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Introduction

In the atom, the nucleus provides the Coulomb field in which, negatively charged electrons ($-1 \times e$) move independently of each other in single-particle orbitals. The filling of these orbitals explains Mendeleev's periodic table. Thus the valence of the chemical elements as well as the particular stability of the noble gases (He, Ne, Ar, Kr, Xe and Ra) associated with the closing of shells (Fig. 1). The dimensions of the atom is measured in angstroms ($\mathring{A} = 10^{-8}$ cm), and typical energies in eV, the electron mass being $m_e \approx 0.5$ MeV (MeV $= 10^6$ eV).

The atomic nucleus is made out of positively charged protons ($1 \times e$) and of (uncharged) neutrons, nucleons, of mass $\approx 10^3$ MeV ($m_p = 938{,}3$ M, $m_n = 939{,}6$ MeV). Nuclear dimensions are of the order of few fermis (fm $= 10^{-13}$ cm). While the stability of the atom is provided by a source external to the electrons, the atomic nucleus, this nucleus is a self-bound system, resulting from the strong interaction of range $a_0 \approx 0.9$ fm and strength $v_0 \approx -100$ MeV acting among nucleons. And again, while most of the atom is empty space, the density of the atomic nucleus is conspicuous ($\rho = 0.17$ nucleon/fm^3). The "close packed" nature of this system implies a short mean free path as compared to nuclear dimensions. This can be estimated from classical kinetic theory $\lambda \approx (\rho\sigma)^{-1} \approx 1$ fm, where $\sigma \approx 2\pi a_0^2$ is the nucleon-nucleon cross section. It seems then natural to liken the atomic nucleus to a liquid drop (Bohr and Kalckar).

This picture of the nucleus provided the basis to describe the basic features of the fission process (Meitner and Frisch, 1939, Bohr and Wheeler 1939).

The leptodermic properties of the atomic nucleus are closely connected with the semi-empirical mass formula (Weizsäcker 1935),

$$M(N,Z) = N m_n + Z m_p - \frac{1}{c^2} B(N,Z), \quad (1)$$

the binding energy being

$$B(N,Z) = \left(b_{vol} A - b_{surf} A^{2/3} - \frac{1}{2} b_{sym} \frac{(N-Z)^2}{A} - \frac{3}{5} \frac{Z^2 e^2}{R_c}\right). \quad (2)$$

The second term in (2) represents the surface energy, while

$$b_{surf} = 4\pi r_0^2 \gamma. \quad (4)$$

The nuclear radius is written as $R = r_0 A^{1/3}$, with $r_0 = 1.2\,fm$, the surface tension energy being $\gamma \approx 0.95\, MeV/fm^2$.

When, in a heavy-ion reaction, the two nuclei come within range of the nuclear forces, the trajectory of relative motion will be changed by the attraction which will act between the nuclear surfaces. This surface interaction is a fundamental quantity in all heavy ion reactions. Assuming two spherical nuclei at a distance $r_{aA} = R_a + R_A$, where R_a and R_A are the corresponding half-density radii, the force acting between the two surfaces is

$$\left(\frac{\partial U^N_{aA}}{\partial r}\right)_{r_{aA}} = 4\pi\gamma \frac{R_a R_A}{R_a + R_A} \quad (5)$$

This result allows for the calculation of the ion-ion (proximity) potential and, supplemented with a position dependent absorption, to accurately describe heavy ion reactions.

In such reactions, not only elastic processes are observed, but also anelastic ones in which one, or both of the nuclear surfaces is set into vibration (Fig. 2). The restoring force parameter associated with oscillations of multipolarity λ is

$$C_\lambda = (\lambda-1)(\lambda+2)R^2\gamma - \frac{3}{2\pi}\frac{\lambda-1}{2\lambda+1}\frac{Z^2e^2}{R} \quad (6)$$

where the second term corresponds to the contribution of the Coulomb energy to C_λ. Assuming the flow associated with surface vibrations to be irrotational, the associated inertia for small amplitude oscillations is,

$$D_\lambda = \frac{3}{4\pi}\frac{1}{\lambda}AMR^2, \quad (7)$$

the energy of the corresponding mode being

$$\hbar\omega_\lambda = \hbar\sqrt{\frac{C_\lambda}{D_\lambda}}. \quad (8)$$

Experimental information associated with low-energy quadrupole vibrations, namely $\hbar\omega_2$ and the electromagnetic transition probabilities $B(E2)$, allow to determine C_2 and D_2. The resulting C_2 values exhibit variations by more than a factor of 10 both above and below the liquid-drop estimate. The observed values of D_2 are large as compared with the mass parameter for irrotational flow.

A picture apparently antithetic to that of the liquid drop, the shell model, emerged from the study of experimental data, plotting them against either the number of protons (atomic number), or the number of neutrons in the nuclei, rather than against the mass number.

One of the main nuclear features which led to the development of the shell model was the study of the stability and abundance of nuclear species and the discovery of what are usually called magic numbers (Elsasser (1933), Mayer (1948), Haxel et al (1949)). What makes a number magic is that a configuration of a magic number of neutrons, or of protons, is unusually stable whatever the associated number of other nucleons (Mayer (1949), Mayer and Teller (1949)).

The strong binding of a magic number of nucleons and weak binding for one more reminds, only relatively much weaker, the results displayed in Fig. 1 concerning the atomic stability of rare gases. In the nuclear case, at variance with the atomic case, the spin-orbit coupling play an important role, as can be seen from the level scheme shown in Fig. 3, obtained by assumming ~~that~~ nucleons move independently of each other in an average potential assumed to have spherical symmetry.

A closed shell, or a filled level, has angular momentum zero. Thus, nuclei with one nucleon outside (missing from) closed shell, should have the spin and parity of the orbital associated with the odd nucleon (-hole), a prediction confirmed by the data (available at that time) throughout the mass table. Such a picture implies that the nucleon mean free path is large compared to nuclear dimensions.

The low-lying states of closed shell nuclei can be interpreted as harmonic quadrupole or octupole collective vibrations (Fig. 4) described by the Hamiltonian

$$H_{coll} = \sum_{\lambda\mu}\left(\frac{1}{2D_\lambda}|\Pi_{\lambda\mu}|^2 + \frac{C_\lambda}{2}|\alpha_{\lambda\mu}|^2\right) \quad (1)$$

Following Dirac (1935) one can quantize the oscillatory motion introducing boson creation (annihilation) operator $\Gamma^{+}_{\lambda\mu}$ ($\Gamma_{\lambda\mu}$) obeying

$$[\Gamma_\alpha, \Gamma^{+}_{\alpha'}] = \delta(\alpha,\alpha'), \quad (2)$$

leading to

$$\hat{\alpha}_{\lambda\mu} = \sqrt{\frac{\hbar\omega}{2C}}\left(\Gamma^{+}_{\lambda\mu} + (-1)^{\mu}\Gamma_{\lambda\mu}\right) \quad (3)$$

and a similar expression for the conjugate momentum variable $\hat{\Pi}_{\lambda\mu}$, resulting in

$$\hat{H}_{coll} = \sum_{\lambda\mu}\hbar\omega\left(\Gamma^{+}_{\lambda\mu}\Gamma_{\lambda\mu} + 1/2\right), \quad (4)$$

The frequency is $\omega_\lambda = (C_\lambda/D_\lambda)^{1/2}$, while $(\hbar\omega_\lambda/2C_\lambda)$ is the amplitude of the zero-point fluctuation of the bosonic vacuum state $|0\rangle_B$, $\Gamma^{+}_{\lambda\mu}|0\rangle_B$ (5) being the one-phonon state.

The ground and low-lying states of nuclei with one nucleon outside closed shell can be described by the Hamiltonial

$$H_{sp} = \sum_{\nu} \varepsilon_\nu \, a^+_\nu a_\nu , \quad (6)$$

where a^+_ν (a_ν) is the single-particle creation (annihilation) operator,

$$|\nu\rangle = a^+_\nu |0\rangle_F , \quad (7)$$

being the single-particle state of quantum numbers $\nu (\equiv n l j m)$ and energy ε_ν, $|0\rangle_F$ being the Fermion vacuum.

Both the existence of drops of nuclear matter (displaying collective surface vibrations) and independent-particle motion in a self-confining mean field are emergent properties not contained in the particles forming the system, neither in the NN-force, but on the fact that these particles are many.

Generalized rigidity as measured by the inertia parameter D_λ, as well as surface tension closely connected to the restoring force C_λ, implies that acting on the system with an external time-depent (nuclear and/or Coulomb) field, the system reacts as a whole. This behavior is to be found nowhere in the properties of the nucleons, nor in the nucleon-nucleon scattering phase shifts at the basis of Yukawa prediction of the existence of a π-meson as the carrier of the strong force acting among nucleons.

Similarly, the fact ~~that~~ nucleons, asked to react collectively on short call ($\approx \hbar/\hbar\omega_\lambda \approx 10^{-21}$ s) under an external sollicitation, move independently of each other filling the pushing and pullings of the other nucleons only when trying to leave the nucleus, as testified by one-particle transfer reactions, is not apparent in the detailed properties of the NN-forces, not even in those carrying the quark-gluon input. Within this context, independent particle motion can be considered a <u>bona fide</u> emergent property.

Collective surface vibrations and independent particle motion are examples of what is called elementary modes of excitation in many-body physics, and collective variables in soft-matter physics.

The oscillation of the nucleus under the influence of surface tension implies that the potential $U(R,r)$ in which nucleons move independently of each other change with time. For low-energy collective vibrations this change is slow as compared with single-particle motion. Within this scenario the nuclear radius can be written as

$$R = R_0\left(1 + \sum_{LM} \alpha_{\lambda\mu} Y^*_{\lambda\mu}\right), \quad (8)$$

Assuming small amplitude motion,

$$U(r,R) = U(r,R_0) + \delta U(r), \quad (9)$$

where

$$\delta U = -\kappa \hat{\alpha} \hat{F}, \quad (10)$$

and

$$\hat{F} = \sum_{\nu_1\nu_2} \langle \nu_1 | F | \nu_2 \rangle a^\dagger_{\nu_1} a_{\nu_2}, \quad (11)$$

while

$$F = \frac{R_0}{\kappa} \frac{\partial U}{\partial r} Y^*_{\lambda\mu}(\hat{r}). \quad (12)$$

The coupling between surface oscillation and single-particle motion, namely the particle vibration coupling (PVC) Hamiltonian δU ($\equiv H_{coupling}$; Fig. 5) is a consequence of the overcompletness of the basis. Taken proper care of δU, that is, diagonalizing the Hamiltonian H_c making use of the rules of Nuclear Field Theory (NFT) to be discussed below, one obtains a solution of the total Hamiltonian,

$$H = H_{sp} + H_{coll} + H_c \qquad (13)$$

In fact, within the NFT framework, single-particles are to be calculated as the Hartree-Fock solution of the NN-interaction (Fig. 6), while vibrations are to be calculated making use of the same interaction, in the Random Phase Approximation (RPA) (Fig. 7). The associated dispersion relation and corresponding wavefunctions provide the unitary transformation leading to,

$$\hat{F} = \hat{\alpha} \qquad (14)$$

Because of quantal zero point fluctuations, a nucleon propagating in the nuclear medium moves through clouds of bosonic and fermionic virtual excitations to which it couple (H_c), becoming dressed and acquiring an effective mass, charge, etc. (Fig. 8). Vice versa, vibrational modes can become renormalized through the coupling to dressed nucleons which, in intermediate virtual states, can exchange the vibrational clothing with the second fermion (hole state) and renormalize the PVC vertex (Fig. 9)

From being antithetic views of the nuclear structure a proper analysis of the experimental data testifies to the fact that the collective and the independent particle picture of the nuclear structure require and support each other (Bohr and Mottelson 1975). To obtain a quantitative description of nucleon motion one needs a proper description of the k- and w-dependent "dielectric" function of the nuclear medium, in a similar way in which a proper description of the reaction processes used as probes of the nuclear structure, require the use of the optical potential (continuum "dielectric" function). Not only the solution of (13) plus the so called four-point vertices associated the the NN-interaction provide all the elements to calculate the optical potential, but both single-particle and vibrational elementary modes of excitation emerge from the same properties of the NN-interaction.

The development of experimental techniques and associated hardware has allowed for the identification of a rich variety of elementary modes of excitation aside from collective surface vibrations and of independent particle motion: quadrupole and octupole rotational bands, giant resonance of varied multipolarity and isospin, as well as pairing vibrations and rotation, together with giant pairing vibrations of transfer quantum number ±2. Modes which can be specifically excited in inelastic and Coulomb excitation processes, ...

charge exchange, and one-and two-particle transfer reactions.

One can choose to priviledge one among this rich variety of elementary modes of excitation, for example, independent particle motion and, making use of the shell model, eventually the so-called no core shell model, together with the best NN- interaction, attempt at describing the whole of structure and reaction.

Another possibility is to use the elementary modes of excitation basis states and nuclear field theory to deal with the overcompletness and Pauli principle violations of the basis states.

From a systematic collaboration between the two approaches and stong experimental input, it is likely that shell model calculations can help at individuating the proper interaction leading to realistic Hartree-Fock mean fields and collective RPA particle-hole and pairing vibrational modes. As a possible return of such input, nuclear field theory will eventually be able to provide shell model friendly microscopic collective modes of excitation.

The possible outcome could be that of being able to coin into few physical concepts the elements needed to carry out ab initio calculations which are largely independent of the basis chosen, and of truly predictive theories of structure and reactions, in which the physical content is simple to apprehend and visualize.

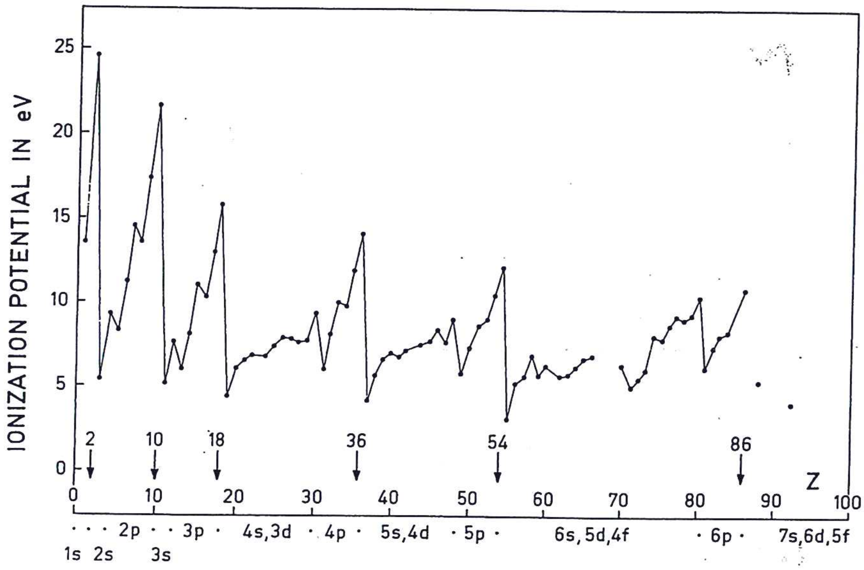


Fig. 1

The values of the atomic ionization potentials. The dots unde the abcissa indicate closed shells, corresponding to electron numbers; 2 (He), 10 (Ne), 18 (Ar), 36 (Kr), 54 (Xe) and 86 (Ra). Afer Bohr and Mottelson (1969).

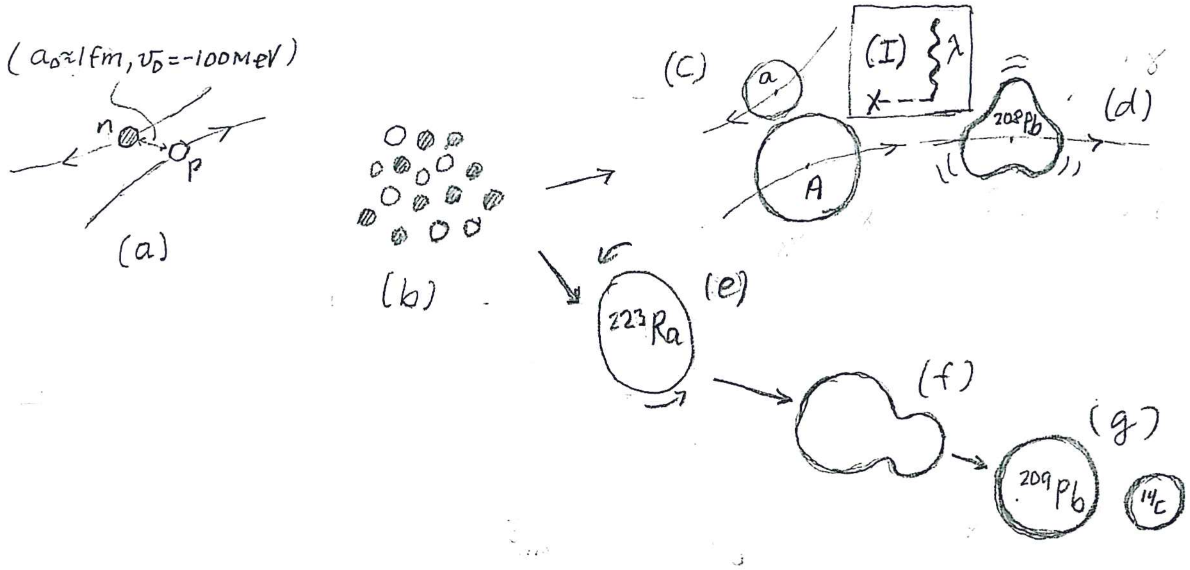
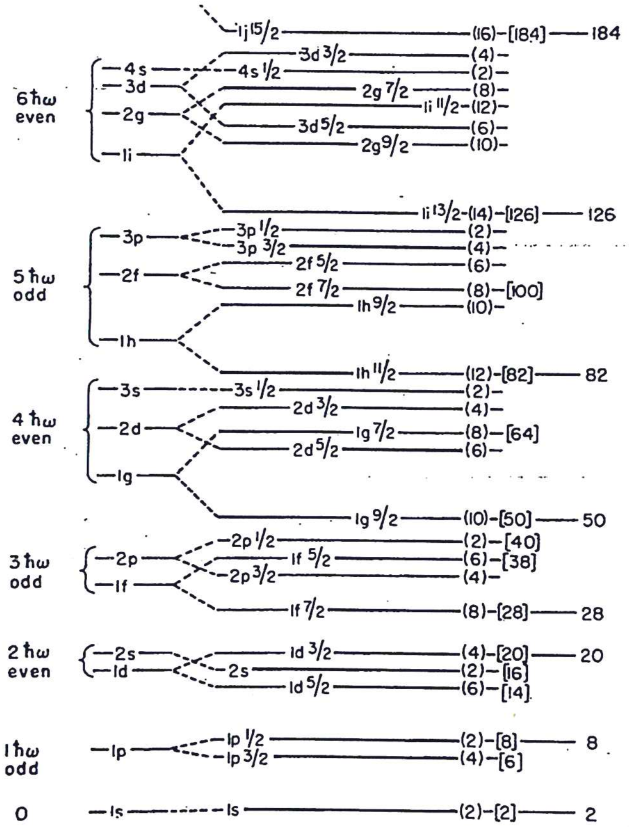


Fig. 2

Emergent properties (collective nuclear modes)

(a) Nucleon-Nucleon (NN) interaction in a scattering experiment; (b) assembly of a swarm of nucleons condensing into drops of nuclear matter, examples shown in (c) and (e); (c) anelastic heavy ion reaction $a + A \rightarrow a + A^*$ setting the nucleus A into an octupole surface oscillation (d); in inset (I) the time-dependent nuclear plus Coulomb fields associated with the reaction (c) is represented by a cross followed by a dashed line, while the wavy line labeled λ described the propagation of the surface vibration shown in (d), time running upwards; (e) another possible outcome of nucleon condensation, the (weakly) quadrupole deformed nucleus ^{223}Pb which can rotate as a whole with a moment of inertia smaller than the rigid moment of inertia, but much larger than the irrotational one; (f) the surface of a quantal drop fluctuates (zero point fluctuations), with the variety of of multipolarities with which the system reacts to time-dependent Coulomb + nuclear external field (quadrupole ($\lambda=2$), octupole ($\lambda=3$), ..), eventually producing a neck-in (saddle conformation) and the exoctic decay ^{223}Ra $\rightarrow$ ^{209}Pb + ^{14}C (g) experimentally observed.



In the inset to the right a schematic representation of the reaction $^{208}Pb(d,p)^{209}Pb(gs)$ which conferms the shell model predictions is displayed.

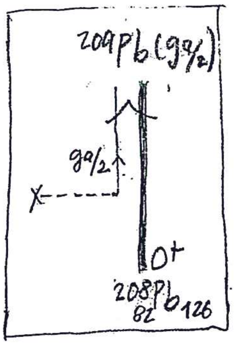


Fig. 3 ~~Fig. 7.~~ Realistic level diagram for protons. To the left (first column), the sequence of levels of the harmonic oscillator potential labeled with the total oscillator quantum number and parity $\pi=(-1)^N$. The next column shows the splitting of major shell degeneracies obtained using a more realistic potential (Woods-Saxon), the quantum number being the number of radial nodes of the associated single-particle wavefunctions. The levels at the center result when a spin-orbit term is considered the quantum numbers nlj characterizing the states of degeneracy $(2j+1)$ $(j=|l\pm 1/2|)$ (After Mayer (1963))

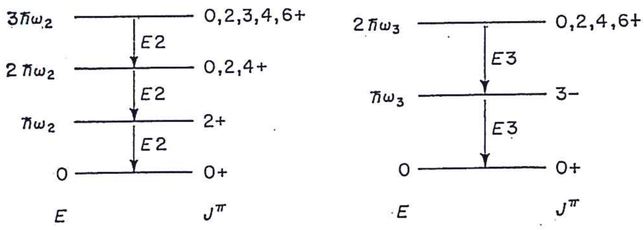


Fig. 4 Harmonic quadrupole and octupole liquiddrop collective surface vibrational modes

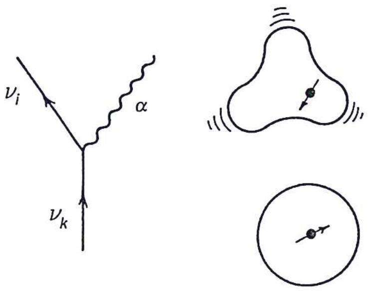


Figure 5 ~~8.10~~. Graphical representation of the process by which a fermion, bouncing inelastically off the surface, sets it into vibration. Particles are represented by an arrowed line, while the vibration is shown by a wavy line. The black dot represents a nucleon moving in a spherical mean field of which it excites an octupole vibration after bouncing inelastically off the surface.

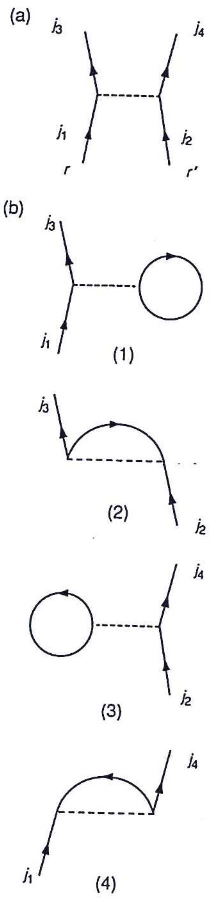


Figure ~~A.1~~ 6. (a) Scattering of two nucleons through the bare NN interaction. (b) (1) and (3): Contributions to the (direct) Hartree potential; ~~(see equations (A.20) and (A.22) as well as (A.28)),~~ (2) and (4): contributions to the (exchange) Fock potential ~~(see equations (A.21), (A.23) and (A.30))~~.

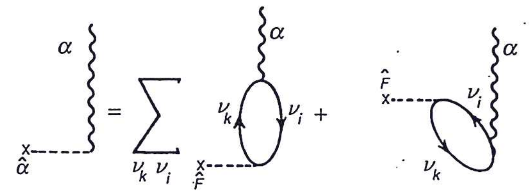


Figure 7. Excitation of the collective vibration in terms of the operators $\hat{\alpha}$ and $\hat{F}$. After Bohr and Mottelson (1975).

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Fig. 8

(a) a nucleon (single arrowed line) moving in presence of the zero point fluctuation of the nuclear ground state associated with a collective surface vibration; (b) Pauli principle leads to a dressing event of the nucleon; (c) time ordering gives rise to the second possible lowest order clothing process (time is assumed to run upwards).

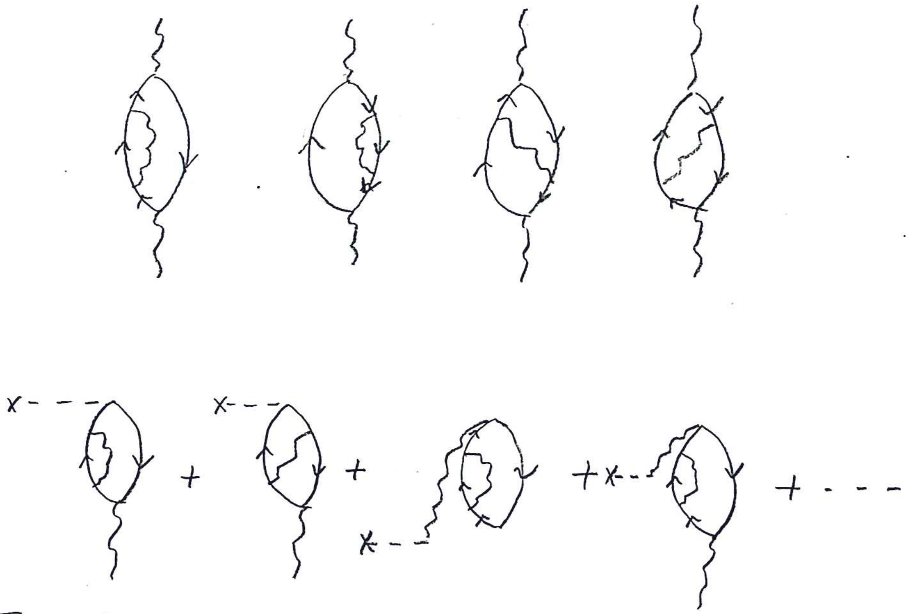

Fig. 9

(Upper part) Examples of renormalization processes dressing a surface collective vibrational state. (Lower part) Intervening with an external electromagnetic field the B(Eλ) transition strength can be measured.

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References

Mayer, M.G. On closed shells in nuclei, Phys. Rev 74, 235 (1948)

Mayer, M.G. On closed shells in nuclei 2, Phys. Rev. 75, 1969 (1949)

Mayer, M.G. and E. Teller, On the origin of elements, Phys. Rev. 76, 1226 (1949)

Elsasser, W. - - - - - - -
J. Phys. Radium 4, 549 (1933)

Haxel, O., Jensen, J.H.D., and Suess, H.E. - - - - - - - - - - - - - - -
Phys. Rev. 75, 1766 (1949)

Mayer M.G, The shell model, Nobel lecture (1963

Bohr N. and Kalchar, F. (1937) Mat. Fys. Medd. Vid. Selsk. 14, no. 10,

Bohr N. and Wheeler J.A. (1939) Phys. Rev. 56, 426

Meitner, L. and Frisch, O.R. Nature 143, 239 (1939)

Weitsäcker, von, C.F. Z. Phys. 96, 431 (1935)

Bohr A. ad Mottelson B.R. Nuclear structure, Vol I Benjamin (1969)

Dirac P.A.M. The principles of quantum mechanics, Oxford: Clarendon Press (1935)

Bohr A. ad Mottelson B.R Nuclear Structure Vol II (1975)