$$
\begin{aligned}
r_{b2} &= m_b^{-1} \left|(m_b+1)\mathbf{r}_{A2} + \mathbf{r}_{A1} - (m_b+2)\mathbf{r}_{Aa}\right| \\
&= m_b^{-1}\Big((m_b+2)^2 r_{Aa}^2 + (m_b+1)^2 r_{A2}^2 + r_{A1}^2 \\
&- 2(m_b+2)(m_b+1)\mathbf{r}_{Aa}\,\mathbf{r}_{A2} - 2(m_b+2)\mathbf{r}_{Aa}\,\mathbf{r}_{A1} + 2(m_b+1)\mathbf{r}_{A2}\mathbf{r}_{A1}\Big)^{1/2},
\end{aligned}
\tag{7.2.102}
$$

$$
\begin{aligned}
r_{Bb} &= \left|\frac{m_b+2}{m_b}\mathbf{r}_{Aa} - \frac{m_A+m_b+2}{(m_A+2)m_b}(\mathbf{r}_{A1}+\mathbf{r}_{A2})\right| \\
&= \Bigg[\left(\frac{m_b+2}{m_b}\right)^2 r_{Aa}^2 + \left(\frac{m_A+m_b+2}{(m_A+2)m_b}\right)^2 (r_{A1}^2 + r_{A2}^2 + 2\mathbf{r}_{A1}\mathbf{r}_{A2}) \\
&- 2\frac{(m_b+2)(m_A+m_b+2)}{(m_A+2)m_b^2}\mathbf{r}_{Aa}(\mathbf{r}_{A1}+\mathbf{r}_{A2})\Bigg]^{1/2},
\end{aligned}
\tag{7.2.103}
$$

$$
\begin{aligned}
r_{Cc} &= \left|\frac{m_b+2}{m_b+1}\mathbf{r}_{Aa} - \frac{m_b+2+m_A}{(m_b+1)(m_A+1)}\mathbf{r}_{A2}\right| \\
&= \Bigg[\left(\frac{m_a}{(m_a-1)}\right)^2 r_{Aa}^2 + \left(\frac{m_A+m_a}{(m_A+1)(m_a-1)}\right)^2 r_{A2}^2 \\
&- 2\frac{m_A m_a + m_a^2}{(m_A+1)(m_a-1)^2}\mathbf{r}_{Aa}\mathbf{r}_{A2}\Bigg]^{1/2},
\end{aligned}
\tag{7.2.104}
$$

$$
\cos\omega_b = \frac{\mathbf{r}_{b1}\mathbf{r}_{b2}}{r_{b1}r_{b2}}, \tag{7.2.105}
$$

$$
\cos\omega_{if} = \frac{\mathbf{r}_{Aa}\mathbf{r}_{Bb}}{r_{Aa}r_{Bb}}, \tag{7.2.106}
$$

with

$$
\mathbf{r}_{Aa}\mathbf{r}_{A1} = r_{Aa}r_{A1}\cos\alpha, \tag{7.2.107}
$$

$$
\mathbf{r}_{Aa}\mathbf{r}_{A2} = r_{Aa}r_{A2}(\sin\alpha\cos\gamma\sin\omega_A + \cos\alpha\cos\omega_A), \tag{7.2.108}
$$

$$
\mathbf{r}_{A1}\mathbf{r}_{A2} = r_{A1}r_{A2}\cos\omega_A. \tag{7.2.109}
$$

7.2.6 Successive transfer

The successive two–neutron transfer amplitudes can be written as (Bayman and Chen (1982)):

$$\begin{aligned} T_{2NT}^{2step} = & \frac{4\mu_{Cc}}{\hbar^2} \sum_{\substack{\sigma_1\sigma_2 \\ \sigma_1'\sigma_2' \\ KM}} \int d^3r_{Cc} d^3r_{b1} d^3r_{A2} d^3r'_{Cc} d^3r'_{b1} d^3r'_{A2} \chi^{(-)*}(\mathbf{k}_{Bb}, \mathbf{r}_{Bb}) \\ & \times \left[\psi^{j_f}(\mathbf{r}_{A1},\sigma_1)\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\right]_0^{0*} v(r_{b1}) \left[\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\psi^{j_i}(\mathbf{r}_{b1},\sigma_1)\right]_M^K \\ & \times G(\mathbf{r}_{Cc},\mathbf{r}'_{Cc}) \left[\psi^{j_f}(\mathbf{r}'_{A2},\sigma'_2)\psi^{j_i}(\mathbf{r}'_{b1},\sigma'_1)\right]_M^{K*} v(r'_{c2}) \\ & \times \left[\psi^{j_i}(\mathbf{r}'_{b1},\sigma'_1)\psi^{j_i}(\mathbf{r}'_{b2},\sigma'_2)\right]_0^0 \chi^{(+)}(\mathbf{r}'_{Aa}). \end{aligned} \tag{7.2.110}$$

It is of notice that the time–reversal phase convention is used throughout. Expanding the Green function and the distorted waves in a basis of angular momentum eigenstate one can write,

$$\chi^{(-)*}(\mathbf{k}_{Bb},\mathbf{r}_{Bb}) = \sum_{\tilde{l}} \frac{4\pi}{k_{Bb}r_{Bb}} i^{-\tilde{l}} e^{i\sigma_f^{\tilde{l}}} F_{\tilde{l}} \sum_m Y_m^{\tilde{l}}(\hat{r}_{Bb}) Y_m^{\tilde{l}*}(\hat{k}_{Bb}), \tag{7.2.111}$$

the sum over m being

$$\sum_m (-1)^{\tilde{l}-m} Y_m^{\tilde{l}}(\hat{r}_{Bb}) Y_{-m}^{\tilde{l}}(\hat{k}_{Bb}) = \sqrt{2\tilde{l}+1}\left[Y^{\tilde{l}}(\hat{r}_{Bb})Y^{\tilde{l}}(\hat{k}_{Bb})\right]_0^0, \tag{7.2.112}$$

where we have used (7.K.2) and (7.K.18), so

$$\chi^{(-)*}(\mathbf{k}_{Bb},\mathbf{r}_{Bb}) = \sum_{\tilde{l}} \sqrt{2\tilde{l}+1}\frac{4\pi}{k_{Bb}r_{Bb}} i^{-\tilde{l}} e^{i\sigma_f^{\tilde{l}}} F_{\tilde{l}}(r_{Bb}) \left[Y^{\tilde{l}}(\hat{r}_{Bb})Y^{\tilde{l}}(\hat{k}_{Bb})\right]_0^0 \tag{7.2.113}$$

Similarly,

$$\chi^{(+)}(\mathbf{r}'_{Aa}) = \sum_l i^l \sqrt{2l+1}\frac{4\pi}{k_{Aa}r'_{Aa}} e^{i\sigma_i^l} F_l(r'_{Aa}) \left[Y^l(\hat{r}'_{Aa})Y^l(\hat{k}_{Aa})\right]_0^0 \tag{7.2.114}$$

where we have taken into account the choice $\hat{k}_{Aa} \equiv \hat{z}$. The Green function can be written as

$$G(\mathbf{r}_{Cc},\mathbf{r}'_{Cc}) = i\sum_{l_c} \sqrt{2l_c+1}\frac{f_{l_c}(k_{Cc},r_<)P_{l_c}(k_{Cc},r_>)}{k_{Cc}r_{Cc}r'_{Cc}} \left[Y^{l_c}(\hat{r}_{Cc})Y^{l_c}(\hat{r}'_{Cc})\right]_0^0. \tag{7.2.115}$$

Finally

$$
\begin{aligned}
T_{2NT}^{2step} =& \frac{4\mu_{Cc}(4\pi)^2 i}{\hbar^2 k_{Aa}k_{Bb}k_{Cc}} \sum_{l,l_c,\bar{l}} e^{i(\sigma_i^l+\sigma_f^{\bar{l}})} i^{l-\bar{l}} \sqrt{(2l+1)(2l_c+1)(2\bar{l}+1)} \\
&\times \sum_{\substack{\sigma_1\sigma_2\\ \sigma_1'\sigma_2'}} \int d^3r_{Cc}d^3r_{b1}d^3r_{A2}d^3r'_{Cc}d^3r'_{b1}d^3r'_{A2}v(r_{b1})v(r'_{c2}) \left[Y^{\bar{l}}(\hat{r}_{Bb})Y^{\bar{l}}(\hat{k}_{Bb})\right]_0^0 \\
&\times \left[Y^{l}(\hat{r}'_{Aa})Y^{l}(\hat{k}'_{Aa})\right]_0^0 \left[Y^{l_c}(\hat{r}_{Cc})Y^{l_c}(\hat{r}'_{Cc})\right]_0^0 \frac{F_{\bar{l}}(r_{Bb})}{r_{Bb}} \frac{F_l(r'_{Aa})}{r'_{Aa}} \\
&\times \frac{f_{l_c}(k_{Cc},r_<)P_{l_c}(k_{Cc},r_>)}{r_{Cc}r'_{Cc}} \left[\psi^{j_f}(\mathbf{r}_{A1},\sigma_1)\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\right]_0^{0*} \\
&\times \left[\psi^{j_i}(\mathbf{r}'_{b1},\sigma'_1)\psi^{j_i}(\mathbf{r}'_{b2},\sigma'_2)\right]_0^0 \sum_{KM} \left[\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\psi^{j_i}(\mathbf{r}_{b1},\sigma_1)\right]_M^K \\
&\times \left[\psi^{j_f}(\mathbf{r}'_{A2},\sigma'_2)\psi^{j_i}(\mathbf{r}'_{b1},\sigma'_1)\right]_M^{K*}.
\end{aligned}
\tag{7.2.116}
$$

Let us now perform the integration over $\mathbf{r}_{A2}$,

$$
\begin{aligned}
&\sum_{\sigma_1,\sigma_2} \int d\mathbf{r}_{A2} \left[\psi^{j_f}(\mathbf{r}_{A1},\sigma_1)\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\right]_0^{0*} \left[\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\psi^{j_i}(\mathbf{r}_{b1},\sigma_1)\right]_M^K \\
&= \sum_{\sigma_1,\sigma_2} (-1)^{1/2-\sigma_1+1/2-\sigma_2} \int d\mathbf{r}_{A2} \left[\psi^{j_f}(\mathbf{r}_{A1},-\sigma_1)\psi^{j_f}(\mathbf{r}_{A2},-\sigma_2)\right]_0^0 \left[\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\psi^{j_i}(\mathbf{r}_{b1},\sigma_1)\right]_M^K \\
&= -\sum_{\sigma_1,\sigma_2} (-1)^{1/2-\sigma_1+1/2-\sigma_2} \int d\mathbf{r}_{A2} \left[\psi^{j_f}(\mathbf{r}_{A2},-\sigma_2)\psi^{j_f}(\mathbf{r}_{A1},-\sigma_1)\right]_0^0 \left[\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\psi^{j_i}(\mathbf{r}_{b1},\sigma_1)\right]_M^K \\
&= -((j_fj_f)_0(j_fj_i)_K|(j_fj_f)_0(j_fj_i)_K)_K \sum_{\sigma_1,\sigma_2} (-1)^{1/2-\sigma_1+1/2-\sigma_2} \\
&\times \int d\mathbf{r}_{A2} \left[\psi^{j_f}(\mathbf{r}_{A2},-\sigma_2)\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\right]_0^0 \left[\psi^{j_f}(\mathbf{r}_{A1},-\sigma_1)\psi^{j_i}(\mathbf{r}_{b1},\sigma_1)\right]_M^K \\
&= \frac{1}{2j_f+1}\sqrt{2j_f+1}((l_f\tfrac{1}{2})_{j_f}(l_i\tfrac{1}{2})_{j_i}|(l_fl_i)_K(\tfrac{1}{2}\tfrac{1}{2})_0)_K \\
&\times u_{l_f}(r_{A1})u_{l_i}(r_{b1})\left[Y^{l_f}(\hat{r}_{A1})Y^{l_i}(\hat{r}_{b1})\right]_M^K \sum_{\sigma_1}(-1)^{1/2-\sigma_1}\left[\chi^{1/2}(-\sigma_1)\chi^{1/2}(\sigma_1)\right]_0^0 \\
&= -\sqrt{\frac{2}{2j_f+1}}((l_f\tfrac{1}{2})_{j_f}(l_i\tfrac{1}{2})_{j_i}|(l_fl_i)_K(\tfrac{1}{2}\tfrac{1}{2})_0)_K \left[Y^{l_f}(\hat{r}_{A1})Y^{l_i}(\hat{r}_{b1})\right]_M^K u_{l_f}(r_{A1})u_{l_i}(r_{b1}),
\end{aligned}
\tag{7.2.117}
$$

where we have evaluated the $9j$–symbol

$$
((j_fj_f)_0(j_fj_i)_K|(j_fj_f)_0(j_fj_i)_K)_K = \frac{1}{2j_f+1}, \tag{7.2.118}
$$

as well as (7.K.19). We proceed in a similar way to evaluate the integral over $\mathbf{r}'_{b1}$,

$$
\begin{aligned}
\sum_{\sigma'_1,\sigma'_2} \int d\mathbf{r}'_{b1} & \left[\psi^{j_i}(\mathbf{r}'_{b1},\sigma'_1)\psi^{j_i}(\mathbf{r}'_{b2},\sigma'_2)\right]^0_0 \left[\psi^{j_f}(\mathbf{r}'_{A2},\sigma'_2)\psi^{j_i}(\mathbf{r}'_{b1},\sigma'_1)\right]^{K*}_M \\
&= -(-1)^{K-M} \sum_{\sigma'_1,\sigma'_2} \int d\mathbf{r}'_{b1} \left[\psi^{j_f}(\mathbf{r}'_{A2},-\sigma'_2)\psi^{j_i}(\mathbf{r}'_{b1},-\sigma'_1)\right]^K_{-M} \\
&\times \left[\psi^{j_i}(\mathbf{r}'_{b2},\sigma'_2)\psi^{j_i}(\mathbf{r}'_{b1},\sigma'_1)\right]^0_0 (-1)^{1/2-\sigma'_1+1/2-\sigma'_2} \\
&= -(-1)^{K-M}((j_f j_i)_K (j_i j_i)_0 | (j_f j_i)_K (j_i j_i)_0)_K (-\sqrt{2j_i+1}) \\
&\times ((l_f \tfrac{1}{2})_{j_f} (l_i \tfrac{1}{2})_{j_i} | (l_f l_i)_K (\tfrac{1}{2}\tfrac{1}{2})_0)_K (-\sqrt{2}) u_{l_f}(r'_{A2}) u_{l_i}(r'_{b2}) \left[Y^{l_f}(\hat{r}'_{A2}) Y^{l_i}(\hat{r}'_{b2})\right]^K_{-M} \\
&= -\sqrt{\frac{2}{2j_i+1}} ((l_f \tfrac{1}{2})_{j_f} (l_i \tfrac{1}{2})_{j_i} | (l_f l_i)_K (\tfrac{1}{2}\tfrac{1}{2})_0)_K \left[Y^{l_f}(\hat{r}'_{A2}) Y^{l_i}(\hat{r}'_{b2})\right]^{K*}_M u_{l_f}(r'_{A2}) u_{l_i}(r'_{b2}).
\end{aligned}
\tag{7.2.119}
$$

Setting the different elements together one obtains

$$
\begin{aligned}
T^{2step}_{2NT} =& \frac{4\mu_{Cc}(4\pi)^2 i}{\hbar^2 k_{Aa} k_{Bb} k_{Cc}} \frac{2}{\sqrt{(2j_i+1)(2j_f+1)}} \sum_{K,M} ((l_f \tfrac{1}{2})_{j_f} (l_i \tfrac{1}{2})_{j_i} | (l_f l_i)_K (\tfrac{1}{2}\tfrac{1}{2})_0)^2_K \\
&\times \sum_{l_c,l,\bar{l}} e^{i(\sigma^l_i+\sigma^l_f)} \sqrt{(2l_c+1)(2l+1)(2\bar{l}+1)}\, i^{l-\bar{l}} \\
&\times \int d^3 r_{Cc} d^3 r_{b1} d^3 r'_{Cc} d^3 r'_{A2} v(r_{b1}) v(r'_{c2}) u_{l_f}(r_{A1}) u_{l_i}(r_{b1}) u_{l_f}(r'_{A2}) u_{l_i}(r'_{b2}) \\
&\times \left[Y^{l_f}(\hat{r}'_{A2}) Y^{l_i}(\hat{r}'_{b2})\right]^{K*}_M \left[Y^{l_f}(\hat{r}_{A1}) Y^{l_i}(\hat{r}_{b1})\right]^K_M \frac{F_l(r'_{Aa}) F_{\bar{l}}(r'_{Bb}) f_{l_c}(k_{Cc}, r_<) P_{l_c}(k_{Cc}, r_>)}{r'_{Aa} r_{Bb} r_{Cc} r'_{Cc}} \\
&\times \left[Y^{\bar{l}}(\hat{r}_{Bb}) Y^{\bar{l}}(\hat{k}_{Bb})\right]^0_0 \left[Y^{l}(\hat{r}'_{Aa}) Y^{l}(\hat{k}_{Aa})\right]^0_0 \left[Y^{l_c}(\hat{r}_{Cc}) Y^{l_c}(\hat{r}'_{Cc})\right]^0_0 .
\end{aligned}
\tag{7.2.120}
$$

We now proceed to write this expression in a more compact way. For this purpose one writes

$$
\begin{aligned}
\left[Y^{\bar{l}}(\hat{r}_{Bb}) Y^{\bar{l}}(\hat{k}_{Bb})\right]^0_0 & \left[Y^{l}(\hat{r}'_{Aa}) Y^{l}(\hat{k}_{Aa})\right]^0_0 = \\
& ((l\,l)_0 (\bar{l}\,\bar{l})_0 | (l\,\bar{l})_0 (l\,\bar{l})_0)_0 \left[Y^{\bar{l}}(\hat{r}_{Bb}) Y^{l}(\hat{r}'_{Aa})\right]^0_0 \left[Y^{\bar{l}}(\hat{k}_{Bb}) Y^{l}(\hat{k}_{Aa})\right]^0_0 \\
&= \frac{\delta_{\bar{l}l}}{2l+1} \left[Y^{l}(\hat{r}_{Bb}) Y^{l}(\hat{r}'_{Aa})\right]^0_0 \left[Y^{l}(\hat{k}_{Bb}) Y^{l}(\hat{k}_{Aa})\right]^0_0 .
\end{aligned}
\tag{7.2.121}
$$

Taking into account the relation

$$
\left[Y^{l}(\hat{k}_{Bb}) Y^{l}(\hat{k}_{Aa})\right]^0_0 = \frac{(-1)^l}{\sqrt{4\pi}} Y^l_0(\hat{k}_{Bb}) i^l, \tag{7.2.122}
$$

and

$$\left[Y^{l}(\hat{r}_{Bb})Y^{l}(\hat{r}'_{Aa})\right]_0^0 \left[Y^{l_c}(\hat{r}_{Cc})Y^{l_c}(\hat{r}'_{Cc})\right]_0^0 =$$
$$((l\,l)_0(l_c\,l_c)_0|(l\,l_c)_K(l\,l_c)_K)_0 \left\{\left[Y^{l}(\hat{r}_{Bb})Y^{l_c}(\hat{r}_{Cc})\right]^K \left[Y^{l}(\hat{r}'_{Aa})Y^{l_c}(\hat{r}'_{Cc})\right]^K\right\}_0^0$$
$$= \sqrt{\frac{2K+1}{(2l+1)(2l_c+1)}}$$
$$\times \sum_{M'} \frac{(-1)^{K+M'}}{\sqrt{2K+1}} \left[Y^{l}(\hat{r}_{Bb})Y^{l_c}(\hat{r}_{Cc})\right]_{-M'}^K \left[Y^{l}(\hat{r}'_{Aa})Y^{l_c}(\hat{r}'_{Cc})\right]_{M'}^K$$
$$= \sqrt{\frac{1}{(2l+1)(2l_c+1)}}$$
$$\times \sum_{M'} \left[Y^{l}(\hat{r}_{Bb})Y^{l_c}(\hat{r}_{Cc})\right]_{M'}^{K*} \left[Y^{l}(\hat{r}'_{Aa})Y^{l_c}(\hat{r}'_{Cc})\right]_{M'}^K. \tag{7.2.123}$$

It is of notice that the integrals

$$\int d\hat{r}_{Cc} d\hat{r}_{b1} \left[Y^{l}(\hat{r}_{Bb})Y^{l_c}(\hat{r}_{Cc})\right]_M^{K*} \left[Y^{l_f}(\hat{r}_{A1})Y^{l_i}(\hat{r}_{b1})\right]_M^K, \tag{7.2.124}$$

and

$$\int d\hat{r}'_{Cc} d\hat{r}'_{A2} \left[Y^{l}(\hat{r}'_{Aa})Y^{l_c}(\hat{r}'_{Cc})\right]_M^K \left[Y^{l_f}(\hat{r}'_{A2})Y^{l_i}(\hat{r}'_{b2})\right]_M^{K*}, \tag{7.2.125}$$

over the angular variables do not depend on M. Let us see why this is so with the help of (7.2.124),

$$\left[Y^{l}(\hat{r}_{Bb})Y^{l_c}(\hat{r}_{Cc})\right]_M^{K*} \left[Y^{l_f}(\hat{r}_{A1})Y^{l_i}(\hat{r}_{b1})\right]_M^K = (-1)^{K-M} \left[Y^{l}(\hat{r}_{Bb})Y^{l_c}(\hat{r}_{Cc})\right]_{-M}^K$$
$$\times \left[Y^{l_f}(\hat{r}_{A1})Y^{l_i}(\hat{r}_{b1})\right]_M^K = (-1)^{K-M} \sum_J \langle K\;K\;M\;-M|J\;0\rangle$$
$$\times \left\{\left[Y^{l}(\hat{r}_{Bb})Y^{l_c}(\hat{r}_{Cc})\right]^K \left[Y^{l_f}(\hat{r}_{A1})Y^{l_i}(\hat{r}_{b1})\right]^K\right\}_0^J. \tag{7.2.126}$$

After integration, only the term

$$(-1)^{K-M}\langle K\;K\;M\;-M|0\;0\rangle \left\{\left[Y^{l}(\hat{r}_{Bb})Y^{l_c}(\hat{r}_{Cc})\right]^K \left[Y^{l_f}(\hat{r}_{A1})Y^{l_i}(\hat{r}_{b1})\right]^K\right\}_0^0 = .$$
$$\frac{1}{\sqrt{2K+1}} \left\{\left[Y^{l}(\hat{r}_{Bb})Y^{l_c}(\hat{r}_{Cc})\right]^K \left[Y^{l_f}(\hat{r}_{A1})Y^{l_i}(\hat{r}_{b1})\right]^K\right\}_0^0 \tag{7.2.127}$$

corresponding to $J = 0$ survives, which is indeed independent of M. We can thus omit the sum over M in (7.2.120) and multiply by $(2K + 1)$, obtaining

$$\begin{aligned}T^{2step}_{2NT} =& \frac{64\mu_{Cc}(\pi)^{3/2} i}{\hbar^2 k_{Aa} k_{Bb} k_{Cc}} \frac{i^{-l}}{\sqrt{(2j_i+1)(2j_f+1)}} \\ &\times \sum_K (2K+1)((l_f \tfrac{1}{2})_{j_f} (l_i \tfrac{1}{2})_{j_i} | (l_f l_i)_K (\tfrac{1}{2}\tfrac{1}{2})_0)^2_K \\ &\times \sum_{l_c, l} \frac{e^{i(\sigma^l_i + \sigma^l_f)}}{\sqrt{(2l+1)}} Y^l_0(\hat{k}_{Bb}) S_{K,l,l_c},\end{aligned} \tag{7.2.128}$$

where

$$\begin{aligned}S_{K,l,l_c} =& \int d^3 r_{Cc} d^3 r_{b1} v(r_{b1}) u_{l_f}(r_{A1}) u_{l_i}(r_{b1}) \frac{s_{K,l,l_c}(r_{Cc})}{r_{Cc}} \frac{F_l(r_{Bb})}{r_{Bb}} \\ &\times \left[Y^{l_f}(\hat{r}_{A1}) Y^{l_i}(\hat{r}_{b1})\right]^K_M \left[Y^{l_c}(\hat{r}_{Cc}) Y^{l}(\hat{r}_{Bb})\right]^{K*}_M ,\end{aligned} \tag{7.2.129}$$

and

$$\begin{aligned}s_{K,l,l_c}(r_{Cc}) =& \int_{r_{Cc} fixed} d^3 r'_{Cc} d^3 r'_{A2} v(r'_{c2}) u_{l_f}(r'_{A2}) u_{l_i}(r'_{b2}) \frac{F_l(r'_{Aa})}{r'_{Aa}} \frac{f_{l_c}(k_{Cc}, r_<) P_{l_c}(k_{Cc}, r_>)}{r'_{Cc}} \\ &\times \left[Y^{l_f}(\hat{r}'_{A2}) Y^{l_i}(\hat{r}'_{b2})\right]^{K*}_M \left[Y^{l_c}(\hat{r}'_{Cc}) Y^{l}(\hat{r}'_{Aa})\right]^K_M .\end{aligned} \tag{7.2.130}$$

It can be shown that the integrand in (7.2.129) is independent of M. Consequently, one can sum over M and divide by $(2K + 1)$, to get

$$\begin{aligned}&\frac{1}{2K+1} v(r_{b1}) u_{l_f}(r_{A1}) u_{l_i}(r_{b1}) \frac{s_{K,l,l_c}(r_{Cc})}{r_{Cc}} \frac{F_l(r_{Bb})}{r_{Bb}} \\ &\qquad \times \sum_M \left[Y^{l_f}(\hat{r}_{A1}) Y^{l_i}(\hat{r}_{b1})\right]^K_M \left[Y^{l_c}(\hat{r}_{Cc}) Y^{l}(\hat{r}_{Bb})\right]^{K*}_M .\end{aligned} \tag{7.2.131}$$

This integrand is rotationally invariant (it is proportional to a T^L_M spherical tensor with $L = 0$, $M = 0$), so one can evaluate it in the "standard" configuration in which $\mathbf{r}_{Cc}$ is directed along the z–axis and multiply by $8\pi^2$ (see Bayman and Chen (1982)), obtaining the final expression for S_{K,l,l_c}:

$$\begin{aligned}S_{K,l,l_c} =& \frac{4\pi^{3/2}\sqrt{2l_c+1}}{2K+1} i^{-l_c} \\ &\times \int r^2_{Cc} dr_{Cc} r^2_{b1} dr_{b1} \sin\theta \, d\theta \, v(r_{b1}) u_{l_f}(r_{A1}) u_{l_i}(r_{b1}) \\ &\times \frac{s_{K,l,l_c}(r_{Cc})}{r_{Cc}} \frac{F_l(r_{Bb})}{r_{Bb}} \\ &\times \sum_M \langle l_c \, 0 \, l \, M | K \, M \rangle \left[Y^{l_f}(\hat{r}_{A1}) Y^{l_i}(\theta+\pi, 0)\right]^K_M Y^{l*}_M(\hat{r}_{Bb}).\end{aligned} \tag{7.2.132}$$

Similarly, one has

$$
\begin{aligned}
s_{K,l,l_c}(r_{Cc}) =& \frac{4\pi^{3/2}\sqrt{2l_c+1}}{2K+1} i^{l_c} \\
&\times \int r'^2_{Cc}\, dr'_{Cc}\, r'^2_{A2}\, dr'_{A2}\, \sin\theta'\, d\theta'\, v(r'_{c2}) u_{l_f}(r'_{A2}) u_{l_i}(r'_{b2}) \\
&\times \frac{F_l(r'_{Aa})}{r'_{Aa}} \frac{f_{l_c}(k_{Cc}, r_<) P_{l_c}(k_{Cc}, r_>)}{r'_{Cc}} \\
&\times \sum_M \langle l_c\ 0\ l\ M | K\ M\rangle \left[Y^{l_f}(\hat{r}'_{A2}) Y^{l_i}(\hat{r}'_{b2})\right]^{K*}_M Y^l_M(\hat{r}'_{Aa}).
\end{aligned}
\tag{7.2.133}
$$

Introducing the further approximations $\mathbf{r}_{A1} \approx \mathbf{r}_{C1}$ and $\mathbf{r}_{b2} \approx \mathbf{r}_{c2}$, one obtains the final expression

$$
\begin{aligned}
T^{2step}_{2NT} =& \frac{1024\mu_{Cc}\pi^{9/2} i}{\hbar^2 k_{Aa} k_{Bb} k_{Cc}} \frac{1}{\sqrt{(2j_i+1)(2j_f+1)}} \\
&\times \sum_K \frac{1}{2K+1} ((l_f \tfrac{1}{2})_{j_f} (l_i \tfrac{1}{2})_{j_i} | (l_f l_i)_K (\tfrac{1}{2}\tfrac{1}{2})_0)^2_K \\
&\times \sum_{l_c, l} e^{i(\sigma^l_i + \sigma^l_f)} \frac{(2l_c+1)}{\sqrt{2l+1}} Y^l_0(\hat{k}_{Bb}) S_{K,l,l_c},
\end{aligned}
\tag{7.2.134}
$$

with

$$
\begin{aligned}
S_{K,l,l_c} =& \int r^2_{Cc}\, dr_{Cc}\, r^2_{b1}\, dr_{b1}\, \sin\theta\, d\theta\, v(r_{b1}) u_{l_f}(r_{C1}) u_{l_i}(r_{b1}) \\
&\times \frac{s_{K,l,l_c}(r_{Cc})}{r_{Cc}} \frac{F_l(r_{Bb})}{r_{Bb}} \\
&\times \sum_M \langle l_c\ 0\ l\ M | K\ M\rangle \left[Y^{l_f}(\hat{r}_{C1}) Y^{l_i}(\theta+\pi, 0)\right]^K_M Y^{l*}_M(\hat{r}_{Bb}),
\end{aligned}
\tag{7.2.135}
$$

and

$$
\begin{aligned}
s_{K,l,l_c}(r_{Cc}) =& \int r'^2_{Cc}\, dr'_{Cc}\, r'^2_{A2}\, dr'_{A2}\, \sin\theta'\, d\theta'\, v(r'_{c2}) u_{l_f}(r'_{A2}) u_{l_i}(r'_{c2}) \\
&\times \frac{F_l(r'_{Aa})}{r'_{Aa}} \frac{f_{l_c}(k_{Cc}, r_<) P_{l_c}(k_{Cc}, r_>)}{r'_{Cc}} \\
&\times \sum_M \langle l_c\ 0\ l\ M | K\ M\rangle \left[Y^{l_f}(\hat{r}'_{A2}) Y^{l_i}(\hat{r}'_{c2})\right]^{K*}_M Y^l_M(\hat{r}'_{Aa}).
\end{aligned}
\tag{7.2.136}
$$

7.2.7 Coordinates for the successive transfer

In the standard configuration in which the integrals (7.2.135) and (7.2.136) are to be evaluated, we have

draw figure ??

$$
\mathbf{r}_{Cc} = r_{Cc}\,\hat{\mathbf{z}}, \qquad \mathbf{r}_{b1} = r_{b1}(-\cos\theta\,\hat{\mathbf{z}} - \sin\theta\,\hat{\mathbf{x}}). \tag{7.2.137}
$$

Now,

$$\begin{aligned}\mathbf{r}_{C1} &= \mathbf{r}_{Cc} + \mathbf{r}_{c1} = \mathbf{r}_{Cc} + \frac{m_b}{m_b+1}\mathbf{r}_{b1} \\ &= \left(r_{Cc} - \frac{m_b}{m_b+1} r_{b1}\cos\theta\right)\hat{\mathbf{z}} - \frac{m_b}{m_b+1} r_{b1}\sin\theta\hat{\mathbf{x}},\end{aligned} \tag{7.2.138}$$

and

$$\mathbf{r}_{Bb} = \mathbf{r}_{BC} + \mathbf{r}_{Cb} = -\frac{1}{m_B}\mathbf{r}_{C1} + \mathbf{r}_{Cb}. \tag{7.2.139}$$

Substituting the relation

$$\mathbf{r}_{Cb} = \mathbf{r}_{Cc} + \mathbf{r}_{cb} = \mathbf{r}_{Cc} - \frac{1}{m_b+1}\mathbf{r}_{b1}, \tag{7.2.140}$$

in (7.2.139) one gets

$$\mathbf{r}_{Bb} = \left(\frac{m_B-1}{m_B} r_{Cc} + \frac{m_b+m_B}{m_B(m_b+1)} r_{b1}\cos\theta\right)\hat{\mathbf{z}} + \frac{m_b+m_B}{m_B(m_b+1)} r_{b1}\sin\theta\hat{\mathbf{x}}. \tag{7.2.141}$$

The primed variables are arranged in a similar fashion,

$$\mathbf{r}'_{Cc} = r'_{Cc}\,\hat{\mathbf{z}}, \qquad \mathbf{r}'_{A2} = r'_{A2}(-\cos\theta'\,\hat{\mathbf{z}} - \sin\theta'\,\hat{\mathbf{x}}). \tag{7.2.142}$$

Thus,

$$\mathbf{r}'_{c2} = \left(-r'_{Cc} - \frac{m_A}{m_A+1} r'_{A2}\cos\theta'\right)\hat{\mathbf{z}} - \frac{m_A}{m_A+1} r'_{A2}\sin\theta'\hat{\mathbf{x}}, \tag{7.2.143}$$

and

$$\mathbf{r}'_{Aa} = \left(\frac{m_a-1}{m_a} r'_{Cc} - \frac{m_A+m_a}{m_a(m_A+1)} r'_{A2}\cos\theta'\right)\hat{\mathbf{z}} - \frac{m_A+m_a}{m_a(m_A+1)} r'_{A2}\sin\theta'\hat{\mathbf{x}}. \tag{7.2.144}$$

7.2.8 Simplifying the vector coupling

We will now turn our attention to the vector–coupled quantities in (7.2.135) and (7.2.136),

$$\sum_M \langle l_c\ 0\ l\ M|K\ M\rangle \left[Y^{l_f}(\hat{r}_{C1})Y^{l_i}(\theta+\pi,0)\right]^K_M Y^{l_*}_M(\hat{r}_{Bb}), \tag{7.2.145}$$

and

$$\sum_M \langle l_c\ 0\ l\ M|K\ M\rangle \left[Y^{l_f}(\hat{r}'_{A2})Y^{l_i}(\hat{r}'_{c2})\right]^{K*}_M Y^l_M(\hat{r}'_{Aa}). \tag{7.2.146}$$

We can express them both as

$$\sum_M f(M), \tag{7.2.147}$$

where e.g. in the case of (7.2.145), one has

$$f(M) = \langle l_c\ 0\ l\ M|K\ M\rangle \left[Y^{l_f}(\hat{r}_{C1})Y^{l_i}(\theta+\pi,0)\right]^K_M Y^{l_c*}_M(\hat{r}_{Bb}). \qquad (7.2.148)$$

Note that all the vectors that come into play in the above expressions are in the (x,z)–plane. Consequently, the azimuthal angle ϕ is always equal to zero. Under these circumstances and for time–reversed phases, $(Y^{L*}_M(\theta,0) = (-1)^L Y^L_M(\theta,0))$ one has

$$f(-M) = (-1)^{l_c+l_f+l_i+l} f(M). \qquad (7.2.149)$$

Consequently,

$$\sum_M \langle l_c\ 0\ l\ M|K\ M\rangle f(M) = \langle l_c\ 0\ l\ 0|K\ 0\rangle f(0) + \sum_{M>0} \langle l_c\ 0\ l\ M|K\ M\rangle f(M)\left(1+(-1)^{l_c+l+l_i+l_f}\right). \qquad (7.2.150)$$

Consequently, in the case in which $l_c + l + l_i + l_f$ is odd, we have only to evaluate the $M = 0$ contribution. This consideration is useful to restrict the number of numerical operations needed to calculate the transition amplitude.

7.2.9 non–orthogonality term

We write the non–orthogonality contribution to the transition amplitude (see Bayman and Chen (1982)):

$$\begin{aligned} T^{NO}_{2NT} =& 2\sum_{\substack{\sigma_1\sigma_2\\ \sigma'_1\sigma'_2\\ KM}} \int d^3r_{Cc} d^3r_{b1} d^3r_{A2} d^3r'_{b1} d^3r'_{A2} \chi^{(-)*}(\mathbf{k}_{Bb},\mathbf{r}_{Bb}) \\ &\times \left[\psi^{j_f}(\mathbf{r}_{A1},\sigma_1)\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\right]^{0*}_0 v(r_{b1}) \left[\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\psi^{j_i}(\mathbf{r}_{b1},\sigma_1)\right]^K_M \\ &\times \left[\psi^{j_f}(\mathbf{r}'_{A2},\sigma'_2)\psi^{j_i}(\mathbf{r}'_{b1},\sigma'_1)\right]^{K*}_M \left[\psi^{j_i}(\mathbf{r}'_{b1},\sigma'_1)\psi^{j_i}(\mathbf{r}'_{b2},\sigma'_2)\right]^0_0 \chi^{(+)}(\mathbf{r}'_{Aa}). \end{aligned} \qquad (7.2.151)$$

This expression is equivalent to (7.2.110) if we make the replacement

$$\frac{2\mu_{Cc}}{\hbar^2} G(\mathbf{r}_{Cc},\mathbf{r}'_{Cc}) v(r'_{A2}) \rightarrow \delta(\mathbf{r}_{Cc}-\mathbf{r}'_{Cc}). \qquad (7.2.152)$$

Looking at the partial–wave expansions of $G(\mathbf{r}_{Cc},\mathbf{r}'_{Cc})$ and $\delta(\mathbf{r}_{Cc}-\mathbf{r}'_{Cc})$ (see Section ??), we find that we can use the above expressions for the successive transfer with the replacement

$$i\frac{2\mu_{Cc}}{\hbar^2}\frac{f_{l_c}(k_{Cc},r_<)P_{l_c}(k_{Cc},r_>)}{k_{Cc}} \rightarrow \delta(r_{Cc}-r'_{Cc}). \qquad (7.2.153)$$

We thus have

$$
\begin{aligned}
T_{2NT}^{NO} =& \frac{512\pi^{9/2}}{k_{Aa}k_{Bb}} \frac{1}{\sqrt{(2j_i+1)(2j_f+1)}} \\
&\times \sum_K ((l_f \tfrac{1}{2})_{j_f} (l_i \tfrac{1}{2})_{j_i} | (l_f l_i)_K (\tfrac{1}{2}\tfrac{1}{2})_0)_K^2 \\
&\times \sum_{l_c,l} e^{i(\sigma_i^l+\sigma_f^l)} \frac{(2l_c+1)}{\sqrt{2l+1}} Y_0^l(\hat{k}_{Bb}) S_{K,l,l_c},
\end{aligned}
\tag{7.2.154}
$$

with

$$
\begin{aligned}
S_{K,l,l_c} =& \int r_{Cc}^2\, dr_{Cc}\, r_{b1}^2\, dr_{b1}\, \sin\theta\, d\theta\, v(r_{b1}) u_{l_f}(r_{C1}) u_{l_i}(r_{b1}) \\
&\times \frac{s_{K,l,l_c}(r_{Cc})}{r_{Cc}} \frac{F_l(r_{Bb})}{r_{Bb}} \\
&\times \sum_M \langle l_c\; 0\; l\; M | K\; M \rangle \left[Y^{l_f}(\hat{r}_{C1}) Y^{l_i}(\theta+\pi, 0) \right]_M^K Y_M^{l_c *}(\hat{r}_{Bb}),
\end{aligned}
\tag{7.2.155}
$$

and

$$
\begin{aligned}
s_{K,l,l_c}(r_{Cc}) =& r_{Cc} \int dr'_{A2}\, r'^2_{A2}\, \sin\theta'\, d\theta'\, u_{l_f}(r'_{A2}) u_{l_i}(r'_{c2}) \frac{F_l(r'_{Aa})}{r'_{Aa}} \\
&\times \sum_M \langle l_c\; 0\; l\; M | K\; M \rangle \left[Y^{l_f}(\hat{r}'_{A2}) Y^{l_i}(\hat{r}'_{c2}) \right]_M^{K*} Y_M^l(\hat{r}'_{Aa}).
\end{aligned}
\tag{7.2.156}
$$

7.2.10 Arbitrary orbital momentum transfer

We will now examine the case in which the two transferred nucleons carry an angular momentum Λ different from 0. Let us assume that two nucleons coupled to angular momentum Λ in the initial nucleus a are transferred into a final state of zero angular momentum in nucleus B. The transition amplitude is given by the integral

$$
\begin{aligned}
2\sum_{\sigma_1\sigma_2} \int & d\mathbf{r}_{cC} d\mathbf{r}_{A2} d\mathbf{r}_{b1} \chi^{(-)*}(\mathbf{r}_{bB}) \left[\psi^{j_f}(\mathbf{r}_{A1}, \sigma_1) \psi^{j_f}(\mathbf{r}_{A2}, \sigma_2) \right]_0^{0*} \\
&\times v(r_{b1}) \Psi^{(+)}(\mathbf{r}_{aA}, \mathbf{r}_{b1}, \mathbf{r}_{b2}, \sigma_1, \sigma_2).
\end{aligned}
\tag{7.2.157}
$$

If we neglect core excitations, the above expression is exact as long as $\Psi^{(+)}(\mathbf{r}_{aA}, \mathbf{r}_{b1}, \mathbf{r}_{b2}, \sigma_1, \sigma_2)$ is the exact wavefunction. We can instead obtain an approximation for the transfer amplitude using

$$
\begin{aligned}
\Psi^{(+)}(\mathbf{r}_{aA}, \mathbf{r}_{b1}, \mathbf{r}_{b2}, \sigma_1, \sigma_2) \approx& \chi^{(+)}(\mathbf{r}_{aA}) \left[\psi^{j_{i1}}(\mathbf{r}_{b1}, \sigma_1) \psi^{j_{i2}}(\mathbf{r}_{b2}, \sigma_2) \right]_\mu^\Lambda \\
&+ \sum_{K,M} \mathcal{U}_{K,M}(\mathbf{r}_{cC}) \left[\psi^{j_f}(\mathbf{r}_{A2}, \sigma_2) \psi^{j_{i1}}(\mathbf{r}_{b1}, \sigma_1) \right]_M^K
\end{aligned}
\tag{7.2.158}
$$

as an approximation for the incoming state. The first term of (7.2.158) gives rise to the simultaneous amplitude, while from second one leads to both the successive and the non-orthogonality contributions. To extract the amplitude $\mathcal{U}_{K,M}(\mathbf{r}_{cC})$, we define $f_{KM}(\mathbf{r}_{cC})$ as the scalar product

$$f_{KM}(\mathbf{r}_{cC}) = \left\langle \left[\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\psi^{j_{i1}}(\mathbf{r}_{b1},\sigma_1)\right]^K_M \middle| \Psi^{(+)}(\mathbf{r}_{aA},\mathbf{r}_{b1},\mathbf{r}_{b2},\sigma_1,\sigma_2)\right\rangle \quad (7.2.159)$$

for fixed $\mathbf{r}_{cC}$, which can be seen to obey the equation

$$\begin{aligned}\left(\frac{\hbar^2}{2\mu_{cC}}k^2_{cC} + \frac{\hbar^2}{2\mu_{cC}}\nabla^2_{r_{cC}} - U(r_{cC})\right) f_{KM}(\mathbf{r}_{cC})\\ = \left\langle \left[\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\psi^{j_{i1}}(\mathbf{r}_{b1},\sigma_1)\right]^K_M \middle| v(r_{c2}) \middle| \Psi^{(+)}(\mathbf{r}_{aA},\mathbf{r}_{b1},\mathbf{r}_{b2},\sigma_1,\sigma_2)\right\rangle.\end{aligned} \quad (7.2.160)$$

The solution can be written in terms of the Green function $G(\mathbf{r}_{cC},\mathbf{r}'_{cC})$ defined by

$$\left(\frac{\hbar^2}{2\mu_{cC}}k^2_{cC} + \frac{\hbar^2}{2\mu_{cC}}\nabla^2_{r_{cC}} - U(r_{cC})\right) G(\mathbf{r}_{cC},\mathbf{r}'_{cC}) = \frac{\hbar^2}{2\mu_{cC}}\delta(\mathbf{r}_{cC}-\mathbf{r}'_{cC}). \quad (7.2.161)$$

Thus,

$$\begin{aligned}f_{KM}(\mathbf{r}_{cC}) &= \frac{2\mu_{cC}}{\hbar^2}\int d\mathbf{r}'_{cC} G(\mathbf{r}_{cC},\mathbf{r}'_{cC}) \left\langle \left[\psi^{j_f}(\mathbf{r}'_{A2},\sigma'_2)\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\right]^K_M \middle| v(r_{C2}) \middle| \Psi^{(+)}(\mathbf{r}'_{aA},\mathbf{r}'_{b1},\mathbf{r}'_{b2},\sigma'_1,\sigma'_2)\right\rangle\\ &\approx \frac{2\mu_{cC}}{\hbar^2}\sum_{\sigma'_1\sigma'_2}\int d\mathbf{r}'_{cC}d\mathbf{r}'_{A2}d\mathbf{r}'_{b1} G(\mathbf{r}_{cC},\mathbf{r}'_{cC}) \left[\psi^{j_f}(\mathbf{r}'_{A2},\sigma'_2)\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\right]^{K*}_M\\ &\times v(r'_{c2})\chi^{(+)}(\mathbf{r}'_{aA})\left[\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\psi^{j_{i2}}(\mathbf{r}'_{b2},\sigma'_2)\right]^{\Lambda}_{\mu} = \mathcal{U}_{K,M}(\mathbf{r}_{cC})\\ &+ \left\langle \left[\psi^{j_f}(\mathbf{r}'_{A2},\sigma_2)\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma_1)\right]^K_M \middle| \chi^{(+)}(\mathbf{r}'_{aA})\left[\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\psi^{j_{i2}}(\mathbf{r}'_{b2},\sigma'_2)\right]^{\Lambda}_{\mu}\right\rangle.\end{aligned} \quad (7.2.162)$$

Therefore

$$\begin{aligned}\mathcal{U}_{K,M}(\mathbf{r}_{cC}) &= \frac{2\mu_{cC}}{\hbar^2}\sum_{\sigma'_1\sigma'_2}\int d\mathbf{r}'_{cC}d\mathbf{r}'_{A2}d\mathbf{r}'_{b1} G(\mathbf{r}_{cC},\mathbf{r}'_{cC}) \left[\psi^{j_f}(\mathbf{r}'_{A2},\sigma'_2)\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\right]^{K*}_M\\ &\times v(r'_{c2})\chi^{(+)}(\mathbf{r}'_{aA})\left[\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\psi^{j_{i2}}(\mathbf{r}'_{b2},\sigma'_2)\right]^{\Lambda}_{\mu}\\ &- \left\langle \left[\psi^{j_f}(\mathbf{r}'_{A2},\sigma_2)\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma_1)\right]^K_M \middle| \chi^{(+)}(\mathbf{r}'_{aA})\left[\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\psi^{j_{i2}}(\mathbf{r}'_{b2},\sigma'_2)\right]^{\Lambda}_{\mu}\right\rangle.\end{aligned} \quad (7.2.163)$$

When we substitute $\mathcal{U}_{K,M}(\mathbf{r}_{cC})$ into (7.2.158) and (7.2.157), the first term gives rise to the successive amplitude for the two–particle transfer, while the second term is responsible for the non–orthogonal contribution.

7.2.11 Successive transfer contribution

We need to evaluate the integral

$$\begin{aligned}T_\mu^{succ} =& \frac{4\mu_{cC}}{\hbar^2}\sum_{\sigma_1\sigma_2}\sum_{KM}\int d\mathbf{r}_{cC}d\mathbf{r}_{A2}d\mathbf{r}_{b1}d\mathbf{r}'_{cC}d\mathbf{r}'_{A2}d\mathbf{r}'_{b1}\left[\psi^{j_f}(\mathbf{r}_{A1},\sigma_1)\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\right]_0^{0*}\\ &\times\chi^{(-)*}(\mathbf{r}_{bB})G(\mathbf{r}_{cC},\mathbf{r}'_{cC})\left[\psi^{j_f}(\mathbf{r}'_{A2},\sigma'_2)\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\right]_M^{K*}\chi^{(+)}(\mathbf{r}'_{aA})v(r'_{c2})v(r_{b1})\\ &\times\left[\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\psi^{j_{i2}}(\mathbf{r}'_{b2},\sigma'_2)\right]_\mu^\Lambda\left[\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\psi^{j_{i1}}(\mathbf{r}_{b1},\sigma_1)\right]_M^K,\end{aligned} \tag{7.2.164}$$

where we must substitute the Green function and the distorted waves by their partial wave expansions (see App.7.K). The integral over $\mathbf{r}'_{b1}$ is:

$$\begin{aligned}&\sum_{\sigma'_1}\int d\mathbf{r}'_{b1}\left[\psi^{j_f}(\mathbf{r}'_{A2},\sigma'_2)\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\right]_M^{K*}\left[\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\psi^{j_{i2}}(\mathbf{r}'_{b2},\sigma'_2)\right]_\mu^\Lambda\\ &=\sum_{\sigma'_1}\int d\mathbf{r}'_{b1}(-1)^{-M+j_f+j_{i1}-\sigma_1-\sigma_2}\left[\psi^{j_{i1}}(\mathbf{r}'_{b1},-\sigma'_1)\psi^{j_f}(\mathbf{r}'_{A2},-\sigma'_2)\right]_{-M}^{K}\left[\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\psi^{j_{i2}}(\mathbf{r}'_{b2},\sigma'_2)\right]_\mu^\Lambda\\ &=\sum_{\sigma'_1}\int d\mathbf{r}'_{b1}(-1)^{-M+j_f+j_{i1}-\sigma_1-\sigma_2}\sum_P\langle K\;\Lambda\;-M\;\mu|P\;\mu-M\rangle((j_{i1}j_f)_K(j_{i1}j_{i2})_\Lambda|(j_{i1}j_{i1})_0(j_fj_{i2})_P)_P\\ &\qquad\times\left[\psi^{j_{i1}}(\mathbf{r}'_{b1},-\sigma'_1)\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\right]_0^0\left[\psi^{j_f}(\mathbf{r}'_{A2},-\sigma'_2)\psi^{j_{i2}}(\mathbf{r}'_{b2},\sigma'_2)\right]_{\mu-M}^P\\ &\quad=(-1)^{-M+j_f+j_{i1}}\sqrt{2j_{i1}+1}\,u_{l_f}(r_{A2})u_{l_{i2}}(r'_{b2})\sum_P\langle K\;\Lambda\;-M\;\mu|P\;\mu-M\rangle\\ &\qquad\times((j_{i1}j_f)_K(j_{i1}j_{i2})_\Lambda|(j_{i1}j_{i1})_0(j_fj_{i2})_P)_P((l_f\tfrac{1}{2})_{j_f}(l_{i2}\tfrac{1}{2})_{j_{i2}}|(l_fl_{i2})_P\,(\tfrac{1}{2}\tfrac{1}{2})_0)_P\\ &\qquad\qquad\times\left[Y^{l_f}(\hat{\mathbf{r}}'_{A2})Y^{l_{i2}}(\hat{\mathbf{r}}'_{b2})\right]_{\mu-M}^P u_{l_f}(r_{A2})u_{l_{i2}}(r_{b2}).\end{aligned} \tag{7.2.165}$$

Integrating ~~Integral~~ over $\mathbf{r}_{A2}$ (see (7.2.117)) leads to,

$$\begin{aligned}&\sum_{\sigma_2}\int d\mathbf{r}_{A2}\left[\psi^{j_f}(\mathbf{r}_{A1},\sigma_1)\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\right]_0^{0*}\left[\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\psi^{j_{i1}}(\mathbf{r}_{b1},\sigma_1)\right]_M^K\\ &=-\sqrt{\frac{2}{2j_f+1}}((l_f\tfrac{1}{2})_{j_f}(l_{i1}\tfrac{1}{2})_{j_{i1}}|(l_fl_{i1})_K\,(\tfrac{1}{2}\tfrac{1}{2})_0)_K\left[Y^{l_f}(\hat{\mathbf{r}}_{A1})Y^{l_{i1}}(\hat{\mathbf{r}}_{b1})\right]_M^K u_{l_f}(r_{A1})u_{l_{i1}}(r_{b1}).\end{aligned} \tag{7.2.166}$$

Let us examine the term

$$\sum_M(-1)^M\,\langle K\;\Lambda\;-M\;\mu|P\;\mu-M\rangle\left[Y^{l_f}(\hat{\mathbf{r}}_{A1})Y^{l_{i1}}(\hat{\mathbf{r}}_{b1})\right]_M^K\left[Y^{l_f}(\hat{\mathbf{r}}'_{A2})Y^{l_{i2}}(\hat{\mathbf{r}}'_{b2})\right]_{\mu-M}^P. \tag{7.2.167}$$

Making use of the relation

$$\langle l_1\;l_2\;m_1\;m_2|L\;M_L\rangle=(-1)^{l_2-m_2}\sqrt{\frac{2L+1}{2l_1+1}}\langle L\;l_2\;-M_L\;m_2|l_1\;-m_1\rangle, \tag{7.2.168}$$

the expression (7.2.168) is equivalent to,

$$(-1)^K \sqrt{\frac{2P+1}{2\Lambda+1}} \left\{ \left[Y^{l_f}(\hat{\mathbf{r}}'_{A2}) Y^{l_{i2}}(\hat{\mathbf{r}}'_{b2}) \right]^P \left[Y^{l_f}(\hat{\mathbf{r}}_{A1}) Y^{l_{i1}}(\hat{\mathbf{r}}_{b1}) \right]^K \right\}^\Lambda_\mu . \tag{7.2.169}$$

We now recouple the term

$$\left[Y^{l_a}(\hat{\mathbf{r}}'_{aA}) Y^{l_a}(\hat{\mathbf{k}}_{aA}) \right]^0_0 \left[Y^{l_b}(\hat{\mathbf{r}}_{bB}) Y^{l_b}(\hat{\mathbf{k}}_{bB}) \right]^0_0 , \tag{7.2.170}$$

arising from the partial wave expansion of the incoming and outgoing distorted waves to have,

$$((l_a l_a)_0 (l_b l_b)_0 | (l_a l_b)_\Lambda (l_a l_b)_\Lambda)_0 \left\{ \left[Y^{l_a}(\hat{\mathbf{r}}'_{aA}) Y^{l_b}(\hat{\mathbf{r}}_{bB}) \right]^\Lambda \left[Y^{l_a}(\hat{\mathbf{k}}_{aA}) Y^{l_b}(\hat{\mathbf{k}}_{bB}) \right]^\Lambda \right\}^0_0 . \tag{7.2.171}$$

The only term which does not vanish upon integration is

$$\frac{(-1)^{\Lambda-\mu}}{\sqrt{(2l_a+1)(2l_b+1)}} \left[Y^{l_a}(\hat{\mathbf{r}}'_{aA}) Y^{l_b}(\hat{\mathbf{r}}_{bB}) \right]^\Lambda_{-\mu} \left[Y^{l_a}(\hat{\mathbf{k}}_{aA}) Y^{l_b}(\hat{\mathbf{k}}_{bB}) \right]^\Lambda_\mu . \tag{7.2.172}$$

Again, the only term surviving

$$\left\{ \left[Y^{l_f}(\hat{\mathbf{r}}'_{A2}) Y^{l_{i2}}(\hat{\mathbf{r}}'_{b2}) \right]^P \left[Y^{l_f}(\hat{\mathbf{r}}_{A1}) Y^{l_{i1}}(\hat{\mathbf{r}}_{b1}) \right]^K \right\}^\Lambda_\mu \left[Y^{l_a}(\hat{\mathbf{r}}'_{aA}) Y^{l_b}(\hat{\mathbf{r}}_{bB}) \right]^\Lambda_{-\mu} \tag{7.2.173}$$

is

$$\begin{aligned} \frac{(-1)^{\Lambda+\mu}}{\sqrt{2\Lambda+1}} \Bigg[\Bigg\{ & \left[Y^{l_f}(\hat{\mathbf{r}}'_{A2}) Y^{l_{i2}}(\hat{\mathbf{r}}'_{b2}) \right]^P \\ & \left[Y^{l_f}(\hat{\mathbf{r}}_{A1}) Y^{l_{i1}}(\hat{\mathbf{r}}_{b1}) \right]^K \Bigg\}^\Lambda \left[Y^{l_a}(\hat{\mathbf{r}}'_{aA}) Y^{l_b}(\hat{\mathbf{r}}_{bB}) \right]^\Lambda \Bigg]^0_0 . \end{aligned} \tag{7.2.174}$$

We now couple this last term with the term $\left[Y^{l_c}(\hat{\mathbf{r}}'_{cC})Y^{l_c}(\hat{\mathbf{r}}_{cC})\right]^0_0$, arising from the partial wave expansion of the Green function. That is,

$$
\begin{aligned}
&\left[\left\{\left[Y^{l_f}(\hat{\mathbf{r}}'_{A2})Y^{l_{i2}}(\hat{\mathbf{r}}'_{b2})\right]^P\left[Y^{l_f}(\hat{\mathbf{r}}_{A1})Y^{l_{i1}}(\hat{\mathbf{r}}_{b1})\right]^K\right\}^\Lambda\left[Y^{l_a}(\hat{\mathbf{r}}'_{aA})Y^{l_b}(\hat{\mathbf{r}}_{bB})\right]^\Lambda\right]^0_0\left[Y^{l_c}(\hat{\mathbf{r}}'_{cC})Y^{l_c}(\hat{\mathbf{r}}_{cC})\right]^0 \\
&= ((l_a l_b)_\Lambda (l_c l_c)_0|(l_a l_c)_P (l_b l_c)_K)_\Lambda \left[\left\{\left[Y^{l_f}(\hat{\mathbf{r}}'_{A2})Y^{l_{i2}}(\hat{\mathbf{r}}'_{b2})\right]^P\left[Y^{l_f}(\hat{\mathbf{r}}_{A1})Y^{l_{i1}}(\hat{\mathbf{r}}_{b1})\right]^K\right\}^\Lambda\right. \\
&\left.\left\{\left[Y^{l_a}(\hat{\mathbf{r}}'_{aA})Y^{l_c}(\hat{\mathbf{r}}'_{cC})\right]^P\left[Y^{l_b}(\hat{\mathbf{r}}_{bB})Y^{l_c}(\hat{\mathbf{r}}_{cC})\right]^K\right\}^\Lambda\right]^0_0 = ((l_a l_b)_\Lambda (l_c l_c)_0|(l_a l_c)_P (l_b l_c)_K)_\Lambda \\
&\times ((PK)_\Lambda (PK)_\Lambda|(PP)_0 (KK)_0)_0 \left\{\left[Y^{l_f}(\hat{\mathbf{r}}'_{A2})Y^{l_{i2}}(\hat{\mathbf{r}}'_{b2})\right]^P\left[Y^{l_a}(\hat{\mathbf{r}}'_{aA})Y^{l_c}(\hat{\mathbf{r}}'_{cC})\right]^P\right\}^0_0 \\
&\times \left\{\left[Y^{l_f}(\hat{\mathbf{r}}_{A1})Y^{l_{i1}}(\hat{\mathbf{r}}_{b1})\right]^K\left[Y^{l_b}(\hat{\mathbf{r}}_{bB})Y^{l_c}(\hat{\mathbf{r}}_{cC})\right]^K\right\}^0_0 = ((l_a l_b)_\Lambda (l_c l_c)_0|(l_a l_c)_P (l_b l_c)_K)_\Lambda \\
&\times \sqrt{\frac{2\Lambda+1}{(2K+1)(2P+1)}}\left\{\left[Y^{l_f}(\hat{\mathbf{r}}'_{A2})Y^{l_{i2}}(\hat{\mathbf{r}}'_{b2})\right]^P\left[Y^{l_a}(\hat{\mathbf{r}}'_{aA})Y^{l_c}(\hat{\mathbf{r}}'_{cC})\right]^P\right\}^0_0 \\
&\times \left\{\left[Y^{l_f}(\hat{\mathbf{r}}_{A1})Y^{l_{i1}}(\hat{\mathbf{r}}_{b1})\right]^K\left[Y^{l_b}(\hat{\mathbf{r}}_{bB})Y^{l_c}(\hat{\mathbf{r}}_{cC})\right]^K\right\}^0_0 .
\end{aligned}
\tag{7.2.175}
$$

Collecting all the contributions (including the constants and phases arising from the partial wave expansion of the distorted waves and the Green function), we get

$$
\begin{aligned}
T^{succ}_\mu =& (-1)^{j_f+j_{i1}}\frac{2048\pi^5\mu_{Cc}}{\hbar^2 k_{Aa}k_{Bb}k_{Cc}}\sqrt{\frac{(2j_{i1}+1)}{(2\Lambda+1)(2j_f+1)}}\sum_{K,P}((l_f\tfrac{1}{2})_{j_f}(l_{i2}\tfrac{1}{2})_{j_{i2}}|(l_f l_{i2})_P(\tfrac{1}{2}\tfrac{1}{2})_0)_P \\
&\times ((l_f\tfrac{1}{2})_{j_f}(l_{i1}\tfrac{1}{2})_{j_{i1}}|(l_f l_{i1})_K(\tfrac{1}{2}\tfrac{1}{2})_0)_K\ ((j_{i1}j_f)_K(j_{i1}j_{i2})_\Lambda|(j_{i1}j_{i1})_0(j_f j_{i2})_P)_P \\
&\times \frac{(-1)^K}{(2K+1)\sqrt{2P+1}}\sum_{l_c,l_a,l_b}((l_a l_b)_\Lambda (l_c l_c)_0|(l_a l_c)_P (l_b l_c)_K)_\Lambda e^{i(\sigma_i^{l_a}+\sigma_f^{l_b})}i^{l_a-l_b} \\
&\times (2l_c+1)^{3/2}\left[Y^{l_a}(\hat{\mathbf{k}}_{aA})Y^{l_b}(\hat{\mathbf{k}}_{bB})\right]^\Lambda_\mu S_{K,P,l_a,l_b,l_c},
\end{aligned}
\tag{7.2.176}
$$

with (note that we have reduced the dimensionality of the integrals in the same fashion as for the $L=0$–angular momentum transfer calculation, see (7.2.132))

$$
\begin{aligned}
S_{K,P,l_a,l_b,l_c} =& \int r_{Cc}^2\,dr_{Cc}\,r_{b1}^2\,dr_{b1}\ \sin\theta\,d\theta\,v(r_{b1})u_{l_f}(r_{C1})u_{l_i}(r_{b1}) \\
&\times \frac{s_{P,l_a,l_c}(r_{Cc})}{r_{Cc}}\frac{F_{l_b}(r_{Bb})}{r_{Bb}} \\
&\times \sum_M (l_c\ 0\ l_b\ M|K\ M)\left[Y^{l_f}(\hat{r}_{C1})Y^{l_{i1}}(\theta+\pi,0)\right]^K_M Y^{l_b}_{-M}(\hat{r}_{Bb}),
\end{aligned}
\tag{7.2.177}
$$