where use was made of the relations

$$\mathcal{G}(\phi) a_\nu^\dagger \mathcal{G}^{-1}(\phi) = e^{-i\phi} a_\nu^\dagger = a_\nu'^\dagger, \tag{1.4.50}$$

and

$$\mathcal{G}(\phi) P_\nu^\dagger \mathcal{G}^{-1}(\phi) = e^{-2i\phi} P_\nu^\dagger = P_\nu'^\dagger. \tag{1.4.51}$$

It is to be noted that $\mathcal{G}$ induces a counter clockwise rotation, *)

$$\mathcal{G}(\chi)\,\hat{\phi}\,\mathcal{G}^{-1}(\chi) = \hat{\phi} - \chi. \tag{1.4.52}$$

As a consequence, to rotate $|BCS\,(\phi = 0)\rangle_{\mathcal{K}'}$ back into the laboratory system, use has to be made of the clockwise rotation of angle ϕ induced by $\mathcal{G}^{-1}(\phi)$,

$$\begin{aligned}
\mathcal{G}^{-1}(\phi)\,|BCS\,(\phi = 0)\rangle_{\mathcal{K}'} &= \prod_{\nu>0}\left(U_\nu' + V_\nu' \mathcal{G}^{-1}(\phi) P_\nu'^\dagger\right)|0\rangle_F \\
&= \prod_{\nu>0}\left(U_\nu' + e^{2i\phi} V_\nu P_\nu^\dagger\right)|0\rangle_F = |BCS\,(\phi)\rangle_{\mathcal{K}}
\end{aligned} \tag{1.4.53}$$

where use was made of (1.4.49)

$$\mathcal{G}^{-1}(\phi)\left(\mathcal{G}(\phi) P_\nu^\dagger \mathcal{G}^{-1}(\phi)\right)\mathcal{G}(\phi) = \mathcal{G}^{-1}(\phi) P_\nu'^\dagger \mathcal{G}(\phi). \tag{1.4.54}$$

We note furthermore

$$|BCS\,(\phi = 0)\rangle_{\mathcal{K}'} = \prod_{\nu>0}\left(U_\nu' + V_\nu' P_\nu'^\dagger\right)|0\rangle_F = \prod_{\nu>0}\left(U_\nu + V_\nu P_\nu^\dagger\right)|0\rangle_F. \tag{1.4.55}$$

Spontaneous broken symmetry in nuclei is, as a rule associated with the presence of rotational bands, as already found in the case of qudrupole deformed nuclei. Consequently, one expects in nuclei with $\Delta \neq 0$ rotational bands in which particle number plays the role of angular momentum. That is pairing rotational bands.

In what follow we will discuss the structure of H_{fluct} and single out the term responsible for restoring gauge invariance to the BCS mean field solution giving thus, rise to pairing rotational bands. In terms of quasiparticles, H_{fluct} can be expressed as

$$H_{fluct} = H_p' + H_p'' + C \tag{1.4.56}$$

where

$$H_p' = -\frac{G}{4}\left(\sum_{\nu>0}\left(U_\nu^2 - V_\nu^2\right)\left(\Gamma_\nu^\dagger + \Gamma_\nu\right)\right)^2 \tag{1.4.57}$$

and

$$H_p'' = \frac{G}{4}\left(\sum_\nu\left(\Gamma_\nu^\dagger - \Gamma_\nu\right)\right)^2, \tag{1.4.58}$$

København 22/01/2020

p.18 ~~Bohr + Ulfbeck~~

~~eventually~~

footnote *)

(antes de Eq. (1,4,52)

p. 42

$[N, \phi] = -i$

$\mathcal{G}(\chi) = e^{-iN\chi}$ $\qquad \chi$: c-number

$$[e^{-iN\chi}, \phi] = e^{-iN\chi}\phi - \phi e^{-iN\chi}$$

$$[e^{-iN\chi}, \phi] e^{iN\chi} = e^{-iN\chi}\phi e^{iN\chi} - \phi$$

$$e^{-iN\chi} = 1 - iN\chi \qquad \chi^2 \ll \chi$$

$$[1 - iN\chi, \phi] = -i[N\chi, \phi] = -i(N[\chi,\phi] + [N,\phi]\chi)$$

$$= -i(0 - i\chi) = -\chi$$

$$[e^{-iN\chi}, \phi] = -\chi e^{-iN\chi}$$

$$[e^{-iN\chi}, \phi] e^{iN\chi} = -\chi e^{-iN\chi} e^{iN\chi} = -\chi$$

$$-\chi = \mathcal{G}(\chi)\phi\mathcal{G}^{-1}(\chi) - \phi$$

$$\mathcal{G}(\chi)\phi\mathcal{G}^{-1}(\chi) = \phi - \chi$$

Graphs (a) and (b) and (c) and (d) describe the changes in the density operator and in the single-particle potential respectively. This can be seen from the insets (I) and (II). The dashed horizontal line starting with a cross and ending at a hatched circle represents the renormalized density operator. This phenomenon is similar to that encountered in connection with vertex renormalization in Fig. 1.2.5, that is the renormalization of the particle-vibration coupling (insets (I) and (I')). Concerning potential renormalization, the bold face arrowed line shown in inset (II) of Fig. 1.8.1 represents the motion of a renormalized nucleon due to the self-energy process induced by the coupling to vibrational modes. A phenomenon which can be described at profit through an effective mass, the so called ω-mass m_ω, in which case particle motion is described by the Hamiltonian[20] $(\hbar^2/2m_\omega)\nabla^2 + \left(\frac{m}{m_\omega}\right)U(r)$. The ω-mass can be written as $m_\omega = (1+\lambda)m$, where λ is the so called mass enhancement factor $\lambda = N(0)\Lambda$, where $N(0)$ is the density of levels at the Fermi energy, and Λ the PVC vertex strength, typical values being $\lambda = 0.4$.

~~where~~ m is the nucleon mass and

The fact that in calculating $\delta\rho$, that is, the correction to the nuclear density distribution (renormalization of the density operator), one finds to the same order of perturbation a correction to the potential, is in keeping with the self consistency existing between the two quantities (Eq. (1.2.6)). Now, what changes is not only the single-particle energy, but also the single-particle content as well as the radial dependence of the wavefunctions of the states measured by $Z_\omega = m/m_\omega$. It is of notice that the effective mass approximation, although being quite useful, cannot take care of the energy dependence of the renormalization process which leads, in the case of single-particle motion to renormalized energies, spectroscopic amplitudes and wavefunctions. The analytic expressions associated with diagrams (a) and (c) of Fig. 1.8.1 are

$$\delta\rho(r)_{(a)} = \frac{(2\lambda+1)}{4\pi}\sum_{\nu_1\nu_2 n}\left[Y_n(\nu_1\nu_2;\lambda)\right]^2 R_{\nu_1}(r)R_{\nu_2}(r), \tag{1.8.7}$$

and

$$\delta\rho(r)_{(b)} = (2\lambda+1)\Lambda_n(\lambda)\sum_{\nu_1\nu_2\nu_3}\frac{M(\nu_1,\nu_3;\lambda)}{\epsilon_{\nu_1}-\epsilon_{\nu_2}}(2j_1+1)^{-1/2}$$
$$\times Y_n(\nu_3\nu_2;\lambda)\times R_{\nu_1}(r)R_{\nu_2}(r), \tag{1.8.8}$$

The functions R(r) are the radial wavefunctions associated with the states ν_1 and ν_2.

where M is the matrix element of $\frac{R_0}{\kappa}\frac{\partial U}{\partial r}Y_{\lambda\mu}(\hat{r})$ and $n = 1, 2\ldots$ the first, second, etc vibrational modes as a function of increasing energy, and Λ_n is the strength of the particle-vibration coupling associated with the n-mode of multipolarity λ. While $\delta\rho_{(a)}$ can be written in terms of the RPA Y-amplitudes which are directly associated with the zero point fluctuations of harmonic motion (Fig. 1.2.3 (c)), $\delta\rho_{(c)}$ contains a scattering vertex not found in RPA – that is going beyond the harmonic approximation– and essential to describe renormalization processes of the different degrees of freedom, namely single-particle (energy, single-particle

[20]Brink, D. and Broglia (2005) and refs. therein).

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(P) The ratio (m/m_ω) gives a measure of the single-particle energy content (Mahaux et al (1985), Eq. (3.5.18)), while m_ω is proportional to the (energy-) slope of the (single-particle) self-energy dispersion relation (Sect. 5.A.1; see also Brink and Broglia (2005) and refs. therein). (P) (50)

to p. 50 footnote 20.

content and radial dependence of the wavefunction) and collective motion, as well as interactions. In particular the pairing interaction.

In Fig. 1.8.2 we show the results of calculations of $\delta\rho$ carried out for the closed shell nucleus ^{40}Ca. The vibrations were calculated by diagonalizing separable interactions of multipolarity λ in the RPA. All the roots of multipolarity and parity $\lambda^\pi = 2^+, 3^-, 4^+$ and 5^- which exhaust the EWSR were included in the calculations. Both isoscalar and isovector degrees of freedom were included, and low-lying and giant resonances.

From the point of view of the single-particle motion the vibrations associated with low-lying modes display very low frequency ($\hbar\omega_\lambda/\epsilon_F \approx 0.1$) and lead to an ensemble of deformed shapes. Nucleons can thus reach to distances from the nuclear center which are considerably larger than the radius R of the static spherical potential. Because the frequency of the giant resonances are of similar magnitude to those corresponding to the single-particle motion, the associated surface deformations average out.

Said it differently, the low-lying vibrational modes account for most of the contributions to the changes in the density distribution[21]. Making use of the corresponding $(\delta\rho)_{low-lying}$, the mean square radius of ^{40}Ca was calculated[22], leading to $\langle r^2\rangle = (3/5)R_0^2 = 10.11$ fm^2($R_0 = 1.2A^{1/3}$ fm $= 4.1$ fm), in overall agreement with the experimental findings. Similar calculations to the ones discussed above, but in this case taking into account only the contributions of the low-lying octupole vibration[23] indicate that nucleons are to be found a reasonable part of the time in higher shells than those assigned to them by the shell model. The average number of "excited" particles being ≈ 2.4. If these are present, pickup reactions such as (p, d) and (d, t) will show them. From the nature of the correlations, the pickup of such a particle will leave a hole and a vibration. That is, the final nucleus will be in one of the states which can be related by coupling the hole and the vibration. Conversely, because of the presence of hole states in the closed shell nucleus, one can transfer a nucleon to states below the Fermi energy in, for example, (d, p) or (^{3}He,d) one-neutron or one-proton stripping reactions respectively, leaving the final nucleus with one-nucleon above closed shell coupled to the vibrations.

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Systematic studies of such multiplets have been carried out throughout the mass table. In particular around the closed shell nucleus $^{208}_{82}$Pb$_{126}$ (Fig. 1.8.3). Within this context it is not only quite natural but also necessary, to deal with structure and reactions on equal footing. This is one of the main goals of the present monograph, as will become clear already from the next chapter.

[21]Another example of the recurrent central role played by low-frequency modes in determining the properties and behavior of systems at all levels of organization, from the atomic nucleus to the Casimir effect in QED, to phonons in superconductors as well as to the folding of proteins and brain activity ($\nu < 0.1$ Hz) (Mitra et al. (2018)).

[22]Barranco and Broglia (1987).

[23]Brown and Jacob (1963).

Insert here 1.9 Interactions
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1.9 Hindsight

In order for a nucleon moving in a level close to the Fermi energy to display a mean free path larger than nuclear dimensions and to be reflected at the nuclear surface through an elastic process, all other nucleons must move in a rather ordered, correlated fashion. Within this context to posit that single-particle motion is the most collective of all nuclear motions (Mottelson (1962)), seems natural.

Associated with mean field and single-particle motion one finds the typical bunching of the associated levels closely connected with major shells lying above and below the Fermi energy. A fact which determines the parity (and angular momentum) of the lowest energies needed to promote a nucleon across the Fermi surface and, ~~which~~ profiting of the large phase space, allows for the correlation among them – through the same components of the NN-interaction leading to the single-particle bunching as testified by the self-consistent relations (1,2,6) and (1,2,7) – and thus to the existence of low-lying collective multipole vibrations of particle-hole type, interweaved with single-particles (PVC).

Similar arguments result in the presence of multipole pairing vibrations in the low-energy spectrum, and in their coupling to single-particle motion.

It is then natural to consider shell model and collective states on equal footing as basis states of a physical description of the atomic nucleus, and thus to include the three point vertex (PVC) together with the four-point vertex (v), eventually taking also 3N terms into account, in solving the nuclear many-body problem.

Such an approach also provides direct indication of the minimum set of experiments needed to obtain a "complete" picture (test) of the atomic nucleus (of the theoretical description). These are the specific probe of each of the basis states (elementary modes of excitation). Namely inelastic scattering and Coulomb excitation (particle-hole collective vibrations and rotations), one-particle transfer processes (independent-particle motion) and two-particle transfer reactions (pairing vibrations and rotations).

Summing up, because of the interweaving existing between the variety of elementary modes of excitation, experimental probes associated with fields which carry transfer quantum number $\beta = 0, \pm 1$ and ± 2, and different multipolarities are needed to characterize the structure of nuclei. This is what we attempt at explaining and formulating in the following chapters, setting special empha-sis in transfer processes, in which case the relative motion of the reacting nuclei and the intrinsic motion of the nucleons in target and projectile cannot be separated, and one is forced to treat structure and reactions in a unified way.

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Ver comentario
p. 56