Nuclear Structure and Reactions

pairing in nuclei with Cooper pair transfer

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COOPER
ONE
KNOCK
INELASTIC
COULOMB

Chapter 1

Preface

The elementary modes of nuclear excitation are vibrations and rotations, single–particle (quasiparticle) motion, and pairing vibrations and rotations. The specific reactions probing these modes are inelastic and Coulomb excitation, and single– and two– particle transfer processes respectively.

Pairing vibrations and rotations, closely connected with nuclear superfluidity are, arguably, a paradigm of quantal nuclear phenomena. They thus play a central role within the field of nuclear structure. It is only natural that two–nucleon, Cooper pair, transfer plays a similar role concerning direct nuclear reactions. In fact, this is the central subject of the present monograph.

At the basis of pairing phenomena one finds Cooper pairs, weakly bound, extended, strongly overlapping bosonic entities, made out of pairs of nucleons dressed by collective vibrations and interacting through the exchange of these vibrations as well as through the bare NN–interaction. (correlation length)

Cooper pairs not only change the statistics of the nuclear stuff around the Fermi surface and, condensing, the properties of nuclei close to their ground state. They also display a rather remarkable mechanism of tunnelling between the weakly interacting nuclei acting as target and projectile in a direct two–nucleon transfer reaction. In fact, displaying correlations over distances much larger than nuclear dimensions, Cooper pairs are forced to be confined within such dimensions by the action of the average potential, which can be viewed as an external field as far as Cooper pairs are concerned.

The correlation length paradigm comes into evidence, for example,

~~The situation is radically altered~~ when two nuclei are set into weak contact in a direct reaction. In this case, each of the partner nucleons of a pair has a finite probability to be confined within the mean field of the target and of the projectile. It is then natural that a Cooper pair can tunnel, equally well correlated, between target and projectile, through a simultaneous than through a successive transfer process. In particular, in this last case, making use of virtual states which, if forced to become real by intervening the reaction with an external mean field, will lead to single–nucleon transfer processes. The above mentioned weak coupling Cooper pair tunnelling ~~sounds quite similar to~~ reminds the tunnelling mechanism of electron Cooper pairs across a barrier (e.g. a dioxide layer) separating two superconductors, known as Josephson junction. The main difference is that, as a rule, in the nuclear time dependent junction provided by a direct two–nucleon transfer process, only one or even none of the two weakly interacting nuclei are superfluid (or superconducting). Now, in nuclei, ~~paradigm of~~ paradigmatic example of fermionic finite many–body system, zero point fluctuation (ZPF) in general, and those associated with pair addition and pair substraction modes known as pairing vibrations, (in particular,) are much stronger than in condensed matter.

Consequently, and in keeping with the fact that

for example

Cooper pair tunneling has played and is playing a central role in the probing

even

~~Because~~ pairing vibrations are the nuclear embodiment of Cooper pairs in nuclei, pairing correlations based on a single Cooper pair lead to clearly observable effects. In ~~the~~ s~~a~~ome cases, like ~~e.g.~~ in connection with the exotic nucleus ^{11}Li, to a tenuous halo extending much beyond standard nuclear dimensions. ~~Realization~~ of these subtle quantal phenomena in the case of both exotic nuclei as well as nuclei lying along the stability valley have been instrumental in shedding light on the subject of pairing in nuclei at large, and ~~in~~ on nuclear superfluidity in particular. Consequently, ~~they occupy~~ a central place in the present monograph, both concerning the conceptual and the computational aspects of the description of ~~the associated phenomena~~, as well as regarding the quantitative confrontation of the results and predictions with the experimental findings.

and

of

nuclear pairing

the subject of two-nucleon transfer reactions occupies

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Paris, November 2013

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Milan, November 2013

Because the interweaving of the variety of elementary modes of nuclear excitation, the study of Cooper pair tunneling in nuclei involves also the description of one-nucleon transfer as well as knock out processes, let alone inelastic and Coulomb excitation processes.

The corresponding softwares COOPER, ONE, KNOCK, INELASTIC AND COULOMB are briefly presented, referring ~~for referring~~ to the enclosed CD for the corresponding files and input-output examples.

Summing up, general physical arguments and technical computational details, as well as the software used in the description and calculation of the absolute two-nucleon transfer cross sections, making use of state of the art nuclear structure information, are provided.

As a consequence, theoretical and experimental practioners, as well as PhD students could use the present monograph at profit.

One-nucleon transfer

Corrections introduced directly in the text
Febr 11, 2013 Gregory Potel Aguilar

Chapter 6

One-particle transfer

C-C → handwritten pp 1 and 2 19/05/13

nuclei ~~nucleus~~ A and b respectively

and $\vec{r}_{An}$ and $\vec{r}_{bn}$ the relative coordinate of the transferred nucleon with respect to the CM of.

Denoting ξ_A and ξ_b the intrinsic wavefunctions describing the structure of nuclei A and b respectively, one can write

6.1 General derivation

We want to derive the transition amplitude for the reaction

$$A + a(= b+1) \longrightarrow B(= A+1) + b. \tag{6.1}$$

Let us assume that the nucleon bounde~~d~~ ~~initially~~ is to the core b is in a single–particle state with orbital and total angular momentum l_i and j_i respectively, and that in the final state (bounded to core A) it ~~will be~~ in a l_f, j_f state. The total spin and magnetic quantum numbers of nuclei A, a, B, b are $\{J_A, M_A\}$; $\{J_a, M_a\}$; $\{J_B, M_B\}$; $\{J_b, M_b\}$. ~~Thus,~~ the ~~initial~~ wavefunctions of the colliding nuclei A, a ~~are~~ as

$$\begin{aligned}
&\phi^{J_A}_{M_A}(\xi_A),\\
&\Psi(\xi_b, \mathbf{r}_{b1}) = \sum_{m_i} \langle J_b\ j_i\ M_b\ m_i | J_a\ M_a\rangle \phi^{J_b}_{M_b}(\xi_b)\psi^{j_i}_{m_i}(\mathbf{r}_{bn}, \sigma),
\end{aligned} \tag{6.2}$$

the intrinsic describing the structure of nuclei

while the wavefunctions ~~for~~ B and b ~~will be~~ are of the residual nuclei

$$\begin{aligned}
&\phi^{J_b}_{M_b}(\xi_b),\\
&\Psi(\xi_A, \mathbf{r}_{A1}) = \sum_{m_f} \langle J_A\ j_f\ M_A\ m_f | J_B\ M_B\rangle \phi^{J_A}_{M_A}(\xi_A)\psi^{j_f}_{m_f}(\mathbf{r}_{An}, \sigma).
\end{aligned} \tag{6.3}$$

assuming that

For ~~an~~ unpolarized incident beam (sum over M_A, M_a and ~~division~~ dividing by $(2J_A + 1)(2J_a + 1)$) and ~~if~~ ~~we do~~ one does not detect the final polarization (sum over M_B, M_b), ~~the differential cross section is~~ the differential cross section in the DWBA can be written as

$$\begin{aligned}
\frac{d\sigma}{d\Omega} &= \frac{k_f}{k_i}\frac{\mu_i\mu_f}{4\pi^2\hbar^4}\frac{1}{(2J_A+1)(2J_a+1)}\\
&\times \sum_{\substack{M_A,M_a\\M_B,M_b}} \left| \sum_{m_i,m_f} \langle J_b\ j_i\ M_b\ m_i | J_a\ M_a\rangle \langle J_A\ j_f\ M_A\ m_f | J_B\ M_B\rangle T_{m_i,m_f}\right|^2.
\end{aligned} \tag{6.4}$$

The transition amplitude T_{m_i,m_f} is

$$T_{m_i,m_f} = \sum_\sigma \int d\mathbf{r}_f d\mathbf{r}_{bn} \chi^{(-)*}(\mathbf{r}_f)\psi^{j_f*}_{m_f}(\mathbf{r}_{An}, \sigma)V(r_{bn})\psi^{j_i}_{m_i}(\mathbf{r}_{bn}, \sigma)\chi^{(+)}(\mathbf{r}_i). \tag{6.5}$$

The incoming and outgoing distorted waves are

$$\chi^{(+)}(\mathbf{k}_i, \mathbf{r}_i) = \frac{4\pi}{k_i r_i}\sum_{l'} i^{l'} e^{i\sigma^{l'}_i} g_{l'}(\hat{r}_i)\left[Y^{l'}(\hat{r}_i)Y^{l'}(\hat{k}_i)\right]^0_0, \tag{6.6}$$

and

$$\chi^{(-)*}(\mathbf{k}_f, \mathbf{r}_f) = \frac{4\pi}{k_f r_f}\sum_{l} i^{-l} e^{i\sigma^{l}_f} f_{l}(\hat{r}_f)\left[Y^{l}(\hat{r}_f)Y^{l}(\hat{k}_f)\right]^0_0, \tag{7}$$

where

$$\psi^{j_i}_{m_i}(r_{bn}, \sigma) = u_{j_i}(r_{bn})\left[{}^3Y^{l_i}(\hat{r}_i)\chi(\sigma)\right]_{j_i m_i}$$

coupled channel Hamamoto Jens Bang

In what follows we present a derivation of the one-particle transfer differential cross section within the framework of the DWBA (cf. Eqs. (6.4) and (6.21)-(6.25)).

The structure input for the calculations are mean field potentials and single-particle states dressed through the coupling with the variety of collective, (quasi) bosonic nuclear degrees of freedom. With the help of these elements, and of optical potentials, one can calculate the absolute differential cross sections, [redacted] quantities which can be directly compared with the experimental findings.

In this way one avoids to introduce, let alone use spectroscopic factors, quantities which are rather elusive to define. This is in keeping with the fact that as a nucleon moves through the nucleus it feels [redacted] the presence of the other nucleons whose configurations change as time proceeds. It takes time for this information to be fed back on the nucleon. This renders the average potential nonlocal in time. A time-dependent operator can always be transformed into an energy dependent operator, [redacted] implying an ω-dependence of the properties which are usually adscribed to particles like (effective) mass, charge, etc. [redacted] Furthermore, due to the Pauli principle, the average potential is also non local in space (cf. App. 6.A and 6.B)

August Schwenk fellow TRIUMPH

Consequently, one is forced to deal with nucleons which carry around a cloud of (quasi) bosons, aside from continuously and instantaneously exchanging its position with that of the other nucleons. ~~Consequently~~

It is of notice that the above ~~issues~~ questions are ~~common~~ not only found within the realm of nuclear physics, but are common within the framework of many-body systems as well a field theories like quantum electrodynamic (QED). In fact, a basic result of such theories is ~~that~~ nothing is free. A textbook example ~~being~~ is provided by the ~~from~~ Lamb shift, resulting from the dessing of the hydrogen atom electron, as a result of ~~(Pauli principle)~~ the exchange of such electron with those participating in the spontaneous virtual excitation (ZPF) of the QED vaccum (Cf. app 6.C).

Within this context see Sect. 6.3 (applications) concerning the phenomenon of parity inversion in N=6 ~~nuclei~~ (closed shell) exotic halo nuclei.

c

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respectively. Now,

$$\left[Y^l(\hat{r}_f)Y^l(\hat{k}_f)\right]_0^0\left[Y^{l'}(\hat{r}_i)Y^{l'}(\hat{k}_i)\right]_0^0=\sum_K((ll)_0(l'l')_0|(ll')_K(ll')_K)_0$$
$$\times\left\{\left[Y^l(\hat{r}_f)Y^{l'}(\hat{r}_i)\right]^K\left[Y^l(\hat{k}_f)Y^{l'}(\hat{k}_i)\right]^K\right\}_0^0. \tag{8}$$

notation

The $9j$ symbol can be explicitly evaluated to be

?

$$((ll)_0(l'l')_0|(ll')_K(ll')_K)_0=\sqrt{\frac{2K+1}{(2l+1)(2l'+1)}}, \tag{9}$$

and the angular momenta coupling is

$$\left\{\left[Y^l(\hat{r}_f)Y^{l'}(\hat{r}_i)\right]^K\left[Y^l(\hat{k}_f)Y^{l'}(\hat{k}_i)\right]^K\right\}_0^0=\sum_M\langle K\;K\;M\;-M|0\;0\rangle\left[Y^l(\hat{r}_f)Y^{l'}(\hat{r}_i)\right]_M^K$$
$$\times\left[Y^l(\hat{k}_f)Y^{l'}(\hat{k}_i)\right]_{-M}^K=\sum_M\frac{(-1)^{K+M}}{\sqrt{2K+1}}\left[Y^l(\hat{r}_f)Y^{l'}(\hat{r}_i)\right]_M^K\left[Y^l(\hat{k}_f)Y^{l'}(\hat{k}_i)\right]_{-M}^K. \tag{10}$$

Thus,

$$\left[Y^l(\hat{r}_f)Y^l(\hat{k}_f)\right]_0^0\left[Y^{l'}(\hat{r}_i)Y^{l'}(\hat{k}_i)\right]_0^0=$$
$$\sum_{K,M}\frac{(-1)^{K+M}}{\sqrt{(2l+1)(2l'+1)}}\left[Y^l(\hat{r}_f)Y^{l'}(\hat{r}_i)\right]_M^K\left[Y^l(\hat{k}_f)Y^{l'}(\hat{k}_i)\right]_{-M}^K. \tag{11}$$

For the angular integral to be different from zero, the integrand must be coupled to zero angular momentum (scalar). Noting that the only integrated variables in the above expression are $\hat{r}_i, \hat{r}_f$, we have to couple the remaining functions of the angular variables, namely the wavefunctions $\psi^{j_f*}_{m_f}(\mathbf{r}_{An},\sigma)=(-1)^{j_f-m_f}\psi^{j_f}_{-m_f}(\mathbf{r}_{An},-\sigma)$ and $\psi^{j_i}_{m_i}(\mathbf{r}_{bn},\sigma)$ to angular momentum K, as well as to fulfill $M=m_f-m_i$. Let us then consider

$$(-1)^{j_f-m_f}\psi^{j_f}_{-m_f}(\mathbf{r}_{An},-\sigma)\psi^{j_i}_{m_i}(\mathbf{r}_{bn},\sigma)=(-1)^{j_f-m_f}u_{j_f}(r_{An})u_{j_i}(r_{bn})$$
$$\times\sum_P\langle j_f\;j_i\;-m_f\;m_i|P\;m_i-m_f\rangle\left\{\left[Y^{l_f}(\hat{r}_{An})\chi^{1/2}(-\sigma)\right]^{j_f}\left[Y^{l_i}(\hat{r}_{bn})\chi^{1/2}(\sigma)\right]^{j_i}\right\}^P_{m_i-m_f}. \tag{12}$$

?

Recoupling ~~Now we recouple together~~ the spherical harmonics to angular momentum K and the spinors to S, ~~so the term that survives the integral is~~ ~~one obtains~~ only one term survives the angular integral in (5), namely

$S=0$

$$(-1)^{j_f-m_f}u_{j_f}(r_{An})u_{j_i}(r_{bn})((l_f\tfrac{1}{2})_{j_f}(l_i\tfrac{1}{2})_{j_i}|(l_fl_i)_K(\tfrac{1}{2}\tfrac{1}{2})_0)_K$$
$$\times\langle j_f\;j_i\;-m_f\;m_i|K\;m_i-m_f\rangle\left[Y^{l_f}(\hat{r}_{An})Y^{l_i}(\hat{r}_{bn})\right]^K_{m_i-m_f}[\chi(-\sigma)\chi(\sigma)]^0_0. \tag{13}$$

? χ

$\boxed{\chi_{m_s}(\sigma)}$?

$\sigma\equiv m_s$

Making use of the fact that the

. ~~The~~ sum over spins yields a factor $-\sqrt{2}$, ~~and so far we have (remember that $M = m_f - m_i$)~~ and in keeping with the fact that $M = m_f - m_i$, one obtains,

$$
\begin{aligned}
T_{m_i,m_f} = (-1)^{j_f-m_f}\frac{-16\sqrt{2}\pi^2}{k_f k_i}\sum_{ll'} i^{l'-l} e^{\sigma^l_f+\sigma^{l'}_i}\sum_K ((l_f\tfrac{1}{2})_{j_f}(l_i\tfrac{1}{2})_{j_i}|(l_f l_i)_K(\tfrac{1}{2}\tfrac{1}{2})_0)_K \\
\times\langle j_f\; j_i\; -m_f\; m_i|K\; m_i-m_f\rangle \left[Y^l(\hat{k}_f)Y^{l'}(\hat{k}_i)\right]^K_{m_i-m_f}\int d\mathbf{r}_f d\mathbf{r}_{bn}\frac{f_l(r_f)g_{l'}(r_i)}{r_f r_i} \\
\times u_{j_f}(r_{An})u_{j_i}(r_{bn})V(r_{bn})(-1)^{K+m_f-m_i}\left[Y^l(\hat{r}_f)Y^{l'}(\hat{r}_i)\right]^K_{m_f-m_i}\left[Y^{l_f}(\hat{r}_{An})Y^{l_i}(\hat{r}_{bn})\right]^K_{m_i-m_f}.
\end{aligned}
\tag{14}
$$

Again, ~~note that~~ the only term ~~that survives~~ of the expression

$$
\begin{aligned}
&(-1)^{K+m_f-m_i}\left[Y^l(\hat{r}_f)Y^{l'}(\hat{r}_i)\right]^K_{m_f-m_i}\left[Y^{l_f}(\hat{r}_{An})Y^{l_i}(\hat{r}_{bn})\right]^K_{m_i-m_f} = \\
&(-1)^{K+m_f-m_i}\sum_P\langle K\;K\;m_f-m_i\;m_i-m_f|P\;0\rangle\left\{\left[Y^l(\hat{r}_f)Y^{l'}(\hat{r}_i)\right]^K\left[Y^{l_f}(\hat{r}_{An})Y^{l_i}(\hat{r}_{bn})\right]^K\right\}^P_0
\end{aligned}
$$

which survives after angular integration is the one with $P = 0$, that is,

$$
\begin{aligned}
&\frac{1}{\sqrt{(2K+1)}}\left\{\left[Y^l(\hat{r}_f)Y^{l'}(\hat{r}_i)\right]^K\left[Y^{l_f}(\hat{r}_{An})Y^{l_i}(\hat{r}_{bn})\right]^K\right\}^0_0 = \\
&\qquad\frac{1}{\sqrt{(2K+1)}}\sum_{M_K}\langle K\;K\;M_K\;-M_K|0\;0\rangle\left[Y^l(\hat{r}_f)Y^{l'}(\hat{r}_i)\right]^K_{M_K} \\
&\times\left[Y^{l_f}(\hat{r}_{An})Y^{l_i}(\hat{r}_{bn})\right]^K_{-M_K} = \frac{1}{\sqrt{(2K+1)}}\sum_{M_K}\frac{(-1)^{K+M_K}}{\sqrt{(2K+1)}}\left[Y^l(\hat{r}_f)Y^{l'}(\hat{r}_i)\right]^K_{M_K} \\
&\qquad\times\left[Y^{l_f}(\hat{r}_{An})Y^{l_i}(\hat{r}_{bn})\right]^K_{-M_K} = \\
&\qquad\frac{1}{2K+1}\sum_{M_K}(-1)^{K+M_K}\left[Y^l(\hat{r}_f)Y^{l'}(\hat{r}_i)\right]^K_{M_K}\left[Y^{l_f}(\hat{r}_{An})Y^{l_i}(\hat{r}_{bn})\right]^K_{-M_K},
\end{aligned}
$$

in particular setting

~~This last~~ (an) expression (which) is spherically symmetric. ~~We~~ One can evaluate it for a particular configuration, ~~taking, e.g.,~~ $\hat{r}_f = \hat{z}$ and the ~~three bodies~~ center of mass of A, b, n ~~lying~~, in the $x-z$
6.1 plane (see Fig. (1)). Once the orientation in space of this "standard" configuration is specified (with, for example, a rotation $0 \leq \alpha \leq 2\pi$ around $\hat{z}$, a rotation $0 \leq \beta \leq \pi$ around the new x axis and a rotation $0 \leq \gamma \leq 2\pi$ around $\hat{r}_{bB}$), the only remaining angular coordinate is ~~the angle~~ θ, while the integral over the other three angles yields a factor $8\pi^2$. ~~With $\hat{r}_f = \hat{z}$ we have~~ one obtains

$$
\left[Y^l(\hat{r}_f)Y^{l'}(\hat{r}_i)\right]^K_{M_K} = \langle l\; l'\; 0\; M_K|K\; M_K\rangle\sqrt{\frac{2l+1}{4\pi}}\,Y^{l'}_{M_K}(\hat{r}_i). \tag{15}
$$

~~In terms of~~ $M = m_i - m_f$ and $m = m_f$ ~~we get the following expression for $T_{m,M}$~~ $\equiv$

setting

Because

(~~14) can be written as~~ $T_{m_i,m_f} \equiv T_{m,M}$ where

$$T_{m,M} = (-1)^{j_f - m} \frac{-64\sqrt{2}\pi^{7/2}}{k_f k_i} \sum_{ll'} i^{l'-l} e^{\sigma_f^l + \sigma_i^{l'}} \sqrt{2l+1} \sum_K \frac{(-1)^K}{2K+1} ((l_f \tfrac{1}{2})_{j_f} (l_i \tfrac{1}{2})_{j_i} | (l_f l_i)_K (\tfrac{1}{2}\tfrac{1}{2})_0)_K$$
$$\times \langle j_f \; j_i \; -m \; M+m | K \; M \rangle \left[Y^l(\hat{k}_f) Y^{l'}(\hat{k}_i) \right]_M^K \int d\mathbf{r}_f d\mathbf{r}_{bn} \frac{f_l(r_f) g_{l'}(r_i)}{r_f r_i}$$
$$\times u_{j_f}(r_{An}) u_{j_i}(r_{bn}) V(r_{bn}) \sum_{M_K} (-1)^{M_K} \langle l \; l' \; 0 \; M_K | K \; M_K \rangle \left[Y^{l_f}(\hat{r}_{An}) Y^{l_i}(\hat{r}_{bn}) \right]_{-M_K}^K Y_{M_K}^{l'}(\hat{r}_i). \quad (16)$$

We now turn our attention to the sum

$$\sum_{\substack{M_A, M_a \\ M_B, M_b}} \left| \sum_{m,M} \langle J_b \; j_i \; M_b \; m_i | J_a \; M_a \rangle \langle J_A \; j_f \; M_A \; m_f | J_B \; M_B \rangle T_{m,M} \right|^2, \quad (17)$$

found in the expression for the differential cross section ~~(see (4))~~ (4). For any given value m', M' of m, M, the sum will be

$$\sum_{M_a, M_b} |\langle J_b \; j_i \; M_b \; m_i | J_a \; M_a \rangle|^2 \sum_{M_A, M_B} \left| \langle J_A \; j_f \; M_A \; m_f | J_B \; M_B \rangle \right|^2 \left| T_{m',M'} \right|^2 =$$
$$\frac{(2J_a+1)(2J_B+1)}{(2j_i+1)(2j_f+1)} \sum_{M_a, M_b} |\langle J_b \; J_a \; M_b \; -M_a | j_i \; m_i \rangle|^2$$
$$\times \sum_{M_A, M_B} \left| \langle J_A \; J_B \; M_A \; -M_B | j_f \; m_f \rangle \right|^2 \left| T_{m',M'} \right|^2, \quad (18)$$

by virtue of the symmetry property of Clebsch–Gordan coefficients

$$\langle J_b \; j_i \; M_b \; m_i | J_a \; M_a \rangle = (-1)^{J_b - M_b} \sqrt{\frac{(2J_a+1)}{(2j_i+1)}} \langle J_b \; J_a \; M_b \; -M_a | j_i \; m_i \rangle. \quad (19)$$

The sum over the Clebsch–Gordan coefficients in (18)is one, so (17) is just

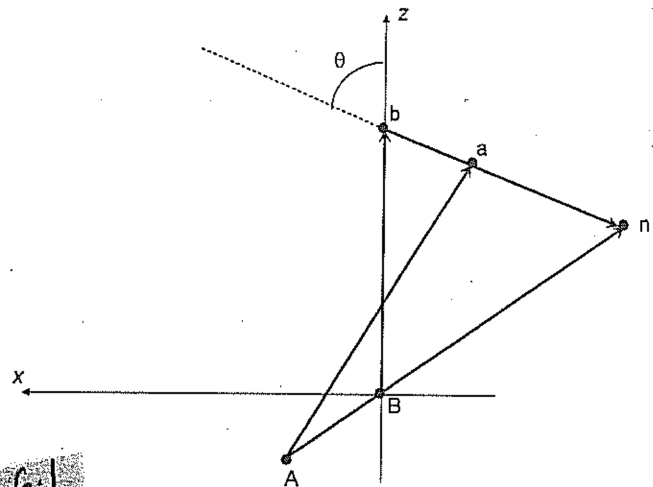
$$\frac{(2J_a+1)(2J_B+1)}{(2j_i+1)(2j_f+1)} \sum_{m,M} \left| T_{m,M} \right|^2, \quad (20)$$

and the differential cross section turns out to be

$$\frac{d\sigma}{d\Omega} = \frac{k_f}{k_i} \frac{\mu_i \mu_f}{4\pi^2 \hbar^4} \frac{(2J_B+1)}{(2j_i+1)(2j_f+1)(2J_A+1)} \sum_{m,M} \left| T_{m,M} \right|^2. \quad (21)$$

where ~~We can write~~

$$T_{m,M} = \sum_{Kll'} (-1)^{-m} \langle j_f \; j_i \; -m \; M+m | K \; M \rangle \left[Y^l(\hat{k}_f) Y^{l'}(\hat{k}_i) \right]_M^K t_{ll'}^K. \quad (22)$$



6.1

Figure ~~6~~: Coordinate system in the "standard" configuration. Note that $\mathbf{r}_f \equiv \mathbf{r}_{Bb}$, and $\mathbf{r}_i \equiv \mathbf{r}_{Aa}$.

Orienting ~~If~~ $\hat{k}_i$ ~~is taken to be in~~ along the the incident z direction,

$$\left[Y^{l}(\hat{k}_f)Y^{l'}(\hat{k}_i)\right]^K_M = \langle l\; l'\; M\; 0|K\; M\rangle \sqrt{\frac{2l'+1}{4\pi}}\, Y^l_M(\hat{k}_f), \tag{23}$$

and

$$T_{m,M} = \sum_{Kll'} (-1)^{-m} \langle l\; l'\; M\; 0|K\; M\rangle \langle j_f\; j_i\; -m\; M+m|K\; M\rangle Y^l_M(\hat{k}_f)\, t^K_{ll'}, \tag{24}$$

with

$$\begin{aligned} t^K_{ll'} = (-1)^{K+j_f} \frac{-32\sqrt{2}\pi^3}{k_f k_i} i^{l'-l} e^{\sigma^l_f+\sigma^{l'}_i} \frac{\sqrt{(2l+1)(2l'+1)}}{2K+1} ((l_f \tfrac{1}{2})_{j_f}(l_i \tfrac{1}{2})_{j_i}|(l_f l_i)_K(\tfrac{1}{2}\tfrac{1}{2})_0)_K \\ \times \int dr_f\, dr_{bn} d\theta r^2_{bn} \sin\theta\, r_f \frac{f_l(r_f) g_{l'}(r_i)}{r_i} u_{j_f}(r_{An}) u_{j_i}(r_{bn}) V(r_{bn}) \\ \times \sum_{M_K} (-1)^{M_K} \langle l\; l'\; 0\; M_K|K\; M_K\rangle \left[Y^{l_f}(\hat{r}_{An}) Y^{l_i}(\hat{r}_{bn})\right]^K_{-M_K} Y^{l'}_{M_K}(\hat{r}_i). \end{aligned} \tag{25}$$

1.1 Coordinates

~~In order to be able~~ To perform the integral in (25), one ~~we~~ needs the expression of $r_i, r_{An}, \hat{r}_{An}, \hat{r}_{bn}, \hat{r}_i$ in term of the integration variables r_f, r_{bn}, θ. ~~We recall that we want to evaluate~~ Because one is interested in evaluating these quantities in the particular ~~orientation corresponding to the~~,

It is of notice that

6.1

configuration depicted in Fig. ~~8~~ 6, ~~we~~ one has

$$
\begin{aligned}
\mathbf{r}_f &= r_f\,\hat{z}, && (26)\\
\mathbf{r}_{bn} &= -r_{bn}(\sin\theta\,\hat{x} + \cos\theta\,\hat{z}), && (27)\\
\mathbf{r}_{Bn} &= \mathbf{r}_f + \mathbf{r}_{bn} = -r_{bn}\sin\theta\,\hat{x} + (r_f - r_{bn}\cos\theta)\,\hat{z}. && (28)
\end{aligned}
$$

~~Now, in function of the number of nucleons of the different nuclei involved in the reaction, we can write~~
One can then write

$$
\begin{aligned}
\mathbf{r}_{An} &= \frac{A+1}{A}\mathbf{r}_{Bn} = -\frac{A+1}{A}r_{bn}\sin\theta\,\hat{x} + \frac{A+1}{A}(r_f - r_{bn}\cos\theta)\,\hat{z}, && (29)\\
\mathbf{r}_{an} &= \frac{b}{b+1}\mathbf{r}_{bn} = -\frac{b}{b+1}r_{bn}(\sin\theta\,\hat{x} + \cos\theta\,\hat{z}), && (30)
\end{aligned}
$$

and

$$
\mathbf{r}_i = \mathbf{r}_{An} - \mathbf{r}_{an} = -\frac{2A+1}{(A+1)A}r_{bn}\sin\theta\,\hat{x} + \left(\frac{A+1}{A}r_f - \frac{2A+1}{(A+1)A}r_{bn}\cos\theta\right)\hat{z}, \quad (31)
$$

where A, b are the number of nucleons of nuclei A and b respectively.

6.2 Zero range approximation

In the zero range approximation,

$$
\int dr_{bn} r_{bn}^2 u_{j_i}(r_{bn})V(r_{bn}) = D_0; \quad u_{j_i}(r_{bn})V(r_{bn}) = \delta(r_{bn})/r_{bn}^2. \quad (32)
$$

It can be shown (see Fig. ~~6~~ 6.1) that for $r_{bn} = 0$

$$
\begin{aligned}
\mathbf{r}_{An} &= \frac{m_A+1}{m_A}\mathbf{r}_f\\
\mathbf{r}_i &= \frac{m_A+1}{m_A}\mathbf{r}_f.
\end{aligned} \quad (33)
$$

One then ~~We~~ obtains

$$
\begin{aligned}
t_{ll'}^K =& \frac{-16\sqrt{2}\pi^2}{k_f k_i}(-1)^K\frac{D_0}{\alpha}i^{l'-l}e^{\sigma_f^l+\sigma_i^{l'}}\,\frac{\sqrt{(2l+1)(2l'+1)(2l_i+1)(2l_f+1)}}{2K+1}((l_f\tfrac{1}{2})_{j_f}(l_i\tfrac{1}{2})_{j_i}|(l_f l_i)_K(\tfrac{1}{2}\tfrac{1}{2})_0)_K\\
&\times\langle l\,l'\;0\;0|K\;0\rangle\langle l_f\;l_i\;0\;0|K\;0\rangle\int dr_f\;f_l(r_f)g_{l'}(\alpha r_f)u_{j_f}(\alpha r_f),
\end{aligned} \quad (34)
$$

with

$$
\alpha = \frac{A+1}{A}. \quad (35)
$$

coupled channel Hamamoto Jens Bang transfer ffs

6.3 Examples

In this section we ~~present examples~~ use the above formalism

$^{120}Sn(d,p)^{121}Sn$ $\quad$ $^{120}Sn(p,d)^{119}Sn$

$^{132}Sn(d,p)^{133}Sn$

Appendix 6.A

Minimal requirements for a consistent mean field theory

In what follows the question of why, rigorously speaking, one cannot talk about single-particle motion, let alone spectroscopic factor, not even within the framework of Hartree-Fock theory, is briefly touched upon.

a - a from p. 2

to p. 4, caption

19/05/13

2

(b) nucleons lead, through processes like the one depicted in (g), eventually propagated to all orders, to: (h) softening of the discontinuity of the occupancy of levels at ε_F, as well as to: (i) generalization of the static selfconsistency into a dynamic relation encompassing also collective vibrations (Time-dependent HF solutions of the nuclear Hamiltonian, conserving energy weighted sum rules (EWSR)). (b)

(a) As can be seen from Fig. A1 the minimum requirements of selfconsistency to be imposed upon single-particle motion requires both non-locality, in space (HF) and in time (TDHF),

$$i\hbar \frac{\partial \varphi_\nu(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2 \varphi_\nu(x,t) + \int dx' dt' U(x-x', t-t')\varphi_\nu(x',t') \quad (1)$$

and consequently also of collective vibrations and, ~~consequently~~, from their interweaving to dressed single-particles (quasiparticles), let alone renormalized collective modes. Assuming for simplicity infinite nuclear matter (confined by a constant potential of depth V_0), and thus plane wave solutions, the above time-dependent Schrödinger equation leads to the quasiparticle dispersion relation (cf. e.g. Brink and Broglia, Nuclear Superfluidity Cambridge University Press, Cambridge (2005) App. B)

$$\hbar\omega = \frac{\hbar^2 k^2}{2m^*} + \frac{m}{m^*}V_0, \quad (2)$$

where the effective mass

$$m^* = \frac{m_k m_\omega}{m}, \quad (3)$$

is the product of the k-mass

$$m_k = m\left(1 + \frac{m}{\hbar^2 k}\frac{\partial U}{\partial k}\right)^{-1} \quad (4)$$

closely connected with the Pauli principle $\left(\frac{\partial U}{\partial k} \approx \frac{\partial U_x}{\partial k}\right)$, while the ω-mass

$$m_\omega = m\left(1 - \frac{\partial U}{\partial(\hbar\omega)}\right) \quad (5)$$

results from the dressing of the nucleon through the coupling with the (quasi) bosons.

Because typically $m_k \approx 0.7m$ and $m_\omega \approx 1.4m$ $m^* \approx m$, one could be tempted to conclude that the results embodied in the dispersion relation (2) reflects but the well known empirical fact ~~that~~ the distribution of levels around the Fermi energy can be desribed in terms of the solutions of a Schröding equation in which nucleons of mass equal to the bare nucleon mass m move in a Saxon-Woods potential of depth V_0.

Now, it can be shown ~~that~~ the occupancy of levels around ε_F is given by Z_ω (cf. Fig A1(h)) a quantity which equal to $m/m_\omega \approx 0.7$. This, in keeping with the fact ~~that~~ the time the nucleon is coupled to the vibrations it cannot behave as a single-particle and can thus not contribute to e.g. the single-particle pickup cross section.

It is of notice ~~that~~ the selfconsistent requirements for the iterative solution of (1) (see Figs. A1(d) and A1(i)) remind very much those associated with the solution of the Kohn-Sham equations

$$H^{KS} \varphi_j(\vec{r}) = \lambda_j \varphi_j(\vec{r}),$$

where

$$H^{KS} = -\frac{\hbar^2}{2m_e}\nabla^2 + v_H(\vec{r}) + V_{ext}(\vec{r}) + v_{xc}(\vec{r}),$$

H^{KS} being known as the Kohn-Sham Hamiltonian, $V_{ext}(\vec{r})$ being the field created by the ions and acting on the electrons

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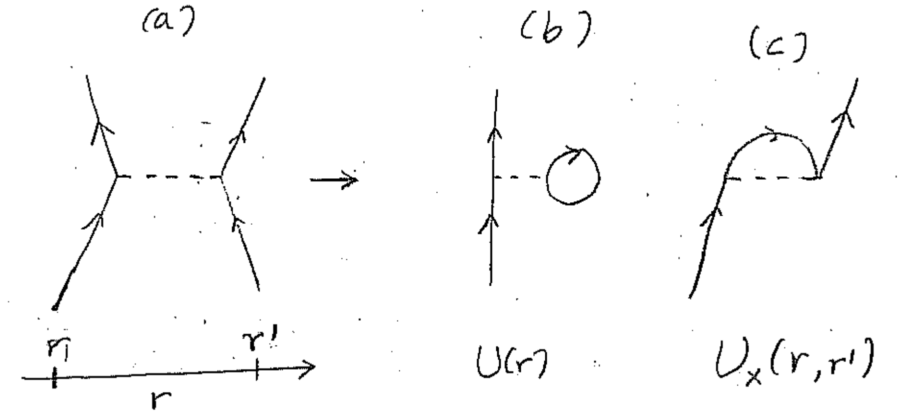
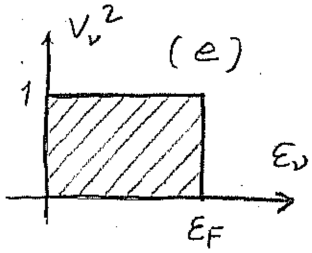
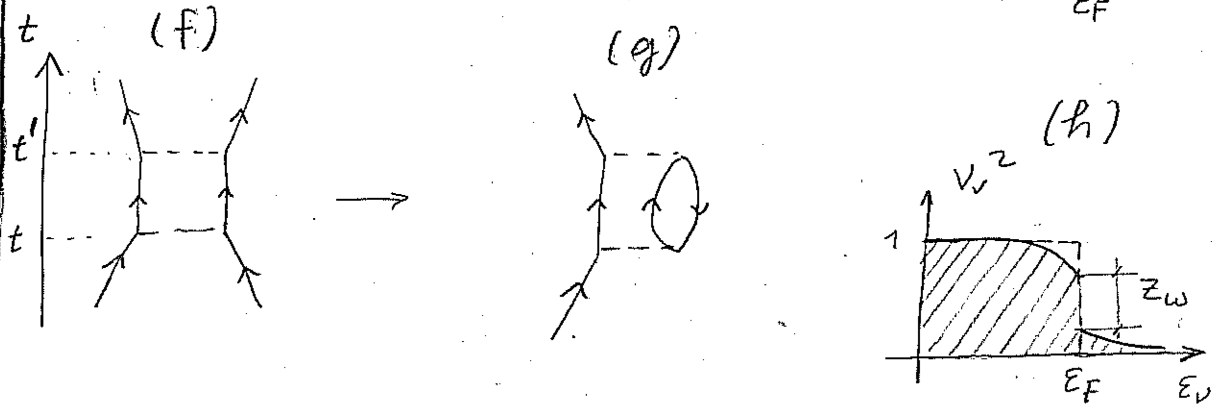
Both the Hartree and the exchange-correlation potentials $v_H(\vec{r})$ and $v_{xc}(\vec{r})$ depend on the (local) density, hence on the whole set of wavefunction $\varphi_j(\vec{r})$. Thus, the set of KS-equations must be solve selfconsistently. (Broglia, Colò, Onida and Roman Solid State Physics of Finite Systems, Springer-Verlag Heidelberg (2004) Ch. 3)



(d) $U(r) = \int d^3r\, \rho(r')\, v(|\vec{r}-\vec{r}'|)$





(i) $\delta U(r) = \int d^3r\, \delta\rho(r')\, v(|\vec{r}-\vec{r}'|)$

Fig. A (a) scattering of two nucleons through the bare NN interaction $v(|\vec{r}-\vec{r}'|)$, (b) contribution to the direct (U, Hartree) and (c) to the exchange (U_x, Fock) potential, resulting in (d) the (static) selfconsistent relation between potential and density, which (e) uncouples occupied ($\varepsilon_\nu \leq \varepsilon_F$) from empty states ($\varepsilon_\nu > \varepsilon_F$). (f) multiple scattering of two

b - b from p. 2 before

Appendix 6.B
Model for single-particle strength function: Dyson equation

In the previous Section we introduce the argument of the impossibility of defining a "bona fide" single-particle spectroscopic factor. It was done with the help of Feynman (NFT) diagrams. In what follows we we essentially repeat the arguments, but this time in terms of Dyson's, (Schwinger) language.

For simplicity, we consider a two-level model where the pure single-particle state $|a\rangle$ couples to a more complicated state $|\alpha\rangle$, made out of a fermion (particle or hole), couple to a particle-hole excitation which, if iterated to all orders can give rise to a collective state (cf. Fig. B1). The Hamiltonian describing the system is

see also Brink and Broglia, Nuclear superfluidity, Cambridge University Press, Cambridge, (2005) App. D.

$$H = H_0 + v \quad (6)$$

where

$$H_0|a\rangle = E_a|a\rangle, \quad (7)$$

and

$$H_0|\alpha\rangle = E_\alpha|\alpha\rangle. \quad (8)$$

Let us call $\langle a|v|\alpha\rangle = v_{a\alpha}$ and assume $\langle a|v|a\rangle = \langle\alpha|v|\alpha\rangle = 0.$

From the secular equation associated with H, namely

$$\begin{pmatrix} E_\alpha - E_i & v_{a\alpha} \\ v_{a\alpha} & E_a - E_i \end{pmatrix} \begin{pmatrix} C_\alpha(i) \\ C_a(i) \end{pmatrix} = 0, \quad (9)$$

and associated normalization condition

$$C_a^2(i) + C_\alpha^2(i) = 0, \quad (10)$$

one obtains

$$C_a^2(i) = \left(1 + \frac{v_{a\alpha}^2}{(E_\alpha - E_i)^2}\right)^{-1}, \quad (11)$$

and

$$\Delta E_a(E) = E_a - E = \frac{v_{a\alpha}^2}{E_a - E}. \quad (12)$$

The relations (11) and (12) allows one to write the correlated state

$$|\tilde{a}\rangle = C_a(i)|a\rangle + C_\alpha(i)|\alpha\rangle, \quad (13)$$

the corresponding energy being provided by the (iterative) solution of the Dyson equation (12), which propagate the bubble diagrams shown in Figs. B.1(a) and (b) to infinite order, leading to collective vibrations (see Fig. B.1(c))

With the help of the definition (5), and making use of the fact that in the present case, $U \equiv \Delta E_a(E)$, one obtains

$$Z_\omega = C_a^2(i) = \frac{m_\omega}{m} \quad (14)$$

The solution of (12) together with the relations (10) and (11) lead to the quasiparticle ~~state~~ (13), to be employed in the calculation of the one-particle transfer transition amplitudes (cf. e.g. Eqs (6.6) and (6.25)).

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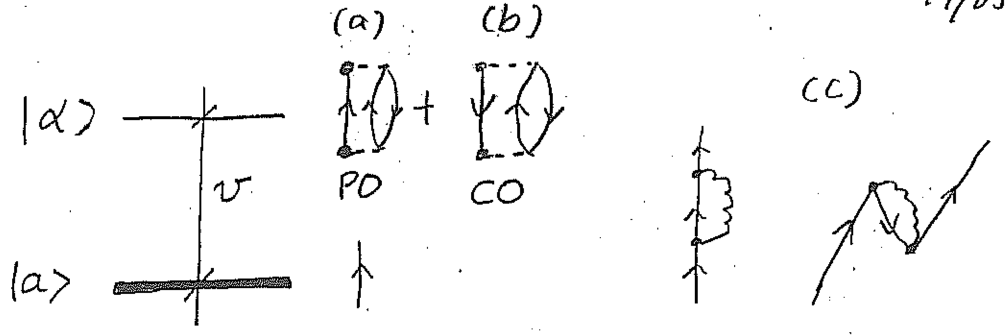


Fig. B.1

Two state schematic model describing the breaking of the strength of the pure single-particle state |a>, through the coupling to collective vibrations (wavy line) associated with polarization (PO) and correlation (CO) processes.

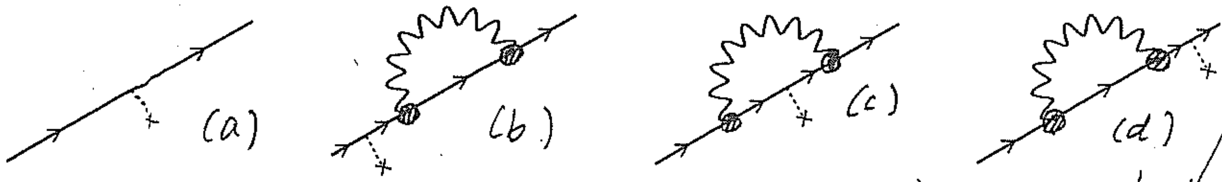
Appendix 6.C

(the modulus squared of the sum of the amplitudes associated with all four diagrams)

Self-Energy (effective mass) processes

Nothing is free

R.P. Feynman *Theory of Fundamental Processes*

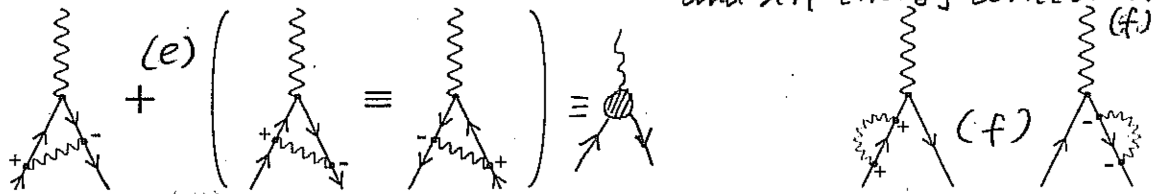


The result of (e.g. a single-particle pick-up experiment) is given by

Vertex corrections

(P.W. Anderson *Basic Notions of Condensed Matter Physics*)

In fact, there ~~is~~ exists, as a rule, an important cancellation between vertex and self-energy corrections (see (e), (f))...



Pines - dettached vertex diagrams

These are triple-interaction ~~vertecees~~ in which none of the incoming lines can be from either of the other two by cutting one line. Midgal's (1958) theorem states that, for phonons and electrons (Bardeen-Frölich mechanism to break gauge invariance), vertex corrections can be neglected, but usually they are not negligible, any case not in nuclei.

Lamb shift

Fig. 1 Broglia More is different: 50 Years of Nuclear BCS

p. 648 book

Two-Nucleon transfer

(A) - (A) Ch. 7 p. 1 below

Ch. 7 React

7.1 Summary of 2nd order DWBA

~~0.1 Details of the Calculation~~ the theory of second order DWBA two-nucleon transfer with the discussed in 7.2.10 subsect.

Let us illustrate the calculation with ~~a~~ $A+t \rightarrow B(\equiv A+2)+p$ reaction, in which $A+2$ and A are even nuclei in their 0^+ ground state. The extension of the following expressions to the transfer of pairs coupled to arbitrary angular momentum is ~~straightforward~~. The wavefunction of the nucleus $A+2$ ~~is~~ can be written as

$$\Psi_{A+2}(\xi_A, \mathbf{r}_{A1}, \sigma_1, \mathbf{r}_{A2}, \sigma_2) = \psi_A(\xi_A) \sum_{l_i, j_i} [\phi^{A+2}_{l_i, j_i}(\mathbf{r}_{A1}, \sigma_1, \mathbf{r}_{A2}, \sigma_2)]^0_0, \tag{1}$$

where

$$\phi^{A+2}_{l_i, j_i}(\mathbf{r}_{A1}, \sigma_1, \mathbf{r}_{A2}, \sigma_2) = \sum_{nm} a_{nm} \left[\varphi^{A+2}_{n, l_i, j_i}(\mathbf{r}_{A1}, \sigma_1)\varphi^{A+2}_{m, l_i, j_i}(\mathbf{r}_{A2}, \sigma_2)\right]^0_0, \tag{2}$$

while ~~and~~ the wavefunctions $\varphi^{A+2}_{n, l_i, j_i}(\mathbf{r})$ are eigenfunctions of a $\boxed{\text{Woods-}}\boxed{\text{Saxon}}$ potential

$$U(r) = -\frac{V_0}{1 + \exp\left[\frac{r-R_0}{a}\right]}, \qquad R_0 = r_0 A^{1/3}, \tag{3}$$

of ~~The~~ depth V_0 ~~is~~ adjusted to reproduce the experimental single–particles energies. The ~~spatial part of~~ radial dependence of the wavefunction of the two neutrons in the ~~tritium~~ triton is $\phi_t(\mathbf{r}_{p1}, \mathbf{r}_{p2}) = \rho(r_{p1})\rho(r_{p2})\rho(r_{12})$, where r_{p1}, r_{p2}, r_{12} are the distances between neutron 1 and the proton, neutron 2 and the proton and between neutrons 1 and 2 respectively, ~~and~~ while $\rho(r)$ is ~~a~~ written as the ($r_{core} = 0.45$ fm) hard core potential wavefunction ~~with hard core at $r = 0.45$ fm, as~~ depicted in Fig. 7.1.

The differential cross section is written as,

$$\frac{d\sigma}{d\Omega} = \frac{\mu_i \mu_f}{(4\pi\hbar^2)^2} \frac{k_f}{k_i} \left|T^{(1)} + T^{(2)}_{succ} - T^{(2)}_{NO}\right|^2, \tag{4}$$

definition of all quantities

where ~~the three amplitudes contributing to the transfer are~~ two-nucleon transfer (see also [1]) ~~is~~,

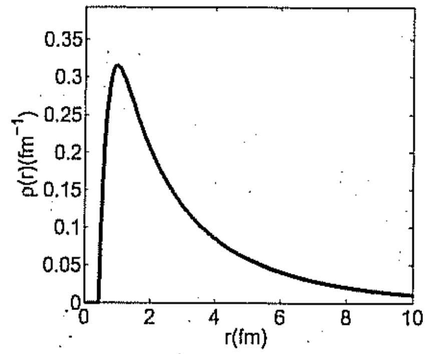
$$T^{(1)} = 2 \sum_{l_i, j_i} \sum_{\sigma_1 \sigma_2} \int d\mathbf{r}_{tA} d\mathbf{r}_{p1} d\mathbf{r}_{A2} [\phi^{A+2}_{l_i, j_i}(\mathbf{r}_{A1}, \sigma_1, \mathbf{r}_{A2}, \sigma_2)]^{0*}_0 \chi^{(-)*}_{pB}(\mathbf{r}_{pB})$$
$$\times v(\mathbf{r}_{p1}) \phi_t(\mathbf{r}_{p1}, \mathbf{r}_{p2}) \chi^{(+)}_{tA}(\mathbf{r}_{tA}), \tag{5a}$$

$$T^{(2)}_{succ} = 2 \sum_{l_i, j_i} \sum_{l_f, j_f, m_f} \sum_{\substack{\sigma_1 \sigma_2 \\ \sigma_1' \sigma_2'}} \int d\mathbf{r}_{dF} d\mathbf{r}_{p1} d\mathbf{r}_{A2} [\phi^{A+2}_{l_i, j_i}(\mathbf{r}_{A1}, \sigma_1, \mathbf{r}_{A2}, \sigma_2)]^{0*}_0 \chi^{(-)*}_{pB}(\mathbf{r}_{pB}) v(\mathbf{r}_{p1})$$
$$\times \phi_d(\mathbf{r}_{p1}) \varphi^{A+1}_{l_f, j_f, m_f}(\mathbf{r}_{A2}) \int d\mathbf{r}'_{dF} d\mathbf{r}'_{p1} d\mathbf{r}'_{A2} G(\mathbf{r}_{dF}, \mathbf{r}'_{dF})$$
$$\times \phi_d(\mathbf{r}'_{p1})^* \varphi^{A+1*}_{l_f, j_f, m_f}(\mathbf{r}'_{A2}) \frac{2\mu_{dF}}{\hbar^2} v(\mathbf{r}'_{p2}) \phi_d(\mathbf{r}'_{p1}) \phi_d(\mathbf{r}'_{p2}) \chi^{(+)}_{tA}(\mathbf{r}'_{tA}); \tag{5b}$$

Sabama BenBayman

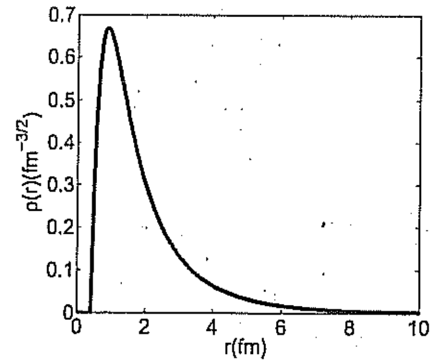
$$T^{(2)}_{NO} = 2 \sum_{l_i, j_i} \sum_{l_f, j_f, m_f} \sum_{\substack{\sigma_1 \sigma_2 \\ \sigma_1' \sigma_2'}} \int d\mathbf{r}_{dF} d\mathbf{r}_{p1} d\mathbf{r}_{A2} [\phi^{A+2}_{l_i, j_i}(\mathbf{r}_{A1}, \sigma_1, \mathbf{r}_{A2}, \sigma_2)]^{0*}_0 \chi^{(-)*}_{pB}(\mathbf{r}_{pB}) v(\mathbf{r}_{p1})$$
$$\times \phi_d(\mathbf{r}_{p1}) \varphi^{A+1}_{l_f, j_f, m_f}(\mathbf{r}_{A2}) \int d\mathbf{r}'_{p1} d\mathbf{r}'_{A2} d\mathbf{r}'_{dF}$$
$$\times \phi_d(\mathbf{r}'_{p1})^* \varphi^{A+1*}_{l_f, j_f, m_f}(\mathbf{r}'_{A2}) \phi_d(\mathbf{r}'_{p1}) \phi_d(\mathbf{r}'_{p2}) \chi^{(+)}_{tA}(\mathbf{r}'_{tA}). \tag{5c}$$

The quantities μ_i, μ_f (k_i, k_f) are the reduced masses (relative linear momenta) in both entrance (initial, i) and ~~exit~~ exit (final, f) channels, respectively.



7.1

Figure ~~1~~: Tritium wavefunction



7.2

Figure ~~2~~: Deuteron wavefunction

Although the are a number of ways to exactly treat such states, discretization processes may be sufficiently accurate. They can be implemented by, for example,

the above   being   lie   7.2

sufficiently

In ~~these~~ expressions, $\varphi^{A+1}_{l_f,j_f,m_f}(\mathbf{r}_{A1})$ are the wavefunctions describing the intermediate states of the nucleus $F(\equiv A+1)$, generated as solutions of a Woods–Saxon potential, and $\phi_d(\mathbf{r}_{p2})$ ~~is~~ the wavefunction ~~of the~~ deuteron bound ~~state~~ (see Fig. ~~2~~). Note that some or all of the $\varphi^{A+1}_{l_f,j_f,m_f}(\mathbf{r}_{A1})$ may ~~be~~ in the continuum ~~for unbound or loosely bound)~~ ~~and some discretization procedure is required in order to deal with these states. In this case, they are generated by~~ embedding the Woods–Saxon potential in a spherical box of large ~~enough~~ radius. In actual calculations we got convergence ~~with less than~~ 20 continuum states ~~in a~~ and a box of 30 fm radius ~~box~~. ~~As for the wavefunction of the neutrons in the tritium, it is~~ generated with the $p-n$ Tang–Herndon interaction

(case in which the nucleus F is loosely bound or unbound)

(3)

single-particle states described by the wavefunctions

$$v(r) = -v_0 \exp(-k(r - r_c)) \qquad r > r_c \tag{6}$$

$$v(r) = \infty \qquad r < r_c, \tag{7}$$

Concerning the component   so as

involving the halo nucleus ^{11}Li, and where $|F\rangle = |^{10}\text{Li}\rangle$, one achieved convergence making use of about

where $k = 2.5$ fm^{-1} and $r_c = 0.45$ fm, ~~and~~ the depth v_0 ~~is~~ adjusted to reproduce the experimental separation energies. The positive–energy wavefunctions $\chi^{(+)}_{tA}(\mathbf{r}_{tA})$ and $\chi^{(-)}_{pB}(\mathbf{r}_{pB})$ are the ingoing distorted wave in the initial channel and the outgoing distorted wave in the final channel respectively. They are continuum solutions of the Schrödinger equation associated with the corresponding optical potentials. being

The transition potential responsible for the transfer of the pair is, in the *post* representation,

$$V_\beta = v_{pB} - U_\beta, \tag{8}$$

where v_{pB} is the interaction between the proton and nucleus B, and U_β is the optical potential in the final channel. We make the assumption that v_{pB} can be decomposed into a term containing the interaction between A and p and the potential describing the interaction between the proton and each of the transferred nucleons, namely

of the the triton wavefunction describing the relative motion of the dineutron, it was

$$v_{pB} = v_{pA} + v_{p1} + v_{p2}, \tag{9}$$

where v_{p1} and v_{p2} is the hard–core potential (6). The transition potential is

$$V_\beta = v_{pA} + v_{p1} + v_{p2} - U_\beta. \tag{10}$$

Assuming that $\langle\beta|v_{pA}|\alpha\rangle \approx \langle\beta|U_\beta|\alpha\rangle$ (i.e, assuming that the matrix element of the core–core interaction between the initial and final states is very similar to the matrix element of the real part of the optical potential), one obtains the final expression of the transfer potential in the *post* representation, namely,

$$V_\beta \simeq v_{p1} + v_{p2} = v(\mathbf{r}_{p1}) + v(\mathbf{r}_{p2}). \tag{11}$$

???

We make the further approximation of using the same interaction potential in all (e.g. initial, intermediate and final) the channels.

and if not?

The extension to a heavy–ion reaction $A + a(\equiv b+2) \longrightarrow B(\equiv A+2) ~~+ b~~$ imply no essential modifications in the formalism. The deuteron and triton ~~states~~ wavefunctions appearing in ~~(5a,5b,5c)~~ Eqs. 5(a), 5(b) ad 5(c) ~~must~~ can to be substituted with the corresponding wavefunctions $\Psi_{b+2}(\xi_b, \mathbf{r}_{b1}, \sigma_1, \mathbf{r}_{b2}, \sigma_2)$, constructed in a similar way ~~as in~~ those introduced in Eqs. (1) ad (2). The interaction potential used in ~~(5a,5b,5c)~~ Eqs. 5(a), 5(b) ad 5(c) will now be the Woods–Saxon used to define the initial (final) state in the post (prior) representation, instead of the proton–neutron interaction (6).

The Green function $G(\mathbf{r}_{dF}, \mathbf{r}'_{dF})$ appearing in 5(b) propagates the intermediate channel d, F. ~~and can~~ It can be expanded in partial waves as

$$G(\mathbf{r}_{dF}, \mathbf{r}'_{dF}) = i \sum_l \sqrt{2l+1} \frac{f_l(k_{dF}, r_<)g_l(k_{dF}, r_>)}{k_{dF} r_{dF} r'_{dF}} \left[Y^l(\hat{r}_{dF}) Y^l(\hat{r}'_{dF})\right]^0_0. \tag{12}$$

g ?

g for a

The $f_l(k_{dF}, r)$ and ~~$F_l(k_{dF}, r)$~~ are the regular and the irregular solutions of a Schrödinger equation ~~with a~~ suitable optical potential and an energy equal to the kinetic energy ~~in~~ the intermediate state. In most cases of interest, the result is hardly altered if we use the same energy of ~~the~~ relative motion ~~between nuclei~~ for all the intermediate states. This representative energy is calculated when both intermediate nuclei are in their corresponding ground states. ~~However,~~ the validity of this approximation can break down in some particular cases. If, for example, some relevant intermediate state become off shell, its contribution is significantly quenched. An interesting situation can arise when this happens to all possible intermediate states, so they can only be virtually populated.

of

It is of notice that

COOP, KNOCK...

software ~~COOPS is~~ used in the applications (cf. also app. A Ch. 8)

A @–@ from p.1

gregory has to write

~~7.1 simultaneous transfer~~

~~7.1.1 distorted waves~~

7.2 Detailed derivation of 2nd order DWBA

7.2.1 Simultaneous transfer: distorted waves

For a (t, p) reaction, the triton is represented by an total incoming wave. We make the assumption that the two transferred neutrons are in the $S = 0$ singlet state and that the triton has orbital angular momentum $L = 0$, so the spin is entirely due to the spin of the proton. We will explicitly treat it as, unlike in [?], we will consider a spin–orbit term in the optical potential ~~between the triton and the heavy ion~~ in the triton channel. We use the notation of [?].

After (??) we can write the triton distorted wave as

$$\psi^{(+)}_{m_t}(\mathbf{R}, \mathbf{k}_i, \sigma_p) = \sum_{l_t} \exp\left(i\sigma^t_{l_t}\right) g_{l_t j_t} Y^{l_t}_0(\hat{\mathbf{R}}) \frac{\sqrt{4\pi(2l_t+1)}}{k_i R} \chi_{m_t}(\sigma_p), \tag{7.1}$$

where we have used $Y^{l_t}_0(\hat{\mathbf{k}}_i) = i^{l_t}\sqrt{\frac{2l_t+1}{4\pi}}\delta_{m_t,0}$, as $\mathbf{k}_i$ is oriented along the z–axis. Note the phase difference with eq. (7) of [?], due to the use of time–reversal rather than Condon–Shortley phase convention. If we write

$$Y^{l_t}_0(\hat{\mathbf{R}})\chi_{m_t}(\sigma_p) = \sum_{j_t} \langle l_t\ 0\ 1/2\ m_t | j_t\ m_t\rangle \left[Y^{l_t}(\hat{\mathbf{R}})\chi(\sigma_p)\right]^{j_t}_{m_t}, \tag{7.2}$$

we have

$$\begin{aligned}\psi^{(+)}_{m_t}(\mathbf{R}, \mathbf{k}_i, \sigma_p) = \sum_{l_t, j_t} &\exp\left(i\sigma^t_{l_t}\right) \frac{\sqrt{4\pi(2l_t+1)}}{k_i R} g_{l_t j_t}(R) \\ &\times \langle l_t\ 0\ 1/2\ m_t | j_t\ m_t\rangle \left[Y^{l_t}(\hat{\mathbf{R}})\chi(\sigma_p)\right]^{j_t}_{m_t}.\end{aligned} \tag{7.3}$$

We now turn our attention to the outgoing proton distorted wave, which, after (??), is

$$\psi^{(-)}_{m_p}(\zeta, \mathbf{k}_f, \sigma_p) = \sum_{l_p j_p} \frac{4\pi}{k_f \zeta} i^{l_p} \exp\left(-i\sigma^p_{l_p}\right) f^*_{l_p j_p}(\zeta) \sum_m Y^{l_p}_m(\hat{\zeta}) Y^{l_p *}_m(\hat{\mathbf{k}}_f)\chi_{m_p}(\sigma_p). \tag{7.4}$$

Now,

$$\begin{aligned}\sum_m Y^{l_p}_m(\hat{\zeta}) Y^{l_p *}_m(\hat{\mathbf{k}}_f)\chi_{m_p}(\sigma_p) &= \sum_{m, j_p} Y^{l_p *}_m(\hat{\mathbf{k}}_f)\langle l_p\ m\ 1/2\ m_p | j_p\ m+m_p\rangle \\ &\times \left[Y^{l_p}(\hat{\zeta})\chi_{m_p}(\sigma_p)\right]^{j_p}_{m+m_p} \\ &= \sum_{m, j_p} Y^{l_p *}_{m-m_p}(\hat{\mathbf{k}}_f)\langle l_p\ m-m_p\ 1/2\ m_p | j_p\ m\rangle \left[Y^{l_p}(\hat{\zeta})\chi_{m_p}(\sigma_p)\right]^{j_p}_m,\end{aligned} \tag{7.5}$$

and, finally,

$$\begin{aligned}\psi^{(-)}_{m_p}(\zeta, \mathbf{k}_f, \sigma_p) = \frac{4\pi}{k_f \zeta} \sum_{l_p j_p, m} &i^{l_p} \exp\left(-i\sigma^p_{l_p}\right) f^*_{l_p j_p}(\zeta) Y^{l_p *}_{m-m_p}(\hat{\mathbf{k}}_f) \\ &\times \langle l_p\ m-m_p\ 1/2\ m_p | j_p\ m\rangle \left[Y^{l_p}(\hat{\zeta})\chi(\sigma_p)\right]^{j_p}_m.\end{aligned} \tag{7.6}$$