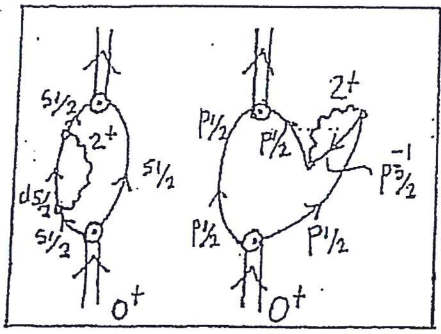
Fig. 10

^{11}Li - halo



input

MeV
$\varepsilon_{s1/2} = 1.5$
$\varepsilon_{p3/2} = 4.7$
$\varepsilon_{d5/2} = 3.5$
$\varepsilon_{p1/2} = -1.2$

bare sp

WS potential $R_0 = 1.2 A^{1/3}\,fm = 2.7\,fm$

① $U_0 = V_0 + 0.4\,E$ (exchange; Pauli)

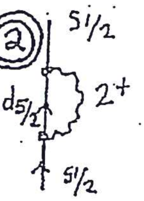
$\boxed{R(^{11}Li) = 4.58 \pm 0.13\,fm}$ $\quad \theta = \left(\frac{R_0}{R}\right)^3 = \left(\frac{2.7}{4.58}\right)^3 \approx 0.2$

$$m_k = \frac{m}{(1+\theta \times 0.4)} \approx \frac{m}{1.08} \approx 0.93\,m$$

clothing sp: $\boxed{\hbar\omega_{2^+}(^9Li) = 3.3\,MeV;\ \beta_2 \approx 0.66}$

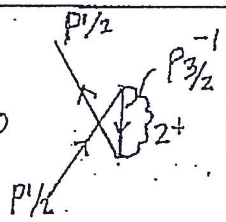
$$\langle H_c \rangle = \beta_2 \left\langle \frac{R_0}{\sqrt{5}} \frac{\partial U}{\partial r} \right\rangle \theta \approx 0.66 \left(-\frac{50 \times 0.2}{\sqrt{5}} MeV\right) \approx -3\,MeV$$

②



$$\begin{pmatrix} (6.8-\lambda) & -3 \\ -3 & (1.5-\lambda) \end{pmatrix} = 0$$

MeV



$$\begin{pmatrix} (-3.8-\lambda) & -3 \\ -3 & (-1.2-\lambda) \end{pmatrix} = 0;$$

MeV

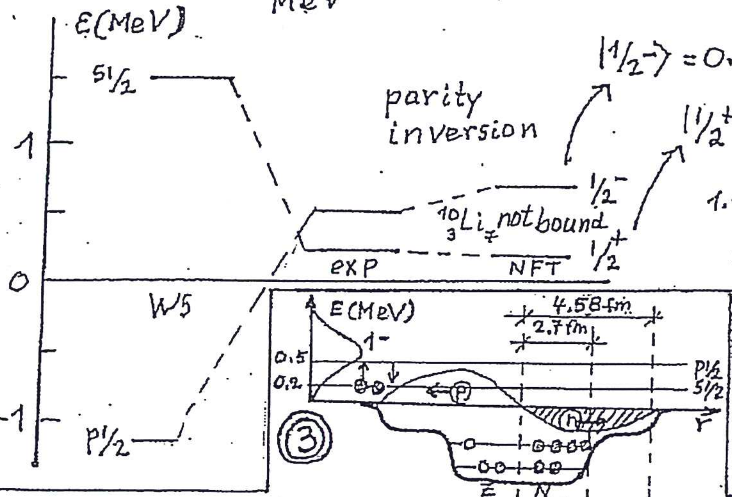
diagonalization

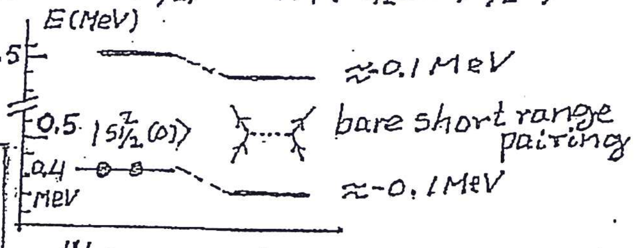
$\tilde{\varepsilon}_{s1/2} = 0.15\,MeV$

$\tilde{\varepsilon}_{p1/2} = 0.77\,MeV$

$|1/2^-\rangle = 0.84\,|p_{1/2}\rangle + 0.55\,|((p_{1/2}, p_{3/2}^{-1})_{0^+} \otimes 2^+)\, p_{1/2}; 1/2^-\rangle$

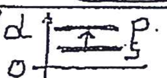
$|1/2^+\rangle = 0.91\,|s_{1/2}\rangle + 0.41\,|(d_{5/2} \otimes 2^+)\, 1/2^+\rangle$





$^{11}_{3}Li_8$ not bound $\quad \langle V_p \rangle = 0.048 \times \frac{25\,MeV}{A} \approx \frac{1\,MeV}{A}$ (screening) $\approx 0.1\,MeV$

halo-anti pairing effect s,p at threshold



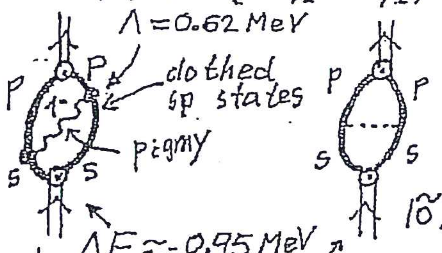
scenario pigmy

$H_D = K_1 \vec{D} \cdot \vec{D}$ $\quad K_1 = s K_1^0$; $\quad K_1^0 \sim 5 V_1 = 165\,MeV$

(screening) $\quad K_1 \sim 12\,MeV\ (s \approx 0.07)$; $\quad \boxed{(8\%)\,TRK = \frac{9}{4\pi} \frac{\hbar^2}{2M} \frac{NZ}{A} e^2}$

$$\hbar\omega_{pigmy} = \left((\varepsilon_{1/2^+} - \varepsilon_{1/2^-})^2 + K_1 \times (2 \times 0.06\,TRK)^2\right)^{1/2} \approx 0.8\,MeV$$

$\Lambda^2 \approx 0.4\,MeV^2$



$E_{corr} \approx |2\tilde{\varepsilon}_{s1/2} + \Delta E_b + \Delta E_{ind}| = |0.4 - 0.1 - 0.95|\,MeV \approx 0.7\,MeV$

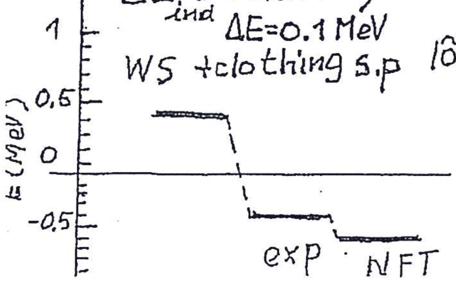
$(E_{corr})_{exp} \approx 0.380\,MeV$

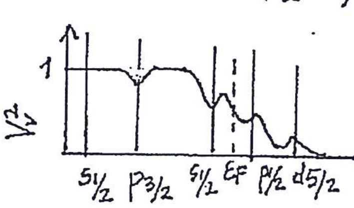
$\Delta E_{ind} \approx -0.95\,MeV \quad \Delta E = 0.1\,MeV$

$|\tilde{0}\rangle = |0\rangle_\nu + 0.7\,|(p_{1/2}, p_{3/2}^{-1})_{1^-} \otimes 1^-; 0\rangle + 0.1\,|(s_{1/2}, d_{5/2})_{2^+} \otimes 2^+; 0\rangle$

$|\tilde{0}\rangle = \quad |0\rangle_\nu = 0.45\,|s_{1/2}^2(0)\rangle + 0.55\,|p_{1/2}^2(0)\rangle + 0.04\,|d_{5/2}^2(0)\rangle$

WS + clothing s.p





QRPA symbiotic bootstrap

halo pairaddition mode

back to ①

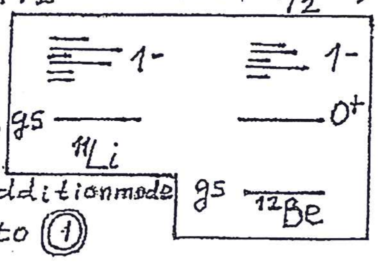


Fig. 10

Fig. 11 Nuclear structure

(1) Starting with well defined elements: Woods-Saxon (WS) potential, and the parameters characterizing the low-lying quadrupole vibration of the core (<u>input</u>), calculate the single-particle levels and collective vibration (separable interaction) and determine the corresponding scattering vertices (strength and form factors). From the ratio of the WS radius (R_0) and of the observed one ($R(^{11}Li)$ <u>input</u>) determine the overlap θ. Because $\theta \ll 1$, the contribution of the exchange (Fock) potential to the empirical WS potential is small (energy (k-) dependent term $U \approx -50\,\mathrm{Me} + \theta U_x$, $U_x = 0{,}4E\,(=\hbar^2k^2/2m)$, $m_k/m = (1+\theta \times (m/\hbar^2 k)\partial U_x/\partial k)^{-1}$) concerning the halo neutrons, essentially blurring the emergent new Pauli quantum number one (single occupancy) closely related to the many-body Dirac interpretation of the stability of the fully occupied vacuum (Pauli, Dirac, Nobel lectures). Consequently the neutron halo k-mass m_k has a value close to the bare mass m.

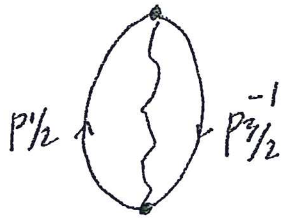
(2) Making use of the above elements one can cloth the bare single-particle states, in particular the $s_{1/2}$ and $p_{1/2}$ states. Parity inversion ensues, with $1/2^+$ and $1/2^-$ at threshold. As a consequence the $N=8$ shell closure melts away, $N=6$ becoming a new magic number, testifying to the fact that large amplitude fluctuations are as important or even more important than static mean field effects. As a result $^{10}_{3}Li_7$ is not bound. Adding one more neutron and switching on the bare pairing interaction (e.g. a contact force V_p with constant matrix element $G = 1{,}2\,\mathrm{fm}^{-3}\frac{V_0}{A} \approx (28/A)\,\mathrm{MeV}$, Brink and Broglia (2005), pp 40-42), the screening resulting from the ratio $r = \frac{(M_j)\,\text{halo}}{(M_j)\,\text{core}} \approx \frac{2}{2j+1}\left(\frac{R_0}{R}\right)^3 \approx 0.048$ makes $(G)_{scr} = rG$ subcritical, resulting in an unbound system.

(3) Considering the sloshing back and forth of the halo neutron (with a small contribution from the core neutrons) against the core protons, leads to a dipole mode feeling a strongly screened (repulsive) symmetry potential $s = \theta(R_0/R)^2 \approx 0.07$ in keeping with the fact $\kappa_1 \sim 1/R^2$. In other words, while

the price to pay to separate protons from neutrons in the core is $5V_1 = 5 \times 33\,MeV = 165\,MeV$, when referring to halo neutrons this price is reduced to for halo neutrons $5 \times 5V = 11{,}6\,MeV$ ($V_1 = 0{,}07 \times 33\,MeV = 2{,}3\,MeV$). This fact is at the basis that $\approx 8\%$ of the Thomas-Reiche-Kuhn sum rule (input) gets down to $\approx 0{,}6\,MeV$. Another way to say the same thing is that $(V_1)_{screened} = 5V_1$ is at the basis of the fact ~~that~~ the E1 transition $s_{1/2} \to p_{1/2}$ ($\Delta E \approx 0{,}3\,MeV$) ($\nu \approx 10^{21}\,Hz$) is only increased by a modest value ($\hbar\omega_{pygmy} \approx 0{,}6\,MeV$), while the E1 single-particle strength remains essentially unchanged (typical values in the case of stable nuclei being $\approx 10^{-4} B_{sp}(E1)$ for pure low-energy ($\lesssim 1\,MeV$) single-particle transitions). Now, the two halo neutrons dressed by the vibrations of the core (heavy arrowed lines) and interacting through the bare NN-pairing force are not bound. Consequently, the pygmy resonance will fade away almost as soon as it is generated (essentially lasting the neutron transversal time $\approx 10^{-21}\,s$), unless... Unless it is exchanged between the two neutron configuration $s_{1/2}^2(0)$ at threshold ($2\varepsilon_{s_{1/2}} \approx 0{,}4\,MeV$) making it jump into the $p_{1/2}^2(0)$, also close to threshold ($2\varepsilon_{p_{1/2}} \approx 1\,MeV$). As an intermediate boson, the pygmy resonance which couples to the halo neutron with a strength $\Lambda \approx 0.62\,MeV$ (QRPA calculation), contributes to the gluing of the neutron halo Cooper pair with about 1 MeV binding. The corresponding correlation energy $E_{corr} \approx 0{,}7\,MeV$ being mainly due to the pygmy exchange process. The symbiotic halo pair addition mode – pygmy DR of ^{11}Li can, in principle be used as a building block of the nuclear spectrum, which can be moved around. A possible candidate being the first excited 0^+ state of ^{12}Be, together with the associate dipole state. Because to calculate the giant dipole pygmy resonance (GDR) of ^{11}Li one needs to know the ground state of this nucleus (halo-pair addition mode) so as to be able to determine microscopically the occupations factor the $1s_{1/2}$, $1p_{3/2}$, $s_{1/2}$, $p_{1/2}$, $d_{5/2}$.., etc states to carry out a QRPA calculation of the mode, but to do so one needs to know the pygmy, once arrived ~~to~~ this point, one needs to go back to 1 and repeat the whole procedure so as to eventually reach convergence.

<u>appendix</u>: shift of $p_{1/2}$ state up in ^{10}Li (blocking ZPF)

contribution ZPF 2^+ to the binding energy of ^9Li.



$$\Lambda = -\frac{\beta_2}{\sqrt{5}} \times 0.1(2j+1)\left\langle R_0 \frac{\partial U}{\partial r} \right\rangle \approx -\frac{1}{\sqrt{5}} \times 0.1 \times 2 \times 50\,\text{MeV}$$

$$\approx -4.5\ \text{MeV}$$

$$ZPF = -\frac{(-4.5)^2\ \text{MeV}^2}{(3.5+3.3)\ \text{MeV}} \approx -3\ \text{MeV}$$

<u>appendix</u>: shift downwards $s_{1/2}$ in ^{10}Li (increase radius)

Bruch + Broglia p. 296

$$\langle r^2 \rangle = \frac{\hbar}{m^* \omega_0}\left(N + \frac{3}{2}\right)$$

$$(\hbar\omega_0)_{halo} \sim \frac{1}{\langle r^2 \rangle_{halo}} \quad ; \quad (\hbar\omega_0)_{syst} \sim \frac{1}{\langle r^2 \rangle_{syst}}$$

$$(\hbar\omega_0)_{halo} \sim \frac{1}{(4.6)^2} \qquad (\hbar\omega_0)_{syst} \sim \frac{1}{(2.7)^2} = 0.14$$

$$(\hbar\omega_0)_{syst} = c\ \overset{0.14}{\cancel{\quad}} = 1\,\text{MeV}\ (\text{w-s of B+M})\ ;$$

$$c \approx (0.14)^{-1} = 7.1$$

$$(\hbar\omega_0)_{halo} = \overset{7.1\ \text{MeV}}{\cancel{7\ \text{MeV}}}\ \frac{1}{(4.6)^2} \approx 0.3\ \text{MeV}$$

$$\varepsilon_{s_{1/2}} = 1\ \text{MeV} \qquad \tilde{\varepsilon}_{s_{1/2}} \approx 0.3\ \text{MeV}$$