in many–body physics, and collective variables in soft–matter physics.

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0.2 The particle-vibration coupling

The oscillation of the nucleus under the influence of surface tension implies that the potential $U(R, r)$ in which nucleons move independently of each other change with time. For low–energy collective vibrations this change is slow as compared with single–particle motion. Within this scenario the nuclear radius can be written as

$$R = R_0\left(1 + \sum_{LM} \alpha_{\lambda\mu} Y^*_{\lambda\mu}\right)(\hat{r}) \qquad (0.1.18)\ \cancel{(0.1.14)}\qquad (0,2,1)$$

Assuming small amplitude motion,

$$U(r, R) = U(r, R_0) + \delta U(r), \qquad 19\ \cancel{(0.1.15)}\qquad (0,2,2)$$

where

$$\delta U = -\kappa\hat{\alpha}\hat{F}, \qquad (0.1.\cancel{16})\ 20\qquad (0,2,3)$$

and

$$\hat{F} = \sum_{\nu_1\nu_2}\langle\nu_1|F|\nu_2\rangle a^\dagger_{\nu_1}a_{\nu_2}, \qquad (0.1.\cancel{17})\ 21\qquad (0,2,4)$$

*) It is of ~~notice that~~ $[H_{coll}, \Gamma^\dagger_{\lambda'\mu'}] = \hbar\omega_{\lambda'}\Gamma^\dagger_{\lambda'\mu'}$ and $[H_{sp}, a^\dagger_{\nu'}] = \varepsilon_{\nu'} a^\dagger_{\nu'}$. This is an obvious ~~result~~ outcome resulting from the bosonic ($[\Gamma_\alpha, \Gamma^\dagger_{\alpha'}] = \delta(\alpha,\alpha')$) and fermionic ($\{a_\nu, a^\dagger_{\nu'}\} = \delta(\nu,\nu')$) commutation rules ($[A,B]=AB-BA$ (commutator), $\{A,B\}=AB+BA$, anticommutator.

(a) to p. (4) manuscript

(1)

(4) a

It is of notice that

$$[H_{coll}, \Gamma^{+}_{\lambda'\mu'}] = \hbar\omega_{\lambda'} \Gamma^{+}_{\lambda'\mu'} \qquad (0.1.14)$$

and

$$[H_{sp}, a^{+}_{\nu'}] = \varepsilon_{\nu'} a^{+}_{\nu'} \qquad (0.1.15)$$

This is an obvious outcome resulting from the bosonic

$$[\Gamma_{\alpha}, \Gamma^{+}_{\alpha'}] = \delta(\alpha, \alpha') \qquad (0.1.16)$$

and fermionic

$$\{a_{\nu}, a^{+}_{\nu'}\} = \delta(\nu, \nu') \qquad (0.1.17)$$

commutation relations. (a) p. (4)

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(b) An alternative procedure to the diagramatic one to obtain the HF and RPA solutions associated with the bare NN-interaction v is provided by the relations (0.1.15) and (0.1.14) respectively, replacing the Hamiltonians by $(T+v)$, where T is the kinetic energy operator. The phonon operator associated with surface vibrations is defined through

$$\Gamma^{+}_{\alpha} = \sum_{ki} X^{\alpha}_{ki} \Gamma^{+}_{ki} + Y^{\alpha}_{ki} \Gamma_{ki}, \qquad (0.1.26)$$

and the normalization condition

The label α stands for the quantum numbers characterizing the vibrational mode

$$[\Gamma_{\alpha}, \Gamma^{+}_{\alpha}] = \sum_{ki} (X^{\alpha\,2}_{ki} - Y^{\alpha\,2}_{ki}) = 1. \qquad (0.1.27)$$

The operator $\Gamma^{+}_{ki} = a^{+}_{k} a_{i}$ $(\varepsilon_{k} > \varepsilon_{F};\ \varepsilon_{i} < \varepsilon_{F})$ creates a particle-hole excitation acting on the HF vacuum state $|0\rangle_{F}$.

It is assume that

$$[\Gamma_{ki}, \Gamma^{+}_{k'i'}] = \delta(k,k')\,\delta(i,i') \qquad (0.1.28)$$

Within this context, RPA is a harmonic, quasi-boson approximation (b) and leading to the dispersion relation (Fig 0.4.7)

0.2.3

$$\sum_{ki} \frac{2(\epsilon_k - \epsilon_i)|\langle \tilde{i}|F|k\rangle|^2}{(\epsilon_k-\epsilon_i)^2 - (\hbar\omega_\alpha)^2} = \frac{1}{\kappa},$$

and amplitudes X^{α}_{ki} and Y^{α}_{ki} display in the caption to

insert eq. in caption of here check what is better

Concerning the rules of NFT, they codify the way in which H_c and v are to be treated to all orders of perturbation theory. Also which processes (diagrams) are not allowed because they will imply overcounting of correlations already included included in the basis ~~state~~,*) (single-particle HF, collective vibrations, RPA).

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while

$$F = \frac{R_0}{\kappa}\frac{\partial U}{\partial r} Y^*_{\lambda\mu}(\hat{r}). \qquad (0.1.\cancel{18}) \; 22 \; (0.2.5)$$

0.2.1

The coupling between surface oscillation and single–particle motion, namely the particle vibration coupling (PVC) Hamiltonian δU (~~Fig. 0.1.5~~) is a consequence of the overcompleteness of the basis. Diagonalizing δU making use of the graphical (Feynman) rules of Nuclear Field Theory (NFT) to be discussed ~~below~~ in following Chapter 20/6/19, one obtains structure results which can be used in the calculation of transition probabilities and reaction cross sections, quantities which can be compared with experimental findings.

In fact, within the framework of NFT, single–particle s are to be calculated as the Hartree–Fock solution of the NN–interaction $v(|\mathbf{r}-\mathbf{r}'|)$ (Fig. 0.1.6), in particular 0.2.2

$$U(r) = \int d\mathbf{r}'\rho(r')v(|\mathbf{r}-\mathbf{r}'| \qquad (0.1.\cancel{19}) \; 23 \; (0.2.6)$$

0.2.2

being the Hartree field[2] expressing the selfconsistency between density ρ and potential U (Fig. 0.1.6 (b) (1) and (3)), while vibrations are to be calculated in the Random Phase Approximation (RPA) making use of the same interaction[3] (Fig. ~~0.1.7~~), extending the selfconsistency to fluctuations $\delta\rho$ of the density and δU of the mean field, that is,

inset

0.2.3

$$\delta U(r) = \int d\mathbf{r}'\delta\rho(r')v(|\mathbf{r}-\mathbf{r}'|. \qquad (0.1.20) \; 24 \; (0.2.7)$$

Making use of the selfconsistent solution of the relation (0.1.~~20~~ 24), one obtains the transition density $\delta\rho$. The matrix elements $\langle\nu_i|\delta\rho|\nu_k\rangle$ $\langle n_\lambda=1,\nu_i|\delta\rho|\nu_k\rangle$ provide the particle–vibration coupling strength to work out the variety of coupling processes between single–particle and collective motion (Fig. ~~0.1.5~~ 0.2.1). That is, the matrix element of the PVC Hamiltonian H_c. Diagonalizing

$$H = H_{HF} + H_{RPA} + H_c + v, \qquad (0.1.\cancel{21}) \; 25 \; (0.2.8)$$

applying ~~making use of~~ the rules of NFT, ~~to be discussed below~~, in the basis of single–particle and collective modes, that is solutions of H_{HF} and of H_{RPA} respectively, one obtains a solution of the total Hamiltonian. ¶ Because of quantal zero point fluctuations, a nucleon propagating in the nuclear medium moves through clouds of bosonic and fermionic virtual excitations to which it couples (~~H_c+v~~), becoming dressed and acquiring an effective mass, charge, etc. (Fig. ~~0.1.8~~ 0.2.4). Vice versa,

0.2.2

[2]To this potential one has to add the ~~Fock potential resulting from the fact that nucleons are~~ fermions. This exchange potential (Fig. ~~0.1.6~~ 0.2.3 (2 and 4)) is essential in the determination of single–particle energies and wavefunctions. ~~Among other things, it takes care of eliminating the nucleon self interaction from the Hartree field.~~

[3]The sum of the so called ladder diagrams (see Fig. ~~0.1.7~~) are taken into account to infinite order in RPA. This is the reason why bubble contributions in the diagonalization of Eq. (0.1.~~21~~ 25) are not allowed in NFT, being already contained in the basis states.

*) a simpler example is provided by Eq (2A-31) of Bohr and Mottelson (1969) i.e. although not directly related but only in a general way,

$$G = \frac{1}{4}\sum_{\nu_1\nu_2\nu_3\nu_4}\langle\nu_3\nu_4|G|\nu_1\nu_2\rangle_a a^+(\nu_4)a^+(\nu_3)a(\nu_1)a(\nu_2)$$

$$= \frac{1}{2}\sum_{\nu_1\nu_2\nu_3\nu_4}\langle\nu_3\nu_4|G|\nu_1\nu_2\rangle a^+(\nu_4)a^+(\nu_3)a(\nu_1)a(\nu_2)$$

where $\langle\;\rangle_a$ is the antisymmetric matrix element.

5 a

footnote p. 6

0,2,4

9 Fig. ~~01.9~~ ~~20/6/19~~

*) It is to be noted that in the case in which the renormalized vibrational modes, i.e. the initial and final wavy lines in Fig. 0.1.19 have angular momentum and parity $\lambda^{\pi}=0^{+}$, and one uses a model in which ~~assumes~~ ~~that~~ there is symmetry between the particle and hole subspaces, the four diagrams sum identically to ~~zer~~ zero, because of particle conservation

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vibrational modes can become renormalized through the coupling to dressed nucleons which, in intermediate virtual states, can exchange the ~~vibration~~al clothing with the second fermion (hole state)~~, and renormalize~~ the PVC vertex (Fig. 0.1.9) (Barranco et al. (2004)), as well as the bare NN–interaction.

From being antithetic views of the nuclear structure a proper analysis of the experimental data testifies to the fact that the collective and the independent particle picture of the nuclear structure require and support each other (Bohr, A. and Mottelson (1975)). To obtain a quantitative description of nucleon motion and nuclear phonons (vibrations), one needs a proper description of the k– and ω–dependent "dielectric" function of the nuclear medium, in a similar way in which a proper description of the reaction processes used as probes of the nuclear structure requires the use of the optical potential (continuum "dielectric" function). The NFT solution of (0.1.21) provide all the elements to calculate the ~~nuclear~~ structure properties of nuclei, and also the optical potential needed to describe nucleon–nucleus scattering. It furthermore shows that both single–particle and vibrational elementary modes of excitation emerge from the same properties of the NN–interaction.

The development of experimental techniques and associated hardware has allowed for the identification of a rich variety of elementary modes of excitation aside from collective surface vibrations and of independent particle motion: quadrupole and octupole rotational bands, giant resonance of varied multipolarity and isospin, as well as pairing vibrations and rotations together with giant pairing vibrations of transfer quantum number ±2. Modes which can be specifically excited in inelastic and Coulomb excitation processes, ~~charge exchange~~, and one– and two–particle transfer reactions.

~~One can choose to privilege one among this variety of elementary modes of excitation, for example, independent particle motion. Making use of the shell model, eventually the so called no core shell model, understood within this context as a full diagonalization of the NN–interaction in the single–particle basis, attempt at describing the whole of structure and reactions. Another possibility is to use the elementary modes of excitation basis states to describe both structure and reactions and nuclear field theory to deal with the overcompleteness and Pauli principle violations of the basis states.~~

~~From a systematic collaboration between the two approaches and of strong experimental input, it is likely that shell model calculations can help at individuating~~

~~A schematic separable force leading to surface vibrations can be written as $-\kappa\hat{F}^\dagger\hat{F}$, where $\hat{F}$ is defined in Eq. (0.1.17). The resulting collective modes can thus be viewed as correlated particle–hole ($p-h$) excitations ($a^\dagger a$), a process in which the number of nucleons (fermions) does not change. One speaks in this case of a mode with transfer quantum number $\beta = 0$. In connection with the pairing energy mentioned in relation with the inset to Fig. 0.1.1 and its connection to the theory of superconductivity, it is of notice that this theory is based on the concept of Cooper pairs, that is pairs of fermions moving in time reversal states which interact through $H_p = -G\hat{P}^\dagger\hat{P}$, where $\hat{P}^\dagger = \sum_{\nu>0} a^\dagger_\nu a^\dagger_{\bar{\nu}}$. Consequently, in this case the concept of independent particle field $\hat{F}$ has to be generalized to include $\hat{P}^\dagger$ and $\hat{P}$. The resulting collective modes, pair addition and pair substraction modes (pairing vibrations), can be viewed as correlated ($p-p$) and ($h-h$) modes, changing the number of nucleons in ±2. One the speaks of vibrations with transfer quantum number $\beta = \pm 2$.~~

of

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such a process leads to a renormalization of

vibrations which produce their

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can exchange the vibrations which produce their clothing,

Bertsch et al (1983)

(b)-(b)

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0.2.5

and reaction processes.

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The question of relating it in a direct fashion with the absolute differential cross sections (observables) remains open, in keeping with the important role played by the many-body renormalization processes and associated emergent properties.

$\beta =$

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interaction

$\hat{Q}^\dagger\hat{Q}$ $\hat{Q}$

eliminate

pairing vibrations,

$a^\dagger_\nu$

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starts here
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© 0.3 Pairing vibrations

Let us introduce this new type of elementary mode of excitation by making a parallel with quadrupole surface vibrations, within the framework of the RPA, namely

$$[(H_{sp}+H_i),\Gamma^+_\alpha]=\hbar\omega_\alpha\Gamma^+_\alpha, \qquad (0.3.1)\ \cancel{(0.1.29)}$$

where for simplicity we use, instead of v, a quadrupole-quadrupole separable interaction $(i=QQ)$ defined as

$$H_{QQ}=-\kappa\hat{Q}^+\hat{Q} \qquad (0.3.2)\ \cancel{(0.1.30)}$$

with

$$\hat{Q}^+=\sum_{k,i}\langle k|r^2Y_{2\mu}|i\rangle a^+_k a_i, \quad (\varepsilon_k>\varepsilon_F,\ \varepsilon_i<\varepsilon_F) \qquad (0.1.31)$$

while H_{sp} and Γ^+_α are defined in (0.1.12) and (0.1.26) supplemented by (0.1.27).

In connection with the pairing energy mentioned in relation with the inset of Fig. 0.1.1, it is a consequence of correlation of pairs of like nucleons moving in time reversal states. A similar phenomenon found in metals at low temperatures and giving rise to superconductivity. The pairing interaction $(i=p)$ can be written, within the approximation (0.1.30) used in the case of the quadrupole-quadrupole force as

$$H_p=-\hat{P}^+\hat{P}, \qquad (0.1.32)$$

where

$$\hat{P}^+=\sum_{\nu>0}a^+_\nu a^+_{\bar{\nu}}. \qquad (0.1.33)$$

consequently, in this case the concept of independent-particle field $\hat{Q}$ (see also (0.1.21)) associated with particle-hole (ph) excitations and carrying transfer quantum number $\beta=0$, has to be generalized to include fields describing independent pair motion, in which case $\alpha\equiv(\beta=+2, J^\pi=0^+)$

$$\Gamma^\dagger_\alpha = \sum_k X^\alpha_{kk}\Gamma^\dagger_k + \sum_i Y^\alpha_{ii}\Gamma_i \quad (0.1.\cancel{30}\ 34)$$

with

$$\Gamma^\dagger_k = a^\dagger_k a^\dagger_{\bar{k}}\ (\varepsilon_k > \varepsilon_F), \qquad \Gamma^\dagger_i = a^\dagger_i a^\dagger_{\bar{i}}\ (\varepsilon_i < \varepsilon_F), \quad (0.1.\cancel{31}\ 35)$$

and

$$\sum_k X^{\alpha\,2}_{kk} - \sum_i Y^{\alpha\,2}_{ii} = 1 \quad (0.1.\cancel{32}\ 36)$$

for the pair addition ((pp), $\beta=+2$) mode, and a similar expression for the pair removal ((hh), $\beta=-2$) mode. In Fig. ~~0.1.10~~ 0.3.1 the NFT graphical representation of the RPA equations for the pair addition mode is given. The state $\Gamma^\dagger_\alpha(\beta=+2)|\tilde{0}\rangle$, where $|\tilde{0}\rangle$ is the correlated ground of a closed shell nucleus, can be viewed as the nuclear embodiment of a Cooper pair found at the basis of the microscopic theory of superconductivity.

Fig. 0.1. prelim. nara versio

While surface vibrations are associated with the normal ($\beta=0$) nuclear density, paring vibrations are connected with the so called abnormal ($\beta=\pm2$) nuclear density (density of Cooper pairs), both static and dynamic.

Similarly to the quadrupole and octupole vibrational bands built out of n_α phonons of quantum numbers $\alpha \equiv (\beta=0, \lambda^\pi = 2^+, 3^-)$ schematically shown in Fig. 0.1.4 and experimentally observed in inelastic and Coulomb excitation and associated γ-decay processes, pairing vibrational bands build of n_α phonons of quantum numbers $\alpha \equiv (\beta = \pm 2, \lambda^\pi, 0^+, 2^+)$ have been identified around closed shells in terms of two-nucleon transfer reactions trhoghout the mass table (Fig. ~~0.1.11~~ 0.3.2). ~~Fig 16 p.324 Adv. in NP or three phonon Fig nn.~~

~~fig 3, 2016119~~

0.4 Spontaneous broken symmetry

Because empty space is homogeneous and isotropic, the nuclear Hamiltonian is translational and rotational invariant. It also conserves particle number and is thus gauge invariant. According to quantum mechanics, the corresponding wavefunctions transform in an irreducible way under the corresponding group of transformations.

When the solution of the Hamiltonian does not have some of these symmetries, for example defines a priviledged direction in space violating rotational invariance, one is confronted with the phenomenon of spontaneous broken symmetry. Strictly speaking, this can take place only

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for idealized systems ~~that~~ are infinitely large.

0.4.1 Quadrupole deformations in 3D-space

A nuclear embodiment of the sponteneous symmetry breaking phenomenon is provided by a quadrupole deformed mean field. A situation one is confronted with, when the value of the lowest quadrupole frequency ω_2 of the RPA solution (0.1.14) (see also (0.1.26) and (0.1.27)) tends to zero ($C_2 \to 0$, D_2 finite). A phenomenon resulting from the interplay of the interaction v (H_{QQ} in (0.1.29)), and of the nucleons outside closed shell, leading to tidal-like polarization of the spherical core.

Coordinate and linear momentum ((x, p_x) single-particle motion) as well as Euler angles and angular momentum ((φ, I_z) rotation in two-dimensional (2D)-space) are conjugate variables. Similarly, the gauge angle and the number of particles ((ϕ, N) rotation in gauge space), fulfill $[\phi, N] = i$. The operators $e^{-ip_x x}$, $e^{-i\varphi I_z}$ and $e^{-iN\phi}$ induce Galilean transformation and rotations in 2D- and in gauge space respectively.

New numeration

Making again use, for didactical purposes, of H_{QQ} instead of v, and calling $|N\rangle$ the eventual mean field solution of the Hamiltonian $T+H_{QQ}$, one expects

$$\langle N|\hat{Q}|N\rangle = Q_0 , \qquad (0.4.1)\ (\cancel{0.1.37})$$

where, for simplicity, we assumed axial symmetry ($\lambda=2, \mu=0$). That is, the emergence of a static quadrupole deformation.

Rewritting H_{QQ} in terms of $(\hat{Q}^{+}-Q_0+Q_0)$ and its Hermitian conjugate, one obtains

$$H = H_{sp} + H_{QQ} = H_{MF} + H_{fluct}, \qquad (0.4.2)\ (\cancel{0.1.38})$$

where

$$H_{MF} = H_{sp} - \kappa(\hat{Q}^{+}+Q), \qquad 0.4.3\ (\cancel{0.1.39})$$

is the mean field, and

$$H_{fluct} = -\kappa(\hat{Q}^{+}-Q_0)(\hat{Q}-Q_0) \qquad (0.1.40)$$

the residual interaction inducing fluctuations around Q_0.

Assuming $Q_0 \gg (\hat{Q}^{+}-Q_0)(\hat{Q}-Q_0)$, we concentrate on HMF. The original realization of it is known as the Nilsson Hamiltonian (Nilsson (1955)). It describes the motion of nucleons in a single-particle potential of radius $R=R_0(1+\beta_2 Y_{20}(\hat{r}))$, with β_2 proportional to the intrinsic quadrupole moment Q_0.

The reflection invariance and axial symmetry of the Nilsson Hamiltonian implies that parity π and projection Ω of the total angular momentum along the symmetry axis are constants of motion for the one-particle Nilsson states. These states are two-fold degenerate, since two orbits that differ only in the sign of Ω represent the same motion, appart from the clockwise and anticlockwise sense of revolution around the symmetry axis. One can thus write the Nilsson creation operators in terms of a linear combination of creation operators carrying good total angular momentum j,

$$\gamma^{+}_{a\Omega} = \sum_{j} A^{a}_{j}\, a^{+}_{aj\Omega} , \qquad (A.1.41)$$

where the label a stands for all the quantum numbers aside from Ω, which specify the orbital.

Expressed in the intrinsic, body-fixed, system of coordinates K' where the 3 (z') axis lies along the symmetry axis and the 1 and 2 (x', y') axis lie in a plane perpendicular to it, namely

$$\gamma'^{+}_{a\Omega} = \sum_{j} A^{a}_{j} \sum_{\Omega'} \mathcal{D}^{j}_{\Omega'\Omega}(\omega)\, a^{+}_{aj\Omega'} \qquad (A.1.42)$$

one can write the Nilsson state as

$$|N(\omega)\rangle_{\mathcal{K}'} = \prod_{a\Omega>0} \gamma'^{+}_{a\Omega}\, \gamma'^{+}_{a\tilde{\Omega}} |0\rangle, \quad (0.1.43)$$

where ω represent the Euler angles, $|0\rangle_F$ is the particle vacuum, and $|a\tilde{\Omega}\rangle = \gamma'^{+}_{a\tilde{\Omega}}|0\rangle$ is the state time-reversed to $|a\Omega\rangle$. For well deformed nuclei, a convenient description of the one-particle motion is based on the similarity of the nuclear potential to that of an anisotropic nuclear potential,

$$V = \frac{1}{2} M (\omega_3^2 x_3^2 + \omega_\perp^2 (x_1^2 + x_2^2)) = \frac{1}{2} M\omega_0 r^2 \left(1 - \frac{4}{3}\delta P_2(\cos\theta)\right), \quad (0.1.44)$$

with $\omega_3 \omega_\perp^2 = \omega_0^3$. That is a volume which is independent of the deformation $\delta \approx 0{,}95\,\beta_2$.

The corresponding single-particle states have energy

$$\varepsilon(n_3 n_\perp) = (n_3 + \tfrac{1}{2})\hbar\omega_3 + (n_\perp + \tfrac{1}{2})\hbar\omega_\perp, \quad (0.1.44)$$

where n_3 and $n_\perp = n_1 + n_2$ are the number of quanta along and perpendicular to the symmetry axis. The degenerate states with the same value of $n_\perp$ can be specified by the component Λ of the orbital angular momentum along the symmetry axis,

$$\Lambda = \pm n_\perp, \pm(n_\perp - 2), \ldots, \pm 1 \text{ or } 0 \qquad (0.1.45)$$

One can then label the Nilsson levels in terms of the asymptotic quantum numbers $[N n_3 \Lambda \Omega]$, where $N = n_3 + n_\perp$, is the total oscillator quantum number.

The complete expression of the Nilsson potential includes, aside from the central term discussed above, a spin-orbit and a term proportional to the the orbital angular momentum quantity squared, so as to make the shape of the oscillator to resemble more that of a Saxon-Woods potential. The resulting levels provide an overall account of the experimental findings, providing detailed evidence in terms of individual states of the interplay between the single-particle and the collective aspects of nuclear structure. (0.4.1) An example of relevance for light nuclei (N and Z<20) is given in Fig. ~~0.1.12~~ Fig. 5-1 B+M p. 221 Vol II

The Nilsson intrinsic state (0.1.43) does not have a definite angular momentum but is rather a superposition of such states,

$$|N(\omega)\rangle_{\cancel{\mathcal{K}'}\,z'} = \sum_I c_I |I\rangle, \qquad (0.1.46)$$

The symmetry axis z' defines a priviledged direction in space, thus breaking rotational invariance.