gauge invariance.

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Furthermore, the fact that these results follow the use of QRPA*) in the calculation of the ZPF of the collective solutions of the pairing Hamiltonian indicates the importance of conserving approximations to describe many-body problems in general, and the finite size many-body problem (FSMB) of which the nuclear case represents a paradigmatic example.

to p. 6 manuscript

C

*) Using the Tamm-Dancoff approximation, i.e. setting $Y \equiv 0$ (and thus $\sum X^2 = 1$) in the QRPA approximation does not lead to particle number conservation, in keeping with the fact ~~that~~ the amplitudes Y are closely connected with ZPF.

Continuation right after ©-© p. 7 of the manuscript. 2

Aside from low-lying collective states, that is rotation and low-energy vibrations, nuclei also display high-lying collective modes known as giant resonances.

0.5 Giant dipole resonance

If one shines a beam of photons on a nucleus it is observed that the system absorbs energy resonantly essentially at a single frequency, of the order of*) $\nu = 5 \times 10^{21}$ Hz, corresponding to an energy of $h\nu \approx 20$ MeV.

It is not difficult to understand how γ-rays excite a nuclear dipole vibration. A photon carries with it an oscillating electric field. Although

*) Making use of $h = 4{,}1357 \times 10^{-15}$ eV$\times$s one obtains for $h\nu = 1$ eV the frequency $\nu = 2.42 \times 10^{14}$ Hz and thus $\nu = 4.8 \times 10^{21}$ Hz for $h\nu = 20$ MeV. The wavelength of a photon of energy E is $\lambda = hc/E \approx 2\pi \times 200$ MeV$\times$fm$/E$, which for $E = 20$ MeV leads to $\lambda \approx 63$ fm.

the wavelength of a 20 MeV γ-ray is smaller than that of other forms of electromagnetic radiation such as visible light, it is still large ($\lambda \approx 63$ fm) compared to the dimensions of e.g. ^{40}Ca ($R_0 \approx 4.1$ fm). As a result the photon electric field is nearly uniform across the nucleus at any time. The field excerts a force on the positively charged protons. Consequently, it can set the center of mass into an antenna like, dipole-oscillation (Thompson scattering), in which case no photon is absorbed. Another possibility is that it leads to an internal excitation of the system. In this case because the center of mass of the system does not move, the neutrons have to oscillate against the protons, again in an antenna-like motion. The restoring force of the vibration, known as the giant dipole resonance (GDR), is provided by the attractive force between protons and neutrons.

The connotation of giant is in keeping with the fact that it essentially carries ~~100% of the TRK sum rule~~, and resonance because it displays a Lorentzian-like shape with a full width at half maximum of few MeV ($\lesssim 5$ MeV), considerably smaller than the energy centroid $\hbar\omega_{GDR} \approx 80/A^{1/3}$ MeV.

Microscopically, the GDR can be viewed as a correlated particle-hole excitation, that is, a state made out of a linear combination

(the full ~~of the~~ photo absorption cross section (sum rule, see below)), of proton and neutron particle-hole excitations with essentially $\Delta N = 1$, as well as small $\Delta N = 3, 5, \ldots$ components (Fig 0.1.3). Because the difference in energy between major shells is $\hbar\omega_0 \approx 41/A^{1/3}$ MeV, one expects that about half of the contribution to the energy arises from the neutron-proton interaction. More precisely, from the the so called symmetry potential V_1 (see (0.1.2)), which measures the energy price the system has to pay to separate protons from neutrons. Theoretical estimates lead to

$$\hbar\omega_{GDR} = (\hbar\omega_0)^2 + \frac{3\hbar^2 V_1}{m\langle r^2\rangle} \qquad (0.1.111)$$

$$= \frac{1}{A^{2/3}}\left[(41)^2 + (60)^2\right] \text{MeV}^2,$$

resulting in

$$\hbar\omega_{GDR} \approx \frac{73}{A^{1/3}} \approx \frac{87}{R}\ \text{MeV}, \quad (0.1.112)$$

where $R = 1.2 A^{1/3}$ is the numerical value of the nuclear radius when measured in fm. The above quantity is to be compared with the empirical value $\hbar\omega_{GDR} \approx (80/A^{1/3})$ MeV $\approx (95/R)$ MeV.

It is of notice that the elastic vibrational frequency of a spherical solid made out of particles of mass m can be written as

$\omega_{el}^2 \sim \mu/(m\rho R^2) \sim v_t^2/R^2$, where R is the radius, ρ the density and v_t the transverse sound velocity proportional to the Lamé shear 5 modulus of elasticity μ.

In other words, giant resonances in general, and the GDR in particular, are embodiments of the elastic response of the nucleus, to impulsive external fields, like that provided by a photon. The nuclear rigidity to sudden sollicitations is provided by the shell structure, quantitatively measured by the energy separation between major shells.

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0.6 Giant pairing vibrations

Due to spatial quantization, in particular to the existence of major shells of pair degeneracy $\Omega\ (\equiv (2j+1)/2)$, and separated by an energy $\hbar\omega_0 \approx 41/A^{1/3}$ MeV, the Cooper pair model can be extended to encompass pair addition and pair substraction modes across major shells.*)

Assuming both ε_k and ε_i appearing in Fig. 0.3.1 to be equal to $\hbar\omega_0$, one obtains the dispersion relation

$$-\frac{1}{G} = \frac{\Omega}{W - 2\hbar\omega_0} - \frac{\Omega}{W + 2\hbar\omega_0}, \qquad (0.6.1)$$

leading to

$$(2\hbar\omega_0)^2 - W^2 = 4\hbar\omega_0 \Omega G, \qquad (0.6.2)$$

and implying a high lying pair addition mode of energy

$$W = 2\hbar\omega_0 \left(1 - \frac{G\Omega}{\hbar\omega_0}\right)^{1/2}. \qquad (0.6.3)$$

The forwards (backwards) going RPA amplitudes are, in the present case

$$X = \frac{\Lambda_0 \Omega^{1/2}}{2\hbar\omega_0 - W} \quad \text{and} \quad Y = \frac{\Lambda_0 \Omega^{1/2}}{2\hbar\omega_0 + W}, \qquad (0.6.4)$$

normalized according to the relation

$$1 = X^2 - Y^2 = \Lambda_0^2 \Omega \frac{8\hbar\omega_0 W}{((2\hbar\omega_0)^2 - W^2)^2}, \qquad (0.6.5)$$

where Λ_0 stands for the particle-pair vibration coupling vertex. Making use of (0.6.2) one obtains

$$\left(\frac{\Lambda_0}{G}\right)^2 = \Omega \left(1 - \frac{G\Omega}{\hbar\omega_0}\right)^{-1/2} \qquad (0.6.6)$$

quantity corresponding, within the framework

*) R.A. Broglia and D.R. Bes, Phys. Lett. 69B, 129 (1977).

of the simplified model used, to the two-nucleon transfer cross section.

Summing up, the monópole giant pairing vibration has an energy close to $2\hbar\omega_0$, and is expected to be populated in two-particle transfer processes with a cross section of the order of that associated with the low-lying pair addition mode, being this one of the order of Ω.

Simple estimates of (0.6.3) and (0.6.6) can be obtained making use of $\Omega \approx \frac{2}{3} A^{2/3}$, and $G \approx 17/A$ MeV leading to

$$W = 0{,}85 \times 2\hbar\omega_0 \quad , \quad (\Lambda_0/G)^2 \approx 1{,}2\,\Omega \quad . \quad (0.6.7)$$

Experimental evidence of GPV in light nuclei, have been reported*).

*) F. Cappuzzello et al Nature Communications (2015); see also Bortignon and Broglia, EPJA (2016).

New numbering

0.7 Sum rules

There are important operator identities which restrict the possible matrix elements in a physical system. Let us calculate the double commutator of the Hamiltonian describing the system and a single-particle operator F. That is

$$[\hat{F},[H,\hat{F}]] = (2\hat{F}H\hat{F} - \hat{F}^2 H - H\hat{F}^2) \qquad (0.7.1) \; \cancel{(0.1.103)}$$

Let us assume that $\hat{F} = \sum_k F(\vec{r}_k)$ and $H = T + v(\vec{r},\vec{r}')$, where $v(\vec{r},\vec{r}') = -K_1 \hat{F}(\vec{r})\hat{F}(\vec{r}')$.

Thus
$$[\hat{F},[H,\hat{F}]] = \sum_k \frac{\hbar^2}{m}\left(\vec{\nabla}_k F(\vec{r}_k)\right)^2 \qquad \cancel{(0.1.104)} \; (0.7.2)$$

Let us take the average value on the correlate ground state

$$S(F) = \sum_{\alpha'} |\langle\alpha'|F|\tilde{0}\rangle|^2 (E_{\alpha'} - E_0) = \frac{\hbar^2}{2m}\int d^3r\, |\vec{\nabla}F|^2 \rho(\vec{r}), \qquad (0.1.105)$$

where we have used $H|\alpha\rangle = E_\alpha$ and $H|\tilde{0}\rangle = E_0|\tilde{0}\rangle$, and the sum $\sum_{\alpha'}$ is over the complete of eigenstates of the system. The above result describes the reaction of a system at equilibrium to which one applies an impulsive field, which which gives the particles a momentum $\vec{\nabla}F$. On the average, the particles ~~started~~ at rest so their average energy after the sudden impulse is $\hbar^2|\vec{\nabla}F|^2/2m$, a result which is model independent not depending on the interaction among the nucleons, the energy being absorbed from the (instantaneous) external field before the system is disturbed from equilibrium.

The result (0.1.105) is known as the energy weighted sum rule (EWSR). 7

An important application of (0.1.105) is to the case of a constant force field, ~~that~~ is in the case where F has a constant gradient. Insert $F = z$ in this relation, the integral simplifies because $\vec{\nabla} F = 1$ and the integral leads just to the number of particles,

$$\sum_{\alpha} |\langle\alpha'|F|\tilde{0}\rangle|^2 (E_\alpha - E_0) = \frac{\hbar^2 N}{2m} \quad (0.1.106)$$

The electric field of a photon ~~is, in the dipole approximation, of this form. The dipo~~ is of this form in the dipole approximation, which is valid when the size of the system is small ~~com~~ compared to the wavelength of the photon, the single-particle field being

$$F(\vec{r}_k) = e\left[\frac{N-Z}{A} - t_z(k)\right] r_k Y_{1\mu}(\hat{r}_k), \quad (0.1.107)$$

with $t_z = -1/2$ for protons and $+1/2$ for neutrons. For the dipole operator referred to the nuclear center of mass one obtains

$$\sum_{\alpha'} |\langle\alpha'|F|\tilde{0}\rangle|^2 (E_{\alpha'} - E_0) = \frac{9}{4\pi} \frac{\hbar^2 e^2}{2m} \frac{NZ}{A} \quad (0.1.108)$$

which reduces to $S(F) = (\hbar^2 e^2/2m)\frac{NZ}{A}$, replacing $r_k Y_{1\mu}(\hat{r}_k)$ by z_k.

The above relation is know as the Thomas-Reiche-Kuhn (TRK) sum rule, and is equal to the maximum energy a system can absorb from the dipole field.

The RPA solution respect the EWSR, while the Tamm-Dancoff approximation (TDA), resulting by setting $Y^{\alpha}_{ki} = 0$ and normalising the X-components ($\sum_{ki} X^{\alpha\,2}_{ki} = 1$) fulfill the non-energy weighted sum rules. A fact which testifies to the important role ZPF play in nuclei.

0.8 Ground state correlations

footnote *) p. $\boxed{10}$

The zero point fluctuations associated with collective vibrations of protons and of neutrons affect the nuclear mean field static properties. In particular concerning the nuclear density *) ρ and radius R.

According to the indeterminacy relations,

New numbering

$$\Delta\alpha_{\lambda\mu}^{(n)}\,\Delta\Pi_{\lambda\mu}^{(n)} \geq \frac{\hbar}{2}\,. \quad \cancel{(0.1.109)} \quad (0.8.1)$$

Making use of the virial theorem $(\Delta\Pi_{\lambda\mu}^2/D_\lambda = C_\lambda\,\alpha_{\lambda\mu}^2)$ one can write

$$\Delta\alpha_{\lambda\mu}^{2(n)} \geq \frac{\hbar\omega_\lambda}{2C_\lambda}\,. \quad \cancel{(0.1.110)} \quad (0.8.2)$$

Let us compare this relation with the expectation value of $\Delta\alpha_{\lambda\mu}^2$ in the ground state of the collective Hamiltonian (0.1.8) described by the wavefunction $\Psi_0(\alpha_{\lambda\mu}^{(n)}) = \left(\frac{D_\lambda^{(n)}\omega_\lambda^{(n)}}{\hbar\pi}\right)^{1/4}\exp\left(-\frac{D_\lambda^{(n)}\omega_{\lambda}^{(n)}}{2\hbar}\alpha_{\lambda\mu}^2\right)$. The result coincides with the lowest limit of (0.1.110) in keeping with the fact ~~that~~ $|\Psi_0|^2$ is mathematically a Poisson distribution.**)

**) The same result is found for Ψ_n describing a state with n-quanta, and at the basis that solutions with $n \gg 1$ behave as "quasiclassical" or "coherent" states of the harmonic oscillator (R.J. Glauber, Quantum Theory of Optical Coherence, Wiley, Weinheim (2007)) in keeping with the fact ~~that~~ the contribution of the zero point energy is negligible in such case $((n+1/2)\hbar\omega \approx n\hbar\omega)$ and ~~that~~ the many quanta wavepacket allways attain the lower limit of (0.1.109) (Basdevant and Delibard (2005) pp. 153, 465 (discussions with Pier Francesco Bortignon in March 2018 concerning coherent states are gratefully acknowledged). Schrödinger was the first to find this result which he used in a paper (Schrödinger, Naturw. 14, 644 (1926)) to suggest that waves (material waves) described by his wave function are the only reality, particles being only derivative things. In support of his view he considered a superposition of linear harmonic oscillator wave functions and showed ~~that~~ the wave group holds permanently, together in the course of time. And he

go to p. $\boxed{10}$ bottom

*) D. Gogny, in Nuclear Physics with Electromagnetic Interactions, eds. H. Arenhövel and D. Drechsel, Lecture Notes in Physics, Vol. 108, Springer-Verlag, Heidelberg (1978) p. 88; H. Esbensen and G. F. Bertsch, Phys. Rev. C 28, 355 (1983); P. Reinhard and D. Drechsel, Z. Phys. A 290, 85 (1979); V. A. Khodel, A. P. Platanov and E. E. Saperstein, J. Phys. G 8, 967 (1982); F. Barranco and R. A. Broglia, Phys. Lett. 151 B, 90 (1985); F. Barranco and R. A. Broglia, Phys. Rev. Lett. 59, 2724 (1987); see also G. E. Brown and G. Jacob, Zero-point vibrations and the nuclear surface, Nucl. Phys. 42, 177 (1963) and P. W. Anderson and D. J. Thouless, Phys. Lett. 1, 155 (1962). It is likely a coincidence in connection with this inaugural issue of Phys. Lett. that short of ~~hundred~~ pages after, one finds the paper of B. D. Jophson, Possible new effects in superconducting tunneling, Phys. Lett. 1, 251 (1982).

(Continuation footnote **) p. 9) adds that the same will be true for the electron as it moves in high orbits of the hydrogen atom, hoping that wave mechanics would turn out to be a branch of classical physics (Pais 1986). It was Born who first provided the correct interpretation of Schrödinger's wavefunction (modulus square) in his paper "Quantum mechanical collisions phenomena" (Born (1926)). In it it is stated ~~that~~ the result of solving with wave mechanics the process of elastic scattering of a beam of particles by a static potential is not what the state after the collision is, but how probable is a given effect of the collision.

The fact that $\Delta\alpha^2_{\lambda\mu}(n) = \hbar\omega_\lambda(n)/2C_\lambda(n)$ implies that the mean square radius will be modified from its mean field value R_0 (Eq. (0.1.18)) and thus also the nuclear density.

The value of $\hbar\omega_\lambda(n)/2C_\lambda(n)$ can be determined by calculating the collective mode $|n_\lambda(n)=1\rangle = \Gamma^{\dagger}_{\lambda\mu}(n)|\tilde{0}\rangle$ in RPA. As seen from the caption to Fig. (0.1.7) the zero point fluctuation of the mode enter the definition of the X, Y-amplitudes of the mode.

Let us start by calculating the effect of the zero point fluctuations on the nuclear density. The corresponding operator can be written as

$$\hat{\rho}(\vec{r}) = a^{\dagger}(\vec{r})\, a(\vec{r}), \qquad (0.2.3)$$

where $a^{\dagger}(\vec{r})$ is the creation operator of a nucleon at point $\vec{r}$. It can be expressed in terms of the phase space creation operators $a^{\dagger}_\nu$ $(\nu \equiv n, l, j, m)$ as

$$a^{\dagger}(\vec{r}) = \sum_\nu \varphi^{*}_\nu(\vec{r})\, a^{\dagger}_\nu,$$

where $\varphi_\nu(\vec{r})$ are the single-particle wavefunctions. Thus

$$\hat{\rho}(\vec{r}) = \sum_{\nu\nu'} \varphi^{*}_\nu(\vec{r})\, \varphi_{\nu'}(\vec{r})\, a^{\dagger}_\nu a_{\nu'}.$$

The matrix element in the HF ground state is (Fig. 0.1.6 (8)),

$$\rho_0(\vec{r}) = {}_F\langle 0|\hat{\rho}(\vec{r})|0\rangle_F = \sum_{\substack{i \\ (\varepsilon_i \le \varepsilon_F)}} |\varphi_i(\vec{r})|^2 . \qquad (0.5.1)$$

To lowest order of perturbation theory in the particle-vibration coupling vertex, the changes in ρ_0 due to ZPF are shown in Fig. (0.1.17). Graphs (a) and (b) and (c) and (d) describe the changes in the density operator and in the single-particle potential, respectively.

This can be seen from the insets (I) and (II) to Fig. 0.1.~~16~~17. The dashed horizontal line starting with a cross and ending at a hatched circled represents the renormalized density operator. This phenomenon is similar to that encountered in connection with vertex renormalization in ~~connection with~~ Fig. 0.1.9, that is the renormalization of the particle-vibration coupling (insets (I) and (I′)). 0.2.5

Concerning potential renormalization, the bold face arrowed line shown in inset (II) of 0.5.1 Fig. 0.1.~~16~~17 represents the motion of a renormalised nucleon due to the self-energy process induced by the coupling to vibrational modes. A phenomenon which can be described at profit through an effective mass, the so called ω-mass m_ω, and thus the motion is described by the Hamiltonian*) $(\hbar^2/2m_\omega)\nabla^2+\left(\frac{m}{m_\omega}\right)U(r)$. The ω-mass can be written as $m_\omega=(1+\lambda)\,m$, where λ is the so called mass enhancement factor $\lambda=N(0)\Lambda$, where $N(0)$ is the density of levels at the Fermi energy, and Λ the PVC vertex strength, typical values being $\lambda=0.4$.

The fact that in calculating $\delta\rho$, that is the correction to the nuclear density (renormalization of the density operator), one finds to the same order of perturbation a correction to the potential, is in keeping with the selfconsistency existing between these two quantities (Eq. (0.1.23)). Now, what changes is not only the single-particle energy, but also the single-particle content of the state, given by $Z_\omega=m/m_\omega$. Now, even so, the effective mass approximation although being quite useful, cannot take care of the energy dependence of the renormalization process which leads, in the case of single-particle motion to renormalized energies, spectroscopic amplitudes and wavefunctions.

*) Brink and Broglia (2005) App. B

The analytic expressions associated with diagrams (a) and (c) of Fig. ~~0.1.7~~ (0.5.1) are

$$\delta\rho(r)_{(a)} = \frac{2\lambda+1}{4\pi} \sum_{\nu_1\nu_2,n} [Y_n(\nu_1\nu_2;\lambda)]^2 R_{\nu_1}(r) R_{\nu_2}(r), \quad (0.1.109)$$

and

$$\delta\rho(r)_{(b)} = (2\lambda+1)\Lambda_n(\lambda) \times \sum_{\nu_1\nu_2\nu_3} \frac{M(\nu_1,\nu_3;\lambda)}{\epsilon_{\nu_1}-\epsilon_{\nu_2}} (2j_1+1)^{-1/2}$$
$$\times Y_n(\nu_3\nu_2;\lambda) R_{\nu_1}(r) R_{\nu_2}(r), \quad (0.1.110)$$

wheve M is the matrix element of $\frac{R_0}{K}\frac{\partial U}{\partial r} Y_{\lambda\mu}(\hat{r})$ and $n=1,2,\ldots$ the first, second, etc vibrational modes as a function of increasing energy.

While $\delta\rho_{(a)}$ can be written in terms of the RPA Y-amplitudes which are directly associated with the zero point fluctuations of harmonic motion (Fig. 0.1.7(c)), $\delta\rho_{(c)}$ contains a scattering vertex not found in RPA, and essential to describe renormalization processes of the different degrees of freedom, namely single-particle and collective motion, and interactions. In particular the pairing interaction.

0.5.2

Fig. 2
F.B+RAB
PRL

In Fig ~~0.1.18~~ we show the results of calculations of $\delta\rho$ carried out for the closed shell nucleus (^{40}Ca). The vibrations were calculated by diagonalizing separable interactions of multipolarity λ in the RPA. All the roots of multipolarity and parity $\lambda^{\pi} = 2^+, 3^-, 4^+$ and 5^- which exhaust the EWSR were included in the calculations. Both isoscalar and isovector degrees of freedom were include, and low-lying and giant resonances.

From the point of view of the single-particle motion the vibrations associated with low-lying modes display very low frequency ($\hbar\omega_\lambda/\epsilon_F \approx 0.1$) and lead to an ensemble of deformed shapes. Nucleons can thus reach to distances from the nuclear center which are considerably larger than the radius R of the static spherical potential.

Because the frequency of the giant resonances are of similar magnitude to those corresponding to the single-particle motion, the associated surface deformations average out.

(said it differently)

~~In different terms~~, are the low-lying vibrational modes which account for most of the contributions to the density. Making use of the corresponding $(\delta\rho)_{\text{low-lying}}$, the mean square radius of ^{40}Ca was calculated*) leading to $\delta\langle r^2\rangle = 0.494\ fm^2$, which amounts to a 5% of $\langle r^2\rangle = (3/5)R_0^2 = 10.11\ fm^2$ ($R_0 = 1.2A^{1/3}\ fm = 4.1\ fm$) in overall agreement with the experimental findings.

Similar calculations to the ones discussed above, but in this case taking into account only the contributions of the low-lying octupole vibration**) indicate that nucleons are to be found a reasonable part of the time in higher shells than those assigned to them by the shell model, the average number of "excited" particles being ≈ 2.4. If these are present, pickup reactions such as (p,d) and (d,t) will show them. From the nature of the correlations, the pickup of such a particle will leave a hole and a vibration. That is,

*) Barranco and Broglia PRL (1987)

**) Brown and Jacob (1963)

the final nucleus will be in one of ~~all~~ the states which can be reached by coupling the hole and the vibration.

Conversely, because of the presence of hole states in the closed shell nucleus, one can transfer a nucleon to ~~empty~~ states lying below the Fermi energy in, for example, (d,p) and (^{3}He,d) one-neutron and one-proton stripping reactions respectively, leaving the final nucleus with one-nucleon above closed shell couple to the vibrations.

Systematic studies of such multiplets have been carried out thought the mass table. In particular around the closed shell nucleus $^{208}_{82}Pb_{126}$ (Fig. 0.1.19). then

Within this context it is quite natural to deal with structure and reactions on equal footing, this being one of the main goals of the present monograph, as will become clear already from the next Chapter.

0.5.3