Appendix 3.B Cooper pair: radial dependence Kblum 7/5/96 1

A – A p. 1a

The problem that Cooper solved was that of a pair of electrons which interact above a quiescent Fermi sphere with an interaction of the kind ~~that~~ might be expected due to phonon and screened Coulomb field.[1] What he showed approximating this retarded interaction by a non-local one, active on a thin energy shell near (above) the Fermi surface[2], was that the resulting spectrum has an eigenvalue $E = 2\epsilon_F - 2\Delta$, regardless how weak the interaction is (and as a consequence the binding energy 2Δ of the pair), so long as the interaction is attractive. This result is a consequence of the Fermi statistic and of the existence of a Fermi sea background – the two electrons interact with each other but not with those in the sea, except via the Pauli principle – since it is well known ~~that~~ binding does not ordinarily occur in the two-body problem in three dimensions, until the strength of the attraction exceeds a finite threshold value.

The wavefunction of the two electrons can be written as

$$\psi(\vec{r}_1,\vec{r}_2) = \phi_q(r)\, e^{i\vec{q}\cdot\vec{R}}\, \chi(\sigma_1,\sigma_2) \quad (1)$$

where $\vec{R} = (\vec{r}_1+\vec{r}_2)/2$, $\vec{r} = \vec{r}_1 - \vec{r}_2$, and σ_1 and σ_2 denote the spins.[3]

---

1) Cooper (1956)

2) States below the Fermi surface are frozen because of Pauli principle.

3) b – b p. 1b

-1. (A) København, 8/05/16 (1)a

The fact that one is still trying to understand (BCS-like) pairing (abnormal density phenomena) in nuclei is, to a non negligible extent, due to the fact that, as a rule, pairing (in these systems) is constrained to manifest itself subject to a very strong "external" (mean, normal density) field (A). Also, to same extent, due to the fact that the analysis of two-nucleon transfer data was made in terms of relative cross sections and not absolute ones as done now (B). Within this context, Cooper pair transfer was viewed as simultaneous transfer, successive implying a breaking or, at least an anti-pairing disturbance of the pair.

There exist a number of evidences which testify to the fact that the picture in which nucleon Cooper pairs are viewed as independent correlated entities over distances of the order of tens of fm, contains a number of correct elements (0) (Fig. 3.2.1).

In this appendix an attempt at summarizing these evidences, already mentioned or partially discussed, in the first three chapters is attempted.

(A)

p. (1)

(0) Of course such manifestation will be latent, expressing itself indirectly. In other words, abnormal density can only be present when normal density, at ever so low values already is. The pairing field does not have within this context an existence by itself uncoupled from the normal density. On the other hand this, in most cases latent (more than virtual), and in only few cases factual existence, has important consequence on nuclear properties.

A) cf. e.g Matsuo (2013) and references therein ← 50 Years WS

B) See Potel et al (2013) and references therein ← review

(b) Kblvm 8/5/16 (1)b

In the limit $q \to 0$ the relative coordinate problem is spherically symmetric so that $\phi_0(\vec{r})$ is an eigenfunction of angular momentum and can be labeled by the angular momentum quantum numbers l and m. (Schrieffer (1964), p. 28). (b)

to p. (1)

(c) In other words, one expands the $l=0$ wavefunction ϕ_0 in terms of S-states of relative momentum k and total momentum zero. (c)

to p. (2)

København 7/5/16

(2)

One expects the lowest state to have zero total momentum, so that the two electrons must have equal and opposite momenta. Anticipating an attractive interaction one can also expect ~~that~~ it is the singlet state, with

$$\chi = \frac{1}{\sqrt{2}}\left[\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} - \begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}\right], \quad (2)$$

to have lower energy, in keeping with the fact that this gives a larger probability amplitude for the symmetric orbital wavefunction describing the relative motion of the two electrons to allow them to be near to each other. We have thus a pair of electrons moving in time reversal states and can write, 4)

$$\varphi_0(\vec{r}) = \sum_{k>k_F} g(\vec{k})\, e^{i\vec{k}.\vec{r}} . \quad (3)$$

In the above wavefunction Pauli principle ($k>k_F$) and translational invariance (dependence on the relative coordinate $\vec{r}$) are apparent.

from p. 3 ⓐ-ⓐ the wavefunction of a Cooper pair represents a bound s-state, the motion it describes is a periodic back and forth movement of the two electrons in directions which are uniformly distributed, covering a relative distance ($\delta x \delta p = \hbar$)

$$\xi = \delta x = \frac{\hbar}{\delta p} = \frac{\hbar v_F}{2\Delta}, \quad (6)$$

as schematically shown in Fig. ~~1~~(i) 3.B.1(i)

4) ⓒ-ⓒ from p. 1b

(a) The pair wavefunction is likely a superposition of one-electron levels with energies of the order of 2Δ close to E_F, since tunneling experiments indicate that for higher energies the one-electron density is little altered from the form it has in normal metals. The spread in momenta of the single-electron levels entering (3) is thus fixed by the condition

$$2\Delta \approx \delta E \approx \delta\left(\frac{p^2}{2m}\right)_{E_F} \approx v_F\, \delta p \,. \quad (4)$$

Consequently

$$\frac{\delta p}{p_F} = \frac{2\Delta}{m v_F^2} = \frac{\Delta}{\epsilon_F} \ll 1 \,. \quad (5)$$

Thus, $\phi_0(\vec{r})$ consists mainly of waves of wavenumber k_F. Now, because (a)

to p (2)

It is analogous to the motion of the two nucleons in a deuteron. The hydrogen atom in s-state is also an example; in that case it is the electron ~~that~~ does most of the back and forth moving, whereas the proton only recoils slightly.

In keeping with the above arguments and with (5), $\phi_0(r)$ will look like $e^{ik_F \cdot r}$ for $r \ll \xi$, while for $r \gg \xi$ the waves $e^{i\vec{k}_i \cdot \vec{r}}$ weighted by $g(k)$ will destroy temselve by interference. This is in keeping with the fact that ξ is the relative distance between Cooper pair partners where the different k-waves contributing to $\phi_0(\vec{r})$ start interfering" (Fig. 2). (Fig. 3.B.2)

references quoted in the caption to ~~the~~ Fig. ~~1~~ 3.B.1 (i)

V. F. Weisskopf, "The formation of Superconducting Pairs and the Nature of superconducting currents", Contemporary Physics, 22, 375 (1981);
A. M. Kadin "Spatial Structure of Cooper Pairs", J. Supercond. and Novel Magn. 20:258 (2007),

From the above physical arguments, $\Phi_0(\vec{r})$ will look like $e^{i\vec{k}_F \cdot \vec{r}}$ for $r \ll \xi$, while for $r \gtrsim \xi$ one can approximate the weighting function as,

$$g(k) \sim \delta\left(k, \vec{k}_F + i\frac{\hat{k}}{\xi}\right), \quad (7)$$

resulting in

$$\Phi_0(\vec{r}) \sim e^{-r/\xi} e^{ik_F r}, \quad (8)$$

Because we are dealing with a singlet state, and the total wavefunction has to be antisymmetric,

$$\Phi_0(r) \sim e^{-r/\xi} \cos k_F r, \quad (9)$$

giving maximum probability amplitude for the electrons close to each other.

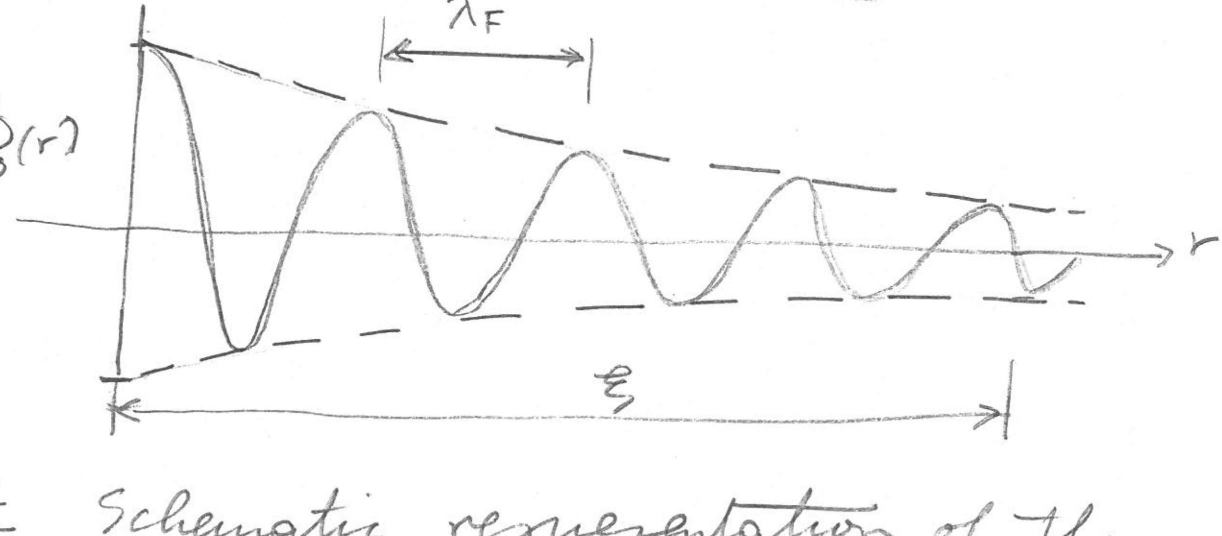


Fig. 2 Schematic representation of the cooper pair wavefunction. Indicated are the coherence length and the Fermi wavelength $\lambda_F = h/p_F = 2\pi/k_F$. In the nuclear case $\lambda_F \approx 4.6$ fm and $\xi \approx \hbar v_F/2\Delta \approx 30$ fm ($v_F/c = 0.3$, $\Delta = 1$ MeV). Thus $\xi/\lambda_F \approx 7$

A more proper solution of the Cooper pair problem leads to 5)

$$\Phi_0(r) \sim K_0(r/\pi\xi)\cos k_F r, \quad (10)$$

where K_0 is the zeroth-order modified Bessel function. For $x \gg 0$, $K_0(x) \sim (\pi/2x)^{1/2}\exp(-x)$, where $x = r/\pi\xi$.

e-e ⟶ from p. 6a

Number of overlapping pairs

The coherence length for low-temperature superconductors is of the order of 10^4 Å. In fact, in the case of e.g. Pb, for which 6) $\Delta = 0.62$ meV and $v_F = 1.83 \times 10^8$ cm/s one obtains $\xi \approx 10^{-4}$ cm, where use of $c = 3\times 10^{10}$ cm/s and $\hbar c \approx 2\times 10^3$ Å eV has been made.

Since electrons in metals typically occupy a volume of the order of $(2\text{Å})^3$ (Wigner-Seitz cell), there would be of the order of (β) $\xi^3/(2\text{Å})^3 \approx 10^{11}$ other electrons within a "coherence volume". Eliminating the electrons deep within the Fermi sea as they behave essentially as if the metal was in the normal phase, one gets (α) $\approx 10^6$. In other words, about a million of other Cooper pairs have their center ~~center~~ of mass falling inside the coherence volume of a pair. Thus, the isolated pair picture is not correct.

5) Kadin (2007) (see refs footnote p. 4)

6) The standard quoted value is $\Delta_0 = 7.19$ K. Making use of the conversion factor $1\text{K} \to 8.6217 \times 10^{-5}$ eV one obtains 0.62 meV.

α Schrieffer (1964) p. 43

β Ketterson and Song (1999) p. 198

(e) Kbhvn 8/6/16 (6a)

A wavefunction which extends over distances much larger than the binding potential is a well-known phenomenon when the binding energy is small. For example, in the case of the deuteron. In any case, it is of notice that here we are discussing a rather subtle phenomenon, pairing or better Cooper pairing, which has to express itself in the presence of a very strong "external" field. Unles one does not relate the NN interaction binding the deuteron to proton-neutron pairing.

Be as it may, the large size of the Cooper pair wave function also explains why the electrostatic repulsion between electron pair does not appreciably influence the binding. The repulsion acts only over distances of the order of the Debye length, in keeping with the fact that one has to do with a screened Coulomb field.

Within the nuclear scenario, ~~pairs to~~ to interact at profit through long wavelength medium polarization pairing, pairs of nucleons have to have low momentum. To do so they have to reduce the effect of the strong external (mean) field by moving away from it, possible mechanisms being, among other: halo (Fig. 3.B.1), transfer processes (see e.g. Fig. 3.4.1), exotic decay (E) ~~(see Fig. 3.B.3)~~ (see Fig. 3.B.3)

(E) In Fig. 3.B.3, a parallel is made between correlation lengths between pairing particle-particle modes and particle-hole vibrations, (See also Tables 3.B.1 and 3.B.2) modes which also display a consistent spatial correlation (see e.g. Broglia et al (1971))

(R.A. Broglia, C. Riedel and T. Udagawa, Coherence properties of two-neutron transfer reactions and their relation to inelastic scattering, Nucl. Phys. A169, 225 (1971)).

(e) to p. 6

F)

Kbhvn 17/05/16

7a

Making use of the harmonic oscillator, one can write $\Omega = \frac{1}{2}(N+1)(N+2) \sim A^{2/3}$, where the proportionality constant has a value between 1/2 and 2/3.

to p. 7

Kbhvn 7/5/16. (7)

In the nuclear case, the number of Cooper pairs participating in the condensate is

$$\alpha_0 = \langle BCS | P^{\dagger} | BCS \rangle = \sum_j \frac{(2j+1)}{2} U_j' V_j' \qquad (11)$$

A simple estimate of this number can be made with the help of the single j-shell model, in which case $V_j = (N/2\Omega)^{1/2}$ and $U_j = (1 - N/2\Omega)^{1/2}$, where $\Omega = (2j+1)/2$. For a half-filled shell ($N = \Omega$) one obtains (F) — see p. (7)$_a$ $\alpha_0' = \Omega/2$. ~~With the help of the approximate expression (harmonic oscillator), $\Omega \approx (2/3) A^{2/3}$, one obtains~~ In the case of ^{120}Sn, $\alpha_0' = 6-8$. In keeping with the fact that $\xi > R_0$, in the nuclear case one has a complete overlap between all Cooper pairs participating in the condensate. This, together with the fact that the nuclear Cooper pairs press against the nuclear surface in an attempt to expand and are forced to bounce elastically off from it, receive strong circumstantial evidence from the following experimental results: 1) while the moment of inertia of rotational bands is $\mathcal{I}_r/2$, it is 5 $\mathcal{I}_{rirrot}$. In other words, while pairing in nuclei is important its role is only partially exhausted, and certainly strongly distorted. (f) - (f) from p. (8) Within the above context, exotic halo nuclei open new possibilities to understand the physics at the basis of pairing in nuclei (Fig. 3)

(Bohr and Mottelson (1975))

2) One and two nucleon transfer reactions in pairing correlated nuclei have the same order of magnitude. For example

$\sigma(^{120}Sn(p,d)\,^{119}Sn(5/2^+;\,1.09\,MeV)) = 5.35\,mb$

$(2° < \theta_{cm} < 55°)$, while $\sigma(^{120}Sn(p,t)\,^{118}Sn(gs)) = 2.25\,mb$ $(7.6° < \theta_{cm} < 69.7°)$. In this last reaction Cooper pair partners can be as far as 12-13 fm. In the case of a heavy ion reaction this distance becomes almost double (Fig. ~~8~~) 3.4.1

3) The decay constant of the exotic decay

$^{223}_{88}Ra_{135} \to \,^{14}_{6}C_{8} + \,^{209}_{82}Pb_{127}$ has been measured

to be $\lambda_{exp} = 4.3\times10^{-16}\,sec^{-1}$. For theoretical purposes it can be written as $\lambda = PfT$, product of the formation probability P of ^{14}C in ^{223}Ra (saddle configuration, see Fig. ~~B~~ 3.B.3), the knocking rate f and the tunneling probability T. These two last quantities hardly depend on pairing. On the other hand P changes from $\approx 2\times10^{-76}$ to 2.3×10^{-10}, and consequently the associated lifetimes from 10^{75}y to the observed value of 10^{8}y by allowing Cooper pairs to be correlated over distances which can be as large as 20 fm. ~~(Fig. 3)~~.

(h) ~~This is in keeping with the fact that the actual rotation patern is a factor of 2 different from rigid ($\mathcal{J}_r$, no pairing), but a much larger factor of 5 from full superfluidity ($\mathcal{J}_{irrot}$, irrotational motion).~~

p. 7 (h)

Fussion below barrier

Kbhvn 8/5/16

$\boxed{\alpha_0}$ special role ~~definite der gauge geom.~~

Tunneling probabilities

In general, the coefficients U_ν, V_ν entering the BCS wavefunction $\prod_{\nu>0}(U_\nu + V_\nu a^\dagger_\nu a^\dagger_{\bar\nu})|0\rangle$ are complex. Let us employ the standard phasing $U_\nu = U'_\nu e^{i\phi}$, $V_\nu = V'_\nu e^{-i\phi}$, where U'_ν and V'_ν are real, and define the state,

$$
\begin{aligned}
|BCS(\phi)\rangle_{\mathcal{K}} &= \mathcal{G}(\phi)\prod_{\nu>0}(U_\nu + V_\nu a^\dagger_\nu a^\dagger_{\bar\nu})|0\rangle \\
&= e^{\frac{iN}{2}\phi}\prod_{\nu>0}(U'_\nu + V'_\nu e^{-2i\phi} a^\dagger_\nu a^\dagger_{\bar\nu})|0\rangle \quad (12) \\
&= e^{i\frac{N}{2}\phi}\prod_{\nu>0}(U'_\nu + V'_\nu a'^\dagger_\nu a'^\dagger_{\bar\nu})|0\rangle = |BCS(\phi)\rangle_{\mathcal{K}'}
\end{aligned}
$$

where use has been made of the gauge transformation $a'^\dagger_\nu = \mathcal{G}(\phi)\, a^\dagger_\nu\, \mathcal{G}^{-1}(\phi) = e^{-i\phi} a^\dagger_\nu$, $\mathcal{G}(\phi) = e^{-iN\phi}$ inducing a rotation in gauge space. The labels $\mathcal{K}$ and $\mathcal{K}'$ indicate the laboratory and the body-fixed reference frame respectively.

The state (12) displays off-diagonal-long-range-order (ODLRO), because each pair is in a state $(U'_\nu + V'_\nu e^{-2i\phi} a^\dagger_\nu a^\dagger_{\bar\nu})|0\rangle$ with the same phase as all the others. In fact, the wavefunction (12) leads to a two-particle density matrix with the property $\lim_{\vec r_1,\vec r_2;\vec r_3,\vec r_4\to\infty}\Phi(\vec r_1,\vec r_2;\vec r_3,\vec r_4)\neq 0$, under the assumption ~~that~~ $r_{12}, r_{34} < \xi$, $(\vec r_1,\vec r_2)$ and $(\vec r_3,\vec r_4)$ being the coordinates of a Cooper pair, r_{ij} the relative modulus of it and ξ the coherence length (7) (see e.g. Ambegaokar (1969) and refs. therein).

(7)

V. Ambegaokar, The Green's function method, in superconductivity, ed. R.D. Parks, Vol I, Marcel Dekker, New York, p. 259 (1969).

~~Bruno~~ Klichum 8/5/16 (11)

Let us bring this structure result into reaction. The fact that the wavefunction of the nucleons in the pair are phase-coherent ($(U'_\nu + V'_\nu e^{-2i\phi} a^\dagger_\nu a^\dagger_{\bar\nu})|0\rangle$) implies that to calculate the probability of two-nucleon transfer, one has to add the amplitudes of one-nucleon transfer before taking modulus squared, that is,

$$P_2 = \lim_{\epsilon\to 0} \left|\frac{1}{\sqrt{2}}\left(e^{i\phi'}\sqrt{P_1} + e^{i\phi}\sqrt{P_1}\right)\right|^2 \quad (13) \quad (\epsilon = \phi - \phi')$$

$$= P_1 \lim_{\epsilon\to 0} (1 + \cos\epsilon) = P_1 . \quad (13)$$

(see Fig. 3.B.4) (Table Eliashberg-Migdal)

In keeping with the parallel made with superconductors one can mention that Josephson showed that at very low temperatures, the pair current is equal to the single-particle current at an equivalent voltage (8) $\pi\Delta/2e$. How conclusive this result is concerning the mechanism at the basis of Cooper pair transfer is connected with the fact ~~that~~ the probability of one-electron-tunneling across a typical dioxide layer giving rise to a weak S-S coupling is 10^{-10}. Consequently, simultaneous pair transfer between two superconductors (S), with a probability $(10^{-10})^2$ cannot be observed (γ).

(γ) See e.g. Mc Donald (2001) D. McDonal The Nobel Laureate versus Physics Today July 2001 p. 46

One could argue that in the reaction $^{120}Sn(p,t)^{118}Sn(gs)$ one can hardly consider the triton as a condensate. While this is correct one can hardly claim either that 6 Cooper pairs make a <u>bona fide</u> one. In any case, when one experimentally observes such unexpected behaviour ($\sigma_{2n} \approx \sigma_{1n}$) one is likely somewhat authorized at using similar concepts (9).

(8) In the case of Pb this voltage is $(\pi \times 0.62/2)(eV/e) \approx 1\,V$ (see e.g. McDonald (2001))

(9) P.W. Anderson (1972) more is different

footnote

Correlation energy

The BCS mean field can be written as[1]

$$H_{MF} = U + H_{11} \qquad (1)$$

where

$$U = 2\sum_{\nu>0} (\varepsilon_\nu - \varepsilon_F) V_\nu^2 - G\alpha_0^2 \qquad (2)$$

while

$$H_{11} = \sum_{\nu>0} E_\nu (\alpha_\nu^+ \alpha_\nu + \alpha_{\bar\nu}^+ \alpha_{\bar\nu}), \qquad (3)$$

E_ν being the quasiparticle energy, and α_ν^+ the quasi-particle creation operator. The pair-correlation energy is the difference between the energies with and without pairing. The energy including pair correlations is

$$E_p = 2\sum_{\nu>0} |V_\nu|^2 \varepsilon_\nu - G|\alpha_0|^2, \qquad (4)$$

while the energy without correlation is

$$E_0 = 2\sum_{\nu>0} |V_\nu^0|^2 \varepsilon_\nu . \qquad (5)$$

The occupation probabilities $|V_\nu^0|^2$ are unity below the Fermi energy level and zero above. In both Eqs. (4) and (5) the Fermi energy has to be chosen to give the correct number of particles. The pairing correlation energy is

$$E_{corr} = E_s - G|\alpha_0|^2 , \qquad (6)$$

where

$$E_s = \sum_{\nu>0} 2(|V_\nu|^2 - |V_\nu^0|^2)\varepsilon_\nu , \qquad (7)$$

---

1) Brink and Broglia (2005), Appendix G

The total pairing energy $-G|\alpha_0|^2$ is partially cancelled by the first term describing the fact ~~that~~, in the BCS ground state, particles moving in levels close to the Fermi energy are partially excited across the Fermi surface, in keeping with the fact that V_ν^2 changes smoothly from 1 to 0 around ε_F, being 1/2 at the Fermi energy.

In other words, the energy gain resulting from the potential energy term, where G is the pairing coupling constant while $|\alpha_0|$ measures the number of Cooper pairs is partially compensated by a quantal, zero point fluctuation like term. It can, in principle, be related to the Cooper pain kinetic energy of confinement $T_\xi = \frac{\hbar^2}{2m}\frac{1}{\xi^2}$ already discussed in connection with the generalized quantality parameter according to $2|\alpha_0|T_\xi$ (for one type of nucleons), in keeping with the fact ~~that~~ (7) is expressed in term of single nucleons energies.

Let us make a simple estimate which can help at providing a qualitative example for the purpose of the above argument, and consider the nucleus ^{223}Ra and $G \approx (22/A)$ MeV, $|\alpha_0| \approx 6$ and $\xi \approx 10$ fm: $T_\xi \approx 0.2$ MeV, $2 \times (2|\alpha_0| \times T_f) = 4.8$ MeV, $2 \times (-G|\alpha_0|^2) = -7.2$ MeV[2] (factors of 2, both protons and neutrons). The resulting

2) This quantity, but divided by 2, i.e. -3.6 MeV can be compared with the effective pairing matrix element $v = \left(\frac{\Delta_\pi^2 + \Delta_\nu^2}{4}\right) \approx -2.9$ MeV operative at ~~the~~ level crossing in the calculation of the inertia of the exotic decay $^{223}Ra \rightarrow {}^{14}C + {}^{209}Pb$, cf. Brink and Broglia (2005) p. 159 and

pairing correlation energy thus being $E_{corr} = -2.6$ MeV. (3) ~~Kblhm 16~~

This number can be compared with a "realistic" estimate provided by the relation[3]

$$E_{corr} = -\frac{g\Delta^2}{4}, \qquad (3)$$

where $g_n = N/16$ MeV^{-1} and $g_p = Z/16$ MeV^{-1}. Taking into account both types of particles, $g = g_n + g_p = A/16$ MeV^{-1} and making use of $\Delta = 12/\sqrt{A}$ MeV, one obtains $E_{corr} = -\frac{144}{64}$ MeV $= -2.25$ MeV.

With the help of E_{corr} and T_ξ, one can estimate the generalized quantality parameter, $q_\xi = T_\xi/|E_{corr}| = 0.2/2.6 \approx 0.08$, as well as make a consistency check on the value of ξ used, namely $\hbar v_F/(2|E_{corr}|) \approx 11.5$ fm.

3) Brink and Broglia (2005) p. 64

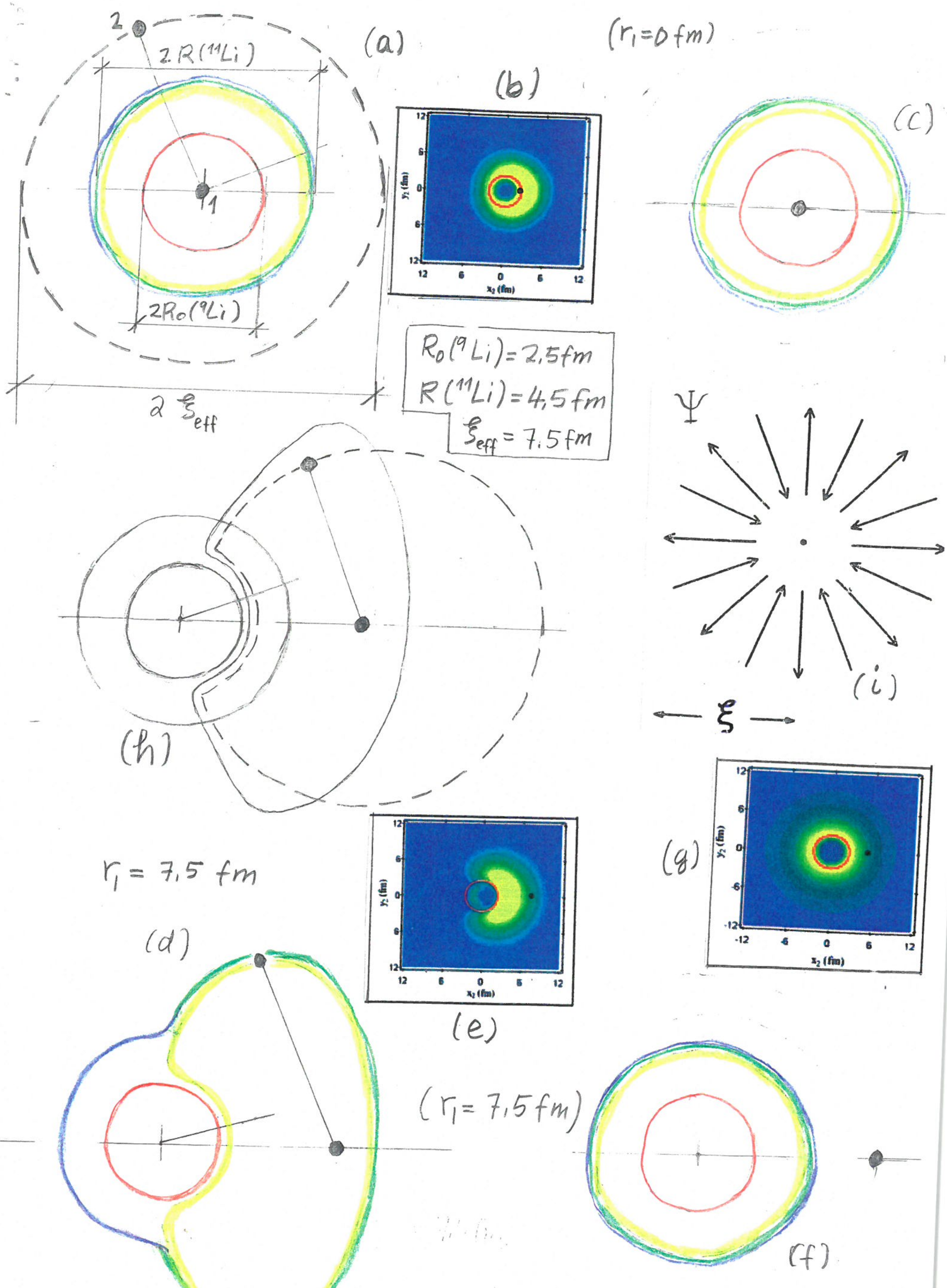


Fig. ~~Cooper~~ Fig. 3.B.1

Caption to Fig. Cooper

Synthesis of the spatial structure of ^{11}Li neutron halo Cooper pair calculated in NFT (Barranco et al (2001)).

To make more direct the comparison between the simple estimates and the results of the above reference, it is assumed that $\xi = 7.5$ fm (dashed circle), instead of 10-11 fm as obtained from $\xi = \hbar v_F / \pi |E_{corr}|$ ($v_F/c \approx 0.08$, $|E_{corr}| = 0.4$ MeV).

bold (a) The continues line circles correspond to the relative distance r at the radius of the ^{9}Li core and of ^{11}Li. The Cooper pair "intrinsic coordinate" r_{12} is also shown. Particle 1 of the Cooper pair is assume to occupy the center of the nucleus (r=0). (b) result of NFT calculation for a situation similar to the above. (c) schematic representation of an Uncorrelated pair in a potential weakly binding the pure configuration $p^2_{1/2}(0)$. (d) Same as (a) but for r = 7.5 fm. (e) result of the NFT calculation for this configuration, (f) schematic representation of a Pure configuration $p^2_{1/2}(0)$, (g) the result of the microscopic calculations for a weakly bound $p^2_{1/2}(0)$ configuration. (h) The variety of situations found in (a) and (d) in comparison to each other in a single schematic representation. (i)

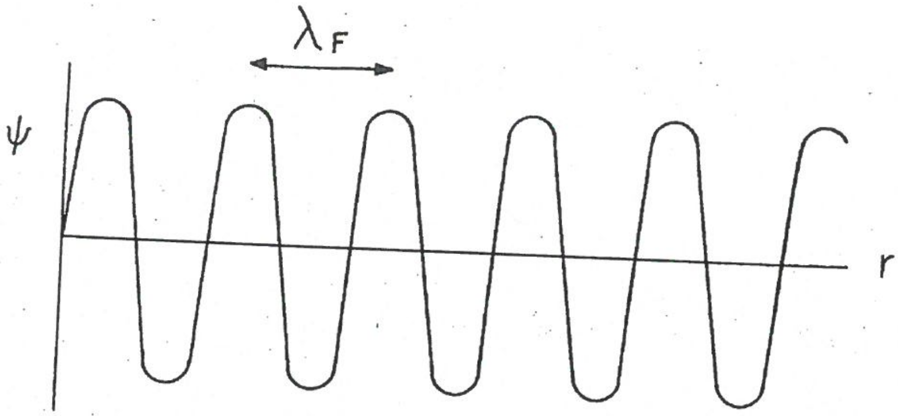
V.F. Weisskopf The formation of Cooper pairs and the nature of the superconducting currents Contemp. Phys. 22, 375 (1981) α Schematic picture of the dynamics in the quantum state of the Cooper pair. It is a linear combination of motions away and towards one another. The electrons stay within a distance of order ξ, root mean square radius of the Cooper pair (after Weisskopf (1981); see also Kadin (2007) and van Witsen (2014)). C2

to p. C1 α

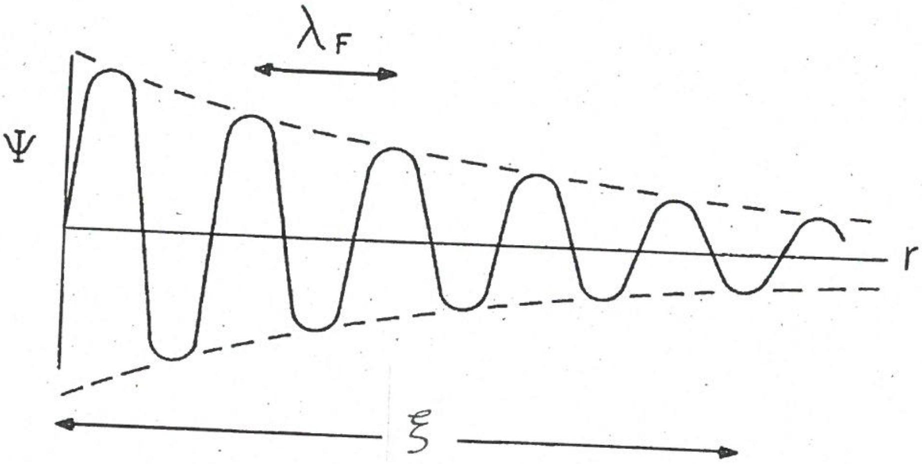
Caption Fig. 3.B.1

Weisskopf (1981) and Kadin (2007) see p. 4 app. 3B Cooper pair: radial dependence

van Witsen (2014): M. van Witsen, Superconductivity in real space, University of Twente, ~~[illegible]~~ (2014) (unpublished)



(a)



(b)

3.B.2

~~Fig.~~ 4. (a) Shows the function ψ of two non-interacting electrons near the Fermi surface in a relative S-state as function of their distance r without any interaction. (b) Shows the wave function Ψ of two electrons with interaction. This is the wave function of a Cooper pair. λ_F is about twice the lattice distance, ρ is the coherence length. Note that, actually, ρ is about 10^4 lattice distances.

ξ ξ (From Weisskopf (1981))

(22), $\Delta\varepsilon$ is about Δ. Hence we get

$$\Delta = \frac{p_F}{m}\Delta p \tag{26}$$

Since $\varepsilon_F = p_F^2/2m$, we get $\Delta p/p_F \sim \Delta/\varepsilon_F$, an extremely small quantity. Thus Ψ consists mainly of waves of wave number p_F/h. It will look like sin $(p_F r/h)$ for $r \ll \rho$, where $\rho \sim h/\Delta p$. For $r > \rho$ the waves (9) will destroy themselves by interferences (see fig. 4). Thus Ψ will have a finite extension (see fig. 4). The distance ρ is the 'size' of the wave function Ψ. We therefore get

$$\rho \sim \frac{h}{\Delta p} \sim \frac{hp_F}{m\Delta} \sim \frac{p_F^2}{m\Delta}d \sim \frac{\varepsilon_F}{\Delta}d \tag{27}$$

where the third '$\sim$' sign comes from $p_F \sim h/d$, and the fourth from $(p_F^2/m) \sim \varepsilon_F$. Here we see that ρ is very large compared to the lattice distance. According to equation vibrations

(ph) | (pp), (hh)

correlated excitations (E_{corr})
with transfer quantum number

$\alpha = 0$ | $\alpha = \pm 2$

waves on

the nuclear | the Fermi

(density) surface

correlation length
infinite medium ($|E_{corr}| = \frac{\hbar^2 k^2}{2m}$)

$$\lambda\!\!\!^{-} = \frac{1}{k} \approx \frac{\langle k \rangle_F}{2m} \frac{1}{|E_{corr}|} = \frac{\hbar V_F}{\pi |E_{corr}|}$$

typical values (finite nuclei), $E_{corr} = \cancel{-1.2\,\text{MeV}}$ -2.0 MeV (-0.4 MeV, ^{11}Li), $V_F/c \approx 0.3$ (0.1, ^{11}Li)

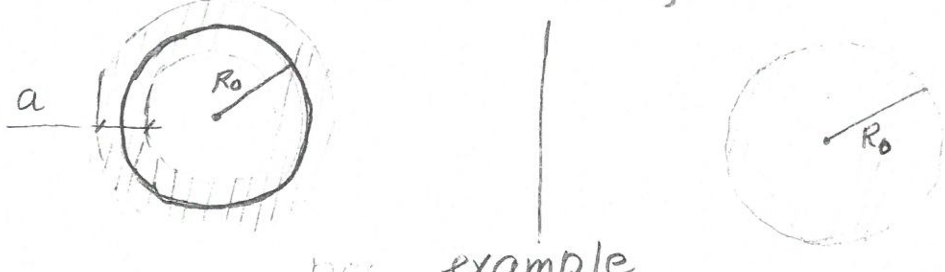
$$\xi \approx \cancel{25}\ 10$$

generalized quantality parameter

$$q_\xi = \frac{\hbar^2}{2m(\xi)^2} \frac{1}{|E_{corr}|} \approx \cancel{0.03}\ 0.1\ (0.08,\ ^{11}Li)$$

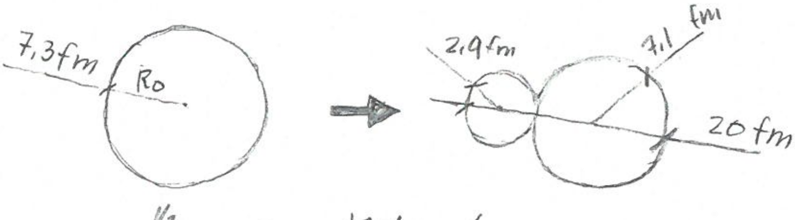
strongly correlated (cluster like $q_\xi \ll 1$), weakly bound ($|E_{corr}|/\epsilon_F \lesssim \cancel{0.03}\ 0.06$)
very extended ($\xi/d \approx \cancel{12}\ 5$, $d = \left(\frac{\frac{4\pi}{3} R^3}{A}\right)^{1/3}$) objects

subject to a strong external field



example

$$^{223}Ra \rightarrow\ ^{14}C +\ ^{209}Pb \quad (\lambda = \text{PFT})$$



$$P = \begin{cases} 10^{-76} & (\Delta = 0) \\ 10^{-10} & \Delta_{exp} \end{cases}$$

$$\langle r^2 \rangle^{1/2}_{cooper} = \xi = \frac{\hbar V_F}{\pi \Delta} \quad (\approx 24\ \text{fm};\ \Delta = 0.8\ \text{MeV})$$

Fig. 3.B.3

Caption to Fig. vibrations Kbhvn 15/05/16

Vibrations can be clasified by the transfer quantum number α. Collective modes with $\alpha=0$ correspond to correlated particle-hole (ph) excitation. For example low-lying quadrupole or octupole (surface and/or density) vibrations. Modes with $\alpha=\pm 2$ are known as pairing vibrations. They can be viewed as correlated (pp) or (hh) modes, ~~that is~~, pair addition and pair substraction modes.

Thinking ~~that~~ these modes propagate in uniform nuclear matter, the reduced wavelength $\lambda\!\!\!^{-} = \lambda/2\pi = 1/k$ is estimated in terms of the correlation energy E_{corr}. The (generalized) quantality parameter, been the ratio of the quantal kinetic energy of localization and the correlation energy, and gives a measure of the tendency to independent particle ($q_\xi \approx 1$) or pair ($q_\xi \ll 1$) motion, in keeping with the fact that potential energy is best profitted by special arrangements betwen the nucleons and thus lower symmetry than the original Hamiltonian, while fluctuations favor symmetry. In going from the infinite to the finite nuclear system, e.g. example density becoming surface modes, they are strongly distorted by the mean field with acts as a strong external field (see also Fig. cooper).

A concrete example which testifies to the fact ~~that~~ (ph) excitations (large amplitude surface distortions) and independent (pp) motion (superfluidity) are correlated over dimensions larger than the nuclear size is provided by e.g. fission and exotic decay, in particular $^{223}Ra \rightarrow {}^{14}C + {}^{209}Pb$ exotic decay.

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In keeping with the uncertainties affecting the above simple estimates (factor 2 or π in the denominator of ξ, $\langle r^2\rangle^{1/2}_{Cooper}$ or $\sqrt{\frac{3}{5}}\langle r^2\rangle^{1/2}_{Cooper}$, etc), it seems fair to conclude ~~that~~ $10 \lesssim \xi \lesssim 20$. Thus, one is likely faced with an intermediate situation in which $1.3 \lesssim \xi/R \lesssim 2.6$.

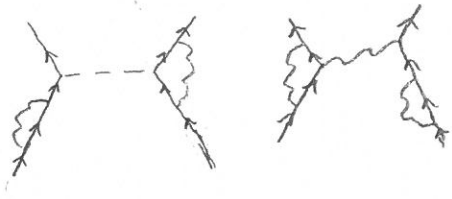
The parallel which can be traced between Cooper pairs and correlated particle-hole excitations is further testified by the fact that two-nucleon transfer reaction do excite quite strongly also these mods (see Tables A and B).

Metals | Nuclei

dressed

electrons ($m = 0{,}5$ MeV) | nucleons ($m = 10^3$ MeV)

--- effect. Coul.



--- $^1S_0$ bare NN-int.

} lattice phonons

} surface vibrations spin modes

spontaneous breaking of gauge symmetry

$(U_\nu + V_\nu \underbrace{a^\dagger_\nu a^\dagger_{\bar\nu}})|0\rangle$ $\quad U'_\nu + e^{-2i\phi} V'_\nu a^\dagger_\nu a^\dagger_{\bar\nu}$

independent pair motion

ξ (ξ/d)

10^4 Å (10^4) | 20 fm (5)

Δ (Δ/ϵ_F)

$\overrightarrow{\text{\# overlapping pairs}}$

\# of pairs

≈ 1 meV (10^{-4}) | ≈ 1 MeV $(\approx 10^{-2})$

generalized quantality parameter

$$q_\xi = \frac{\hbar^2}{2m\,\xi^2}\,\frac{1}{\Delta}$$

10^{-5} | 10^{-2}

probing of gauge deformation

observation of currents between two weakly coupled superconductors (barrier allows essentially for single electron tunneling, with 2e carriers (Josephson effect)

Single Cooper pair tunneling mainly as successive transfer between member of a pairing rotational band fulfilling,

$$\frac{\sigma(gs(N) \to gs(N+2))}{\sum_{exc} \sigma(gs(N) \to 0^+_{exc}(N+2))} \gg 1$$

$(N = N_0, N_0+2, N_0+4 \ldots N_0+14 \ldots (N_0 = 10)$

$P_2 = P_1$

Fig. 3.B.4