The low-lying states of closed shell nuclei can be interpreted as harmonic quadrupole or octupole collective vibrations (Fig. 4) described by the Hamiltonian

$$H_{coll} = \sum_{\lambda\mu}\left(\frac{1}{2D_\lambda}|\Pi_{\lambda\mu}|^2 + \frac{C_\lambda}{2}|\alpha_{\lambda\mu}|^2\right) \quad (1)$$

Following Dirac (1935) one can quantize the oscillatory motion introducing boson creation (annihilation) operator $\Gamma^+_{\lambda\mu}$ ($\Gamma_{\lambda\mu}$) obeying

$$[\Gamma_\alpha, \Gamma^+_{\alpha'}] = \delta(\alpha,\alpha'), \quad (2)$$

leading to

$$\hat{\alpha}_{\lambda\mu} = \sqrt{\frac{\hbar\omega_\lambda}{2C_\lambda}}\left(\Gamma^+_{\lambda\mu} + (-1)^\mu \Gamma_{\lambda\mu}\right) \quad (3)$$

and a similar expression for the conjugate momentum variable $\hat{\Pi}_{\lambda\mu}$, resulting in

$$\hat{H}_{coll} = \sum_{\lambda\mu} \hbar\omega\left(\Gamma^+_{\lambda\mu}\Gamma_{\lambda\mu} + 1/2\right), \quad (4)$$

The frequency is $\omega_\lambda = (C_\lambda/D_\lambda)^{1/2}$, while $(\hbar\omega_\lambda/2C_\lambda)$ is the amplitude of the zero-point fluctuation of the bosonic vacuum state $|0\rangle_B$, $\Gamma^+_{\lambda\mu}|0\rangle_B$ being the one-phonon state. (5)

The ground and low-lying states of nuclei with one nucleon outside closed shell

The coupling between surface oscillation and single-particle motion, namely the particle vibration coupling (PVC) Hamiltonian $\delta U$ ($\equiv H_{coupling}$; Fig. 5) is a consequence of the overcompletness of the basis. Taken proper care of $\delta U$, that is, diagonalizing the Hamiltonian $H_c$ making use of the rules of Nuclear Field Theory (NFT) to be discussed below, one obtains a a solution of the total Hamiltonian,

$$H = H_{sp} + H_{coll} + H_c \qquad (13)$$

In fact, within the framework of NFT, single-particle are to be calculated as the Hartree-Fock solution of the NN-interaction $v(|\vec{r}-\vec{r}'|)$ (Fig. 6), in particular

$$U(r) = \int d^3r' \rho(r')\, v(|\vec{r}-\vec{r}'|) \qquad (14)$$

being the Hartree field expressing the selfconsistency between density $\rho$ and potential $U$ (Fig. 6(b)(1) and (3)), while vibrations are to be calculated in the Random Phase approximation (Fig. 7), extending this selfconsistency to fluctuations $\delta\rho$ of the density and $\delta U$ of the mean field, that is,

$$\delta U = \int d^3r' \delta\rho(r')\, v(|\vec{r}-\vec{r}'|). \qquad (15)$$

The associated dispersion relation and corresponding wavefunctions provide the unitary transformation leading to,

$$\hat{F} = \hat{\alpha} \qquad (16)$$

Because of quantal zero point fluctuations, a nucleon propagating in the nuclear medium moves through clouds of bosonic and fermionic virtual excitations to which it couple ($H_c$), becoming dressed and acquiring an effective mass, charge, etc. (Fig. 8). Vice versa, vibrational modes can become renormalized through the coupling to dressed nucleons which, in intermediate virtual states, can exchange the vibrational clothing with the second fermion (hole state) and renormalize the PVC vertex (Fig. 9) (Barranco et al (2004)) charge exchange, and one-and two-particle transfer reactions.

One can choose to priviledge one among this rich variety of elementary modes of excitation, for example, independent particle motion and, making of the shell model, eventually the so called no core shell model, understood within this context as a full diagonalization of the NN-interaction in the single-particle basis, attempt at describing the whole of structure and reaction.

Another possibility is to use the elementary modes of excitation basis states and nuclear field theory to deal with the overcompletness and Pauli principle violations of the basis states.

From a systematic collaboration between the two approaches and stong experimental input, it is likely ~~that~~ shell model calculations can help at individuating the proper interaction leading to realistic Hartree-Fock mean fields and collective RPA particle-hole and pairing vibrational modes. As a possible return of such input, nuclear field theory will eventually be able to provide shell model friendly microscopic collective modes of excitation.

The possible outcome could be that of being able to coin into few physical concepts the elements needed to carry out ab initio calculations which are largely independent of the basis chosen, and of truly predictive theries of structure and reactions, in which the physical content is simple to apprehend and visualize.

Milano 1/2/18

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