```
0
 App. 3, E Coherent state (PRC 27, 054321 (2013))
The BCS ground state can be written as,
                   |B(S(\emptyset))\rangle_{\mathcal{K}} = \prod_{\nu \geqslant 0} (U_{\nu} + V_{\nu} a_{\nu}^{\dagger} a_{\nu}^{\dagger}) |0\rangle = \prod_{\nu \geqslant 0} U_{\nu} (1 + \frac{V_{\nu}}{U_{\nu}} a_{\nu}^{\dagger} a_{\nu}^{\dagger}) |0\rangle
                                      =(\prod_{\nu > 0} U_{\nu}) (\prod_{\nu > 0} (1 + C_{\nu} P_{\nu}^{\dagger}) | 0) ; c_{\nu} = \stackrel{\vee}{V_{\nu}} ; P_{\nu}^{\dagger} = a_{\nu}^{\dagger} a_{\nu}^{\dagger}
                                                       C_{\nu} = \frac{U_{\nu}}{V_{\nu}} and P_{\nu}^{+} = \alpha_{\nu}^{+} \alpha_{\nu}^{+}
   V=1,2 (two pairs);

\Pi (1+C_{\nu}P_{\nu}^{t}) = (1+C_{i}P_{i}^{t})(1+C_{z}P_{z}^{t}) = 1+C_{i}P_{i}^{t} + C_{z}P_{z}^{t} + C_{i}C_{z}P_{1}^{t}P_{z}^{t}

(3. E. 3)
          = 1 + \sum_{\nu \neq 0} C_{\nu} P_{\nu}^{\dagger} + \frac{1}{2!} \left( \sum_{\nu \neq 0} C_{\nu} P_{\nu}^{\dagger} \right)^{2},
where use the bean made of and of the fact that
\left( C_{1} P_{1}^{\dagger} + C_{2} P_{2}^{\dagger} \right)^{2} = 2C_{1} C_{2} P_{1}^{\dagger} P_{2}^{\dagger} [3, \mathcal{E}, \mathcal{Y}] \left( P_{1}^{\dagger} \right)^{2} = \left( P_{2}^{\dagger} \right)^{2} = 0 \quad , \quad \left[ P_{i}^{\dagger}, P_{j}^{\dagger} \right] = 0 \quad (3, \mathcal{E}, 5)
      V=1,2,3 (3 pairs)
   T(1+c_{2}P_{2}^{t})=(1+c_{3}P_{3}^{t})(1+c_{2}P_{2}^{t})(1+c_{1}P_{1}^{t})=(1+c_{3}P_{3}^{t})(1+c_{1}P_{1}^{t}+c_{2}P_{2}^{t}+c_{1}c_{2}P_{1}^{t}P_{2}^{t})
       = 1 + (c1P1+c2P2+c3P+3) + (c1c2P+P2+c1c3P+P3+c2c3P2P3)+c1c2c3P1+P2+P3
        = 1+ \(\sum_{\nu_{20}} C_{\nu} P_{\nu}^{\pm} + \frac{1}{2!} \left( \sum_{\nu_{20}} C_{\nu} P_{\nu}^{\pm} \right)^{2} + \frac{1}{3!} \left( \sum_{\nu} C_{\nu} P_{\nu}^{\pm} \right)^{3}
                                                                                                                                               (3, E, 6),
where use has been made of the relations (3, 5,5)
[(a+b+c)(a+b+c)] = ab+ac+ba+bc+ca+cb=2ab+2ac+2bc
a^2=b^2=c^2=0
   (a+b+c)[(a+b+c)(a+b+c)] = zabc + zbac + zcab= 6 abc.
          Making use of (\sum_{v=0}^{\infty} C_{v}P_{v}^{+})^{3} = 6 C_{1}C_{2}C_{3}P_{1}^{+}P_{2}^{+}P_{3}^{+} = 3! C_{1}C_{2}C_{3}P_{1}^{+}P_{2}^{+}P_{3}^{+} (3.E,8)
                         e^{x} = 1 + x + \frac{x^{2}}{2!} + \frac{x^{3}}{3!} + \dots; C_{\nu} = e^{-7i\phi}C_{\nu}
one canwrite
     |BCS(\phi)\rangle_{\mathcal{X}} = (\mathcal{T}_{\nu,0} \cup_{\nu}) \left\{ 1 + \frac{1}{n} \left( \sum_{\nu > 0} c_{\nu} P_{\nu}^{+} \right) + \frac{1}{2!} \left( \sum_{\nu > 0} c_{\nu} P_{\nu}^{+} \right)^{2} + \frac{1}{3!} \left( \sum_{\nu < 0} c_{\nu} P_{\nu}^{+} \right)^{3} + \dots \right\} |0\rangle
                       = \left( \frac{\pi U_{\nu}'}{v_{70}} \right) \left( 1 + \frac{e^{-2i\phi}}{1!} \left( \sum_{\nu 70} c_{\nu}' P_{\nu}^{\dagger} \right) + \frac{e^{-4i\phi}}{2!} \left( \sum_{\nu 70} c_{\nu}' P_{\nu}^{\dagger} \right)^{2} + \frac{\bar{e}^{6i\phi}}{3!} \left( \sum_{\nu 70} c_{\nu}' P_{\nu}^{\dagger} \right)^{3} \right) v_{70} 
 (3, E. 10)
                     C_{\nu} = e^{-2i\phi} C_{\nu}, \quad C_{\nu} = V_{\nu}/U_{\nu}. \quad (3.E.11)
  tems,
                         1B(S(Φ))χ = (Πυ) Σ (3.Ε./2)
               |N_0\rangle = \int d\phi e^{iN_0\phi} |B(S(\phi))\rangle_{\mathcal{K}} = \left(\prod_{\nu,n} U_{\nu}'\right) \sum_{Neven} \int d\phi e^{iN_0\phi} \frac{e^{iN_0\phi} - iN\phi}{(N/2)} \left(\sum_{\nu,n} C_{\nu}' P_{\nu}^{\dagger}\right)^{\frac{N}{2}} |0\rangle
                                     ~ ( I c', Pt) No/2 10>
     is the member with No particles of the pairing
```

notational bound, while

is the Cooper pair state, Because U' +0 for Ex « Ex, (3. E.14) is to be integreted to be valid for values of Ex close to EF.

Making use of the single J-shell model  $V' = \sqrt{\frac{N}{2}}$ ,  $V = \sqrt{1 - \frac{N}{2}}$ , (3, E.15)

and

 $\frac{V'}{U'} = \sqrt{\frac{N}{2Q - N}} \approx U'V'$ (3,E,16)

for a number of particle, considerably smaller than the full degeneracy of the myle-particle subspace in which mucleons can correlate, that is for N&ZSZ. Consequently

(3.E.17). 18>= \[ \( \alpha' \), P\$10>,

Where

(3, E, 18) (do), = (BCSIP+1BCS) = (2/V)

- and  $\mathcal{N} = \sum_{\nu > 0} (\alpha'_{o})_{\nu}^{2} ,$ (3.E,18)

```
Kelsahem 11/03/2018
   App. 3, E Coherent state (PRC 27, 054321 (2013))
The BCS ground state can be written as,
                                                    |B(S(\phi))\rangle_{\mathcal{K}} = \prod_{\nu \geqslant 0} (\mathcal{V}_{\nu} + \mathcal{V}_{\nu} a_{\nu}^{\dagger} a_{\nu}^{\dagger}) |0\rangle = \prod_{\nu \geqslant 0} \mathcal{V}_{\nu} (1 + \frac{\mathcal{V}_{\nu}}{\mathcal{V}_{\nu}} a_{\nu}^{\dagger} a_{\nu}^{\dagger}) |0\rangle
                                                                                                         =(\prod_{\nu > 0} U_{\nu}) (\prod_{\nu > 0} (1 + C_{\nu} P_{\nu}^{\dagger}) |0\rangle ; C_{\nu} = \frac{v_{\nu}}{|1|_{\nu}} ; P_{\nu}^{\dagger} = a_{\nu}^{\dagger} a_{\nu}^{\dagger}
                                                                                                                                                         Cu = Uv and Pr = atats
          V=1,2 (two pairs);
                                \Pi (1+C_{\nu}P_{\nu}^{\dagger}) = (1+C_{i}P_{i}^{\dagger})(1+C_{z}P_{z}^{\dagger}) = 1+C_{i}P_{i}^{\dagger} + C_{z}P_{z}^{\dagger} + C_{i}C_{z}P_{1}^{\dagger}P_{z}^{\dagger} 
(3,E,3)
                            = 1 + \sum_{\nu \neq 0} C_{\nu} P_{\nu}^{+} + \frac{1}{2!} \left( \sum_{\nu \neq 0} C_{\nu} P_{\nu}^{+} \right)^{2},
where use ther bean made of:
  \left( \sum_{\nu \neq 0} C_{\nu} P_{\nu}^{+} + \sum_{\nu \neq 0} C_{\nu} P_{\nu}^{+} \right)^{2} = 2C_{1}C_{2} P_{1}^{+} P_{2}^{+} (3.E,4) \left( P_{1}^{+} \right)^{2} = \left( P_{2}^{+} \right)^{2} = 0 , \quad [P_{1}^{+}, P_{2}^{+}] = 0 
  \left( (3.E,5) \right)^{2} = 2C_{1}C_{2} P_{1}^{+} P_{2}^{+} (3.E,4) \left( P_{1}^{+} \right)^{2} = \left( P_{2}^{+} \right)^{2} = 0 , \quad [P_{1}^{+}, P_{2}^{+}] = 0 
                   V=1,2,3 (3 pairs)

\Pi(1+c_{2}P_{2}^{\dagger}) = (1+c_{3}P_{3}^{\dagger})(1+c_{2}P_{2}^{\dagger})(1+c_{1}P_{1}^{\dagger}) = (1+c_{3}P_{3}^{\dagger})(1+c_{1}P_{1}^{\dagger}+c_{2}P_{2}^{\dagger}+c_{1}c_{2}P_{1}^{\dagger}P_{2}^{\dagger})

                   = 1 + (c1P1+c2Pt+c3Pt3) + (c1c2P1Pt+c1c3P1Pt3+c2c3P2Pt3)+c1c2c3P1+PtPt3
  = 1 + \sum_{\nu > 0} C_{\nu} P_{\nu}^{+} + \frac{1}{2!} \left( \sum_{\nu > 0} C_{\nu} P_{\nu}^{+} \right)^{2} + \frac{1}{3!} \left( \sum_{\nu > 0} C_{\nu} P_{\nu}^{+} \right)^{3} (3, E, b), where use has been made of the relations (3, E, 5) 

\left[ (a + b + c)(a + b + c) \right] = ab + ac + ba + bc + ca + cb = 2ab + 2ac + 2bc \quad a^{2} = b^{2} = c^{2} = 0
         (a+b+c)[(a+b+c)(a+b+c)] = zabc + zbac + zcab = 6 abc.
                                                                                                 ( \( \subsect C_0 P_0^+ \)^3 = 6 \( c_1 C_2 C_3 P_1^+ P_2^+ P_3^+ = 3! \) \( c_1 C_2 C_3 P_1^+ P_2^+ P_3^- (3. E, a) \)
                                                                     e^{x} = 1 + x + \frac{x^{2}}{2!} + \frac{x^{3}}{3!} + \dots; c_{v} = e^{-zi\phi}c_{v}
one can write
              |B(S(Φ)) x = (T, U,) {1+1/2 (Σ c ν P+ )+ 1/2! (Σ c ν P+ )2 (Σ c ν P+ )3 (Σ c ν P+ )3 +... >10)
                                                                  = \left( \frac{1}{\nu_{70}} U_{\nu}^{'} \right) \left( 1 + \frac{e^{-2i\phi}}{1!} \left( \sum_{\nu_{70}} c_{\nu}^{'} P_{\nu}^{+} \right) + \frac{e^{-4i\phi}}{2!} \left( \sum_{\nu_{70}} c_{\nu}^{'} P_{\nu}^{+} \right)^{2} + \frac{\bar{e}^{6i\phi}}{3!} \left( \sum_{\nu_{70}} c_{\nu} P_{\nu}^{+} \right)^{3} \right) \left( 1 + \frac{\bar{e}^{-2i\phi}}{1!} \left( \sum_{\nu_{70}} c_{\nu}^{'} P_{\nu}^{+} \right) + \frac{\bar{e}^{-4i\phi}}{2!} \left( \sum_{\nu_{70}} c_{\nu}^{'} P_{\nu}^{+} \right)^{2} + \frac{\bar{e}^{6i\phi}}{3!} \left( \sum_{\nu_{70}} c_{\nu}^{'} P_{\nu}^{+} \right)^{3} \right) \left( 1 + \frac{\bar{e}^{-2i\phi}}{1!} \left( \sum_{\nu_{70}} c_{\nu}^{'} P_{\nu}^{+} \right) + \frac{\bar{e}^{-4i\phi}}{2!} \left( \sum_{\nu_{70}} c_{\nu}^{'} P_{\nu}^{+} \right)^{3} \right) \left( 1 + \frac{\bar{e}^{-2i\phi}}{1!} \left( \sum_{\nu_{70}} c_{\nu}^{'} P_{\nu}^{+} \right) + \frac{\bar{e}^{-4i\phi}}{2!} \left( \sum_{\nu_{70}} c_{\nu}^{'} P_{\nu}^{+} \right)^{3} \right) \left( \sum_{\nu_{70}} c_{\nu}^{'} P_{\nu}^{+} \right)^{3} \left( \sum_{\nu_{70}} c_{\nu}^{'} P_{\nu}^{+} \right)^{3} \left( \sum_{\nu_{70}} c_{\nu}^{'} P_{\nu}^{+} \right)^{3} \right) \left( \sum_{\nu_{70}} c_{\nu}^{'} P_{\nu}^{+} \right)^{3} \left( \sum_{\nu_{70}} c_{\nu}^{'} P_{\nu}^{+}
         where C_{\nu} = e^{-2i\phi} C_{\nu}, C_{\nu} = V_{\nu}/U_{\nu}. (3.E.11)

Thus, E_{\nu} = (E_{\nu}, E_{\nu}) \sum_{N \in V \in N} \frac{e^{-iN\phi}}{(N/2)!} (E_{\nu}, E_{\nu})^{N/2} = (E_{\nu}, E_{\nu}, E_{\nu}, E_{\nu})^{N/2} = (E_{\nu}, E_{\nu}, E_{\nu}, E_{\nu}, E_{\nu})^{N/2} = (E_{\nu}, E_{\nu}, E_{
    Tems,
                                           |N_0\rangle = \int d\phi e^{iN_0\phi} |B(S(\phi))_{\chi} = (\prod_{\nu,n} U_{\nu}') \sum_{Neven} \int d\phi e^{iN_0\phi} \frac{e^{-iN\phi}}{(N/2)} (\sum_{\nu,n} C_{\nu}' P_{\nu}^{\dagger})^{\frac{N}{2}} |0\rangle
                                                                                                       ~ ( Z C' Pt) No/2 10>
                                                                                                                                                                                                                                                                                                                                                                                                                                    (3, E, 13)
                is the member with No particles of the pairing
```

notational band, while

is the Cooper pair state, Because U' +0 for Ex « Ex, (3. E.14) is to be interpreted to be valid for values of Ex close to EF.

Making use of The single J-shell model  $V' = \sqrt{\frac{N}{252}}$ ,  $U' = \sqrt{1 - \frac{N}{252}}$ , (3, E. 15)

and

 $\frac{V'}{U'} = \sqrt{\frac{N}{2Q - N}} \approx U'V'$ 

for a number of particle, considerably smaller than the full degeneracy of the myle-particle subspace mobuch mucleons can correlate, that is for NKZSZ. Consephently

(3.E.17). 18 >= 10 / P\$10>,

Where

(do) = (BCS) P+ 1BCS) = (LVV) (3, E, 18)

and .  $\mathcal{N} = \sum_{\nu > 0} (\alpha_0^{\prime})_{\nu}^2 ,$ (3.E,18)