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lished also by Cambridge University Press can be considered companion volumes to the present one. Volume which shares with those a similar aim: to provide a broad physical view of central issues in the study of finite quantal many-body nuclear systems accessible to motivated students and practitioners. However, neither the present one, nor the other two are introductory texts. In particular the present one in which an attempt at unifying structure and reactions as it happens in nature is made. On the other hand, unifying discrete (mainly structure) and continuum (reactions), implies that we will be dealing with those structure results which can be tested by means of experiment. A fact which makes the subject of the present monograph a chapter of quantum mechanics, and thus close to what fourth year students have been learning.*)

Concerning the notation, we have divided each chapter into sections. Each section may, in turn, be broken down into subsections. Equations and Figures are identified by the number of the chapter and that of the section. Thus (6.1.33) labels the thirtythird equation of section 1 of chapter 6. Similarly, Fig. 6.1.2 labels the second figure of section 1 of chapter 6. Concerning the Appendices, they are labelled by the chapter number and by a Latin letter in alphabetical order, e.g. App. 6.A, App. 6.B, etc. Concerning equations and Figures, a sequential number is added. Thus (6.E.15) labels the fifteenth equation of Appendix E of chapter 6, while Fig. 6.F.1 labels the first figure of Appendix F of Chapter 6. References are referred to in terms of the author's surname and publication year and are found in alphabetic order in the bibliography.

A methodological approach used in the present monograph concerns a certain degree of repetition. Similar, but not the same issues are dealt with more than once using different but equatable terminologies. This approach reflects the fact that useful concepts like reaction channels, or correlation length, let alone elementary modes of excitation, are easy to understand but difficult to define.***) This is because their validity is not exhausted in a single perspective[1]. But even more important, because their power in helping at connecting[2] seemingly unrelated results and phenomena is difficult to be fully appreciated the first time around, spontaneous symmetry breaking and associated emergent properties providing an example of this fact.

see footnote ***) handwritten p. 5a

Throughout, a number of footnotes are found. This is in keeping with the fact that footnotes can play a special role within the framework of an elaborated presentation. In particular, they are useful to emphasize relevant issues in an economic way. Being outside the main text, they give the possibility of stating eventual im-

[1] This is also a consequence of the fact that physically correct concepts are forced to be expressed, to become precise, in an axiomatic fashion, a style foreign to the one used here.

[2] "The concepts and propositions get "meaning" viz. "content", only through their connection with sense–experience. . . The degree of certainty with which this connection, viz., intuitive combination, can be undertaken, and nothing else, differentiates empty fantasy from scientific "truth". . . A correct proposition borrows its "truth" from the truth-content of the system to which it belongs" (A. Einstein, Autobiographical notes, in Albert Einstein, Ed. P. A. Schlipp, Harper, New York (1951)) p.1, Vol I.

**) footnote p. 5 Preface Within this context let us mention the intimately correlated subjects of Random Phase Approximation (RPA) and Particle Vibration Coupling (PVC) not found in a fourth year curriculum. They are explained and refer to in a number of places throughout the present monograph, starting from a pedestrian level and for both surface (particle-hole) and pairing (particle-particle and hole-hole) vibrations (pp 21-28), and then extended to include further details and facets (see pp 47-48, 75-76, 78 (Fig. 2.3.1), 85, 87, 157-158 and 188-189). Furthermore, in the case of RPA of pairing vibrations around the closed shell system ^{208}Pb, one provides in pp. 202-215, detailed documentation of the numerical calculation of the associated dispersion relation solutions and associated wavefunctions (X- and Y-amplitudes) at the level of an excersize in a fourth yearcourse.

A similar situation is ecountered in connection with the subject of the Distorted Wave Born Approximation (DWBA), again a subject not found in fourth yearcurricula. It is treated at the pedestrian level in pp 147-152, 392-398 and 464-469 in connection with inelastic, one-particle and two-particle transfer reactions respectively. And once more in full detail, without eschewing complexities, but again in the style of an example-excersize**) in pp. 362-370 and 430-461 for one- and two-particle transfer reactions respectively.

***) "Gentagelsen den er virkeligheden, og Tilværelsens Alvor" (Repetition is the reality, and life's seriousness; S. Kierkegaard Gjentagelsen (1843))
ПОВТОРЕНИЕ МАТЬ ОБУЧЕНИЕ: repetition is learning's mother.

**) "Sometimes one has to say difficult things, but one ought to say them as simply as one know how" (G. H. Hardy, A mathematician's apology, Cambridge University Press, Cambridge (1969)).

portant results, without the need of elaborating on the proof. Within this context, and keeping the natural distances, one can mention that in the paper in which Born[3] introduces the probabilistic interpretation of Schrödinger's wavefunction, the fact that this probability is connected with its modulus squared and not with the wavefunction itself, is only referred to in a footnote.

Most of the material contained in this monograph have been the subject of lectures of the four year course "Nuclear Structure Theory" which RAB delivered throughout the years at the Department of Physics of the University of Milan, as well as at the Niels Bohr Institute and at Stony Brook (State University of New York). It was also presented by the authors in the course Nuclear Reactions held at the PhD School of Physics of the University of Milan.

GP wants to thank the tutoring of Ben Bayman concerning specific aspects of two–particle transfer reactions. Discussions with Ian Thompson and Filomena Nunes on a variety of reaction subjects are gratefully acknowledged. RAB acknowledges the essential role the collaboration with Francisco Barranco and Enrico Vigezzi has played concerning nuclear structure aspects of the present monograph. Its debt with the late Aage Winther regarding the reaction aspects of it is difficult to express in words. The overall contributions of Daniel Bès, Ben Bayman and Pier Francesco Bortignon[4] are only too explicitly evident throughout the text and constitute a daily source of inspiration. G. P. and R. A. B. have received important suggestions and comments regarding concrete points and the overall presentation of the material discussed below from Ben Bayman, Pier Francesco Bortignon, David Brink, Willem Dickhoff and Vladimir Zelevinsky and are here gratefully acknowledged.

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Bibliography

M. Born. Zur Quantenmechanik der Stoßvorgänge. *Zeitschr. f. Phys.*, 37:863, 1926.

W. Pauli. Über gasentartung und paramagnetismus. *Zeitschrift für Physik*, 41(2): 81, Jun 1927.

especially

We are beholden to Elena Litvinova and Horst Lenske for much constructive criticism and suggestions.

[3] Born (1926). Within this context, it is of notice that the extension of Born probabilistic interpretation to the case of many-partice systems is also found in a footnote (Pauli (1927), footnote on p. 83 of the paper).

[4] Deceased August 27, 2018.

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I

Chapter 1

Introduction

(2(He), 10(Ne), 18(Ar), 36(Kr), 54(Xe), 86(Ra))

1.1 Views of the nucleus

In the atom, the nucleus provides the Coulomb field in which negatively charged electrons ($-e$) move independently of each other in single–particle orbitals. The filling of these orbitals explains Mendeleev's periodic table. Thus the valence of the chemical elements as well as the particular stability of the noble gases (~~He, N, Ar, Kr, Xe and R~~a) associated with the closing of shells (~~Fig. 1.1.1~~). The dimension of the atom is measured in angstroms (Å$=10^{-8}$cm), and typical energies in eV, the electron mass being $m_e \approx 0.5$ MeV (MeV$=10^6$eV).

The atomic nucleus is made out of positively charged protons ($+e$) and of (uncharged) neutrons, nucleons, of mass $\approx 10^3$ MeV (m_p = 938.3 MeV, m_n = 939.6 MeV). Nuclear dimensions are of the order of few fermis (fm= 10^{-13} cm). While the stability of the atom is provided by a source external to the electrons, namely the atomic nucleus, this system is self–bound as a result of the strong interaction of range $a_0 \approx 0.9$ fm and strength $v_0 \approx -100$ MeV acting among nucleons.

1.1.1 The liquid drop and the shell model

While most of the atom is empty space, the density of the atomic nucleus is conspicuous (ρ = 0.17 nucleon/fm^3). The "closed packed" nature of this system implies a short mean free path as compared to nuclear dimensions. This can be estimated from classical kinetic theory $\lambda \approx (\rho\sigma)^{-1} \approx 1$ fm, where $\sigma \approx 2\pi a_0^2$ is the nucleon–nucleon cross section. It seems then natural to liken the atomic nucleus to a liquid drop (Bohr and Kalckar). This picture of the nucleus provided the framework to describe the basic features of the fission process (Meitner and Frisch (1939); Bohr and Wheeler (1939)).

The leptodermic properties of the atomic nucleus are closely connected with the semi–empirical mass formula (Weizsäcker (1935))

$$m(N,Z) = (Nm_n + Zm_p) - \frac{1}{c^2}B(N,Z), \tag{1.1.1}$$

associated inertia for small amplitude oscillations is,

$$D_\lambda = \frac{3}{4\pi}\frac{1}{\lambda}AMR^2, \tag{1.1.7}$$

the energy of the corresponding mode being

$$\hbar\omega_\lambda = \hbar\sqrt{\frac{C_\lambda}{D_\lambda}}. \tag{1.1.8}$$

The label λ stands for the angular momentum of the vibrational mode, μ being its third component (see Eq. (1.2.1)). Aside from λ, μ, surface vibrations can also be characterized by an integer $n(= 1, 2, \dots)$, an ordering number indicating increasing energy. For simplicity, a single common label α will also be used.

Experimental information associated with low–energy quadrupole vibrations, namely $\hbar\omega_2$ and the electromagnetic transition probabilities $B(E2)$, allow to determine C_2 and D_2. The resulting C_2 values exhibit variations by more than a factor of 10 both above and below the liquid–drop estimate. The observed values of D_2 are large as compared with the mass parameter for irrotational flow, a fact connected with with the role played by pairing in nuclei (see footnote 2; see also Bohr and Mottelson (1975 p. 75)

A picture apparently antithetic to that of the liquid drop, the shell model, emerged from the study of experimental data, plotting them against either the number of protons (atomic number), or the number of neutrons in the nuclei, rather than against the mass number. One of the main nuclear features which led to the development of the shell model was the study of the stability and abundance of nuclear species and the discovery of what are usually called magic numbers (Elsasser (1933); Mayer (1948); Haxel et al. (1949)). What makes a number magic is that a configuration of a magic number of neutrons, or of protons, is unusually stable whatever the associated number of other nucleons is (Mayer (1949); Mayer and Teller (1949)).

The strong binding of a magic number of nucleons and weak binding for one more reminds, only relatively weaker, the results ~~displayed in Fig. 1.1.1~~ concerning the atomic stability of rare gases. In the nuclear case, at variance with the atomic case, the spin–orbit coupling play an important role, as can be seen from the level scheme shown in Fig. 1.1.3, obtained by assuming that nucleons move independently of each other in an average potential of spherical symmetry.

A closed shell, or a filled level, has angular momentum zero. Thus, nuclei with one nucleon outside (missing from) closed shell, should have the spin and parity of the orbital associated with the odd nucleon (–hole), a prediction confirmed by the data (available at that time) throughout the mass table. Such a picture implies that the nucleon mean free path is large compared to nuclear dimensions.

The systematic studies of the binding energies leading to the shell model found also that the relation (1.1.2), has to be supplemented to take into account the fact that nuclei with both odd number of protons and of neutrons are energetically unfavored compared with even–even ones ~~(inset Fig. 1.1.1)~~ by a quantity of the order of $\delta \approx 33 MeV/A^{3/4}$ called the pairing energy[2].

[2]Connecting with further developments associated with the BCS theory of superconductivity and at the basis of the odd-even staggering effect.

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The low–lying excited state of closed shell nuclei can be interpreted as a rule, as harmonic quadrupole or octupole collective vibrations (Fig. 1.1.4) described by the Hamiltonian[3] (see e.g. Bohr and Mottelson (1975)),

$$H_{coll} = \sum_{\lambda\mu}\left(\frac{1}{2D_\lambda}|\Pi_{\lambda\mu}|^2 + \frac{C_\lambda}{2}|\alpha_{\lambda\mu}|^2\right) \tag{1.1.9}$$

Following Dirac (1930) one can describe the oscillatory motion introducing boson creation (annihilation) operator $\Gamma^\dagger_{\lambda\mu}$ $(\Gamma_{\lambda\mu})$ obeying

$$\left[\Gamma_\alpha, \Gamma^\dagger_{\alpha'}\right] = \delta(\alpha, \alpha'), \tag{1.1.10}$$

leading to

$$\hat{\alpha}_{\lambda\mu} = \sqrt{\frac{\hbar\omega_\lambda}{2C_\lambda}}\left(\Gamma^\dagger_{\lambda\mu} + (-1)^\mu\Gamma_{\lambda-\mu}\right), \tag{1.1.11}$$

and a similar expression for the conjugate momentum variable $\hat{\Pi}_{\lambda\mu}$, resulting in

$$\hat{H}_{coll} = \sum \hbar\omega_\lambda\left((-1)^\mu\Gamma^\dagger_{\lambda\mu}\Gamma_{\lambda-\mu} + 1/2\right). \tag{1.1.12}$$

The frequency is $\omega_\lambda = (C_\lambda/D_\lambda)^{1/2}$, while $(\hbar\omega_\lambda/2C_\lambda)^{1/2}$ is the amplitude of the zero–point fluctuation of the vacuum state $|0\rangle_B$, $|n_{\lambda\mu} = 1\rangle = \Gamma^\dagger_{\lambda\mu}|0\rangle_B$ being the one–phonon state. To simplify the notation, in many cases one writes $|n_\alpha = 1\rangle$.

The ground and low–lying states of nuclei with one nucleon outside closed shell can be described by the Hamiltonian

$$H_{sp} = \sum_\nu \epsilon_\nu a^\dagger_\nu a_\nu, \tag{1.1.13}$$

where $a^\dagger_\nu (a_\nu)$ is the single–particle creation (annihilation) operator,

$$|\nu\rangle = a^\dagger_\nu|0\rangle_F, \tag{1.1.14}$$

being the single-particle state of quantum numbers $\nu(\equiv nljm)$ and energy ϵ_ν, while $|0\rangle_F$ is the Fermion vacuum. It is of notice that

$$\left[H_{coll}, \Gamma^\dagger_{\lambda'\mu'}\right] = \hbar\omega_{\lambda'}\Gamma^\dagger_{\lambda'\mu'} \tag{1.1.15}$$

and

$$\left[H_{sp}, a^\dagger_{\nu'}\right] = \epsilon_{\nu'}a^\dagger_{\nu'}. \tag{1.1.16}$$

(Bardeen et al. (1957a,b)) and its extension to the atomic nucleus (Bohr et al. (1958)), the quantity δ is identified with the pairing gap Δ parametrized according to $\Delta = 12\text{MeV}/\sqrt{A}$ (Bohr and Mottelson (1969)). It is of notice that for typical superfluid nuclei like ^{120}Sn, the expression of δ leads to a numerical value which can be parametrized as $\delta \approx 10\ \text{MeV}/\sqrt{A}$.

[3] Classically $\Pi_{\lambda\mu} = D_\lambda\dot{\alpha}_{\lambda\mu}$.

(W) 1.1.2 Nuclear excitations

In addition to the quantum numbers λ and μ one can characterize nuclear excitations by additional quantum numbers such as parity π, isospin τ and spin σ. Furthermore one can assign a particle (baryon or transfer) quantum number, α. For a nucleon moving above the Fermi surface one has $\beta=+1$, while for a hole in the Fermi sea $\beta=-1$. For bosonic excitations $\alpha=0$ for a mode associated with e.g. surface oscillation, which can also be viewed as a correlated particle-hole excitation (within this context see Fig. 1.2.3). For modes which involve the addition or substraction of two correlated nucleons to the nucleus $\beta=+2$ (Fig. 1.3.1) ~~and~~ $\alpha=-2$ respectively. The excitation which connects the ground state of an even nucleus, to the ground state of the next even nucleus, i.e. monopole pairing vibration (e.g. $\lambda^\pi=0^+$, $\alpha=+2$) is of this type.

The structure of excitations with $|\beta|>2$, expresses the various possible clustering effects of corresponding order. ~~For example α-vibrations~~. (~~see Fig.~~ ~~see figure α-vibs~~) ~~Macchiavelli Frauendorf~~ ~~refs not treated~~ ~~p-n " "~~

In particular, the low-lying quadrupole vibrations of even-even nuclei have quantum numbers $\lambda^\pi=2^+$, $\tau=0$ (protons and neutrons oscillate in phase) and $\sigma=0$ (no spin-flips in the excitation).

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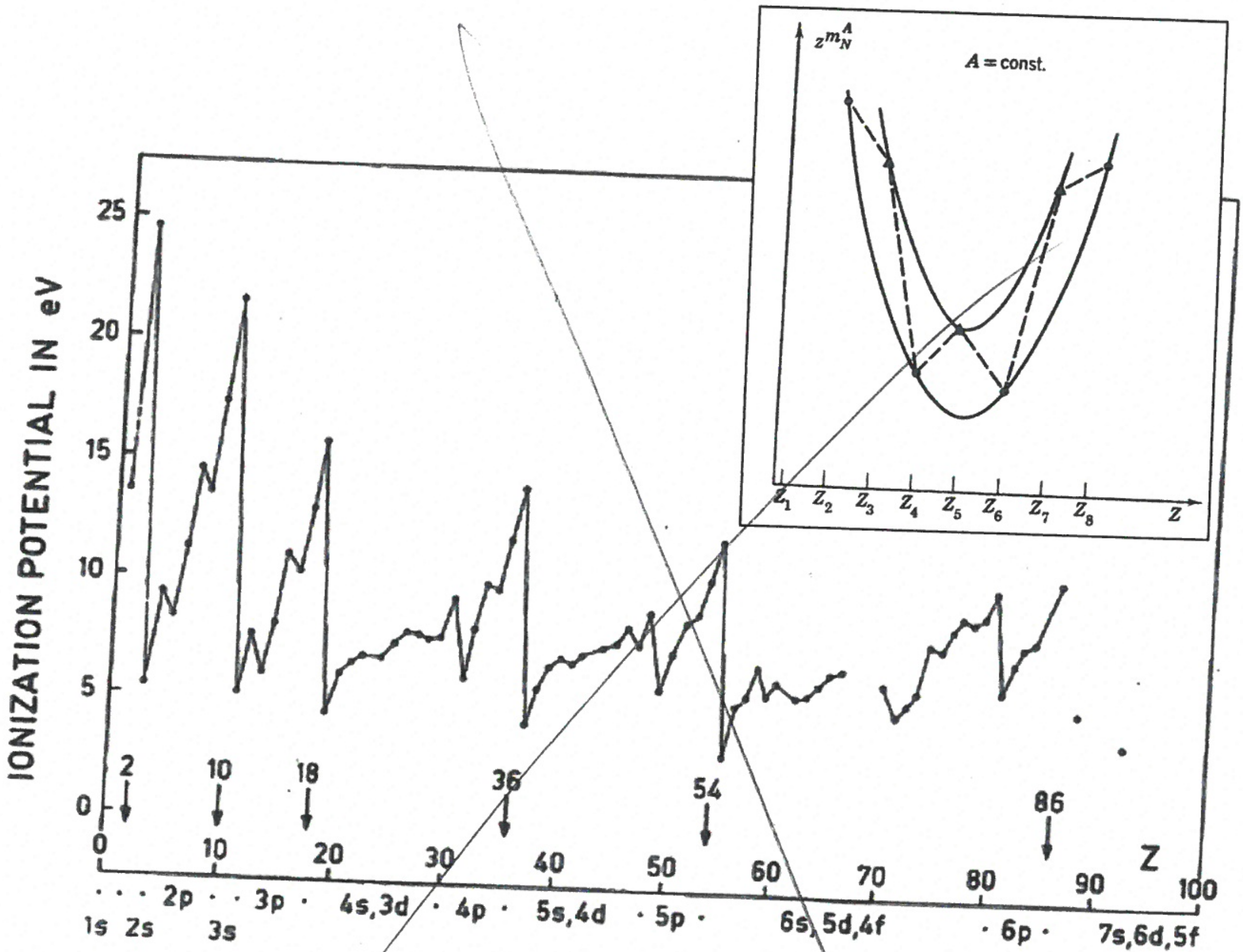

Figure 1.1.1: The values of the atomic ionization potentials. The closed shells, corresponding to electron number 2(He), 10(Ne), 18(Ar), 36(Kr), 54(Xe), and 86(Ra), are indicated. After Bohr and Mottelson (1969). In the inset, masses of nuclei with even A are shown (after Mayer and Jensen (1955)).

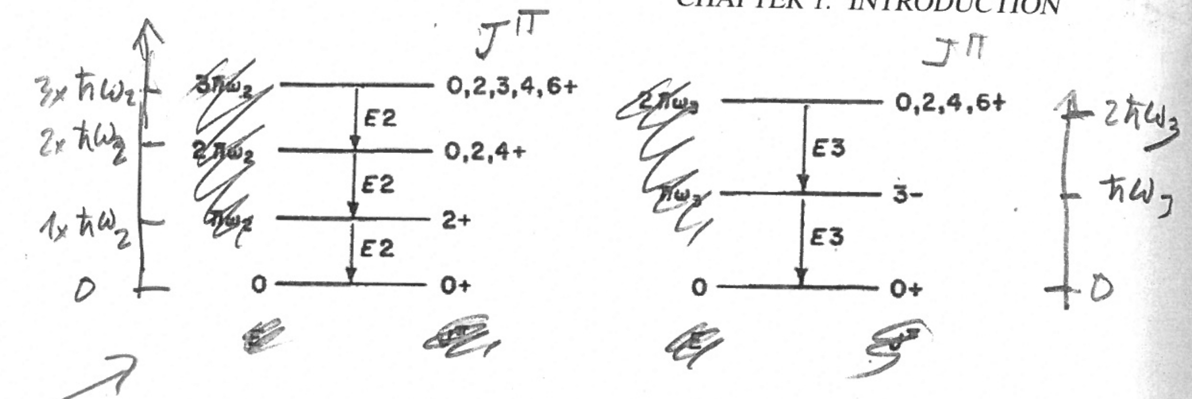

Figure 1.1.4: Schematic representation of harmonic quadrupole and octupole liquid drop collective surface vibrational modes ~~(after Rowe (1970))~~.

This is an obvious outcome resulting from the bosonic

$$\left[\Gamma_\alpha, \Gamma^\dagger_{\alpha'}\right] = \delta(\alpha, \alpha') \tag{1.1.17}$$

and fermionic

$$\left\{a_\nu, a^\dagger_{\nu'}\right\} = \delta(\nu, \nu') \tag{1.1.18}$$

commutation (anti-commutation) relations.

Both the existence of drops of nuclear matter displaying collective surface vibrations, and of independent–particle motion in a self–confining mean field are emergent properties not contained in the particles forming the system, neither in the NN–force, but on the fact that these particles behave according to the rules of quantum mechanics, move in a confined volume and that there are many of them.

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Generalized rigidity as measured by the inertia parameter D_λ, as well as surface tension closely connected to the restoring force C_λ, implies that acting on the system with an external time-dependent (nuclear and/or Coulomb) field, the system reacts as a whole. ~~This behavior is to be found nowhere~~ in the properties of the nucleons, nor in the nucleon-nucleon scattering phase shifts consistent with Yukawa's predictions of the existence of a π-meson as the carrier of the strong force acting among nucleons.

Similarly, ~~the fact that~~ nuclei probed through fields which change in one unit particle number (e.g. (d, p) and (p, d) reactions) react in term of independent particle motion, feeling the pushings and pullings of the other nucleons only when trying to leave the nucleus, is not apparent in the detailed properties of the NN–forces, ~~not even in those carrying the quark–gluon input~~. Within this context, independent particle motion can be considered a *bona fide* emergent property.

Collective surface vibrations and independent particle motion are examples of what are called elementary modes of excitation in many–body physics, and collective variables in soft–matter physics.

L to p. 20

Expressed it differently, generalized rigidity closely connected to the inertial parameter D_λ implies that acting on a nucleus with an external $\beta=0$, time-dependent (nuclear / Coulomb) field, the systems reacts as a whole (collective vibrations; also rotations see Sect. 1.4), while acting with fields which change particle number by one ($\beta=\pm1$; e.g. (d,p) and (p,d) reactions) the system reacts in terms of independent particle motion, felling the pushings and pullings of the other nucleons when trying to leave the nucleus. Such a behaviour can hardly be inferred from the study of the NN-forces in free space, being truly emergent properties of the finite, quantum many-body nuclear system. L to p. 20

~~*) Nuclear excitation can be characterized by a variety of quantum numbers as angular momentum and parity (denoted λ, J or j and π respectively), isospin τ, and transfer quantum number β, $\beta=0$~~

~~*) In other words, in order for a nucleon moving in a level close to the Fermi energy to display a mean free path larger than nuclear dimensions and to be reflected at the nuclear surface through an elastic process, all other nucleons must move in a rather ordered, correlated fashion. Within this context to posit that single-particle motion is the most collective of all nuclear motions (Mottelson (1962)) seems justified.~~

1.2 The particle-vibration coupling $U(r,R)$

The oscillation of the nucleus under the influence of surface tension implies that the potential $U(R,r)$ in which nucleons move independently of each other change with time. For low–energy collective vibrations this change is slow as compared with single–particle motion. Within this scenario the nuclear radius can be written as

$$R = R_0\left(1 + \sum_{LM} \alpha_{\lambda\mu} Y^*_{\lambda\mu}(\hat{r})\right) \quad \lambda\mu \tag{1.2.1}$$

Assuming small amplitude motion,

$$U(r,R) = U(r,R_0) + \delta U(r), \tag{1.2.2}$$

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where

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$$\delta U = -\kappa\hat{\alpha}\hat{F}, \tag{1.2.3}$$

and

$$\hat{F} = \sum_{\nu_1\nu_2} \langle \nu_1|F|\nu_2\rangle a^\dagger_{\nu_1} a_{\nu_2}, \tag{1.2.4}$$

while (see also Sect. 2.3)

$$F \stackrel{?}{=} -\frac{R_0}{K}\frac{\partial U}{\partial r} Y^*_{\lambda\mu}(\hat{r})$$

$$F = \frac{R_0}{\kappa}\frac{\partial U}{\partial r} Y^*_{\lambda\mu}(\hat{r}). \tag{1.2.5}$$

The coupling between surface oscillation and single–particle motion, namely the particle vibration coupling (PVC) Hamiltonian δU (Fig. 1.2.1) is a consequence of the overcompleteness of the basis. Diagonalizing δU making use of the graphical (Feynman) rules of nuclear field theory (NFT) to be discussed in the following Chapter, one obtains structure results which can be used in the calculation of transition probabilities and reaction cross sections, quantities which can be compared with experimental findings.

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In fact, within the framework of NFT, single–particles are to be calculated as the Hartree–Fock solution of the NN–interaction $v(|\mathbf{r}-\mathbf{r}'|)$ (Fig. 1.2.2), in particular to

$$U(r) = \int d\mathbf{r}'\rho(r')v(|\mathbf{r}-\mathbf{r}'|) \tag{1.2.6}$$

— e.g. a regularized NN-bare interaction in terms of renormalization group methods or similar techniques (V_{low-k}), taking eventually also 3N terms into account — leading,

that is, ~~being~~ the Hartree field[4] expressing the selfconsistency between density ρ and potential U (Fig. 1.2.2 (b) (1) and (3)), while vibrations are to be calculated in the

[4] To this potential one has to add the Fock potential resulting from the fact that nucleons are fermions. This exchange potential (Fig. 1.2.2 (b) (2 and 4)) is essential in the determination of single–particle energies and wavefunctions. Among other things, it takes care of eliminating the nucleon self interaction from the Hartree field.

where

$$\delta U = -\kappa \hat{\alpha} \hat{F} = \Lambda_\alpha \left(\Gamma^{+}_{\lambda\mu} + (-1)^{\mu} \Gamma_{\lambda-\mu} \right) \hat{F} \quad (1,2,3)$$

and

$$\Lambda_\alpha = -\kappa \sqrt{\frac{\hbar\omega_\lambda}{2C_\lambda}}, \quad (1.2.4)$$

is the particle-vibration coupling (PVC) strength, product of the dynamic deformation

$$\beta_\lambda = \sqrt{2\lambda+1} \sqrt{\frac{\hbar\omega_\lambda}{2C_\lambda}}, \quad (1,2,5)$$

and of the strength κ, while

$$\hat{F} = \sum_{\nu_1\nu_2} \langle \nu_1 | F | \nu_2 \rangle a^{+}_{\nu_1} a_{\nu_2}, \quad (1,26)$$

with the dimensionless single-particle field,

$$F = \frac{R_0}{\kappa} \frac{\partial U}{\partial r} Y^{*}_{\lambda\mu}(\hat{r}), \quad (1,2,7)$$

An estimate of κ is provided below (Eq. (1,2,11))

The coupling between surface oscillations and single-particle motion reflect, in the nuclear case, the ~~fact that~~ collective vibrations (bosons) are composites of the nucleons (fermions) degrees of freedom*).

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*) The associated overcompletness does not imply particular worries as the elimination of the spurious degrees of freedom can be carried out in a systematic fashion (see Sect. 2,7, in particular 2,7,2 and 2,7,3).