Nuclear Structure and Reactions
superfluidity in nuclei with Cooper pair transfer

G. Potel and R. A. Broglia

June 15, 2017

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Nuclear Structure and Reactions

Pairing in nuclei with Cooper pair Transfer

Preface

, which occupies a large fraction of the present monograph,

This is even more so concerning the study of exotic nuclei, where the study of halo nuclei has blurred almost completely the distinction between bound and continuum states.

are

The elementary modes of nuclear excitation are vibrations and rotations, single–particle (quasiparticle) motion, and pairing vibrations and rotations. The specific reactions probing these modes are inelastic, single– and two– particle transfer processes respectively. Within this context one can posit that nuclear structure (bound) and reactions (continuum) are but two aspects of the same physics. This is the reason why they ~~can be~~ treated on equal footing in terms of elementary modes of excitation, within the framework of nuclear field theory ~~(NFT)~~. This theory provides the rules to diagonalize in a compact and economic way the nuclear Hamiltonian for both bound and continuum states correcting for overcompletness of the basis (particle–vibration coupling (structure), non–orthogonality (reaction)), and for Pauli principle violation.

the unified

of structure and reactions (NFT) (r+s)

Pairing vibrations and rotations, closely connected with nuclear superfluidity are, arguably, paradigms of quantal nuclear phenomena. They thus play an important role within the field of nuclear structure. It is only natural that two–nucleon transfer plays a similar role concerning direct nuclear reactions. In fact, this is the central subject of the present monograph.

At the basis of fermionic pairing phenomena one finds Cooper pairs, weakly bound, extended, strongly overlapping (quasi–) bosonic entities, made out of pairs of nucleons dressed by collective vibrations and interacting through the exchange of these vibrations as well as through the bare NN–interaction, eventually corrected by $3N$ contributions. Cooper pairs not only change the statistics of the nuclear stuff around the Fermi surface and, condensing, the properties of nuclei ~~close~~ to their ground state. They also display a rather remarkable mechanism of tunnelling between target and projectile in direct two–nucleon transfer reaction. In fact, being weakly bound ($\ll \epsilon_F$, Fermi energy) ~~they display correlations~~ over distances (correlation length) much larger than nuclear dimensions ($\gg R$, nuclear radius). On the other hand, Cooper pairs are forced to be confined within such dimensions by the action of the average potential, which can be viewed as an external field as far as these pairs are concerned.

Cooper pair partners are correlate

The correlation length paradigm comes into evidence, for example, when two nuclei are set into weak contact in a direct reaction. In this case, each of the partner nucleons of a Cooper pair has a finite probability to be confined within the mean field of each of the two nuclei. It is then natural that a Cooper pair can tunnel, equally well correlated, between target and projectile, through simultaneous transfer

than through successive transfer processes. Consequently, although one does not expects supercurrents in nuclei, one can study long–range pairing correlations in terms of individual quantal state. The above mentioned weak coupling Cooper pair ~~tunnelling~~ reminds the tunnelling mechanism of electronic Cooper pairs across a barrier (e.g. a dioxide layer) separating two superconductors, known as Josephson junction. The main difference is that, as a rule, in the nuclear time dependent junction efimerely established in direct two–nucleon transfer process, only one or even none of the two weakly interacting nuclei are superfluid (or superconducting). Now, in nuclei, paradigmatic example of fermionic finite many–body system, zero point fluctuations (ZPF) in general, and those associated with pair addition and pair substraction modes known as pairing vibrations in particular, are much stronger than in condensed matter. Consequently, and in keeping with the fact that pairing vibrations are the nuclear embodiment of Cooper pairs in nuclei, pairing correlations based on even a single Cooper pair can lead to clear~~ly observable~~ effects in two–nucleon transfer processes.

pairing correlation

Nucleonic Cooper pair tunnelling has played and is playing a central role in the probing of these subtle quantal phenomena, both in the case of exotic nuclei as well as of nuclei lying along the stability valley, and have been instrumental in shedding light on the subject of pairing in nuclei at large, and on nuclear superfluidity in particular. Consequently, the subject of two–nucleon transfer occupies a central place in the present monograph both concerning the conceptual and the computational aspects of the description of nuclear pairing, as well as regarding the quantitative confrontation of the results and predictions with the experimental findings, in terms of absolute cross sections.

e

Because of the central role the interweaving of the variety of elementary modes of nuclear excitation, namely single particle motion and collective vibrations play in nuclear superfluidity, the study of Cooper pair tunnelling ~~in nuclei~~ aside from requiring a consistent description of nuclear structure in terms of dressed quasiparticles and vibrations resulting from both bare and induced interactions, also involves the description of one–nucleon transfer ~~as well as knock-out~~ processes. Consequently, in the present monograph the general physical arguments and technical computational details concerning the calculation of absolute one–and two nucleon transfer cross sections, making use of state of the art nuclear structure information, are discussed in detail. As a consequence, theoretical and experimental nuclear practitioners, as well as fourth year and PhD students can use the present monograph at profit. To make simpler this use, the basic nuclear structure formalism, in particular that associated with pairing and with collectives modes in nuclei, is economically introduced through general physical arguments. This is also in keeping with the availability in the current literature, of detailed discussions of such material.

Within this context, the monographs *Nuclear Superfluidity* by Brink and Broglia and *Oscillations in Finite Quantum Systems* by Bertsch and Broglia, published also by Cambridge University Press can be considered companion volumes to the present one.

Throughout the text detailed references to these texts is found in connection with specific topics, where the interested reader can find relevant supplementary material.

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Concerning the notation, we have divided each chapter into sections. Each ~~sub~~section may, in turn, be broken down into subsections. Equations and Figures are identified by the number of the chapter and that of the section. Thus (6.1.33) labels the thirtythird equation of section 1 of chapter 6. Similarly, Fig. 6.1.2 labels the second figure of section 1 of chapter 6. Concerning the Appendices, they are labelled by the chapter number and by a Latin letter in alphabetical order, e.g. App. 6.A, App. 6.B, etc. Concerning equations and Figures, a sequential number is added. Thus (6.E.15) labels the fifteenth equation of Appendix E of chapter 6, while Fig. 6.F.1 labels the first figure of Appendix F of Chapter 6. References are referred to in terms of the author's surname and publication year and are found in alphabetic order in the bibliography.

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Throughout, a number of footnotes (they are useful) are found. This is in keeping with the fact that footnotes can play a special role within the framework of an elaborated presentation. In particular, ~~they allow one~~ to emphasize relevant issues in an economic way. Being outside the main text, they give the possibility of stating eventual important results, without the need of elaborating on the proof. Within this context, (one can mention that) in the paper Born (1926), introducing the probabilistic interpretation of Schrödinger's wavefunction, the fact that this probability is connected with its modulus squared and not with the wavefunction itself, is only ~~explained~~ referred to in a footnote.

Most of the material contained in this monograph have been the subject of lectures of the four year course "Nuclear Structure Theory" which RAB delivered throughout the years at the Department of Physics of the University of Milan, as well as at the Niels Bohr Institute and at Stony Brook (State University of New York). It was also presented by the authors in the course Nuclear Reactions held ~~in the academic year 2009~~ at the PhD School of Physics of the University of Milan.

RAB wants to acknowledge the central role the collaboration with Francisco Barranco and Enrico Vigezzi has played concerning the nuclear structure aspects of the present monograph. Its debt with the late Aage Winther regarding the reaction aspects of the present volume is difficult to express in words. The overall contributions of Daniel Bès, Ben Bayman and Pier Francesco Bortignon are only too explicitly evident throughout the text and constitute a daily source of inspiration. G.P. and R.A.Broglia ← see version April 26, 2016

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M. Born. Zur Quantenmechanik der Stoßvorgänge. *Zeitschr. f. Phys.*, 37:863, 1926.

GP ...

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(B) A methodological approach used in the present monography concerns repetition. Similar, if not the same issues are dealt with more than once, and similar things are being said a number of times. Hopefully not in contradiction with each other, but neither totally consistent between themselves either. This approach reflects the fact ~~that~~ useful concepts like reaction channels, or correlation length, let alone elementary modes of excitation, are easy to understand but difficult to define. This is because their validity is not exhausted in a single perspective.*) But even more important, because their power in helping at connecting**) seemingly unrelated results and phenomena is difficult to be fully appreciated the first time around, spontaneous symmetry breaking and associated emergent properties providing an example.

**) "The concepts and propositions get "meaning" viz., "content", only through their connection with sense-experience... The degree of certainty with which this connection, viz., intuitive combination, can be undertaken, and nothing else, differentiates empty phantasy from scientific "truth"... A correct proposition borrows its "truth" from the truth-content of the system to which it belongs" (A. Einstein, Autobiographical notes, in Albert Einstein, Ed. P.A. Schilpp, Harper, New York (1951) p. 1, Vol I.

*) This is also a consequence of the fact ~~that~~ physically correct concepts are forced to be expressed, to become precise, in mathematical terms.

p. 5 (B)
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invariants ~~emergent~~ ~~properties~~ ~~pp 103–104~~ Born

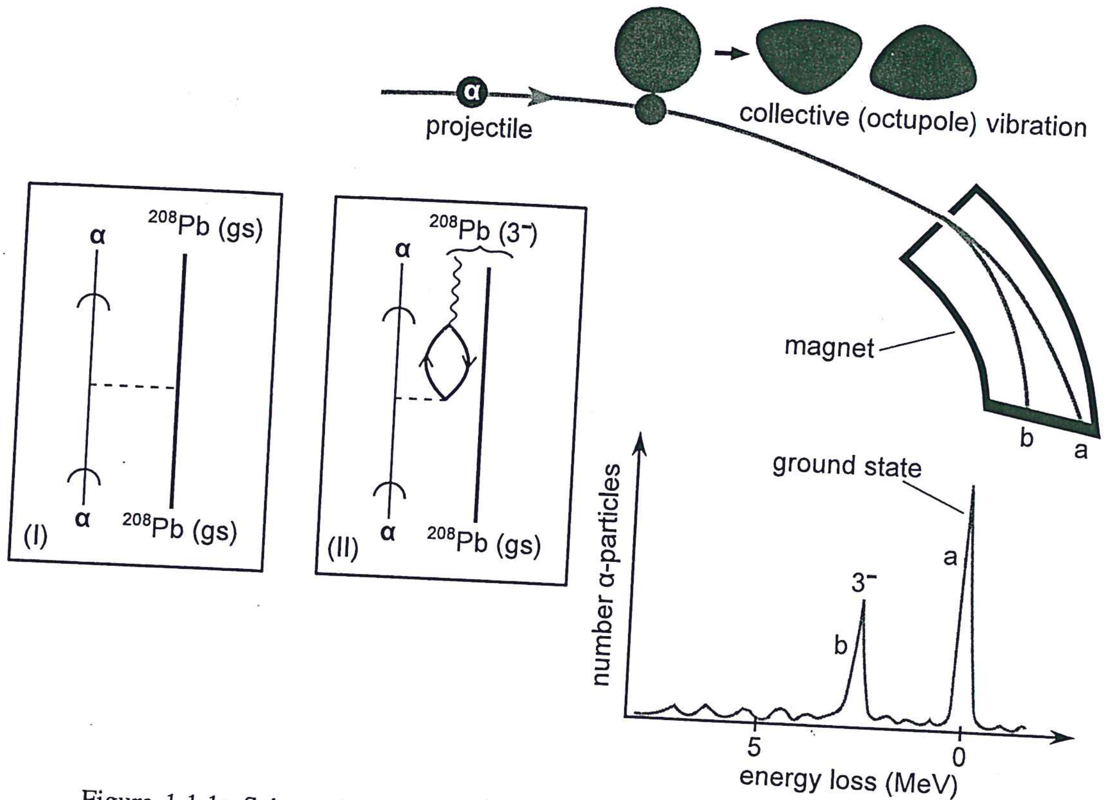

Figure 1.1.1: Schematic representation of: **a) elastic** (population of the ground state), and **inelastic b)** (population lowest octupole vibration 2.62 MeV) processes associated with the reaction $^{208}\text{Pb}(\alpha, \alpha')^{208}\text{Pb}$ (for more details see Sect. 1.3). In the inset (II) one of the NFT(r+s) diagrams describing the excitation of the low–lying octupole vibration (wavy line), of ^{208}Pb is given. A pointed (curved) arrow on a line indicates propagation of a nucleon in the target nucleus (in the continuum). The horizontal dashed line represents the action of the mean field (see Fig. 1.2.1) while the solid dot stands for the particle–vibration coupling vertex (Sect. 1.3). In the inset (I) a NFT(r+s) diagram describing the elastic process (potential scattering) is displayed. From the measurement of the differential cross section one can deduce the partial wave phase shifts (Appendix 1.D) Outgoing particles are deflected in a spectograph and recorded in a detector. The corresponding excitation function is given in the lowest part of the figure (after Mottelson (1976b)).

Concerning the first one an up (down) pointed arrowed line indicates a nucleon (nucleon-hole) moving above (in) the Fermi sea.

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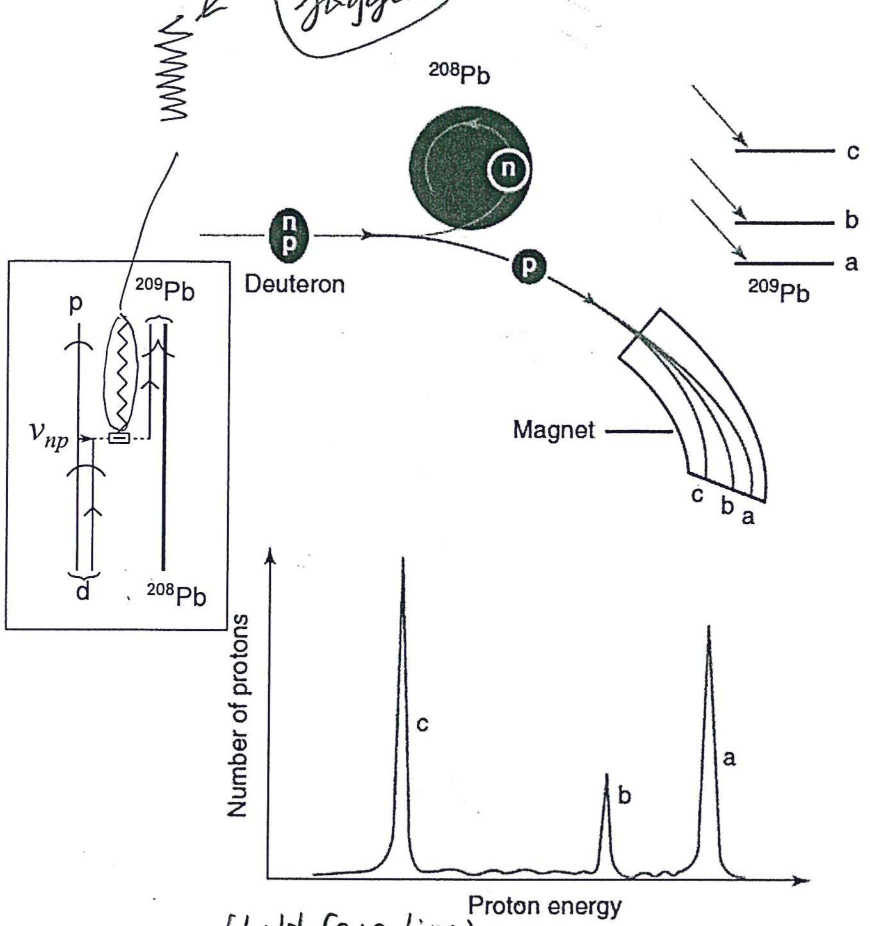

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Figure 1.1.2: Schematic cartoon representation of the one–nucleon transfer reaction ^{208}Pb$(d,p)^{209}$Pb populating the single–particle states of ^{209}Pb. In the inset a NFT(r+s) diagram describing the process is shown. A standard (pointed) arrowed curve indicates the transferred nucleon, moving with the proton in the deuteron (double, curve arrowed, line), or around the (assumed, for simplicity, inert) ^{208}Pb core. The energy of the outgoing proton reflects both the Q–value of the reaction and the excitation energy of the final state. In the inset, a NFT(r+s) diagram describing one of the possible transfer processes is schematically shown. An arrowed curve indicates the transferred nucleon. The jagged curve represents the recoil elementary mode which couples (dashed square) the relative motion (reaction) to the intrinsic nucleonic degrees of freedom (structure) through a Galilean operator transformation. For more details see caption to Fig. 1.1.1 and 1.9.2, see also Ch. 4 and Fig. 4.1.1 as well as App. 1.D and Sect. 5.C (after Mottelson (1976b)).

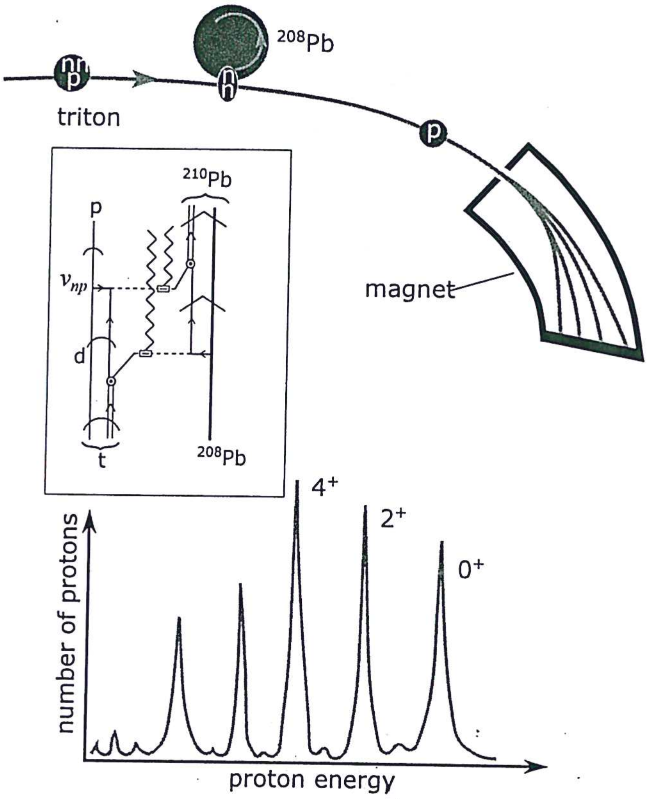

Figure 1.1.3: Schematic representation of the two–nucleon transfer reaction ^{208}Pb$(t, p)^{210}$Pb populating the ground state 0^+, and two particle excited states 2^+ and 4^+ (monopole, quadrupole and hexadecapole pair addition modes of ^{208}Pb, i.e. multipole pariring vibrations, see Brink, D. and Broglia (2005) Sect. 5.3.1 p. 108, Broglia et al. (1974), Ragnarsson and Broglia (1976), Broglia, R. A. et al. (1971a), Broglia, R. A. et al. (1971b), Bès and Broglia (1971b), Bès and Broglia (1971a), Flynn, E.R. and Igo, G. and Barnes, P.D. and Kovar, D. and Bès, D. R. and Broglia, R.A. (1971), Bès et al. (1972), Broglia (1981), Bohr and Mottelson (1974), Flynn, E. R. et al. (1972), Bortignon et al. (1976); see also Kubo et al. (1970)). In the inset a NFT(r+s) diagram of the transfer process is displayed. In selecting it, it was assumed, as detailed calculations indicate, that the main contribution to the process arises from the successive transfer of the nucleons. The jagged curves represent the recoil mode coupling the intrinsic and the relative motion, thus accounting for the mass partition associated with the different channels and the change in scaling between entrance and exit channel distorted waves. The corresponding momentum mismatch being taken care by Galilean transformations (recoil effects) (see also caption to Fig. 1.1.2 and App. 1.D; also Ch. 5, in particular Figs. 5.C.1 and 5.C.2). As discussed in the following Chapters (see in particular Chs. 2 and 3), Cooper pairs are extended objects, the fermionic partners being correlated over distances much larger than nuclear dimensions (correlation length $\xi \approx 36$ fm$\gg 2R_0 \approx 14.2$ fm ($A = 208$)). Because the single particle potential acts on these pairs as a rather strong external field, this feature is not obvious in structure calculations, becoming apparent in reaction calculations. The dineutron moving in the triton and around the ^{208}Pb, (assumed for simplicity to be inert) is represented by a double arrowed line. Each individual transferred neutron is indicated with a single arrowed line. The curved arrows on the triton and on the proton indicate motion in the continuum with outgoing and incoming asymptotic waves, respectively. The arrow encompassing the pair addition mode and the core ^{208}Pb, indicate intrinsic (structure) motion (after Mottelson (1976b)).

core

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describe the NFT diagrams shown in Figs. 1.1.1, 1.1.2 and 1.1.3

are boxed insets Figs. 1.1.1–1.1.3

out the reaction mechanism (see ~~Ch.~~ 4), or the confrontation between theoretical predictions and experimental observation would unlikely be fruitful[2]. ~~Arguably, an~~ An example of such an approach is given in Sect. 1.10 of this Chapter. 1.9 and

Echoing Heisenberg's requirement[3] that no concept enters the quantal description of a physical system which has no direct relation to experiments, and Landau's result that ~~any~~ weakly excited state of a quantal many–body system may be regarded as a gas of ~~weakly~~ interacting elementary modes of excitation[4], Bohr, Mottelson and coworkers[5] developed a unified ~~field theoretical~~ description of the nuclear structure (Nuclear Field Theory, NFT) in terms of quasiparticles, vibrations and rotations, both in 3D– as well as in gauge– and other "abstract" spaces, ~~with~~ which had a close connections with direct nuclear reactions[6]. Within this context also given in Figs. 1.1.1–1.1.3 (insets) are *unified NFT diagrams of structure and reactions (NFT(s+r))*[7], which microscopically describe the variety of processes in terms of elementary modes of excitation. That is, in the present case in which the target is a closed shell system, a particle–hole (inelastic scattering), one–particle (single–particle stripping) and two–particle (Cooper pair transfer) modes. structure and reaction

In keeping with the fact that all the nuclear degrees of freedom are exhausted by those of the nucleons, and that the different reactions, that is Coulomb, inelastic and one– and two– particle transfer reactions project particular, but somewhat overlapping components of the total wavefunction, the nuclear elementary modes of excitation give rise to an overcomplete, non orthogonal, Pauli principle violating basis, both concerning structure as well as reactions. The coupling between unperturbed fermionic and bosonic degrees of freedom is proportional to this overlap between single–particle and collective modes. Nuclear Field Theory[8], provides the conserving sum rules protocol to diagonalize these couplings to any order of perturbation theory, also infinite if so required for specific processes (see Sect. 1.7). The ~~clothed~~ dressed physical elementary modes resulting from the interweaving of the bare modes are orthogonal to each other, fulfill Pauli principle, and behave like a non interacting

[2] *Structure and Reactions*. Within this context one can ask how one understands what the correct elements are to describe a reaction process, if one does not know in detail the structure of the initial and final states? In a nutshell: how can one understand reaction without knowing structure (eyes without object)?.

Vice versa, how can one understand what the elements needed (specific probe) for a correct description of the structure of levels is, if one does not know how to observe them, how to bring that information to the detector? In other words, how can one understand structure without knowing reaction (object without eyes)? The answer to both questions is that you cannot. As simple as that.

OK

OK

[3] Heisenberg (1949).

[4] Landau (1941)).

[5] Bohr and Mottelson (1969), Bohr (1976), Mottelson (1976a), Bohr, A. and Mottelson (1975), Bohr et al. (1958), Belyaev (1959), Nilsson (1955), Bès, D. R. and Broglia (1966).

[6] Alder et al. (1956), Alder and Winther (1975), Broglia and Winther (2004), Austern (1970), Glendenning, N. K. (2004), Satchler (1980), Satchler (1983), see also Potel, G. et al. (2013).

Potel (2013) review paper Cooper pair transfer

[7] Broglia, R. A. (1979); Broglia and Winther (2004); Broglia et al. (2016)

[8] Bès et al. (1976b), Bès et al. (1976c), Bès et al. (1976a), Bès and Broglia (1975), Broglia et al. (1976), Bès, D. R. and Broglia, R. A. and Dussel, G. G. and Liotta (1975), Mottelson (1976a), Bès and Broglia (1977), Bortignon, P. F. et al. (1977), Bortignon, P. F. et al. (1978), Broglia and Winther (2004), Reinhardt (1975), Reinhardt (1978a), Reinhardt (1978b), Reinhardt (1980).

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The NFT(s+r)

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gas, providing a microscopic solution to the many–body nuclear problem. ~~Its~~ predictions, embodied in absolute cross sections and transition probabilities, can be directly compared with the observables whose values are obtained by studying the nuclear system with the variety of ever more precise and varied arsenal of experimental probes[a]. and

vibrational modes

At this point a proviso or two are in place. The original elementary modes of nuclear excitation melt together, due to their interweaving, into effective fields. Each of them display properties which reflects that of all the others, their individuality resulting from the actual relative importance of each one of them. What one calls physically a (clothed) particle is only partially to be associated with that particle field alone. It is also partially to be associated with the vibrational fields (surface, density, spin,[9] pairing, etc.), because they are in interaction through the particle–vibration coupling vertices. And conversely, what one calls a nuclear vibration can couple to particle–hole (in the case of a surface vibration), two–particle (in the case of a pair addition) or a two–hole (in the case of a pair removal) configurations[9,10]. The associated fermions can couple to other vibrational modes and, eventually in the course of time, recombine to reform the original vibration. The outcome of such processes, namely the dressed physical elementary modes of excitation, is closely connected with the renormalization program of quantum electrodynamics (QED)[10][11] implemented, in NFT, in terms of Feynman diagrams[11][12] and made operative in a number of cases in term of ~~empirical~~ the renormalization prescription[b].

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9,10 Brink D. and Broglia (2005) (Ch. 5)

The specific experimental probes of the bare elementary modes of nuclear excitation reveal only one, in most cases likely the most important aspects of the physical (clothed) elementary modes. ~~Renormalization (NFT program)~~ Renormalized NFT reflects the physical unity of low–energy nuclear research requiring the melting not only of elementary modes of excitation but also of structure and reaction theory, let alone of the different experimental techniques developed to study the atomic nucleus (Within this context cf. Ch. Fig. 1.10.1).

¶

As can be seen from the contents of the present monograph, the accent is set on relating theoretical predictions with experimental findings, through the unification of structure and reactions. In particular the unification of pairing and two–nucleon transfer, where the two subjects are blended together, which is what happens in nature. The theory of direct reaction for two–particle transfer is discussed in Ch. 5 (see summary Sect. 5.1). To be operative, two–nucleon spectroscopic amplitudes and optical potentials are needed. One–particle transfer processes are at the basis of the above derivations, and are summarized in Sect. 4.1. Once the NFT rules to work the variety of elements (spectroscopic and, with the help of them,

[a] What NFT cannot do, is to solve problems regarding ill behaved bare forces, or the consequences of associated particle–vibration coupling vertices which eventually lead to divergences. Within this context, *empirical renormalization* has proved to be a powerful and physically consistent prescription to implement the NFT and to make connection with experimental data (see e.g. Sect. 4.2, see also Barranco et al. (2004) and Broglia et al. (2016), and refs. therein)

[11] Schwinger (2001).

[12] Feynman (1975).

to text and explain

In other words the need for a complete set of experimental probes to reveal the multi facetic properties of clothed elementary modes of excitation resulting from the implementation of the (NFT) ren. program of structure and reactions.

ref. [11] see e.g.

9 Bertsch and Broglia (2005) Chs. 6, 7, 8

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16b

@ The outcome of such processes, namely the dressed physical elementary modes of excitation, is closely connected with the renormalization program of quantum electrodynamics (QED) (11,12) implemented in NFT in terms of Feynman diagrams. Renormalized NFT, i.e $(NFT)_{ren}$, implies that the intermediate, virtual states clothing the elementary modes of excitation, are supposed to be fully dressed. Now, because these virtual states can be forced, by acting with an external field, to become real real and thus observable, implies that the intermediate fully dressed states should coincide with the experimental ones. Thus, $(NFT)_{ren}$ is not a calculational ansatz but a quantal requirement.(*) We return back to this point in sections 4.2.1 (see also Fig. 1.10.1) and 4.2.2 in connection with superfluid nuclei lying along the stability valley, and (Sn-isotopes) and light exotic one-neutron halo systems.

@ to p. 16

(*) Broglia et al (2016), Barranco et al (2017) ← (paper 11Be)
Idini et al (2015)

reaction amplitudes) have been laid out and/or the pertinent literature refer to, concrete embodiments are provided and eventual absolute cross sections calculated and confronted with the experimental data (for details see App. 1.A). Within this scenario, once the NFT renormalization program associated with a characterising set of observables, e.g. the energy ($\hbar\omega_{2^+}$) and electromagnetic transition probability ($B(E2)$) of the collective, low–lying quadrupole vibration of ^{120}Sn has been carried out, a decision is to be taken. Unless one is not specifically studying such type of nuclear excitations, e.g. through inelastic scattering, but just using them to e.g. cloth single–particles, one turns to the *empirical renormalization* program[12]. Within the framework of the particular example, one calculates the $\hbar\omega_{2^+}$ and $B(E2)$ values making use of the harmonic approximation to diagonalize a separable quadrupole–quadrupole interaction adjusting its strength k_2 so as to reproduce the experimental findings. As a rule, the resulting k_2 has a value close to that required by self consistency[13] (see also Sect. 1.3). Within the properties mentioned, i.e. single–particle self energy, the final outcome differs little from that obtained from a full implementation of the NFT renormalization program. On the other hand, the use of the *empirical renormalization* program, aside from being an economic procedure for self consistency in phonon renormalization, allows to avoid difficulties associated with the zero–range character (ultraviolet divergences) and finite size instabilities of many effective interactions.

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1.2 Sum rules

physical sum rule

A quantitative measure of the overcompleteness mentioned above and concerning the elementary modes of excitation basis is provided by the use of exact and approximate sum rules that the observables (cross sections) associated with the variety of probes to which the nucleus is subject, have to fulfill. An example of the first type (exact) is provided by the Thomas–Reiche–Kuhn (TRK) sum rule. (a, b) Of the second type (approximate) by some of the two–nucleon transfer (TNTR) sum rules[14]. Others, which relate one– with two–particle transfer processes[15] being exact. In both cases they embody particle (pair) number conservation. Charged particles in the first case (electrons in atoms and molecules, effective charges of neutrons and protons in nuclei). Number of Cooper pairs in nuclei in the second. Physically, they provide information concerning: 1) the maximum amount of energy which the quantal system can absorb from a beam of light (γ–rays) shined on it; 2) the total two–nucleon transfer cross section (ring area fraction of the total (geometrical) reaction cross section) exhausted by the final ($A \pm 2$) states populated in the transfer process.

In other words, these sum rules provide: a quantitative measure of the single–

(a)

~~[12] Barranco, F. et al. (2001); Idini, A. et al. (2014); Idini et al. (2015); Broglia et al. (2016)~~

[13] Bohr, A. and Mottelson (1975), Sect. 6-4.

[14] Broglia, R. A. et al. (1972)

[15] Bayman, B. F. and Clement (1972); Lanford (1977)

(b) Bertsch and Broglia (2005), Chapter 3, in particular Sect. 3.3

particle subspace the quantal system under study, in particular the nucleus, uses to correlate particle–hole excitations and thus induce the antenna–like motion of protons against neutrons or, to correlate pairs of nucleons moving in time reversal states around the Fermi energy, leading to a sigmoidal distribution of the associated level occupancy[16].

The TRK sum rule can, in the nuclear case, be written as[17]

$$S(E1) = \sum_\alpha \left|\langle\alpha|F|\tilde{0}\rangle\right|^2 (E_\alpha - E_0) = \frac{9}{4\pi}\frac{\hbar^2 e^2}{2m}\frac{NZ}{A}, \tag{1.2.1}$$

where $|\alpha\rangle$ labels the complete set of excited dipole states which can be reached ~~operating~~ acting with the dipole operator F on the initial correlated vacuum state $|\tilde{0}\rangle$. Within this context, each elementary mode of excitation, provides a specific contribution to the total zero point fluctuations of the ground state (ZPF, see Sect. 1.7), that is,

$$\langle\tilde{0}|F^2|\tilde{0}\rangle = \frac{\hbar\omega}{2C_\alpha} = \frac{\hbar^2}{2D_\alpha}\frac{1}{\hbar\omega_\alpha}. \tag{1.2.2}$$

In other words, they perturb the static nucleon Fermi sea, that is the set of occupied levels of the mean field potential (Fig. 1.2.1)

$$U(r) = \int d\mathbf{r}'\rho(r')v(|\mathbf{r}-\mathbf{r}'|), \tag{1.2.3}$$

inducing virtual particle–hole excitations (k, i, i.e. $\epsilon_i \leq \epsilon_F$ and $\epsilon_k > \epsilon_F$, see Eqs. (1.2.6) and (1.4.1) (see Fig. 1.2.2). In the above equation, $\rho(r)$ is the nuclear density and v is the nuclear two–body interaction.

In Eq. (1.2.1), the quantity

$$F = e\sum_n \left(\left(\frac{N-Z}{2A} - t_z(n)\right) r_n Y_{1\mu}(\hat{r}_n)\right), \tag{1.2.4}$$

is the dipole operator acting both on the protons ($t_z = -1/2$) and on the neutrons ($t_z = 1/2$) of mass m, as indicated by the α–sum over all nucleon states ($A = N + Z$, mass number).

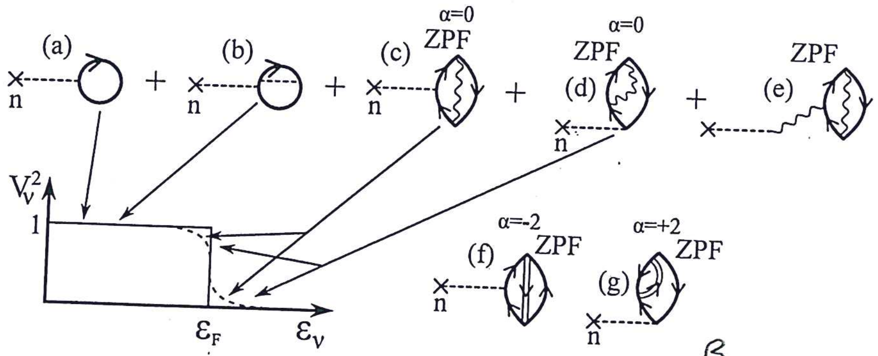
Because $|\langle\alpha|F|\tilde{0}\rangle|^2$ measures the probability with which the state $|\alpha\rangle$ is populated, the α–sum in (1.2.1) gives a measure of the maximum energy that the nucleus can absorb from the γ–beam, as can be seen by measuring $|\langle\alpha|F|0\rangle|^2$ in single–particle (sp) units (Weisskopf (W) units)

$$\begin{aligned} B_{sp}(E1; j_1 \to j_2) &= \frac{3}{4}e_{E1}^2\langle j_1\, \tfrac{1}{2}\, 1\, 0|j_2\, \tfrac{1}{2}\rangle^2 \times \langle j_2|r|j_1\rangle^2, \\ &\approx \frac{1}{4\pi}A^{2/3}e_{E1}^2\ \mathrm{fm}^2 = B_W(E1), \end{aligned} \tag{1.2.5}$$

[16] Within this context, the absolute two–nucleon transfer cross section populating the ground state of a superfluid nucleus is proportional to the number of Cooper pairs contributing to the nuclear condensate. This quantity is rather stable along a pairing rotational band, in keeping with the fact that the "intrinsic" $|BCS\rangle$–state of the deformed system in gauge space, is essentially the same for all members of the band. This fact is at the basis of an approximate, physically sum rule. Potel et al (2017)

[17] Bohr, A. and Mottelson (1975); Bertsch and Broglia (2005); Bortignon, P. F. et al. (1998)

Potel, G, A. Idini, F. Barranco, E. Vigezzi and R. A. Broglia, From bare to renormalized order parameter in gauge space: structure and reactions, submitted (2017).



β

Figure 1.2.2: Schematic representation of the Fermi distribution. The sharp, heavy line drawn step function schematically represents the Hartree–Fock occupation numbers. The associated nuclear density measured with the help of an external field (cross attached to a dashed line) through processes of type (a) (Hartree: H) and (b) (Fock: F) is expected to display a diffusivity of the order of the strong force. Zero Point Fluctuations (ZPF) associated with collective particle–hole state, i.e. processes with transfer quantum number a) $\alpha = 0$ and shown in (c), (d) and (e), and with pairing vibrations, i.e. pair addition (graph (f)) and pair removal (graph (g)), smooth the occupation numbers around the Fermi energy (dashed curve) and leads to a nuclear density of larger diffusivity than that associated with HF. One– and two–particle strengths which in this (mean field) approximation are found in a single A–mass system, are a result of ZPF ($\alpha = 0, \pm 2$) distributed over a number of nuclei $(A, A \pm 2)$.

ℓ

β

where $(e)_{E1} = (N/A)e$ for neutrons and $(e)_{E1} = -(Z/A)e$ for protons, in keeping with the fact that the motion of a nucleon is associated with a recoil of the rest of the nucleus, since the total center of mass remains at rest in an intrinsic excitation.

Within this context is that independent–particle motion in general and the existence of a mean field ((MF), Hartree–Fock (HF) solution) in particular can be viewed as the most collective of all nuclear phenomena[18]. It is then not surprising that

n — like

$$S(E1) = \sum_n |\langle \alpha | F | \tilde{0} \rangle|^2 (E_\alpha - E_0)$$

n

$$= \sum_{k,i} |\langle k, i | F | \mathrm{gs(MF)} \rangle|^2 (\epsilon_k - \epsilon_i), \qquad (1.2.6)$$

provided $|\tilde{0}\rangle$ contains the ground state correlations mentioned in connection with Eq. (1.2.2), and that $|\mathrm{gs(MF)}\rangle$ those associated with $\Delta x \Delta p \geq \hbar$ (see Fig. 1.2.1). In other words, provided

$$|\mathrm{HF}\rangle = |\mathrm{gs(MF)}\rangle = \prod_{i \in occup} a_i^\dagger |0\rangle \qquad (1.2.7)$$

[18] Mottelson (1962).

a) Bohr (1964)

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where $|0\rangle$ is the particle vacuum ($a_j|0\rangle = 0$), and $\Gamma_\alpha|\tilde{0}\rangle = 0$, $\Gamma_\alpha^\dagger$ being the creation operator of a dipole Random Phase Approximation (RPA–) correlated particle–hole like mode ($\Gamma_\alpha^\dagger = \sum_{ki} X_{ki}^\alpha a_k^\dagger a_i + Y_{ki}^\alpha (a_k^\dagger a_i)^\dagger$)[19]. Relation (1.2.6) is a consequence of the fact that $S(E1)$ is proportional to the average value of the double commutator $[[H,F],F]$in the ground state of the system ($|\tilde{0}\rangle$ or $|HF\rangle$). Because F is a function of only the nucleon coordinates, and assuming $v(|\mathbf{r}-\mathbf{r}'|)$ to be velocity independent, the only contribution to the double commutator arises from the (universal) kinetic energy term of the Hamiltonian. Thus, the value (1.2.1) is model independent. In other words, this value does not depend on the correlations acting among the nucleons, but on the number of them participating in the motion and on their mass (inertia) as testified by the fact that $\sum_\alpha \hbar\omega_\alpha \left(\frac{\hbar\omega_\alpha}{2C_\alpha}\right) = \sum_\alpha \left(\frac{\hbar^2}{2D_\alpha}\right)$. It is then not surprising that the TRK sum rule was used in the early stages of quantum mechanics, to determine the number of electrons in atoms.

Let us now go back to two–nucleon transfer (pairing) processes. The absolute cross sections associated with the population of the final states can be set essentially on equal footing with respect to each other concerning Q–value and recoil effects, with the help of empirically determined global functions[20]. In this way, the theoretical absolute cross section associated with e.g. the $A(t,p)A+2$ population (we assume N to be even) of the nth final state of spin J and parity $(-1)^J$ can be written as

$$\sigma^{(n)}(J=L,Q_0) = \left|\sum_{j_1\geq j_2} B(j_1j_2;J_n)S(j_1j_2;L,Q_0)\right|^2, \tag{1.2.8}$$

where

$$S^2(j_1j_2;L,Q_0) = \sigma(j_1,j_2;L,Q_0), \tag{1.2.9}$$

while

$$B(j_1j_2;J_n) = \left\langle \Phi_{J_n}(\xi_{A+2}) \middle| \Phi_{J_i=0}(\xi_A)\, \frac{\left[a_{j_1}^\dagger a_{j_2}^\dagger\right]_J}{[1+\delta(j_1,j_2)]^{1/2}} \right\rangle, \tag{1.2.10}$$

is the two–nucleon spectroscopic amplitude, $\Phi_{J_i=0}(\xi_A)$ being the wavefunction describing the ground state of the initial nucleus, $\Phi_{J_n}(\xi_{A+2})$ that of the final state, ξ labeling the relative radial and spin intrinsic coordinates. Assuming A to be a closed shell system, and $J=0$, one can write

$$|0_n^+\rangle = \sum_{j_1\geq j_2} c^{(n)}(j,j;J=0)|j,j;J=0\rangle, \tag{1.2.11}$$

where $n = 1,2,3,\dots$ labels the final nucleus states of spin and parity $J^\pi = 0^+$ in increasing energy order. Making use of the completeness relation of the coefficients

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19 Bertsch and Broglia (2005), Ch. 4, Brink D. and Broglia (2005) Ch. 8, Sect. 8.3, Bohr and Mottelson (1975) Sect. 6-5 h.

20 See Broglia, R. A. et al. (1972)

$c^{(n)}(j, j; J = 0)$ one obtains,

$$\sum_n \sigma^{(n)}(J = L = 0, Q_0) = \sum_j \sigma(j, j; L = 0, Q_0). \qquad (1.2.12)$$

The above equation is rather similar to (1.2.6), aside from the fact that the Q–value effect can, in connection with (1.2.6), be analytically dealt with, while $\sigma(\{Q\})$ is a functional of Q. Furthermore, the complete separation of the relative and intrinsic motion coordinates taking place in e.g. (1.2.6), is in keeping with the fact that in elastic and inelastic processes the mass partition is equal in both entrance and exit channels. Thus, the intrinsic (structure) and the relative motion (reaction) coordinates can be treated separately. This is not the case for transfer processes, both intrinsic and reaction coordinates being interweaved through the recoil process (particle–recoil mode coupling, see jagged curve Figs 1.1.2 and 1.1.3; see also Sect. 1.6).

within this context see also Sect. 1.6.

A parallel with the discussion carried out in connection with (1.2.5) regarding the TRK sum rule, can be drawn defining two–particle units as,

$$\sigma_{2pu}^{max}(A, L, Q_0) = \max\left[\sigma(j_1, j_2; L, Q_0)\right], \qquad (1.2.13)$$

where max[ ] indicates that the largest two–particle absolute cross section in the single–particle subspace considered (hot orbital), is to be considered. In this way one can write the relation (1.2.12) in dimensionless units. Furthermore, one can define enhancement factors. Another quite useful two–particle transfer sum rule has been introduced in the literature[21], which relates the differences between two–nucleon stripping and pick–up reactions cross sections, with single–particle transfer processes[22].

The above arguments carried out for nuclei around closed shells, can be equally well be applied to the case of open shell nuclei, making use of the corresponding two–nucleon spectroscopic amplitudes[23]. In particular, in the case of independent pair motion, i.e. the BCS mean field solution of the pair problem, the pair transfer amplitude is given by

$$\begin{aligned}\alpha_0' &= \langle BCS\,(N+2)|P^\dagger|BCS\,(N)\rangle \\ &= \sum_j \frac{2j+1}{2} U_j'(N)V_j'(N+2) \qquad (1.2.14).\end{aligned}$$

where

$$P^\dagger = \sum_{m>0} a_{jm}^\dagger a_{\bar{jm}}^\dagger, \qquad (1.2.15)$$

In this connection, and within the context of a schematic model, see Eq.(1,7,80) and subsequent discussion

creates two nucleons in time reversal states.

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[21] Bayman, B. F. and Clement (1972).

[22] Lanford (1977)

[23] See App. 2, Broglia, R.A. et al. (1973), ~~Yoshida~~

S. Yoshida (1962)

In such a case it

The ZPF associated with pairing vibrations, similar to those associated with particle-hole like excitations,

~~Pairing, similar to ZPF (Fig. 1.2.2)~~ smooth out the sharp HF Fermi surface. The number of pairs in each level participating in this smoothing is $(2j+1)/2$, their occupancy being measured by the simultaneous, and apparently contradictory, property of being a particle (amplitude V'_j) and a hole (amplitude U'_j). In other words α'_0 measures the number of pairs of nucleons participating in the smoothing of the Fermi surface and thus can be viewed as the spectroscopic amplitude associated with the population of pairing rotational bands in two–nucleon transfer processes. ~~It~~ is expected that α'_0 ~~does not depend~~ on N and is about conserved along a pairing rotational band. Because $d\sigma(gs(N) \to gs(N+2))/d\Omega \approx |\alpha'_0|$, conservation is also expected for these absolute cross section. But in this case, it is a conservation of physical character, and not mathematical one. If one finds that at the angle where $L = 0$ two–nucleon transfer cross sections have the first maximum, as a rule close to 0°, the two nucleon strength function is dominated by a single peak, that associated with the ground state, and this is so for a number of isotopes differing by two nucleons, one can conclude one is in presence of a pairing rotational band. This is why (1.2.14) can be properly viewed as the order parameter of the nuclear superfluid phase and, in keeping with (1.2.15), two–nucleon transfer reaction is the specific tool to probe pairing in nuclei.

(Fig. 1.2.2)

depends weakly

(see Sect. 6.2.3)

(see Figs. 2.1.3 and 2.1.4)

~~rewrite better expand or quote From bare to renorm. and write in a short app.~~

1.3 Particle–vibration coupling

The Hamiltonian describing a system of independent particles and of collective surface vibrations can be written as[24]

$$H = H_M + H_{coupl} + H_{coll}, \quad (1.3.1)$$

H_c

where

$$H_M = T + U \quad (1.3.2)$$

is the mean field Hamiltonian, sum of the single–particle kinetic energy and of the self–consistent potential $U = f(\rho_A)$, functional of the density. That is,

$$U = U_H + U_x, \quad (1.3.3)$$

where

$$U_H = \int d\mathbf{r}'\rho(r')v(|\mathbf{r} - \mathbf{r}'|), \quad (1.3.4)$$

is the Hartree potential, and

$$U_x = -\sum_{i(\epsilon_i \leq \epsilon_F)} \varphi_i^*(\mathbf{r}')v(|\mathbf{r} - \mathbf{r}'|)\varphi_i^*(\mathbf{r}) \quad (1.3.5)$$

is the exchange (Fock) potentil. It is well established that the nucleus can react collectively to external sollicitations. In particular the nuclear surface[25] can vibrate

[24] Bohr, A. and Mottelson (1975); Brink, D. and Broglia (2005)

[25] We consider in the present section only this type of collective modes

in certain normal modes which, in the harmonic approximation can be described as

$$H_{coll} = \frac{\hat{\Pi}_\alpha^2}{2D_\alpha} + \frac{C_\alpha}{2}\hat{\alpha}^2, \tag{1.3.6}$$

where

$$\hat{\alpha} = \sqrt{\frac{\hbar\omega_\alpha}{2C_\alpha}}\left(\Gamma_\alpha^\dagger + \Gamma_\alpha\right), \tag{1.3.7}$$

ing

is the collective coordinate and $\hat{\Pi}_\alpha$ being the correspond~~ent~~ conjugate momentum. $\Gamma_\alpha^\dagger(\Gamma)$ is the creation (annihilation) operator of the corresponding quanta. Microscopically, these modes can be calculated in the RPA as correlated particle–hole excitations[26].

Γ_α

The particle–vibration coupling Hamiltonian can be written as, -e

H_c

$$H_{coupl} = \kappa\hat{\alpha}\hat{F}, \tag{1.3.8}$$

with

$$\hat{F} = \sum_{\nu_1\nu_2}\langle\nu_1|F|\nu_2\rangle a_{\nu_1}^\dagger a_{\nu_2}, \tag{1.3.9}$$

and

$$F = -\frac{1}{\kappa}R_0\frac{\partial U(r)}{\partial r}Y_{LM}^*(\hat{r}). \tag{1.3.10}$$

It is of notice that κ characterizes the relationship between potential and density, of the mode considered. The self–consistent value is ($\langle F\rangle = \alpha$ cf. Bohr, A. and Mottelson (1975)),

$$\kappa = \int R_0\frac{\partial U}{\partial r}R_0\frac{\partial \rho}{\partial r}r^2 dr. \tag{1.3.11}$$

Both the coupling constant and the potential U are negative, for attractive fields. H_{coupl} embodies the coupling of the motion of a single-nucleons with the collective vibrations of the surface, with a matrix element (see Fig.1.3.1)

$$V_{\nu,\alpha'} = \langle n_\alpha = 1, \nu'|H_{coupl}|\nu\rangle = \langle n_\alpha = 1, \nu\nu'|H_{coupl}|0\rangle = \Lambda_\alpha\langle\nu'|F|\nu\rangle, \tag{1.3.12}$$

where H_c H_c

$$\Lambda_\alpha = \kappa\sqrt{\frac{\hbar\omega_\alpha}{2C_\alpha}} \sim \frac{\kappa\beta_\alpha}{\sqrt{2L_\alpha+1}}, \tag{1.3.13}$$

is the particle-vibration coupling strength, while β_α is the (dynamic) deformation parameter. Assuming $\beta_L^2 \ll \beta_L$, one can treat the particle-vibration coupling in the weak coupling approximation. Consequently H_{coupl}, can be diagonalized perturbatively[27].

[26] Bohm and Pines (1951); Pines and Bohm (1952); Bohm and Pines (1953); Bertsch and Broglia (2005), Ch. 4;

[27] For more details we refer to Bohr, A. and Mottelson (1975); Brink, D. and Broglia (2005) ~~and~~ [redacted] and refs. therein.

Sect 6-5h and Sect. 8,3

Making the ansatz that the physical (clothed) single–particle states results from the coupling to only surface vibrations, the Hamiltonian (1.3.1) can be regarded as being complete to describe the elementary modes of excitation and their couplings. Adding to (1.3.1) the terms describing the spin, spin–isospin, etc. particle–hole modes, as well as those associated with multipole pairing vibrations (see Sect. 2.5 as well as caption to Fig. 1.1.3), i.e. pair addition and pair subtraction modes (with $\lambda^\pi = 0^+, 2^+, 4^+ \ldots$, and eventually $1^-, 3^- \ldots$), and the corresponding coupling terms, and diagonalizing perturbatively the resulting Hamiltonian, will lead to the physical single–particle states of spherical normal systems (nuclei around close shells). For spherical open–shell nuclei effects arising from the coupling to the condensate[28] will somewhat affect the actual value of the results, e.g. the energy of the two–quasiparticle phonon[29] states.

1.3.1 Fluctuation and damping

To second order one finds[30],

$$\begin{aligned}\Big(-\frac{\hbar^2}{2m}\nabla_r^2 &+ U_H(r)\Big)\varphi_j(r) + \int d^3r'\, U_x(\vec{r},\vec{r}')\varphi_j(\vec{r}'),\\ &+ (\Delta E_j + iW_j)\varphi_j(\vec{r})\\ &\approx \left(-\frac{\hbar^2}{2m_k}\nabla_r^2 + U_H''(r) + \Delta E_j'' + iW_j''\right)\varphi_j(\vec{r}), \qquad (1.3.14)\\ &= \varepsilon_j\varphi_j(\vec{r}), \qquad \left(U_H'' = \frac{m}{m_k}U \text{ and similarly for } \Delta E'' \text{ and} W''\right),\end{aligned}$$

where $m_k = \left(1 + \frac{m}{\hbar^2 k}\partial U_x/\partial k\right)^{-1} \approx 0.7m$ is the k–mass[31], while

$$\Delta E_j^{(\omega)} \cancel{= \mathrm{Re}\sum_j(\omega)} = \lim_{\Delta\to 0}\sum_{\alpha'}\frac{V_{\nu,\alpha'}^2(\omega - E_{\alpha'})}{(\omega - E_{\alpha'})^2 + (\frac{\Delta}{2})^2}, \qquad (1.3.15)$$

and

$$W_j^{(\omega)} \cancel{= \mathrm{Im}\sum_j(\omega)} = \lim_{\Delta\to 0}\sum_{\alpha'}\frac{V_{\nu,\alpha'}^2}{(\omega - E_{\alpha'})^2 + (\frac{\Delta}{2})^2}, \qquad (1.3.16)$$

are the real and imaginary contributions to the self-energy calculated in second order perturbation theory[32] (see Fig. 1.3.2; note $E_{\alpha'} = \epsilon_{\nu'} + \hbar\omega_\alpha$). ~~and $V_{\nu,\alpha'}$ have been defined in (1.3.12)~~.

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[28] see Bès, D. R. and Kurchan (1990)
[29] see Barranco et al. (2004)
[30] see e.g. Brink, D. and Broglia (2005), see also Mahaux, C. et al. (1985) and references therein. see also Bernard and Giai (1981)
[31] This is in keeping with the fact that the non–local component of the mean field can be parametrized at profit as $0.4E$, where $E = |\hbar^2k^2/2m - \epsilon_F|$ (Perey and Buck (1962)) see also Sect. 1.9.
[32] Given a Hamiltonian H_{coupl}, the contribution to the energy in second order perturbation theory is

$E = |(\hbar^2k^2/2m) - \epsilon_F|$