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THE NUCLEAR COOPER PAIR

Structure and Reactions

G. Potel
R.A. Broglia

scattering amplitude,

the variety of

In other words (NFT)_(r+s) focus on the scattering amplitude, which directly determine the absolute cross sections, for various physical processes, involving also those in which bosons and fermions are created or annihilated, connecting such processes to form factors and transition densities. Processes where one set of particles with given energies, momenta, angular momenta, etc. go in and another group (or the same), comes out. That is, what happens in the laboratory, let alone in nature.

Preface

(B) - (B) from p. ③ a handwritten

in the present monograph

(NFT)_(s+r)

The elementary modes of nuclear excitation are vibrations and rotations, single-particle (quasiparticle) motion, and pairing vibrations and rotations. The specific reactions probing these modes are inelastic and Coulomb excitation, single- and two-particle transfer processes respectively. Within this context one can posit that nuclear structure (bound) and reactions (continuum) are but two aspects of the same physics. This is even more so concerning the study of exotic nuclei, where the study of halo nuclei, which occupies a large fraction of the present monograph, has blurred almost completely the distinction between bound and continuum states. This is the reason why they are treated on equal footing in terms of elementary modes of excitation, within the framework of the unified nuclear field theory of structure and reactions (NFT)_(r+s). This theory provides the rules to diagonalize in a compact and economic way the nuclear Hamiltonian for both bound and continuum states, correcting for overcompleteness of the basis (particle-vibration coupling (structure), non-orthogonality (reaction)), and for Pauli principle violation. The outcome connects directly with observables; absolute reaction cross sections and decay probabilities.

Pairing vibrations and rotations, closely connected with nuclear superfluidity are paradigms of quantal nuclear phenomena. They thus play an important role within the field of nuclear structure. It is only natural that two-nucleon transfer plays a similar role concerning direct nuclear reactions. At the basis of fermionic pairing phenomena one finds Cooper pairs, weakly bound, extended, strongly overlapping (quasi-) bosonic entities, made out of pairs of nucleons dressed by collective vibrations and interacting through the exchange of these vibrations as well as through the bare NN -interaction, eventually corrected by $3N$ contributions. Cooper pairs not only change the statistics of the nuclear stuff around the Fermi surface and, condensing, the properties of nuclei close to their ground state. They also display a rather remarkable mechanism of tunnelling between target and projectile in direct two-nucleon transfer reaction. In fact, being weakly bound ($\ll \epsilon_F$, Fermi energy), Cooper pair partners are correlated over distances (correlation length) much larger than nuclear dimensions ($\gg R$, nuclear radius). On the other hand, Cooper pairs are forced to be confined within regions in which normal density is present and thus, within nuclear dimensions, by the action of the average potential, which can be viewed as an external field as far as these pairs are concerned. Nonetheless, the correlation length paradigm comes into evidence, for example, when two nuclei are set into weak contact in a direct reaction. In this

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completely
as well as blurred

reaction cross
sections and
decay probabili-
ties.

the probing of
the nucleus

Within this context

(infinite)

It does so in terms
of Feynman diagrams which
describe the interweaving
of elementary modes of
excitation,

strong

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strongly distorting the
spatial structure of
a Cooper pair in nuclear
matter!

long range correlations. Also of

(3) to p. 3

The interweaving of the elementary modes of excitation lead to renormalization of the energy, wavefunction and particle content of the single-particle, as well as of the energy, width and collectivity of vibrations. This implies renormalization of the formfactors and transition densities, Q-value and effective deformation parameters both in 3D- and in gauge-space, and state and mass number dependence of the optical potential. As a consequence, the emergence of resonance phenomena ~~-in the absolute cross sections-~~ as a function of the bombarding energy of the projectile inducing the anelastic and/or transfer processes, implying ^{also} the need to go beyond lowest order distorted wave **Born** approximation (DWBA).

to p. 3 (3)

Born

As a consequence, the emergence of long range correlations. Also of resonance phenomena

thus correlated over distances much larger than nucleardimensions.

of dimensions much smaller
than the correlation length

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and of the tunneling of single
Cooper pairs

Q5

case, each of the partner nucleons of a Cooper pair has a finite probability to be confined within the mean field of each of the two nuclei. It is then natural that a Cooper pair can tunnel, equally well correlated, between target and projectile, through simultaneous than through successive transfer processes. Although one does not expect supercurrents in nuclei, one can study long-range pairing correlations in terms of individual quantal state. The above mentioned weak coupling Cooper pair transfer reminds the tunnelling mechanism of electronic Cooper pairs across a barrier (e.g. a dioxide layer) separating two superconductors, known as a Josephson junction. The main difference is that, as a rule, in the nuclear time dependent junction is merely established in direct two-nucleon transfer process, only one or even none of the two weakly interacting nuclei are superfluid. Now, on the other hand → in nuclei, paradigmatic example of fermionic finite many-body system, zero point fluctuations (ZPF) in general, and those associated with pair addition and pair subtraction modes known as pairing vibrations in particular, are much stronger than in condensed matter. Consequently, and in keeping with the fact that pair addition and subtraction modes are nuclear embodiments of Cooper pairs in nuclei, pairing correlations based on even a single Cooper pair can lead to distinct pairing correlation effects in two-nucleon transfer processes. medium heavy

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Nucleonic Cooper pair tunnelling has played and is playing a central role in the probing of these subtle quantal phenomena, both in the case of exotic nuclei as well as of nuclei lying along the stability valley, and have been instrumental in shedding light on the subject of pairing in nuclei at large, and on nuclear superfluidity in particular. Consequently, the subject of two-nucleon transfer occupies a central place in the present monograph, both concerning the conceptual and the computational aspects of the description of nuclear pairing, as well as regarding the quantitative confrontation of the results and predictions with the experimental findings in terms of absolute cross sections. B-B from p ④ a + ④ b

They
single Cooper
pair

B

NFT
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Thus,

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and,
making use
of the resulting
renormalized
wavefunctions
(form factors),

Because of the central role the interweaving of the variety of elementary modes of nuclear excitation, namely single-particle motion and collective vibrations play in nuclear superfluidity, the study of Cooper pair tunnelling in nuclei aside from requiring a consistent description of nuclear structure in terms of dressed quasiparticles and vibrations resulting from both bare and induced interactions, also involves the description of one-nucleon transfer as well as knock out processes. Consequently, in the present monograph the general physical arguments and technical computational details concerning the calculation of absolute one-and two nucleon transfer cross sections, making use of state of the art nuclear structure information, are discussed in detail. As a consequence, theoretical and experimental nuclear practitioners, as well as fourth year and PhD students can use the present monograph at profit. To help this use, the basic nuclear structure formalism, in particular that associated with pairing and with collective modes in nuclei, is economically introduced through general physical arguments. This is also in keeping with the availability in the current literature, of detailed discussions of the corresponding material. Within this context, the monographs *Nuclear Superfluidity* by Brink and Broglia and *Oscillations in Finite Quantum Systems* by Bertsch and Broglia, pub-

differen-
tial

result of this
approach,

This is similar to the situation encountered in superconductors, in connection with strongly coupled systems, and experimentally studied through one- and two-electron tunneling experiments. ***)

**) J. Giaever,
Nobel lecture

from exper-
iment

*) Within this context one recognizes the difficulties and risks of extracting spectroscopic factors, in terms of single-particle transmission sections calculated making use of mean field wavefunctions

(B) top. 4

Concerning exotic nuclei, recent experiments⁽⁴⁾ carried out at TRIUMF (Canada) have provided, through the magnifying ~~glass~~^{lens} of (NFT)_{STR}, a microscopic view of what can be considered, a ~~the first~~^{unique} embodiment of Cooper's pair model^{*)}: a pair of fermions (neutrons) moving in time reversal states on top of a quiescent Fermi surface and interacting through the exchange of a long wavelength vibration (phonon)^{**)}, leading to a barely bound system. The two neutrons give rise to an isotropic halo. Because the vibration (phonon) results from the sloshing back and forth of the neutron halo against the core nucleons, one is in presence of a realization of Nambu's tumbling^{***)} or, more precisely, symbiotic mechanism of spontaneously broken symmetry in gauge space.

Concerning the case of medium heavy nuclei lying along the stability valley, recent studies of heavy ion reactions between superfluid nuclei carried out at energies below the Coulomb barrier at the National Laboratory of Legnaro (Italy) have provided, again with the help of (NFT)_{STR} analysis, a measure of the neutron, Cooper pair correlation length. Within this context, in the present monograph interdisciplinarity is used as a tool to attack concrete nuclear problems. In return, making

*) L.N. Cooper
**) Frölich, Bardera and Pines, BCS I + II
***) Y. Nambu

use of the unique laboratory provided by ④, the finite quantum many-body system of which the atomic nucleus is a paradigmatic example*), to shed light on condensed matter results, in terms of individual, quantal single-particle states, let alone tunneling of single Cooper pairs. 17...

(B) to p. 4

* A. Bohr, Selected papers in The Finite Quantum Many-Body Problem, Ed. R. AlBruglia, World Scientific, Singapore (2019),

lished also by Cambridge University Press can be considered companion volumes to the present one. Volume which shares with those a similar aim: to provide a broad physical view of central issues in the study of finite quantal many-body nuclear systems accessible to motivated students and practitioners. However, neither the present one, nor the other two are introductory texts. In particular the present one in which an attempt at unifying structure and reactions as it happens in nature is made. On the other hand, unifying discrete (mainly structure) and continuum (reactions), implies that we will be dealing with those structure results which can be tested by means of experiment. A fact which makes the subject of the present monograph a chapter of quantum mechanics, and thus close to what fourth year students have been learning.

Concerning the notation, we have divided each chapter into sections. Each section may, in turn, be broken down into subsections. Equations and Figures are identified by the number of the chapter and that of the section. Thus (8.1.33) labels the thirtythird equation of section 1 of chapter 8. Similarly, Fig. 8.1.2 labels the second figure of section 1 of chapter 8. Concerning the Appendices, they are labelled by the chapter number and by a Latin letter in alphabetical order, e.g. App. 8.A, App. 8.B, etc. Concerning equations and Figures, a sequential number is added. Thus (6E.15) labels the ^{second} fifteenth equation of Appendix E of chapter 6, while Fig. 6.E.1 labels the first figure of Appendix E of Chapter 6. References are referred to in terms of the author's surname and publication year and are found in alphabetic order in the bibliography and the end of each chapter, as well as in the complete bibliography at the end of the monograph.

2.B.1
called

A methodological approach used in the present monograph concerns a certain degree of repetition. Similar, but not the same issues are dealt with more than once using different but equatable terminologies. This approach reflects the fact that useful concepts like reaction channels, or correlation length, let alone elementary modes of excitation, are easy to understand but difficult to define. This is because their validity is not exhausted in a single perspective. But even more important, because their power in helping at connecting² seemingly unrelated results and phenomena is difficult to be fully appreciated the first time around, spontaneous symmetry breaking and associated emergent properties providing an example of this fact.

Throughout, a number of footnotes are found. This is in keeping with the fact that footnotes can play a special role within the framework of an elaborated presentation. In particular, they are useful to emphasize relevant issues in an economic way. Being outside the main text, they give the possibility of stating eventual im-

¹This is also a consequence of the fact that physically correct concepts are forced to be expressed, to become precise, in an axiomatic fashion, a style foreign to the one used here.

²"The concepts and propositions get "meaning" viz. "content", only through their connection with sense-experience... The degree of certainty with which this connection, viz., intuitive combination, can be undertaken, and nothing else, differentiates empty fantasy from scientific "truth"... A correct proposition borrows its "truth" from the truth-content of the system to which it belongs" (A. Einstein, Autobiographical notes, in Albert Einstein, Ed. P. A. Schilpp, Harper, New York (1951)) p.1, Vol I.

(but referring to the corresponding sources)
 portant results, without the need of elaborating on the proof. Within this context, and keeping the natural distances, one can mention that in the paper in which Born³ introduces the probabilistic interpretation of Schrödinger's wavefunction, the fact that this probability is connected with its modulus squared and not with the wavefunction itself, is only referred to in a footnote.

Most of the material contained in this monograph have been the subject of lectures of the four year course "Nuclear Structure Theory" which RAB delivered throughout the years at the Department of Physics of the University of Milan, as well as at the Niels Bohr Institute and at Stony Brook (State University of New York). It was also presented by the authors in the course Nuclear Reactions held at the PhD School of Physics of the University of Milan.

GP wants to thank the tutoring of Ben Bayman concerning specific aspects of two-particle transfer reactions. Discussions with Ian Thompson and Filomena Nuncs on a variety of reaction subjects are gratefully acknowledged. RAB acknowledges the essential role the collaboration with Francisco Barranco and Enrico Vigezzi has played concerning nuclear structure aspects of the present monograph. Its debt with the late Aage Winther regarding the reaction aspects of it is difficult to express in words. The overall contributions of Daniel Bès, Ben Bayman and Pier Francesco Bortignon are only too explicitly evident throughout the text and constitute a daily source of inspiration. G. P. and R. A. B. have received important suggestions and comments regarding concrete points and the overall presentation of the material discussed below from Ben Bayman, Pier Francesco Bortignon⁴, David Brink, Willem Dickhoff and Vladimir Zelevinsky and are here gratefully acknowledged.

Gregory Potel Aguilar
 East Lansing

Ricardo A. Broglia
 Copenhagen

³Born (1926). Within this context, it is of notice that the extension of Born probabilistic interpretation to the case of many-particle systems is also found in a footnote (Pauli (1927), footnote on p. 83 of the paper).

⁴Deceased August 27, 2018.

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Model for
single-particle
strength function;
Dyson equation

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