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(remember that $\mathbf{R} = R\,\hat{\mathbf{z}}$, see last section). $\mathbf{R}$ being the center of mass coordinate ~~we~~ one ~~have~~ can write

$$\mathbf{R} = \frac{1}{3}\left(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_p\right) = \frac{1}{3}\left(\mathbf{R} + \mathbf{d}_1 + \mathbf{R} + \mathbf{d}_2 + \mathbf{R} + \mathbf{d}_p\right), \tag{7.2.49}$$

so

$$\mathbf{d}_1 + \mathbf{d}_2 + \mathbf{d}_p = 0. \tag{7.2.50}$$

Together with

$$\mathbf{d}_1 + \mathbf{r}_{12} = \mathbf{d}_2 \qquad \mathbf{d}_2 + \mathbf{r}_{2p} = \mathbf{d}_p, \tag{7.2.51}$$

we find

$$\mathbf{d}_1 = \frac{1}{3}\left(2\mathbf{r}_{12} + \mathbf{r}_{2p}\right), \tag{7.2.52}$$

~~so~~ and

$$d_1^2 = \frac{1}{9}\left(4r_{12}^2 + r_{2p}^2 + 4\mathbf{r}_{12}\mathbf{r}_{2p}\right). \tag{7.2.53}$$

~~But~~ Making use of

$$\begin{aligned} &\mathbf{r}_{12} + \mathbf{r}_{2p} = \mathbf{r}_{1p} \\ &r_{1p}^2 = r_{12}^2 + r_{2p}^2 + 2\mathbf{r}_{12}\mathbf{r}_{2p} \\ &2\mathbf{r}_{12}\mathbf{r}_{2p} = r_{1p}^2 - r_{12}^2 - r_{2p}^2. \end{aligned} \tag{7.2.54}$$

one obtains
~~Finally,~~

$$d_1 = \frac{1}{3}\sqrt{2r_{12}^2 + 2r_{1p}^2 - r_{2p}^2}. \tag{7.2.55}$$

Similarly, ~~we find~~

$$d_2 = \frac{1}{3}\sqrt{2r_{12}^2 + 2r_{2p}^2 - r_{1p}^2} \qquad d_p = \frac{1}{3}\sqrt{2r_{2p}^2 + 2r_{1p}^2 - r_{12}^2}. \tag{7.2.56}$$

We now express the angle α between $\mathbf{d}_1$ and $\mathbf{r}_{12}$. We have

$$-\mathbf{d}_1\mathbf{r}_{12} = r_{12}d_1\cos(\alpha), \tag{7.2.57}$$

and

$$\begin{aligned} &\mathbf{d}_1 + \mathbf{r}_{12} = \mathbf{d}_2 \\ &d_1^2 + r_{12}^2 + 2\mathbf{d}_1\mathbf{r}_{12} = d_2^2 \end{aligned} \tag{7.2.58}$$

Consequently, ~~so~~

$$\cos(\alpha) = \frac{d_1^2 + r_{12}^2 - d_2^2}{2r_{12}d_1}. \tag{7.2.59}$$

The complete determination of $\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_{12}$ can be made by writing their expression in a simple configuration, in which the triangle lies in the xz–plane with $\mathbf{d}_1$ pointing along the positive z–direction, and $\mathbf{R} = 0$. Then, a first rotation $\mathcal{R}_z(\gamma)$ of an angle γ around the z–axis, a second rotation $\mathcal{R}_y(\beta)$ of an angle β around the y–axis, and a translation along $\mathbf{R}$ will bring the vectors to the most general configuration. In other words,

$$\begin{aligned} \mathbf{r}_1 &= \mathbf{R} + \mathcal{R}_y(\beta)\mathcal{R}_z(\gamma)\mathbf{r}'_1, \\ \mathbf{r}_{12} &= \mathcal{R}_y(\beta)\mathcal{R}_z(\gamma)\mathbf{r}'_{12}, \\ \mathbf{r}_2 &= \mathbf{r}_1 + \mathbf{r}_{12}, \end{aligned} \tag{7.2.60}$$

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with

$$\mathbf{r}'_1 = \begin{bmatrix} 0 \\ 0 \\ d_1 \end{bmatrix}, \tag{7.2.61}$$

$$\mathbf{r}'_{12} = r_{12} \begin{bmatrix} \sin(\alpha) \\ 0 \\ -\cos(\alpha) \end{bmatrix}, \tag{7.2.62}$$

and the rotation matrixes are

$$\mathcal{R}_y(\beta) = \begin{bmatrix} \cos(\beta) & 0 & \sin(\beta) \\ 0 & 1 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) \end{bmatrix}, \tag{7.2.63}$$

and

$$\mathcal{R}_z(\gamma) = \begin{bmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{7.2.64}$$

We ~~obtain~~ then

$$\mathbf{r}_1 = \begin{bmatrix} d_1 \sin(\beta) \\ 0 \\ R + d_1 \cos(\beta) \end{bmatrix}, \tag{7.2.65}$$

$$\mathbf{r}_{12} = \begin{bmatrix} r_{12}\cos(\beta)\cos(\gamma)\sin(\alpha) - r_{12}\sin(\beta)\cos(\alpha) \\ r_{12}\sin(\gamma)\sin(\alpha) \\ -r_{12}\sin(\beta)\cos(\gamma)\sin(\alpha) - r_{12}\cos(\alpha)\cos(\beta) \end{bmatrix}, \tag{7.2.66}$$

$$\mathbf{r}_2 = \begin{bmatrix} d_1\sin(\beta) + r_{12}\cos(\beta)\cos(\gamma)\sin(\alpha) - r_{12}\sin(\beta)\cos(\alpha) \\ r_{12}\sin(\gamma)\sin(\alpha) \\ R + d_1\cos(\beta) - r_{12}\sin(\beta)\cos(\gamma)\sin(\alpha) - r_{12}\cos(\alpha)\cos(\beta) \end{bmatrix}. \tag{7.2.67}$$

We also need $\cos(\theta_{12})$, ζ and $\cos(\theta_\zeta)$, θ_{12} being the angle between $\mathbf{r}_1$ and $\mathbf{r}_2$, $\zeta = \mathbf{r}_p - \frac{\mathbf{r}_1+\mathbf{r}_2}{A+2}$ the position of the proton with respect to the final nucleus, and θ_ζ the angle between ζ and the z–axis:

$$\cos(\theta_{12}) = \frac{\mathbf{r}_1\mathbf{r}_2}{r_1 r_2}, \tag{7.2.68}$$

and

$$\zeta = 3\mathbf{R} - \frac{A+3}{A+2}(\mathbf{r}_1 + \mathbf{r}_2), \tag{7.2.69}$$

where we have used (7.2.49).

For heavy ions, we find instead

$$\mathbf{R} = \frac{1}{m_a}(\mathbf{r}_{A1} + \mathbf{r}_{A2} + m_b \mathbf{r}_{Ab}), \tag{7.2.70}$$

$$\mathbf{d}_1 = \frac{1}{m_a}(m_b \mathbf{r}_{b2} - (m_b + 1)\mathbf{r}_{12}), \tag{7.2.71}$$

$$d_1 = \frac{1}{m_a}\sqrt{(m_b+1)r_{12}^2 + m_b(m_b+1)r_{b1}^2 - m_b r_{b2}^2}, \tag{7.2.72}$$

$$d_2 = \frac{1}{m_a}\sqrt{(m_b+1)r_{12}^2 + m_b(m_b+1)r_{b2}^2 - m_b r_{b1}^2}, \tag{7.2.73}$$

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and

$$\zeta = \frac{m_a}{m_b}\mathbf{R} - \frac{m_B + m_b}{m_b m_B}(\mathbf{r}_{A1} + \mathbf{r}_{A2}). \tag{7.2.74}$$

The rest of the formulae are identical to the (t, p) ones. We list them for ~~reference~~ convenience,

$$\mathbf{r}_{A1} = \begin{bmatrix} d_1 \sin(\beta) \\ 0 \\ R + d_1 \cos(\beta) \end{bmatrix}, \tag{7.2.75}$$

$$\mathbf{r}_{A2} = \begin{bmatrix} d_1 \sin(\beta) + r_{12}\cos(\beta)\cos(\gamma)\sin(\alpha) - r_{12}\sin(\beta)\cos(\alpha) \\ r_{12}\sin(\gamma)\sin(\alpha) \\ R + d_1\cos(\beta) - r_{12}\sin(\beta)\cos(\gamma)\sin(\alpha) - r_{12}\cos(\alpha)\cos(\beta) \end{bmatrix}. \tag{7.2.76}$$

We we also find

$$\mathbf{r}_{b1} = \frac{1}{m_b}(\mathbf{r}_{A2} + (m_b + 1)\mathbf{r}_{A1} - m_a\mathbf{R}), \tag{7.2.77}$$

and

$$\mathbf{r}_{b2} = \frac{1}{m_b}(\mathbf{r}_{A1} + (m_b + 1)\mathbf{r}_{A2} - m_a\mathbf{R}). \tag{7.2.78}$$

One can readily obtain ~~We easily obtain~~

$$\cos\theta_{12} = \frac{r_{A1}^2 + r_{A2}^2 - r_{12}^2}{2 r_{A1} r_{A2}}, \tag{7.2.79}$$

and

$$\cos\theta_i = \frac{r_{b1}^2 + r_{b2}^2 - r_{12}^2}{2 r_{b1} r_{b2}}. \tag{7.2.80}$$

7.2.4 Matrix element for the transition amplitude (alternative derivation)

~~The simultaneous amplitude can be written as, see~~ it can be written as Bayman and Chen (1982),

$$\begin{aligned} T^{1step}_{2NT} =& 2\frac{(4\pi)^{3/2}}{k_{Aa}k_{Bb}} \sum_{l_p j_p m l_t j_p} i^{-l_p} \exp[i(\sigma^p_{l_p} + \sigma^t_{l_t})]\sqrt{2l_t + 1} \\ & \times \langle l_p\ m - m_p\ 1/2\ m_p | j_p\ m\rangle \langle l_t\ 0\ 1/2\ m_t | j_t\ m_t\rangle Y^{l_p}_{m-m_p}(\hat{\mathbf{k}}_{Bb}) \\ & \times \sum_{\sigma_1\sigma_2\sigma_p} \int d\mathbf{r}_{Cc} d\mathbf{r}_{b1} d\mathbf{r}_{A2} \left[\psi^{j_f}(\mathbf{r}_{A1}, \sigma_1)\psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)\right]^{0*}_0 \\ & \times v(r_{b1}) \left[\psi^{j_i}(\mathbf{r}_{b1}, \sigma_1)\psi^{j_i}(\mathbf{r}_{b2}, \sigma_2)\right]^0_0 \frac{g_{l_t j_t}(r_{Aa}) f_{l_p j_p}(r_{Bb})}{r_{Aa} r_{Bb}} \\ & \times \left[Y^{l_t}(\hat{\mathbf{r}}_{Aa})\chi(\sigma_p)\right]^{j_t}_{m_t} \left[Y^{l_p}(\hat{\mathbf{r}}_{Bb})\chi(\sigma_p)\right]^{j_p*}_{m}. \end{aligned} \tag{7.2.81}$$

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t a(=b+2)

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As ~~we have~~ shown ~~before~~ above, ~~we~~ one can write,

$$\sum_{\sigma_p} \langle l_p\ m - m_p\ 1/2\ m_p | j_p\ m\rangle \langle l_t\ 0\ 1/2\ m_t | j_t\ m_t\rangle \left[Y^{l_t}(\hat{\mathbf{r}}_{Aa})\chi(\sigma_p)\right]^{j_t}_{m_t} \left[Y^{l_p}(\hat{\mathbf{r}}_{Bb})\chi(\sigma_p)\right]^{j_p*}_{m}$$

$$= -\frac{\delta_{l_p,l_t}\delta_{j_p,j_t}\delta_{m,m_t}}{\sqrt{2l+1}} \left[Y^l(\hat{\mathbf{r}}_{Aa})Y^l(\hat{\mathbf{r}}_{Bb})\right]^0_0 \begin{cases} \dfrac{l}{2l+1} & \text{if } m_t = m_p \\ -\dfrac{\sqrt{l(l+1)}}{2l+1} & \text{if } m_t = -m_p \end{cases} \tag{7.2.82}$$

In what follows we work an alternative derivation of T^{1step}_{2NT}, more closely related to ~~[illegible]~~ heavy ion reactions. Following

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when $j = l - 1/2$ and

$$\sum_{\sigma_p} \langle l_p\ m-m_p\ 1/2\ m_p | j_p\ m\rangle \langle l_t\ 0\ 1/2\ m_t | j_t\ m_t\rangle \left[Y^{l_t}(\hat{\mathbf{r}}_{Aa})\chi(\sigma_p)\right]^{j_t}_{m_t} \left[Y^{l_p}(\hat{\mathbf{r}}_{Bb})\chi(\sigma_p)\right]^{j_p *}_{m}$$
$$= -\frac{\delta_{l_p,l_t}\delta_{j_p,j_t}\delta_{m,m_t}}{\sqrt{2l+1}} \left[Y^l(\hat{\mathbf{r}}_{Aa})Y^l(\hat{\mathbf{r}}_{Bb})\right]^0_0 \begin{cases} \dfrac{l+1}{2l+1} & \text{if } m_t = m_p \\ \dfrac{\sqrt{l(l+1)}}{2l+1} & \text{if } m_t = -m_p \end{cases} \tag{7.2.83}$$

if $j = l + 1/2$. ~~We get~~ One gets

$$\begin{aligned} T^{1step}_{2NT} =& 2\frac{(4\pi)^{3/2}}{k_{Aa}k_{Bb}} \sum_l i^{-l} \frac{\exp[i(\sigma^p_l + \sigma^t_l)]}{2l+1} Y^l_{m_t-m_p}(\hat{\mathbf{k}}_{Bb}) \\ &\times \sum_{\sigma_1\sigma_2} \int \frac{d\mathbf{r}_{Cc}d\mathbf{r}_{b1}d\mathbf{r}_{A2}}{r_{Aa}r_{Bb}} \left[\psi^{j_f}(\mathbf{r}_{A1},\sigma_1)\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\right]^{0*}_0 \\ &\times v(r_{b1}) \left[\psi^{j_i}(\mathbf{r}_{b1},\sigma_1)\psi^{j_i}(\mathbf{r}_{b2},\sigma_2)\right]^0_0 \left[Y^l(\hat{\mathbf{r}}_{Aa})Y^l(\hat{\mathbf{r}}_{Bb})\right]^0_0 \\ &\times \Big[\big(f_{ll+1/2}(r_{Bb})g_{ll+1/2}(r_{Aa})(l+1) + f_{ll-1/2}(r_{Bb})g_{ll-1/2}(r_{Aa})l\big)\delta_{m_p,m_t} \\ &+\big(f_{ll+1/2}(r_{Bb})g_{ll+1/2}(r_{Aa})\sqrt{l(l+1)} - f_{ll-1/2}(r_{Bb})g_{ll-1/2}(r_{Aa})\sqrt{l(l+1)}\big)\delta_{m_p,-m_t}\Big]. \end{aligned} \tag{7.2.84}$$

~~Now~~ Making use of the relations,

$$\begin{aligned} &\left[\psi^{j_f}(\mathbf{r}_{A1},\sigma_1)\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\right]^{0*}_0 \\ &= ((l_f\tfrac{1}{2})_{j_f}(l_f\tfrac{1}{2})_{j_f}|(l_fl_f)_0(\tfrac{1}{2}\tfrac{1}{2})_0)_0 u_{l_f}(r_{A1})u_{l_f}(r_{A2}) \\ &\times \left[Y^{l_f}(\hat{\mathbf{r}}_{A1})Y^{l_f}(\hat{\mathbf{r}}_{A2})\right]^{0*}_0 [\chi(\sigma_1)\chi(\sigma_2)]^{0*}_0 \\ &= \sqrt{\frac{2j_f+1}{2(2l_f+1)}} u_{l_f}(r_{A1})u_{l_f}(r_{A2}) \\ &\times \left[Y^{l_f}(\hat{\mathbf{r}}_{A1})Y^{l_f}(\hat{\mathbf{r}}_{A2})\right]^{0*}_0 [\chi(\sigma_1)\chi(\sigma_2)]^{0*}_0 \\ &= \sqrt{\frac{2j_f+1}{2}} \frac{u_{l_f}(r_{A1})u_{l_f}(r_{A2})}{4\pi} P_{l_f}(\cos\omega_A) [\chi(\sigma_1)\chi(\sigma_2)]^{0*}_0, \end{aligned} \tag{7.2.85}$$

and

$$\begin{aligned} &\left[\psi^{j_i}(\mathbf{r}_{b1},\sigma_1)\psi^{j_i}(\mathbf{r}_{b2},\sigma_2)\right]^0_0 \\ &= ((l_i\tfrac{1}{2})_{j_i}(l_i\tfrac{1}{2})_{j_i}|(l_il_i)_0(\tfrac{1}{2}\tfrac{1}{2})_0)_0 u_{l_i}(r_{b1})u_{l_i}(r_{b2}) \\ &\times \left[Y^{l_i}(\hat{\mathbf{r}}_{b1})Y^{l_i}(\hat{\mathbf{r}}_{b2})\right]^0_0 [\chi(\sigma_1)\chi(\sigma_2)]^0_0 \\ &= \sqrt{\frac{2j_i+1}{2(2l_i+1)}} u_{l_i}(r_{b1})u_{l_i}(r_{b2}) \\ &\times \left[Y^{l_i}(\hat{\mathbf{r}}_{b1})Y^{l_i}(\hat{\mathbf{r}}_{b2})\right]^0_0 [\chi(\sigma_1)\chi(\sigma_2)]^0_0 \\ &= \sqrt{\frac{2j_i+1}{2}} \frac{u_{l_i}(r_{b1})u_{l_i}(r_{b2})}{4\pi} P_{l_i}(\cos\omega_b) [\chi(\sigma_1)\chi(\sigma_2)]^0_0, \end{aligned} \tag{7.2.86}$$

where ω_A is the angle between $\mathbf{r}_{A1}$ and $\mathbf{r}_{A2}$, and ω_b is the angle between $\mathbf{r}_{b1}$ and $\mathbf{r}_{b2}$. ~~So~~ Consequently

$$\begin{aligned}T_{2NT}^{1step} =&(4\pi)^{-3/2}\frac{\sqrt{(2j_i+1)(2j_f+1)}}{k_{Aa}k_{Bb}}\sum_l i^{-l}\frac{\exp[i(\sigma_l^p+\sigma_l^t)]}{\sqrt{2l+1}}Y^l_{m_t-m_p}(\hat{\mathbf{k}}_{Bb})\\ &\times\int\frac{d\mathbf{r}_{Cc}d\mathbf{r}_{b1}d\mathbf{r}_{A2}}{r_{Aa}r_{Bb}}P_{l_f}(\cos\omega_A)P_{l_i}(\cos\omega_b)P_l(\cos\omega_{if})\\ &\times v(r_{b1})u_{l_i}(r_{b1})u_{l_i}(r_{b2})u_{l_f}(r_{A1})u_{l_f}(r_{A2})\\ &\times\Big[\Big(f_{ll+1/2}(r_{Bb})g_{ll+1/2}(r_{Aa})(l+1)+f_{ll-1/2}(r_{Bb})g_{ll-1/2}(r_{Aa})l\Big)\delta_{m_p,m_t}\\ &+\Big(f_{ll+1/2}(r_{Bb})g_{ll+1/2}(r_{Aa})\sqrt{l(l+1)}-f_{ll-1/2}(r_{Bb})g_{ll-1/2}(r_{Aa})\sqrt{l(l+1)}\Big)\delta_{m_p,-m_t}\Big],\end{aligned}\tag{7.2.87}$$

where ω_{if} is the angle between $\mathbf{r}_{Aa}$ and $\mathbf{r}_{Bb}$. For heavy ions, we can consider that the the optical potential does not have a spin–orbit term, and the distorted waves are independent of j. We thus have

$$\begin{aligned}T_{2NT}^{1step} =&(4\pi)^{-3/2}\frac{\sqrt{(2j_i+1)(2j_f+1)}}{k_{Aa}k_{Bb}}\sum_l i^{-l}\exp[i(\sigma_l^p+\sigma_l^t)]Y_0^l(\hat{\mathbf{k}}_{Bb})\sqrt{2l+1}\\ &\times\int\frac{d\mathbf{r}_{Cc}d\mathbf{r}_{b1}d\mathbf{r}_{A2}}{r_{Aa}r_{Bb}}P_{l_f}(\cos\omega_A)P_{l_i}(\cos\omega_b)P_l(\cos\omega_{if})\\ &\times v(r_{b1})u_{l_i}(r_{b1})u_{l_i}(r_{b2})u_{l_f}(r_{A1})u_{l_f}(r_{A2})f_l(r_{Bb})g_l(r_{Aa}).\end{aligned}\tag{7.2.88}$$

~~We change the variables:~~ Changing variables one obtains,

$$\begin{aligned}T_{2NT}^{1step} =&(4\pi)^{-1}\frac{\sqrt{(2j_i+1)(2j_f+1)}}{k_{Aa}k_{Bb}}\sum_l \exp[i(\sigma_l^p+\sigma_l^t)]P_l(\cos\theta)(2l+1)\\ &\times\int dr_{1A}\,dr_{2A}\,dr_{Aa}\,d(\cos\beta)\,d(\cos\omega_A)\,d\gamma\,r_{1A}^2r_{2A}^2r_{Aa}^2\\ &\times P_{l_f}(\cos\omega_A)P_{l_i}(\cos\omega_b)P_l(\cos\omega_{if})v(r_{b1})\\ &\times u_{l_i}(r_{b1})u_{l_i}(r_{b2})u_{l_f}(r_{A1})u_{l_f}(r_{A2})f_l(r_{Bb})g_l(r_{Aa}).\end{aligned}\tag{7.2.89}$$

7.2.5 Coordinates used to derive (7.2.89)

We determine the relation between the integration variables in (7.2.87) and the coordinates needed to evaluate the quantities in the integrand. Noting that

$$\mathbf{r}_{Aa}=\frac{\mathbf{r}_{A1}+\mathbf{r}_{A2}+m_b\mathbf{r}_{Ab}}{m_b+2},\tag{7.2.90}$$

one has ~~We have~~

$$\mathbf{r}_{b1}=\mathbf{r}_{bA}+\mathbf{r}_{A1}=\frac{(m_b+1)\mathbf{r}_{A1}+\mathbf{r}_{A2}-(m_b+2)\mathbf{r}_{Aa}}{m_b},\tag{7.2.91}$$

$$\mathbf{r}_{b2}=\mathbf{r}_{bA}+\mathbf{r}_{A2}=\frac{(m_b+1)\mathbf{r}_{A2}+\mathbf{r}_{A1}-(m_b+2)\mathbf{r}_{Aa}}{m_b},\tag{7.2.92}$$

and

$$\begin{aligned}\mathbf{r}_{Cc}=\mathbf{r}_{CA}+\mathbf{r}_{A1}+\mathbf{r}_{1c}&=-\frac{1}{m_A+1}\mathbf{r}_{A2}+\mathbf{r}_{A1}-\frac{m_b}{m_b+1}\mathbf{r}_{b1}\\ &=\frac{m_b+2}{m_b+1}\mathbf{r}_{Aa}-\frac{m_b+2+m_A}{(m_b+1)(m_A+1)}\mathbf{r}_{A2}\end{aligned}\tag{7.2.93}$$

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~~Now, since~~ Since,

$$\mathbf{r}_{AB} = \frac{\mathbf{r}_{A1} + \mathbf{r}_{A2}}{m_A + 2}, \tag{7.2.94}$$

one obtains

$$\mathbf{r}_{Bb} = \mathbf{r}_{BA} + \mathbf{r}_{Ab} = \frac{m_b + 2}{m_b}\mathbf{r}_{Aa} - \frac{m_A + m_b + 2}{(m_A + 2)m_b}(\mathbf{r}_{A1} + \mathbf{r}_{A2}). \tag{7.2.95}$$

Using ~~We use~~ the same rotations those used as in Section 7.2.3 ~~to get~~ one gets,

$$\mathbf{r}_{A1} = r_{A1}\begin{bmatrix}\sin\alpha \\ 0 \\ \cos\alpha\end{bmatrix}, \tag{7.2.96}$$

and

$$\mathbf{r}_{A2} = r_{A2}\begin{bmatrix}-\cos\alpha\cos\gamma\sin\omega_A + \sin\alpha\cos\omega_A \\ -\sin\gamma\sin\omega_A \\ \sin\alpha\cos\gamma\sin\omega_A + \cos\alpha\cos\omega_A\end{bmatrix}, \tag{7.2.97}$$

with

$$\cos\alpha = \frac{r_{A1}^2 - d_1^2 + r_{Aa}^2}{2r_{A1}r_{Aa}}, \tag{7.2.98}$$

and

$$d_1 = \sqrt{r_{A1}^2 - r_{Aa}^2\sin^2\beta} - r_{Aa}\cos\beta. \tag{7.2.99}$$

Note that though β, r_{1A}, r_{Aa} are independent integration variables, they have to fulfill the condition

$$r_{Aa}\sin\beta \leq r_{A1}, \quad \text{for } 0 \leq \beta \leq \pi. \tag{7.2.100}$$

The expression of the ~~other~~ remaining quantities appearing in the integral (7.2.89) are ~~is~~ now straightforward, namely:

$$\begin{aligned} r_{b1} &= m_b^{-1}\,|(m_b+1)\mathbf{r}_{A1} + \mathbf{r}_{A2} - (m_b+2)\mathbf{r}_{Aa}| \\ &= m_b^{-1}\Big((m_b+2)^2 r_{Aa}^2 + (m_b+1)^2 r_{A1}^2 + r_{A2}^2 \\ &\quad - 2(m_b+2)(m_b+1)\mathbf{r}_{Aa}\,\mathbf{r}_{A1} - 2(m_b+2)\mathbf{r}_{Aa}\,\mathbf{r}_{A2} + 2(m_b+1)\mathbf{r}_{A1}\mathbf{r}_{A2}\Big)^{1/2}, \end{aligned} \tag{7.2.101}$$

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$$\begin{aligned} r_{b2} &= m_b^{-1}\,|(m_b+1)\mathbf{r}_{A2} + \mathbf{r}_{A1} - (m_b+2)\mathbf{r}_{Aa}| \\ &= m_b^{-1}\Big((m_b+2)^2 r_{Aa}^2 + (m_b+1)^2 r_{A2}^2 + r_{A1}^2 \\ &\quad - 2(m_b+2)(m_b+1)\mathbf{r}_{Aa}\,\mathbf{r}_{A2} - 2(m_b+2)\mathbf{r}_{Aa}\,\mathbf{r}_{A1} + 2(m_b+1)\mathbf{r}_{A2}\mathbf{r}_{A1}\Big)^{1/2}, \end{aligned} \tag{7.2.102}$$

$$\begin{aligned} r_{Bb} &= \left|\frac{m_b+2}{m_b}\mathbf{r}_{Aa} - \frac{m_A+m_b+2}{(m_A+2)m_b}(\mathbf{r}_{A1}+\mathbf{r}_{A2})\right| \\ &= \Bigg[\left(\frac{m_b+2}{m_b}\right)^2 r_{Aa}^2 + \left(\frac{m_A+m_b+2}{(m_A+2)m_b}\right)^2 (r_{A1}^2 + r_{A2}^2 + 2\mathbf{r}_{A1}\mathbf{r}_{A2}) \\ &\quad - 2\frac{(m_b+2)(m_A+m_b+2)}{(m_A+2)m_b^2}\mathbf{r}_{Aa}(\mathbf{r}_{A1}+\mathbf{r}_{A2})\Bigg]^{1/2}, \end{aligned} \tag{7.2.103}$$

$$\begin{aligned} r_{Cc} &= \left|\frac{m_b+2}{m_b+1}\mathbf{r}_{Aa} - \frac{m_b+2+m_A}{(m_b+1)(m_A+1)}\mathbf{r}_{A2}\right| \\ &= \Bigg[\left(\frac{m_a}{(m_a-1)}\right)^2 r_{Aa}^2 + \left(\frac{m_A+m_a}{(m_A+1)(m_a-1)}\right)^2 r_{A2}^2 \\ &\quad - 2\frac{m_A m_a + m_a^2}{(m_A+1)(m_a-1)^2}\mathbf{r}_{Aa}\mathbf{r}_{A2}\Bigg]^{1/2}, \end{aligned} \tag{7.2.104}$$

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$$\cos\omega_b = \frac{\mathbf{r}_{b1}\mathbf{r}_{b2}}{r_{b1}r_{b2}}, \tag{7.2.105}$$

$$\cos\omega_{if} = \frac{\mathbf{r}_{Aa}\mathbf{r}_{Bb}}{r_{Aa}r_{Bb}}, \tag{7.2.106}$$

with

Notación para el simultaneous T_{2N}^{1step} Eq. (7.2.44)

no sería mejor para el succ. T_{2N}^{2step}

$$\mathbf{r}_{Aa}\mathbf{r}_{A1} = r_{Aa}r_{A1}\cos\alpha, \tag{7.2.107}$$

$$\mathbf{r}_{Aa}\mathbf{r}_{A2} = r_{Aa}r_{A2}(\sin\alpha\cos\gamma\sin\omega_A + \cos\alpha\cos\omega_A), \tag{7.2.108}$$

$$\mathbf{r}_{A1}\mathbf{r}_{A2} = r_{A1}r_{A2}\cos\omega_A. \tag{7.2.109}$$

7.2.6 ~~s~~Successive transfer

~~Note that we use time-reversed phases for the spherical harmonics (see (8.1.1)) throughout. We write~~ The successive two-nucleon ~~transition amplitude~~ transfer amplitude can be written as (~~see~~ Bayman and Chen (1982)):

$$\begin{aligned}
T_{2NT}^{VV} = \frac{4\mu_{Cc}}{\hbar^2} \sum_{\substack{\sigma_1\sigma_2\\ \sigma_1'\sigma_2'\\ KM}} \int & d^3r_{Cc}d^3r_{b1}d^3r_{A2}d^3r'_{Cc}d^3r'_{b1}d^3r'_{A2}\,\chi^{(-)*}(\mathbf{k}_{Bb},\mathbf{r}_{Bb}) \\
&\times \left[\psi^{j_f}(\mathbf{r}_{A1},\sigma_1)\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\right]_0^{0*} v(r_{b1}) \left[\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\psi^{j_i}(\mathbf{r}_{b1},\sigma_1)\right]_M^K \\
&\times G(\mathbf{r}_{Cc},\mathbf{r}'_{Cc}) \left[\psi^{j_f}(\mathbf{r}'_{A2},\sigma'_2)\psi^{j_i}(\mathbf{r}'_{b1},\sigma'_1)\right]_M^{K*} v(r'_{c2}) \\
&\times \left[\psi^{j_i}(\mathbf{r}'_{b1},\sigma'_1)\psi^{j_i}(\mathbf{r}'_{b2},\sigma'_2)\right]_0^0 \chi^{(+)}(\mathbf{r}'_{Aa})
\end{aligned} \tag{7.2.110}$$

~~Expansion~~ of the Green function and the distorted waves in a basis of angular momentum eigenstates one can write,

Expanding

$$\chi^{(-)*}(\mathbf{k}_{Bb},\mathbf{r}_{Bb}) = \sum_{\tilde l} \frac{4\pi}{k_{Bb}r_{Bb}} i^{-\tilde l} e^{i\sigma_f^l} F_{\tilde l} \sum_m Y_m^{\tilde l}(\hat r_{Bb}) Y_m^{\tilde l *}(\hat k_{Bb}), \tag{7.2.111}$$

~~but~~ the sum over m ~~is~~ being

$$\sum_m (-1)^{\tilde l - m} Y_m^{\tilde l}(\hat r_{Bb}) Y_{-m}^{\tilde l}(\hat k_{Bb}) = \sqrt{2\tilde l + 1}\left[Y^{\tilde l}(\hat r_{Bb}) Y^{\tilde l}(\hat k_{Bb})\right]_0^0, \tag{7.2.112}$$

appendix Eq. (221) (237) gregory check

where we have used (8.1.1) and (8.1.2), so

$$\chi^{(-)*}(\mathbf{k}_{Bb},\mathbf{r}_{Bb}) = \sum_{\tilde l} \sqrt{2\tilde l + 1}\frac{4\pi}{k_{Bb}r_{Bb}} i^{-\tilde l} e^{i\sigma_f^l} F_{\tilde l}(r_{Bb}) \left[Y^{\tilde l}(\hat r_{Bb}) Y^{\tilde l}(\hat k_{Bb})\right]_0^0 \tag{7.2.113}$$

Similarly~~,~~,

$$\chi^{(+)}(\mathbf{r}'_{Aa}) = \sum_l i^l \sqrt{2l+1}\frac{4\pi}{k_{Aa}r'_{Aa}} e^{i\sigma_i^l} F_l(r'_{Aa}) \left[Y^l(\hat r'_{Aa}) Y^l(\hat k_{Aa})\right]_0^0, \tag{7.2.114}$$

where we have taken into account the choice ~~that~~ $k_{Aa} \equiv \hat z$. ~~And~~ The Green function ~~is~~ can be written as

$$G(\mathbf{r}_{Cc},\mathbf{r}'_{Cc}) = i\sum_{l_c} \sqrt{2l_c+1}\frac{f_{l_c}(k_{Cc},r_<)P_{l_c}(k_{Cc},r_>)}{k_{Cc}r_{Cc}r'_{Cc}} \left[Y^{l_c}(\hat r_{Cc}) Y^{l_c}(\hat r'_{Cc})\right]_0^0. \tag{7.2.115}$$

It is of notice ~~that~~ the time-reversal phase convention is used throughout.

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Finally

$$
\begin{aligned}
T^{VV}_{2NT} = & \frac{4\mu_{Cc}(4\pi)^2 i}{\hbar^2 k_{Aa}k_{Bb}k_{Cc}} \sum_{l,l_c,\bar{l}} e^{i(\sigma_l^i+\sigma_f^l)} i^{l-\bar{l}} \sqrt{(2l+1)(2l_c+1)(2\bar{l}+1)} \\
& \times \sum_{\substack{\sigma_1\sigma_2\\ \sigma_1'\sigma_2'}} \int d^3r_{Cc} d^3r_{b1} d^3r_{A2} d^3r'_{Cc} d^3r'_{b1} d^3r'_{A2} v(r_{b1}) v(r'_{c2}) \left[Y^{\bar{l}}(\hat{r}_{Bb}) Y^{\bar{l}}(\hat{k}_{Bb})\right]^0_0 \\
& \times \left[Y^{l}(\hat{r}'_{Aa}) Y^{l}(\hat{k}'_{Aa})\right]^0_0 \left[Y^{l_c}(\hat{r}_{Cc}) Y^{l_c}(\hat{r}'_{Cc})\right]^0_0 \frac{F_{\bar{l}}(r_{Bb})}{r_{Bb}} \frac{F_l(r'_{Aa})}{r'_{Aa}} \\
& \times \frac{f_{l_c}(k_{Cc}, r_<) P_{l_c}(k_{Cc}, r_>)}{r_{Cc} r'_{Cc}} \left[\psi^{j_f}(\mathbf{r}_{A1}, \sigma_1)\psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)\right]^{0*}_0 \\
& \times \left[\psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)\psi^{j_i}(\mathbf{r}'_{b2}, \sigma'_2)\right]^0_0 \sum_{KM} \left[\psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)\psi^{j_i}(\mathbf{r}_{b1}, \sigma_1)\right]^K_M \\
& \times \left[\psi^{j_f}(\mathbf{r}'_{A2}, \sigma'_2)\psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)\right]^{K*}_M \quad o
\end{aligned}
\tag{7.2.116}
$$

Let us now perform the integration over $\mathbf{r}_{A2}$)

$$
\begin{aligned}
& \sum_{\sigma_1,\sigma_2} \int d\mathbf{r}_{A2} \left[\psi^{j_f}(\mathbf{r}_{A1}, \sigma_1)\psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)\right]^{0*}_0 \left[\psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)\psi^{j_i}(\mathbf{r}_{b1}, \sigma_1)\right]^K_M \\
& = \sum_{\sigma_1,\sigma_2} (-1)^{1/2-\sigma_1+1/2-\sigma_2} \int d\mathbf{r}_{A2} \left[\psi^{j_f}(\mathbf{r}_{A1}, -\sigma_1)\psi^{j_f}(\mathbf{r}_{A2}, -\sigma_2)\right]^0_0 \left[\psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)\psi^{j_i}(\mathbf{r}_{b1}, \sigma_1)\right]^K_M \\
& = -\sum_{\sigma_1,\sigma_2} (-1)^{1/2-\sigma_1+1/2-\sigma_2} \int d\mathbf{r}_{A2} \left[\psi^{j_f}(\mathbf{r}_{A2}, -\sigma_2)\psi^{j_f}(\mathbf{r}_{A1}, -\sigma_1)\right]^0_0 \left[\psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)\psi^{j_i}(\mathbf{r}_{b1}, \sigma_1)\right]^K_M \\
& = -((j_f j_f)_0 (j_f j_i)_K | (j_f j_f)_0 (j_f j_i)_K)_K \sum_{\sigma_1,\sigma_2} (-1)^{1/2-\sigma_1+1/2-\sigma_2} \\
& \times \int d\mathbf{r}_{A2} \left[\psi^{j_f}(\mathbf{r}_{A2}, -\sigma_2)\psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)\right]^0_0 \left[\psi^{j_f}(\mathbf{r}_{A1}, -\sigma_1)\psi^{j_i}(\mathbf{r}_{b1}, \sigma_1)\right]^K_M \\
& = \frac{1}{2j_f+1} \sqrt{2j_f+1} ((l_f \tfrac{1}{2})_{j_f} (l_i \tfrac{1}{2})_{j_i} | (l_f l_i)_K (\tfrac{1}{2}\tfrac{1}{2})_0)_K \\
& \times u_{l_f}(r_{A1}) u_{l_i}(r_{b1}) \left[Y^{l_f}(\hat{r}_{A1}) Y^{l_i}(\hat{r}_{b1})\right]^K_M \sum_{\sigma_1} (-1)^{1/2-\sigma_1} \left[\chi^{1/2}(-\sigma_1)\chi^{1/2}(\sigma_1)\right]^0_0 \\
& = -\sqrt{\frac{2}{2j_f+1}} ((l_f \tfrac{1}{2})_{j_f} (l_i \tfrac{1}{2})_{j_i} | (l_f l_i)_K (\tfrac{1}{2}\tfrac{1}{2})_0)_K \left[Y^{l_f}(\hat{r}_{A1}) Y^{l_i}(\hat{r}_{b1})\right]^K_M u_{l_f}(r_{A1}) u_{l_i}(r_{b1}),
\end{aligned}
\tag{7.2.117}
$$

where we have evaluated the 9j-symbol

$$
((j_f j_f)_0 (j_f j_i)_K | (j_f j_f)_0 (j_f j_i)_K)_K = \frac{1}{2j_f+1}, \tag{7.2.118}
$$

gregory (2387 app)

 (22)

as well as ~~and have also used~~ (9?). We proceed in a similar way to evaluate the integral over $\mathbf{r}'_{b1}$,

$$
\begin{aligned}
&\sum_{\sigma'_1,\sigma'_2} \int d\mathbf{r}'_{b1} \left[\psi^{j_i}(\mathbf{r}'_{b1},\sigma'_1)\psi^{j_i}(\mathbf{r}'_{b2},\sigma'_2)\right]^0_0 \left[\psi^{j_f}(\mathbf{r}'_{A2},\sigma'_2)\psi^{j_i}(\mathbf{r}'_{b1},\sigma'_1)\right]^{K*}_M \\
&= -(-1)^{K-M} \sum_{\sigma'_1,\sigma'_2} \int d\mathbf{r}'_{b1} \left[\psi^{j_f}(\mathbf{r}'_{A2},-\sigma'_2)\psi^{j_i}(\mathbf{r}'_{b1},-\sigma'_1)\right]^K_{-M} \\
&\times \left[\psi^{j_i}(\mathbf{r}'_{b2},\sigma'_2)\psi^{j_i}(\mathbf{r}'_{b1},\sigma'_1)\right]^0_0 (-1)^{1/2-\sigma'_1+1/2-\sigma'_2} \\
&= -(-1)^{K-M}((j_f j_i)_K(j_i j_i)_0|(j_f j_i)_K(j_i j_i)_0)_K(-\sqrt{2j_i+1}) \\
&\times ((l_f \tfrac{1}{2})_{j_f}(l_i \tfrac{1}{2})_{j_i}|(l_f l_i)_K(\tfrac{1}{2}\tfrac{1}{2})_0)_K(-\sqrt{2})u_{l_f}(r'_{A2})u_{l_i}(r'_{b2})\left[Y^{l_f}(\hat{r}'_{A2})Y^{l_i}(\hat{r}'_{b2})\right]^K_{-M} \\
&= -\sqrt{\frac{2}{2j_i+1}}((l_f \tfrac{1}{2})_{j_f}(l_i \tfrac{1}{2})_{j_i}|(l_f l_i)_K(\tfrac{1}{2}\tfrac{1}{2})_0)_K \left[Y^{l_f}(\hat{r}'_{A2})Y^{l_i}(\hat{r}'_{b2})\right]^{K*}_M u_{l_f}(r'_{A2})u_{l_i}(r'_{b2}).
\end{aligned}
\tag{7.2.119}
$$

~~Putting all together~~ Setting the different elements together one obtains

$$
\begin{aligned}
T^{VV}_{2NT} &= \frac{4\mu_{Cc}(4\pi)^2 i}{\hbar^2 k_{Aa}k_{Bb}k_{Cc}} \frac{2}{\sqrt{(2j_i+1)(2j_f+1)}} \sum_{K,M}((l_f \tfrac{1}{2})_{j_f}(l_i \tfrac{1}{2})_{j_i}|(l_f l_i)_K(\tfrac{1}{2}\tfrac{1}{2})_0)^2_K \\
&\times \sum_{l_c,l,\tilde{l}} e^{i(\sigma^l_i+\sigma^l_f)} \sqrt{(2l_c+1)(2l+1)(2\tilde{l}+1)}\, i^{l-\tilde{l}} \\
&\times \int d^3r_{Cc}d^3r_{b1}d^3r'_{Cc}d^3r'_{A2}v(r_{b1})v(r'_{c2})u_{l_f}(r_{A1})u_{l_i}(r_{b1})u_{l_f}(r'_{A2})u_{l_i}(r'_{b2}) \\
&\times \left[Y^{l_f}(\hat{r}'_{A2})Y^{l_i}(\hat{r}'_{b2})\right]^{K*}_M \left[Y^{l_f}(\hat{r}_{A1})Y^{l_i}(\hat{r}_{b1})\right]^K_M \frac{F_l(r'_{Aa})F_{\tilde{l}}(r'_{Bb})f_{l_c}(k_{Cc},r_<)P_{l_c}(k_{Cc},r_>)}{r'_{Aa}r_{Bb}r_{Cc}r'_{Cc}} \\
&\times \left[Y^{\tilde{l}}(\hat{r}_{Bb})Y^{\tilde{l}}(\hat{k}_{Bb})\right]^0_0 \left[Y^l(\hat{r}'_{Aa})Y^l(\hat{k}_{Aa})\right]^0_0 \left[Y^{l_c}(\hat{r}_{Cc})Y^{l_c}(\hat{r}'_{Cc})\right]^0_0 .
\end{aligned}
\tag{7.2.120}
$$

~~We can write~~ For this purpose one writes

$$
\begin{aligned}
&\left[Y^{\tilde{l}}(\hat{r}_{Bb})Y^{\tilde{l}}(\hat{k}_{Bb})\right]^0_0 \left[Y^l(\hat{r}'_{Aa})Y^l(\hat{k}_{Aa})\right]^0_0 = \\
&\qquad ((l\,l)_0(\tilde{l}\,\tilde{l})_0|(l\,\tilde{l})_0(l\,\tilde{l})_0)_0 \left[Y^{\tilde{l}}(\hat{r}_{Bb})Y^l(\hat{r}'_{Aa})\right]^0_0 \left[Y^{\tilde{l}}(\hat{k}_{Bb})Y^l(\hat{k}_{Aa})\right]^0_0 \\
&\qquad = \frac{\delta_{\tilde{l}l}}{2l+1} \left[Y^l(\hat{r}_{Bb})Y^l(\hat{r}'_{Aa})\right]^0_0 \left[Y^l(\hat{k}_{Bb})Y^l(\hat{k}_{Aa})\right]^0_0 .
\end{aligned}
\tag{7.2.121}
$$

Taking into account ~~that~~ the relation

$$
\left[Y^l(\hat{k}_{Bb})Y^l(\hat{k}_{Aa})\right]^0_0 = \frac{(-1)^l}{\sqrt{4\pi}} Y^l_0(\hat{k}_{Bb})i^l, \tag{7.2.122}
$$

We now proceed to write the above expression in a compact way.

more

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and

$$\left[Y^{l}(\hat{r}_{Bb})Y^{l}(\hat{r}'_{Aa})\right]^{0}_{0}\left[Y^{l_c}(\hat{r}_{Cc})Y^{l_c}(\hat{r}'_{Cc})\right]^{0}_{0} =$$
$$((l\,l)_0(l_c\,l_c)_0|(l\,l_c)_K(l\,l_c)_K)_0\left\{\left[Y^{l}(\hat{r}_{Bb})Y^{l_c}(\hat{r}_{Cc})\right]^{K}\left[Y^{l}(\hat{r}'_{Aa})Y^{l_c}(\hat{r}'_{Cc})\right]^{K}\right\}^{0}_{0}$$
$$= \sqrt{\frac{2K+1}{(2l+1)(2l_c+1)}}$$
$$\times\sum_{M'}\frac{(-1)^{K+M'}}{\sqrt{2K+1}}\left[Y^{l}(\hat{r}_{Bb})Y^{l_c}(\hat{r}_{Cc})\right]^{K}_{-M'}\left[Y^{l}(\hat{r}'_{Aa})Y^{l_c}(\hat{r}'_{Cc})\right]^{K}_{M'}$$
$$= \sqrt{\frac{1}{(2l+1)(2l_c+1)}}$$
$$\times\sum_{M'}\left[Y^{l}(\hat{r}_{Bb})Y^{l_c}(\hat{r}_{Cc})\right]^{K*}_{M'}\left[Y^{l}(\hat{r}'_{Aa})Y^{l_c}(\hat{r}'_{Cc})\right]^{K}_{M'}. \tag{7.2.123}$$

It is of notice ~~It is important to noto~~ that the integrals

$$\int d\hat{r}_{Cc}d\hat{r}_{b1}\left[Y^{l}(\hat{r}_{Bb})Y^{l_c}(\hat{r}_{Cc})\right]^{K*}_{M}\left[Y^{l_f}(\hat{r}_{A1})Y^{l_i}(\hat{r}_{b1})\right]^{K}_{M}, \tag{7.2.124}$$

and

$$\int d\hat{r}'_{Cc}d\hat{r}'_{A2}\left[Y^{l}(\hat{r}'_{Aa})Y^{l_c}(\hat{r}'_{Cc})\right]^{K}_{M}\left[Y^{l_f}(\hat{r}'_{A2})Y^{l_i}(\hat{r}'_{b2})\right]^{K*}_{M}, \tag{7.2.125}$$

over the angular variables do not depend on M. Let us see why this is so with the help of (7.2.124),

$$\left[Y^{l}(\hat{r}_{Bb})Y^{l_c}(\hat{r}_{Cc})\right]^{K*}_{M}\left[Y^{l_f}(\hat{r}_{A1})Y^{l_i}(\hat{r}_{b1})\right]^{K}_{M} = (-1)^{K-M}\left[Y^{l}(\hat{r}_{Bb})Y^{l_c}(\hat{r}_{Cc})\right]^{K}_{-M}$$
$$\times\left[Y^{l_f}(\hat{r}_{A1})Y^{l_i}(\hat{r}_{b1})\right]^{K}_{M} = (-1)^{K-M}\sum_{J}\langle K\;K\;M\;-M|J\;0\rangle \tag{7.2.126}$$
$$\times\left\{\left[Y^{l}(\hat{r}_{Bb})Y^{l_c}(\hat{r}_{Cc})\right]^{K}\left[Y^{l_f}(\hat{r}_{A1})Y^{l_i}(\hat{r}_{b1})\right]^{K}\right\}^{J}_{0}.$$

After integration, only the term

$$(-1)^{K-M}\langle K\;K\;M\;-M|0\;0\rangle\left\{\left[Y^{l}(\hat{r}_{Bb})Y^{l_c}(\hat{r}_{Cc})\right]^{K}\left[Y^{l_f}(\hat{r}_{A1})Y^{l_i}(\hat{r}_{b1})\right]^{K}\right\}^{0}_{0} = .$$
$$\frac{1}{\sqrt{2K+1}}\left\{\left[Y^{l}(\hat{r}_{Bb})Y^{l_c}(\hat{r}_{Cc})\right]^{K}\left[Y^{l_f}(\hat{r}_{A1})Y^{l_i}(\hat{r}_{b1})\right]^{K}\right\}^{0}_{0} \tag{7.2.127}$$

corresponding to $J = 0$ survives, which is indeed independent of M. We can thus omit the sum over M, and multiply by $(2K+1)$, obtaining

$$T^{VV}_{2NT} = \frac{64\mu_{Cc}(\pi)^{3/2}i}{\hbar^2 k_{Aa}k_{Bb}k_{Cc}}\frac{i^{-l}}{\sqrt{(2j_i+1)(2j_f+1)}}$$
$$\times\sum_{K}(2K+1)((l_f\tfrac{1}{2})_{j_f}(l_i\tfrac{1}{2})_{j_i}|(l_f l_i)_K(\tfrac{1}{2}\tfrac{1}{2})_0)^2_K$$
$$\times\sum_{l_c,l}\frac{e^{i(\sigma^l_i+\sigma^l_f)}}{\sqrt{(2l+1)}}Y^{l}_{0}(\hat{k}_{Bb})S_{K,l,l_c}, \tag{7.2.128}$$

in (7.2.120)

notation

~~with~~ where

$$S_{K,l,l_c} = \int d^3r_{Cc}d^3r_{b1}v(r_{b1})u_{l_f}(r_{A1})u_{l_i}(r_{b1})\frac{s_{K,l,l_c}(r_{Cc})}{r_{Cc}}\frac{F_l(r_{Bb})}{r_{Bb}} \times \left[Y^{l_f}(\hat{r}_{A1})Y^{l_i}(\hat{r}_{b1})\right]^K_M \left[Y^{l_c}(\hat{r}_{Cc})Y^{l}(\hat{r}_{Bb})\right]^{K*}_M, \tag{7.2.129}$$

and

$$s_{K,l,l_c}(r_{Cc}) = \int_{r_{Cc}fixed} d^3r'_{Cc}d^3r'_{A2}v(r'_{c2})u_{l_f}(r'_{A2})u_{l_i}(r'_{b2})\frac{F_l(r'_{Aa})}{r'_{Aa}}\frac{f_{l_c}(k_{Cc},r_<)P_{l_c}(k_{Cc},r_>)}{r'_{Cc}} \times \left[Y^{l_f}(\hat{r}'_{A2})Y^{l_i}(\hat{r}'_{b2})\right]^{K*}_M \left[Y^{l_c}(\hat{r}'_{Cc})Y^{l}(\hat{r}'_{Aa})\right]^{K}_M. \tag{7.2.130}$$

~~The~~ It can be shown that the integrand in (7.2.129) ~~can easily seen to be~~ is independent of M, ~~so we~~ consequently, one can sum over M and divide by $(2K+1)$, to get ~~the integrand~~

$$\frac{1}{2K+1}v(r_{b1})u_{l_f}(r_{A1})u_{l_i}(r_{b1})\frac{s_{K,l,l_c}(r_{Cc})}{r_{Cc}}\frac{F_l(r_{Bb})}{r_{Bb}} \times \sum_M \left[Y^{l_f}(\hat{r}_{A1})Y^{l_i}(\hat{r}_{b1})\right]^K_M \left[Y^{l_c}(\hat{r}_{Cc})Y^{l}(\hat{r}_{Bb})\right]^{K*}_M. \tag{7.2.131}$$

This integrand is rotationally invariant (it is proportional to a T^L_M spherical tensor with $L=0$, $M=0$), so ~~we~~ one can ~~just~~ evaluate it in the "standard" configuration in which $\mathbf{r}_{Cc}$ is directed along the z–axis and multiply by $8\pi^2$ (see Bayman and Chen (1982)), obtaining the final expression for S_{K,l,l_c}:

$$\begin{aligned} S_{K,l,l_c} =& \frac{4\pi^{3/2}\sqrt{2l_c+1}}{2K+1}i^{-l_c} \\ &\times \int r^2_{Cc}\,dr_{Cc}\,r^2_{b1}\,dr_{b1}\,\sin\theta\,d\theta\,v(r_{b1})u_{l_f}(r_{A1})u_{l_i}(r_{b1}) \\ &\times \frac{s_{K,l,l_c}(r_{Cc})}{r_{Cc}}\frac{F_l(r_{Bb})}{r_{Bb}} \\ &\times \sum_M \langle l_c\;0\;l\;M|K\;M\rangle \left[Y^{l_f}(\hat{r}_{A1})Y^{l_i}(\theta+\pi,0)\right]^K_M Y^{l*}_M(\hat{r}_{Bb}). \end{aligned} \tag{7.2.132}$$

Similarly, ~~we have~~ one has

$$\begin{aligned} s_{K,l,l_c}(r_{Cc}) =& \frac{4\pi^{3/2}\sqrt{2l_c+1}}{2K+1}i^{l_c} \\ &\times \int r'^2_{Cc}\,dr'_{Cc}\,r'^2_{A2}\,dr'_{A2}\,\sin\theta'\,d\theta'\,v(r'_{c2})u_{l_f}(r'_{A2})u_{l_i}(r'_{b2}) \\ &\times \frac{F_l(r'_{Aa})}{r'_{Aa}}\frac{f_{l_c}(k_{Cc},r_<)P_{l_c}(k_{Cc},r_>)}{r'_{Cc}} \\ &\times \sum_M \langle l_c\;0\;l\;M|K\;M\rangle \left[Y^{l_f}(\hat{r}'_{A2})Y^{l_i}(\hat{r}'_{b2})\right]^{K*}_M Y^{l}_M(\hat{r}'_{Aa}). \end{aligned} \tag{7.2.133}$$

~~If we do the further~~ Introducing the approximations $\mathbf{r}_{A1} \approx \mathbf{r}_{C1}$ and $\mathbf{r}_{b2} \approx \mathbf{r}_{c2}$, ~~we obtain~~ one obtains the final

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expression

notation

$$
\begin{aligned}
T_{2NT}^{VV} = & \frac{1024\mu_{Cc}\pi^{9/2} i}{\hbar^2 k_{Aa} k_{Bb} k_{Cc}} \frac{1}{\sqrt{(2j_i+1)(2j_f+1)}} \\
& \times \sum_K \frac{1}{2K+1} ((l_f \tfrac{1}{2})_{j_f} (l_i \tfrac{1}{2})_{j_i} | (l_f l_i)_K (\tfrac{1}{2}\tfrac{1}{2})_0)_K^2 \\
& \times \sum_{l_c, l} e^{i(\sigma_i^l + \sigma_f^l)} \frac{(2l_c+1)}{\sqrt{2l+1}} Y_0^l(\hat{k}_{Bb}) S_{K,l,l_c},
\end{aligned}
\tag{7.2.134}
$$

with

$$
\begin{aligned}
S_{K,l,l_c} = & \int r_{Cc}^2\, dr_{Cc}\, r_{b1}^2\, dr_{b1}\, \sin\theta\, d\theta\, v(r_{b1}) u_{l_f}(r_{C1}) u_{l_i}(r_{b1}) \\
& \times \frac{s_{K,l,l_c}(r_{Cc})}{r_{Cc}} \frac{F_l(r_{Bb})}{r_{Bb}} \\
& \times \sum_M \langle l_c\ 0\ l\ M | K\ M \rangle \left[Y^{l_f}(\hat{r}_{C1}) Y^{l_i}(\theta+\pi, 0) \right]_M^K Y_M^{l*}(\hat{r}_{Bb}),
\end{aligned}
\tag{7.2.135}
$$

and

$$
\begin{aligned}
s_{K,l,l_c}(r_{Cc}) = & \int r_{Cc}'^2\, dr_{Cc}'\, r_{A2}'^2\, dr_{A2}'\, \sin\theta'\, d\theta'\, v(r_{c2}') u_{l_f}(r_{A2}') u_{l_i}(r_{c2}') \\
& \times \frac{F_l(r_{Aa}')}{r_{Aa}'} \frac{f_{l_c}(k_{Cc}, r_<) P_{l_c}(k_{Cc}, r_>)}{r_{Cc}'} \\
& \times \sum_M \langle l_c\ 0\ l\ M | K\ M \rangle \left[Y^{l_f}(\hat{r}_{A2}') Y^{l_i}(\hat{r}_{c2}') \right]_M^{K*} Y_M^l(\hat{r}_{Aa}').
\end{aligned}
\tag{7.2.136}
$$

7.2.7 Coordinates for the successive transfer

In the standard configuration in which the integrals (7.2.135) and (7.2.136) are to be evaluated, we have

$$
\mathbf{r}_{Cc} = r_{Cc}\,\hat{\mathbf{z}}, \qquad \mathbf{r}_{b1} = r_{b1}(-\cos\theta\,\hat{\mathbf{z}} - \sin\theta\,\hat{\mathbf{x}}). \tag{7.2.137}
$$

Now,

$$
\begin{aligned}
\mathbf{r}_{C1} &= \mathbf{r}_{Cc} + \mathbf{r}_{c1} = \mathbf{r}_{Cc} + \frac{m_b}{m_b+1}\mathbf{r}_{b1} \\
&= \left(r_{Cc} - \frac{m_b}{m_b+1} r_{b1}\cos\theta \right)\hat{\mathbf{z}} - \frac{m_b}{m_b+1} r_{b1}\sin\theta\hat{\mathbf{x}},
\end{aligned}
\tag{7.2.138}
$$

and

$$
\mathbf{r}_{Bb} = \mathbf{r}_{BC} + \mathbf{r}_{Cb} = -\frac{1}{m_B}\mathbf{r}_{C1} + \mathbf{r}_{Cb}. \tag{7.2.139}
$$

Substituting the relation

$$
\mathbf{r}_{Cb} = \mathbf{r}_{Cc} + \mathbf{r}_{cb} = \mathbf{r}_{Cc} - \frac{1}{m_b+1}\mathbf{r}_{b1}, \tag{7.2.140}
$$

~~eine~~ in (7.2.139) one gets

$$
\mathbf{r}_{Bb} = \left(\frac{m_B - 1}{m_B} r_{Cc} + \frac{m_b + m_B}{m_B(m_b+1)} r_{b1}\cos\theta \right)\hat{\mathbf{z}} + \frac{m_b + m_B}{m_B(m_b+1)} r_{b1}\sin\theta\hat{\mathbf{x}}. \tag{7.2.141}
$$

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The primed variables are arranged in a similar fashion,

$$\mathbf{r}'_{Cc} = r'_{Cc}\,\hat{\mathbf{z}}, \qquad \mathbf{r}'_{A2} = r'_{A2}(-\cos\theta'\,\hat{\mathbf{z}} - \sin\theta'\,\hat{\mathbf{x}}). \tag{7.2.142}$$

Thus,

$$\mathbf{r}'_{c2} = \left(-r'_{Cc} - \frac{m_A}{m_A+1} r'_{A2}\cos\theta'\right)\hat{\mathbf{z}} - \frac{m_A}{m_A+1} r'_{A2}\sin\theta'\hat{\mathbf{x}}, \tag{7.2.143}$$

and

$$\mathbf{r}'_{Aa} = \left(\frac{m_a-1}{m_a} r'_{Cc} - \frac{m_A+m_a}{m_a(m_A+1)} r'_{A2}\cos\theta'\right)\hat{\mathbf{z}} - \frac{m_A+m_a}{m_a(m_A+1)} r'_{A2}\sin\theta'\hat{\mathbf{x}}. \tag{7.2.144}$$

7.2.8 Simplifying the vector coupling

We will now turn our attention to the vector–coupled quantities in (7.2.135) and (7.2.136),

$$\sum_M \langle l_c\ 0\ l\ M|K\ M\rangle \left[Y^{l_f}(\hat{r}_{C1})Y^{l_i}(\theta+\pi,0)\right]^K_M Y^{l*}_M(\hat{r}_{Bb}), \tag{7.2.145}$$

and

$$\sum_M \langle l_c\ 0\ l\ M|K\ M\rangle \left[Y^{l_f}(\hat{r}'_{A2})Y^{l_i}(\hat{r}'_{c2})\right]^{K*}_M Y^{l}_M(\hat{r}'_{Aa}). \tag{7.2.146}$$

We can express them both as

$$\sum_M f(M), \tag{7.2.147}$$

where e.g. in the case of (7.2.145), ~~we have~~ one has

$$f(M) = \langle l_c\ 0\ l\ M|K\ M\rangle \left[Y^{l_f}(\hat{r}_{C1})Y^{l_i}(\theta+\pi,0)\right]^K_M Y^{l*}_M(\hat{r}_{Bb}). \tag{7.2.148}$$

Note that all the vectors that come into play in the above expressions are in the (x,z)–plane. Consequently, the azimuthal angle ϕ is always equal to zero. Under these circumstances and for time–reversed phases, $(Y^{L*}_M(\theta,0) = (-1)^L Y^L_M(\theta,0))$ one has

$$f(-M) = (-1)^{l_c+l_f+l_i+l} f(M). \tag{7.2.149}$$

Consequently,

$$\begin{aligned}\sum_M \langle l_c\ 0\ l\ M|K\ M\rangle f(M) &= \langle l_c\ 0\ l\ 0|K\ 0\rangle f(0) \\ &+ \sum_{M>0} \langle l_c\ 0\ l\ M|K\ M\rangle f(M)\left(1+(-1)^{l_c+l+l_i+l_f}\right).\end{aligned} \tag{7.2.150}$$

Consequently, in the case in which $l_c + l + l_i + l_f$ is odd, we have only to evaluate the $M = 0$ contribution. This consideration is useful to restrict the number of numerical operations needed to calculate the transition amplitude.

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7.2.9 non–orthogonality term

We write the non–orthogonality contribution to the transition amplitude (see Bayman and Chen (1982)):

$$\begin{aligned} T^{NO}_{2NT} =& 2\sum_{\substack{\sigma_1\sigma_2\\ \sigma_1'\sigma_2'\\ KM}} \int d^3r_{Cc}d^3r_{b1}d^3r_{A2}d^3r'_{b1}d^3r'_{A2}\chi^{(-)*}(\mathbf{k}_{Bb},\mathbf{r}_{Bb}) \\ &\times \left[\psi^{j_f}(\mathbf{r}_{A1},\sigma_1)\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\right]^{0*}_0 v(r_{b1})\left[\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\psi^{j_i}(\mathbf{r}_{b1},\sigma_1)\right]^K_M \\ &\times \left[\psi^{j_f}(\mathbf{r}'_{A2},\sigma'_2)\psi^{j_i}(\mathbf{r}'_{b1},\sigma'_1)\right]^{K*}_M \left[\psi^{j_i}(\mathbf{r}'_{b1},\sigma'_1)\psi^{j_i}(\mathbf{r}'_{b2},\sigma'_2)\right]^0_0\chi^{(+)}(\mathbf{r}'_{Aa}). \end{aligned} \tag{7.2.151}$$

This expression is equivalent to (7.2.110) if we make the replacement

$$\frac{2\mu_{Cc}}{\hbar^2}G(\mathbf{r}_{Cc},\mathbf{r}'_{Cc})v(r'_{A2}) \to \delta(\mathbf{r}_{Cc}-\mathbf{r}'_{Cc}). \tag{7.2.152}$$

Looking at the partial–wave expansions of $G(\mathbf{r}_{Cc},\mathbf{r}'_{Cc})$ and $\delta(\mathbf{r}_{Cc}-\mathbf{r}'_{Cc})$ (see Section ??), we find that we can use the above expressions for the successive transfer with the replacement

$$i\frac{2\mu_{Cc}}{\hbar^2}\frac{f_{l_c}(k_{Cc},r_<)P_{l_c}(k_{Cc},r_>)}{k_{Cc}} \to \delta(r_{Cc}-r'_{Cc}). \tag{7.2.153}$$

We thus have

$$\begin{aligned} T^{NO}_{2NT} =& \frac{512\pi^{9/2}}{k_{Aa}k_{Bb}}\frac{1}{\sqrt{(2j_i+1)(2j_f+1)}} \\ &\times\sum_K((l_f\tfrac{1}{2})_{j_f}(l_i\tfrac{1}{2})_{j_i}|(l_fl_i)_K(\tfrac{1}{2}\tfrac{1}{2})_0)^2_K \\ &\times\sum_{l_c,l} e^{i(\sigma_i^l+\sigma_f^l)}\frac{(2l_c+1)}{\sqrt{2l+1}}Y^l_0(\hat{k}_{Bb})S_{K,l,l_c}, \end{aligned} \tag{7.2.154}$$

with

$$\begin{aligned} S_{K,l,l_c} =& \int r^2_{Cc}\,dr_{Cc}\,r^2_{b1}\,dr_{b1}\,\sin\theta\,d\theta\,v(r_{b1})u_{l_f}(r_{C1})u_{l_i}(r_{b1}) \\ &\times\frac{s_{K,l,l_c}(r_{Cc})}{r_{Cc}}\frac{F_l(r_{Bb})}{r_{Bb}} \\ &\times\sum_M\langle l_c\;0\;l\;M|K\;M\rangle\left[Y^{l_f}(\hat{r}_{C1})Y^{l_i}(\theta+\pi,0)\right]^K_M Y^{l*}_M(\hat{r}_{Bb}), \end{aligned} \tag{7.2.155}$$

and

$$\begin{aligned} s_{K,l,l_c}(r_{Cc}) =& r_{Cc}\int dr'_{A2}\,r'^2_{A2}\,\sin\theta'\,d\theta'\,u_{l_f}(r'_{A2})u_{l_i}(r'_{c2})\frac{F_l(r'_{Aa})}{r'_{Aa}} \\ &\times\sum_M\langle l_c\;0\;l\;M|K\;M\rangle\left[Y^{l_f}(\hat{r}'_{A2})Y^{l_i}(\hat{r}'_{c2})\right]^{K*}_M Y^l_M(\hat{r}'_{Aa}). \end{aligned} \tag{7.2.156}$$

7.2.10 Arbitrary orbital momentum transfer

We will now examine the case in which the two transferred nucleons carry an angular momentum Λ different from 0. Let us assume that two nucleons coupled to angular

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momentum Λ in the initial nucleus a are transferred into a final state of zero angular momentum in nucleus B. The transition amplitude is given by the integral

$$2\sum_{\sigma_1\sigma_2}\int d\mathbf{r}_{cC}d\mathbf{r}_{A2}d\mathbf{r}_{b1}\chi^{(-)*}(\mathbf{r}_{bB})\left[\psi^{j_f}(\mathbf{r}_{A1},\sigma_1)\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\right]^{0*}_{0}$$
$$\times\, v(r_{b1})\Psi^{(+)}(\mathbf{r}_{aA},\mathbf{r}_{b1},\mathbf{r}_{b2},\sigma_1,\sigma_2). \tag{7.2.157}$$

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If we neglect core excitations, the above expression is exact as long as $\Psi^{(+)}(\mathbf{r}_{aA},\mathbf{r}_{b1},\mathbf{r}_{b2},\sigma_1,\sigma_2)$ is the exact wavefunction. We can instead obtain an approximation for the transfer amplitude using

$$\Psi^{(+)}(\mathbf{r}_{aA},\mathbf{r}_{b1},\mathbf{r}_{b2},\sigma_1,\sigma_2)\approx\chi^{(+)}(\mathbf{r}_{aA})\left[\psi^{j_{i1}}(\mathbf{r}_{b1},\sigma_1)\psi^{j_{i2}}(\mathbf{r}_{b2},\sigma_2)\right]^{\Lambda}_{\mu}$$
$$+\sum_{K,M}\mathcal{U}_{K,M}(\mathbf{r}_{cC})\left[\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\psi^{j_{i1}}(\mathbf{r}_{b1},\sigma_1)\right]^{K}_{M} \tag{7.2.158}$$

leads to both

as an approximation for the incoming state. The first term of (7.2.158) gives rise to the simultaneous amplitude, while from second one ~~we get~~ the successive and the non-orthogonality contributions. To extract the amplitude $\mathcal{U}_{K,M}(\mathbf{r}_{cC})$, we define $f_{KM}(\mathbf{r}_{cC})$ as the scalar product

$$f_{KM}(\mathbf{r}_{cC})=\left\langle\left[\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\psi^{j_{i1}}(\mathbf{r}_{b1},\sigma_1)\right]^{K}_{M}\middle|\Psi^{(+)}(\mathbf{r}_{aA},\mathbf{r}_{b1},\mathbf{r}_{b2},\sigma_1,\sigma_2)\right\rangle \tag{7.2.159}$$

for fixed $\mathbf{r}_{cC}$, which can be seen to obey the equation

$$\left(\frac{\hbar^2}{2\mu_{cC}}k^2_{cC}+\frac{\hbar^2}{2\mu_{cC}}\nabla^2_{r_{cC}}-U(r_{cC})\right)f_{KM}(\mathbf{r}_{cC})$$
$$=\left\langle\left[\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\psi^{j_{i1}}(\mathbf{r}_{b1},\sigma_1)\right]^{K}_{M}\middle|v(r_{c2})\middle|\Psi^{(+)}(\mathbf{r}_{aA},\mathbf{r}_{b1},\mathbf{r}_{b2},\sigma_1,\sigma_2)\right\rangle. \tag{7.2.160}$$

The solution can be written in terms of the Green function $G(\mathbf{r}_{cC},\mathbf{r}'_{cC})$ defined by

$$\left(\frac{\hbar^2}{2\mu_{cC}}k^2_{cC}+\frac{\hbar^2}{2\mu_{cC}}\nabla^2_{r_{cC}}-U(r_{cC})\right)G(\mathbf{r}_{cC},\mathbf{r}'_{cC})=\frac{\hbar^2}{2\mu_{cC}}\delta(\mathbf{r}_{cC}-\mathbf{r}'_{cC}). \tag{7.2.161}$$

Thus,

$$f_{KM}(\mathbf{r}_{cC})=\frac{2\mu_{cC}}{\hbar^2}\int d\mathbf{r}'_{cC}G(\mathbf{r}_{cC},\mathbf{r}'_{cC})\left\langle\left[\psi^{j_f}(\mathbf{r}'_{A2},\sigma'_2)\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\right]^{K}_{M}\middle|v(r_{c2})\middle|\Psi^{(+)}(\mathbf{r}'_{aA},\mathbf{r}'_{b1},\mathbf{r}'_{b2},\sigma'_1,\sigma'_2)\right\rangle$$
$$\approx\frac{2\mu_{cC}}{\hbar^2}\sum_{\sigma'_1\sigma'_2}\int d\mathbf{r}'_{cC}d\mathbf{r}'_{A2}d\mathbf{r}'_{b1}G(\mathbf{r}_{cC},\mathbf{r}'_{cC})\left[\psi^{j_f}(\mathbf{r}'_{A2},\sigma'_2)\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\right]^{K*}_{M}$$
$$\times\, v(r'_{c2})\chi^{(+)}(\mathbf{r}'_{aA})\left[\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\psi^{j_{i2}}(\mathbf{r}'_{b2},\sigma'_2)\right]^{\Lambda}_{\mu}=\mathcal{U}_{K,M}(\mathbf{r}_{cC})$$
$$+\left\langle\left[\psi^{j_f}(\mathbf{r}'_{A2},\sigma_2)\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma_1)\right]^{K}_{M}\middle|\chi^{(+)}(\mathbf{r}'_{aA})\left[\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\psi^{j_{i2}}(\mathbf{r}'_{b2},\sigma'_2)\right]^{\Lambda}_{\mu}\right\rangle. \tag{7.2.162}$$

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Therefore

$$\mathcal{U}_{K,M}(\mathbf{r}_{cC}) = \frac{2\mu_{cC}}{\hbar^2}\sum_{\sigma_1'\sigma_2'}\int d\mathbf{r}_{cC}'d\mathbf{r}_{A2}'d\mathbf{r}_{b1}'G(\mathbf{r}_{cC},\mathbf{r}_{cC}')\left[\psi^{j_f}(\mathbf{r}_{A2}',\sigma_2')\psi^{j_{i1}}(\mathbf{r}_{b1}',\sigma_1')\right]_M^{K*}$$
$$\times v(r_{c2}')\chi^{(+)}(\mathbf{r}_{aA}')\left[\psi^{j_{i1}}(\mathbf{r}_{b1}',\sigma_1')\psi^{j_{i2}}(\mathbf{r}_{b2}',\sigma_2')\right]_\mu^\Lambda$$
$$-\left\langle\left[\psi^{j_f}(\mathbf{r}_{A2}',\sigma_2)\psi^{j_{i1}}(\mathbf{r}_{b1}',\sigma_1)\right]_M^K\middle|\chi^{(+)}(\mathbf{r}_{aA}')\left[\psi^{j_{i1}}(\mathbf{r}_{b1}',\sigma_1')\psi^{j_{i2}}(\mathbf{r}_{b2}',\sigma_2')\right]_\mu^\Lambda\right\rangle. \tag{7.2.163}$$

When we substitute $\mathcal{U}_{K,M}(\mathbf{r}_{cC})$ into (7.2.158) and (7.2.157), the first term gives rise to the successive amplitude for the two–particle transfer, while the second term is responsible for the non–orthogonal contribution.

7.2.11 Successive ~~Contribution~~ transfer) ← più grande le lettere, come in 10,

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We need to evaluate the integral

$$T_\mu^{succ} = \frac{4\mu_{cC}}{\hbar^2}\sum_{\sigma_1\sigma_2}\sum_{KM}\int d\mathbf{r}_{cC}d\mathbf{r}_{A2}d\mathbf{r}_{b1}d\mathbf{r}_{cC}'d\mathbf{r}_{A2}'d\mathbf{r}_{b1}'\left[\psi^{j_f}(\mathbf{r}_{A1},\sigma_1)\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\right]_0^{0*}$$
$$\times\chi^{(-)*}(\mathbf{r}_{bB})G(\mathbf{r}_{cC},\mathbf{r}_{cC}')\left[\psi^{j_f}(\mathbf{r}_{A2}',\sigma_2')\psi^{j_{i1}}(\mathbf{r}_{b1}',\sigma_1')\right]_M^{K*}\chi^{(+)}(\mathbf{r}_{aA}')v(r_{c2}')v(r_{b1})$$
$$\times\left[\psi^{j_{i1}}(\mathbf{r}_{b1}',\sigma_1')\psi^{j_{i2}}(\mathbf{r}_{b2}',\sigma_2')\right]_\mu^\Lambda\left[\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\psi^{j_{i1}}(\mathbf{r}_{b1},\sigma_1)\right]_M^K, \tag{7.2.164}$$

where we must substitute the Green function and the distorted waves by their partial wave expansions (see Appendix). The integral over $\mathbf{r}_{b1}'$ is ~~:~~ → Gregory ??

$\boxed{\text{app. K}}$

$$\sum_{\sigma_1'}\int d\mathbf{r}_{b1}'\left[\psi^{j_f}(\mathbf{r}_{A2}',\sigma_2')\psi^{j_{i1}}(\mathbf{r}_{b1}',\sigma_1')\right]_M^{K*}\left[\psi^{j_{i1}}(\mathbf{r}_{b1}',\sigma_1')\psi^{j_{i2}}(\mathbf{r}_{b2}',\sigma_2')\right]_\mu^\Lambda$$
$$=\sum_{\sigma_1'}\int d\mathbf{r}_{b1}'(-1)^{-M+j_f+j_{i1}-\sigma_1-\sigma_2}\left[\psi^{j_{i1}}(\mathbf{r}_{b1}',-\sigma_1')\psi^{j_f}(\mathbf{r}_{A2}',-\sigma_2')\right]_{-M}^K\left[\psi^{j_{i1}}(\mathbf{r}_{b1}',\sigma_1')\psi^{j_{i2}}(\mathbf{r}_{b2}',\sigma_2')\right]_\mu^\Lambda$$
$$=\sum_{\sigma_1'}\int d\mathbf{r}_{b1}'(-1)^{-M+j_f+j_{i1}-\sigma_1-\sigma_2}\sum_P\langle K\ \Lambda\ -M\ \mu|P\ \mu-M\rangle((j_{i1}j_f)_K(j_{i1}j_{i2})_\Lambda|(j_{i1}j_{i1})_0(j_fj_{i2})_P)_P$$
$$\times\left[\psi^{j_{i1}}(\mathbf{r}_{b1}',-\sigma_1')\psi^{j_{i1}}(\mathbf{r}_{b1}',\sigma_1')\right]_0^0\left[\psi^{j_f}(\mathbf{r}_{A2}',-\sigma_2')\psi^{j_{i2}}(\mathbf{r}_{b2}',\sigma_2')\right]_{\mu-M}^P$$
$$=(-1)^{-M+j_f+j_{i1}}\sqrt{2j_{i1}+1}\,u_{l_f}(r_{A2})u_{l_{i2}}(r_{b2}')\sum_P\langle K\ \Lambda\ -M\ \mu|P\ \mu-M\rangle$$
$$\times((j_{i1}j_f)_K(j_{i1}j_{i2})_\Lambda|(j_{i1}j_{i1})_0(j_fj_{i2})_P)_P((l_f\tfrac{1}{2})_{j_f}(l_{i2}\tfrac{1}{2})_{j_{i2}}|(l_fl_{i2})_P\ (\tfrac{1}{2}\tfrac{1}{2})_0)_P$$
$$\times\left[Y^{l_f}(\hat{\mathbf{r}}_{A2}')Y^{l_{i2}}(\hat{\mathbf{r}}_{b2}')\right]_{\mu-M}^P u_{l_f}(r_{A2})u_{l_{i2}}(r_{b2}). \tag{7.2.165}$$

Integral over $\mathbf{r}_{A2}$ (see (7.2.117)) ~~is~~ leads to, ←

$$\sum_{\sigma_2}\int d\mathbf{r}_{A2}\left[\psi^{j_f}(\mathbf{r}_{A1},\sigma_1)\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\right]_0^{0*}\left[\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\psi^{j_{i1}}(\mathbf{r}_{b1},\sigma_1)\right]_M^K$$
$$=-\sqrt{\frac{2}{2j_f+1}}((l_f\tfrac{1}{2})_{j_f}(l_{i1}\tfrac{1}{2})_{j_{i1}}|(l_fl_{i1})_K\ (\tfrac{1}{2}\tfrac{1}{2})_0)_K\left[Y^{l_f}(\hat{\mathbf{r}}_{A1})Y^{l_{i1}}(\hat{\mathbf{r}}_{b1})\right]_M^K u_{l_f}(r_{A1})u_{l_{i1}}(r_{b1}). \tag{7.2.166}$$