6.2.3 Structure-reaction stability of the order parameter α_0

The order parameter associated with distortion in gauge space can be written as

$$\alpha_0' = \sum_{J_a} \sqrt{\frac{2J_a+1}{2}}\, B(J_a^2(0), N \to N+2), \qquad (6.2.14)$$

where

$$B(J_a^2(0), N \to N+2) = \sqrt{\frac{2J+1}{2}}\, U'_{J_a}(N)\, V'_{J_a}(N+2) \qquad (6.2.15)$$

is the two-nucleon spectroscopic amplitude associated with Cooper pair transfer between members of a pairing rotational band. Thus,

$$\alpha_0' \approx \sum_{J_a} \left(\frac{2J_a+1}{2}\right) U'_{J_a} V'_{J_a} = e^{-2i\phi} \sum_{J_a} \left(\frac{2J_a+1}{2}\right) U_{J_a} V_{J_a}$$

$$= e^{-2i\phi}\, \alpha_0 \qquad (6.2.15)$$

defining a priviledged orientation in gauge space. Within the unified description of structure and reactions of the present monograph, the quantities (6.2.15) are the weighting factors of the successive, simultaneous and non-orthogonality formfactors involved in the calculation of the corresponding transfer amplitudes.

In what follows we analyze the stability of these quantities, making use

of three schemes to calculate the (390) b
β-amplitude*) associated with the reaction $^{120}Sn(pit)^{118}Sn(gs)$,
The first one corresponds to BCS approximation making use of a pairing interaction of constant matrix elements, starting from the HF solution of a Skyrme interaction, namely SLy4 the gap and number equations are solved in the pairing approximation with G = 0.26 MeV leading to the empirical value of the three-point expression of the pairing gap ($\Delta^{exp} \approx 1.4$ MeV). The U,V factors for the valence orbitals are reported in Table 6.2.3.

In the second calculational ~~R-HFB~~ scheme, and

Making use of the same Skyrme interaction and of the v_{14} Argonne, $^1S_0$ NN-potential and neglecting the influence of the bare pairing force in the mean field, the HFB equation was solved. As a result, this step corresponds to an extended BCS calculation over the HF basis, allowing for the interference between states of equal quantum numbers $a(\equiv lj)$, but different number of nodes (k, k'). ~~We~~ include (N_a) states (for each a) up to ≈1 GeV, to properly take into account the repulsive core of v_{14} and be able to accurately calculate Δ^{HFB}. As a consequence, one obtains a set of quasiparticle energies E_a^μ, with the quasiparticle index $(\mu = 1, 2...N_a)$. To each quasiparticle $\alpha^+_{a,\mu} = \sum_{k=1}^{N_a}(U_a^{\mu,k} a^+_{a,k} - V_a^{\mu,k} a_{a,k})$ is associated an array of quasiparticle amplitudes $U_a^{\mu,k}$ and $V_a^{\mu,k}$ which are the components of the quasiparticles over the HF basis states $\phi_k^a = \langle \vec{r} | a^+_{a,k} | 0 \rangle (\equiv \langle \vec{r} | a, k \rangle)$. Going to the canonical basis, where the density matrix takes a diagonal form, ~~we~~ look for the state having the largest value of the abnormal density, $(UV)_{max}$. As a rule, for a well-bound nucleus such as ^{120}Sn, this canonical state is the quasiparticle state having the lowest value of the quasiparticle energy. The label k then drops because there is only one orbital for a given value of $a(\equiv (lj))$. This implies that the bare quasiparticle amplitudes can be characterised simply by U_a, V_a and ~~the associated state dependent value of the bare pairing gap is equal $\Delta_a^{bare} = 2U_a V_a E_a$. The values of $(E_a)_{min}$ and $(V_a(v_{14}))_{max}$ for the five valence orbitals are reported in Table I.~~

(see Table 6.2.3)

the parameters

~~C—Renormalized NFT and NG~~

One

one

from p. 9 From bare to ren.

*) G. Potel et al (2017)

From bare to renormalized... Phys. Rev. C (submitted).

going beyond mean field and including the particle-vibration coupling leading to retardation phenomena, both in self-energy as well as in ≡ induced pairing interaction process within the framework of (NFT)$_{rm}$ and Nambu-Gorkov (NG) equation leads, in the canonical basis to the re-normalized spectroscopic amplitudes $\tilde{B}(a)_{rm} = (\frac{2j_a+1}{2})\tilde{u}_a\tilde{v}_a$ (Table 6.2.3)

$a \equiv \{ljt\}$	NFT(NG)	HFB(VM)	BCS (G)
$d_{5/2}$	0.22	0.29	0.41
$g_{7/2}$	0.46	0.47	0.57
$s_{1/2}$	0.37	0.34	0.41
$d_{3/2}$	0.59	0.60	0.66
$h_{11/2}$	0.95	1.0	1.03

Table 6.2.3, Two-nucleon spectroscopic amplitudes. Note the small difference between the values of the third column with those reported in the column labelled ^{120}Sn of Table 6.2.1, due to a small difference in the value of G used in Potel G. et al (2013a) and Potel et al (2017).

Making use of the β-coefficients collected in Table 6.2.3, together with the global optical parameters reported in Table 6.2.2, the corresponding absolute differential cross sections of the reaction $^{120}Sn(p,t)^{118}Sn(gs)$ at 21 MeV of bombarding energy were calculated. They are displayed in Fig. 6.2.2 in comparison with the experimental findings.

VII. DISCUSSION

The spectroscopic results reported in Table I testify to the important effects renormalisation of the single-particle states and of the pairing interaction have at the level of quasiparticles. In spite of this, all three approaches (NFT(NG),HFB, BCS), notwithstanding their large differences in terms of many-body facets, predict essentially equally correct absolute two-nucleon transfer cross sections, as testified by the results displayed in Figs. 4 and 5 where theory is compared to experiment,

It seems then fair to conclude that the quantity which controls the specific excitation of pairing rotational bands, namely the order parameter α_0, in the sense of Cooper pair transfer amplitude (Sect. III), is essentially invariant whether calculated within the framework of the simplest one-pole quasiparticle (BCS) approximation, or taking into account the variety of many-body renormalisation effects.

The emergence of a physical sum–rule is apparent (within this context see [56], while for exact sum rules see [55, 57]). Let us elaborate on this point.

Approximating

$$\tilde{u}_{a(n)} = \sqrt{N_{a(n)}} U_a \quad ; \quad \tilde{v}_{a(n)} = \sqrt{N_{a(n)}} V_a, \tag{34}$$

and

$$N_{a(n)} \approx Z_{a(n)} \approx Z_\omega, \quad (\epsilon_a \approx \epsilon_F) \tag{35}$$

one can write,

$$\alpha_0 = \sum_{a,n} \frac{2j_a+1}{2} \tilde{u}_{a(n)} \tilde{v}_{a(n)} = \frac{N(0)}{Z_\omega} \int d\epsilon \frac{2j_\epsilon+1}{2} \tilde{u}_\epsilon \tilde{v}_\epsilon, \tag{36}$$

where $N(0)/Z_\omega$ is the effective density of levels at the Fermi energy [35]. With the help of Eq. (35) one obtains,

$$\alpha_0 = \frac{N(0)}{Z_\omega} Z_\omega \int d\epsilon \frac{2j_\epsilon+1}{2} U_\epsilon V_\epsilon \approx \sum_a \frac{2j_a+1}{2} U_a V_a \tag{37}$$

(NFT(NG))

Using each term of the expressions (36) and (37) as weighting factors of the corresponding two–nucleon transfer formfactors, in keeping with the unified structure–reaction physical interpretation of α_0 (Sect. III), and that (see Figs. 4 and 5) $|\sigma_i - \sigma_{exp}|/\sigma_{exp}$ is equal to 0.09, 0.13 and 0.07 (i = BCS, HFB, NG), the relative errors of the associated two–nucleon transfer amplitudes $\alpha_0 (\sim \sqrt{\sigma})$ are 4.5%, 6.5% and 3.5%. Within this context, it is of notice that the fact that the HFB result lies closer to the NG one than BCS does, is a simple consequence of NG being based on HFB.

Furthermore, because the matrix elements of v_{14} for configurations based on the valence orbitals is essentially state independent together with the fact that $Z^2 \approx 0.5$, setting $v_{ind} = 0$, one expects for the renormalised (NFT(NG)) cross section a value $\approx 1000 \mu b$ ($0.5 \times \sigma_{HFB}$), precluding the above accuracy. Consequently, at the basis of the validity of (36)–(37) and thus of the conservation of two–nucleon transfer amplitudes in going from BCS mean field to NFT(NG) many-body, medium renormalization representations, one also finds the central role played by the induced pairing interaction.

NFT(NG)

the results collected in Figs. 6,2.2 and 6,2,3, namely the fact that the absolute cross section ratio

mentioned after Eq. (6,2.15)

In other words, spontaneous breaking of gauge symmetry, a feature which is embodied in the three descriptions used (BCS, HFB, NFT(NG)) albeit, at very different level of many-body refinement, seems to give rise to a new emergent property: a physical sum rule resulting from the intertwining of structure and reaction aspects of pairing in nuclei.

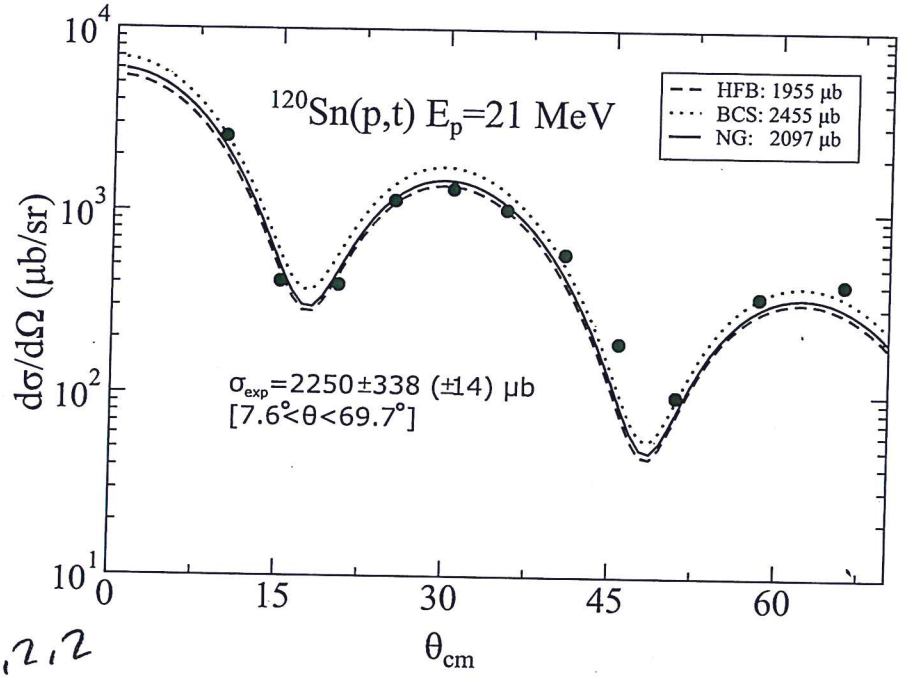

FIG. 2: Absolute differential cross sections associated with the reaction ^{120}Sn(p,t)^{118}Sn(gs) calculated making use of the BCS, HFB and renormalised NFT(NG) spectroscopic amplitudes (~~Table II~~) and global optical parameters (~~Table III~~), in comparison with the experimental findings (solid dots) [~~50~~].

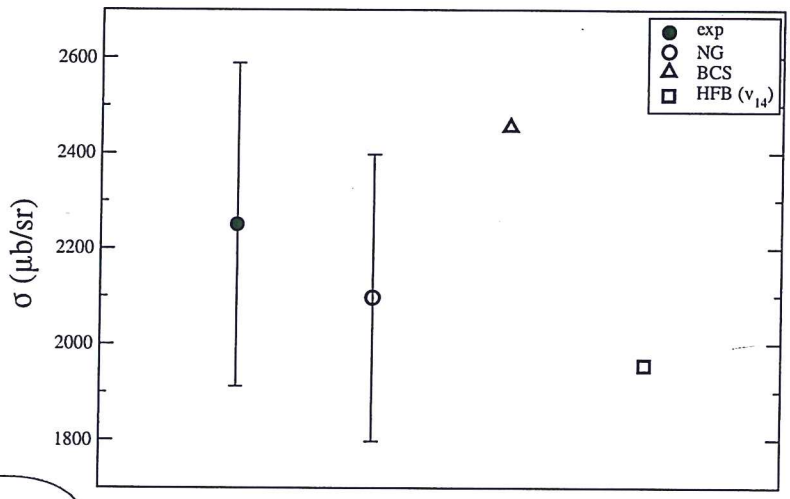

FIG. 5: Integrated absolute cross sections associated with the reaction ^{120}Sn(p,t)^{118}Sn(gs) (see caption to Fig. 4). The error ascribed to the NFT(NG) theoretical results stems from the uncertainties in the calculation of the two-neutron transfer spectroscopic amplitudes estimated from the variations the contribution of spin modes associated with different Skyrme interactions induce in the $B-$coefficients.