give examples heavy ion transfer

~~, arguably, lowest member of the $|2^+({}^8He) \otimes p_{3/2}(\pi); J^\pi\rangle$ ($J^\pi = 7/2^-, 5/2^-, 3/2^-, 1/2^-$) multiplet, resulting from the coupling of the quadrupole vibration of ${}^8_2He$ and of the $p_{3/2}$ proton (π) single-particle state of ${}^9_3Li$ (cf. Fig. 8.1.1).~~

Chapter 8

Nuclear Structure with two–nucleon transfer

cf. App. 8.D, c differential cross sections, associated with reactions

no new line

In what follows, we apply the formalism worked out in the previous Chapter with the help of software edveloped to calculate absolulte two–particle transfer induced by both light and heavy ions (COOPER, ONE). A number of ~~Two~~ examples are treated with special detail. Namely, two–particle transfer in halo unstable nuclei (${}^{11}Li$) ~~and~~ in superfluid medium heavy nuclei lying along the stability valley (Sn-isotopes) and in closed shell heavy systems (Pb), in this case for both light an heavy ion projectiles.

8.1 The H(^{11}Li,^{9}Li)^{3}H reaction: evidence for phonon mediated pairing

We start by discussing Particular

~~In what follows,~~ the analysis of the two–neutron pickup reaction H(^{11}Li,^{9}Li)^{3}H (I. Tanihata et al., 2008) ~~is discussed,~~ setting special emphasis on treating the structure and reaction aspects of the process on equal footing. ~~Special~~ attention is paid to the direct excitation of the $1/2^-$ state of ^{9}Li lying at 2.69 MeV. ~~For this purpose~~ To asses the direct character of the $1/2^-$ excitation process, the importance of inelastic (cf. Ch.5) and knockout (cf. Ch.6) channels is considered let alone successive transfer process. While this ~~process~~ channel is the dominant one, the other mentioned two–step channels are found to contribute little to the absolute differential cross section. These results provide evidence for a new mechanism of pairing correlations in nuclei: pigmy resonance mediated pairing interaction (F. Barranco et al. (2001), see also App. 8.A), which strongly renormalizes the bare, NN–$1S_0$ interaction (Potel et al., 2010). This is but a particular embodiment of phonon mediated pairing interaction found throughout in nuclei (cf. e.g. Barranco et al. (1999); Gori et al. (2004) cf. also Brink and Broglia (2005), Ch. 10 and Ch. 11). The main difference between light halo exotic nuclei and medium heavy superfluid nuclei lying along the valley of stability is the role fluctuations play in dressing particles (quasiparticles) and in renormalizing their properties (mass, charge, etc.) and their interactions. While e.g. in Sn nuclei one can, as a rule, più grande

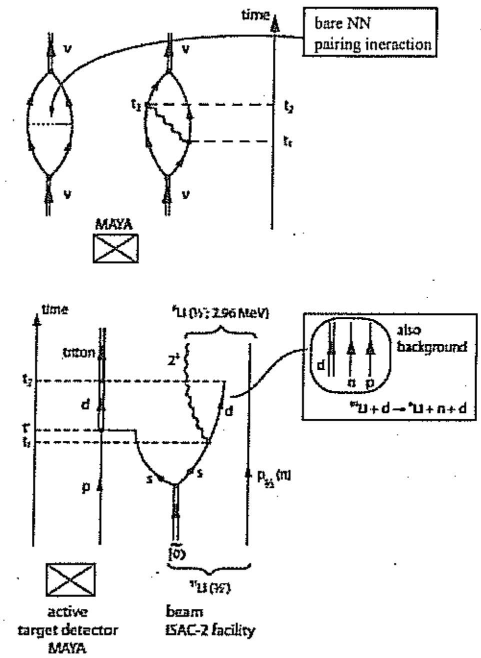

Figure 8.1.1: Schematic representation of the bare nucleon-nucleon and phonon induced pairing correlations (upper part) NFT diagrams, and of the excitation of the final, excited state of ^{9}Li($1/2^-$; 2.69 MeV), in the TRIUMF experiment reported in ref. I. Tanihata et al. (2008)

più grande

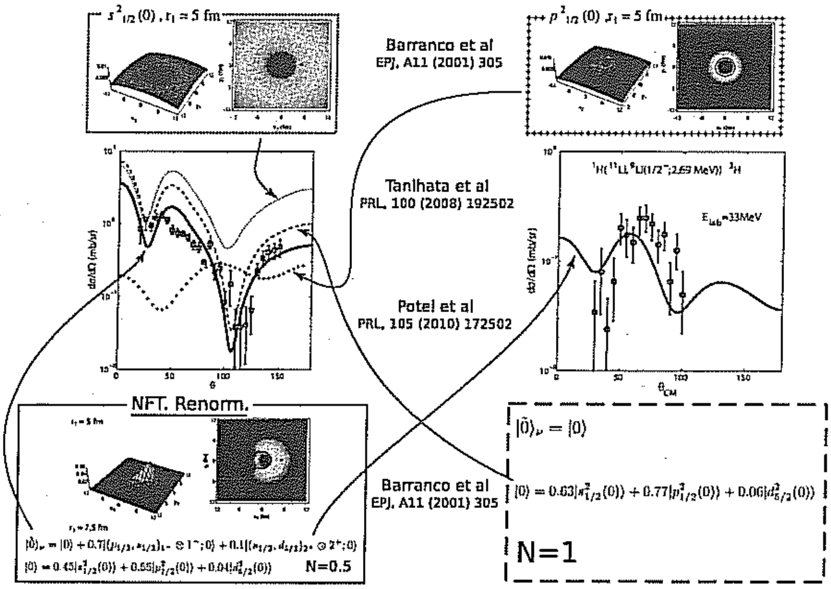

Figure 8.1.2: Absolute, two–nucleon transfer differential cross section associated with the ground state and the first excited state of ^{9}Li, excited ~~(Tanihata et al., 2008)~~ in the reaction ^{1}H(^{11}Li,^{9}Li)^{3}H (I. Tanihata et al., 2008) in comparison with the predicted differential cross sections (Potel et al., 2010) calculated making use of spectroscopic amplitudes and Cooper pair wavefunctions calculated in NFT.

e

and the second the resonant $p_{1/2}$ orbital (cf. Fig. 6.1.3)

(cf. Fig. 6.1.3)

ω- ω-independent

, the situation is quite different in the case of ^{11}Li

explain observables, in particular two–nucleon transfer absolute cross sections in terms of spectroscopic amplitudes obtained solving the BCS equations in terms of an effective, independent coupling constant (see ~~however~~ Sect. 8.2). In particular, the fact that the first empty single–particle state in which one can place a neutron in ^{9}Li is the $s_{1/2}$ virtual state of ^{10}Li implies a pairing–anti halo effect for the lowest energy unperturbed pair state of ^{11}Li, namely $|s^2_{1/2}\rangle$. Because the bare $1S_0$ interaction, namely ~~the short~~ range pairing interaction, builds its strength out of many contributions of ~~different, but ever~~ increasing multipolarities, it is not surprising that it cannot bind an $s^2_{1/2}(0)$ Cooper pair, nor mix it with e.g. the $p^2_{1/2}(0)$ resonant configuration with any probability (F. Barranco et al., 2001). In the following Section we elaborate on this point (see also App. 8.A).

state in space

can

(cf. Hamamoto 2003, 2004 and Bennaceur 2000)

the need for a long range component of the nuclear pairing interaction is apparent

8.1.1 Structure

Sect 6.2 in

Within the scenario presented in connection with the phenomenon of parity inversion and associated dressing of the $s_{1/2}$ and $p_{1/2}$ virtual and resonant states of ^{10}Li, ~~discussed in Sect.6.2, in particular~~ ~~(cf. also App. 8.A, in particular Fig. 8.A.1)~~, ^{11}Li(gs) corresponds to an unbound $s^2_{1/2}(0)$ configuration (see Fig. 8(II)). The bare residual interaction lowers this configuration by less than 100 keV. On the other hand, the exchange of the quadrupole mode of the ^{9}Li core and of the pigmy resonance of ^{11}Li lead to a neutron Cooper pair bound by about ~~300~~ keV, the experimental value being $\approx$ 380 keV. This neutron halo state is the pair addition ~~mode~~ of the N=6, ^{9}Li closed shell system. *Of notice that the pigmy resonance is the result of a delicate (Baron Münchausen–like) bootstrap process, in which an originally extended neutron halo created by the two unbound neutrons passing by ^{9}Li are, quantum mechanically, forced to slosh back and forth with respect to the proton core field (ZPF), leading to a collective mode which ,exchanged between the halo neutrons, binds the Cooper pair to the core.* In other words, the pigmy resonance is in a very real sense a consequence of (translational) symmetry restoration and of a virtual process (vibrations of an extended neutron field) becoming real as a low–lying excitation (App. 8.C pushing model), after having acted as glue to bind the two outer neutrons to the ^{9}Li core thus generating the weakly, but nonetheless bound ground state of ^{11}Li, the resulting Cooper pair wavefunction being

6.1.3

(App. 8.A)

only thing to leave with italics

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$$|0\rangle_\nu = |0\rangle + \alpha|(p_{1/2}, s_{1/2})_{1^-} \otimes 1^-; 0\rangle + \beta|(s_{1/2}, d_{5/2})_{2^+} \otimes 2^+; 0\rangle, \qquad (8.1.1)$$

with

$$\alpha = 0.7, \quad \text{and} \quad \beta = 0.1, \qquad (8.1.2)$$

and

$$|0\rangle = 0.45|s^2_{1/2}(0)\rangle + 0.55|p^2_{1/2}(0)\rangle + 0.04|d^2_{5/2}(0)\rangle. \qquad (8.1.3)$$

are

The states $|1^-\rangle$ and $|2^+\rangle$ ~~being~~ the (RPA) states describing the dipole pigmy resonance of ^{11}Li and the quadrupole vibration of the core. While these states are virtual excitations which, exchanged between the two neutrons bind them to the Fermi surface provided by the ^{9}Li core, they can be forced to become real with the c Figs. 8.1.1 and 8.1.2)

Due to

help of the specific probe of Cooper pairs in nuclei, namely two–particle transfer reactions.

We are then in presence of a paradigmatic nuclear embodiment of Cooper's model which is at the basis of BCS theory: a single weakly bound neutron pair on top of the Fermi surface of the ^{9}Li core. But the analogy goes beyond these aspects, and covers also the very nature of the interaction acting between Cooper pair partners. ~~Because of~~ the high polarizability of the system under study, most of the Cooper pair correlation energy stems, according to NFT, from the exchange of collective vibrations, the role of the bare interaction being, in this case, minor. *In other words, we are in the presence of a new realization of Cooper's model in which a totally novel Bardeen–Pines–Frölich–like phonon induced interaction, is generated by a self induced collective vibration of the nuclear medium.* Because one is in possess of the specific tool to probe pairing correlations in nuclei, namely, two–particle transfer reactions, one can force these virtual processes to become real. Within this context, it is revealing that, the two final states excited in the inverse kinematics, two–neutron pick up reaction ^{1}H(^{11}Li,^{9}Li)^{3}H are, the $|3/2^-\mathrm{gs}(^9\mathrm{Li})\rangle$ and the first excited $|1/2^-, 2.69\mathrm{MeV}\rangle$I. Tanihata et al. (2008). The associated absolute differential cross sections thus probe, within the NFT scenario, the $|0\rangle$ and the $|(s_{1/2}, d_{5/2})_{2^+} \otimes 2^+; 0\rangle$ component of the Cooper pair wavefunction respectively, the $p_{3/2}$ proton acting as a spectator. It is of notice that the $|1/2^-, 2.69\mathrm{MeV}\rangle$ state of ^{9}Li can be viewed as the $1/2^-$ member of the multiplet resulting from the coupling of the ^{8}He core quadrupole vibration and the $p_{3/2}$ proton. Associated modified formfactors were worked out ~~(cf. 8.C)~~ mking use of spectroscopic amplitudes and the optical potentials collected in Table ?. Theory is compared with the experimental findings in Fig. ?. It reproduces the absolute two–particle differential cross section within experimental errors.

8.1.2 8.1.1 8.1.1 Eq.(8.1.2)

noitalics

Gregory, with all the small contribution continuum

(cf. App. 8.C)

(Fig. 8.1.1 cf. also Figs. 6.1.4 and 6.1.5; cf. also Figs. 8.1.2 and 6.1.3)

(Fig. 8.1.3)

given in Eqs.(8.1.1–8.1.3)

8.1.2 Reaction

Because detailed, second order calculations of inelastic, break up and final state interaction channels, which in principle can provide alternative routes to the $|1/2^-, 2.69\mathrm{MeV}\rangle$ state than that predicted by ~~the~~ NFT (β component), lead to absolute cross sections which are smaller by few orders of magnitude than that shown in Fig. 8.B.? (excited state) (Potel et al., 2010), one can posit that quadrupole core polarization effects in $|\mathrm{gs}(^{11}\mathrm{Li})\rangle$, are essential to account for the observation of the $|1/2^-, 2.69\mathrm{MeV}\rangle$ state, thus providing direct evidence for phonon mediated pairing in nuclei ~~(see Fig. 5 of the contribution of Potel and Broglia to this Volume)~~. At variance with the case of the infinite system (e.g. normal superconducting metals) in which there is a bound state for any strength of the interaction, in finite FMBS there is a lower limit for the strength below which the system correlates but does not condense. This is what happens around closed shell nuclei, in which the decoupling between occupied and empty states blocking pair condensation, arizes from the gap in the single–particle spectrum observed at magic numbers, and forced upon the system by the "external" mean field produced by all the nucleons on the motion of each

a 2

Table 8.1.1 from p. 144 half of Table 8.2.2

(attractive)

; see also Fig. 8.B.3 and Fig. 8.1.2 as well as Table 8.B.1

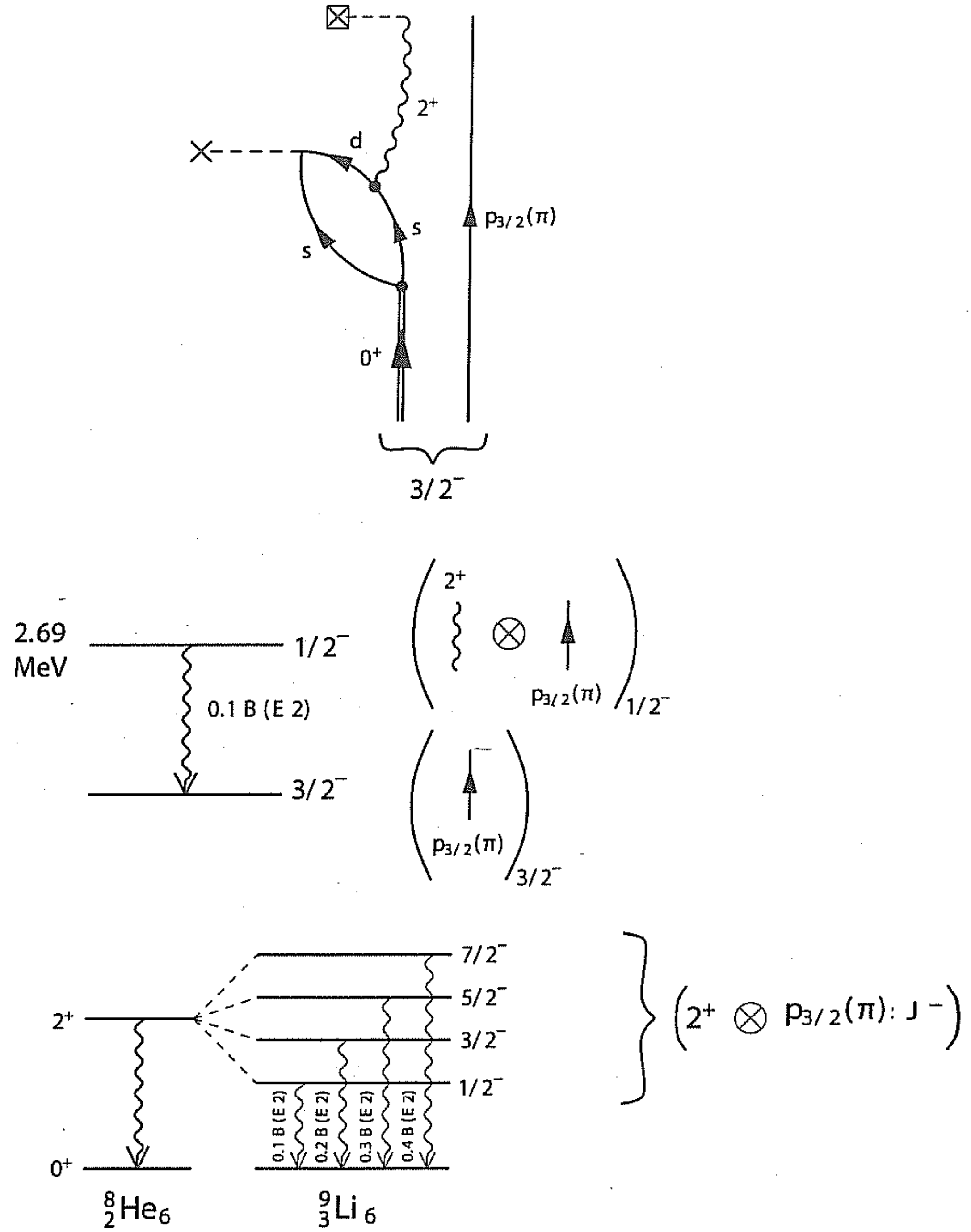


Figure ~~4~~ 8.1.3. *Gedanken* (two–particle transfer)–(γ decay) coincidence experiments aimed at better individuating the couplings involved in the neutron halo Cooper pair correlations in ^{11}Li and of the $1/2^-$ member of ^{9}Li excited in the ^{1}H(^{11}Li,^{9}Li)^{3}H reaction (Barranco et al (2001), Potel et al (2010)). From Potel et al (2014).

within this context think for example on parity inversion

single neutron and proton.

The reason why in the case of ^{11}Li evidence for phonon mediated pairing is, arguably, inescapable (see also Table 8.B.1), is connected with the fact that reaching the limits of stability associated with drip line nuclei, and thus to situations in which medium polarization and spatial quantization effects become overwhelming. In fact one is, in such cases, confronted with elementary modes of nuclear excitation in which dynamic, fluctuation effects are as important as static, mean field effects. Nuclear Field Theory within the Bloch–Horowitz (Dyson) set up which allows one to sum to infinite order little convergent processes are specially suited to study these systems (cf. e.g. F. Barranco et al. (2001) and Gori et al. (2004)). From these studies it emerges a possible new elementary mode of excitation, namely pair addition halo vibration, of which $|gs(^{11}\mathrm{Li})\rangle$ state is a concrete embodiment. They are closely connected with a new mechanism to stabilize Cooper pairs, arising from a (dynamical) breakup of gauge invariance (cf. App 8.A). Their most distinctive feature, namely that of carrying on top a (dipole) pigmy resonance at a relative excitation energy of few MeV, a necessary although not sufficient condition for this new mode to exist, can be instrumental for its characterization. It could thus be directly observed in an $L = 0$, two–particle transfer reaction to excited states, or in terms of $E1$ decay of the pigmy resonance built on top of it. Within this context, it is an open question whether one could expect to find such a halo pair addition mode, for example, as an excited state of ^{12}Be.

Pairing elementary modes of excitation based on $s_{1/2}$ and $p_{1/2}$ states at threshold, having been found to lead, within the bare, short range, pairing interaction scheme to halo anti–pairing effects (cf. Bennaceur et al. (2000), cf. also Hamamoto and Mottelson (2003), Hamamoto and Mottelson (2004)). The fact that the separation energy of the halo neutrons (halo Cooper pair) of ^{11}Li(gs) is $\approx$ 400keV, testifies to the fact that the anti–halo pairing effect is, in that case, overwhelmed by (dynamical) medium polarization effects.

To conclude this section it is of notice that again, the interweaving of the different elementary modes of nuclear excitation, pairing and pigmy resonances in the present case, conditions reaction studies, let alone the possibility to study (pigmy) giant resonances on excited states, (cf. Brink (1955); cf. also e.g. Bortignon et al. (1998)).

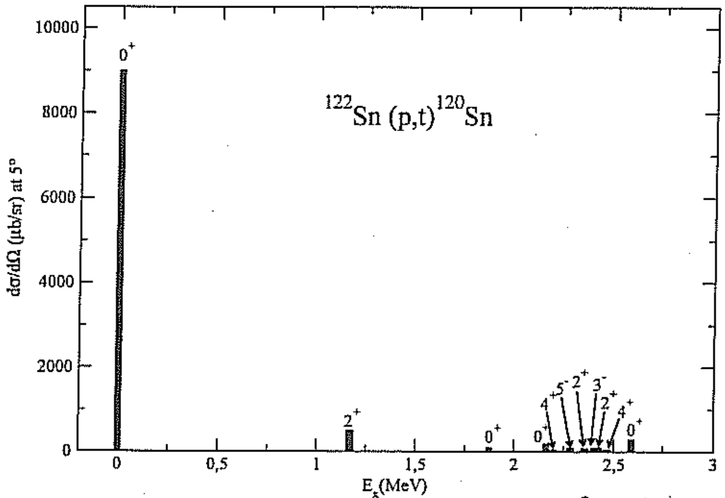
8.2 Pairing rotational band with two–nucleon transfer: Sn–isotopes

Nuclear superfluidity can be studied at profit in terms of the mean field, BCS diagonalization of the pairing Hamiltonian, namely,

$$H = H_{sp} + V_p, \tag{8.2.1}$$

where

$$H_{sp} = \sum_{\nu}(\epsilon_\nu - \lambda)a_\nu^+ a_\nu, \tag{8.2.2}$$



(Guazzoni et al, 1999)

Figure 8.2.1: Excitation function associated with the reaction $^{122}\text{Sn}(p,t)^{120}\text{Sn}(J^\pi)$. The absolute experimental value (~~see ref. g) of Table 3~~) of $d\sigma(J^\pi)/d\Omega|_{5^\circ}$ is given as a function of the excitation energy E_x.

while

$$V_p = -\Delta(P^+ + P) - \frac{\Delta^2}{G}, \tag{8.2.3}$$

and

$$\Delta = G\alpha_0, \tag{8.2.4}$$

is the pairing gap ($\Delta \approx 12\ \text{MeV}/\sqrt{A}$), G (≈ 25 MeV/A) being the pairing coupling constant (Bohr and Mottelson (1975)), and

$$P^+ = \sum_{\nu>0} P^+_\nu = \sum_{\nu>0} a^+_\nu a^+_{\bar\nu}, \tag{8.2.5}$$

$$P = \sum_{\nu>0} a_{\bar\nu} a_\nu, \tag{8.2.6}$$

are the pair addition and pair removal operators, a_ν and a^+_ν being single-particle creation and annihilation operators, $(\nu\bar\nu)$ labeling pairs of time reversal states.

The BCS ground state wavefunction describing the most favorable configuration of pairs to profit from the pairing interaction, can be written in terms of the product of the occupancy probabilities h_ν for individual pairs,

$$|BCS>= \Pi_\nu((1-h_\nu)^{1/2} + h_\nu^{1/2} a^+_\nu a^+_{\bar\nu})|0>, \tag{8.2.7}$$

where $|0>$ is the fermion vacuum.

which in the nuclear case is few units (< 10)

Superfluidity is tantamount to the existence of a finite average value of the operators (8.2.5), (8.2.6) in this state, that is, to a finite value of the order parameter

$$\alpha_0 =< BCS|P^+|BCS >=< BCS|P|BCS >^*, \tag{8.2.8}$$

which is equivalent to Cooper pair condensation. In fact, α_0 gives a measure of the number of correlated pairs in the BCS ground state. While the pairing gap (8.2.4) is an important quantity relating theory with experiment, α_0 provides the specific measure of superfluidity. In fact, the matrix elements of the pairing interaction may vanish for specific regions of space, or in the case of specific pairs of time reversal orbits, but this does not necessarily imply a vanishing of the order parameter α_0, nor the obliteration of superfluidity.

In keeping with the fact that Cooper pair tunneling is proportional to $|\alpha_0|^2$, this quantity plays also the role of a ($L = 0$) two-nucleon transfer sum rule, sum rule which is essentially exhausted by the superfluid nuclear $|BCS >$ ground state (see Fig. 8.2.1). Within the above context, one can posit that two-nucleon transfer reactions are the specific probes of pairing in nuclei.

8.2.1 Fluctuations

The BCS solution of the pairing Hamiltonian was recasted by Bogoliubov ~~&~~ (1958) and ~~Valatin~~ Valatin (1958) in terms of quasiparticles,

$$\alpha_\nu^+ = U_\nu a_\nu^+ - V_\nu a_{\bar{\nu}}, \tag{8.2.9}$$

linear transformation inducing the rotation in (a^+, a)-space which diagonalizes the Hamiltonian (8.2.1).

The variational parameters U_ν, V_ν appearing in the above relation indicate that α_ν^+ acting on $|0>$ creates a particle in the state $|\nu>$ which is empty with a probability ~~$U_\nu^2 (\equiv (1-h_\nu)$~~), and annihilates a particle in the time reversal state $|\bar{\nu}>$ (creates a hole) which is occupied with probability $V_\nu^2(\equiv h_\nu)$ $= \left(1-(\varepsilon_\nu-\lambda)/E_\nu\right)^{1/2}/2$. Thus,

$$|BCS >= \Pi_{\nu>0}(U_\nu + V_\nu a_\nu^+ a_{\bar{\nu}}^+)|0 >, \tag{8.2.10}$$

is the quasiparticle vacuum, as $|BCS >\sim \Pi_\nu \alpha_\nu |0>$, the order parameter being

$$\alpha_0 = \sum_{\nu>0} U_\nu V_\nu. \tag{8.2.11}$$

Making use of these results we collect in Table 8.2.1 the spectroscopic amplitudes associated with the reactions $^{A+2}$Sn(p,t)^A Sn, for A in the interval 112-126. ~~as well as the spectroscopic amplitudes of other pairing vibrational modes (see below as well as F. Barranco et al. (2001), Gori et al. (2004)~~).

$U_\nu^2 (\equiv (1-h_\nu) = \left(1+(\varepsilon_\nu-\lambda)/E_\nu\right)^{1/2}/2)$

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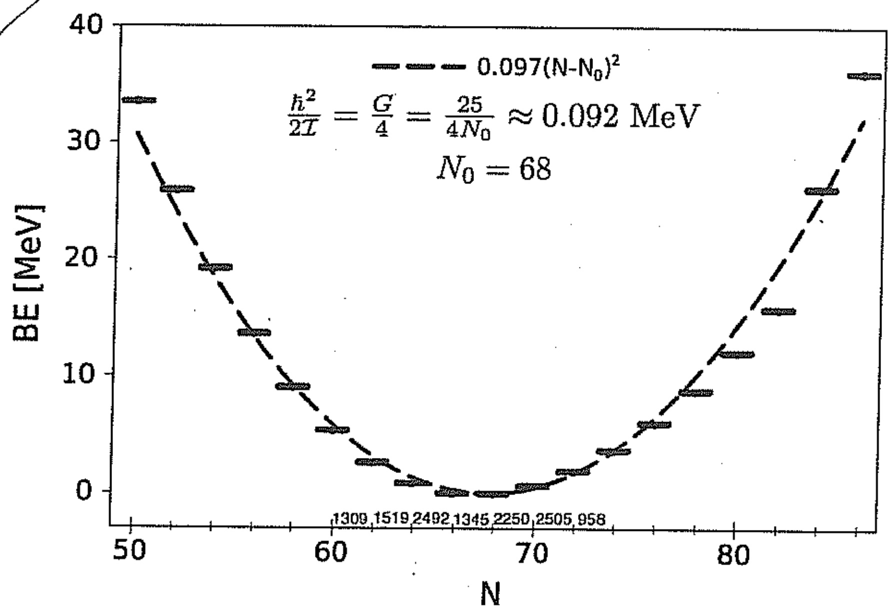

Figure 8.2.2: Pairing rotational band along the tin isotopes. The lines represent the energies calculated according to the expression $BE = B(^{50+N}\mathrm{Sn}) - 8.124N + 46.33$, subtracting the contribution of the single nucleon addition to the nuclear binding energy obtained by a linear fitting of the binding energies of the whole Sn-chain. The estimate of $\hbar/2\mathcal{I}$ was obtained using the single j-shell model (see e.g. Brink and Broglia (2005) App. H). The numbers given on the abscissa are the absolute value of the experimental gs $\rightarrow$ gs cross section (in units of μb; ~~see Table ??~~) .
after Potel etal (2011)

see Fig. 8.2.3

 space

	^{112}Sn	^{114}Sn	^{116}Sn	^{118}Sn	^{120}Sn	^{122}Sn	^{124}Sn
$1d_{5/2}$	0.664	0.594	0.393	0.471	0.439	0.394	0.352
$0g_{7/2}$	0.958	0.852	0.542	0.255	0.591	0.504	0.439
$2s_{1/2}$	0.446	0.477	0.442	0.487	0.451	0.413	0.364
$1d_{3/2}$	0.542	0.590	0.695	0.706	0.696	0.651	0.582
$0h_{11/2}$	0.686	0.720	1.062	0.969	1.095	1.175	1.222

$\sqrt{2j_\nu+1}\, U_\nu(A)V_\nu(A+2)$

Table 8.2.1: ~~Two–nucleon spectroscopic~~ amplitudes $\langle BCS(A)|P_\nu|BCS(A+2)\rangle = \sqrt{(2j_\nu+1)/2}\,U_\nu(A)V_\nu(A+2)$, associated with two–nucleon pick–up reactions connecting the ground states (members of a pairing rotational band) of two superfluid Sn–nuclei $^{A+2}$Sn(p,t)^{A}Sn(gs).

8.2.2 Pairing rotations

( )

The phase of the ground state BCS wavefunction may be chosen so that $U_\nu = |U_\nu| = U'_\nu$ is real and $V_\nu = V'_\nu e^{2i\phi}$ ($V'_\nu \equiv |V_\nu|$). Thus Schrieffer (1964)

$$|BCS(\phi)>_{\mathcal{K}} = \Pi_{\nu>0}(U'_\nu + V'_\nu e^{-2i\phi} a^+_\nu a^+_{\bar\nu})|0> = \Pi_{\nu>0}(U'_\nu + V'_\nu a'^+_\nu a'^+_{\bar\nu})|0> = |BCS(\phi=0)>_{\mathcal{K}'}, \quad (8.2.12)$$

where $a'^+_\nu = e^{-i\phi}a^+_\nu$ and $a'^+_{\bar\nu} = e^{-i\phi}a^+_{\bar\nu}$. This is in keeping with the fact that a^+_ν and $a^+_{\bar\nu}$ are single-particle creation operators which under gauge transformations (rotations in the 2D-gauge space of angle ϕ) induced by the operator $\mathcal{G}(\phi) = e^{-i\hat N\phi}$ and connecting the intrinsic and the laboratory frames of reference $\mathcal{K}$ and $\mathcal{K}'$ respectively, behave according to $a'^+_\nu = \mathcal{G}(\phi)a^+_\nu\mathcal{G}^{-1}(\phi) = e^{-i\phi}a^+_\nu$ and $a'^+_{\bar\nu} = \mathcal{G}(\phi)a^+_{\bar\nu}\mathcal{G}^{-1}(\phi) = e^{-i\phi}a^+_{\bar\nu}$, a consequence of the fact that $\hat N$ is the number operator and that $[\hat N, a^+_\nu] = a^+_\nu$. The fact that the mean field ground state ($|BCS(\phi)\rangle_{\mathcal{K}}$) is a product of oper-

$\mathcal{K}$

$\mathcal{K}'$

	^{A}Sn$(p,t)^{A-2}$Sn											
	V	W	V_{so}	W_d	r_1	a_1	r_2	a_2	r_3	a_3	r_4	a_4
p, ^{A}Sn $^{a)}$	50	5	3	6	1.35	0.65	1.2	0.5	1.25	0.7	1.3	0.6
d, $^{A-1}$Sn $^{b)}$	78.53	12	3.62	10.5	1.1	0.6	1.3	0.5	0.97	0.9	1.3	0.61
t, $^{A-2}$Sn $^{a)}$	176	20	8	8	1.14	0.6	1.3	0.5	1.1	0.8	1.3	0.6
	^{11}Li$(p,t)^{9}$Li											
	V	W	V_{so}	W_d	r_1	a_1	r_2	a_2	r_3	a_3	r_4	a_4
p, ^{11}Li $^{d)}$	63.62	0.33	5.69	8.9	1.12	0.68	1.12	0.52	0.89	0.59	1.31	0.52
d, ^{10}Li $^{b)}$	90.76	1.6	3.56	10.58	1.15	0.75	1.35	0.64	0.97	1.01	1.4	0.66
t, ^{9}Li $^{c)}$	152.47	12.59	1.9	12.08	1.04	0.72	1.23	0.72	0.53	0.24	1.03	0.83

Table 8.2.2:

ators - one for each pair state - acting on the vacuum, implies that (8.2.12) represents an ensemble of ground state wavefunctions averaged over systems with ...$N-2, N, N+2$... even number of particles. In fact, (8.2.12) can also be written in the form

To page 139 Table 8.1.1

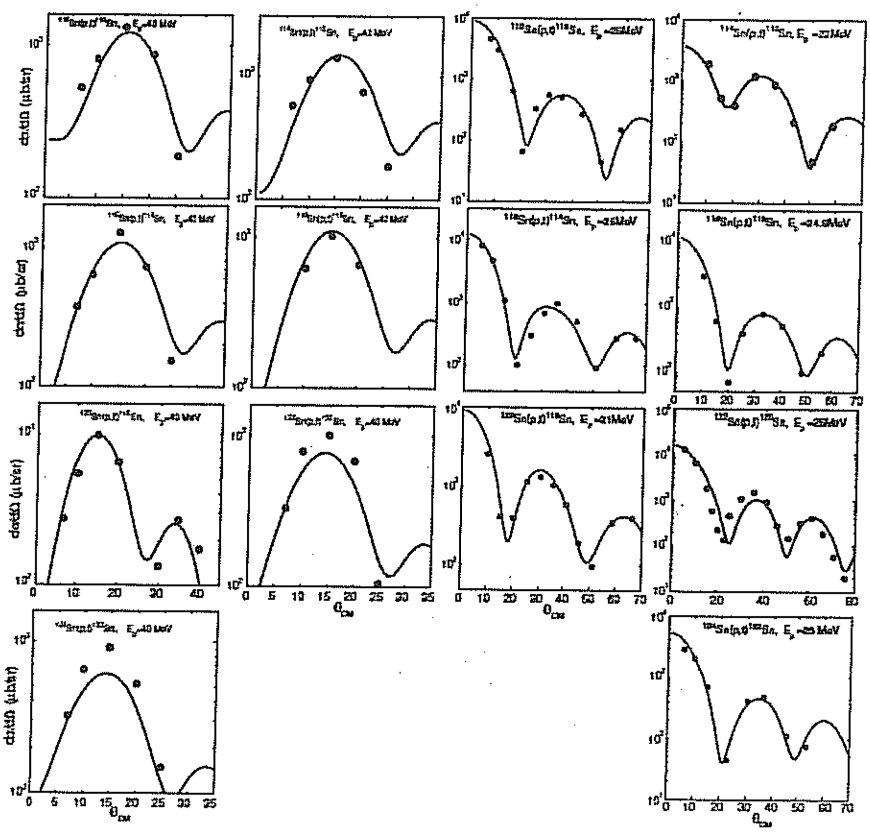

Figure 8.2.3: Predicted absolute differential $^{A+2}$Sn $(p, t)^{A}$Sn(gs) cross sections for bombarding energies E_p=40 MeV (in the two left columns) and 21 MeV $\leq E_p \leq$ 26 MeV (in the two right columns) in comparison with the experimental data(Potel et al (2013 a), (2013 b)).

$$|BCS>_{\mathcal{K}}=(\Pi_{\nu>0}U'_\nu)\,(1+...+\frac{e^{-(N-2)i\phi}}{\left(\frac{N-2}{2}\right)!}\left(\sum_{\nu>0}c_\nu a^+_\nu a^+_{\bar\nu}\right)^{\frac{N-2}{2}}+\frac{e^{-Ni\phi}}{\left(\frac{N}{2}\right)!}\left(\sum_{\nu>0}c_\nu a^+_\nu a^+_{\bar\nu}\right)^{\frac{N}{2}}$$

$$+\frac{e^{-(N+2)i\phi}}{\left(\frac{N+2}{2}\right)!}\left(\sum_{\nu>0}c_\nu a^+_\nu a^+_{\bar\nu}\right)^{\frac{N+2}{2}}+...)|0\rangle, \tag{8.2.13}$$

where $c_\nu = V'_\nu/U'_\nu$.

Adjusting the Lagrange multiplier λ (chemical potential, see Eqs. (8.2.9), (8.2.10) and associated text), one can ensure that the mean number of fermions ~~Eq. (22)~~ has the desired value N_0. Summing up, the BCS ground state is a wavepacket in the number of particles. In other words, it is a deformed state in gauge space defining a privileged orientation in this space, and thus an intrinsic coordinate system $\mathcal{K}'$ (Anderson (1958), Bohr (1964), Bès and Broglia (1966)). The magnitude of this deformation is measured by α_0.

8.2.3 Pairing vibrations

All the above arguments, point to a static picture of nuclear superfluidity which results from BCS theory. This is quite natural, as one is dealing with a mean field approximation. The situation is radically changed taking into account the interaction acting among the Cooper pairs (quasiparticles) which has been neglected until now, that is the term $-G(P^+ - \alpha_0)(P - \alpha_0)$ left out in the mean field (BCS) approximation leading to (8.2.3) ~~Anderson (1958), Bès and Broglia (1966)~~. This interaction can essentially be written as (for details see e.g. Brink and Broglia (2005) App. J)

$$H_{residual} = H'_p + H''_p, \tag{8.2.14}$$

where

$$H'_p = -\frac{G}{4}\left(\sum_{\nu>0}(U^2_\nu - V^2_\nu)(P^+_\nu + P_\nu)\right)^2, \tag{8.2.15}$$

and

$$H''_p = \frac{G}{4}\left(\sum_{\nu>0}(P^+ - P)\right)^2. \tag{8.2.16}$$

The term H'_p gives rise to vibrations of the pairing gap which (virtually) change particle number in $\pm$ 2 units. The energy of these pairing vibrations cannot be lower than 2Δ. They are, as a rule, little collective, corresponding essentially to almost pure two-quasiparticle excitations (see excited 0^+ states of Fig. 8.2.1).

The term H''_p leads to a solution of particular interest, displaying exactly zero energy, thus being degenerate with the ground state. The associated wavefunction is proportional to the particle number operator and thus to the gauge operator inducing an infinitesimal rotation in gauge space. The fluctuations associated with this zero frequency mode diverge, although the Hamiltonian defines a finite inertia. A proper inclusion of these fluctuations (of the orientation angle ϕ in gauge space) restores gauge invariance in the $|BCS(\phi)>_{\mathcal{K}}$ state leading to states with fixed particle number

$$|N_0\rangle \sim \int_0^{2\pi} d\phi e^{iN_0\phi}|BCS(\phi)>_{\mathcal{K}}\sim (\sum_{\nu>0} c_\nu a^+_\nu a^+_{\bar{\nu}})^{N_0/2}|0>. \qquad (8.2.17)$$

Table 8.2.2

These are the members of the pairing rotational band, e.g. ~~the~~ ground states ~~of~~ the superfluid Sn-isotope nuclei. These states provide the nuclear embodiment of Schrieffer's ensemble of ground state wavefunctions which is at the basis of the BCS theory of superconductivity. An example of such a rotational band is provided by the ground states of the Sn–isotopes (cf. Fig. 8.2.2). Making use of COOPER, namely of an implementation of two–nucleon transfer second order DWBA which includes successive and simultaneous transfer, corrected from non–orthogonality, of the spectroscopic amplitudes collected in table 8.2.1, and of global optical parameters from the literature (~~see~~...) the two–nucleon transfer absolute differential cross sections associated with the Sn–isotopes rotational band ~~(cf Fig. 8.2.3)~~ have been calculated. They are compared with the experimental findings in Fig. 8.2.3. ~~In table 8.2.2 the corresponding integrated absoute cross sections are collected in comparison with observations~~. Theory provides an overall account of the observation within experimental errors (cf. also Potel et al (2013a) and (2013b).

Appendix 8.A Bootstrap particle–phonon mechanism to spontaneously break gauge invariance

Arguably, one can posit that one now knows how to calculate the absolute differential two–nucleon transfer cross sections within experimental errors. One can then use this probe to test nuclear structure predictions, by direct comparison with experimental data. In particular a new embodiment of the Bardeen–Pines–Frölich mechanism to bind Cooper pairs [1] "... It has become fashionable... to assert... that once gauge symmetry is broken the properties of superconductors follow, with no need to inquire into the mechanism by which symmetry is broken... in 1957... the major problem was to show... how... gauge–invariant symmetry of the Lagrangian could be spontaneously broken due to interactions which were themselves gauge invariant". ~~L. Cooper in BCS:50 Years~~. Cooper (2011).

8.A.1 Gedanken eksperiment

Let us assume that one shines a very intense low-energy neutron beam on a ^{9}Li target. If these neutrons felt only the associated single-particle mean field, they will go by essentially as fast as they came in. However, part of the time these neutron

[1] ^1S_0NN–force $V(r_{12}) = \sum_\lambda V_\lambda P_\lambda(\cos\theta)$, $V_\lambda = \frac{2\lambda+1}{4\pi r_1^2}\delta(r_1 - r_2)$ contributions come from high λ–terms ($r_{12} < R/\lambda$). In the case of e.g. ^{11}Li small.

check R.

8.3 Heavy ion reactions

will bound themselves in presence of phonon (bosonic) excitations of quadrupole and of (pygmy) dipole character, produced also by the field the two neutron create themselves. The first of these collective modes is associated with vibrations of the (even) ^{8}He core, the $p_{3/2}$ proton being a spectator, the second resulting from the sloshing back and forth of the strongly non-local field of two (passing by) neutrons of the beam, together with the neutrons, and against the protons, of the core. made out of

Such possibility implies that, for a short time, of the order of the traversal time, the two (unbound) neutrons will move in a gas of bosonic excitations, also dipole pigmy resonances. Consequently, they can get dressed becoming heavier (lighter), these bosons as well as getting correlated by exchanging (information). ~~The first phenomenon is~~ associated, as discussed above, with phononic backflow (Pauli principle upflow) leading ~~leads~~ to ^{10}Li-like quasi-bound (s–wave) and resonant (p–wave) dressed single-particle states displaying parity inversion. The second phenomenon, mediated by phonon exchange, contributes in a major way to the glue which binds the neutron halo Cooper pair to the ^{9}Li core. The above described bootstrap phonon-exchange mechanism can be viewed as a novel microscopic embodiment of the Bardeen–Pines–Frölich-like processes to spontaneously break gauge invariance[2].

Appendix 8.B Table 1 PRL

the $1/2^-$ (2.69 MeV) first excited state of ^{9}Li can also be excited through a break up process in which one (see Fig. 8.B.1(f)), or both neutrons (see Fig. 8.B.1(g)) are forced into the continuum for then eventually one of them to fall into the $1p_{3/2}$ orbital of ^{9}Li and excite the quadrupole vibration of the core, in keeping with the fact that the main RPA amplitude of this state is precisely $X(1p_{3/2}^{-1}, 1p_{1/2})(\approx 1)$ (cf. ref F. Barranco et al. (2001)). The remaining channel populating the first excited state of ^{9}Li is associated with an inelastic process (see Fig. 8.B.1(h)): two–particle transfer to the ground state of ^{9}Li and Final State (inelastic scattering) Interaction (FSI) between the outgoing triton and ^{9}Li in its ground state, resulting in the inelastic excitation of the $1/2^-$ state.

Making use of the wavefunctions of reference F. Barranco et al. (2001) and of software developed on purpose to take into account microscopically all the different processes mentioned above, that is 9 different reaction channels and continuum states up to 50 MeV of excitation energy, we have calculated the corresponding transfer amplitude and associated probabilities p_l.

In Table 8.B.1 we display the probabilities $p_l = |S_l^{(c)}|^2$ associated with each of the processes discussed above, where the amplitude $S_l^{(c)}$ is related to the total cross section associated with each of the channels c by the expression (Satchler, 1980;

[2]Bootstrapping or booting. The term is often attributed to Rudolf Erich Raspe's story The surprising Adventures of Baron Münchausen, where the main character pulls himself out of a swamp by his hair. Early 19th century USA: "pull oneself over a fence by one's bootstraps"

as Broglia and Winther (2005)), breakup and inelastic channels were calculated, and the results displayed in Figs. 8.B.2 and 8.B.3 and in Table 8.B.1. Theory provides an overall account of the experimental findings. In particular, in connection with the $1/2^-$ state, this result essentially emerges from cancellations and coherence effects taking place between the three terms contributing to the multistep two–particle transfer cross section (see Fig. 8.B.3), tuned by the nuclear structure amplitudes associated with the process shown in Fig. 8.B.1 (e) as well as Eqs. (7.2.110)–(6.G.6). In fact, and as shown in Figs. 8.B.2 and 8.B.3, the contributions of inelastic and break up processes (Figs. 8.B.1(f),(g) and (h) respectively) to the population of the $1/2^-$ (2.69 MeV) first excited state of ^{9}Li are negligible as compared with the process depicted in Fig. 8.B.1(e). In the case of the breakup channel (Figs. 8.B.1(f) and 8.B.1(g)) this is a consequence of the low bombarding energy of the ^{11}Li beam (inverse kinematics), combined with the small overlap between continuum (resonant) neutron $p_{1/2}$ wavefunctions and bound state wavefunctions. In the case of the inelastic process (Fig. 8.B.1(h)), it is again a consequence of the relative low bombarding energy. In fact, the adiabaticity parameters ξ_C, ξ_N (see eqs. (IV.12) and (IV.14) of ref. Broglia and Winther (2005)) associated with Coulomb excitation and inelastic excitation in the t+^{9}Li channel are larger than 1, implying an adiabatic cutoff. In other words, the quadrupole mode is essentially only polarized during the reaction but not excited. The situation is quite different in the case of the virtual process displayed in Fig. 8.B.1 (e). Being this an off–the–energy shell process, energy is not conserved, and adiabaticity plays no role. It would be very interesting to challenge the results of the above calculations with eventual data for the same reaction at higher incident energy.

Appendix 8.C Modified formfactor associated with the reactions H(^{11}Li,^{9}Li(gs)^{3}H and H(^{11}Li,^{9}Li($1/2^-$; 2.69 MeV)^{3}H state.

(pushing model calculation pigmy centroid (talk Pisa))

Appendix 8.D Software

COOP, ONE, KNOCKOUT software descriptions.

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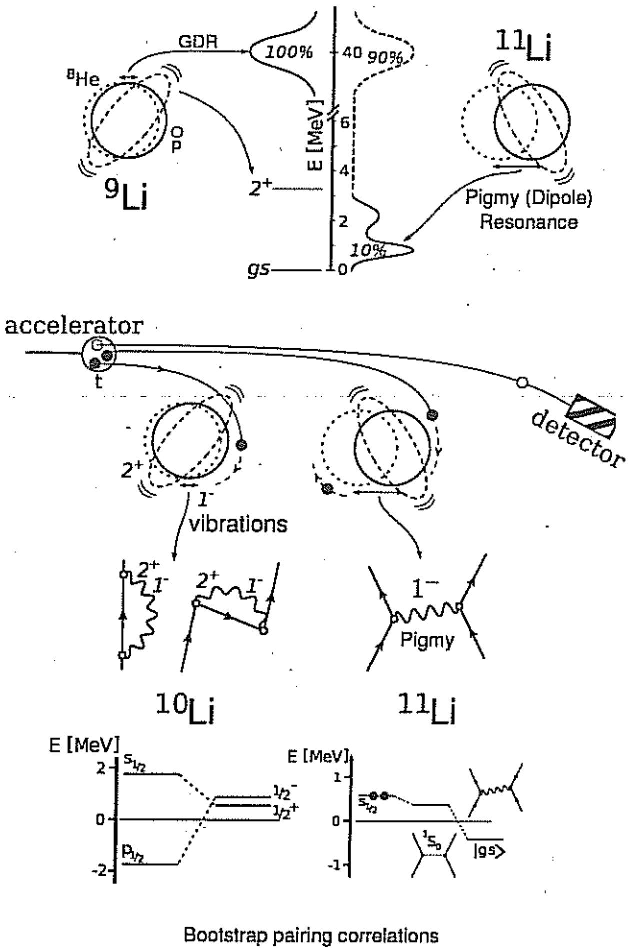

Figure 8.A.1: The dressing of single–particle levels by collective vibrations and the renormalization of the bare NN–interaction, in particular of the pairing interaction through the exchange of these modes between nucleons moving in time reversal states lying close to the Fermi energy, play a central role in nuclear structure. (a) In particular, in the case of the single Cooper pair system ^{11}Li, most of the glue is rovided by the exchange of the pigmy resonance, namely a low–lying isovector dipole vibration. (b)The pigmy resonance is a chunk of the GDR of the core ^{9}Li in which protons and neutrons move out of phase, a mode which is intimately related to the spontaneous symmetry breaking of space inhomogeneity associated with the fact that the center of mass of a finite system like the atomic nucleus, specifies a privileged position in space. While $^9_3$Li$_6$ is bound, $^{10}_3$Li$_7$ is not. (c,d) Through renormalization processes, the $p_{1/2}$ bound state is shifted to higher energies from that predicted by a standard mean field potential, while the $s_{1/2}$ continuum state is lowered to an energy close, but below that of the $p_{1/2}$ state (see also ...)

Fig. 6.1.3