f		$\sigma(gs \to f)/\sigma(gs \to gs)$	Table A
J^π	E_x		
$0^+(gs)$	0	100	pair removal (hh)
3^-	2.62	21	(ph) collective mode
5^-	3.20	45	(ph) collective mode
0^+	4.87	45	pair addition

Table 3.B.1: Relative two–nucleon transfer cross sections $\sigma(^{206}\text{Pb}\ (t,p)^{208}\text{Pb}(f)/\sigma(^{206}\text{Pb}\ (t,p)^{208}\text{Pb(gs)})$ integrated in the range $5° - 175°$ of cm angles. (After Broglia, R.A. et al. (1973), Table A. VIII b)

J^π	$\sigma(gs \to f)$ (mb)	$\sigma(gs \to f)/\sigma(gs \to gs)$
$0^+(gs)$	2250±338	100
2^+	613± 92	27

Table 3.B.2: Absolute cross section associated with the reaction $^{120}\text{Sn}\ (p,t)^{118}\text{Sn}$ to the ground state and first excited state integrated in the range $7.6° < \theta_{cm} < 69.7°$. After Guazzoni, P. et al. (2008).

some typical orders of magnitude: $E_{corr} \approx -1.2$ MeV and $\Delta \approx 1.2$ MeV for medium heavy nuclei lying along the stability valley. Thus

$$\xi = \frac{\hbar v_F}{\pi |E_{corr}|} \approx \frac{\hbar c(v_F/c)}{3.8\text{ MeV}} \approx \frac{200\text{ MeV fm} \times 0.27}{3.8} \approx 14\text{ fm}. \tag{3.B.14}$$

In the case of ^{11}Li, $E_{corr} \approx -500$ keV and $v_F/c \approx 0.16$. Thus

$$\xi \approx \frac{200\text{ MeV fm} \times 0.16}{1.6} \approx 20\text{ fm}. \tag{3.B.15}$$

Generalized quantality parameter

$$q_\xi = \frac{\hbar^2}{2m\xi^2}\frac{1}{|E_{corr}|} = \begin{cases} \frac{20\text{ MeV fm}}{14^2 \times 2 \times 1.2\text{ MeV}} \approx 0.04 \\ \frac{20\text{ MeV fm}}{20^2 \times 0.5\text{ MeV}} \approx 0.10 \quad {}^{11}\text{Li} \end{cases} \tag{3.B.16}$$

The parallel which can be traced between Cooper pairs and correlated particle–hole excitations is further testified by the fact that two–nucleon transfer reaction do excite quite strongly also these modes (see Tables 3.B.1 and 3.B.2).

H – H

3.B.3 tunneling probabilities

The state $|BCS(\phi)\rangle_\mathcal{K} = \prod_{\nu>0}(U'_\nu + V'_\nu e^{-2i\phi}a^\dagger_\nu a^\dagger_{\bar\nu})|0\rangle$ (see Eq. (3.7.17)) displays off–diagonal–long–range–order (ODLRO) because each pair is in a state $(U'_\nu + V'_\nu e^{-2i\phi}a^\dagger_\nu a^\dagger_{\bar\nu})|0\rangle$ with the same phase as all the others. In fact, the above wavefunction leads to a two–particle density matrix with the property $\lim_{\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}_3,\mathbf{r}_4\to\infty}\phi(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}_3,\mathbf{r}_4) \neq$

pp 289a + 289b handwritten

(H) The parallel between $\beta=0$ and $\beta=\pm 2$ modes emerged already in the analysis*) of the $^{206}Pb(t,p)^{208}Pb$ which provided the experimental confirmation of the theoretical predictions concerning pairing vibrations (see Fig. 2.5.1) (see Fig. 2.1.1 (b) transition denoted a)

In particular concerning the population of the pair addition mode $|gs(^{208}Pb)\rangle$ predicted at an excitation energy of 4.9 MeV in**) ^{208}Pb, and observed***) at 4.87 MeV. Now, because the lowest excited state populated with a sizable crossection (21±3%) of ~~that~~ associated with the ground state transition (and thus populating the pair removal mode $|gs(^{206}Pb)\rangle$, see Fig. 2.1.1 (b) transition denoted r) was observed at $E_x = 2.619$ MeV, much effort and time was dedicated to experimentally disentagled an eventually $J^\pi=0^+$, essentially degenerated with the octupole vibration at $E_x = 2.62 \pm 0.01$ MeV. This was in keeping with the idea that around close shell nuclei, only pairing vibration were strongly populated, this not being the case for $\beta=0$ collective surface state.

The analysis of the data*) of the papers reporting the results dispell such believe, also in the title. In fact, collective

*) R. A. Broglia and C. Riedel (1967) **) Bes and Broglia (1966) pairing vibrations

**) see Fig. 5 Bes and Broglia (1966)

***) Bjerregaard et al Nucl. Phys. 89, 337 (1966)

Be $0^{*}+$

$\beta=0$ and $\beta=\pm 2$ are similarly strongly populated in two-nucleon transfer reactions the main difference that while the Y-components (ground state correlations) see Tables 2.5.2 and 2.5.3 contribute constructively coherent to the cross section, the same components of the $\beta=0$ modes give rise to destructive coherence. Again an example of the competition for phase space between pairing and (dynamical) deformation (see e.g. Fig. 1.8.2). These results originally found in Pb were extended to other mass regions and $\beta=0$ modes.*)

Within this scenario one can look forward to the test of the $^{9}Li(t,p)^{11}Li$ predictions, concerning the population of both the ground state and the DPR ($E_x \lesssim 1$ MeV). Also to compare to which extent the 0^{*+} ^{12}Be excited state and 1⁻ excitation on top of it, can be viewed as related elementary modes of excitation (see Fig. 3.8.1)

to p. 289 (H)

*) Broglia et al (1971) Broglia + Riedel + Udagawa null (t,p)

R check

0 under the assumption that $r_{12}, r_{34} < \xi$, $(\mathbf{r}_1, \mathbf{r}_2)$ and $(\mathbf{r}_3, \mathbf{r}_4)$ being the coordinates of a Cooper pair, r_{ij} the relative modulus of it and ξ the coherence length[113].

Bringing the above argument into reaction implies, as shown in Eq. (3.2.19), that $P_2 = P_1$. In keeping with the parallel made with superconductors (see Fig. 3.A.5) one can mention that Josephson showed that at very low temperatures, the pair current is equal to the single–particle current at an equivalent voltage[114] $\pi\Delta/2e$. $\frac{\pi}{4}\frac{2\Delta}{e}$ Experiments observed values of maximum Josephson current of ≈ 1 mA, consistent with junctions resistances of 1 Ω per unit area (see Sect. 3.6). The importance of this result concerning the mechanism at the basis of Cooper pair transfer is connected with the fact that the probability of one–electron–tunneling across a typical dioxide layer giving rise to a weak $S-S$ coupling *1 is of the order of 10^{-10}. Consequently, simultaneous pair transfer between two superconductors (S), with a probability $(10^{-10})^2$ ~~cannot~~ would not be observed[115]. Thus, the Josephson current of carriers of charge $2e$ results from the tunneling of a Cooper pair partner at a time, equally pairing correlated when they are in the same superconductor (S) within the correlation length ξ, than when each of them is on a different of the two weakly coupled S.

One could argue that in the reaction ^{120}Sn$(p,t)^{118}$Sn(gs) one can hardly consider the triton as a pairing condensate. While this is correct one can hardly claim either that six–eight Cooper pairs (^{120}Sn) make a *bona fide* one. In any case, when one experimentally observes such unexpected phenomenon ($\sigma_{2n} \approx \sigma_{1n}$), one is likely somewhat authorized at using similar concepts[116].

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3.B.4 Nuclear correlation (condensation) energy

The BCS mean field can be written as[117]

$$H_{MF} = U + H_{11} \tag{3.B.17}$$

where

$$U = 2\sum_{\nu>0}(\epsilon_\nu - \epsilon_F)V_\nu^2 - G\alpha_0^2 \tag{3.B.18}$$

while

$$H_{11} = \sum_{\nu>0} E_\nu(\alpha_\nu^\dagger\alpha_\nu + \alpha_{\bar\nu}^\dagger\alpha_{\bar\nu}), \tag{3.B.19}$$

E_ν being the quasiparticle energy, and $\alpha_\nu^\dagger$ the quasiparticle creation operator. The pair-correlation energy is the difference between the energy with and without pair-

[113] See e.g. Ambegaokar (1969) and refs. therein, see also Potel et al. (2017).

[114] In the case of Pb $\Delta = 1.4$ meV (see footnote [104]) this voltage is $(\pi \times 1.4/2) \times 10^{-3} \times$eV/e$\approx$ 2mV (see e.g. McDonald (2001)).

[115] See e.g. McDonald (2001).

[116] Anderson (1972).

[117] Brink, D. and Broglia (2005), Appendix G.

check R

*) Pippard (2012)

ing. The energy including pair correlations is

$$E_p = 2\sum_{\nu>0} |V_\nu|^2 \epsilon_\nu - G\,|\alpha_0|^2 \tag{3.B.20}$$

while the energy without correlation is

$$E_0 = \sum_{\nu>0} |V_\nu^0|^2 \epsilon_\nu. \tag{3.B.21}$$

The occupation probabilities $|V_\nu^0|$ are unity below the Fermi energy level and zero above. In both Eqs. (3.B.20) and (3.B.21) the Fermi energy has to be chosen to give the correct number of particles. The pairing correlation energy is

$$E_{corr} = E_S - G\,|\alpha_0|^2, \tag{3.B.22}$$

where

$$E_S = \sum_{\nu>0} 2(|V_\nu|^2 - |V_\nu^0|^2)\,\epsilon_\nu. \tag{3.B.23}$$

The pairing energy $-G|\alpha_0|^2$ is partially canceled by the first term describing the fact that, in the BCS ground state, particles moving in levels close to the Fermi energy are partially excited across the Fermi surface, in keeping with the fact that V_ν^2 changes smoothly from 1 to 0 around ϵ_F, being 1/2 at the Fermi energy.

In other words, the energy gain resulting from the potential energy term, where G is the pairing coupling constant while $|\alpha_0|$ measures the number of Cooper pairs particle is partially compensated by a quantal, zero point fluctuation like term. It can, in principle, be related to the Cooper pair kinetic energy of confinement $T_\xi = \frac{\hbar^2}{2m}\frac{1}{\xi^2}$ already discussed in connection with the generalized quantality parameter, through the relation $2|\alpha_0|T_\xi$ (for one type of nucleons), in keeping with the fact s that (3.B.23) is expressed in terms of single ~~nucleon~~ energies. Let us make a simple estimate which can help at providing a qualitative example of the above argument, and consider for the purpose the nucleus ^{223}Ra and $G \approx (22/A)$ MeV, $|\alpha_0| \approx 5$ and $\xi \approx 20$ fm: $T_\xi \approx 0.1$ MeV, $2\times(2\times|\alpha_0|\times T_\xi) = 2$ MeV, $2\times(-G|\alpha_0|^2) = -5$ MeV[118] (factors of 2,both protons and neutrons). The resulting pairing correlation energy thus being $E_{corr} = -3$ MeV. This number can be compared with a "realistic" estimate provided by the relation[119]

$$E_{corr} = -\frac{g\Delta^2}{4}, \tag{3.B.24}$$

[118]This quantity, but divided by 2, i.e. -2.5 MeV can be compared with the effective pairing matrix element $v = \left(\frac{\Delta_\pi^2+\Delta_\nu^2}{4} \approx -2.9\right)$ MeV, operative at level crossing in the calculation of the inertia of the exotic decay ^{223}Ra$\rightarrow^{14}$Ca+^{209}Pb, cf. Brink, D. and Broglia (2005) p.159 and refs. therein.

[119]Brink, D. and Broglia (2005).

where $g_n = N/16$ MeV^{-1} and $g_p = Z/16$ MeV^{-1}. Taking into account both types of particles $g = g_n + g_p = A/16$ MeV^{-1} and making use of $\Delta = 12/\sqrt{A}$ MeV, one obtains $E_{corr} = -\frac{143}{64}$ MeV =-2.23 MeV. With the help of E_{corr} and T_ξ, one can estimate the generalized quantality parameter, $q_\xi = T_\xi/|E_{corr}| = 0.1/2 \approx 0.05$, as well as make a consistency check on the value of ξ used, namely $\hbar v_F/(2|E_{corr}|) \approx$ 1~~4~~.5 fm.

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3.B.5 Summary

The nuclear Cooper pair not only is forced to exist in a very strong external field, the HF mean field, of very reduced dimensions as compared to the correlation length. Because of spatial quantization, it is also forced to exist on selected orbitals of varied angular momentum and parity[120] (App. 3.D).

Correlations, in particular pairing correlations within such constrains will have opposite and apparently contradictory effects. As an example, let us consider two neutrons moving around ^{9}Li in the $s^2_{1/2}(0)$ or in the $p^2_{1/2}(0)$ (pure) configurations. In such a situation, fixing one of the neutrons of the pair at a radius r_1, the other one will display equal probability to be close or in the opposite side of the nucleus ($\theta_{12} = 0°$ or $\theta_{12} = 180°$), the average distance between neutrons being of the order of $d = \left(\frac{\frac{4\pi}{3}R^3}{A}\right)^{1/3} \approx 2$ fm (3.3 fm using $R(^{11}\text{Li})$=4.6 fm instead of $R = 1.2A^{1/3}$ fm).

By exchanging the PDR between the two outer neutrons, the halo Cooper pair ~~becomes~~ is stabilized, becoming weakly bound (S_{2n} = 380 keV). Assuming that the odd proton $p_{3/2}(\pi)$ plays only a spectator role, the ground state of ^{11}Li can be written as $|^{11}\text{Li}(gs)\rangle = |\tilde{\tilde{0}}\rangle \otimes |p_{3/2}(\pi)\rangle$, the neutron halo Cooper pair state being

$$|\tilde{\tilde{0}}\rangle = |\tilde{0}\rangle + 0.71|(p_{1/2}, s_{1/2})_{1^-} \otimes 1^-; 0\rangle + 0.1|(s_{1/2}, d_{5/2})_{2^+} \otimes 2^+; 0\rangle. \quad (3.B.25)$$

where

$$|\tilde{0}\rangle = 0.45|s^2_{1/2}(0)\rangle + 0.55|p^2_{1/2}(0)\rangle + 0.04|d^2_{5/2}(0)\rangle \quad (3.B.26)$$

Studying this state microscopically one observes two contrasting effects taking place:

1) The two neutrons will switch from a regime of single–particle motion to that of independent pair motion and adopt the configuration proper to a Cooper pair (radial motion against each other) and expand radially consistent with the fact that its mean ~~surface~~ radius, ($\xi = \langle r^2\rangle^{1/2}_{\text{Cooper}} \approx \frac{\hbar v_F}{\pi E_{corr}} \approx \hbar c(v_F/c)/\pi E_{corr} \approx \frac{200 \text{ MeV fm} \times 0.16}{\pi \times 0.5 \text{ MeV}}$ = 20 fm, $\epsilon_F(^{11}\text{Li}) \approx$ 13 MeV $\rightarrow v_F/c \approx 0.16$). This can be appreciated by the fact that

square

is ≈ 20 fm

[120] Some of them allowing for pure $j^2(0)$ configurations, with large (little) probability of $L = 0$ relative motion, which behaves as hot (cold) orbitals, their contribution to pairing correlations and to two–nucleon transfer reactions being very inhomogeneous, at variance of the situation found in solid state (see e.g. Broglia (2005)). This is also the reason why the second–order–like phase transition normal–superfluid taking place in nuclei as e.g. a function of the number of pairs of nucleons moving outside closed shells, affected by strong pairing fluctuations, are conspicuously blurred as compared to the $N \rightarrow \infty$ case.

the radius of ^{11}Li is much larger than that expected from systematic ($R(^{11}\mathrm{Li}) = 4.6$ fm, corresponding to an effective mass number $A_{eff} \approx 60$)).

2) Interference between positive ($(-1)^l = +1$) and negative ($(-1)^l = -1$) single–particle based $|(l, j)^2_0\rangle$ configurations, constructive at $\theta = 0°$ and destructive at $\theta = 180°$, $\theta = r_1\hat{r}_2$ been the relative angle between the coordinates $\mathbf{r}_1$ and $\mathbf{r}_2$ of the Cooper pair partners. In other words the two nucleons will tend to be close to each other, in particular on the nuclear surface. As can be seen from (3.B.26), this effect is extreme in the case of the ground state of ^{11}Li. Now, such an effect has not much to do with pairing, BCS pairing at it and thus superconductivity[121], but with the properties of the nuclear mean field, result of spatial quantization which not only distorts the Cooper pair through isotropic confinement, but through admixtures of odd and even parity states controlled also by the very strong spin orbit term. Within this context see Sect. 6.6.3.

Summing up, the difficulties of understanding pairing in nuclei as compared with condensed matter is (at least threefold): **a**) the bare interaction is attractive, a fact which lead to the idea that pairing force is short range and delayed the discovery of the other half of pairing, namely the retarded, medium polarization interaction, for a long time[122]; **b**) particle number is small, thus pairing vibrations are important, and renormalize in a conspicuous way the variety of nuclear phenomena, in particular single–particle motion. The fact that such effects are still not being really considered is testified by the fact that a serious treatment of multipole pairing vibrations is still missing; **c**) spatial quantization leading to phenomena which by themselves can be very interesting[123], but which are not directly related to pairing correlations ~~but~~ and nonetheless has conditioned nuclear structure research, let alone reaction mechanism studies and the physics emerging from their interweaving.

[121] Within this context it is of notice that in condensed matter literature Cooper pairs are viewed as fragile, extended di–electron entities, overlapping with a conspicuous number of other pairs, and displaying a delicate "rigid" quantal correlation between partners (generalized quantality parameter) and among Cooper pairs (off diagonal long range order). In fact, Weisskopf's representation of the radial (opposite) motion of electrons provides a useful picture of Cooper pair internal dynamics. In other words, approaching to or recessing from each other does not favour a particular anisotropic configuration , the two electrons being at the mean square radius of the Cooper pair, i.e. the coherence length ξ.

[122] A long held picture: pairing plus long range force, i.e. pairing short range, many (high) relative angular momenta contributing (Kisslinger and Sorensen (1963); Soloviev (1965); Mottelson (1962, 1998)).

[123] Bertsch, G. F. et al. (1967); Ferreira, L. et al. (1984); Lotti et al. (1989); Matsuo (2006); Matsuo, M. (2013)

Appendix 3.C Absolute Cooper pair tunneling cross section: quantitative novel physics at the edge between stability and chaos

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In ~~the~~ study of many–body systems, in particular of finite many–body systems (~~F~~MBS) like the atomic nucleus, much can be learned from symmetries (group theory) as well as from the general phenomena of of spontaneous symmetry breaking. However, it is the texture of the associated emergent properties, concrete embodiment of symmetry breaking (potential energy) and of its restoration (fluctuations, collective modes), which provides insight into the eventual new physics. In fact, when one understands the many–body under study in terms of the detailed motion of single–particles (nucleons) and collective motion, taking properly into account their couplings and associated zero point fluctuations, is that one can hope to have reached a solid, quantitative, understanding of the problem and of its solutions. Even more, that these solutions are likely transferable, at profit, to the study of other ~~F~~MBS like e.g. metal clusters, fullerenes[124], quantum dots[125], and eventually proteins[126], let alone the fact that one can make predictions. Predictions ~~which~~, in connection with the study of halo nuclei, in particular of pairing[127] in such exotic, highly extended systems lying at the nucleon drip line, involve true novel physics[128]. Within this context one can quote from Leon Cooper's contribution to the volume[129] BCS: 50 years: "It has become fashionable... to assert... that once gauge symmetry is broken the properties of superconductors follow... with no need to inquire into the mechanism by which the symmetry is broken[130]. This is not ... true, since broken gauge symmetry might led to molecule–like and a Bose–Einstein rather than BCS condensation... in 1957... the major problem was to show... how... an order parameter or condensation in momentum space could come about... to show how... gauge–invariant symmetry of the Lagrangian could

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soft matter, in particular

[124] Cf. e.g. Gunnarsson (2004), Broglia et al. (2004) and refs. therein.

[125] Lipparini (2003).

[126] Broglia, R. A. (2013).

[127] Cf. e.g. Broglia, R. A. and Zelevinsky, V. (2013).

[128] Cf. e.g. Barranco, F. et al. (2001); Tanihata, I. et al. (2008); Potel et al. (2010) and references therein.

add many more refs.

[129] Cooper (2011).

[130] ~~Detailed~~ Quoting (Weinberg (2011)): "... In consequence of this spontaneous symmetry breaking, products of any even number of electron fields have non–vanishing expectation values in a superconductor, though a single electron field does not. All of the dramatic exact properties of superconductors –zero electric resistance, the expelling of the magnetic fields from superconductors known as the Meissner effect, the quantization of magnetic flux through a thick superconducting ring, and the Josephson formula for the frequency of the ac current at a junction between two superconductors with different voltages– follow from the assumption that electromagnetic gauge invariance is broken in this way, with no need to inquire into the mechanism by which the symmetry is broken." The above quotation is similar to saying that once the idea of a double DNA helix was thought, all about inheritance was solved and known, and that one could forget the X–ray plates of Rosalind Franklin, Maurice Wilkins and collaborators, let alone how DNA and proteins interact with each other (cf. e.g. Stent (1980) and references therein).

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be spontaneously broken due to interactions which were themselves gauge invariant".

Nuclear physics has brought this quest a step further. This time in connection with the "extension" of the study of BCS condensation to its origin, a single Cooper pair in the rarified atmosphere resulting from the strong radial (isotropic) deformation observed in light halo, exotic nuclei in general, and in ^{11}Li in particular. During the last few years, the probing of this system in terms of absolute two–nucleon transfer (pick–up) reactions, has made this field a quantitative one, errors below the %10 limit being the rule. This achievement which has its basis on ~~e.g.~~ the remarkable experiments of Tanihata, I. et al. (2008), is also the result of the combined effort made in treating the structure and reaction aspects of the subject, two sides of the same physics, on equal footing.

3.C.1 Saturation density, spill out and halo

In the incipit to the Chapter on bulk properties of nuclei of Bohr and Mottelson[131] one reads: "The almost constant density of nuclear matter is associated with the finite range of nuclear forces; the range of the forces is r_0 (where r_0 enters the nuclear radius in the expression $R = r_0 A^{1/3}$) thus small compared to nuclear size. This "saturation" of nuclear matter is also reflected in the fact that the total binding energy of the nucleus is roughly proportional to A. In a minor way, these features are modified by surface effects and long–range Coulomb forces acting between the protons".

Electron scattering experiments (see the figure 2-1, p. 159 of the above reference) yield

$$\rho(0) = 0.17\,\text{fm}^{-3}. \qquad (3.C.1)$$

Thus, one can posit that

$$\frac{4\pi}{3} R_0^3 \rho(0) = A, \qquad (3.C.2)$$

leading to

$$r_0 = \left(\frac{3}{4\pi}\frac{1}{\rho(0)}\right)^{1/3} \approx 1.12\,\text{fm}. \qquad (3.C.3)$$

Because the above relations imply a step function distribution, we have to add to (3.C.3) the nucleon spill out[132] $(a_0/R_0)\ln 2 \approx (0.5/5.5)\ln 2 \approx 0.06\,(A = 120))$ associated with the fact that a more realistic distribution is provided by a Fermi function of diffusivity $a_0 \approx 0.5$ fm. Thus $r_0 = (1.12 + 0.06)$ fm ≈ 1.2 fm. In the

[131] Bohr and Mottelson (1969) p. 139.

[132] Bertsch and Broglia (2005).

case of the nucleus ^{11}Li, observations indicate a mean square (gyration radius[133]) radius $\langle r^2 \rangle^{1/2} = 3.55 \pm 0.1$ fm[134]. Thus

$$R(^{11}\text{Li}) = \sqrt{\frac{5}{3}} \langle r^2 \rangle^{1/2} \approx 4.58 \pm 0.13 \,\text{fm}. \tag{3.C.4}$$

Making use of the relation $R(= R_0) \approx 1.2A^{1/3}$ fm, the quantity (3.C.4) leads to $(4.58/1.2)^3 \approx 56$, an effective mass number larger five times the actual value $A = 11$. To be noted that the actual mass number predicts a "systematic" value of the nuclear radius $R_0(^{11}\text{Li}) \approx 2.7$ fm.

it

The above results testify to a very large "**isotropic radial deformation**", or halo region (skin), in keeping with the fact[135] that $R(^{11}\text{Li})$-$R_0(^9\text{Li})$=$R_0(^9\text{Li})(\frac{R(^{11}\text{Li})}{R_0(^9\text{Li})} - 1) = 0.83R_0(^9\text{Li})$. In other words, ^{11}Li can be viewed as made out of a normal ^{9}Li core and of a skin made out of two neutrons extending over a shell radius of the order of that of the core. But even more important, is the fashion in which the above mentioned "deformation" affects nuclear matter. Matter which is little complient to undergo either compressions or, for that sake, "depressions", without resulting in nuclear instability. In one case, through a mini supernova. In the second, by obliterating the effect of the short range strong force acting in the $^1S_0$ channel (screening effect of the bare pairing interaction).

In fact, in the case of the halo Cooper pair of ^{11}Li, that is of the last two weakly bound neutrons, one is dealing with a rarefied nuclear atmosphere of density

$$\rho \approx \frac{2.}{\frac{4\pi}{3}(R^3(^{11}\text{Li}) - R_0^3(^9\text{Li}))} \approx 0.6 \times 10^{-2} \,\text{fm}^{-3} \tag{3.C.5}$$

where the value $R_0(^9\text{Li}) \approx 2.5$ fm was used. That is, we are dealing with pairing in a nuclear system at a density which is only 4% of saturation density.

The quest for the long range pairing mechanism which is at the basis of the binding of the halo Cooper pair of ^{11}Li to the ^{9}Li core ($S_{2n} \approx 0.380$ keV, to be compared to typical systematic values of $S_{2n} \approx 16$ MeV), has lead to the discovery of what can be considered a novel nuclear mode of elementary excitation. The symbiotic halo pair addition mode, which has to carry its own source of binding (glue) like the hermit crab who carries a gastropod shell to protect his body. A novel embodiment of the Axel–Brink scenario in which not only the line shape, but the main structure of the resonance depends on the state on which it is built, and to which it is deeply interweaved as to guarantee its stability[136] and thus its own existence. It also provides a novel realization of the Bardeen-Frölich-Pines[137]

[133]The radius of gyration R_g is a measure of an object of arbitrary shape, R_g^2 being the second moment in 3D space. In the case of a sphere of radius R, $R_g^2 = 3R^2/5$.

[134]Kobayashi, T. et al. (1989).

[135]Let us parametrize the radius of ^{11}Li as (see Bohr, A. and Mottelson (1975)), $R = R_0(1 + \alpha_{00}Y_{00}) = R_0(1 + \beta_0 \frac{1}{\sqrt{4\pi}})$. Thus $\beta_0 = \sqrt{4\pi}(\frac{R}{R_0} - 1) \approx 2.5$ which testifies to the extreme "exoticity" of the phenomenon.

[136]Axel (1962); Brink (1955 (unpublished).

[137]Bardeen and Pines (1955); Fröhlich, H. (1952).

microscopic mechanism to break gauge invariance: through the exchange of quite large ZPF which ensures the same symmetries of the original Hamiltonian to a system displaying essentially a permanent dipole moment, as a consequence of the almost degeneracy of the dipole pygmy resonance (**centroid $\lesssim$ 1 MeV**) with the ground state. To our knowledge, this is the first example of a van der Waals Cooper pair, atomic or nuclear (App. 2.A).

eliminate bold face Normal character

The NFT diagram shown in Fig. 2.A.1 describing this binding seems quite involved and high order. Thus unlikely to be at the basis of a new elementary mode of nuclear excitation, if nothing else because of the apparent lack of "elementarity'. This is not the case and, in fact, the physics at the basis of the process depicted by the oyster-like and eagle–like networks displayed in (a) and (b) is quite simple and recurrent throughout nuclear structure and reactions, let alone ~~many-body~~ theories and QED. In fact, it encompasses (see Fig. 2.A.1): (I,II) the changes in energy of single-particle levels as a function of dynamical quadrupole deformations leading to a Jahn-Teller like effect (III) the interaction between particles through the exchange of (bosons) vibrations, (IV,V) Pauli principle, (VI,VII) the softening of collective modes due to ground state correlations ((ZPF)-components, QRPA), ~~and eventually the permanent~~ dipole distortion of the system (phase transition), (VIII) the interaction between two non-polar systems through virtual, ZPF associated dipoles. Referring to general many-degree of freedom systems, (I,II) and (III) are at the basis of the fact that, in QED, the coupling between one and two photons is zero (Furry's theorem). It is also at the basis, through cancellation, of the small width displayed by giant resonances as compared with single-particle widths of similar excitation energies, as well as of inhomogeneous damping in rotational motion at finite temperatures[138], in NMR of molecules and in GDR of atomic nuclei. Concerning (VIII), one can mention resonant interactions between fluctuating systems like e.g. two coupled harmonic oscillators[139]. It is like to find a new particle. Either one is at the right energy (on-shell) or one would not see it.

dynamical

℘

(and $\hbar\omega_{PDR} \lesssim 1$ MeV

In the case of halo Cooper pair binding by PDR in ^{11}Li, the system ~~is essentially on resonance,~~ in keeping with the fact $\epsilon_{p_{1/2}} - \epsilon_{s_{1/2}} \approx 0.3$ MeV, and that independent particle motion emerges from the same properties of the force from which collective modes emerge. In other words the ^{10}Li inverted parity system is poised to acquire a permanent dipole moment or, almost equivalent, to display a large amplitude, dipole mode at very low energy as well as a collective $B(E1)$ to the halo ground state, of the order of a single-particle unit $B_W(E1)$. The PDR (see Fig. 3.C.1, see also Fig. 1.9.1) with centroid about $\lesssim$ 1 MeV, 8% of the EWSR and screened from the GDR through the poor overlap between core and halo single-particle wavefunctions so as to be able to retain essentially all of its B_W, $E1$-strength which can rightly be considered a new mode of excitation. In other words we are faced, already at the level of single-particle spectrum, with the possibility of a plastic large amplitude dipole mode, as it materializes in ^{11}Li. In

although not being on resonance it is, not far from it,

[138]Broglia, R. A. et al. (1987).

[139]See e.g. Born (1969), App. XL p. 471.

this case, and making use of the relation

$$\frac{dn}{d\beta_L} = \frac{1}{4}\sqrt{\frac{2L+1}{3\pi}}A \qquad (3.C.6)$$

defining the number of crossings n in terms of deformation[140], one obtains for $L = 0$ and $\beta_0 = 2.5$, $n \approx 2$. That is, one is in presence of a large amplitude plastic mode.

It is of notice that all of these processes takes place inside the halo neutron pair addition vibrational mode of the closed shell system ${}^{9}_{3}\mathrm{Li}_6$(gs), and thus in terms of virtual states. On the other hand intervening the processes depicted in Fig. 2.A.1 with external fields, e.g. those associated with one–and two–particle transfer processes, can shed light on much of the physics which is at the basis of the exotic properties of ^{10}Li and ^{11}Li (see e.g. Figs. 2.6.3 (I), 1.9.4 and 1.9.5. See also Fig. 6.1.3).

But let us now proceed one step at a time. A simple and economic picture of parity inversion in ^{10}Li, and thus in ^{11}Li and, as a result of the mechanism at the basis of the existence of the dipole pygmy resonance in ^{11}Li was proposed in[141]. To explain parity inversion use is made of the fact that, for large prolate quadrupole deformations ($\beta_2 \approx 0.6-0.7$), the $m = 1/2$ member of the $1d_{5/2}$ and $1p_{1/2}$ orbitals, i.e. [220 1/2] and [101 1/2] in the Nilsson labeling of levels ([$Nn_3\Lambda\Omega$]), cross. This is in keeping with the fact that quadrupole distortion changes the energy of single–particle states; those having orbits lying in a plane containing the poles become, in the case of prolate deformations, lower in energy, while those lying preferentially in a plane perpendicular to the symmetry axis, increase their energy. Now, for such value of deformation parity inversion is observed between the resonant $1/2^-$ (≈ 0.5 MeV) and the virtual $1/2^+$ (≈ 0.2 MeV) states of ^{10}Li ($p_{1/2}$ and $2_{1/2}$ states). Thus, the energy difference of 0.3 MeV is not very different from the value of 0.6-0.7 MeV of the PDR centroid. In any case, adjusting β_2 to the appropriate value this centroid energy is within reach.

Furthermore, the fact that the observed $\approx 8\%$ of the EWSR below ≈ 5 MeV for the PDR corresponds to about $1B_{SP}(E1)$ for a single particle transition, provides another confirmation of the attractiveness of the model.

W

Bw

On the other hand, because the radius is affected by deformation, one can posit that the above model predicts $R = R_0(1 + \frac{\beta_2}{\sqrt{5}}\sqrt{\frac{5}{4\pi}}) = 2.7\,\mathrm{fm} \times 1.2\,\mathrm{fm} \approx 3.2$ fm ($\beta_2 \approx 0.7$), in disagreement with the experimental finding.

Now, static models (including also the group theoretical models like that provided by SU_3) imply that single–particle states are either occupied or empty. Experimentally, this does not seem the case in the reaction ^{9}Li$(d,p)^{10}$Li, although one can argue that the situation is different in the case of the single-particle states in ^{11}Li.

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[140] Brink, D. and Broglia (2005). Eq. (7.35) and refs. therein.

[141] Hamamoto and Shimoura (2007).

*)

Finally the fact that the dipole resonance observed in a (d, d') experiment at $E_x \leq 1$ MeV and width 0.5 MeV displays a single peak testifies against the presence of a static quadrupole deformation.**)

Let us now go back to the calculation of the giant dipole pygmy resonance based on the ground state of ^{11}Li. To do so, one needs to know the occupation factors of the $s_{1/2}$ and $p_{1/2}$ states (Fig. 3.C.1). This has been done microscopically making use of the diagonalization of the NFT diagrams taking into account self–energy and induced interaction (vertex renormalization processes) leading to[142]

$\approx$

$$|\tilde{0}\rangle = |\tilde{0}\rangle + 0.71|(p_{1/2}, s_{1/2})_{1^-} \otimes 1^-; 0\rangle + 0.1|(s_{1/2}, d_{5/2})_{2^+} \otimes 2^+; 0\rangle, \quad (3.C.7)$$

and

$$|\tilde{0}\rangle = 0.45|s^2_{1/2}\rangle + 0.55|p^2_{1/2}\rangle + 0.04|d^2_{5/2}\rangle. \quad (3.C.8)$$

In Eq. (3.C.7), the state $|1^-\rangle$ and $|2^+\rangle$ stand for the ~~giant dipole~~ pygmy dipole resonance, and for the low–lying collective quadrupole vibration of ^{9}Li, respectively. To calculate the microscopic structure of the state $|1^-\rangle$ (both wavefunction and transition density and consequently the particle–vibration coupling vertex) one needs to calculate $|\tilde{0}\rangle$. But to do so one needs to know the same $|1^-\rangle$ state, the vibrational mode which exchanged between the two neutrons of the halo provides most of its glue to the ^{9}Li core. From here, the symbiotic character of the $|\tilde{0}\rangle$ and $|1^-\rangle$ ~~(Now, static models (including also the group theoretical models like that provided by SU_3) imply that single–particle states are either occupied or empty. Experimentally, this does not seem the case in the reaction ^{9}Li$(d, p)^{10}$Li, although one can argue that the situation is different in the case of the single-particle states in ^{11}Li,) entering the $|^{11}\text{Li}(0^+_\nu \otimes p_{3/2}(\pi))_{3/2^-}; gs\rangle$ and $|^{11}\text{Li}(1^-_\nu \otimes p_{3/2}(\pi))_{1/2,3/2,5/2^-}; \approx 0.8\,\text{MeV}\rangle$ states.~~

$\sim$ $|\tilde{0}\rangle$ $|1^-\rangle$

Appendix 3.D pairing spatial correlation: simple estimate

Let us assume two equal nucleons above closed shell as the nuclear embodiment of Cooper's model. The two–particle wave function in configuration space can be written as,

$$\Psi(\mathbf{r}_1\sigma_1, \mathbf{r}_2\sigma_2) = \Psi_0(\mathbf{r}_1, \mathbf{r}_2)\chi_{S=0}(\sigma_1, \sigma_2) + [\Psi_1(\mathbf{r}_1, \mathbf{r}_2)\chi_{S=1}(\sigma_1\sigma_2)]_0, \quad (3.D.1)$$

where $\chi_{S=0}$ and $\chi_{S=1}$ are the singlet and triplet spin wavefunctions, respectively.

In what follows we shall consider a pairing interaction acting on pairs of particles moving in time reversal states. Consequently we shall concentrate in the spin singlet radial component of (3.D.1). In the Tamm–Dancoff approximation (in keeping with Cooper ansatz) one can write

$$\Psi_0(\mathbf{r}_1, \mathbf{r}_2) = \sum_{nn'lj} X_{nn'lj} R_{nl}(r_1) R_{n'l}(r_2) \sqrt{\frac{2j+1}{2(2l+1)}} [Y_l(\hat{r}_1) Y_l(\hat{r}_2)]_0. \quad (3.D.2)$$

[142] Barranco, F. et al. (2001).

*) Kanungo et al PRL (d,d')

**) See e.g. Bohr an Mottelson (1975) Fig. 6-21.

This wave function can be rewritten as

$$\Psi_0(\mathbf{r}_1,\mathbf{r}_2) = \Psi_0(|\mathbf{r}_1|, |\mathbf{r}_2|, \theta) \tag{3.D.3}$$

ℓ

$$= \sum_{n'nlj} X_{nn'lj} R_{nl}(r_1) R_{n'l}(r_2) \sqrt{\frac{2j+1}{2}} \frac{1}{4\pi} P_l(\cos\theta), \qquad \cancel{(3.D.4)}$$

where $\theta = \widehat{r_1 r_2}$. A convenient way to display two–particle correlation is by plotting $|\Psi_0(|\mathbf{r}_1|, |\mathbf{r}_2|, \theta)|^2$ in the $x-z$ plane.

In the case of pure configurations $a \equiv nlj$,

$$\Psi_0(|\mathbf{r}_1|, |\mathbf{r}_2|, \theta) = R_{nl}(r_1) R_{nl}(r_2) \sqrt{\frac{2j+1}{2}} \frac{1}{4\pi} P_l(\cos\theta). \tag{3.D.5}$$

In keeping with the fact that the specific probe of pairing correlation is two–nucleon transfer, a phenomenon which takes place mainly, although not only, on the nuclear surface, we shall set $r_1 = r_2 = R_0$, and use the empirical relation

$$R_{nl}(R_0) = \left(\frac{1.4}{R_0^3}\right)^{1/2}, \tag{3.D.6}$$

Thus

$$\Psi_0(R_0, R_0, \theta) = \left(\frac{1.4}{R_0^3}\right) \sqrt{\frac{2j+1}{2}} \frac{1}{4\pi} P_l(\cos\theta). \tag{3.D.7}$$

and

$$|\Psi_0(R_0, R_0, \theta)|^2 \sim |P_l(\cos\theta)|^2. \tag{3.D.8}$$

It is seen that the two particles have the same probability to be on top of each other ($\theta = 0°$; $P_l(1) = 1$), or on opposite sides of the nucleus ($\theta = 180°$; $P_l(-1) = (-1)^l$). Taking into account the actual radial dependence of $R^2_{nl}(r_1)$ for $r_1 = R_0$, the width of the two probability peaks is found to be ≈ 2 fm i.e. $d \approx \left(\frac{4\pi}{3} R^3/A\right)^{1/3}$, aside from $n=n'$,

Let us now consider the general expression (3.D.3), and assume that ~~we have~~ ~~allowed~~ the two nucleons, to correlate in a phase space composed of N single–particle levels, and that all amplitudes are equal,

are allowed

$$X \approx \frac{1}{\sqrt{N}}. \tag{3.D.9}$$

Thus

$$\Psi_0(R_0, R_0, \theta) = \left(\frac{1.4}{R_0^3}\right) \frac{1}{\sqrt{N}} \frac{1}{4\pi} \sqrt{\frac{2j+1}{2}} \sum_l P_l(\cos\theta). \tag{3.D.10}$$

where again (3.D.6) have been used. One can then write

$$|\Psi_0(R_0, R_0, \theta)|^2 \sim |\sum_l P_l(\cos\theta)|^2. \tag{3.D.11}$$

Assuming the closed shell nucleus to be ^{208}Pb and the N single-particle levels the neutron valence orbitals $2g_{9/2}, 1i_{11/2}, 1j_{15/2}, 3d_{5/2}, 4s_{1/2}, 2g_{7/2}$ and $3d_{3/2}$, one obtains

$$\frac{|\Psi_0(R_0, R_0, \theta = 0^\circ)|^2}{|\Psi_0(R_0, R_0, \theta = 180^\circ)|^2} \approx \left(\frac{7}{5}\right)^2 \approx 2, \tag{3.D.12}$$

in keeping with the fact that there is only a single state of opposite parity (intruder $j_{15/2}$).

Making use of an extended basis, containing a similar amount of positive and negative natural parity states ~~(i.e. $\pi = (-1)^l$)~~, that is taking into account a large number of major shells ($\pi = (-1)^N$, N principal quantum number), one can reduce in a consistent fashion the value of[143] $|\Psi_0(R_0, R_0, \theta = 180^\circ)|^2$. This of course materializes already within the basis of valence states in e.g. ^{11}Li, in keeping with the fact that in this case $s_{1/2}$ and $p_{1/2}$ play a similar role. ℓ

Summing up, the above results have something to do with the Cooper pair problem, but much more with the peculiarities of spatial quantization associated with the nuclear self–bound many–body system. That is, the nuclear Cooper pair phenomenon is to be expressed under the influence of a very strong external field which imposes not only confinement, but also spatial quantization with strong spin orbit effects resulting, among other things, in intruder states and thus parity mixing.

Appendix 3.E Coherent state

The BCS ground state can be written as[144],

$$\begin{aligned} |BCS(\phi)\rangle_{\mathcal{K}} &= \prod_{\nu>0}\left(U_\nu + V_\nu a^\dagger_\nu a^\dagger_{\bar\nu}\right)|0\rangle = \prod_{\nu>0} U_\nu\left(1 + \frac{V_\nu}{U_\nu} a^\dagger_\nu a^\dagger_{\bar\nu}\right)|0\rangle \\ &= \left(\prod_{\nu>0} U_\nu\right)\left(\prod_{\nu>0}\left(1 + c_\nu P^\dagger\right)\right)|0\rangle, \end{aligned} \tag{3.E.1}$$

where

$$c_\nu = \frac{V_\nu}{U_\nu} \quad \text{and} \quad P^\dagger = a^\dagger_\nu a^\dagger_{\bar\nu}. \tag{3.E.2}$$

[143] Ferreira, L. et al. (1984).

[144] See e.g. Potel, G. et al. (2013b), and references therein.

one can write

$$
\begin{aligned}
|BCS(\phi)\rangle_{\mathcal{K}} &= \left(\prod_{\nu>0} U_\nu\right)\left\{1 + \frac{1}{1!}\left(\sum_{\nu>0} c_\nu P_\nu^\dagger\right) + \frac{1}{2!}\left(\sum_{\nu>0} c_\nu P_\nu^\dagger\right)^2 + \frac{1}{3!}\left(\sum_{\nu>0} c_\nu P_\nu^\dagger\right)^3 + \cdots\right\}|0\rangle \\
&= \left(\prod_{\nu>0} U'_\nu\right)\left\{1 + \frac{e^{-2i\phi}}{1!}\left(\sum_{\nu>0} c'_\nu P_\nu^\dagger\right) + \frac{e^{-4i\phi}}{2!}\left(\sum_{\nu>0} c'_\nu P_\nu^\dagger\right)^2 + \frac{e^{-6i\phi}}{3!}\left(\sum_{\nu>0} c'_\nu P_\nu^\dagger\right)^3 + \cdots\right\}|0\rangle
\end{aligned}
\tag{3.E.10}
$$

where

$$
c_\nu = e^{-2i\phi} c'_\nu, \quad c'_\nu = V'_\nu / U'_\nu. \tag{3.E.11}
$$

Thus,

$$
|BCS(\phi)\rangle_{\mathcal{K}} = \left(\prod_{\nu>0} U'_\nu\right) \sum_{N\ \text{even}} \frac{e^{-iN\phi}}{(N/2)!}\left(\sum_{\nu>0} c'_\nu P_\nu^\dagger\right)^{N/2}|0\rangle = \left(\prod_{\nu>0} U'_\nu\right)\exp\left(\sum_{\nu>0} c_\nu P_\nu^\dagger\right)|0\rangle \tag{3.E.12}
$$

(see Eq. (3.7.27)) and

$$
\begin{aligned}
|N_0\rangle &= \int d\phi e^{iN_0\phi}|BCS(\phi)\rangle_{\mathcal{K}} \\
&= \left(\prod_{\nu>0} U'_\nu\right)\sum_{N\ \text{even}} \int d\phi e^{iN_0\phi}\frac{e^{-iN\phi}}{(N/2)!}\left(\sum_{\nu>0} c'_\nu P_\nu^\dagger\right)^{N/2}|0\rangle \sim \left(\sum_{\nu>0} c'_\nu P_\nu^\dagger\right)^{N_0/2}|0\rangle
\end{aligned}
\tag{3.E.13}
$$

is the member with N_0 particles of the pairing rotational band, while

$$
\left(\sum_{\nu>0} c'_\nu P_\nu^\dagger\right)|0\rangle \tag{3.E.14}
$$

is the Cooper pair state. Because $U'_\nu \to 0$ for $\epsilon \ll \epsilon_F$, (3.E.14) is to be interpreted to be valid for values of ϵ_ν close to ϵ_F. Making use of the single j–shell model

$$
V' = \sqrt{\frac{N}{2\Omega}}, \quad U' = \sqrt{1 - \frac{N}{2\Omega}}, \tag{3.E.15}
$$

and

$$
\frac{V'}{U'} = \sqrt{\frac{N}{2\Omega - N}} \approx U'V'\left(1 + O\left(\frac{N}{2\Omega}\right)\right) \tag{3.E.16}
$$

for a number of particles considerably smaller than the full degeneracy of the single–particle subspace in which nucleons can correlate, that is for $N \ll 2\Omega$. Consequently, one can write

$$
|\tilde{0}\rangle \approx \frac{1}{\sqrt{N}}\sum_{\nu>0}(\alpha'_0)_\nu P_\nu^\dagger|0\rangle. \tag{3.E.17}
$$

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