beam (sum over initial spin orientations divided by the number of such orientations) and when we do not detect the final polarizations (sum over final spin orientations),

$$\begin{aligned}\frac{d\sigma}{d\mathbf{k}'_a d\mathbf{k}'_b} &= \frac{k'_a}{k_a}\frac{\mu_{aA}\mu_{ac}}{4\pi^2\hbar^4}\frac{1}{(2J_A+1)(2j_a+1)} \\ &\times \sum_{\substack{m_a,m'_a\\ M_A,m'_b}} \left|\sum_{m_b} \langle j_b\ m_b\ j_c\ M_A-m_b|J_A\ M_A\rangle\, T^{m'_a,m'_b}_{m_a,m_b}(\mathbf{k}'_a,\mathbf{k}'_b)\right|^2 .\end{aligned} \tag{6.\cancel{F}.47}$$

G

The sum above can be simplified a bit. Let us consider a single particular value of m_b in the sum over m_b,

$$\begin{aligned}\sum_{m_a,m'_a,m'_b} \left|T^{m'_a,m'_b}_{m_a,m_b}(\mathbf{k}'_a,\mathbf{k}'_b)\right|^2 \sum_{M_A}\left|\langle j_b\ m_b\ j_c\ M_A-m_b|J_A\ M_A\rangle\right|^2 &= \\ \frac{2J_A+1}{2j_b+1}\sum_{m_a,m_a,m'_b}\left|T^{m'_a,m'_b}_{m_a,m_b}(\mathbf{k}'_a,\mathbf{k}'_b)\right|^2 & \\ \times\sum_{M_A}\left|\langle J_A\ -M_A\ j_c\ M_A-m_b|j_b\ m_b\rangle\right|^2, &\end{aligned} \tag{6.\cancel{F}.48}$$

G

where we have used

$$\langle j_b\ m_b\ j_c\ M_A-m_b|J_A\ M_A\rangle = (-1)^{j_c-M_A+m_b}\sqrt{\frac{2J_A+1}{2j_b+1}}\langle J_A\ -M_A\ j_c\ M_A-m_b|j_b\ m_b\rangle. \tag{6.\cancel{F}.49}$$

G

As

$$\sum_{M_A}\left|\langle J_A\ -M_A\ j_c\ M_A-m_b|j_b\ m_b\rangle\right|^2 = 1, \tag{6.\cancel{F}.50}$$

G

we finally have

$$\frac{d\sigma}{d\mathbf{k}'_a d\mathbf{k}'_b} = \frac{k'_a}{k_a}\frac{\mu_{aA}\mu_{ac}}{4\pi^2\hbar^4}\frac{1}{(2j_b+1)(2j_a+1)}\sum_{m_a,m'_a,m'_b}\left|\sum_{m_b} T^{m'_a,m'_b}_{m_a,m_b}(\mathbf{k}'_a,\mathbf{k}'_b)\right|^2 . \tag{6.\cancel{F}.51}$$

G

Zero range approximation. 6.G.2

The zero range approximation consists in taking $v(r_{ab}) = D_0\delta(r_{ab})$. Then, (see (~~7.2.147~~))

$$\begin{aligned}\mathbf{r}_{aA} &= \frac{c}{A}\mathbf{r}_{bc}, \\ \mathbf{r}_{ac} &= \mathbf{r}_{bc}.\end{aligned} \tag{6.\cancel{F}.52}$$

G G

The angular dependence of the integral can be readily evaluated. From (6.~~F~~.20), noting that $\hat{\mathbf{r}}_{aA} = \hat{\mathbf{r}}_{ac} = \hat{\mathbf{r}}_{bc} \equiv \hat{\mathbf{r}}$,

$$\begin{aligned}\left[Y^{l_a}(\hat{\mathbf{r}})Y^{l'_a}(\hat{\mathbf{r}})\right]^K_M\left[Y^{l_b}(\hat{\mathbf{r}})Y^{l'_b}(\hat{\mathbf{r}})\right]^K_{-M} &= \\ \frac{(-1)^{K-M}}{\sqrt{2K+1}}\left\{\left[Y^{l_a}(\hat{\mathbf{r}})Y^{l'_a}(\hat{\mathbf{r}})\right]^K\left[Y^{l_b}(\hat{\mathbf{r}})Y^{l'_b}(\hat{\mathbf{r}})\right]^K\right\}^0_0 . &\end{aligned} \tag{6.\cancel{F}.53}$$

G

G.G.2

We can as before evaluate this expression in the configuration shown in Fig. (${\bf \hat{r}} = \hat{z}$), but now the multiplicative factor is 4π. The corresponding contribution to the integral is

$$\frac{(-1)^K}{4\pi(2K+1)}\langle l_a\ 0\ l'_a\ 0|K\ 0\rangle\sqrt{(2l_a+1)(2l'_a+1)(2l_b+1)(2l'_b+1)}, \tag{6.F.54}$$

and

$$\begin{aligned}T^{m'_a,m'_b}_{m_a,m_b}({\bf k}'_a,{\bf k}'_b) = &\frac{16\pi^2}{k_a k'_a k'_b}\frac{c}{A}D_0 T_\sigma \sum_{l_a,j_a}\sum_{l'_a,j'_a}\sum_{l'_b,j'_b}\sum_{K,M} e^{i(\sigma^{l_a}+\sigma^{l'_a}+\sigma^{l'_b})}i^{l_a-l'_a-l'_b}(-1)^{l_a+l'_a+l'_b-j'_a-j'_b}\\
&\times\sqrt{(2l_a+1)(2l'_a+1)(2l_b+1)(2l'_b+1)}\,\langle l_a\ 0\ l'_a\ 0|K\ 0\rangle\\
&\times\frac{2l_a+1}{2K+1}((l'_a\tfrac{1}{2})_{j'_a}(l_a\tfrac{1}{2})_{j_a}|(l_a l'_a)_K(\tfrac{1}{2}\tfrac{1}{2})_0)_K\,((l'_b\tfrac{1}{2})_{j'_b}(l_b\tfrac{1}{2})_{j_b}|(l_b l'_b)_K(\tfrac{1}{2}\tfrac{1}{2})_0)_K\\
&\times\langle l'_a\ m_a-m'_a-M\ 1/2\ m'_a|j'_a\ m_a-M\rangle\langle l'_b\ m_b-m'_b+M\ 1/2\ m'_b|j'_b\ M+m_b\rangle\\
&\times\langle l\ 0\ 1/2\ m_a|j\ m_a\rangle Y^{l'_b}_{M+m_b+m'_b}(\hat{\bf k}'_b)Y^{l'_a}_{m_a+m'_a-M}(\hat{\bf k}'_a)\mathcal{I}_{ZR}(l_a,l'_a,l'_b,j_a,j'_a,j'_b),\end{aligned} \tag{6.F.55}$$

where now the 1–dimensional integral to solve is

$$\mathcal{I}_{ZR}(l_a,l'_a,l'_b,j_a,j'_a,j'_b) = \int dr u_{l_b}(r)F_{l_a,j_a}(\tfrac{c}{A}r)F_{l'_a,j'_a}(r)F_{l'_b,j'_b}(r)/r. \tag{6.F.56}$$

6.F.3 One particle transfer

G

It may be interesting to state the expression for the one particle transfer reaction within the same context and using the same elements, in order to better compare these two type of experiments. In particle transfer, the final state of b is a bounded state of the $B(=a+b)$ nucleus, and we can carry on in a similar way as done previously just by substituting the distorted wave (continuum) wave function (6.F.34) with

$$\psi^{l'_b,j'_b*}_{m'_b}({\bf r}_{ab},\sigma_b) = u^*_{l'_b,j'_b}(r_{ab})\left[Y^{l'_b}(\hat{\bf r}_{ab})\phi^{1/2}(\sigma_b)\right]^{j'_b*}_{m'_b}, \tag{6.F.57}$$

so the transition amplitude is now

$$\begin{aligned}T^{m'_a,m'_b}_{m_a,m_b}({\bf k}'_a) = &\frac{8\pi^{3/2}}{k_a k'_a}\sum_{\sigma_a,\sigma_b}\sum_{l_a,j_a}\sum_{l'_a,m_{l'_a},j'_a} e^{i(\sigma^{l_a}+\sigma^{l'_a})}i^{l_a-l'_a}(-1)^{l_a+l'_a-j'_a-j'_b}\\
&\times\sqrt{2l_a+1}\,\langle l'_a\ m_{l'_a}\ 1/2\ m'_a|j'_a\ m_{l'_a}+m'_a\rangle\langle l_a\ 0\ 1/2\ m_a|j_a\ m_a\rangle\\
&\times Y^{l'_a}_{-m_{l'_a}}(\hat{\bf k}'_a)\int d{\bf r}_{aA}d{\bf r}_{bc}\left[Y^{l'_a}(\hat{\bf r}_{Bc})\phi^{1/2}(\sigma_a)\right]^{j'_a}_{-m_{l'_a}-m'_a}\left[Y^{l'_b}(\hat{\bf r}_{ab})\phi^{1/2}(\sigma_b)\right]^{j'_b}_{-m'_b}\\
&\times\frac{F_{l_a,j_a}(r_{aA})F_{l'_a,j'_a}(r_{Bc})}{r_{Bc}r_{aA}}u^*_{l'_b,j'_b}(r_{ab})u_{l_b,j_b}(r_{bc})v(r_{ab})v_\sigma(\sigma_a,\sigma_b)\\
&\times\left[Y^{l_a}(\hat{\bf r}_{aA})\phi^{1/2}(\sigma_a)\right]^{j_a}_{m_a}\left[Y^{l_b}(\hat{\bf r}_{bc})\phi^{1/2}(\sigma_b)\right]^{j_b}_{m_b}.\end{aligned} \tag{6.F.58}$$

no he encontrado a la la referencia a la Figura 6.F.4 en el texto

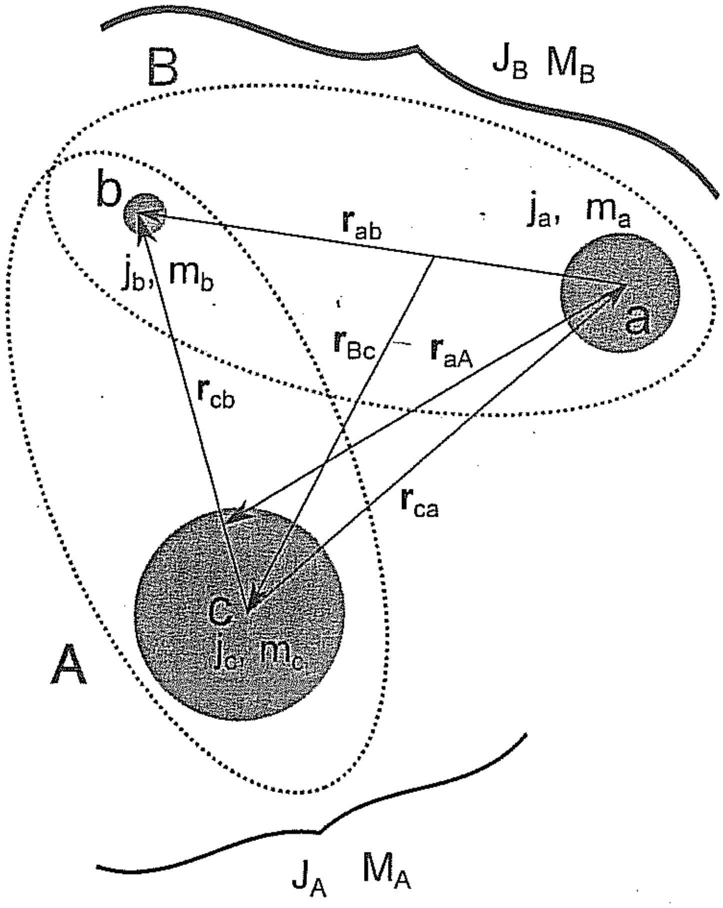

Figure 6.~~F~~6.4: One particle transfer reaction $A(= c + b) + a \rightarrow B(= a + b) + c$.

Using (6.~~F~~G.39), (6.~~F~~G.40), (6.~~F~~G.41), and setting $M = m_a - m'_a - m_{l'_a}$

$$
\begin{aligned}
T^{m'_a,m'_b}_{m_a,m_b}(\mathrm{k}'_a) = {} & \frac{8\pi^{3/2}}{k_a k'_a} T_\sigma \sum_{l_a,j_a}\sum_{l'_a,j'_a}\sum_{K,M} e^{i(\sigma^{l_a}+\sigma^{l'_a})} i^{l_a-l'_a}(-1)^{l_a+l'_a-j'_a-j'_b} \\
& \times ((l'_a \tfrac{1}{2})_{j'_a}(l_a \tfrac{1}{2})_{j_a}|(l_a l'_a)_K(\tfrac{1}{2}\tfrac{1}{2})_0)_K \, ((l'_b \tfrac{1}{2})_{j'_b}(l_b \tfrac{1}{2})_{j_b}|(l_b l'_b)_K(\tfrac{1}{2}\tfrac{1}{2})_0)_K \\
& \times \sqrt{2l_a+1}\,\langle l'_a\ m_a - m'_a - M\ 1/2\ m'_a | j'_a\ m_a - M\rangle \langle l_a\ 0\ 1/2\ m_a | j_a\ m_a\rangle \\
& \times Y^{l'_a}_{m_a-m'_a-M}(\hat{\mathrm{k}}'_a) \int d\mathrm{r}_{aA} d\mathrm{r}_{bc} \frac{F_{l_a,j_a}(r_{aA})F_{l'_a,j'_a}(r_{Bc})}{r_{Bc} r_{aA}} u^*_{l'_b,j'_b}(r_{ab}) u_{l_b,j_b}(r_{bc}) v(r_{ab}) \\
& \times \left[Y^{l_a}(\hat{\mathrm{r}}_{aA}) Y^{l'_a}(\hat{\mathrm{r}}_{Bc})\right]^K_M \left[Y^{l_b}(\hat{\mathrm{r}}_{bc}) Y^{l'_b}(\hat{\mathrm{r}}_{ab})\right]^K_{-M}. \qquad (6.\cancel{F}G.59)
\end{aligned}
$$

(6.G.2)

Aside from (~~[illegible]~~), we also need

G — 6.G.24

$$
\mathrm{r}_{Bc} = \frac{a+B}{B}\mathrm{r}_{aA} + \frac{b}{A}\mathrm{r}_{bc}. \qquad (6.\cancel{F}G.60)
$$

From (6.~~F~~G.20~~8~~ ~~(8.1.2), (8.1.1), (8.1.3), (7.2.121), we get~~ on gets

$$
\begin{aligned}
T^{m'_a,m'_b}_{m_a,m_b}(\mathrm{k}'_a) = {} & \frac{32\pi^3}{k_a k'_a} T_\sigma \sum_{l_a,j_a}\sum_{l'_a,j'_a}\sum_{K,M} e^{i(\sigma^{l_a}+\sigma^{l'_a})} i^{l_a-l'_a}(-1)^{l_a+l'_a-j'_a-j'_b} \\
& \times ((l'_a \tfrac{1}{2})_{j'_a}(l_a \tfrac{1}{2})_{j_a}|(l_a l'_a)_K(\tfrac{1}{2}\tfrac{1}{2})_0)_K \, ((l'_b \tfrac{1}{2})_{j'_b}(l_b \tfrac{1}{2})_{j_b}|(l_b l'_b)_K(\tfrac{1}{2}\tfrac{1}{2})_0)_K \\
& \times \frac{2l_a+1}{2K+1}\langle l'_a\ m_a - m'_a - M\ 1/2\ m'_a | j'_a\ m_a - M\rangle \\
& \times \langle l_a\ 0\ 1/2\ m_a | j_a\ m_a\rangle Y^{l'_a}_{m_a-m'_a-M}(\hat{\mathrm{k}}'_a) I(l_a, l'_a, j_a, j'_a, j'_b, K), \qquad (6.\cancel{F}G.61)
\end{aligned}
$$

with

$$
\begin{aligned}
I(l_a, l'_a, j_a, j'_a, K) = {} & \int dr_{aA} dr_{bc} d\theta\, r_{aA} r^2_{bc} \frac{\sin\theta}{r_{Bc}} \\
& \times F_{l_a,j_a}(r_{aA}) F_{l'_a,j'_a}(r_{ac}) u^*_{l'_b,j'_b}(r_{ab}) u_{l_b,j_b}(r_{bc}) v(r_{ab}) \\
& \times \sum_{M_K} \langle l_a\ 0\ l'_a\ M_K | K\ M_K\rangle \left[Y^{l_b}(\cos\theta, 0) Y^{l'_b}(\cos\theta_{ab}, 0)\right]^K_{-M_K} Y^{l'_a}_{M_K}(\cos\theta_{Bc}, 0), \qquad (6.\cancel{F}G.62)
\end{aligned}
$$

(6.G.2)

where (see (~~[illegible]~~), (6.F.60) and Fig. ~~7.~~ 6.G.2)

$$
\cos\theta_{ab} = \frac{-r_{aA} - \frac{c}{A} r_{bc}\cos\theta}{\sqrt{\left(\frac{c}{A} r_{bc}\sin\theta\right)^2 + \left(r_{aA} + \frac{c}{A} r_{bc}\cos\theta\right)^2}}, \qquad (6.\cancel{F}G.63)
$$

$$
\cos\theta_{Bc} = \frac{\frac{a+B}{B} r_{aA} + \frac{b}{A} r_{bc}\cos\theta}{\sqrt{\left(\frac{b}{A} r_{bc}\sin\theta\right)^2 + \left(\frac{a+B}{B} r_{aA} + \frac{b}{A} r_{bc}\cos\theta\right)^2}}, \qquad (6.\cancel{F}G.64)
$$

and

$$
r_{Bc} = \sqrt{\left(\frac{b}{A} r_{bc}\sin\theta\right)^2 + \left(\frac{a+B}{B} r_{aA} + \frac{b}{A} r_{bc}\cos\theta\right)^2}. \qquad (6.\cancel{F}G.65)
$$

Again, this is nothing new as many codes exist which deal with one particle transfer within the same DWBA formalism we have used here, but it may be useful to have our own code to better compare transfer and knock–out experiments. By the way, (6.~~F~~G.61) can also be used when particle b populates a resonant state in the continuum of nucleus B.

√ Appendix 6.H Modified formfactors

6.H.1 Two-particle transfer

(cf. pp. -11a- and -11b- right after p. 11)

6.H.2 One-particle transfer

6.H.3 One-particle knockout

6.H.4 Inelastic scattering

6.H.5 Elastic scattering

Likely it is better to have it as an Appendix of Ch. 7 (2n-transfer)

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