4/11/13

Simultaneous versus successive Cooper pair transfer in nuclei

Cooper pair transfer is thought to be tantamount to simultaneous transfer. In this process a nucleon goes over through the NN-interaction v, the (Fig. α) second one does it making use of the correlations with its partners. Consequently, in the independent particle limit, simultaneous Cooper pair transfer should not be possible. Nonetheless, it remains operative. This is because the particle transferred by v is followed by a second one which profits of the non-orthogonality of the ~~inter~~ ~~single-particle~~ wave functions describing ~~the~~ single-particle motion in target and projectile (Fig. β, lower part). This is the reason why this ~~two-particle~~ transfer amplitude has to be substracted from the previous one, representing a spurious contribution to simultaneous transfer

In other words $T^{(1)}$ gives the wrong cross section, even at the level of simultaneous transfer, as it violates two-nucleon transfer sum rules.

arising from the overcompleteness of the basis employed.

~~The resulting cancellation, which ensures the correct limit of the simultaneous transfer amplitude in the independent particle limit, is quite conspicuous in actual nuclei. This is because Cooper pairs are weakly bound system, its correlation energy E_{co} being much smaller than typical nuclear energies, as represented by one-nucleon separation energies and/or the Fermi energy E_F.~~

The resulting cancellation (Fig γ and App α) is quite conspicuous in actual nuclei, in keeping with the fact ~~that~~ Cooper pairs are weakly correlated systems. This is the reason why, ~~as a rule~~, ~~the~~ successive transfer processes, in which v acts twice, is the dominant ~~process~~ mechanism ~~of two-nucleon~~ in two-nucleon transfer reactions (Fig. β, upper part).

While this mechanism is ~~viewed as~~ antithetical ~~pairing correlated pair correlated transfer & stron~~ to the transfer of strongly correlated (bosons) fermion pairs, ~~it~~ probes the same ~~nuclear correlati~~ pairing correlations as simultaneous transfer does in the nuclear core (quasi-bosons) (App. α).

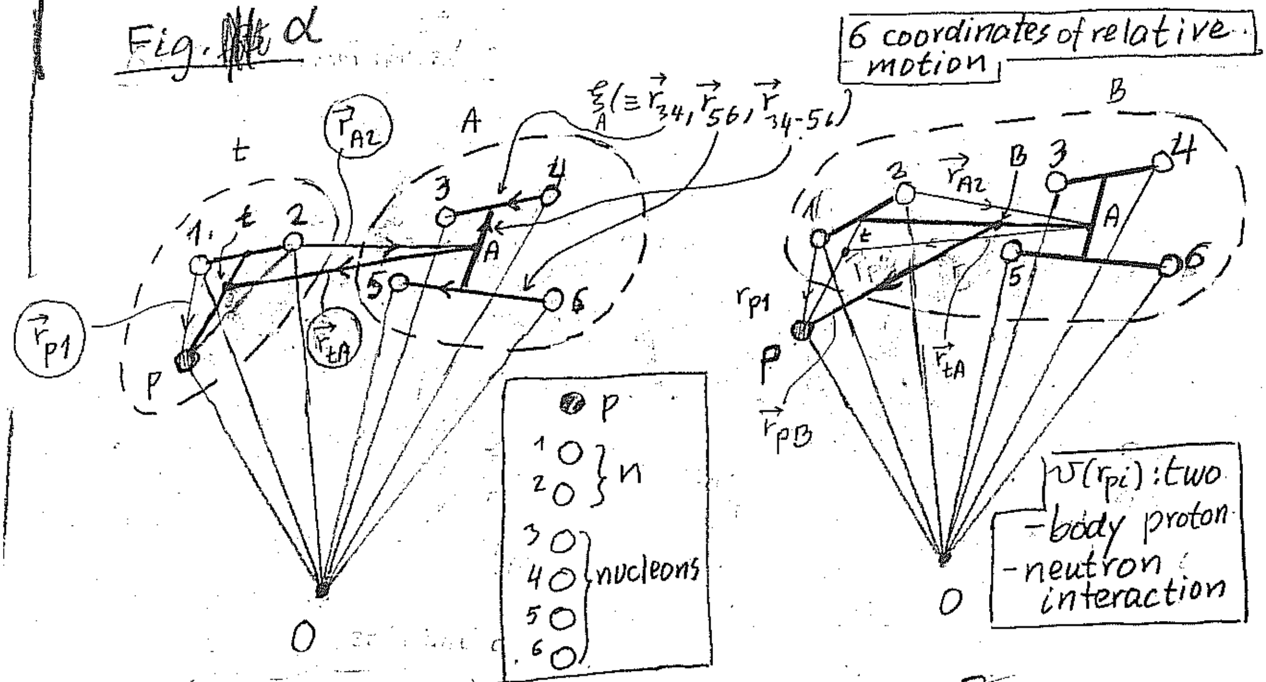
This is because Cooper pairs are ~~quite exte~~ ~~virtually~~ quite extended objects, the two nucleons being (virtually) correlated over distances much larger than typical nuclear dimensions (Fig. γ). ~~a property which becomes real in a reaction processes~~

In a two-nucleon transfer ~~reaction~~ process this virtual property becomes real, the difference between the character of simultaneity and of succession ~~become~~ of the two-nucleon transfer process becoming strongly blurred.

~~two-nucleon transfer~~
~~In a reaction process, , in which the partners of a Cooper pair (F~~

box

Fig. α



$$\underbrace{t \quad + \quad A}_{\text{entrance channel}} \longrightarrow \underbrace{p + B}_{\text{exit channel}}$$

$$\phi_t(\vec r_{p1}, \sigma_1, \vec r_{p2}, \sigma_2)\, \chi^{1/2}_{m_s}(\sigma_p)\, \psi_A(\xi_A)\, \chi^{(+)}_{tA}(\vec r_{tA}) \qquad \chi^{1/2}_{m_s}(\sigma_p)\, \psi_B(\xi_B)\, \chi^{(-)}_{pB}(\vec r_{pB})$$

$$\left(\Phi_d(r_{p1}, \sigma_1)\, \phi_d(r_{p2}, \sigma_2)\, \chi^{1/2}_{m_s}(\sigma_p)\, \psi_A(\xi_A)\, \chi^{(+)}_{tA}(r_{tA})\right)$$

$$\psi_B(\xi_A, \vec r_{A1}, \sigma_1, \vec r_{A2}, \sigma_2) = \psi_A(\xi_A) \sum_{l_1 j_1} \left[\phi^{A+2}_{l_1 j_1}(\vec r_{A1}, \sigma_1, \vec r_{A2}, \sigma_2)\right]^0_0$$

$$= \psi_A(\xi_A) \sum_{nm} a_{nm} \left[\varphi^{A+2}_{n l_1 j_1}(\vec r_{A1}, \sigma_1)\, \varphi^{A+2}_{m l_2 j_2}(\vec r_{A2}, \sigma_2)\right]^0_0$$

$\boxed{\text{First order in } v}$

$$T^{(1)} = 2 \sum_{\sigma_1 \sigma_2 \sigma_p} \int d\xi_A\, d^3 r_{tA}\, d^3 r_{p1}\, d^3 r_{A2}\, \psi^*_A(\xi_A) \sum_{l_1 j_1} \left[\phi^{A+2}_{l_1 j_1}(\vec r_{A1}, \sigma_1, \vec r_{A2}, \sigma_2)\right]^{0*}_0 \chi^{(-)*}(\vec r_{pB})\, \chi^{1/2 *}_{m_s}(\sigma_p)\, v(r_{p1})$$

$$\times\, \phi_t(\vec r_{p1}, \sigma_1, \vec r_{p2}, \sigma_2)\, \chi^{1/2}_{m_s}(\sigma_p)\, \psi_A(\xi_A)\, \chi^{(+)}_t(\vec r_{tA})$$

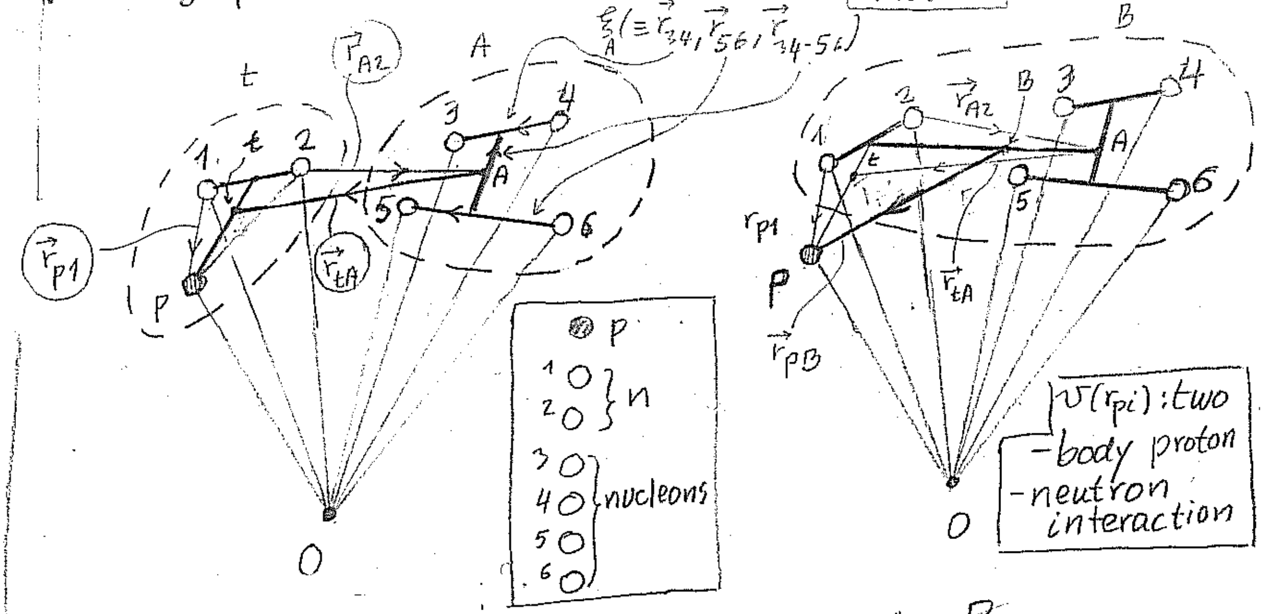
$$= 2 \sum_{\substack{l_1 j_1 \\ \sigma_1 \sigma_2}} \int d\xi_A\, d^3 r_{tA}\, d^3 r_{p1}\, d^3 r_{A2} \left[\phi^{A+2}_{l_1 j_1}(\vec r_{A1}, \sigma_1, \vec r_{A2}, \sigma_2)\right]^{0*}_0 \chi^{(-)*}_{pB}(\vec r_{pB})\, v(r_{p1})$$

$$\phi_t(\vec r_{p1}, \sigma_1, \vec r_{p2}, \sigma_2)\, \chi^{(+)}_t(\vec r_{tA})$$

Of notice that the above expression violates two-nucleon sum rule transfer (i.e. it transfers more than two neutrons) by exactly $T^{(1)}_{NO}$, operative also in the independent particle limit.

box

Fig. ~~β~~ d



$$\underbrace{t \quad + \quad A}_{\text{entrance channel}} \longrightarrow \underbrace{p \quad + \quad B}_{\text{exit channel}}$$

$$\phi_t(\vec{r}_{p1},\sigma_1,\vec{r}_{p2},\sigma_2)\chi^{1/2}_{m_s}(\sigma_p)\psi_A(\xi_A)\chi^{(+)}_{tA}(\vec{r}_{tA}) \qquad \chi^{1/2}_{m_s}(\sigma_p)\,\psi_B(\xi_B)\,\chi^{(-)}_{pB}(\vec{r}_{pB})$$

$$\left(\phi_d(r_{p1},\sigma_1)\,\phi_d(r_{p2},\sigma_2)\,\chi^{1/2}_{m_s}(\sigma_p)\,\psi_A(\xi_A)\,\chi^{(+)}_{tA}(r_{tA})\right)$$

$$\psi_B(\xi_A,\vec{r}_{A1},\sigma_1,\vec{r}_{A2},\sigma_2) = \psi_A(\xi_A)\sum_{l_1 j_1}\left[\phi^{A+2}_{l_1 j_1}(\vec{r}_{A1},\sigma_1,\vec{r}_{A2},\sigma_2)\right]^0_0$$

$$= \psi_A(\xi_A)\sum_{nm} a_{nm}\left[\varphi^{A+2}_{n l_1 j_1}(\vec{r}_{A1},\sigma_1)\,\varphi^{A+2}_{m l_2 j_2}(\vec{r}_{A2},\sigma_2)\right]^0_0$$

First order in v

$$T^{(1)} = 2\sum_{\substack{\sigma_1\sigma_2\\ \sigma_p}}\int d\xi_A\, d^3r_{tA}\, d^3r_{p1}\, d^3r_{A2}\, \psi^*_A(\xi_A)\sum_{l_1 j_1}\left[\phi^{A+2}_{l_1 j_1}(\vec{r}_{A1},\sigma_1,\vec{r}_{A2},\sigma_2)\right]^{0*}_0 \chi^{(-)*}(\vec{r}_{pB})\chi^{1/2\,*}_{m_s}(\sigma_p)\, v(\vec{r}_{p1})$$

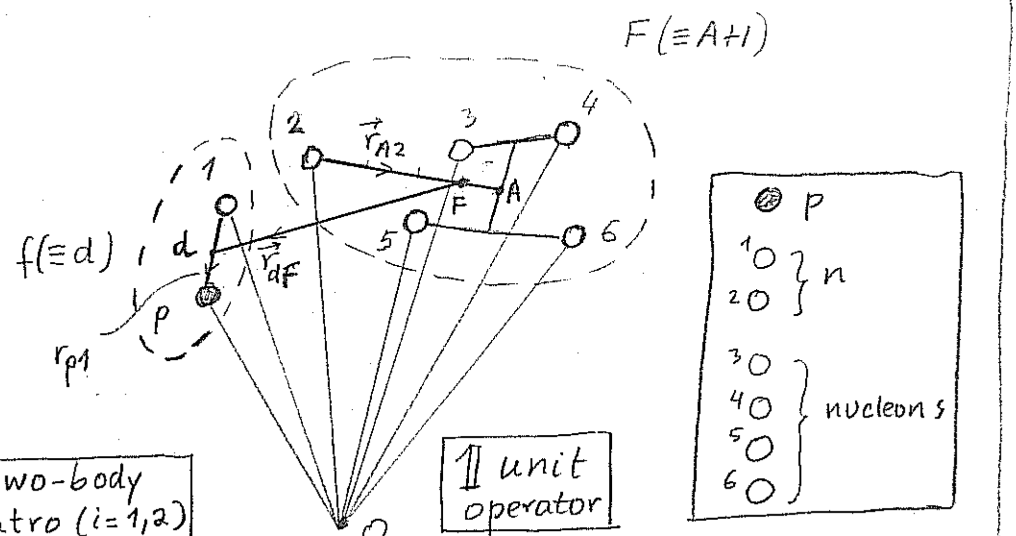
$$\times\, \phi_t(\vec{r}_{p1},\sigma_1,\vec{r}_{p2},\sigma_2)\,\chi^{1/2}_{m_s}(\sigma_p)\,\psi_A(\xi_A)\,\chi^{(+)}_t(\vec{r}_{tA})$$

$$= 2\sum_{\substack{l_1 j_1\\ \sigma_1\sigma_2}}\int d\xi_A\, d^3r_{tA}\, d^3r_{p1}\, d^3r_{A2}\left[\phi^{A+2}_{l_1 j_1}(\vec{r}_{A1},\sigma_1,\vec{r}_{A2},\sigma_2)\right]^{0*}_0 \chi^{(-)*}_{pB}(\vec{r}_{pB})\, v(\vec{r}_{p1})$$

$$\phi_t(\vec{r}_{p1},\sigma_1,\vec{r}_{p2},\sigma_2)\,\chi^{(+)}_t(\vec{r}_{tA})$$

Of notice that the above expression violates two-nucleon sum rule transfer (i.e. it transfers more than two neutrons) by exactly $T^{(1)}_{NO}$, operative also in the independent particle limit.

Fig. ~~*~~ β



$v(r_{pi})$: two-body proton neutro ($i=1,2$) interaction

$\underbrace{f(\equiv d) + F(\equiv A+1)}$
intermediate channel

no distorted wave as (f,F) is a virtual channel with no asymptotic waves, Thus 5 relative coordinates →
$$\chi^{1/2}_{m_s}(\sigma_p)\,\Phi_d(\vec{r}_{p1},\sigma_1)\,\psi(\xi_A)\,\varphi^{A+1}_{\ell_f j_f m_f}(\vec{r}_{A2},\sigma_2)$$

Second order in v

$$T^{(2)}_{succ} = 2\sum_{\substack{l_i j_i,\, \ell_f j_f m_f, m_s, m_s' \\ \sigma_1\sigma_2\sigma_1'\sigma_2'}} \int d\xi_A\, d^3r_{dF}\, d^3r_{p1}\, d^3r_{A2}\, \chi^{(-)*}_{pB}(\vec{r}_{pB})\, \chi^{1/2*}_{m_s}(\sigma_p)\, \chi^{*}_{B}(\xi_B)\, v(r_{p1})\, \Phi_d(r_{p1},\sigma_1)\, \varphi^{A+1}_{\ell_f j_f m_f}(\vec{r}_{A2},\sigma_2)\, \chi^{1/2}_{m_s}(\sigma_p)\, \psi_A(\xi_A)$$
$$\times \frac{2\mu_{dF}}{\hbar^2} \int d\xi_A'\, d^3r_{p1}'\, d^3r_{A2}'\, d^3r_{dF}'\, G(\vec{r}_{dF},\vec{r}_{dF}')\, \chi^{1/2*}_{m_s'}(\sigma_p')\, \Phi^*_d(\vec{r}_{p1}',\sigma_1')\, \psi(\xi_A')\, \varphi^{A+1}_{\ell_f j_f m_f}(\vec{r}_{A2}',\sigma_2')\, v(r_{p2}')$$
$$\times\, \Phi_t(\vec{r}_{p1}'\sigma_1', \vec{r}_{p2}',\sigma_2')\, \chi^{1/2}_{m_s'}(\sigma_p')\, \psi_A(\xi_A')\, \chi^{(+)}_{tA}(\vec{r}_{pA}')$$

$$= 2\sum_{\substack{l_i j_i\, \ell_f j_f m_f \\ \sigma_1\sigma_2\sigma_1'\sigma_2'}} \int d^3r_{dF}\, d^3r_{p1}\, d^3r_{A2}\, \chi^{(-)*}_{pB}(\vec{r}_{pB}) \left[\varphi^{A+2}_{l_i j_i}(\vec{r}_{A1},\sigma_1;\vec{r}_{A2},\sigma_2)\right]^0_0 v(r_{p1})\, \Phi_d(r_{p1},\sigma_1)\, \varphi^{A+1}_{\ell_f j_f m_f}(\vec{r}_{A2},\sigma_2)$$
$$\times \frac{2\mu_{dF}}{\hbar^2} \int d^3r_{p1}'\, d^3r_{A2}'\, d^3r_{dF}'\, G(\vec{r}_{dF},\vec{r}_{dF}')\, \Phi^*_d(\vec{r}_{p1}',\sigma_1')\, \varphi^{A+1}_{\ell_f j_f m_f}(\vec{r}_{A2}',\sigma_2')\, v(r_{p2}')\, \Phi_d(r_{p1}',\sigma_1)\, \Phi_d(r_{p2}',\sigma_2')\, \chi^{(+)}_{tA}(\vec{r}_{pA}')$$

First order in v

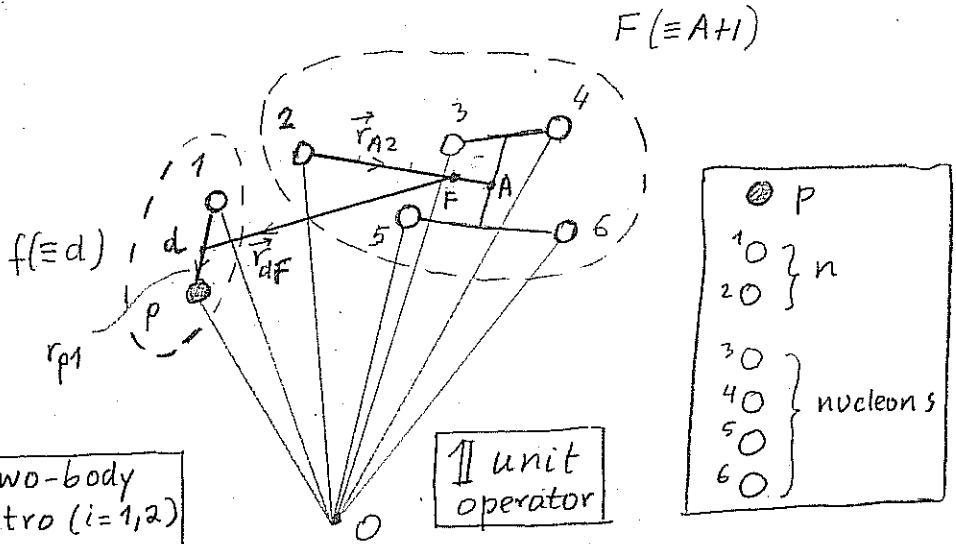
$$T^{(1)}_{NO} = 2\sum_{\substack{l_1 j_1;\, \ell_f j_f \\ m_s m_s'}} \int \overbrace{d\xi_A\, d^3r_{dF}\, d^3r_{p1}}\, d^3r_{A2}\, \chi^{(-)*}_{pB}(\vec{r}_{pB})\, \chi^{1/2*}_{m_s}(\sigma_p)\, \psi^*_B(\xi_B)\, v(r_{p1})\, \Phi_d(\vec{r}_{p1},\sigma_1)\, \varphi^{A+1}_{\ell_f j_f m_f}(\vec{r}_{A2},\sigma_2)\, \chi^{1/2}_{m_s}(\sigma_p)\, \psi_A(\xi_A)$$
$$\times \int \underbrace{d\xi_A\, d^3r_{p1}\, d^3r_{A2}'\, d^3r_{dF}'}_{\text{6 relative coords.}}\, \chi^{1/2*}_{m_s'}(\sigma_p')\, \Phi^*_d(\vec{r}_{p1}',\sigma_1')\, \psi^*(\xi_A)\, \varphi^{A+1}_{\ell_f j_f m_f}(\vec{r}_{A2}',\sigma_2')\, \mathbb{1}\, \Phi_t(\vec{r}_{p1}',\sigma_1',\vec{r}_{p2}',\sigma_2')\, \psi_A(\xi_A)\, \chi^{1/2}_{m_s'}(\sigma_p)\, \chi^{(+)}_{tA}$$

$$= 2\sum_{l_1 j_1, \ell_f j_f} d^3_{dF}\, d^3r_{p1}\, d^3r_{A2}\, \chi^{(-)*}_{pB}(\vec{r}_{pB}) \left[\varphi^{A+2}_{l_1 j_1}(\vec{r}_{A1},\sigma_1,\vec{r}_{A2},\sigma_2)\right]^0_0 v(\vec{r}_{p1})\, \Phi_d(\vec{r}_{p1},\sigma_1)\, \varphi^{A+1}_{\ell_f j_f m_f}(\vec{r}_{A2},\sigma_2)$$
$$\times \int d^3r_{p1}'\, d^3r_{A2}'\, d^3r_{dF}'\, \Phi^*_d(\vec{r}_{p1}',\sigma_1')\, \varphi^{A+1}_{\ell_f j_f m_f}(\vec{r}_{A2}',\sigma_2')\, \mathbb{1}\, \Phi_d(\vec{r}_{p1}',\sigma_1')\, \Phi_d(\vec{r}_{p2}',\sigma_2')\, \chi^{(+)}_{tA}(\vec{r}_{tA}')$$

box

repetido (5)

Fig. β



$\boxed{\mathbb{1}\text{ unit operator}}$

$\boxed{v(r_{pi})\text{: two-body proton neutro } (i=1,2) \text{ interaction}}$

$$\underbrace{f(\equiv d)+F(\equiv A+1)}_{\text{intermediate channel}}$$

no distorted wave as (f,F) is a virtual channel with no asymptotic waves. Thus 5 relative coordinates $\rightarrow$

$$\chi^{1/2}_{m_s}(\sigma_p)\,\phi_d(\vec{r}_{p1},\sigma_1)\,\psi(\xi_A)\,\varphi^{A+1}_{\ell_f j_f m_f}(\vec{r}_{A2},\sigma_2)$$

Second order in v

$$T^{(2)}_{succ}=2\sum_{\substack{l_i j_i,\, \ell_f j_f m_f, m_s, m_s' \\ \sigma_1\sigma_2\sigma_1'\sigma_2'}}\int d\xi_A\, d^3r_{dF}\, d^3r_{p1}\, d^3r_{A2}\, \chi^{(-)*}_{pB}(\vec{r}_{pB})\,\chi^{1/2*}_{m_s}(\sigma_p)\,\chi^{*}_{B}(\xi_B)\,v(r_{p1})\,\phi_d(r_{p1},\sigma_1)\,\varphi^{A+1}_{\ell_f j_f m_f}(\vec{r}_{A2},\sigma_2)\,\chi^{1/2}_{m_s}(\sigma_p)\,\psi_A(\xi_A)$$
$$\times\frac{2\mu_{dF}}{\hbar^2}\times\int d\xi_A'\, d^3r_{p1}'\, d^3r_{A2}'\, d^3r_{dF}'\, G(\vec{r}_{dF},\vec{r}_{dF}')\,\chi^{1/2*}_{m_s'}(\sigma_p')\,\phi^*_d(\vec{r}_{p1}',\sigma_1')\,\psi(\xi_A')\,\varphi^{A+1}_{\ell_f j_f m_f}(\vec{r}_{A2}',\sigma_2')\,v(r_{p2}')$$
$$\times\,\phi_t(\vec{r}_{p1}'\sigma_1',\vec{r}_{p2}',\sigma_2')\,\chi^{1/2}_{m_s'}(\sigma_p')\,\psi_A(\xi_A')\,\chi^{(+)}_{tA}(\vec{r}_{pA}')$$

$$=2\sum_{\substack{l_i j_i\, \ell_f j_f m_f \\ \sigma_1\sigma_2\sigma_1'\sigma_2'}}\int d^3r_{dF}\, d^3r_{p1}\, d^3r_{A2}\,\chi^{(-)*}_{pB}(\vec{r}_{pB})\left[\varphi^{A+2}_{l_i j_i}(\vec{r}_{A1},\sigma_1,\vec{r}_{A2},\sigma_2)\right]^0_0 v(r_{p1})\,\phi_d(r_{p1},\sigma_1)\,\varphi^{A+1}_{\ell_f j_f m_f}(\vec{r}_{A2},\sigma_2)$$
$$\times\frac{2\mu_{dF}}{\hbar^2}\int d^3r_{p1}'\, d^3r_{A2}'\, d^3r_{dF}'\, G(\vec{r}_{dF},\vec{r}_{dF}')\,\phi^*_d(\vec{r}_{p1}',\sigma_1')\,\varphi^{A+1}_{\ell_f j_f m_f}(\vec{r}_{A2}',\sigma_2')\,v(r_{p2}')\,\phi_d(r_{p1}',\sigma_1')\,\phi_d(r_{p2}',\sigma_2')\,\chi^{(+)}_{tA}(\vec{r}_{pA}')$$

First order in v

$$T^{(1)}_{NO}=2\sum_{\substack{l_i j_i;\, \ell_f j_f \\ m_s m_s'}}\int d\xi_A\, d^3r_{dF}\, d^3r_{p1}\, d^3r_{A2}\,\chi^{(-)*}_{pB}(\vec{r}_{pB})\,\chi^{1/2*}_{m_s}(\sigma_p)\,\psi^*_B(\xi_B)\,v(r_{p1})\,\phi_d(\vec{r}_{p1},\sigma_1)\,\varphi^{A+1}_{\ell_f j_f m_f}(\vec{r}_{A2},\sigma_2)\,\chi^{1/2}_{m_s}(\sigma_p)\,\psi_A(\xi_A)$$
$$\times\underbrace{\int d\xi_A\, d^3r_{p1}\, d^3r_{A2}'\, d^3r_{dF}'}_{\text{6 relative coords.}}\,\chi^{1/2*}_{m_s'}(\sigma_p')\,\phi^*_d(\vec{r}_{p1}',\sigma_1')\,\psi^*(\xi_A)\,\varphi^{A+1}_{\ell_f j_f m_f}(\vec{r}_{A2}',\sigma_2')\,\mathbb{1}\,\phi_t(\vec{r}_{p1},\sigma_1',\vec{r}_{p2},\sigma_2)\,\psi_A(\xi_A)\,\chi^{1/2}_{m_s'}(\sigma_p)\,\chi^{(+)}_{tA}(\vec{r}_{tA}')$$

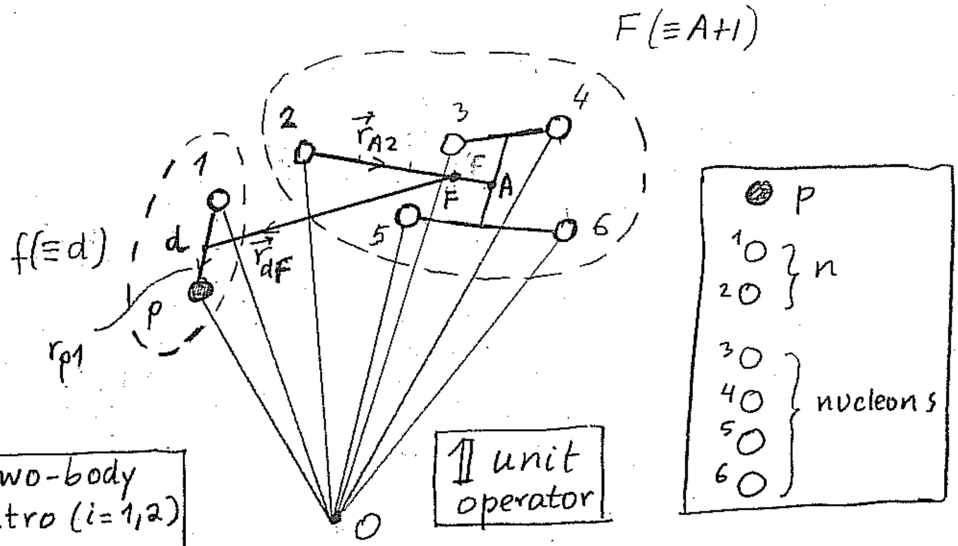
$$=2\sum_{l_i j_i,\, \ell_f j_f} d^3_{dF}\, d^3r_{p1}\, d^3r_{A2}\,\chi^{(-)*}_{pB}(\vec{r}_{pB})\left[\varphi^{A+2}_{l_i j_i}(\vec{r}_{A1},\sigma_1,\vec{r}_{A2},\sigma_2)\right]^0_0 v(r_{p1})\,\phi_d(\vec{r}_{p1},\sigma_1)\,\varphi^{A+1}_{\ell_f j_f m_f}(\vec{r}_{A2},\sigma_2)$$
$$\times\int d^3r_{p1}'\, d^3r_{A2}'\, d^3r_{dF}'\,\phi^*_d(\vec{r}_{p1}',\sigma_1')\,\varphi^{A+1}_{\ell_f j_f m_f}(\vec{r}_{A2}',\sigma_2')\,\mathbb{1}\,\phi_d(\vec{r}_{p1}',\sigma_1')\,\phi_d(\vec{r}_{p2}',\sigma_2')\,\chi^{(+)}_{tA}(\vec{r}_{tA}')$$

box

repetido

5

Fig. β



$v(r_{pi})$: two-body proton neutro $(i=1,2)$ interaction

$\underbrace{f(\equiv d)+F(\equiv A+1)}_{\text{intermediate channel}}$

no distorted ware as (f,F) is a virtual channel with no asymptotic waves. Thus 5 relative coordinates $\rightarrow$ $\chi^{1/2}_{m_s'}(\sigma_p)\,\phi_d(\vec{r}_{p1},\sigma_1)\,\psi(\xi_A)\,\varphi^{A+1}_{\ell_f j_f m_f}(\vec{r}_{A2},\sigma_2)$

Second order in v

$$T^{(2)}_{succ}=2\sum_{\substack{l_i j_i,\ell_f j_f m_f, m_s, m_s'\\ \sigma_1\sigma_2\sigma_1'\sigma_2'}}\int d\xi_A\, d^3r_{dF}\, d^3r_{p1}\, d^3r_{A2}\,\chi^{(-)*}_{pB}(\vec{r}_{pB})\,\chi^{1/2*}_{m_s}(\sigma_p)\,\chi^{*}_{B}(\xi_B)\,v(r_{p1})\,\phi_d(r_{p1},\sigma_1)\,\varphi^{A+1}_{\ell_f j_f m_f}(\vec{r}_{A2},\sigma_2)\,\chi^{1/2}_{m_s}(\sigma_p)\,\psi_A(\xi_A)$$
$$\times\frac{2\mu_{dF}}{\hbar^2}\int d\xi_A'\, d^3r_{p1}'\, d^3r_{A2}'\, d^3r_{dF}'\, G(\vec{r}_{dF},\vec{r}_{dF}')\,\chi^{1/2*}_{m_s'}(\sigma_p')\,\phi^*_d(\vec{r}_{p1}',\sigma_1')\,\psi(\xi_A')\,\varphi^{A+1}_{\ell_f j_f m_f}(\vec{r}_{A2}',\sigma_2')\,v(r_{p2}')$$
$$\times\,\phi_t(\vec{r}_{p1}'\sigma_1',\vec{r}_{p2}',\sigma_2')\,\chi^{1/2}_{m_s'}(\sigma_p')\,\psi_A(\xi_A')\,\chi^{(+)}_{tA}(\vec{r}_{pA}')$$

$$=2\sum_{\substack{l_i j_i\, \ell_f j_f m_f\\ \sigma_1\sigma_2\sigma_1'\sigma_2'}}\int d^3r_{dF}\, d^3r_{p1}\, d^3r_{A2}\,\chi^{(-)*}_{pB}(\vec{r}_{pB})\left[\varphi^{A+2}_{l_i j_i}(\vec{r}_{A1},\sigma_1,\vec{r}_{A2},\sigma_2)\right]^0_0 v(r_{p1})\,\phi_d(r_{p1},\sigma_1)\,\varphi^{A+1}_{\ell_f j_f m_f}(\vec{r}_{A2},\sigma_2)$$
$$\times\frac{2\mu_{dF}}{\hbar^2}\int d^3r_{p1}'\, d^3r_{A2}'\, d^3r_{dF}'\, G(\vec{r}_{dF},\vec{r}_{dF}')\,\phi^*_d(\vec{r}_{p1}',\sigma_1')\,\varphi^{A+1}_{\ell_f j_f m_f}(\vec{r}_{A2}',\sigma_2')\,v(r_{p2}')\,\phi_d(r_{p1}',\sigma_1)\,\phi_d(r_{p2}',\sigma_2')\,\chi^{(+)}_{tA}(\vec{r}_{pA}')$$

First order in v

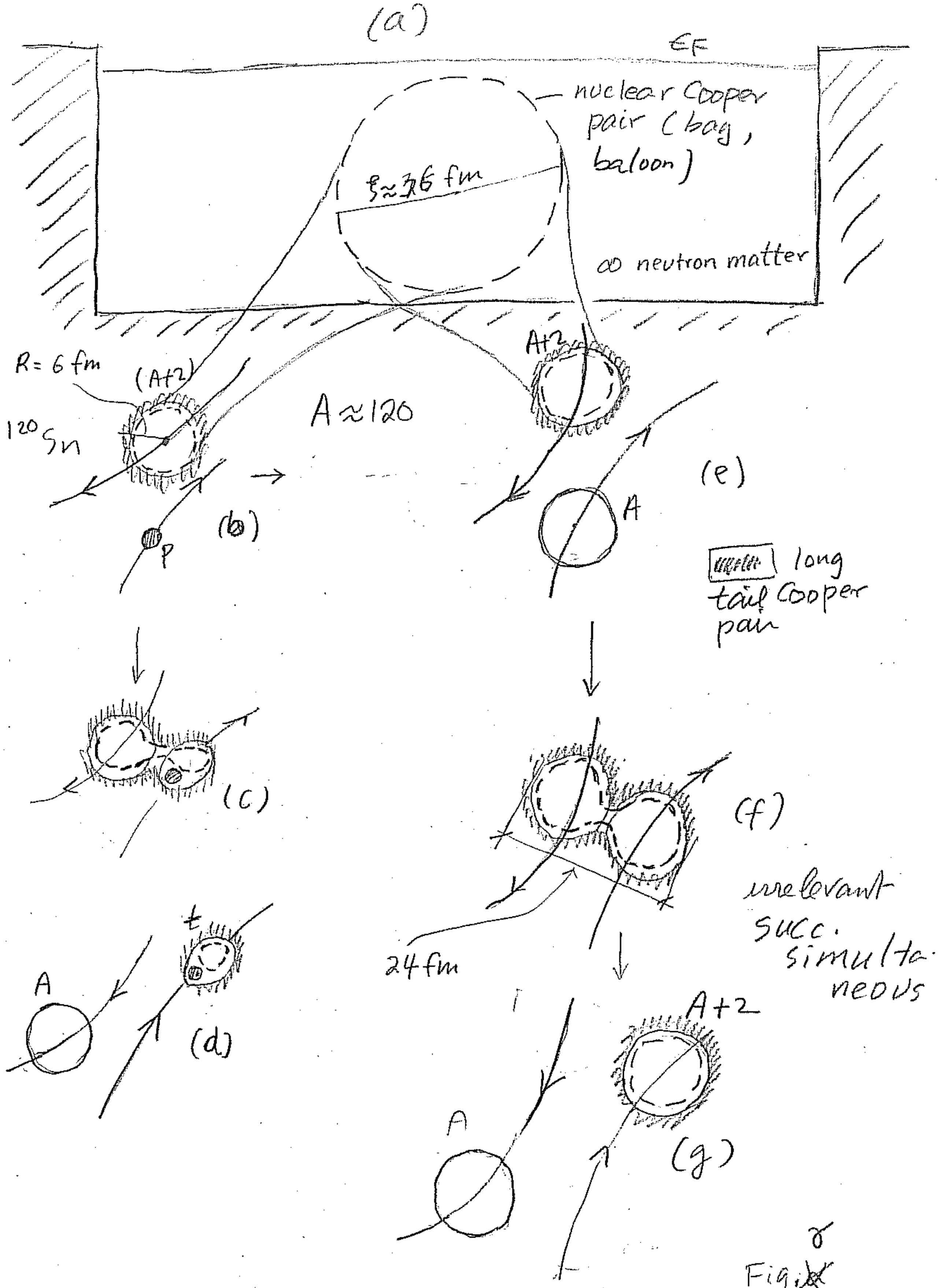
$$\vec{T}^{(1)}_{NO}=2\sum_{\substack{l_1 j_1,\ell_f j_f\\ m_s m_s'}}\int d\xi_A\, d^3r_{dF}\, d^3r_{p1}\, d^3r_{A2}\,\chi^{(-)*}_{pB}(\vec{r}_{pB})\,\chi^{1/2*}_{m_s}(\sigma_p)\,\psi^*_B(\xi_B)\,v(r_{p1})\,\phi_d(\vec{r}_{p1},\sigma_1)\,\varphi^{A+1}_{\ell_f j_f m_f}(\vec{r}_{A2},\sigma_2)\,\chi^{1/2}_{m_s}(\sigma_p)\,\psi_A(\xi_A)$$
$$\times\underbrace{\int d\xi_A\, d^3r_{p1}\, d^3r_{A2}'\, d^3r_{dF}'}_{\text{6 relative coords.}}\,\chi^{1/2*}_{m_s'}(\sigma_p')\,\phi^*_d(\vec{r}_{p1}',\sigma_1')\,\psi^*(\xi_A)\,\varphi^{A+1}_{\ell_f j_f m_f}(\vec{r}_{A2}',\sigma_2')\,\mathbb{1}\,\phi_t(\vec{r}_{p1}',\sigma_1',\vec{r}_{p2}',\sigma_2')\,\psi_A(\xi_A)\,\chi^{1/2}_{m_s'}(\sigma_p')\,\chi^{(+)}_{tA}(\vec{r}_{tA}')$$

$$=2\sum_{l_1 j_1,\ell_f j_f}d^3r_{dF}\, d^3r_{p1}\, d^3r_{A2}\,\chi^{(-)*}_{pB}(\vec{r}_{pB})\left[\varphi^{A+2}_{l_1 j_1}(\vec{r}_{A1},\sigma_1,\vec{r}_{A2},\sigma_2)\right]^0_0 v(\vec{r}_{p1})\,\phi_d(\vec{r}_{p1},\sigma_1)\,\varphi^{A+1}_{\ell_f j_f m_f}(\vec{r}_{A2},\sigma_2)$$
$$\times\int d^3r_{p1}'\, d^3r_{A2}'\, d^3r_{dF}'\,\phi^*_d(\vec{r}_{p1}',\sigma_1')\,\varphi^{A+1}_{\ell_f j_f m_f}(\vec{r}_{A2}',\sigma_2')\,\mathbb{1}\,\phi_d(\vec{r}_{p1}',\sigma_1')\,\phi_d(\vec{r}_{p2}',\sigma_2')\,\chi^{(+)}_{tA}(\vec{r}_{tA}')$$

box

Caption to Fig. ~~X~~ β

Successive and non-orthogonality contributions to the transfer amplitude describing two-nucleon transfer in second order DWBA, entering in the expression of the absolute differential cross secction $d\sigma/d\Omega = \frac{\mu_i \mu_f}{(4\pi\hbar^2)^2} \frac{k_f}{k_i} |T^{(1)} + T^{(2)}_{succ} - T^{(2)}_{NO}|$. Concerning $T^{(1)}$ we refer to Fig. β. In the upper part of the figure the coordinates used to describe the intermediate channel $d + F (\equiv A+1)$ are given, while in the lower part the corresponding expressions are displayed. Schematically, the three contributions $T^{(1)}$, $T^{(2)}_{succ}$ and $T^{(2)}_{NO}$ to the transfer amplitude can be written as $\langle pB|v|tA\rangle$, $\sum \langle pB|v|dF\rangle \langle dF|v|tA\rangle$ and $\sum \langle pB|v|dF\rangle \langle dF|\mathbb{1}|tA\rangle$ respectively, where v is the proton-neutron interaction and $\mathbb{1}$ the unit operator. Within this context, while $T^{(2)}_{NO}$ receives contributions from the intermediate (virtual) close d+F channel as $T^{(2)}_{succ}$, it is first order in v as $T^{(1)}$ (from Potel et al (2013), cf. Caption Fig β).



(a)
ϵ_F
nuclear Cooper pair (bag, baloon)
$\xi \approx 36$ fm
∞ neutron matter
R = 6 fm
(A+2)
^{120}Sn
A ≈ 120
P
(b)
A+2
(e)
A
long tail Cooper pair
(c)
(f)
24 fm
irrelevant succ. simultaneous
t
A
(d)
A+2
A
(g)

Fig. 8

(8)

~~by medium polarization effects,~~

Caption to Fig. ~~6~~ 8

~~and eluding from the NN-1S0 short range force, eventually renormalized~~

The correlation length associate with a nuclear Cooper pair is of the order of $\xi \approx \hbar v_F/\Delta \approx 36$ fm. In other words, in a (a) neutron matter at typical densities of the order of 0.5–0.8 saturation density, the NN-$^1S_0$ short range force, eventually renormalized by medium polarization effect, makes pairs of nucleons moving in time reversal states to communicate over distances of the order of 5–6 times typical nuclear radii. How can one get evidence for such a extended object? Certainly not when the cooper bag (balloon) is introduced in (b) the mean field of a superfluid nuclei which, acting as an external field, constrains the Cooper pair to be within the nuclear radius with some spill out (long tail of Cooper pair, grey shaded area extending outside the nuclear surface). But yes in a (c) two-nucleon transfer process (e.g. (p,t) reaction) in which the absolute cross section can change by orders of magnitude in going from pure two-particle (uncorrelated configurations) to long tail Cooper pair spill outs. This effect being stronger by allowing ~~ex~~ (e), (f), (g) ~~a~~ pair transfer between similar ~~Cooper pairs~~ superfluid nuclei, in which case one profits by the same type of correlations as resulting from identical external (mean) fields.

Within this context, it is apparent that pairs of nucleons will ~~be~~ feel equally ~~the~~ pairing correlations whether they are transferred ~~successful~~ ~~success~~ simultaneously or one after the oter (cf. (c) and (f)).

appendix α

Independent-particle limit (after Potel, G., Idini, A., Barranco, F, Vigezzi, E. and Broglia, R.A., Cooper pair transfer in nuclei, Rep. Prog. Phys. 76 (2013) 106301)

$$a^{(1)}_{sim} = a^{(1)}_{NO}$$

and

$$a^{(2)}_{succ} = a^{(1)}_{one\text{-}part} \times a^{(1)}_{one\text{-}part}$$

$$a + A \to \boxed{f + F} \to b + B$$

Product of two single nucleon transfer processes.

Strong correlation (cluster) limit

post-prior representation

post-prior $\tilde{a}^{(2)}_{succ} = a^{(2)}_{succ} - a^{(1)}_{NO}$

$$\lim_{\substack{E_{corr} \to \infty \\ (\nu_{12})}} \tilde{a}^{(2)}_{succ} = 0$$

all transfer is due to simultaneous.

Actual nuclei close to ind. particle limit (E_{corr} (1-2 MeV) $\ll \epsilon_F$ (37 MeV)). Then successive is the major contribution.

But successive seems to break the pair, right?

Wrong

Cooper pair dimensions

Typical correlation energies of Cooper pairs are 1-2 MeV. Now, such a system (dineutron or diproton) is not bound and needs of an external field to be confined. This is the single-particle field

$$\delta x\, \delta p \geq \hbar \qquad \delta \varepsilon \approx 2 \cdot E_{corr}$$

$$\varepsilon = \frac{p^2}{2m} \; ; \quad \delta\varepsilon = \frac{2p}{m}\delta p \approx v_F\, \delta p$$

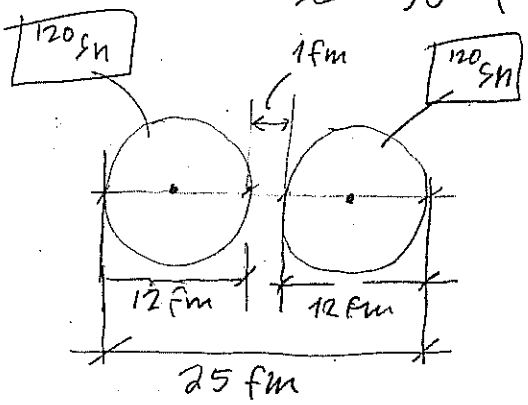
$$\delta\varepsilon \approx 2 E_{corr} \approx v_F\, \delta p$$

$$\xi = \delta x = \frac{\hbar}{\delta p} \approx \frac{\hbar v_F}{2 E_{corr}} \quad \text{(correl. length)}$$

$$\frac{v_F}{c} \approx 0.3 \qquad \hbar c = 200 \text{ MeV fm}$$

$$\xi \approx \frac{200 \text{ MeV} \times \text{fm} \times 0.3}{2 \text{ MeV}} \quad (E_{corr} \approx 1 \text{ MeV})$$

$$\approx 30 \text{ fm}$$



$A \approx 120 \quad (^{120}Sn)$

$R = 1{,}2 \text{ fm } A^{1/3} \approx 6 \text{ fm}$
$(A^{1/3} \approx 5)$

Successive and simultaneous transfer feel equally well the pairing correlations giving rise to long range order.

Objection

What about $v_{pairing}$ (=G) becoming zero, e.g. between the two nuclei

Answer

$$\frac{d\sigma(a(=b+2)+A \to b+B(=A+2))}{d\Omega} \sim |\alpha_0|^2$$

$$\alpha_0 = \langle BCS(A+2)|P^\dagger|BCS(A)\rangle = \sum_{\nu>0} U_\nu(A) V_\nu(A+2) \quad (1)$$

and not to $\Delta = G\alpha_0$.

$$\boxed{\begin{array}{l} \alpha_\nu^\dagger = U_\nu a_\nu^\dagger - V_\nu a_{\bar\nu} \\ a_\nu^\dagger = U_\nu \alpha_\nu^\dagger + V_\nu \alpha_{\bar\nu} \end{array}}$$

Objection

Eq. (1) valid for simultaneous transfer ~~only~~, right? wrong

$$\alpha_0 = \sum_{\nu,\nu'>0} \langle BCS|a_\nu^\dagger|int(\nu')\rangle\langle int(\nu')|a_{\bar\nu}^\dagger|BCS\rangle$$

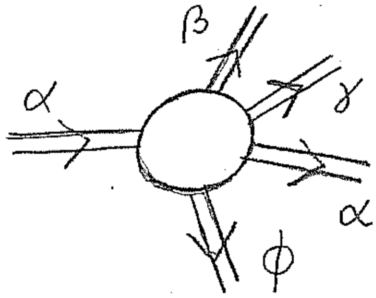
$$\approx \sum_{\nu,\nu'>0} \langle BCS|a_\nu^\dagger \alpha_{\nu'}^\dagger|BCS\rangle\langle BCS|\alpha_{\nu'} a_{\bar\nu}^\dagger|BCS\rangle$$

$$= \sum_{\nu,\nu'>0} \langle \overset{(A+2)}{BCS}|\overset{(A+1)}{V_\nu} \alpha_{\bar\nu}\alpha_{\nu'}^\dagger|\overset{(A+1)}{BCS}\rangle\langle \overset{(A+1)}{BCS}|\alpha_{\nu'} U_\nu^{(A)} \alpha_{\nu'}^\dagger|\overset{(A)}{BCS}\rangle$$

$$= \sum_{\nu,\nu'>0} V_\nu(A+1)\, U(A)$$

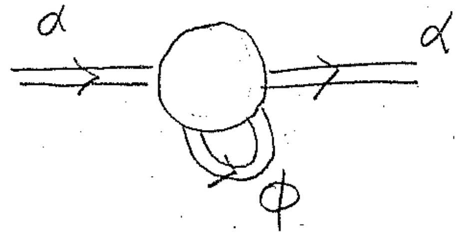
Comment on the optical potential

As a rule, the depopulation of the entrance, elastic channel $\alpha\,(a,A)$ is mainly due to one-particle transfer channels $\phi(f(=a-1), F(=A+1))$,



other channels, like e.g. inelastic ones $\beta(a^*,A)$ $\gamma(a,A^*)$ being operative in particular situation, as a rule, whe deformed nuclei are involved in the reaction process. Let us assume ~~that~~ this is not the case, and ~~that~~ ϕ is the main depopulating channel. In this case, the calculation of the optical potential$^{*)}$,

footnote
See p. (2)



is quite reminiscent to the calculation of two-particle transfer (2nd order process), and can be carried out with essentially the same elements. In fact,

$$T^{(2)}_{succ} \sim \langle f|v|int\rangle\langle int|v|i\rangle \qquad (1)$$
$$T^{(2)}_{NO} \sim \langle f|v|int\rangle\langle int|\mathbb{1}|i\rangle ,$$

where $|i\rangle = |a,A\rangle$, $|int\rangle = |f,F\rangle$ and $|f\rangle = |bB\rangle$

become

$$\langle i|v|int\rangle\langle int|v|i\rangle$$
$$\langle i|v|int\rangle\langle int|\mathbb{1}|i\rangle , \qquad (2)$$

as contributions to the optical potential.

Barranco, Broglia and Bertsch (1988)

footnote

*) It is of notice ~~that~~ the optical potential can be viewed as the complex "dielectric" function of direct nuclear reactions. In other words, the function describing the properties of the medium ("vacuum") in which incoming and outgoing distorted waves propagate, properties which are, as a rule determined through the analysis of elastic scattering processes, under the assumption ~~that~~ the coupling between the relative motion (reaction) and intrinsic (structure) coordinate are only coupled (recoil effect), trough a galilean transformation ~~which~~ smoothly matches the incoming with the outgoing wave (trajectories). Now, within the present context ~~namely that~~ of the microscopic calculation of $U+iW$, non-locality and ω-dependence are microscopically calculated on equal footing with the calculation of structure properties. In particular, within the framework of NFT, taking into account the variety of correlations and coupling between single-particle and collective motion. Such an approach of structure provides the elements and rules for an *ab initio* calculation of the structure and reaction texture of the corresponding vacuum ~~states~~, an thus of the bound and continuum properties of the nuclear quantal system by itself and in interaction.

It is of notice that such a scenario includes also limiting situations like sub-barrier fussion processes (cf. e.g. Sargsyan et al (2013) and refs. therein) and, arguably, also exotic decay (cf. e.g. Barranco et al (1990) and refs. therein) and cold fusion (Barranco et al (1988))

(Barranco, Broglia and Bertsch (1988))

footnote

repetido

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