It is suggestive that the expression (3.6.20) is formally similar to that of the ion–ion potential acting between two heavy ions in weak contact, namely at a distance a diffusivity away from th grazing distance r_g. In this case the role of the reduced gap is played by a quantity closely related to the reduced radius of curvature[49]

$$U^N_{aA}(r_g + a) \sim \gamma \frac{R_a R_A}{R_a + R_A} a, \tag{3.6.22}$$

In the above expression $\gamma \approx 0.9$ MeV/fm^2 is the surface tension, $a = 0.63$ fm the diffusivity of the potential, $R_i(=(1.233A^{1/3} - 0.98A^{-1/3})$ fm) being the radii of nuclei $i = a, A$. For two identical nuclei $R_a = R_A = R$ and $V_0 = 8\pi\gamma R a$. In the case in which the interacting nuclei are two ^{120}Sn systems $U^N_{aA}(r_g + a) = U^N_{aA}(13.43 \text{ fm}) \approx -9$ MeV.

Nuclei being leptodermous systems can be described at profit, concerning a number of properties, with the help of the liquid drop. Because at the grazing distance the two leptodermous objects overlap, although weakly, two "unit" areas disappear. To reconstruct them one has to separate the two nuclei until these areas are reconstructed again. The energy needed to do so has to compensate the value (3.6.22) namely, in the present case it is 6 MeV. Microscopically, the interaction (3.6.22) arises from a kind of, weak, covalent mechanism. The single–particle orbitals of the two individual nuclei a and A, are shared when in contact, leading to a common mean field.

pp. (1') + (2')

(A) - (A)

Similarly, the weak limit (3.6.20) between the two superconductors 1 and 2, is associated with the situations in which each partner of a Cooper pair is in a different superconductor, a kind of covalent phenomenon, in which each Cooper pair is simultaneously shared by the two superconductors.

3.7 Rotation in gauge space

(a) - (a) p. (1)

3.7.1 Phase coherence

3.7.1

The phase of a wavefunction and ~~the number~~ G of nucleons (electrons in condensed matter) are conjugate variables. Gauge invariance, i.e. invariance under phase changes, implies number conservation in the same way that rotational invariance implies angular momentum conservation. Q5

Example: let us introduce the trivially invariant many-body wavefunction,

$$\Psi = a_1^\dagger a_2^\dagger \cdots a_N^\dagger \Psi_{vac}. \tag{3.7.1}$$

(b) - (b)

pp. (1) + (2)

Multiplying the creation operators by a phase factor,

$$a'^\dagger = e^{-i\phi} a^\dagger, \tag{3.7.2}$$

[49]Broglia and Winther (2004) p.114 Eq. (40), $U^N_{aA}(r) = -V_0/(1 + \exp(\frac{r-R_0}{a}))$, $V_0 = 16\pi\gamma R_{aA} a$, $R_0 = R_a + R_A + 0.29$ fm, which for two ^{120}Sn nuclei ($R = 5.883$ fm) leads to $R_0 \approx 12.1$ fm and $V_0 = 83.8$ MeV. For energies somewhat above the Coulomb barrier, the grazing distance (Eq. (25) p. 128) of the above reference is $r_g = r_B - \delta \approx 12.8$ fm ($r_B \approx 13.3$ fm, $\delta \approx 0.5$ fm). Thus $(1 + \exp(\frac{r_g+a-R}{a})) \approx 10.1$.

a ~~planical~~
~~[illegible]~~

A

The occurrence of rotation as a feature of the nuclear spectrum, e.g. of pairing rotational bands, originates in the phenomenon of spontaneous symmetry breaking of rotational invariance in the two dimensional gauge space. In other words, violation of particle number conservation, which introduces a deformation that makes it possible to specify an orientation of the system in gauge space.

The condensate in the nuclear superfluid system involve a deformation of the field ~~that~~ creates the fermion pairs. The process of addition or removal of a Cooper pair constitutes a rotational mode in gauge space in which particle number plays the role of angular momentum. Pairing rotational bands represents the collective mode associated with spontaneous symmetry of particle number conservation (Goldstone mode).

Let us ellaborate on the above points within the framework of a simple model which contains the basic physical features one is interested in discussing.

We consider N nucleons moving in a single j-shell*) of energy ϵ_j and total angular momentum $(ls)j$. The number of pairs moving in time reversal state which can be accomodated in the shell is $\Omega = (2j+1)/2$. Consequently, the value of the BCS occupation parameters is $V = (N/2\Omega)^{1/2}$ and $U = (1-N/2\Omega)^{1/2}$. Setting $\epsilon_j = 0$, the solution of the BCS number and gap equations associated

---

*) For details see e.g. App. H of Brink and Broglia (2005) and refs. therein.

with a pairing force with constant matrix elements G are

$$\lambda = -\frac{G}{2}(\Omega - N), \qquad (3.7.1)$$

and

$$\Delta = \frac{G}{2}\sqrt{N(2\Omega - N)}, \qquad (3.7.2)$$

respectively.

The BCS ground state energy of the superfluid system is

$$U = 2\sum_{\nu>0}(\epsilon_\nu - \lambda)V_\nu^2 - \frac{\Delta^2}{G}. \qquad (3.7.3)$$

Using (3.7.1) and (3.7.2), one can write

$$U \approx \frac{\hbar^2}{2\mathcal{J}}N^2, \qquad (N \ll \Omega),$$

where

$$\mathcal{J} = \frac{2\hbar^2}{G}, \qquad (3.7.4)$$

is the moment of inertia of the associated pairing rotational band. The fact that the single-particle energies are measured with respect to λ implies that the nucleons feel the Coriolis force ($\hbar\dot{\phi} = \lambda$) associated with rotation in gauge space (Eq. (3.7.9) below). In other words, the BCS solution is carried out in the intrinsic system of reference K' (see below). In fact, the ground state energy in the laboratory system K is (see Fig. 2.1.3)

$$E_0 = U + \lambda N = \lambda N + \frac{\hbar^2}{2\mathcal{J}}N^2, \qquad (3.7.5)$$

a quantity which approaches zero linearly in N, as expected for a Goldstone mode.

p. 248 A

a In other words, the gauge and and the particle 1
number operators $\hat{\phi}$ and $\hat{N}$
satisfy the conmutation relation,

$$[\hat{\phi},\hat{N}]=i. \qquad (3.7.6)$$

In the particle number representation

$$\hat{N}=-i\,\partial/\partial\phi\ ,\quad \hat{\phi}=\phi, \qquad (3.7.7)$$

while

$$\hat{N}=N\ ,\quad \hat{\phi}=i\partial/\partial N, \qquad (3.7.8)$$

in the gauge angle representation. The time derivative of the gauge angle is given by the equation of motion*),

$$\dot{\phi}=\frac{i}{\hbar}[H,\phi]=\frac{1}{\hbar}\frac{\partial H}{\partial N}=\frac{1}{\hbar}\lambda, \qquad (3.7.9)$$

where λ is the chemical potential (Fermi energy).

p.248 a

~~Example: let us introduce the trivially invariant many-body wavefunction~~

$$\cancel{|\Psi_N\rangle=a_1^{\dagger}a_2^{\dagger}\ldots a_N^{\dagger}|0\rangle,} \qquad \cancel{(3.7.4)}$$

b and rotate it an angle ϕ making use of the operator

$$\mathcal{G}(\phi)=e^{-i\hat{N}\phi}. \qquad (3.7.\cancel{5}6)$$

One obtains

$$\mathcal{G}(\phi)|\Psi_N\rangle=e^{-iN\phi}|\Psi\rangle=|\Psi'\rangle, \qquad (3.7.\cancel{6}7)$$

where

$$|\Psi_N'\rangle=a_1'^{\dagger}a_2'^{\dagger}\ldots a_N'^{\dagger}|0\rangle, \qquad (3.7.\cancel{7}8)$$

and

$$\mathcal{G}^{-1}(\phi)\,a_\nu^{\dagger}\,\mathcal{G}(\phi)=e^{-i\phi}a_\nu^{\dagger}=a_\nu'^{\dagger}, \qquad (3.7.\cancel{8}9)$$

*) See e.g. Brink and Broglia (2005) App. I

i mayuscula

Thus

$$|\Psi_N\rangle = e^{iN\phi}\,|\Psi_N'\rangle\,, \qquad (3,7,\overset{10}{\cancel{9}})$$

and

$$\hat{N}\,|\Psi_N\rangle = -i\frac{\partial}{\partial N}\,|\Psi_N\rangle = N\,|\Psi_N\rangle\,, \qquad (3,7,\overset{11}{\cancel{10}})$$

b

p. 248

one can rewrite

$$\begin{aligned}\Psi &= (e^{i\phi}a_1'^{\dagger})(e^{i\phi}a_2'^{\dagger})\cdots(e^{i\phi}a_N'^{\dagger})\Psi_{vac}\\ &= e^{iN\phi}\Psi'.\end{aligned} \tag{3.7.3}$$

Thus

$$-i\frac{\partial}{\partial\phi}\Psi = Ne^{iN\phi}\Psi' = N\Psi, \tag{3.7.4}$$

where

$$N = -i\frac{\partial}{\partial\phi}; \qquad \phi = i\frac{\partial}{\partial N} \tag{3.7.5}$$

and

$$[\phi, N] = 1; \quad \Delta\phi\Delta N = 1. \tag{3.7.6}$$

A phase change for a gauge invariant function is just a trivial operation. Like to rotate a rotational invariant function. Quantum mechanically nothing happens rotating a spherical system (in 3D–, gauge, etc.) space.

The situation is very different in the case of the wavefunction

$$\begin{aligned}|BCS(\phi)\rangle_{\mathcal{K}} &= \prod_{\nu>0}\left(U_\nu + V_\nu a_\nu^\dagger a_{\bar\nu}^\dagger\right)|0\rangle,\\ &= \prod_{\nu>0}\left(U'_\nu + e^{-2i\phi}V'_\nu a_\nu^\dagger a_{\bar\nu}^\dagger\right)|0\rangle,\\ &= \prod_{\nu>0}\left(U'_\nu + V'_\nu a_\nu'^\dagger a_{\bar\nu}'^\dagger\right)|0\rangle,\\ &= |BCS(\phi=0)\rangle_{\mathcal{K}'}, \qquad (3.7.\cancel{7}\,12)\end{aligned}$$

where

$$\cancel{a_\nu'^\dagger = G(\phi)a_\nu^\dagger G^{-1}(\phi) = e^{-i\phi}a_\nu^\dagger (a_{\bar\nu}'^\dagger = e^{-i\phi}a_{\bar\nu}^\dagger),} \qquad \cancel{(3.7.8)}$$

$$U_\nu = |U_\nu| = U'_\nu; \quad V_\nu = e^{-2i\phi}V'_\nu \; (V'_\nu = |V_\nu|). \qquad (3.7.\cancel{9}\,13)$$

~~and~~ In fact,

$$|BCS(\phi)\rangle_{\mathcal{K}} = \left(\prod_{\nu>0}U'_\nu\right)\sum_{N}\frac{e^{-iN\phi}}{(N/2)!}\left(\sum_{\nu>0}c'_\nu P_\nu^\dagger\right)^{N/2}|0\rangle, \qquad (3.7.\cancel{10}\,14)$$

with

Neven

$$c'_\nu = \frac{V'_\nu}{U'_\nu}; \qquad P_\nu^\dagger = a_\nu^\dagger a_{\bar\nu}^\dagger, \qquad (3.7.\cancel{11}\,15)$$

$c'_\nu = \dfrac{V'_\nu}{U'_\nu}$ $\quad P^\dagger_\nu = a^\dagger_\nu a^\dagger_{\bar\nu}$ $\quad \left(\prod_{\nu>0} U'_\nu\right)\sum_{N\,\mathrm{even}}\dfrac{e^{-iN\phi}}{(N/2)!}\left(\sum_{\nu>0} c'_\nu P^\dagger_\nu\right)^{N/2}|0\rangle$

$$\left(\sim \sum_N \int_0^{2\pi} d\phi\, e^{-i(N-N')\phi}\, |\Psi_N\rangle \sim |\Psi_{N'}\rangle\right)$$

(c)-(c) p.(3)

is a wavepacket in particle number, which can be written as,

$$|BCS(\phi)\rangle_{\mathcal{K}} = \left(\prod_{\nu>0} U_\nu\right) \sum_N e^{-iN\phi}|N\rangle. \tag{3.7.12}$$

Let us now apply the gauge angle operator

$$\hat{\phi}|BCS(\phi)\rangle_{\mathcal{K}} = i\frac{\partial}{\partial N}|BCS(\phi)\rangle_{\mathcal{K}}$$
$$= \phi\left(\prod_{\nu>0} U_\nu\right) \sum_N e^{-iN\phi}|N\rangle = \phi|BCS(\phi)\rangle_{\mathcal{K}}. \tag{3.7.13}$$

Thus the state $|BCS(\phi = 0)\rangle_{\mathcal{K}'}$ is rigidly aligned in gauge space in which it defines a privileged orientation (z').

An isolated nucleus will not remain long in this product type state. Due to the term[50] $(G/4)\left(\sum_{\nu>0}\left(U_\nu^2 + V_\nu^2\right)\left(\Gamma_\nu^\dagger - \Gamma_\nu\right)\right)^2$ in the residual quasiparticle Hamiltonian it will fluctuate (QM, ZPF, Goldstone mode), and decay into a state

(d)-(d) p.(3)

$$|N\rangle \sim \int d\phi e^{iN_0\phi}|BCS(\phi)\rangle_{\mathcal{K}} \sim \qquad \cancel{(3.7.14)}\ (3.7.21)$$

member of a pairing rotational band around neutron mass number N: for example the ground states of the Sn–isotopes around $N_0 = 68$ (see Fig. 2.1.3). Because $E_R = (\hbar^2/2\mathcal{I})(N - N_0)^2 = (G/4)(N - N_0)^2 = G/4(\frac{1}{\delta\phi})^2$ is the kinetic energy of rotation in (nuclear) gauge space, and $G/4 \approx 25/(4N_0) \approx 0.092$ MeV, the wavepacket (3.7.12) will decay[51] in the state (3.7.14) in a time of the order of[52]

[50] Within BCS theory of pairing, there are two parameters which determines spontaneous symmetry breaking in gauge space. The probability amplitude with which a pair state ($\nu\bar{\nu}$) is occupied, and that with which it is empty. Namely, V_ν and U_ν respectively. As a consequence, there only two fields F which contribute, through terms of type $FF^\dagger$, to the residual interaction H_{res} acting among quasiparticles, which is neglected in the mean field solution of the pairing Hamiltonian. One which is antisymmetric with respect to the Fermi surface namely $(U_\nu^2 - V_\nu^2)$ and which leads to pairing vibrations of the gauge deformed state $|BCS\rangle$. The other one, $(U_\nu^2 + V_\nu^2)$ is symmetric with respect to ϵ_F and leads to fluctuations which diverge in the long wavelength limit ($W_1'' \to 0$) in precisely the right way to set $|BCS\rangle$ into rotation with a finite inertia, and restore symmetry. This term is written as $H_p'' = (G/4)\left(\sum_{\nu>0}\left(U_\nu^2 + V_\nu^2\right)\left(\Gamma_\nu^\dagger - \Gamma_\nu\right)\right)^2 = (G/4)\left(\sum_{\nu>0}\left(\Gamma_\nu^\dagger - \Gamma_\nu\right)\right)^2$, where $\Gamma_\nu^\dagger(\Gamma_\nu)$ is the two quasiparticle creation (annihilation) operator in the harmonic (quasiboson) approximation $[\Gamma_\nu, \Gamma_{\nu'}^\dagger] = \delta(\nu, \nu')$ see Brink, D. and Broglia (2005), eq. 4.24 and Sect. I.4. For more details see App. J of this reference.

[51] Within this context note that setting in phase at $t = 0$ all the states in which a GDR breaks down through the hierarchy of doorway–states–coupling, they would dissipate like a wavepacket of free particles after 10^{-22} sec (assuming $\Gamma_{GDR} \approx 3 - 4$ MeV). It is of notice that the GDR will eventually branch into the ground state, although $\Gamma_\gamma \ll \Gamma_{GDR}$, in keeping with the fact that the $t = 0$ phase coherent states are, individually, stationary. What is not stationary is its phase coherence (see Sect. 1.3). Pushing the analogy a step further, one can say that in quantum mechanics, while the outcome of an experiment is probabilistic the associated probability evolve in a deterministic way (Born (1926)). This is the reason why a large gamma ray detector will reveal a well defined peak of the resonant dipole state long after its lifetime deadline ($\hbar/\Gamma$). Also, one can obtain a ~~completely~~ (classical) picture of a face making use of single photons at a time, provided one waits long enough.

[52] In these estimates the approximated value $\hbar/1$ MeV $\approx (2/3) \times 10^{-21}$ s is used.

not corrected 06/04/18

c

$$|BCS(\phi)\rangle_K = \sum_N f_N(\phi)\,|\Psi_N\rangle \qquad (3.7.16)$$

Let us now apply the gauge angle operator to it,

$$\hat{\phi}\,|BCS(\phi)\rangle_K = \hat{\phi}\sum_N f_N(\phi)\,|\Psi_N\rangle, \qquad (3.7.17)$$

that is,

$$i\frac{\partial}{\partial N} f_N(\phi) = \phi\, f_N(\phi) . \qquad (3.7.18)$$

Thus

$$f_N(\phi) \sim e^{-iN\phi}, \qquad (3.7.19)$$

and

$$|BCS(\phi)\rangle_K \sim \sum e^{-iN\phi}\,|\Psi_N\rangle \qquad (3.7.20)$$

p. 250 c

$$\cancel{|N'\rangle \sim \int_0^{2\pi} d\phi\, e^{iN\phi}\,|BCS(\phi)\rangle_K}$$

$$\cancel{\sim \sum_N \int_0^{2\pi} d\phi\, e^{-i(N-N')\phi}\,|\Psi_N\rangle}$$

$$\sim |\Psi_N\rangle$$

d in keeping with the fact that

$$\int_0^{2\pi} d\phi\, e^{-i(N-N')\phi} = \begin{cases} 2\pi\,\delta(N,N') & (N=N') \\ \dfrac{i}{N-N'}\, e^{-i(N-N)}\Big|_0^{2\pi} = 0 & (N\neq N') \end{cases}$$

The state $|N'\rangle$ is a d p. 250

$\hbar/(\hbar^2/2\mathcal{I}) \approx \hbar/0.092$ MeV) $\approx 10^{-20}$ s. More accurately, because[53] $N = N_0 \pm 2$, $\hbar/E_R \approx \hbar/(4 \times 0.092$ MeV) $\approx 2 \times 10^{-21}$ s. In other words, superfluid nuclei cannot be prepared, in isolation, in states with coherent superposition of different N–values. The common assumption that N is fixed, ϕ meaningless is correct.

This is also the case for real superconductors. In fact, the corresponding state (3.7.12) even if prepared in isolation would dissipate because there is a term in the energy of the superconductor depending on N, namely the electrostatic energy $e(N - N_0)^2/2C = e^2/2C(\partial/\partial\phi)^2$, where C is the electrostatic capacity. The system will dissipate, no matter how small $\delta\phi$ is. In fact, let us assume $\delta\phi = 1$ degree. The kinetic energy of rotation in gauge space is of the order of $(e^2/2C)(1/\delta\phi)^2$ (of notice that $\delta N \delta\phi/2\pi \sim 1$), and

$$\Delta E = \frac{1.44\,\text{fm MeV}}{1\,\text{cm}\,(1^\circ)^2} \sim 1.44 \times 10^{-13}\,\text{MeV}, \tag{3.7.15}$$

which corresponds to an interval of time

$$\Delta t \approx \frac{\hbar}{1\,\text{MeV}}\frac{10^{13}}{1.44} \approx \frac{0.667 \times 10^{-21}\,\text{sec}}{1.44} \times 10^{13} \approx 10^{-9}\,\text{sec}. \tag{3.7.16}$$

The opposite situation is that of the case in which one considers different parts of the same superconductor. In this case one can define relative variables $n = N_1 - N_2$ and $\phi = \phi_1 - \phi_2$ and again $n = -i\partial/\partial\phi$ and $\phi = i\partial/\partial n$. Thus, locally there is a superposition of different n states: ϕ is fixed so n is uncertain. It is clear that there must be a dividing line between these two behaviors, perfect phase coherence and negligible coherence, namely the Josephson effect.

In the nuclear case, one can view the systems $|BCS(A+2)\rangle$ and $|BCS(A)\rangle$ as parts of a fermion superfluid (superconductor) which, in presence of a proton $(p + (A+2))$ are in weak contact to each other, the $d + |BCS(A+1)\rangle$ system (without scattering, running waves, but as a closed, virtual, channel) acting as the dioxide layer of a Josephson junction (Fig. 3.7.1(a);).

~~; within this context see discussion concerning the validity of analogies carried out between the finite many-body nuclear system and the (N ~ Avogadro number) many-particle condensed matter counterpart. see also footnote 45~~

Clearly, again, the total phase of the assembly is not physical. However, the relative phases can be given a meaning when one observes, as one does in e.g. metallic superconductors, that electrons can pass back and forth through the barrier, leading to the possibility of coherence between states in which the total number of electrons is not fixed *locally*. Under such conditions there is, for instance, a coherence between the state with $N/2$ electrons in one half of the block and $N/2$ in the other, and that with $(N/2) + 2$ on one side side and $(N/2) - 2$ on the other.

Under favorable conditions, in particular of Q–value for the different channels involved and, similarly to the so called backwards rise effect, one may, arguably, observe signals of the coherence between systems $(A+2)$ and A in the elastic scattering process $^{A+2}X + p \to (^{A}X + t) \to {}^{A+2}X$, ^{i}X denoting a member of a pairing rotational band (cf. Fig. 3.7.1, see also Fig 2.1.3).

[53]Implying $\delta\phi \approx 1/\delta N \approx 0.5/(\pi/180^\circ) \approx 30^\circ$ a rather large number, in keeping with the small number (5–6) of Cooper pairs participating in the nuclear condensate.

; within this context see the discussion concerning the validity of analogies carried out between phenomena observed in many-particle (N ~ Avogadro number) condensed matter systems and the finite many-body nuclear counterpart. see also footnote 45 as well as last paragraph before Sect. 3.B.4.

the γ-rays 55 54 Bath

Whether an effect which may parallel that shown in (c) (backwards rise) can be seen or not depends on a number of factors, but very likely it is expected to be a weak effect. This was also true in the case of the Josephson effect in its varied versions (AC, DC, etc.). In fact, its observation required to take into account the effect of the earth magnetic field, let alone quantal and thermal fluctuations.

3.8 Hindsight

The formulation of superconductivity (BCS theory) described by Gor'kov[54] allows, among other things for a simple visualization of spatial dependences. In this formulation $F(\mathbf{x}, \mathbf{x}')$ is the amplitude for two Fermions (electrons) at $\mathbf{x}, \mathbf{x}'$, to belong to the Cooper pair (within the framework of nuclear physics cf. e.g. Fig. 2.6.3 $\Psi_0(\mathbf{r}_1, \mathbf{r}_2)$; see also App. 3.B). The phase of F is closely related to the angular orientation of the spin variable in Anderson's quasispin formulation of BCS theory[55]. The gap function $\Delta(x)$ is given by $V(\mathbf{x})F(\mathbf{x}, \mathbf{x})$ where $V(\mathbf{x})$ is the local two–body interaction at the point $\mathbf{x}$. In the insulating barrier between the two superconductors of a Josephson junction, $V(\mathbf{x})$ is zero and thus $\Delta(x)$ is also zero.

The crucial point is that vanishing[56] $\Delta(x)$ does not imply vanishing F, provided, of course, that one has within the insulating barrier, a non–zero particle (electron) density, resulting from the overlap of densities from right (R) and left (L) superconductors. Now, these barriers are such that they allow for one–electron–tunneling with a probability of the order of 10^{-10} and, consequently, the above requirement is fulfilled. Nonetheless, conventional (normal) simultaneous pair transfer, with a probability of $(10^{-10})^2$ will not be observed[57]. But because one electron at a time can tunnel profiting of the small, but finite electron density within the layer, $F(\mathbf{x}, \mathbf{x}')$ can have large amplitude for electrons, on each side of the barrier (i.e. L and R), separated by distances $|\mathbf{x} - \mathbf{x}'|$ up to the coherence length. Hence, for barriers thick to only allow for essentially the tunneling of one electron at a time, but thin compared with the coherence length, two electrons on opposite sides of the barrier can still be correlated and the pair current can be consistent. An evaluation of its value shows that, at zero temperature, the pair current is equal to the single particle current at an equivalent voltage[58] $\pi\Delta/2e$.

The translation of the above parlance to the language of nuclear physics has to come to terms with the basic fact that nuclei are self–bound, finite many–body systems in which the surface, as well as space quantization, play a very important

[54] Gor'kov (1958); Gor'kov, L.P. (1959).

[55] Anderson (1958); within the framework of nuclear physics cf. e.g. Bohr and Ulfbeck (1988), Potel, G. et al. (2013b) and references therein.

[56] This point was not understood by Bardeen who writes"... In my view, virtual pair excitations do not extend across the layer...", see McDonald (2001), see also Bardeen (1961) and Bardeen (1962).

[57] Pippard (2012) see also McDonald (2001).

[58] In the case of Pb at low temperatures ($\approx$ 7.19 K (0.62 meV)) this voltage is $\approx$ 1 meV/e = 1 mV leading to $\approx$2mA for a barrier resistance of $R \sim 1\Omega$ (Ambegaokar and Baratoff (1963); McDonald (2001); Tinkham (1996)).

A direct observation of weak coupling coherent phenomena between ~~states~~ $|BCS(A+2)\rangle$ and $|BCS(A)\rangle$ in $A+a \to A+a$ process could, arguably, be achieved in e.g. the virtual transfer of two proton and of the associated γ-rays of frequency $\nu = Q_{2p}/h$.