One- and two-neutron halo at the dripline: from Be to Li and back

$^{11}_{4}Be_7$ one-neutron halo systems[a,b] Københvn 10/12/17

1^-	320 keV
E1	$B(E1) = 0.102 \pm 0.002$ e^2fm^2
gs	10% TRK $\approx 1\, B_W(E1)$

Parity inversion

materialised $1/2^+, 1/2^-$

$\varepsilon_{s1/2} = 0.07\,\text{MeV}$; $s1/2$, $d5/2$, 2^+, $s1/2$ $(-570\,\text{keV})$ $\quad \tilde{\varepsilon}_{s1/2} = (0.070 - 0.570)\,\text{MeV} = -0.500\,\text{MeV}$

$\varepsilon_{p1/2} = -3.04\,\text{MeV}$; $p1/2$, 2^+, $p1/2$, $p3/2$ $(+2.86)$ $\quad \tilde{\varepsilon}_{p1/2} = (-3.04 + 2.86)\,\text{MeV} = -0.180\,\text{MeV}$

closed call

$\varepsilon_{d5/2} = 7.30\,\text{MeV}$ $\quad d5/2$, $s1/2$, 2^+, $d5/2$ $(-1.77\,\text{MeV})$ $\quad d5/2$, $d5/2$, 2^+, $d5/2$ $(-4.08\,\text{MeV})$

$\tilde{\varepsilon}_{d5/2} \approx (7.30 - 1.77 - 4.08)\,\text{MeV} = 1.45\,\text{MeV}$

i	$\tilde{\varepsilon}_i$ (MeV) Theory a)	Exp.
$\tilde{s}_{1/2}$	-0.5	-0.5
$\tilde{p}_{1/2}$	-0.180	-0.180
$\tilde{d}_{5/2}$	1.45	1.28

a) Barranco et al PRL <u>119</u>, 082501 (2017)

b) Barranco et al PL Eur. Phys. J A 11, 385 (2001) (see p. ②)

$\tilde{d}_{5/2}$ 3×3 matrix representation

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$^{10}_{3}Li_7$

ℓj	$\varepsilon_{\ell j}$ (MeV)
$d5/2$	3,5
$s1/2$	1,5
$p1/2$	−1,2
$p3/2$	−4,7

WS potential $\quad a = 0{,}65\,\text{fm} \quad R_0 = 1{,}2\,A^{1/3}$

$$U(r) = U f(r) \qquad f(r) = \frac{1}{1+\exp\left(\frac{r-R_0}{a}\right)}$$

$$U = U_0 + 0.4E\ ;\ U_0 = V_0 + 30\,\frac{N-Z}{A}\,\text{MeV}\ ;\ V_0 = -51\,\text{MeV}$$

0.4 E term taken care of by k-mass.

<u>core</u> $\ ^{9}_{3}Li_6$ $\qquad U_0 = \left(-51 + 30\,\frac{6-3}{9}\right)\text{MeV} = -41\,\text{MeV}$

$$\left\langle R_0 \frac{\partial U}{\partial r}\right\rangle \approx 1.44 \times U_0 \approx -60\,\text{MeV}$$

<u>Input</u>

$^{8}_{2}He_6(p_{3/2}(\pi), {}^{9}Li$ spectator) $\quad \boxed{\hbar\omega_{2^+} \approx 3.3\,\text{MeV}\ ;\ \beta_2 = 0.66}$

core $\quad 1/2^- \quad 2.69$ MeV

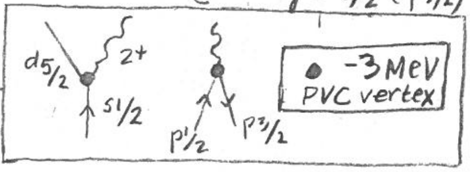
$\boxed{m_k = (1+0.4\times\theta)^{-1} m \approx 0.93\,m}$

Making use of $R_{eff}(^{11}Li) \approx 4.8$ fm (^{11}Li, p.②) and of $R_0 = 1.2\,A^{1/3}$fm, $R_0 = 2.7$fm ($A=11$) one can estimate

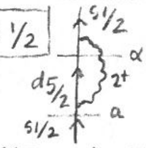
$$\theta = \left(\frac{2.7}{4.8}\right)^3 \approx 0.2$$

$$\langle H_c \rangle = \frac{\beta_2}{\sqrt{5}} \left\langle R_0 \frac{\partial U}{\partial r} \right\rangle \theta \langle j \| Y_2 \| 1/2 \rangle\ ;\ \langle j \| Y_2 \| 1/2 \rangle \approx \sqrt{\frac{2j+1}{4\pi}} = \begin{cases} 0.7 & j = 5/2\ (d5/2) \\ 0.6 & j = 3/2\ (p3/2) \end{cases}$$

$$v = \langle H_c \rangle \approx \frac{0.7}{\sqrt{5}}(-60\,\text{MeV}) \times 0.2 \times 0.7 \approx -3\,\text{MeV}$$



$\boxed{s\,1/2}$



$$M^{(2+)}_{s1/2} = \frac{v^2}{\varepsilon_{s1/2} - (\varepsilon_{d5/2} + \hbar\omega_{2+})} = \frac{9\,\text{MeV}^2}{(1.5-(3.5+3.3))\,\text{MeV}}$$

$$= -\frac{9\,\text{MeV}}{5.3} = -1.7\,\text{MeV}\ ;\ \tilde{\varepsilon}_{s1/2} = \varepsilon_{s1/2} + M^{(2+)}_{s1/2} = (1.5-1.7)\,\text{MeV} = -0.2\,\text{MeV (perturb.)}$$

$E_a = \varepsilon_{s1/2} = +1.5$ MeV ; $E_\alpha = \varepsilon_{d5/2} + \hbar\omega_{2^+} = 3.5 + 3.3 = 6.8$ MeV

$$\begin{pmatrix} (E_a-\lambda) & v \\ v & (E_\alpha-\lambda) \end{pmatrix} = \begin{pmatrix} (1.5-\lambda) & -3 \\ -3 & (6.8-\lambda) \end{pmatrix} = (1.5-\lambda)(6.8-\lambda) - 9 = 0\ ;\ \lambda^2 - 8.3\lambda + (1.5\times 6.8 - 9) = \lambda^2 - 8.3\lambda + 1.2 = 0$$

$$\lambda = \frac{8.3 \pm \sqrt{(8.3)^2 - 4.8}}{2} = \frac{8.3 \pm 8.0}{2} = \{\ 0.15\,\text{MeV lowest root}$$

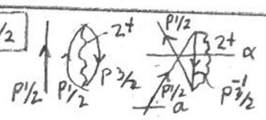
$$\boxed{\tilde{\varepsilon}_{s1/2} = E_{s1/2}(1) = 0.15\,\text{MeV}}$$

$$C^2_{s1/2}(1) = \left(1 + \frac{v^2}{(E_\alpha - E_1)^2}\right)^{-1} = \left(1 + \frac{9\,\text{MeV}^2}{(6.8-0.15)\,\text{MeV}^2}\right)^{-1} = (1+0.20)^{-1} = 0.83\ ;$$

$$\boxed{|\widetilde{1/2^+}; 0.15\,\text{MeV}\rangle = 0.91\,|s1/2\rangle + 0.41\,|(d_{5/2}\otimes 2^+); 1/2^+\rangle}$$

$m_\omega = 1.20\,m\ ;\ \lambda = 0.20\ ;\ Z_\omega = 0.83$

$\boxed{p\,1/2}$



$$M^{(2+)}_{p1/2} = \frac{(-1)^1(-3\,\text{MeV})^2}{\varepsilon_{p1/2} - ((2\varepsilon_{p1/2} - \varepsilon_{p3/2}) + \hbar\omega_{2^+}]} = -\frac{9\,\text{MeV}^2}{-[(\varepsilon_{p1/2} - \varepsilon_{p3/2}) + \hbar\omega_{2^+}]} = +\frac{9\,\text{MeV}}{[(-1.2-(-4.7))+3.3]} = \frac{9}{6.8} = 1.3\,\text{MeV}$$

$(-1)^1$: line crossing $\quad \tilde{\varepsilon}_{p1/2} = \varepsilon_{p1/2} + M^{(2+)}_{p1/2} = -1.2\,\text{MeV} + 1.3\,\text{MeV} = 0.1\,\text{MeV}$ (perturb.)

$$E_a = \varepsilon_{p1/2} = -1.2\,\text{MeV}\ ;\ E_\alpha = 2\varepsilon_{p1/2} - \varepsilon_{p3/2} + \hbar\omega_{2^+} = (-2.4-(-4.7)+3.3)\,\text{MeV} = 5.6\,\text{MeV}$$

$$\begin{pmatrix} (-1.2-\lambda) & -3i \\ -3i & (5.6-\lambda) \end{pmatrix} = (-1.2-\lambda)(5.6-\lambda) + 9 = (-1.2\times 5.6) + \lambda(1.2-5.6) + 9 + \lambda^2 = \lambda^2 - 4.4\lambda + 2.28 = 0$$

i: imaginary unit $\ i^2 = -1$, to that care of Pauli principle (line crossing).

$$\lambda^2 - 4.4\lambda + 2.28 = 0\ ;\ \lambda = \frac{4.4 \pm \sqrt{(4.4)^2 - 4\times 2.28}}{2} = \frac{4.4 \pm 3.2}{2} = \{\ 0.6\,\text{MeV lowest root}$$

$$\boxed{\tilde{\varepsilon}_{p1/2} = E_{p1/2}(1) = 0.60\,\text{MeV}}$$

$$C^2_{p1/2}(1) = \left(1 + \frac{(-3\,\text{MeV})^2}{(5.6-0.6)^2}\right)^{-1} = (1+0.36)^{-1} = 0.74\ ;\ C_{p1/2}(1) = 0.86$$

$$|\widetilde{1/2^-; 0.6\,\text{MeV}}\rangle = 0.86\,|p1/2\rangle + 0.51\,|((p1/2, p^{-1}_{3/2})_{2^+} \otimes 2^+)_{0^+}\,p1/2; 1/2^-\rangle\ ;\ m_\omega = (1.36)m\ ;\ \lambda = 0.36\ ;\ Z_\omega = 0.74$$

i	$\tilde{\varepsilon}_i$ (MeV) Theory	$\tilde{\varepsilon}_i$ (MeV) Exp[a,b]
$\widetilde{s1/2}$	0.15	0.1−0.25
$\widetilde{p1/2}$	0.60	0.5−0.6

a) Zinser et al PRL <u>75</u>, 1719 (1995)

b) See however Cavallaro et al PRL <u>118</u>, 012701 (2017).

$\boxed{{}^{11}_{3}Li_{8}}$ $\quad \xi = \frac{\hbar v_F}{\pi \Delta} = \frac{(\hbar c)(v_F/c)}{\pi \Delta} \approx 14\ fm;\ (v_F/c) \approx 0.3;\ \Delta \approx 1.4\ MeV\ ({}^{120}Sn)$

Single halo Cooper pair: ansatz $\Delta \to |E_{corr}|$ $(v_F/c) \to \frac{1}{2}\left(\frac{v_F}{c}\right) \approx 0.15$

Input $\boxed{E_{corr} \approx -0.5\ MeV}$ $\quad \xi = \frac{200\ fm\ MeV \times 0.15}{\pi\ 0.5\ MeV} \approx 20\ fm\ \ (\xi/2 = 10\ fm)$

$R_0 = 1.2 A^{1/3} fm;\ R_0 = 2.5\ fm\ ({}^{9}Li);\ R_0 = 2.7\ fm\ ({}^{11}Li),\ R_{eff}({}^{11}Li) = \left((2.5)^2 \frac{9}{11} + \left(\frac{\xi}{2}\right)^2 \frac{2}{11}\right)^{1/2} \approx 4.8\ fm$

$\langle r^2 \rangle^{1/2} = \sqrt{\frac{3}{5}}\, R_{eff} \approx 3.7\ fm$ to be compared with $\langle r^2 \rangle^{1/2}_{exp} \approx 3.55 \pm 0.1\ fm.$

$\widetilde{|1/2^+; 0.15\ MeV\rangle}, \widetilde{|1/2^-; 0.60\ MeV\rangle}$ is the scenario of soft E1-modes.

Making use of the ${}^{11}Be$ result, namely the E1-transition between parity inverted states*) $B(E1) = 0.102 \pm 0.002\ e^2 fm^2$ as compared to**) $(TRK/\hbar\omega_D) \approx 1 e^2 fm^2$. Thus, one can assume that a soft E1-mode in ${}^{11}Li$ will also carry $\approx 10\%$ TRK sum rule.

Pygmy RPA $\quad W(E) = \sum_{ki} \frac{2(\varepsilon_k - \varepsilon_i)|\langle \tilde{i}|F|k\rangle|^2}{(\varepsilon_k - \varepsilon_i)^2 - E^2} = \frac{1}{K_1}$; $K_1 = -\frac{5 V_1}{A R^2}$; $V_1 = 25\ MeV$

$(\tilde{\varepsilon}_{p_{1/2}} - \tilde{\varepsilon}_{s_{1/2}})^2 - \hbar\omega_{pygmy} = K_1 \times 2 \times 10\%\ TRK$ $\quad TRK = \frac{3\hbar^2}{2m}\frac{NZ}{A} \approx \frac{3 \times 20 \times 3 \times 8}{11} MeV fm^2 = 131\ MeV fm^2$ (see footnote 4*)

$K_1 = -\frac{5 \times 25\ MeV}{11\ (4.8)^2 fm^2} \approx -0.03\ MeV\ fm^{-2}$

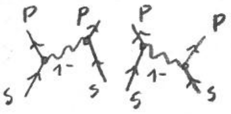
$\hbar\omega_{pygmy} = \left((0.6 - 0.15)^2 MeV^2 - (-0.03\ MeV fm^{-2}) \times 2 \times 0.1 \times 131\ MeV fm^2)\right) \approx ((0.45)^2 + (0.9)^2) MeV \approx 1\ MeV.$

Let us now calculate the PVC strength of this mode with the nucleons.

$\Lambda^2 = \left(\partial W(E)/\partial E \Big|_{\hbar\omega_{pygmy}}\right)^{-1};\ \Lambda^2 = \left\{ 2\hbar\omega_{pygmy} \frac{2 \times 0.1\ TRK/R_{eff}^2}{[(\tilde{\varepsilon}_{p_{1/2}} - \tilde{\varepsilon}_{s_{1/2}})^2 - (\hbar\omega_{pygmy})^2]} \right\}^{-1}$

$\Lambda^2 = \left\{ 2 \times 1\ MeV \frac{2 \times 0.1 \times 131\ MeV fm^2/(4.8\ fm)^2}{[(0.45)^2 - (1\ MeV)^2]^2 MeV^4} \right\}^{-1} = \left(\frac{2.3}{0.64\ MeV^2}\right)^{-1}$

$\Lambda^2 = 0.28\ MeV^2\ (\Lambda = 0.53\ MeV);\ M_{ind} = -\frac{2 \times \Lambda^2}{\hbar\omega_{pymy}} = 0.6\ MeV$



Let us now calculate the bare pairing interaction***)

$(G)_{scr} = \frac{(2j+1)_{halo}}{(2j+1)_{core}} \left(\frac{R_0}{R_{eff}}\right)^3 G \approx \frac{2}{8}\left(\frac{2.7}{4.8}\right)^3 \frac{25}{A} MeV \approx \frac{1\ MeV}{A};\ (G)_{scr} = 0.1\ MeV$

$E_{corr} = 2\tilde{\varepsilon}_{s_{1/2}} - (G)_{scr} + M_{ind} \approx (0.3 - 0.1 - 0.6)\ MeV \approx -0.4\ MeV.$

(MeV)	Th	Exp
E_{corr}	-0.4	-0.38
$\hbar\omega_{pygmy}$	1.0	1.0

4*) Associated with the operator $F(r_k) = e\left[\frac{N-Z}{A} - t_z(k)\right] r_k$; no spher. harm. $Y_{1M}(\hat{r}_k)$.

***) The screening factor is the ratio between the matrix element of a δ-force in a $j^2(0)$ core and halo configurations, j being a single j-shell representation of the phase space available for the pair to correlate. In the halo case $j = 1/2$. In the case of the core one can estimate it as $k_F R_0 \approx 1.36\ fm^{-1} \times 2.7\ fm \approx 3.7$ and thus $2j+1 \approx 8$.

**) The discrete E1-transition $1/2^-(-0.18\ MeV) \to 1/2^+(-0.5\ MeV)$ can be viewed as the barely bound analogue to the E1-soft mode (giant dipole pygmy resonance, GDPR) of ${}^{11}Li$.

*) $TRK = \frac{9}{4\pi}\frac{\hbar^2 e^2}{2m}\frac{NZ}{A} \approx 14.8 \frac{NZ}{A} e^2 fm^2 MeV \approx 37.7\ e^2 fm^2 MeV\ ({}^{11}_{4}Be_7)$, $\hbar\omega_D = 80\ MeV/(11)^{1/3} \approx 36\ MeV$

$TRK/\hbar\omega_D \approx 1 e^2 fm^2$.

$^{12}_{4}Be_{8}$