Similarly,

$$\Gamma_i^\dagger = a_{\bar{i}} a_i, \quad (\epsilon_i \leq \epsilon_F).$$

The relations

$$[H, \Gamma_a^\dagger(n)] = \hbar W_n(\beta = +2),$$

and

$$[H, \Gamma_r^\dagger(n)] = \hbar W_n(\beta = -2),$$

where β is the transfer quantum number, while n labels the roots of the corresponding dispersion relations[53],

$$\frac{1}{G(\pm 2)} = \sum_k \frac{(\Omega_k/2)}{2\epsilon_k \mp W_n(\pm 2)} + \sum_i \frac{(\Omega_i/2)}{2\epsilon_i \pm W_n(\pm 2)},$$

n labeling the corresponding solutions in increasing order of energy. In the above equation, $\Omega_j = j + 1/2$ is the pair degeneracy of the orbital with total angular momentum j.

For the case of the (neutron) pair addition and pair substraction modes of ^{208}Pb the above equations are graphically solved in Fig 2.5.1 (see also Table 2.5.1). The minimum of the dispersion relation defines the Fermi energy of the system under study. This is in keeping with the fact that in the case in which $W_1(\beta = +2) = W_1(\beta = -2) = 0$, situation corresponding to the transition between normal and superfluid phases, the energy value at which the dispersion relation touches for the first time the energy axis, coincides with the BCS λ variational parameter. It is of notice that, as a rule, the Fermi energy of closed shell nuclei is empirically defined as half the energy difference between the last occupied and the first empty single particle state[54]. Making use of the values (see Fig. 2.5.1)

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and Table 2,5,1 see attachment

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$$\begin{cases} E_{corr}(+2) = B(208) + B(210) - 2B(209) = 1.248\,\text{MeV}, \\ E_{corr}(-2) = B(208) + B(206) - 2B(207) = 0.630\,\text{MeV}, \end{cases}$$

one obtains $W_1(+2) + W_1(-2) = (B(208) - B(206)) - (B(210) - B(208)) = 14.11 - 9.115 = 4.995\,\text{MeV}$. Notice that in the above calculations all energies differences are positive. In particular (see Table 2.5.1)

$$\epsilon_i < \epsilon_F \Rightarrow \epsilon_F - \epsilon_i = -|\epsilon_F| + |\epsilon_i| = |\epsilon_i| - |\epsilon_F| > 0,$$

and

$$\epsilon_k > \epsilon_F \Rightarrow \epsilon_k - \epsilon_F = -|\epsilon_k| + |\epsilon_F| = |\epsilon_F| - |\epsilon_k| > 0.$$

Thus,

$$\begin{cases} 2(\epsilon_F - \epsilon_{p_{1/2}}) = W_1(-2) + E_{corr}(-2) > 0, \\ 2(\epsilon_{g_{9/2}} - \epsilon_F) = W_1(+2) + E_{corr}(+2) > 0. \end{cases}$$

[53] cf. Bès, D. R. and Broglia (1966).
[54] cf. e.g. Mahaux, C. et al. (1985).

A	BE/A (MeV)	BE (MeV)
206	7875,365	1622,32519
207	7869,869	1629,06288
208	7867,456	1636,43085
209	7848,652	1640,36827
210	7835,968	1645,55328

Table 2.5.1

Binding energy BE in MeV for a number of the ^{A}Pb-isotopes. In the first column the mass number, sum of neutron and proton number is given ($A = N + Z$), while in the second column the binding energy per nucleon is reported (Iterative chart nuclear data center).

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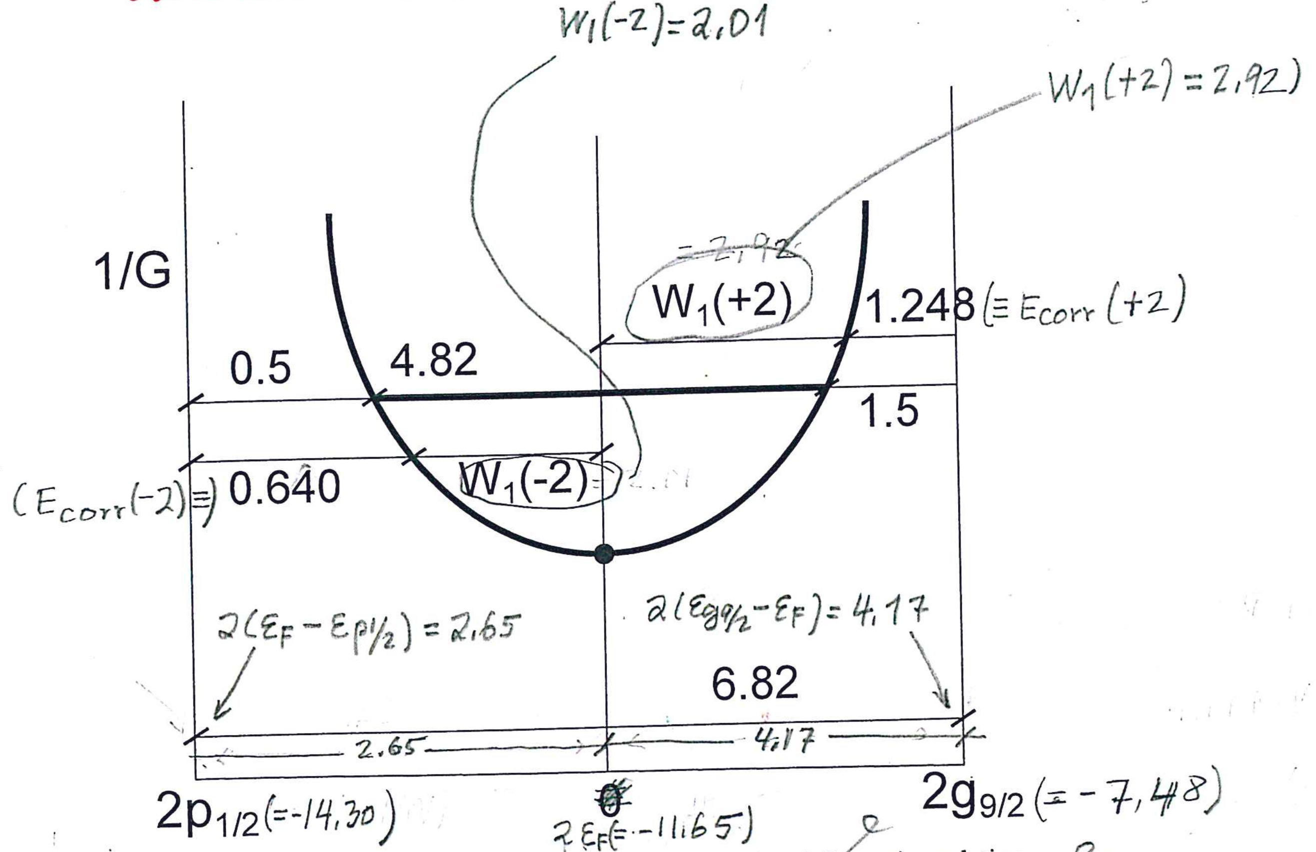

Figure 2.5.1: The right hand side of the RPA pairing vibrational dispersion relation for neutrons in the case of the closed shell system ^{208}Pb (cf. Bès, D. R. and Broglia (1966)) in the region between the two neighboring shells ($p_{1/2}$ and $g_{9/2}$). All quantities are in MeV. For each G there is a straight horizontal line, which is divided by the the curve in three sections. The first one from the left corresponds to the pairing correlation energy of the nucleus ^{206}Pb (two correlated neutron hole states) while the last segment to the right measures the pairing correlation energy of ^{210}Pb (two correlated neutrons above closed shell) the intermediate segment measures the energy of the two phonon (correlated $(2p - 2h)$) pairing vibrational state of ^{208}Pb.

The corresponding energies ($\varepsilon_{p1/2} = -7{,}15$ and $\varepsilon_{g9/2} = -3{,}74$) are taken from Table 2.5.1; as well as the Fermi energy $\varepsilon_F = -5{,}825$ MeV, define by the minimum of the dispersion relation.