(From III written in Copenhagen)

11/7/14

~~mean field~~

1

1. Introduction

optical ~~potl~~.

1.1 Elementary modes of excitation

12/09/14 12:04

Subject to ~~an~~ external probes which couple weakly to the nucleus, ~~that is~~ in such away that the system can be expressed in terms of the properties of the excitation in the absence of probes, (Pines and Nozieres (1966)) the nucleus reacts (Bohr and Mottelson 1975) in terms of single-particle (-hole) motion (one-particle transfer), vibrations (surface, spin, etc) and rotations (Coulomb excitation and inelastic scattering) and pairing vibrations and rotation (two-nucleon transfer reactions) (cf. Figs (1,2 and 3) 1.1, 1.2 and 1.3.

; cf. also Broglia and Zelevinsky, eds. (2013) and refs. therein)

Echoing Heisenberg's requirement that no concept enters the quantal description of a physical systems which has not direct relation to experiments (Heisenberg 1949) and Landau's result that any weakly excited state of a quantal many-body system may be regarded as an ensemble of weakly interacting elementary modes of excitation (Landau, 1941), Bohr, Mottelson and coworkers developed a unified description of the nuclear structure in terms of quasiparticles, vibrations and rotations both in 3D- as well as in gauge space, which was eventually extended to direct nuclear

Bohr (1976), Mottelson (1976), Bohr and Mottelson 1975; Bohr, Mottelson and Pines (1958), Belyaev (1959), Nilsson, S.G. (1955), Bes and Broglia 1966, 1977 and refs. therein;

B+M Vol II, Nobel Lectures, Bes + Broglia etc.

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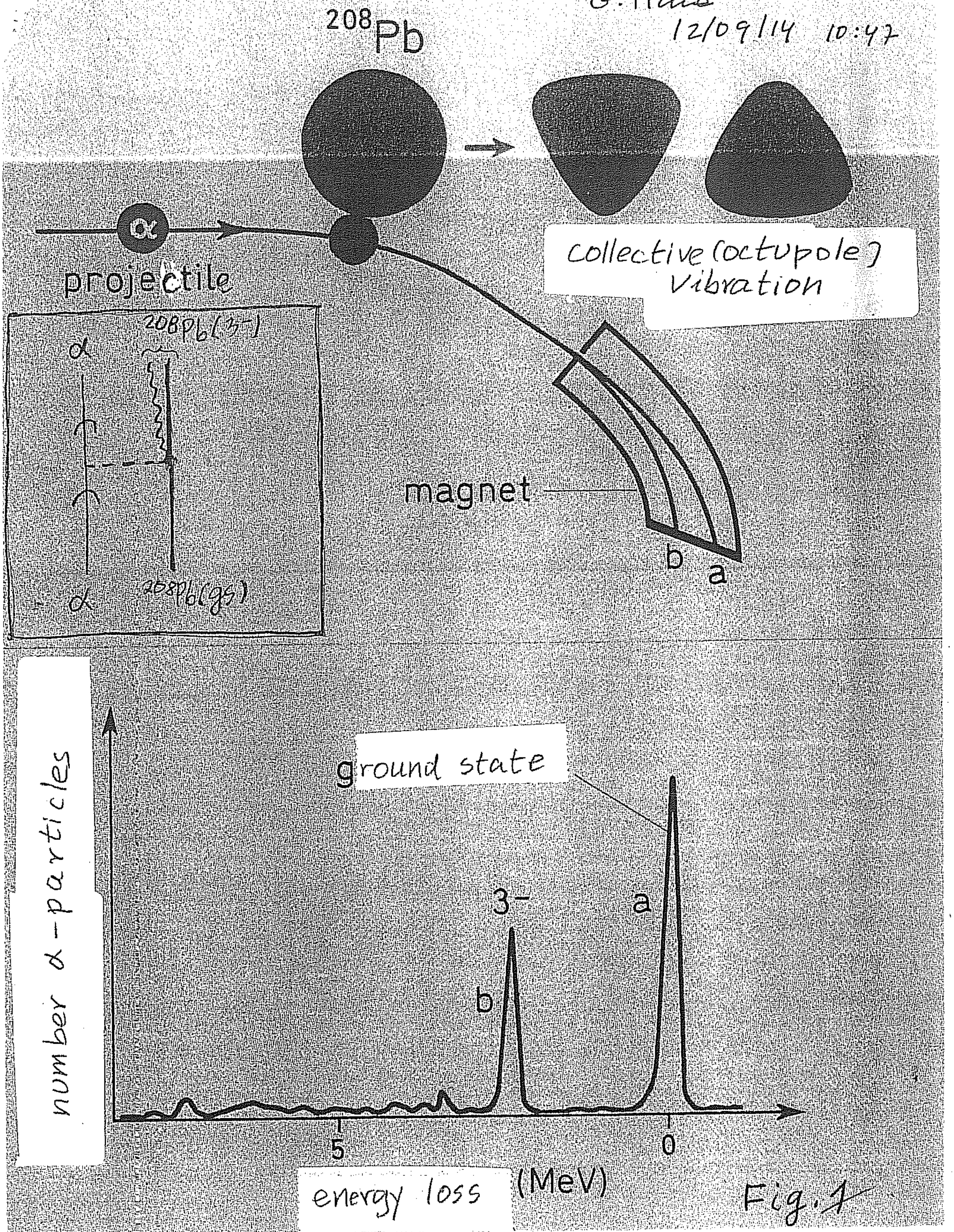

Fig. ~~1~~ 1.1

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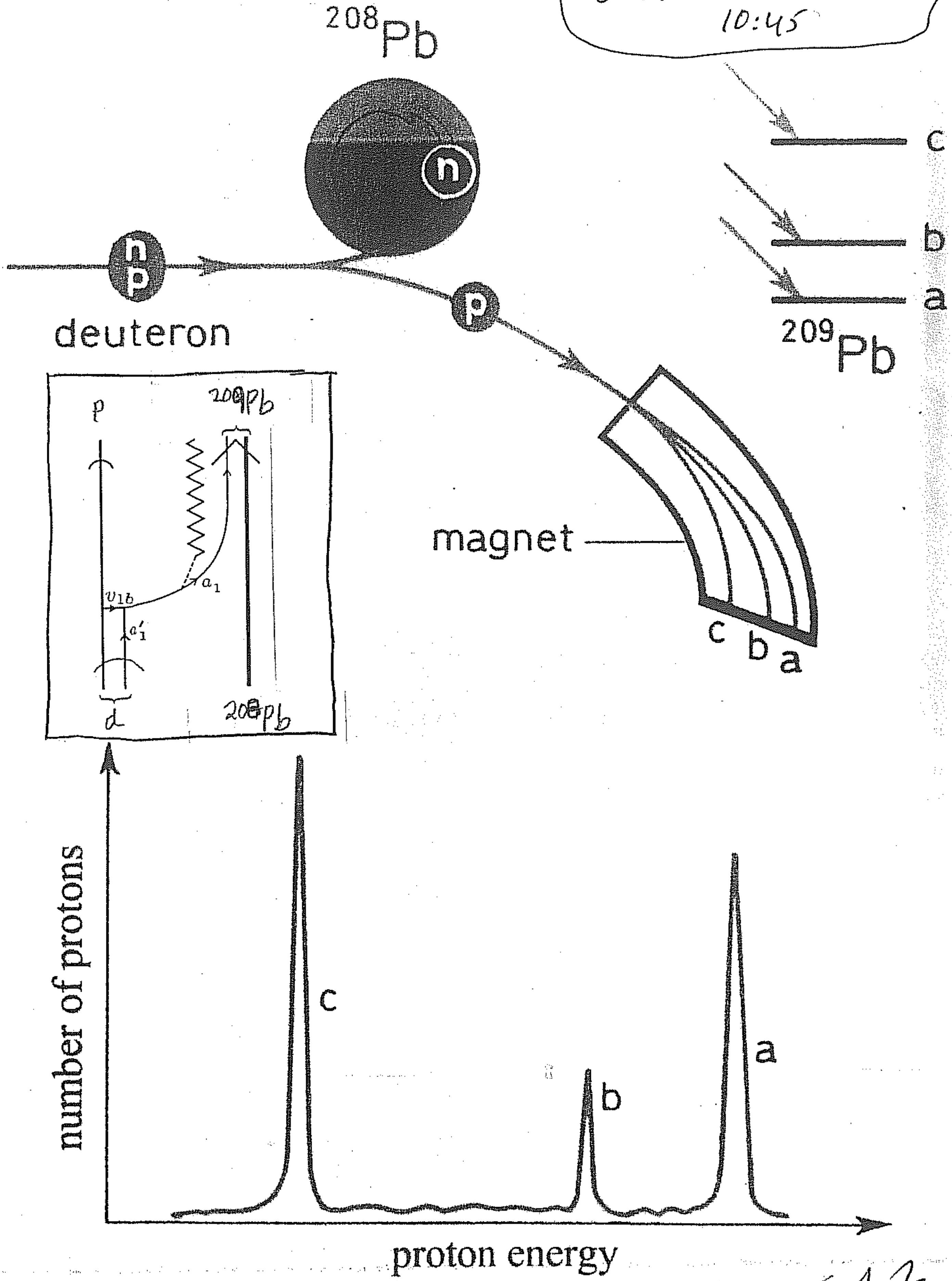


Fig. ~~2~~ 1.2

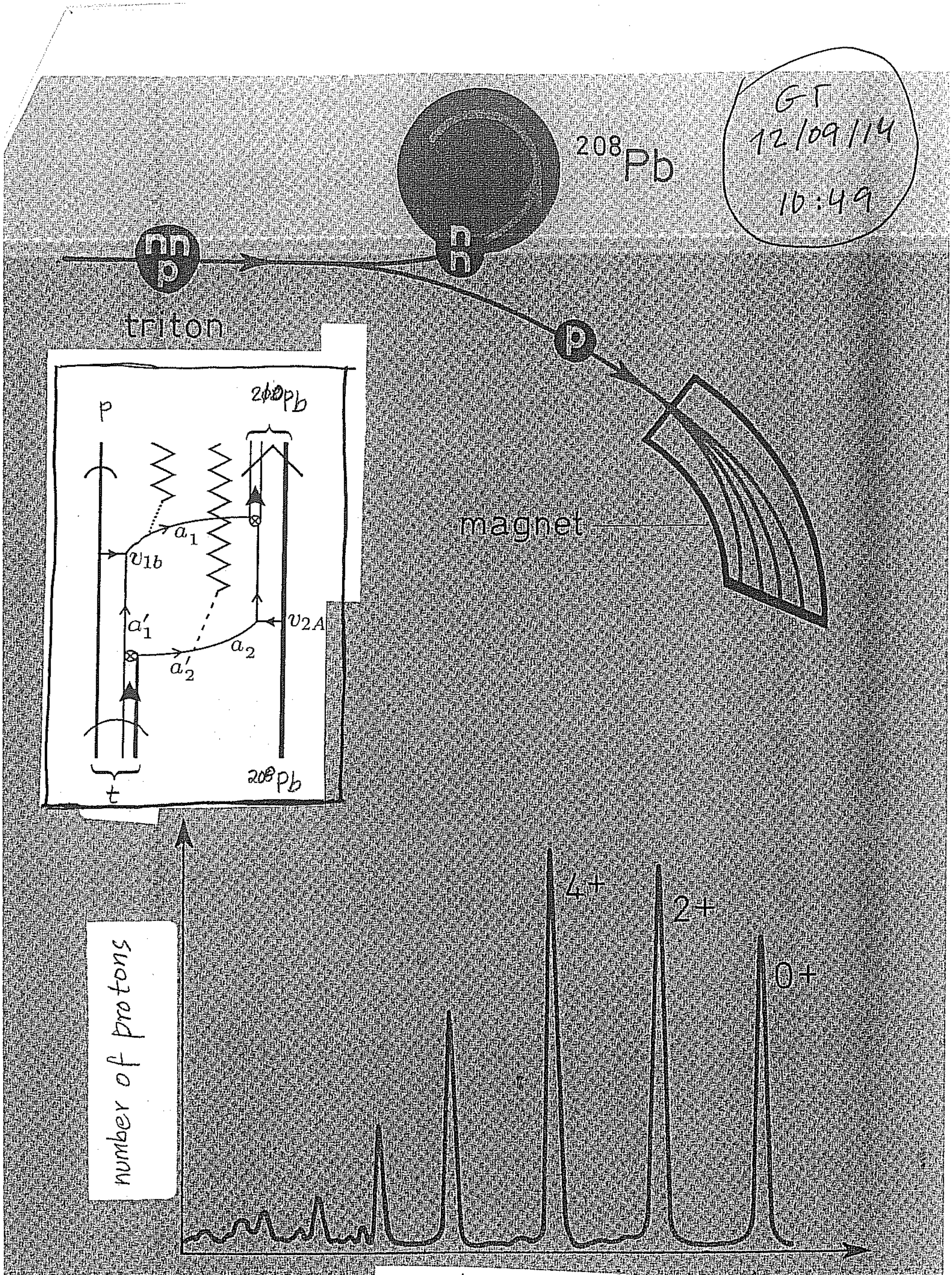

GT
12/09/14
10:49
208Pb
triton
magnet
p
208Pb
t
4+
2+
0+
number of protons
proton energy

Fig. ~~3~~ 1.3

Fig. 1

Schematic representation of
elastic (population of the ground state
a) an inelastic (lowest octupole
vibration 2.65 MeV) processes associa-
ted with the reaction $^{208}Pb(\alpha,\alpha')^{208}Pb$.
In the inset the Nuclear Field Theory
(NFT) diagram associated with the
octupole excitation (wavy curve) is given. a
(curved) arrow on a line indicates
propagation in the continuum. Outgoing
particles are deflected in a spectrograph
and recorded in a detector. The corres-
ponding excitation function is
given in the lowest part of the Figure.

Fig. 2

Schematic representation of the one-nu
cleon transfer reaction $^{208}Pb(d,p)^{209}Pb$
populating the single-particle state of ^{209}Pb.
The energy of the outgoing proton reflects
both the Q-value of the reaction and the
excitation energy of the final state. For
more detail cf. caption to Fig. 1.1
(cf. also ~~Fig. 6.1.1~~ Ch. 6 and Fig. 6.1.1)

Fig. 3

Schematic representation of the two-nucleon transfer reaction $^{208}Pb(t,p)^{210}Pb$, populating the ground state 0^+, and two particle excited states 2^+ ad 4^+. In the inset it is assume ~~that~~ the main contribution to the ~~trans~~ process arises from the succesive transfer of the nucleons, the jagged curves representing the recoil elementary mode taken care of recoil. effects (cf. Ch. 7 ad Figs 7.C.1 and 7.C.2)

13
24

(2) a

1.2 Sum rules (These exact and approximate sum rules were)

A quantitative measure of this over-completness is provided by exact and approximate sum rules that the diffe-rent observables (cross sections) [redacted] ~~recorded~~ obtained ~~making use of~~ related to ~~sulting from~~ the variety of probes to which the nucleus is subject [redacted] have to fullfil. An example of the first type (exact) is the Thomas-Reiche-Kuhn [redacted] (TRK) sum rule. Of the second (approximate) two-nucleon transfer (TNTR) sum rules (cf. Broglia et al 1972; Bayman and Clement 1972). In both cases they embody particle (pair) number conserva-tion. Charged (particle, in the first case) (electrons in atoms and molecules, effective charge of neutrons and protons in nuclei). Number of Cooper pairs in nuclei in the second.

(A)-(A) from p. (2) b

Physically, they [redacted] ~~give~~ provide with: the 1) maximum amount of photons (cross section) which the quantal system can absorb from a beam of light (γ-rays) shined on it; 2) the [redacted] ~~area~~ total [redacted] maximum value of the total two-nucleon transfer cross section (~~fraction (ring~~ ring area fraction) of the (geometrical) reaction cross section) exhausted by the final (A±2) states populated in the process.

total

... transfer)

(A) 1.2 Sum rules (2)b

A quantitative measure of the above mentioned overcompletness is provided by exact and approximate sum rules the observables (cross sections) associated with the variety of probes to which the nucleus is subject, have to fullfil. An example of the first type (exact) is provided by the Thomas-Reiche-Kuhn (TRK) sum rule. Of the second type (approximate) by two-nucleon transfer (TNTR) sum rules (cf. Broglia et al 1972; Bayman and (A)

to p. (2)a

see refs Ch. 2

In other words, these sum rules provide:
a quantitative measure of ~~the~~ single-particle subspace the quantal system under study, in particular the nucleus, uses to induce the antenna-like motion of protons against neutrons; or,: to correlate pairs of nucleons moving in time reversal states around the Fermi energy level, they ~~and~~ providing a sigmoidal distribution of the associated occupancy.
The TRK sum rule can be written as (, in the nuclear case,) model independent

$$S(E1) = \sum_n |\langle \alpha | F | \tilde{0} \rangle|^2 (E_\alpha - E_0) = \frac{9}{4\pi} \frac{\hbar^2 e^2}{2m} \frac{NZ}{A}, \quad \cancel{(1)}\ 1.1$$

moving, # ~~nucleo~~ or electr.

where $|\alpha\rangle$ labels the complete set of excited dipole states which can be reached operating with the dipole operator F (see Eq.(1.4)) on the initial state $|\tilde{0}\rangle$. Within this context, each elementary mode of excitation, in the present case of dipole type, defines a (, cf. app. A) ground state. This is in keeping with the fact ~~that~~ they induce specific quantal Zero Point Fluctuations (ZPF) measured by (harmonic approximation)

$$\langle \tilde{0} | F^2 | \tilde{0} \rangle = \frac{\hbar\omega}{2C_\alpha} = \frac{\hbar^2}{2D_\alpha} \frac{1}{\hbar\omega_\alpha}. \quad \cancel{(2)}\ 1.2$$

(nucleon

In other words, they perturb the static Fermi sea, ~~that~~ is the set of occupied levels of the mean field potential

$$U(r) = \int d^3r' \rho(r') v(|\vec{r}-\vec{r}'|), \quad (1.3) \ \cancel{(3)}$$

(k, i, i.e. $\varepsilon_i \leq \varepsilon_F$ and $\varepsilon_k > \varepsilon_F$; see App A)

inducing virtual particle-hole excitations: In the above equation, $\rho(r)$ is the nuclear density and v is the nuclear two-body interaction.

In Eq. (1), the quantity

$$(1.1) \quad F = \cancel{e} \sum_n{}' \left(\left(\frac{N-Z}{2A} - t_z \right) r_n Y_{1\mu}(\hat{r}_n) \right), \quad (1.4) \ \cancel{(4)}$$

is the dipole operator acting both on the Z protons ($t_z = -\tfrac{1}{2}$) and on the N neutrons ($t_z = +\tfrac{1}{2}$) of mass m, as indicated by the n-sum over all nucleon states ($A = N+Z$, mass number).

Because $|\langle\alpha|F|0\rangle|^2$ measures the probability with which ~~the state~~ $|\alpha\rangle$ is populated, the n-sum in (1.1) gives a measure of the maximum energy ~~that~~ the nucleus can absorb from the γ-beam, as can be seen by measuring $|\langle 0|F|\alpha\rangle|^2$ in single-particle (sp) units (Weisskopf (W) units)

$$B_{sp}(E1; j_1 \to j_2) = \frac{3}{4} e_{E1}^2 \langle j_1 \tfrac{1}{2} 1 0 | j_2 \tfrac{1}{2} \rangle^2 \times \langle j_2 | r | j_1 \rangle, \quad (1.5) \ \cancel{(5)}$$

$$\approx \frac{1}{4\pi} A^{2/3} e_{E1}^2 \, fm^2 = B_W(E1),$$

~~$\frac{0.81}{4\pi}$~~

see Eq. 36-38 BLM p 389

where $(e)_{E1} = (N/A)e$ for neutrons, and $(e)_{E1} = -(Z/A)e$ for protons, in keeping with the fact ~~that~~ the motion of a nucleon is associated with a recoil of the rest of the nucleus, since the total center of mass remains at rest in an intrinsic excitation.

(MF), Within this context, independent particle motion in general and the existence of a mean field (Hartree-Fock (HF) solution) in particular can be viewed as the most collective of all nuclear phenomena.

It is then not surprising that

$$S(E1) = \sum_n |\langle\alpha|F|\tilde{0}\rangle|^2 (E_\alpha - E_0), \qquad 1.6 \; \cancel{(6)}$$

$$= \sum_{ki} |\langle k,i|F|gs(MF)\rangle (\varepsilon_{ki} - \varepsilon_0),$$

provided $|\tilde{0}\rangle$ contains the ground state correlations mentioned in connection with Eq(2), and that $|gs(MF)\rangle$ those associated with $\Delta x_n \Delta p_{x_n} \geq \hbar$ (see Fig. 4), in other words, provided 1.4

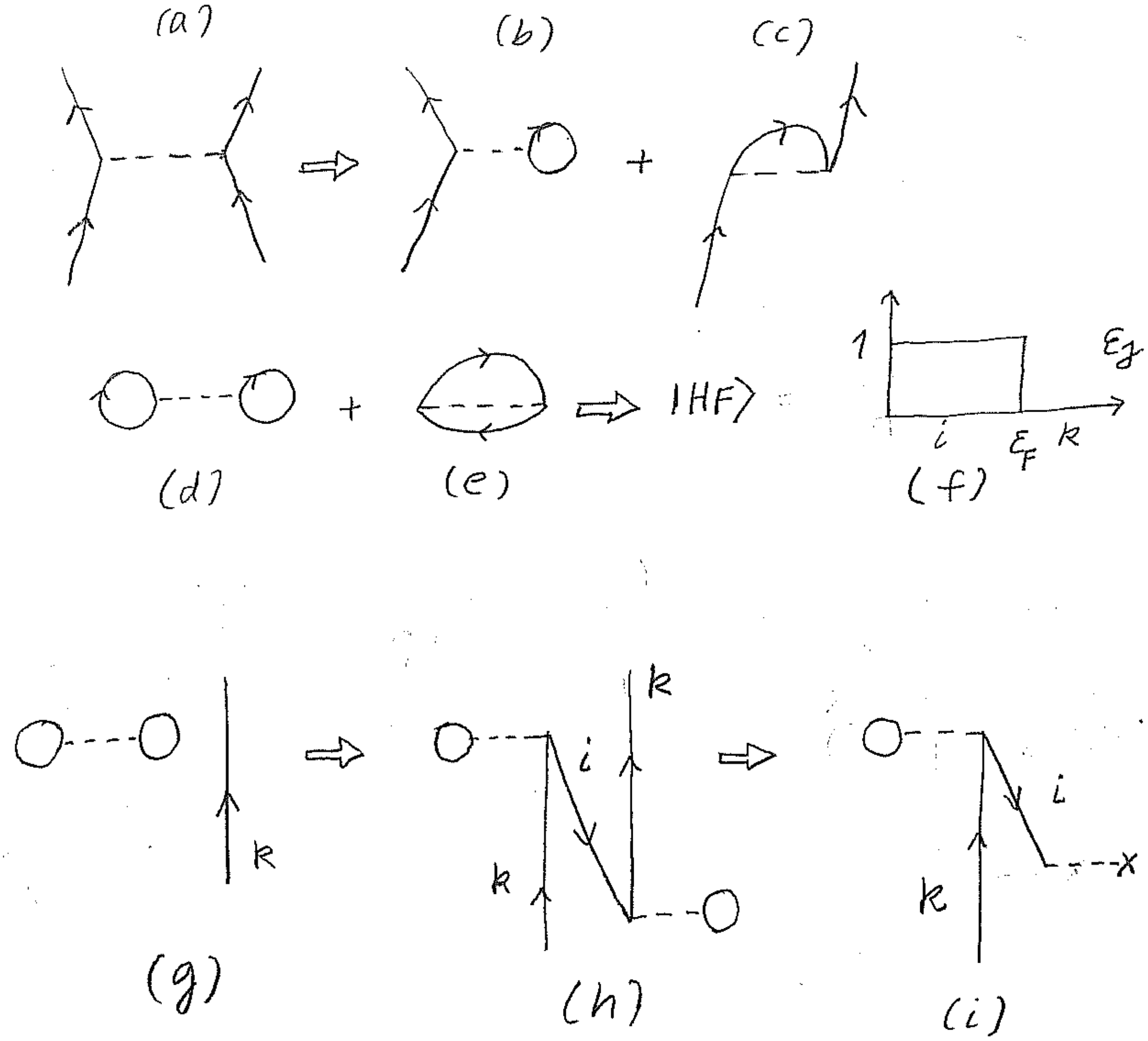
$$|HF\rangle = |gs(MF)\rangle = \prod_{i \in occup.} a_i^\dagger |0\rangle \qquad 1.7 \; \cancel{(7)}$$

where $|0\rangle$ is the particle vacuum ($a_j|0\rangle = 0$), and that $\Gamma_\alpha|\tilde{0}\rangle = 0$, $\Gamma^\dagger_\alpha$ being the creation operator of a dipole correlated particle-hole like mode ($\Gamma^\dagger_\alpha = \sum_{ki} X^\alpha_{ki} a^\dagger_k a_i + Y^\alpha_{ki} (a^\dagger_k a_i)^\dagger$),

$$E_{corr} = -\sum_\alpha \hbar\omega_\alpha (Y^2_\alpha) = -\sum \hbar\omega_\alpha \frac{\hbar\omega_\alpha}{2C_\alpha} = -\sum \hbar\omega_\alpha \frac{\hbar^2}{2D_\alpha}\frac{1}{\hbar\omega_\alpha} = \boxed{\frac{\hbar^2}{2D_\alpha}}$$

1 coll. state

model indep



$$\alpha_\nu |HF\rangle = 0 \quad ; \quad \alpha_\nu = \begin{cases} a_k & (\varepsilon_k > \varepsilon_F) \\ b_i = (-1)^{\text{phase}} a_{\tilde{i}} & (\varepsilon_i \le \varepsilon_F) \end{cases}$$

(j)

Fig. 1.4

Fig. ~~2~~ 4 ⑤b

Schematic representation of the processes characterizing the Hartree-Fock ground state (single-particle vacuum), in terms of Feynman-NFT diagrams. (a) nucleon-nucleon interaction through the bare (instantaneous) NN-potential. (b) Hartree mean field contribution. (b) Fock meanfiel contribution. (d,e) ground state correlations (ZPF) associated with Hartree and Fock processes. (f) decoupling between occupied and empty state operated by the HF field. (g) nucleon in presence of Hartree ZPF. (h) modification of single-particle state k due to Hartree ZPF and implying the existence of hole (antiparticle) states ($b_i|HF\rangle$) with quantum numbers time reversed to that of particle states ($b_i = (-1)^{phase} a_{\tilde{i}}$). In other words, the $|HF\rangle$ ground (vacuum) state is filled to the rim (ε_F) with N nucleons. The system with (N-1) nucleons can, within the lenguage of (Feynman's) field theory, be described in terms of the degrees of freedom of ~~that~~ of the missing nucleon (hole-, antiparticle state). ~~much~~ Such a description is obviously considerably more economic than that corresponding to a wavefunction with N-1 spatial and spin coordinates pairs ($\vec{r}_i, \sigma_i$), Relation (6) is a consequence of the fact ~~that~~ $S(E1)$ is proportional to the average value of the double commutator $[[H,F],F]$ in the ground ~~state~~ of the system ($|\tilde{0}\rangle$ or $|HF\rangle$). Because F is a function of only the nucleon coordinates, and assuming $v(|\vec{r}-\vec{r}'|)$ to be velocity independent, the only contribution to the double commutator arizes from the (universal) ~~kinetic energy~~ term of the Hamiltonian. Thus, the value (1.1) ~~(1)~~ is model independent. In other words, this value does not depend on the correlations acting among the nucleons, but on the number of them participating in the motion and thus on the corresponding effective inertia. In fact $\sum_\alpha \hbar\omega_\alpha\left(\frac{\hbar\omega_\alpha}{2C_\alpha}\right) = \sum_\alpha \left(\hbar^2/2D_\alpha\right)$. It is then not surprising that the TRK sum rule was used in the early stages of quantum mechanics, to determine the number of electrons of atoms.

Let us now go back to two-nucleon transfer (pairing) processes. The associated absolute cross sections can be set on essentially ~~equal~~ footing with respect to Q-value effects with the help of empirically determined ~~determined~~ global functions (cf. Broglia et al 1972)).

In this way, the theoretical absolute cross sections associated with e.g. the $A(t,p)A+2$ population (we assume ~~A to be even~~ N to be even) of ~~the~~ the n^{th} final state of spin J and parity $(-1)^J$ can be written as

$$\sigma^{(n)}(J=L,Q_0) = \left|\sum_{j_1 \geq j_2} B(j_1 j_2; J_n) S(j_1, j_2; L, Q_0)\right|^2, \qquad \cancel{(8)}\ 1.8$$

where

$$S^2(j_1, j_2; L, Q_0) = \sigma(j_1 j_2; L, Q_0) \qquad \cancel{(9)}\ 1.9$$

and

$$B(j_1 j_2; J_n) = \left\langle \Phi_{J_n}(\xi_{A+2}) \right| \left[\Phi_{J_i=0}(\xi_A) \frac{[a^{\dagger}_{j_1} a^{\dagger}_{j_2}]_J}{[(1+\delta(j_1,j_2))]^{1/2}} \right\rangle \qquad \cancel{(10)}\ 1.10$$

is the two-nucleon spectroscopic amplitude, ~~[illegible]~~ $\Phi_{J_i=0}(\xi_A)$ being the wavefunction describing the ground state of the initial ~~state~~ ~~state (ground state of the system~~ intrinsic nucleus, Φ_{J_n} that of the final state, ~~[illegible]~~ ξ labeling the radial ~~and spin~~ relative, J coordinates. Assuming A to be a closed shell system, at least in neutrons, and $J_i=0$ one can write

$$|0^+_n\rangle = \cancel{|\ldots\rangle} = \sum_{j_1 \geq j_2} c^{(n)}(j_1, j_2; J=0)\,|j_1, j_2; J=0\rangle, \qquad \cancel{(11)}\ 1.11$$

where $n = 1, 2, 3, \ldots$ labels the ~~final~~ final nucleus ~~[illegible]~~ states of spin $J=0$ in increasing energy order, making use of the completness relation of the coefficents $c^{(n)}(j,j;J=0)$ one [redacted] obtains,

$$\sum_n \sigma^{(n)}(J=L=0,Q_0) = \sum_j \sigma(j,j;L=0,Q_0). \quad 1.12 \ \cancel{(12)}$$

the similarity becomes even deeper

The above equation is quite simi-lar to 1.6 ~~(6)~~, aside from the fact that the Q-value effect in 1.6 ~~(6)~~ can be analitically dealt with, [redacted] while $\sigma(\{Q\})$ is a functional of Q. This is in keeping with the fact that in elastic and inelastic processes the mass partition is equal in both entrance and exit channels. Thus, the intrinsic [redacted] (intrinsic structure) and the [redacted] ~~reaction~~ [redacted] relative motion (reaction) coordinates [redacted] can be treated separately. This is not the case for transfer processes, both intrinsic and reaction ~~reaction~~ coordinates being interweaved through the recoil process (particle - recoil mode coupling). [redacted]

jaggy

The analogy becomes (even) richer (from a physics point of view) ~~physically~~, [redacted] by remembering the fact that in the case of ~~independent particle~~

, cf. Figs. 1.3, 1.5 and 1.6 ~~#5 and #6~~; cf. also App. B)

pp 27 - 30

pure two-particle configurations

$$|0^+_{n=1}\rangle = |j^2(0)\rangle = [a^\dagger_j a^\dagger_j]_0 |0\rangle, \quad \text{1.13} \; \cancel{(11)} \qquad a_j|0\rangle = 0$$

the vacuum $|0\rangle$ contains ~~only~~ independent-particle ZPF ($\Delta x_k \Delta p_{x_k} \geq \hbar_i$), while in the case of collective pairing vibrations, i.e. vibrations which ~~change~~ particle number in two units, 1.14 ~~(14)~~

$$|0^+_{n=1}\rangle = \Gamma^{\dagger(\beta=+2)}_{n=1}|\tilde{0}\rangle = \sum_{k>0} X^{n=1}_k [a^\dagger_k a^\dagger_k]_0 - \sum_{i\leq 0} Y^{n=1}_i [a_i a_i]_0,$$

being ~~where~~ β the transfer quantum number

and $\Gamma_{n=1}(\beta=+2)|\tilde{0}\rangle = 0$. The corresponding ZPF are ~~given~~ measured by $\left(\frac{\hbar\omega_{n=1}}{2C_{n=1}}\right)^{1/2} = \left(\frac{\hbar^2}{2D_{n=1}}\frac{1}{\hbar\omega_{n=1}}\right)^{1/2}$. ~~collectivity~~

A further parallel can be achieved by ~~introducing~~ defining two-particle units (1.15) ~~(15)~~

$$\sigma^{max}_{2pu}(A,L,Q_0) = \max[\sigma(j_1,j_2;L,Q_0)],$$

where max[ ] indicates that the largest two-particle absolute cross section in the single-particle subspace considered (hot orbital), is to be considered. In this way one can write the relation 1.12 ~~(12)~~ in dimensionless units. Furthermore, one can define enhancement factors.

~~period Ø sound ku dx~~

Another (two-particle transfer) sum rule has been introduced in the literature (Bayman and Clement 1972), which relate the difference between two-nucleon stripping and pick-up reactions cross sections, with single-particle transfer processes. (cf. also ~~Landford~~ and

~~let us now return~~

1.3 Couplings between elementary modes

Pauli principle

Let us now return to the subject of the finite overlap existing between the elementary modes of nuclear excitation. That is, to the ~~fact that~~ one is working in a ~~physical~~ basis of states ~~which~~ contains much of the physics one likes to describe, but which has ~~having~~ the shortcoming of ~~being non~~ ~~[illegible]~~ being overcomplete. An orthogonalization ~~[illegible]~~ protocol ~~procedure~~, like a generalized Gram-Schmidt procedure, but leading to an effective field theory, where the different modes melt together, is called for (cf. App. C)

Because the overlaps between elementary modes of excitation is proportional to their coupling, and in keeping with the fact that mean field theory is the natural starting point of ~~[illegible]~~ nuclear

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Pauli principle

Pauli principle

Because the overlaps between elementary modes of excitation is proportional to their coupling, and in keeping with the fact that mean field theory is the natural starting point of ~~[struck]~~ nuclear structure calculations, overcompletness of the basis is tantamount to the appearance of linear couplings between quasiparticles and collective modes (cf. App. 1.D and 1.E) (old ch. 5 inelastic scattering)

Basis orthogonalization thus implies the diagonalization of the associated particle-vibration coupling Hamiltonian H_C. The rules to do so have been cast into a (graphical) effective field theory, namely the Nuclear Field Theory (NFT). In this theory ~~particle~~ the free fields are ~~allowed to interact through four-point vertices (bare interaction acting among particle lines) and the particle-vibration vertices (between particle lines and~~

to be calculated in the HF (HFB) approximation (particle lines) and in the RPA (QRPA) (vibrations). These elementary modes of excitation interact through four-point vertices (nucleon-nucleon bare interaction), and through the particle-vibration coupling vertices.

The NFT rules for evaluating the effect of these couplings between fermions and bosons involve a number of restrictions, ~~in keeping with the fact that the collective~~ concerning initial and intermediate ~~states~~ as compared with the usual rules of perturbation theory ~~that~~ are to be used in evaluating the effect of the original nucleon-nucleon interaction acting in the fermion space. This is in keeping with the fact ~~that~~ the collective modes contain the correlations arising from forwards and backwards going particle-hole ($\beta=0$) as well as particle-particle ($\beta=+2$) ~~and~~ hole-hole ($\beta=-2$) bubbles. Furthermore, this is because these (quasi) bosons are not elementary but composite fields, made out of pairs of fermions, and thus subject to the Pauli principle. ~~The general validity~~

~~Nuclear Field Theory~~

The general validity of the NFT rules have been demonstrated different ground states

by proving the equivalence existing, to each order of perturbation theory, between the many-body finite nuclear system propagator calculated ~~in~~ terms of only Feynman diagrams involving only the fermionic degrees of freedom(s), i.e. explicitly respecting Pauli in a complete and not overcomplete basis, and the propagator constructed in terms of Feynman diagram involving fermion and phonon degrees of freedom (NFT Feynman diagrams) in the case of a general two-body interaction and an arbitrary distribution of single-particle levels.

(the practical difficulty of recognized)

Concerning the actual embodiment of NFT one can ~~that~~ ~~it is short of impossible~~ respecting in keeping with the fact that the corresponding rule, there is not a single bare NN-force (eventually with 3N and higher order corrections) with which it is possible to generate a mean field ~~and coll~~ (cf. Eq. (3)) and collective modes, that is, Concerning the actual embodiment of NFT one can recognize the practical difficulties of respecting the corresponding rules. This is in keeping with the fact ~~that~~ there is not a single bare NN-force (eventually with 3N and higher order corrections) with which it is possible to generate a mean field (Eq(3)) [1,3] to determine the single-particle states and, by introducing a periodic time-dependence with the constrain [1,16]

$$\delta U(r) = \int d^3 r\, \delta\rho(r')\, v(|\vec{r}-\vec{r}'|), \quad (16)$$

calculate the collective modes associated with the variety of particle-hole ($\beta=0$; density, spin, isospin, etc) and pairing ($\beta=\pm 2$; monopole an multipole pair addition and pair substraction) channels. If such a low-k well behaved bare force was available one could then diagonalize, within the framework of NFT and to any desired order of perturbation theory the Hamiltonian H_c, sum

#5

Schwenk

One could argue ~~that~~ one can reach a similar goal by making use of effective interactions, each tailored to provide a sensible description of each of the channels considered. For example a Skyrme interaction (like SLy4) to calculate mean field, single-particle modes, and eventual some $\beta=0$ modes.

1.16 14

$$\delta U(r) = \int d^3r' \, \delta\rho(r') \, v(|\vec{r} - \vec{r}'|), \quad \cancel{(16)}$$

~~renormalizing~~

for all channels (density, spin, isospin, etc) which, once the variety of couplings are carried out to the needed order, as well as to dressed modes, in some cases infinite ~~for~~ ~~processes, if needed~~, provide a quantitative account of the data. In other words, a real ab initio ~~is ruled out~~ ~~and thus to~~ and thus a common ground state ~~corrected~~ which corrected with the corresponding homogeneous ZPF lead eventually to the "exact" ground state (cf. App. A)

~~On the other hand empirical~~

On the other hand, empirical implementation of the NFT rules have been carried out, making use of the bare Argonne v_{14} potential, and of Skyrme forces (or Saxon-Woods ~~metrization~~ parametrizations) to determine the mean field and spin vibrational channels, and multipole-multipole forces with self-consistent coupling constants for the variety of density vibration channels.

The resulting predictions are, as a rule, able to provide ~~an overall account~~, together with specific reaction software, in particular COOPER, of ~~the~~ "complete" an overall account

dual origin
+ Sn(p,t) etc.

15

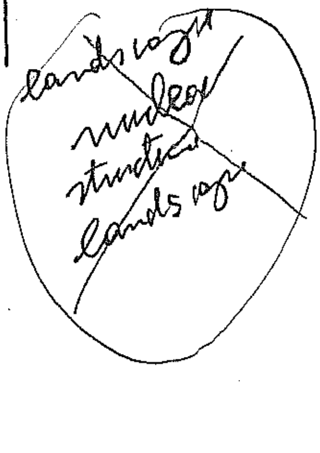
[redacted] sets of experimental data, [redacted] obtained with the help of Coulomb, inelastic and one- and two-nucleon transfer data, (as can be seen from Figs. ....)

dual
^{11}Li
^{12}Be
Lworm
2^+
Sn
article
Colò

Summing up, [redacted] the nuclear structure description ~~provided~~ given by ~~t~~ the elementary modes of nuclear excitation approach within the framework of NFT, provides a ~~complete~~ ~~tc~~ unified description of the variety of observables. At the same time, each cross section or transition probability is connected essentially to all others (see Fig 1, 6)

iEqps
β_2

able to map out the nuclear structure-reaction landscape



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p. 25

those associated with

competition between
the variety of ZPF,
in particular density
($\beta=0$) and pairing vibrational
($\beta=\pm 2$) modes

Non-orthogonality of the NFT basis made out of local elementary modes of excitation   Appendix C   30/07/14   ①

cf. end of Sect 1.1 (2.1)

The ground state of $^{210}_{84}Po_{126}$ can be (16) viewed as the proton pair addition mode of the doubly closed shell nucleus $^{208}_{82}Pb_{126}$, mode displaying $J^\pi = 0^+$ and $\beta = +2$ (transfer-) quantum numbers. Within this framework $^{209}_{83}Bi_{126}$ is expected to be a <u>bona fide</u> proton single-particle system ($\beta = +1$), in which the $g_{7/2}$, $d_{5/2}$, $h_{11/2}$, $d_{3/2}$ and $s_{1/2}$ are occupied, the odd proton occupying, in the ground state, a substate of the $h_{9/2}$ orbital.

This picture can be specifically probed through one-proton stripping and pick up reactions, e.g. with the help of $^{210}Po(t,\alpha)^{209}Bi$ and $^{208}Pb(^3He,d)^{209}Bi$ transfer processes.[1] footnote

It is of notice that in the present case, at variance with the rest of the monograph, use will be made of spectroscopic factors.

While essentially the ~~reaction cross~~ pick-up reaction ~~cross~~ section consistent with full (2j+1) occupancy ~~(2j+1)~~ is found in a single peak in the case of ~~the~~ states $1/2^+$ ($s_{1/2}$) and $11/2^-$ ($h_{11/2}$), $3/2^+$ two states with essentially equal strength exhausting the $(2j+1)(=4)$ strength ~~and~~ lying close to the expected (independent-particle) energy, are observed. These four peaks are essentially not excited in the stripping process (see Table C.1). ~~testifying to their essential hole character~~

see Table C.1

It is of notice that

30/07/14 (2)

17

In an attempt to further clarify the structure of the two $3/2^+$ recourse is made to the inelastic process $^{209}Bi(d,d')$. Both states are excited in the ~~process~~

~~with a summed cross section consistent with that expected for the $3/2^+$ member of the $(h_{9/2}(\pi) \otimes 3^-(^{208}Pb))_J$ positive parity septuplet ($J = 3/2 - 15/2$)~~

~~process through~~ process, the angular distribution of the associated cross sections revealing the $L=3$, octupole character (Tables C2 and C3) with centroid around 2.6 MeV and

(Tables C2 and C3) In keeping ~~with~~ the ~~fact~~ that the same experiment reveals a multiplet (septuplet) of states with summed $L=3$ inelastic cross section ~~consistent~~ with ~~that~~ of the lowest collective (2.615 MeV, $B(E3)/B_{sp} \approx 32$) octupole vibration of ^{208}Pb, one can posit that the two $3/2^+$ states are a linear combination of the unperturbed ~~states~~ (2p-1h) states, ~~state~~ (two-particle)-(one hole)

$$|\alpha\rangle = |d^{-1}_{3/2} \otimes gs(^{210}Pb); 3/2^+\rangle \equiv |\!\downarrow\Uparrow\rangle, \quad (1.C.1)$$

and

$$|\beta\rangle = |h_{9/2} \otimes 3^-(^{208}Pb); 3/2^+\rangle \equiv |\!\uparrow \wr\rangle. \quad (1.C.2)$$

Because these states lie very close in energy they mix. According to NFT the most important contribution to this mixing arises ~~arising~~ from the process given in Fig. C.1

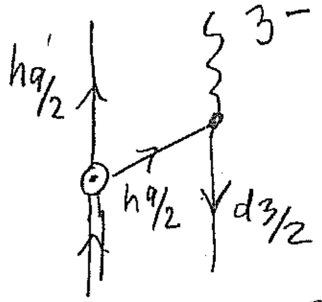


Fig. 1.C.1 Diagram describing one of the most important processes coupling the 2p-1h ~~states~~ (1.C.1) and (1.C.2).

The resulting physical (mixed) ~~state~~ can be approximately written (see, Fig 4.7 in p. 344 Bortignon et al 1977 for details) as,

$$|I\rangle = -0{,}53\,|\alpha\rangle + 0{,}76\,|\beta\rangle, \quad (1,C,3)$$

and

$$|II\rangle = \overset{1,02}{\cancel{+0{,}92}}\,|\alpha\rangle + \overset{0,80}{\cancel{0{,}71}}\,|\beta\rangle. \quad (1,C,4)$$

ⓐ – ⓐ ⟶ (from p. 19a)

Let us now calculate the overlap $\mathcal{O} = \langle\alpha|\beta\rangle$ between the basis states $|\alpha\rangle$ and $|\beta\rangle$, ~~that is~~ $\mathcal{O} = \cos\chi$ (cf. Fig. 1, C, 2 p. 19b). Following this figure (cf. also caption) one can write,

$$\sqrt{\sigma_I^{tr}} = \cos\phi \quad ; \quad \sqrt{\sigma_{II}^{tr}} = \cos\left(\frac{\pi}{2} - \phi\right) = \sin\phi, \quad (1,C,5)$$

where

$$\sigma^{tr} = \sigma_I^{tr} + \sigma_{II}^{tr} = 1, \qquad (1,C,6)$$

in keeping with the fact that the absolute cross sections to the states $|I\rangle$ and $|II\rangle$ are normalized in terms of the total cross section.

In the same way

$$\sqrt{\sigma_I^{oct}} = \cos(\chi - \phi) = \cos\chi\cos\phi + \sin\chi\sin\phi, \quad (1,C,7)$$

and

$$\sqrt{\sigma_{II}^{oct}} = -\cos\left(\pi - \left(\frac{\pi}{2} - \phi + \chi\right)\right) = -\cos\left(\frac{\pi}{2} + (\phi - \chi)\right),$$
$$= \sin\phi\cos\chi + \sin\chi\cos\phi. \quad (1,C,8)$$

(a) as resulting from the calculation of the diagram displayed in Fig. 1C.1 to all orders with the help of Brillouin-Wigner perturbation theory (diagonalization of the corresponding effective Hamiltonian; cf. p. 316 Bortignon et al 1977). (a)

to p. (19)

(b) A simple estimate of the NFT prediction can be made making use of the relations

$\langle I|I\rangle = (-0.53)^2 + (0.76)^2 - 2\times 0.53\times 0.76\,\theta = 1$,

$\langle II|II\rangle = (1.02)^2 + (0.80)^2 + 2\times 1.02\times 0.80\,\theta = 1$

and $\langle I|II\rangle = -0.53\times 1.02 + 0.76\times 0.80$
$+ (-0.53\times 0.80 + 0.76\times 1.02)\,\theta = 0$ leading to $\theta = -0.18$, -0.42 and -0.19 respectively and, thus, to the average value of -0.26. (b)

to p. (20)

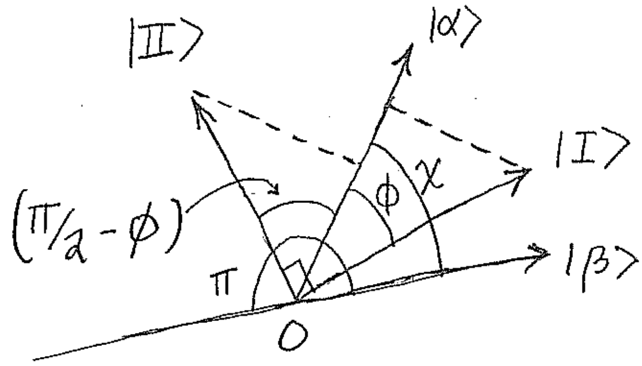


Fig. 1.C.2

schematic representation of the $3/2^+$ states entering the NFT calculation of the process displayed in Fig. 1.C.1. The ~~state~~ basis $|\alpha\rangle$ carries the full (t,α) transfer strength (tr.) σ^{tr}, while the basis state $|\beta\rangle$ the full octupole (oct.) strength σ^{oct} (see Tables 1.C.1 – 1.C.3). The overlap between these states is denoted $\cos\chi$. The physical ~~states~~ obtained through the (Feynman) diagramatic "orthogonalization process" are denoted $|I\rangle$ and $|II\rangle$ (cf. Eqs. (1.C.3) and (1.C.4)

$$|I\rangle = -0.53\,|\alpha\rangle + 0.76\,|\beta\rangle \;, \quad (1.C.3)$$

~~(19)~~ (2) ~~(11)~~

and

$$|II\rangle = -0.92\,|\alpha\rangle - 0.71\,|\beta\rangle \;. \quad (1.C.4)$$

Let us now calculate the overlap $\mathcal{O} = \langle \alpha | \beta \rangle$, ~~overlap~~ between the basis states $|\alpha\rangle$ and $|\beta\rangle$, that is $\mathcal{O} = \cos\chi$ (cf. Fig. 1.C.2)

$$|\alpha\rangle \equiv |d^{-1}_{3/2} \otimes gs(^{210}Po); 3/2^+\rangle = |\,\!\!\!\!\!\!\!\!\!\!\!\!\!\!\;\rangle$$

$$|\beta\rangle \equiv |h_{9/2} \otimes 3^-(^{208}Pb); 3/2^+\rangle = |\,\rangle$$

The ~~ov~~ overlap between the states is denoted $\cos\chi$. The ~~ov~~ physical states obtained through the NFT "orthogonalization process" are denoted $|I\rangle$ and $|II\rangle$ (cf. Eqs. (1.C.3) and (1.C.4))

Schematic representation of the ~~basis~~ $3/2^+$ states entering the

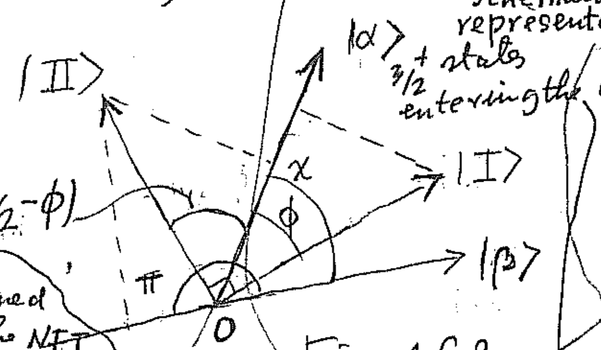


Fig. 1.C.2

The state $|\alpha\rangle$ has the full (t,α) transfer strength (tr.) σ^{tr}, while $|\beta\rangle$ the full octupole (oct.) strength σ^{oct} (see Table 1-3) caption.

NFT calculation of the process displayed in Fig. 1.C.1

One can then write

$$\sqrt{\sigma^{tr}_{I}} = \cos\phi \;; \quad \sqrt{\sigma^{tr}_{II}} = \cos\left(\frac{\pi}{2}-\phi\right) = \sin\phi \quad (1.C.5)$$

where

$$\sigma^{tr.} = \sigma^{tr.}_{I} + \sigma^{tr.}_{II} = 1 \quad (1.C.6)$$

in keeping with the fact that the absolute cross sections to states $|I\rangle$ and $|II\rangle$ are normalized in terms of the total cross section.

In the same way

$$\sqrt{\sigma^{oct}_{I}} = \cos(\chi-\phi) = \cos\chi\cos\phi + \sin\chi\sin\phi \quad (1.C.7)$$

and

$$\sqrt{\sigma^{oct}_{II}} = -\cos\left(\pi - \left(\frac{\pi}{2}-\phi+\chi\right)\right) = -\cos\left(\frac{\pi}{2}+(\phi-\chi)\right)$$

$$= -\sin\phi\cos\chi + \sin\chi\cos\phi \quad (1.C.8)$$

$\sin(a\pm b) = \sin a\cos b \pm \cos a \sin b \qquad \cos(-x) = \cos x$

$\cos(a\pm b) = \cos a\cos b \mp \sin a \sin b \qquad \sin(-x) = -\sin x$

~~33~~ 20

Thus

$$\sqrt{\sigma^{oct}_{I}} = \cos\chi \sqrt{\sigma^{tr}_{I}} + \sin\chi \sqrt{\sigma^{tr}_{II}} \quad (1.C.9)$$

and

$$\sqrt{\sigma^{oct}_{II}} = -\cos\chi \sqrt{\sigma^{tr}_{II}} + \sin\chi \sqrt{\sigma^{tr}_{I}}, \quad (1.C.10)$$

Multiplying the above relations by $\sqrt{\sigma^{tr}_{I}}$ and $\sqrt{\sigma^{tr}_{II}}$ respectively one obtains,

$$\sqrt{\sigma^{tr}_{I}\sigma^{oct}_{I}} = \cos\chi \times \sigma^{tr}_{I} + \sin\chi \sqrt{\sigma^{tr}_{I}\sigma^{tr}_{II}}, \quad (1.C.11)$$

and

$$\sqrt{\sigma^{tr}_{II}\sigma^{oct}_{II}} = -\cos\chi\, \sigma^{tr}_{II} + \sin\chi \sqrt{\sigma^{tr}_{I}\sigma^{tr}_{II}}, \quad (1.C.12)$$

Substracting the above relations leads to the overlap expression

$$\cos\chi = \frac{\sqrt{\sigma^{tr}_{I}\sigma^{oct}_{I}} + \sqrt{\sigma^{tr}_{II}\sigma^{oct}_{II}}}{\sigma^{t}_{I} + \sigma^{t}_{II}} \quad (1.C.13)$$

Making use of the values of Tables 1.C.1 – 1.C.3 (cf. Table 4.7 of (Bortignon el 1977)) one obtains

$$\cos\chi = \frac{\sqrt{1{,}8 \times 4{,}2} - \sqrt{2{,}2 \times 1.1}}{4} = -0{,}298 \quad (1.C.14)$$

(see Table 4.6 Bortignon et al (1977), $M^{(3/2)_1 (3/2)_2}_{(3^-\otimes h_{9/2}) (0^+\otimes d^{-1}_{3/2})} = -0{,}271$)

in overall agreement with the NFT predictions ((~~−0,18~~ −0,27 − ~~0,16~~)/3 ≈ −~~0,20~~
− ~~0,~~42 − 0,19 − 0,26
(b) − (b) from p (19) a

(22)

~~4~~ ~~6~~ 21

Of course we can hardly expect to obtain the sign to agree with the NFT as it is associated with a free choice of the axis of reference (Fig. 1, C, 2) (note also the ~~fact~~ ~~that~~ σ is well defined, while $\sqrt{\sigma}$ is undetermined by an overall sign).

~~Non-orthogonality NFT (elementary modes of excitation) basis~~

(1) ~~7~~ (22)

$$|3/2_1\rangle \approx -0.53\, |\!\uparrow\!\downarrow\!\downarrow^{\,d_{3/2}}\rangle + 0.76\, |3^- \uparrow^{h_{9/2}}\rangle$$

of

$$|3/2_2\rangle = -0.92\, |\!\uparrow\!\downarrow\!\downarrow\rangle - 0.71\, |3^- \uparrow^{h_{9/2}}\rangle$$

of $d_{3/2}$

alternative

another way to work out the overlap $O = \langle\alpha|\beta\rangle$ can be determined from the ~~normali~~ orthonormalization conditions ~~of the states~~ fulfilled by the states $|I\rangle$ and $|II\rangle$.

$$\langle I|I\rangle \approx (0.53)^2 + (0.76)^2 - 2\times 0.53\times 0.76\, O = 1 \quad (1.C.15)$$

Thus,

$$0.859 - 0.806\, O = 1 \quad (1.C.16)$$

and

$$O = -\frac{0.141}{0.806} \approx -0.18 \quad (1.C.17)$$

norm

~~like~~

NMR

Similarly,

$$\langle II|II\rangle \approx (0.92)^2 + (0.71)^2 + 2\times 0.92\times 0.71\, O = 1 \quad (1.C.18)$$

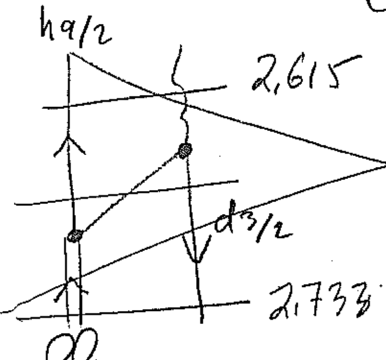
leading to

$$1.351 + 1.306\, O = 1 \quad (1.C.19)$$

and

$$O = \frac{-0.306}{1.351} \approx -0.27. \quad (1.C.20)$$

~~From state~~ In fact, neglecting the term proportional to O one obtains,



~~$M_{1,2} = -0,271$ see Table 4.6~~

~~[Phys. Rep.]~~

Finally,

$$\langle I|II\rangle \approx \langle 3/2_1|3/2_2\rangle = 0 = (0.53\times 0.92) - (0.76\times 0.71) \quad (1.C.21)$$

$$+\,2(-0.53\times(-0.71) - 0.92\times 0.76)\, O$$

$$= -0.052 - 0.32300\, O = 0 \quad (1.C.22)$$

leading to

$$O = -\frac{0.052}{0.323} \approx -0.16 \quad (1.C.23)$$

orth.

8

23

	E_x (MeV)	$S(t,\alpha)$ $(2j+1)$		$S(^3He,d)$
$3/2^+$	2.49	1.8 ± 0.3	(4)	<0.01
$3/2^+$	2.95	2.2 ± 0.3		<0.01
$1/2^+$	2.43	1.8	(2)	<0.02
$11/2^-$	3.69	10.0	(12)	<0.05

87.8

single-particle transfer

Table ~~A~~1.C.1 ~~Relative~~ Experimental value of the energy and of the relative cross sections associated with low-lying states ex. of ^{209}Bi populated in the reaction $^{210}Po(t,\alpha)^{209}Bi$

		$\dfrac{\sigma(^{209}Bi(9/2^-;gs) \to {}^{209}Bi(3/2^+;E))}{\sigma(^{208}Pb(gs) \to (3^-;2.615\ MeV))}$
3/2	2.49	0.041 ± 0.003
3/2	2.95	0.011 ± 0.002

Table ~~A~~ 1.C.2 Relative ~~inelastic~~ octupole cross sections associated with the inelastic excitation of the two lowest-lying $3/2^+$ states of ^{209}Bi (after Bortignon et al 1977 and refs. therein)

	E_n (MeV)		$\frac{\sigma(h9/2 \to 3/2^+)}{\sigma(0^+ \to 3^-)}$ (%)		$S(t,\alpha)$		$S(^3He,d)$	
	Theory	Ex	Theory	Exp	Theory	Exp.	Theory	Exp
3/2	~~3.76~~ 2.480	2.494	3.76	4.2 ± 0.3	1.83	1.8 ± 0.3	0.02	<0.01
3/2	~~1.56~~ 3.125	2.95	1.56	1.1 ± 0.2	2.25	2.2 ± 0.3	10^{-5}	<0.01

Table. ~~A~~1.C.3

Resumé of NFT prediction of the structure and reaction properties of the lowest-lying $3/2^+$ state of ^{209}Bi in comparison with the experimental data (after Bortignon et al 1977 and refs therein; see also Tables 1.C.1 and 1.C.2).

V ~~Table 2~~ C.2

(total

The ~~full~~ inelastic cross section σ^{oct} (9) (24) associated with the octupole vibrational state of ^{208}Pb can be written in terms of that associated with a single magnetic substate σ' as $\sigma^{oct}_{3^-} = 7\sigma'$. That associated with the multiplet $(h_{9/2} \otimes 3^-)_{J^+}$ $(J = 3/2 - 15/2)$ as $\sigma^{oct} = 70\sigma'$, in keeping with the fact tha the $h_{9/2}$ state has 10 magnetic substates. Thus, the strength associate with the 3/2 channel is $4/70 = 0.057$, to be compared with the observed (percentage) summed strength 0.053 ± 0.005 $(= 0.042 \pm 0.003 + 0.011 \pm 0.002$ associated with the 2.45 MeV and the 2.95 MeV $3/2^+$ state; see Bortignon et al 1977). (Table 4.11

V ~~Table 1~~ C.1

~~[illegible]~~ ~~[illegible]~~ Single-particle strength associated with the single-particle transfer reactions $^{210}Po(t,\alpha)^{209}Bi$ and $^{208}Pb(^3He,d)^{209}Bi$ (cf. Bortignon et al 1977). Table 4.11)

V ~~Table 3~~ C.3

Sumary of NFT ~~calculations~~ predictions ~~and~~ ~~for the two lowest $3/2^+$~~ concerning the structure of the two lowest $3/2^+$ state of ^{209}Bi, in comparison with the experimental data (see Table 4.7 of Bortignon et al 1977)

$\boxed{X = \frac{G\,2\Omega}{D} = G N(0) = \lambda}$

appendix ?

Competition between the variety of those associated ZPF, in particular density ($\beta=0$) and pairing ($\beta=\pm 2$) vibrational modes

16 25

~~The single-particle dressing through the vibrations~~

~~Vibra~~

Particle-hole like (Nilsson-like) vibrations, like e.g. quadrupole vibrations, induce dynamical distortions of the mean field which break virtually the $(2j+1)$ magnetic degeneracy of levels with two-fold (Kramer's) degenerate levels, thus effectively reducing the Density of states (DOS) around the Fermi energy.

Fig 1

?

In Fig. 1 is given a schematic representation of the subtle effects the interweaving of single-particle motion and collective vibrations has on pairing correlations

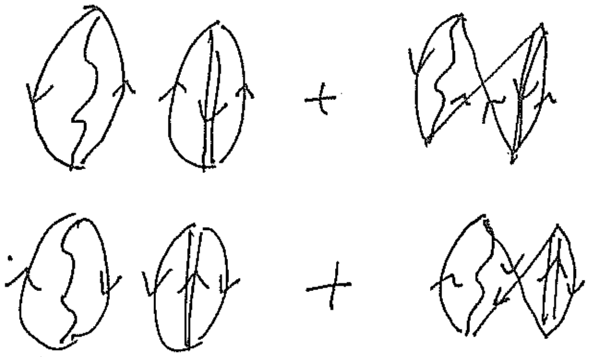
~~too large~~

6 Pairing vibrations smooth out the sharp discontinuity at the Fermi energy displayed by ~~non-interacting~~ closed shell systems, thus effectively concentrating in an effective single j-shell through dynamical $U_j V_j$ weighting factors, the global degeneracy of levels ~~around~~ in the ~~interval~~ energy region

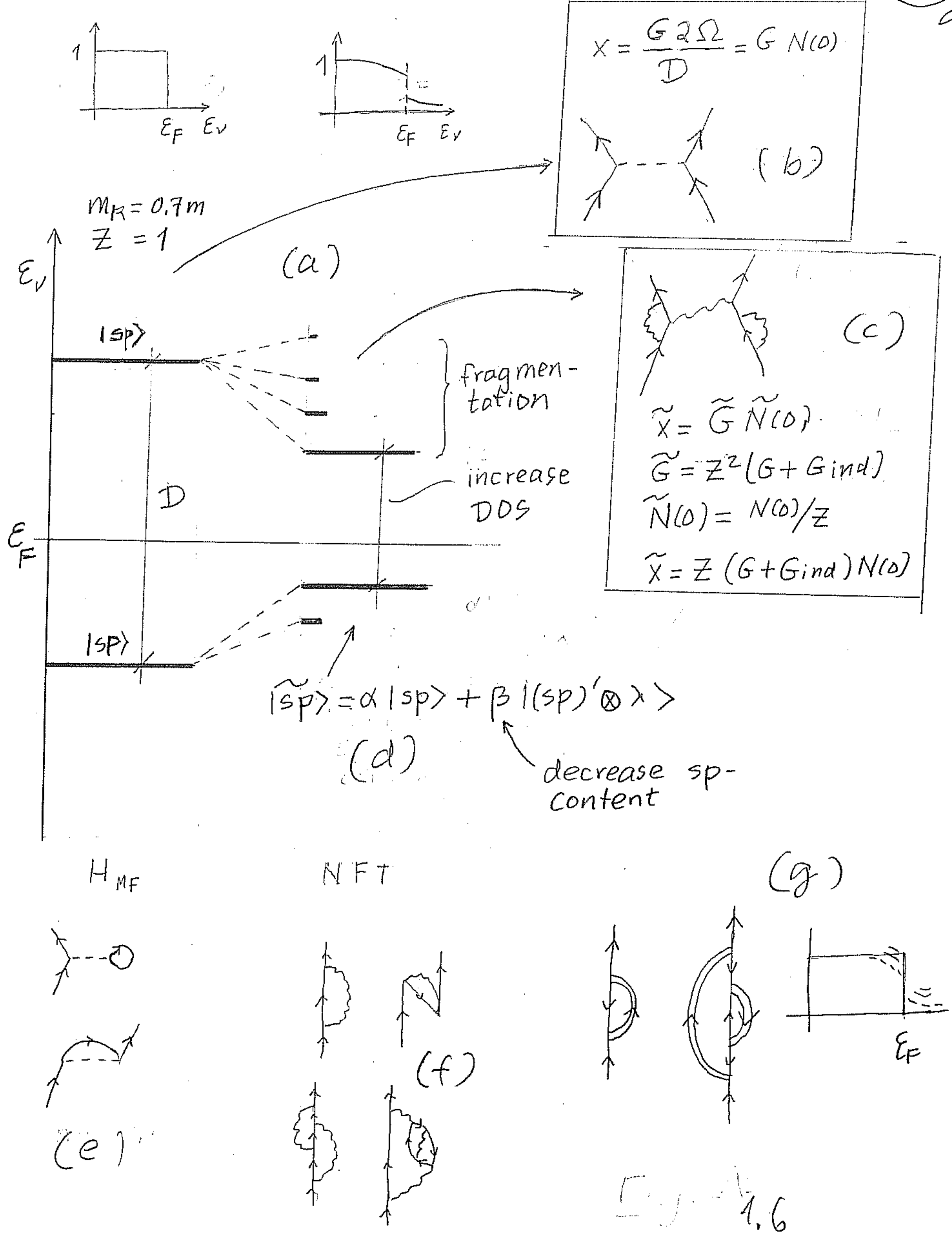
$\epsilon_F \pm E_{corr}(\beta = \pm 2)$ (Fig 1(g)) 1.6

17 26

ZPF ~~of~~ induced by ~~p-h~~ particle-hole like and pairing modes compete with each other, through Pauli principle



Thus leading to a single ground ~~state~~ containing all the dressed, renormalized ZPF (cf. app. A)



1
ε_F ε_ν
1
ε_F ε_ν
$x = \frac{G\, 2\Omega}{D} = G\, N(0)$
(b)
$m_k = 0.7m$
$Z = 1$
(a)
ε_ν
$|sp\rangle$
fragmentation
D
increase DOS
ε_F
$|sp\rangle$
(c)
$\tilde{x} = \tilde{G}\,\tilde{N}(0)$
$\tilde{G} = Z^2(G + G_{ind})$
$\tilde{N}(0) = N(0)/Z$
$\tilde{x} = Z\,(G + G_{ind})\,N(0)$
$|\widetilde{sp}\rangle = \alpha\,|sp\rangle + \beta\,|(sp)' \otimes \lambda\rangle$
(d)
decrease sp-content
H_{MF}
NFT
(g)
(e)
(f)
ε_F

Fig. ~~1~~ 1.6

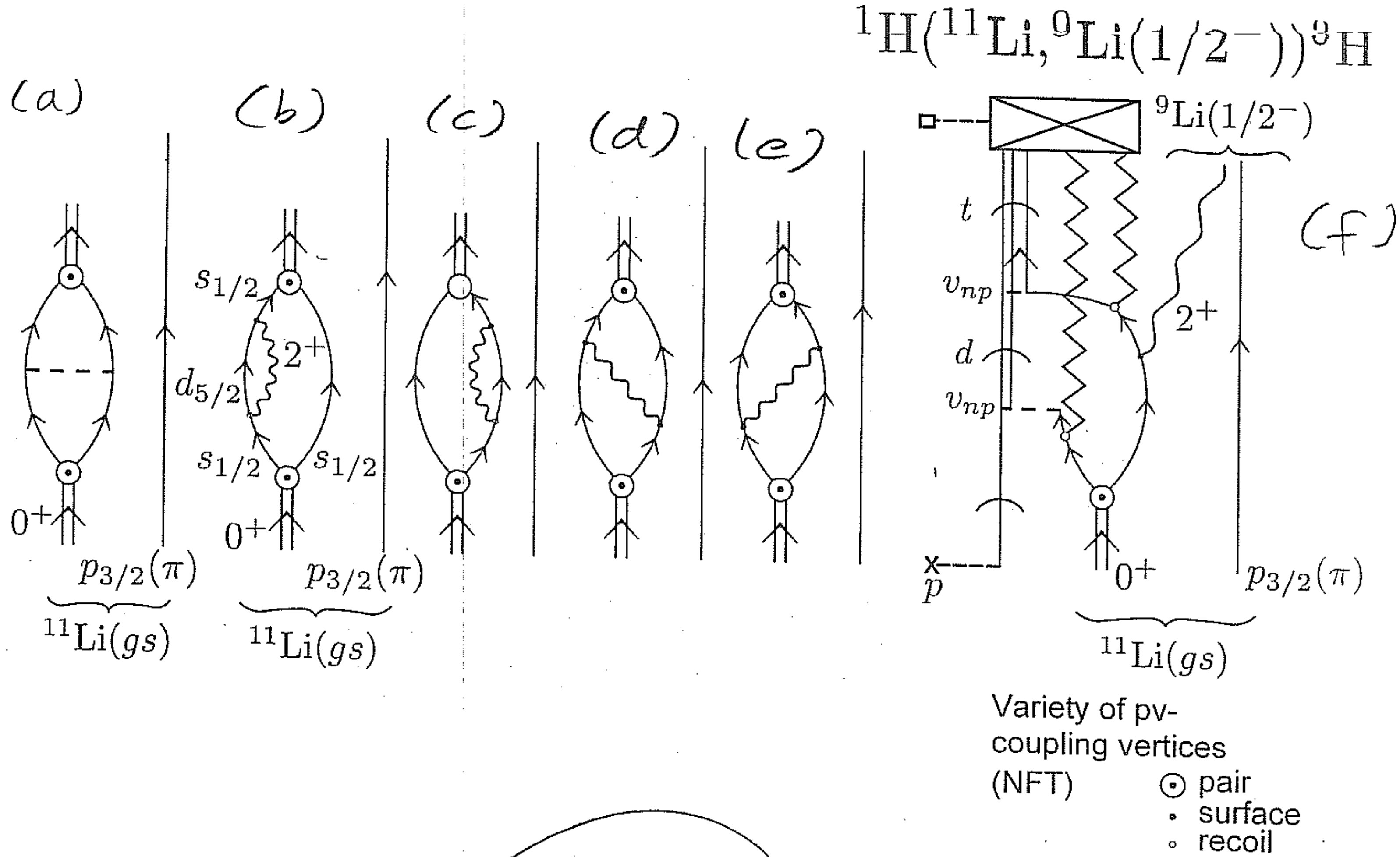
Fig. A 1.6

26

Schematic representation ; (e), (f) of some of the consequences the interweaving of the elementary modes of excitation with varied transfer quantum number ($\beta = 0, \pm 1, \pm 2$) have in the (mainly single-particle) nuclear spectrum,(a) in particular pair correlations,(b,c) as measured by the (two-level) dimensionless parameter $x = G 2\Omega / D = G N(0)$, product of the bare coupling constant G and the density of states (DOS) at the Fermi energy (ratio of ~~the~~ single-particle degeneracy $2\Omega = (2j+1)$, and the single-particle energy separation). Coupling with surface modes (f) reduce the effective value of D leading to an increase of $N(0)$ (as measured by $1/Z$) but, at the same time decreases, through the breaking of the single-particle strength, the single-particle content(d) of each level (as measured by Z). The eventual increase of x, as reflected by $\tilde{x}$, results from a delicate balance of the two effects eventually overwhelmed by the induced pairing interaction resulting from the exchange of collective ($\beta = 0$) vibration between pair of nucleons

bold face

moving in time reversal states,
and by the dynamical smoothing 26
C
of the Fermi energy by the coupling
of single-particle states to $\beta = \pm 2$
pairing vibration (g).



1,5

≠

Fig. ~~4~~

Testo

Fig 1,5

27

; ^{9}Li pair addition mode

Caption to Fig. ~~4~~ 5

(28)

(doubled arrowed line)

Lowest order, NFT diagrams associated with the processes gluing the neutron halo Cooper pair of ^{11}Li to the core ^{9}Li through the exchange of the core quadrupole phonon (wavy line); single arrowed lines describe the nucleon independent-particle motion of neutrons ($1s_{1/2}$, $d_{5/2}$, etc) as well as of protons ($p_{3/2}(\pi)$).

(a) Bare interaction, four-point vertex (horizontal dashed line); (b,c) self energy, effective mass process dressing the $s_{1/2}(\nu)$ single-particle ~~state~~; (d,e) vertex correction (induced interaction) renormalizing the pair addition mode coupling vertex with which it couples to the fermion (dotted open circle); (f) NFT diagrams describing the reaction $^{1}H(^{11}Li, ^{9}Li(1/2^-; 2.69\,MeV))^{3}H$ populating the first excited ~~state~~ of ^{9}Li.

The jagged line is the recoil phonon carrying asymptotically to the detector the momentum missmatch associate with the transfer process. In this case of successive transfer, one for each transferred neutron ($^{11}Li(gs)+p \to {}^{10}L + d \to {}^{9}Li(1/2^-)+t$)

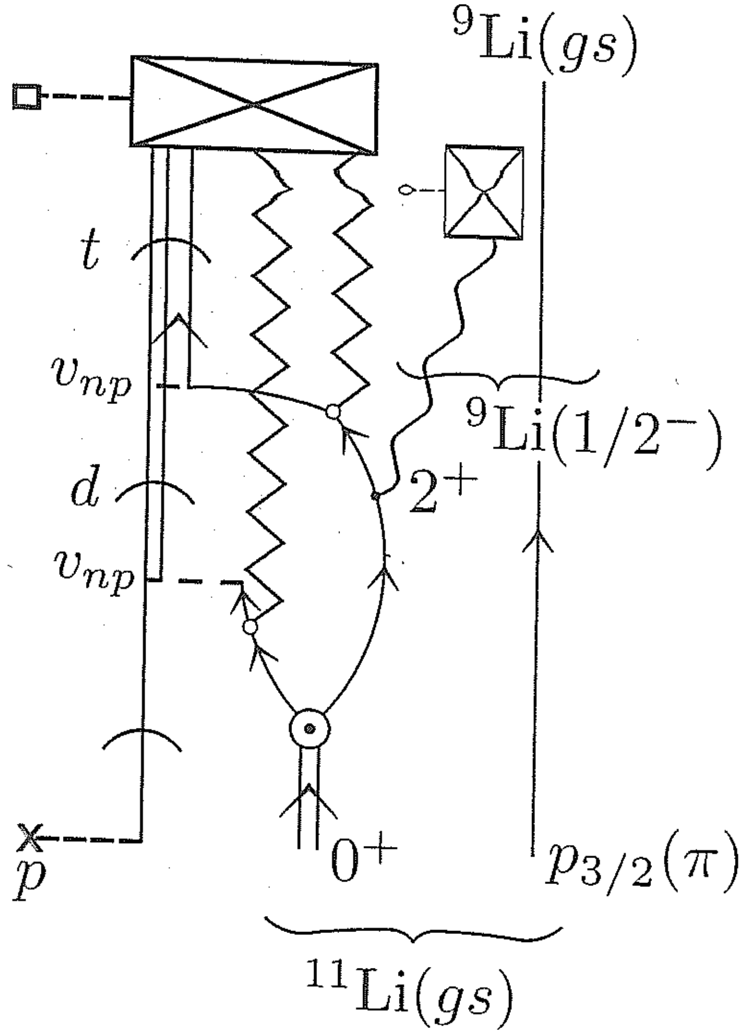


Fig. 1.6

~~5~~ 6

Fig~~5~~

Variety of pv-
coupling vertices
(NFT)

⊙ pair
• surface
∘ recoil

Testo

29

Fig ~~13~~ ~~4~~ 6

(30)

Gedanken γ-ray coincidence experiment $^1H(^{11}Li, ^9Li)^3H$ and $^9Li(gs) + \gamma(E2; 2.69\,MeV)$. In this case, the virtual quadrupole phonon associated with ~~self~~-energy and vertex correction processes ~~due to the~~ becomes real ~~(asymptot by~~ ~~by intervening the~~ through the action of the (pit) external field. Thus, it is not only the recoil phonons measured by particle detectors ~~(external)~~ which have asymptotic wavefunction, but also the quadrupole vibration, which is ~~det~~ ~~reveal~~ measured by the γ-detector. For detail see Caption Fig. ~~d~~ 4.