~~Appendix 0~~ A: Specific probes and elementary modes of (nuclear) excitation

In a classical world empty space, is the absence of physics, and the existence of something, e.g. of light or of an electron is only a clue to eventually learn what the "object" is. Think only on all the work on vibrations of an hypothetical "aether", or concerning the "divergent" mass (energy) of the electron. Within this last context, the same is somewhat true in quantum mechanics, with a small difference. We do not need to think whether light is a particle or a wave. But yes, whether it is a composite particle or less, in both the sense of finite lifetime or coupling to other particles. Namely what are the bare properties of the different particles, and what the measured, dressed observables are.

Within this context, think only on the discussions concerning spectroscopic factors, RPA vibrations (sharp states, no coupling to 2p–2h states), role of induced interaction in pairing correlations in nuclei, etc.

Now, all these doubts vanish by acting with the variety of specific probes on the quantal vacuum. Namely, by making virtual processes associated with ZPF become real, taking properly into account Pauli principles (see e.g. Fig. ~~2~~ 1.A.1),

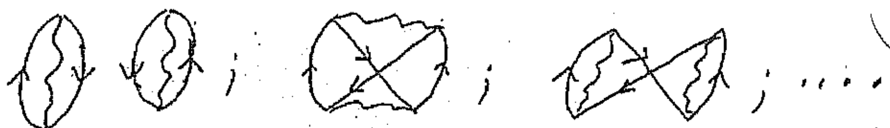

Fig. 1.A.1 Oyster diagrams describing the correlation of the ground state on which a collective particle-hole excitation ~~Figure 2.~~ is built, an Pauli principle correction processes in which fermions are exchanged

As seen from Fig. 3, acting with an external, time dependent hadronic field (e.g. p_1p'), one excites properly dressed $(p-h)$–like vibrations. 1.A.2

In other words, if one is in doubt of what properly dressed (nuclear) elementary modes of excitation are, do not study them, or calculate them and then compare the results with the experimental data. This comes after. One should first find out the specific mode out of ZPF of the vacuum, and then carry out a gedanken, NFT experiment as in Fig. ~~3~~. Because the vacuum contains all the information (right physical degrees of freedom) of each quantal system (also of the whole Universe), by forcing virtual processes associated with vacuum ZPF to become real, one is sure to get, in each instance, the real, dressed, physical particle, an example of which is given in Fig. ~~3~~. Of course, once the gedanken experiment has

1.A.2 italics

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provided this information, one should use such properly renormalized modes, in all the rest of the calculations, at the risk of neglecting relevant physics.

Pauli Principle between oyster diagrams

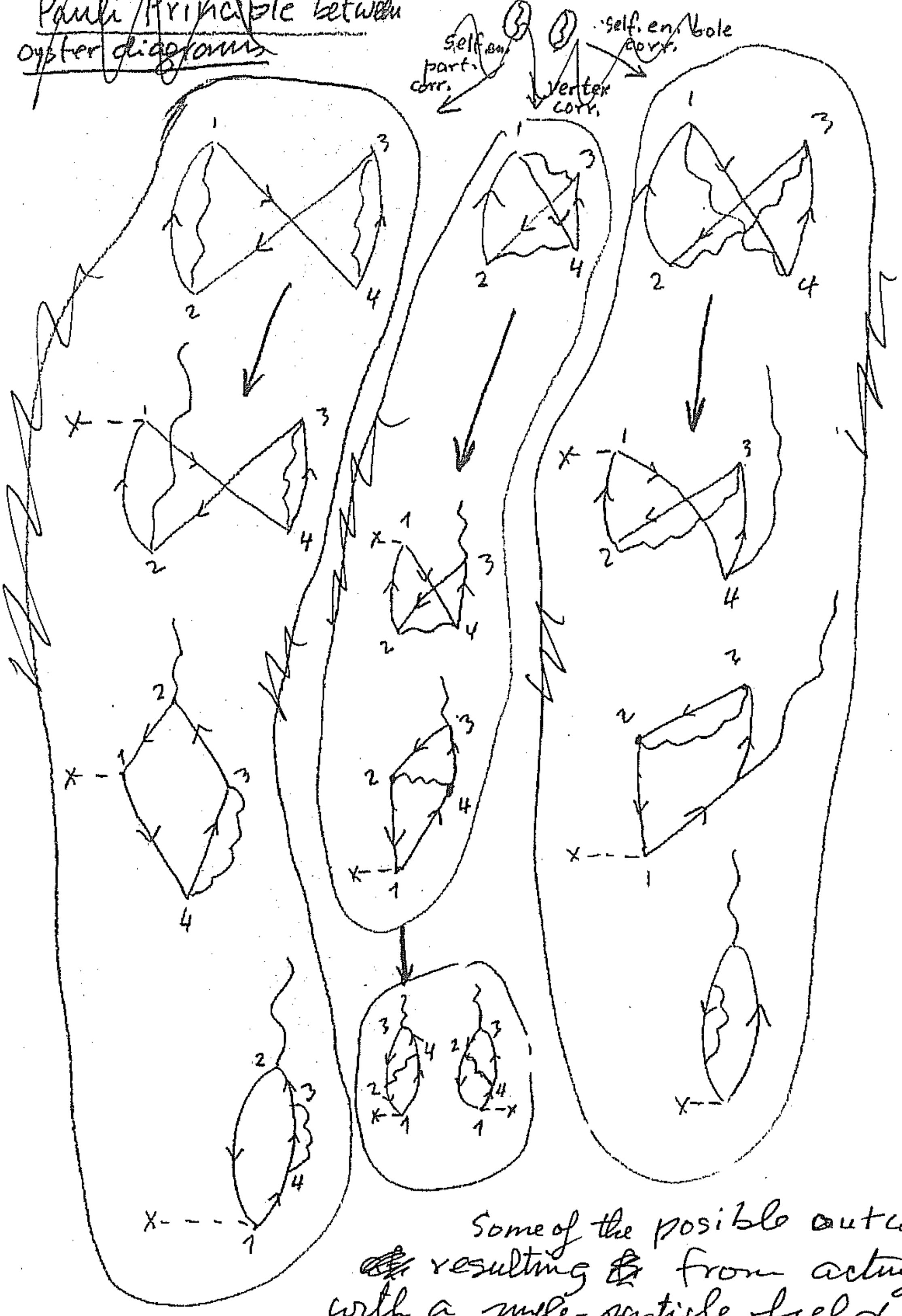

Figure ~~3.~~ 1.A.2

Some of the posible outcomes resulting from acting with a single-particle field, e.g. that associated with inelastic processes, on the Pauli corrected ZPF oyster diagrams associated with collective p-h excitation (see Fig. 1.A.1)

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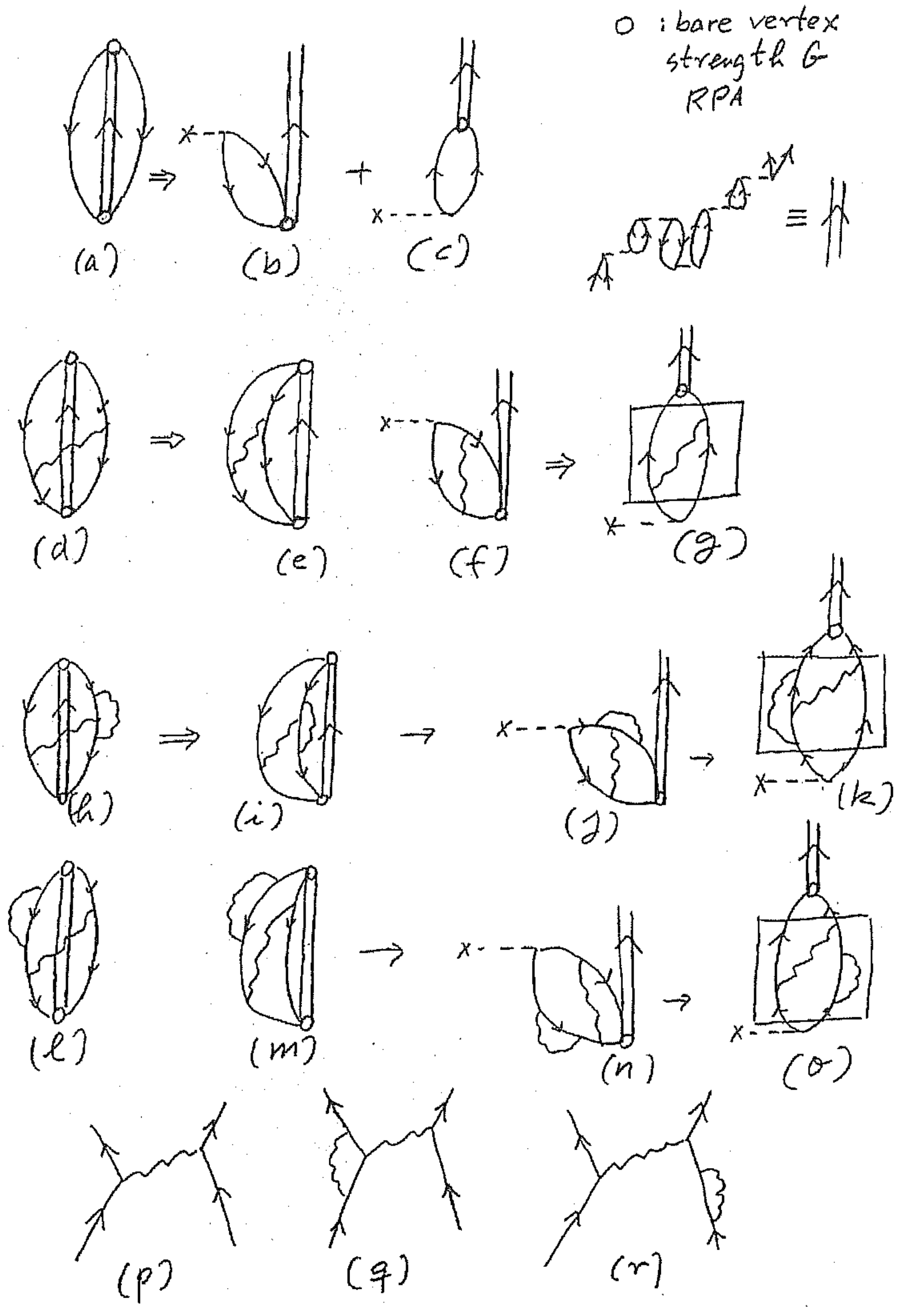

Figure ~~4~~ 1.A.3. ZPF associated with the pair addition mode taking into account the interweaving of nucleons with density modes. The process boxed in ~~(f) and (g)~~ (g), (k) and (o), are associated with the induced pairing interaction (medium polarization effects; cf. (p), (q), (r)) associated with the exchange of density modes between nucleons moving in time reversal states.

Appendix ~~TWO~~ - nucleon transfer sum rules 15/7/14 ①

~~We~~ B. consider the reaction

$$a + A \to b + B.$$ $\boxed{\text{Appendix Introduction}}$

In the center-of-mass system, the total Hamiltonian may be written App B

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$$H = T_{aA} + H_a + H_A + V_{aA}$$
$$= T_{bB} + H_b + H_B + V_{bA},$$

[redacted] in keeping with energy conservation. Within this context other, mixed, representations are posible

One then solves the time-dependent Schrödinger equation

$$i\hbar \frac{\partial \Psi}{\partial t} = H\Psi,$$

with the initial conditions that the nuclei a and A are in their ground states, and where the relative motion is described by a narrow wavepacket of rather well-defined ~~par~~ impact parameter and velocity.

We expand Ψ on (stationary) ~~the~~ channel wavefunctions

$$\Psi = \sum_\beta c_\beta((\vec{r}_\beta - R_\beta))\,\Psi_\beta(t)\, e^{-iE_\beta t/\hbar},$$

where

$$\Psi_\beta(t) = \Psi^b_m(\xi_b)\,\Psi^B_n(\xi_B)\,\exp(i\delta_\beta)$$

The index β labels both the partition of nucleon (b,B), as well as the quantal states of the two nucleons (m,n).

The phase δ_β is defined by

$$\delta_\beta = \frac{1}{\hbar}\Big\{ m_\beta \vec{v}_\beta(t)\cdot(\vec{r}_\beta - \vec{R}_\beta(t)) - \int_0^t \Big(U_\beta(R_\beta(t')) - \frac{1}{2} m_\beta (\vec{v}_\beta(t'))^2\Big)\, dt' \Big\}.$$

(cf. jagged "phonon" Figs. 2 and 3)

3b

The phase factor $\exp(i\delta_\beta)$ in the channel wavefunction is essentially a Galilean transformation where an additional phase has been added to eliminate, as far as possible, the diagonal matrix elements in the coupled equations.

The function c_β can be expressed as

$$c_\beta = a_\beta(t)\, \chi_\beta(\vec{r}_\beta - \vec{R}_\beta(t), t)$$

product of an amplitude a_β of normalized asymptotic value ($t = \pm\infty$, 0 or 1), and a shape (wave-~~packet~~ packet) function, $R_\beta(t)$ being the relative motion elastic trajectory.

Properly combining the above quantities and making use of the time-dependent Schrödinger equation one obtains

$$i\hbar \sum_\beta \dot{a}_\beta(t)\, \langle \Psi_\xi | \Psi_\beta \rangle_{\vec{R}_{\xi\gamma}}\, e^{-iE_\beta t/\hbar}$$

$$= \sum_\gamma \langle \Psi_\xi | V_\gamma - U_\gamma(r_\gamma) | \Psi_\gamma \rangle_{\vec{R}_{\xi\gamma}}\, a_\gamma(t)\, e^{-iE_\gamma t/\hbar},$$

where

where

$$f(\vec{R}) = \langle \psi_\xi | V_\gamma - U_\gamma(\vec{r}_\gamma) | \psi_\gamma \rangle_{\vec{R}}$$

are the formfactor, and

$$g(\vec{R}) = \langle \psi_\xi | \psi_\beta \rangle_{\vec{R}}$$

the overlaps between the intrinsic channel wavefunctions

The coupled equations can be written in a more compact ~~form~~ form by introducing the adjoint channel wavefunctions

$$\omega_\xi = \sum_\gamma g^{-1}_{\xi\gamma} \psi_\gamma ,$$

where g^{-1} is the reciprocal of the overlap matrix

$$g_{\xi\gamma} = \langle \psi_\xi | \psi_\gamma \rangle .$$

Thus

$$(\omega_\xi , \psi_\beta) = \delta(\xi, \beta) ,$$

and

$$i\hbar \dot{a}_\beta(t) = \sum_\gamma \langle \omega_\beta | V_\gamma - U_\gamma | \psi_\gamma \rangle_{\vec{R}_{\beta\gamma}} e^{i(E_\beta - E_\gamma)t/\hbar} \, a_\gamma(t).$$

Consequently, the proper tunneling Hamiltonian is obtained by a basis orthogonalization process.

These coupled equations, being first order in time, can be solved knowing the initial conditions at time $t=-\infty$, (5)

$$a_\gamma(-\infty)=\delta(\gamma,\alpha),$$ (39)

where α labels the entrance channel, that is, the nuclei a and A in their ground state. The cross section for the reaction $\alpha\to\beta$ is

$$\left(\frac{d\sigma}{d\Omega}\right)_{\alpha\to\beta}\sim|a_\beta(t=+\infty)|^2$$

Let us consider a two-nucleon transfer process, and solved the coupled equations in lowest order of $(V-U)$. One obtains

$$a_\beta(t)=\frac{1}{i\hbar}\int_{-\infty}^{t}\langle\omega_\beta|V_\alpha-U_\alpha|\Psi_\alpha\rangle_{\vec{R}_{\beta\alpha}(t)}\times\exp\{i(E_\beta-E_\alpha)t'/\hbar\}\,dt'$$

$$=\frac{1}{i\hbar}\int_{-\infty}^{t}dt'\langle\Psi_\beta|V_\alpha-U_\alpha|\Psi_\alpha\rangle_{\vec{R}_{\beta\alpha}(t)}\,e^{\frac{i(E_\beta-E_\alpha)t'}{\hbar}}$$

$$-\frac{1}{i\hbar}\int_{-\infty}^{t}dt'\,\langle\Psi_\beta|\mathbb{1}|\Psi_\alpha\rangle\langle\Psi_\alpha|V_\alpha-U_\alpha|\Psi_\alpha\rangle\times\exp\{i(E_\beta-E_\alpha)t'/\hbar\}$$

This is in keeping with the fact ~~that~~ global optical potential (U : real part), stand nucleon-nucleon interaction V fullfil the relation

$$\langle \Psi | V_\gamma - U_\gamma | \Psi \rangle = 0 .$$

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Within this scenario, and interpreting $\langle \Psi_\beta | \mathbb{1} | \Psi_\alpha \rangle$ as an effective (dimensionless) formfactor ($\mathbb{1}$ being the unit operator), one can posit ~~that~~ the minimal description of two-nucleon transfer reactions is second (non-orthogonality $(V-U)\times \mathbb{1}$; successive $(V-U)\times(V-U)$).

Let us now return to the sum-rule subject. For simplicity, we deal only with

$$a^{(1)}(t=+\infty) = \frac{1}{i\hbar}\int_{-\infty}^{\infty} dt \, \exp\left[\frac{i}{\hbar}(E^{bB} - E^{aA})t + \gamma_{\beta\alpha}(t)\right]$$

$$\left(\phi^{B(A)}(\vec{r}_{1A}, \vec{r}_{2A})\, U(r_{1b})\, e^{i\sigma_{\beta,\alpha}} \phi^{a(b)}(\vec{r}_{1b}, \vec{r}_{2b})\right)_{\vec{R}_{\alpha\beta}(t)}$$

where

$$\sigma_{\beta,\alpha} = \frac{1}{\hbar}\frac{m_n}{m_A}\left(m_{aA}\vec{v}_{aA}(t) + m_{bB}\vec{v}_{bB}(t)\right)\cdot(\vec{r}_\alpha - \vec{r}_\beta)$$

takes care of recoil effects, the phase factor $e^{i\sigma_{\beta,\alpha}}$ bein a generalized Galilean transformation associated (41) with the mismatch between entrance and exit channel, recoil effects), the phase

$$\gamma_{\beta\alpha}(t) = \int_0^t dt' \{ U_\beta(\vec{R}_\beta(t')) - \frac{1}{2} m_\beta v_\beta^2(t')$$
$$- U_\alpha(\vec{R}_\alpha(t')) + \frac{1}{2} m_\alpha v_\alpha^2(t') \}$$
$$+ \frac{1}{2\hbar} (m_\alpha \vec{v}_\alpha(t) + m_\beta \vec{v}_\beta(t)) \cdot (\vec{R}_\beta(t) - \vec{R}_\alpha(t))$$

being related to the effective Q-value of the reaction.

The rate of change of the formfactor $\langle \phi^{B(A)}, U(r_{1b}) e^{i\sigma_{\beta\alpha}} \phi^{a(b)} \rangle$ with time is slow, being completely overshadowed by the rapidly varying phase factor $\exp[\frac{i}{\hbar}(E^{bB} - E^{aA})t + \gamma_{\beta\alpha}(t)]$.

Consequently to compare two-nucleon transfer cross sections on equal structural footing, one has to eliminate the kinematical oscillating phase which can completely distort the "intrinsic" (reduced matix element) value of the two-nucleon cross section.

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Let us make a parallel with the sum rule associated with electromagnetic decay (Coulomb excitation, inelastic scattering). The absolute transition probability for absorption (emission) of a photon from nuclear dipole state is measured in sec^{-1} by (Bohr and Mottelson 1969)

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$$T(E1) = 1.59 \times 10^{15} \times (E)^3 \times B(E1),$$

where E is the energy of the transition, and $B(E1)$ the reduced transition probability. It is this quantity that enters the TRK-sum rule, and not $T(E1)$. Now, in this particular case Q-value dependence of the observed absolute transition probabilities can be eliminated analytically (E^3 dependence), as well as the overall ~~factor~~ (1.59×10^5), in keeping with ~~the fact that~~ the mass partition $(a+A \to a+A^*)$ or equivalent, coordinate of relative motion $\vec{R}_{\alpha,\alpha'}(t)$ does not change, thus allowing for a complete separation between structure and reaction (kinematics), explicit in the general expression of $T(E\lambda)$, that is,

$$T(E\lambda; I_1 \to I_2) = \left(\frac{8\pi(\lambda+1)}{\lambda[(2\lambda+1)!!]^2}\frac{1}{\hbar}q^{2\lambda+1}\right) \times \left(B(E\lambda; I_1 \to I_2)\right)$$

where $\hbar q$ is the momentum of the photon,

$(S(E1) = \langle 0|[H, \mu(E1)]\,\mathcal{M}(E1)|0\rangle/2)$

and

$$B(E\lambda; I_1 \to I_2) = \frac{\langle I_2 \| \mathcal{M}(E\lambda) \| I_1 \rangle}{\sqrt{2I_1+1}}$$

is the reduced transition probability. (43)
Thus, the first factor in the expression of $T(E\lambda)$ contains all the kinematics (reaction) of the process, the second one the nuclear structure of it.

In the above relation, the multipole tensor is defined as,

$$\mathcal{M}(E\lambda, \mu) = \int \rho(\vec{r})\, r^\lambda Y_{\lambda\mu}(\hat{r})\, d^3r.$$

It is of noticle that the nondiagonal elements (transition moments) are involved in electric (ρ_e) and nuclear (ρ) multipole processes (γ decay, Coulomb excitation, inelastic scattering, etc).

Returning to the expression of the first order (simultaneous) two-nuckon transfer amplitude $a^{(1)}(t=+\infty)$ one can only devise empirical protocols to try to extract the γ and σ - dependence from it. In other words, to set all absolute two-nucleon transfer

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cross sections $d\sigma/d\Omega \sim |a|^2$ on equal footing regarding kinematics, so as to allow to compare the intrinsic, reduced transition probabilities (structure). In other words, extract the structure information contained in, e.g.,

$$\phi^{B(A)}(\vec{r}_{1A},\vec{r}_{2A}) = \langle \vec{r}_{1A},\vec{r}_{2A} | \Gamma_1^+(\beta=+2)|\tilde{0}\rangle,$$

where

$$\Gamma_n^+(\beta=+2) = \sum_k X_k^n [a_k^+ a_{\bar{k}}^+]_0 - \sum_i Y_i^n [a_i a_{\bar{i}}]_0,$$

is pair addition mode of a closed shell system $|\tilde{0}\rangle$, as well as in

$$\phi^{B(A)}(\vec{r}_{1A},\vec{r}_{2A}) = [a_k^+ a_{\bar{k}}^+]_0 |0\rangle,$$

that is, in the pure two-particle configuration $|j_k^2(0)\rangle$ describing to nucleons moving on time reversal states on a single valence orbital of the close shell system $|0\rangle$.

If one were able to disentangle the γ and σ dependence of $a^{(1)}$ from its formfactor dependence, the comparison between relation $\sum_n |a_n|^2$, $\sum_k |a_k|^2$ and $\sum_i |a_i|^2$ could be eventually be phrased in terms of sum rules. This not being the case; one has to deal with approximate TNTR sum rules. With this proviso, ~~they~~ these sum rules are nontheless quite useful.

Reference

Bohr, A. and Mottelson, B.R. (1969) Nuclear Structure, Vol I, Benjamin, Reading, Mass.

Broglia, R.A. and Winther, A. (1991) Heavy Ion Reactions, Addison-Wesley, Reading, Mass.

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App. D

Fluct. dissipation

~~Chapter 4~~

Beyond mean field: particle-vibration coupling

The Hamiltonian describing a system of independent particles and of collective vibrations

~~Inserting the expression given in Eq.(3.7) in the expression of the empirical potential given in Eq.(2.2) and expanding to lowest order in α_{LM} (note that $\beta_L^2 \ll \beta_L$, cf. Eq.(3.3)) one obtains~~

can be written as

$$H = H_M + H_{coupl} + H_{coll}, \qquad 1.D.\ (\cancel{4}.1)$$

where

$$H_{coupl} = -\kappa\hat{\alpha}\hat{F}, \qquad 1.D\ (\cancel{4}.2)$$

with

$$\hat{F} = \sum_{\nu_1\nu_2} \langle \nu_1|F|\nu_2\rangle a^\dagger_{\nu_1} a_{\nu_2}, \qquad 1.D\ (\cancel{4}.3)$$

and

$$F = -\frac{1}{\kappa} R_0 \frac{\partial U(r)}{\partial r} Y^*_{LM}(\hat{r}). \qquad 1.D\ (\cancel{4}.4)$$

The Hamiltonian H_{coupl} thus couples the motion of a single-nucleons with the collective vibrations of the surface, with a matrix element (cf. Fig.~~4.1~~ 1.D1)

$$\langle n_\alpha = 1, \nu'|H_{coupl}|\nu\rangle = \Lambda_\alpha \langle \nu'|F|\nu\rangle = \langle n_\alpha = 1, \nu\nu'|H_{coupl}|0\rangle, \qquad 1.D.5\ (\cancel{4.5})$$

where

$$\Lambda_\alpha = -\kappa\sqrt{\frac{\hbar\omega_\alpha}{2C_\alpha}} \sim -\frac{\kappa\beta_\alpha}{\sqrt{2L_\alpha+1}}, \qquad (4.6)$$

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is the particle-vibration coupling strength. Because $\beta_L^2 \ll \beta_L$, one can usually treat the particle-vibration coupling in the weak coupling situation. Consequently H_{coul}, can be treated in perturbation theory. To second order one finds ~~(cf. Appr. F)~~

$$
\begin{aligned}
(-\frac{\hbar^2}{2m}\nabla_r^2 &+ U_H(r))\varphi_\gamma(r) + \int d^3r' U_x(\vec{r},\vec{r'})\varphi_j(\vec{r'}) \\
&+ (\Delta E + iW_j)\varphi_j(\vec{r}) \\
&\approx \left(-\frac{\hbar^2}{2m_k}\nabla_r^2 + U_H^{"}(r) + \Delta E_j + iW_j\right)\varphi_j(\vec{r}) \\
&= \varepsilon_j\varphi_j(\vec{r}), \qquad \left(U_H^{"} = \frac{m}{m_k}U\right)
\end{aligned}
\tag{4.8}
$$

?Well?

1.D.7 ~~(4.7)~~

1.D.2

where (cf. Fig ~~4.2~~)

$$
\Delta E_j^{(\omega)} = \text{Re}\sum_j(\omega) = \lim_{\Delta\to 0}\sum_{\alpha'}\frac{V_{\nu,\alpha'}^2(\omega - E_{\alpha'})}{(\omega - E_{\alpha'})^2 + (\frac{\Delta}{2})^2} \tag{4.9}
$$

and

$$
W_j^{(\omega)} = \text{Im}\sum_j(\omega) = \lim_{\Delta\to 0}\sum_{\alpha'}\frac{V_{\nu,\alpha'}^2 \cdot \frac{\Delta}{2}}{(\omega - E_{\alpha'})^2 + (\frac{\Delta}{2})^2}
$$

1.D.10 ~~(4.10)~~

are the real and imaginary contributions to the self-energy calculated in second order perturbation theory [1].

[1]Given a Hamiltonian H_{coupl}, the contribution to the energy in second order perturbation theory is

$$
\Sigma_\nu(\omega) = \sum_{\alpha'}\frac{V_{\nu,\alpha'}^2}{\omega - E_{\alpha'}},
$$

(1.D.11) ~~(4.11)~~

where $|\alpha'\rangle \equiv |n_\alpha = 1, \nu'\rangle$ are the intermediate states which can couple to the initial single-particle state ν. Note that the expression above is not well defined, in that the energy denominator may vanish. As a rule, textbooks in quantum mechanics deal with such a situation by stating that accidental degeneracies are to be eliminated by diagonalization. Now, this is not a real solution of the problem, because it does not contemplate the case where there are many intermediate state with $E_{\alpha'} \approx \omega$, in other words, where the particle can decay into a more complicated state, starting in the single-particle level ν of energy ω, without changing its energy (real process). This is a typical dissipative (diffusion) process, and has to be solved by direct diagonalization (cf. Fig. ~~4.5~~). Another way around, is to

1.D.5

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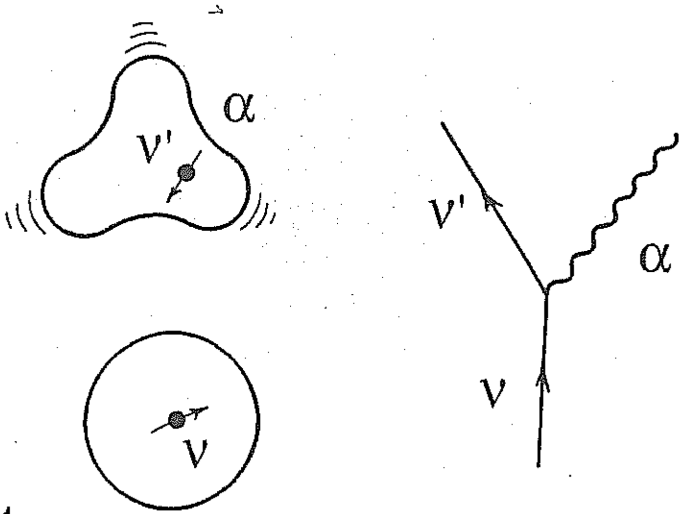
Furthermore,

$$E_{\alpha'} = \varepsilon_{\nu'} + \hbar\omega_\alpha,$$

and

$$V_{\nu,\alpha'} = \langle n_\alpha = 1, \nu' | H_{coupl} | \nu \rangle.$$



1.D.1

Figure ~~4.1~~: Schematic representation of the process by which a nucleon excites the vibrations of the surface.

For most purposes ΔE can be treated in terms of an effective mass ~~(cf. App. C, H and Z)~~

$$m_\omega = m(1+\lambda), \tag{4.12}$$

where

$$\lambda = -\frac{\partial \Delta E}{\partial \omega}, \tag{4.13}$$

is the *mass enhancement factor*, while

---

extend the function $\sum_\nu(\omega)$ into the complex plane ($E_{\alpha'} \to E_{\alpha'} + \frac{i\Delta}{2}$) thus *regularizing the divergence*, determining the finite contributions and then taking the limit for $\Delta \to 0$ (Eqs. ~~(4.9)~~ and ~~(4.10)~~). The resulting complex potential (*optical potential* from the *complex dielectric function* of optics), parametrizes in simple terms the shift of the centroid of the single-particle state and its finite lifetime.

1.D.9 1.D.10

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1.D.2

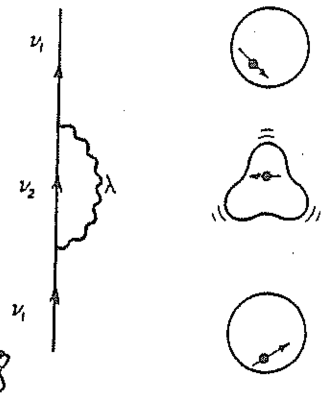

Figure ~~4.2~~: Self-energy graph for a single-particle

(1.D.8)

$$Z_\omega = m/m_\omega,$$

is the spectroscopic factor (discontinuity of the Fermi energy).

Consequently, Eq.(~~4.8~~) can be rewritten as

1.D.14

$$\left(-\frac{\hbar^2}{2m^*}\nabla_r^2 + U'_H + iW(\omega)\right)\varphi_j(\vec{r}) = \varepsilon_j\varphi_j(\vec{r}), \qquad \cancel{(4.14)}$$

with

1.D.3 1.D.4

$$m^* = \frac{m_k m_\omega}{m}. \qquad (4.15)$$

and $U'_H = (m/m^*)U$. Because $\lambda \approx 0.5$ (i.e. the dressed single-particle m_ω is heavier than the bare nucleon, as it has to carry a phonon along), $m^* \approx m$ and $Z_\omega \approx 0.7$. Furthermore, due to the fact that $\hbar\omega_\alpha \approx 2 - 2.5MeV$, *the range* of single-particle energy $E = \varepsilon - \varepsilon_F$ over which the particle-vibration coupling process displayed in Fig.4.2 effective is $\approx \pm 2\hbar\omega_\alpha \approx 4 - 5$ around the Fermi energy (cf. Figs.~~4.3~~ and ~~4.4~~)

1.D.5

To be noted that ΔE_j (Eq.(4.9)) indicates the shift in energy of the energy centroid of the "dressed" single-particle state due to the coupling to the intermediate (more complex states) $\alpha' \equiv (\nu', \alpha)$, while $\Gamma = 2W$ measures the energy range over which the single-particle state spreads due to the coupling (cf. Fig.~~4.5~~). While all states contribute to ΔE ("off the eneregy shell process", i.e. processes which do not conserve the energy), essentially only "on the energy processes", that is processes which conserve the energy, contribute to Γ. In fact

$$\lim_{\Delta\to 0}\frac{1}{(\omega - E_{\alpha'})^2 + \left(\frac{\Delta}{2}\right)^2} = 2\pi\delta(\omega - E_{\alpha'}),$$

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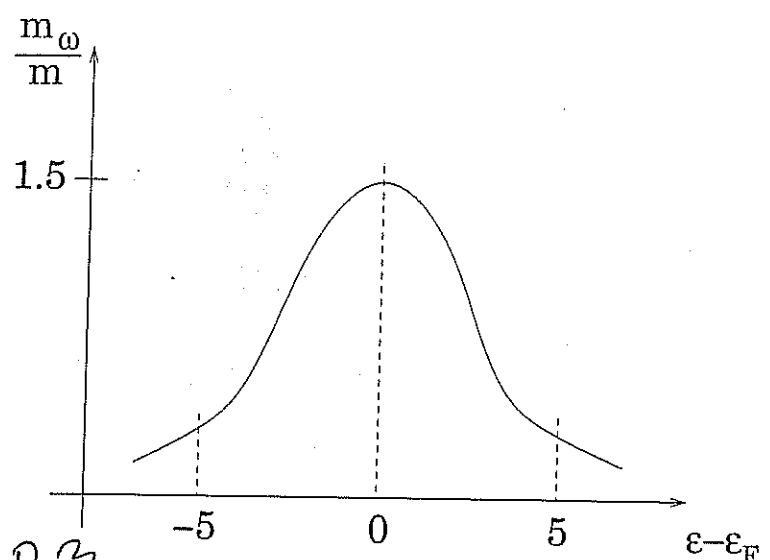
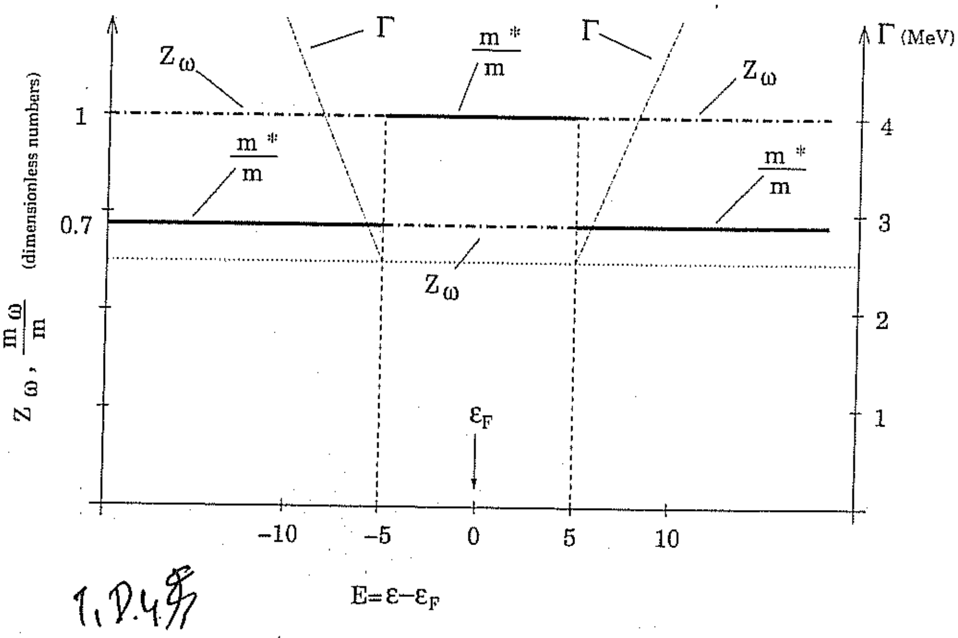


1.D.3

Figure ~~4.3~~: Schematic representation of the ω-mass as a function of the single-particle energy.



1.D.4

Figure ~~4.4~~: Schematic representation of the behaviour of m_ω/m, $Z_\omega = (m_\omega/m)^{-1}$ and Γ as a function of $E = \varepsilon - \varepsilon_F$.

(51)

and

$$\Gamma(\omega) \approx 2\pi \bar{V}^2 n(\omega),$$ (1.D.16) ~~(4.16)~~

where $\bar{V}^2$ is the average value of $V^2_{\nu,\alpha'}$, while

$$n(\omega) = \sum_{\alpha'} \delta(\omega - E_{\alpha'}),$$ (1.D.17) ~~(4.17)~~

is the density of energy-conserving states α'. Eq.(4.16) is known as *the Golden rule*.

On the other hand, assuming the distribution of single-particle levels is symmetric with respect to the Fermi energy

$$\Delta E(\omega) = \lim_{\Delta\to 0} \sum_{\alpha'} \frac{V^2_{\nu_1\alpha'}(\omega - E_{\alpha'})}{(\omega - E_{\alpha'})^2 + \left(\frac{\Delta}{2}\right)^2} = 0$$ (1.D.5)

as there are equally many states pushing the state down than up (cf. Fig.~~4.5 and App. C)~~. 1.D.5

Quantum mechanically there cannot be imaginary potentials, and the breaking of a stationary state into many, more complicate stationary states (Fig. ~~4.~~5(b)) is the only correct description to describe the coupling of a nucleon moving in a single-particle state with more complicate configurations [2]. However, such a description is quite involved. On the other hand, to account for the change of the centroid energy and of its spreading width in terms of an *optical potential* $\Delta E + iW$ is very economic and convenient. In any case Γ measures the range of energy over which the "pure" single-particle state $|a\rangle$ spreads due to the coupling to the more complicated states $|\alpha'\rangle$. In other words, a stationary state

$$\varphi_\nu(\vec{r_i}t) = e^{\frac{i\omega t}{\hbar}} \varphi_\nu(\vec{r}), \tag{4.18}$$

has a probability density

$$\int \mathrm{d}^3r |\varphi_\nu(\vec{r_i}t)|^2 = \int \mathrm{d}^3r |\varphi_\nu(\vec{r})| = 1, \tag{4.19}$$

which does not depend on time. In other words, if at $t = 0$, the probability that the particle is in a state ν is 1, it will have this probability also at $t = \infty$, implying an infinite lifetime. If however (cf. footnote 2),

[2]To be noted that if we spread the strength of a stationary quantal state over an energy range Γ, and set all components in phase at $t = 0$, they will essentially be out of phase at $t = \tau = \hbar/\Gamma$. In other words, each component will behave independent of each other and the state, created at $t = 0$ with probability 1 essentially ceases to exist at $t = \tau$.

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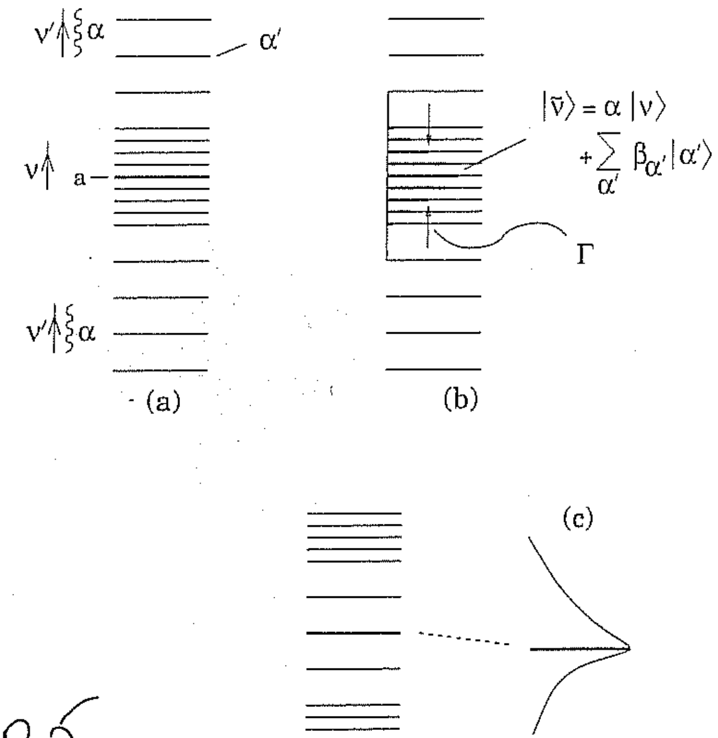

1.D.5

Figure ~~4.5~~: Schematic representation of the diagonalization of H_{coupl} in a basis consisting of the single-particle states $|\nu\rangle$ and the $|\alpha'\rangle = |\nu'_\alpha\rangle$ states. In (c) we show a situation where there are more states $|\alpha'\rangle$ above $|a\rangle$ than below.

$$\omega \quad = \varepsilon_\nu^{(0)} + \Delta E_\nu(\omega) + i\tfrac{\Gamma}{2}\nu(\omega), \quad = \varepsilon_\nu + i\frac{\Gamma_\nu}{2}(\omega) \qquad , (\varepsilon_\nu = \varepsilon_\nu^{(0)} + \Delta E_\nu)$$

then

$$\varphi_\nu(\vec{r_i}t) = e^{i\frac{\varepsilon_\nu t}{\hbar}} e^{-\frac{\Gamma_\nu t}{2\hbar}},$$

and

$$\int d^3r |\varphi_\nu(\vec{r_i}t)|^2 = e^{-\frac{\Gamma_\nu t}{\hbar}}, \qquad 1.D.20 \ \cancel{(4.20)}$$

implying a lifetime of the single-particle state

$$\tau = \Gamma/\hbar. \qquad 1.D.21 \ \cancel{(4.21)}$$

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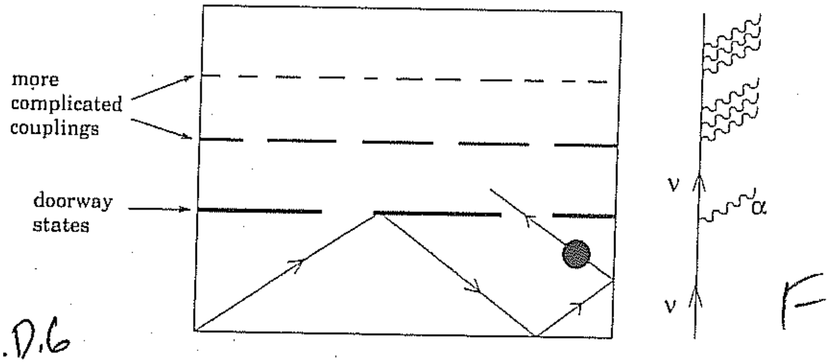
One may ask, how it is possible that the coupling to complicate (but still simple) states like $|\alpha'\rangle = |n_\alpha = 1, \nu'\rangle$ can explain the full damping of a single-particle state $8 - 10MeV$ from the Fermi energy ε_F, where the density of levels of all types is very large. This is because the Hamiltonian given in Eq. (4.1) contains all the basic physics to describe the single-particle motion. Any coupling to more complicated states will go through a hierarchy of couplings. In particular,



Fluctuation
dissipation

Figure ~~4.6~~ 1.D.6: Schematic representation of the different levels of couplings leading to the damping of a single-particle state. It is essentially the first doorway coupling which controls the probability the ball (black dot) reflecting elastically on the walls of the box has to remain in the first compartment.

all couplings, even the most complicate, should go through the coupling to states of type $|\nu', \alpha'\rangle$. In other words, $|\alpha'\rangle$ is a doorway state (cf. Fig. ~~4.6~~ 1.D.6).

In the nuclear case, the *doorway coupling provides the basic breaking* of the single-particle motion, while higher-order couplings only *fill in valleys* (cf. Fig. ~~4.7~~ 1.D.7). In other words, the quantity Γ (Eq. (~~4~~ 1.D.21)), gives the range over which the single-particle state is spread due to all the couplings ~~(cf. also Fig. 2.11)~~.

5 In the case of the $^1s_{1/2}$ orbital of ^{40}Ca ($\varepsilon - \varepsilon_F = -8$ MeV), simple estimates lead to $\bar{V}^2 \approx 0.3MeV$ for the coupling to an $L = 2$ phonon, and $n \approx 2MeV^{-1}$ ~~(cf. App. F and P)~~. Consequently

$$\Gamma \approx 4MeV, \qquad (\text{~~4~~ 1.D.22})$$

in overall agreement with the experimental findings (cf. Fig. ~~4.8~~ 1.D.8).

The result given in Eq. (4.22) is a particular example of the general (empirical) result (cf. Fig. 4.4)

$$\Gamma_{sp}(E) = \begin{cases} 0.5E & E > 5 \text{ MeV}, \\ 0 & E \leq 5 \text{ MeV}, \end{cases} \qquad (\text{~~4~~ 1.D.23})$$

54

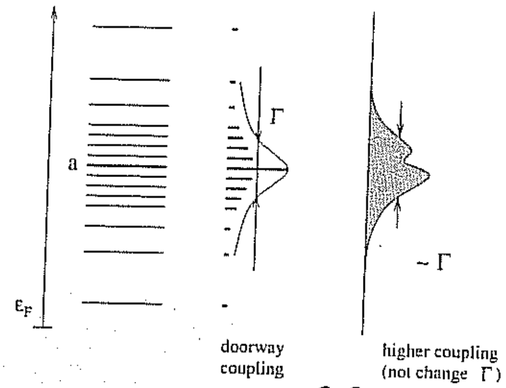


Figure ~~4.7~~ 1.D.7: .

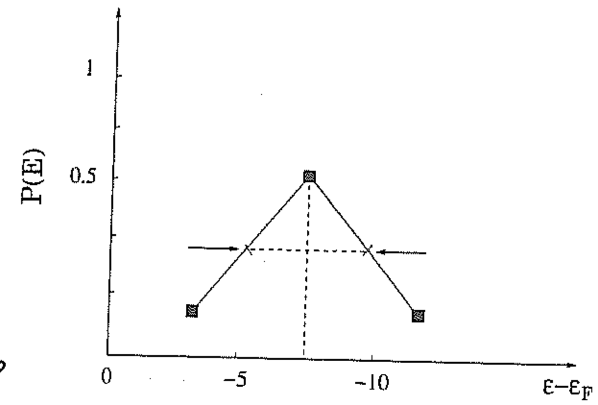


Figure ~~4.8~~ 1.D.8: Schematic representation of the experimental strength function (solid squares) associated to the $1s$ state of ^{40}Ca. Also indicated is the full width at half maximum (FWHM) (after [7]).

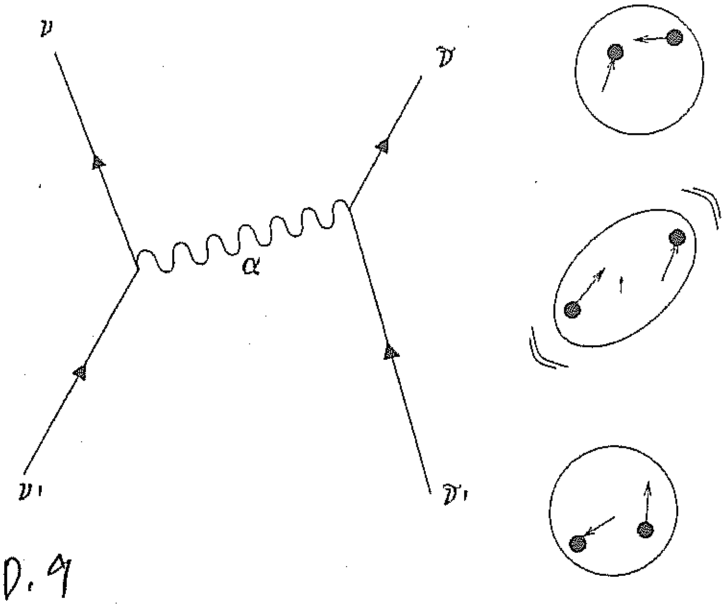
where

$$E = |\varepsilon - \varepsilon_F|. \qquad 1(D.24)\ \ \cancel{(4.24)}$$

~~4.4~~ 1.D.1 Induced interaction

OK

A nucleon at the Fermi energy which creates, by bouncing inelastically off the nuclear surface, has no other choice but to reabsorb it at a later instant of time (virtual process, Fig. ~~4.2~~ 1.D.2). In the presence of another nucleon, the vibration excited by one nucleon may be absorbed by the second one (~~Fig. 4.9~~ 1.D.9), the exchange of a vibration leading to an (induced) interaction.



1.D.9

Figure ~~4.9~~: Schematic representation of the exchange of phonons between nucleons.

Simple estimates of this induced interaction lead to values of the matrix element for pairs of particles coupled to angular momentum $J^{\pi} = 0^{+}$ of $-1.5MeV$, when summed over all the different multipolarities α ($L^{\Pi} = 2^{+}, 3^{-}, 5^{-}$) (~~cf. App. F~~). The fact that one considers particles coupled to anguler momentum zero is because the associated orbitals have maximum overlap, thus profiting at best from the (pairing) interaction [3]. In the case of two particles outside closed shell one would then expect the ground state to display, due to this mechanism, a correlation energy of $1.5MeV$ larger than that predicted by the independent particle model (cf. Fig.~~4.10~~ 1.D.10), a prediction which is confirmed by the experimental findings. From this result one can conclude that the pairing interaction induced by the process depicted in Fig.~~4.9~~ 1.D.9, renormalize in an important way the properties of the nuclear ground state of open shell nuclei.

[3]Note that quadrupole pairing correlations are also important, although weaker.

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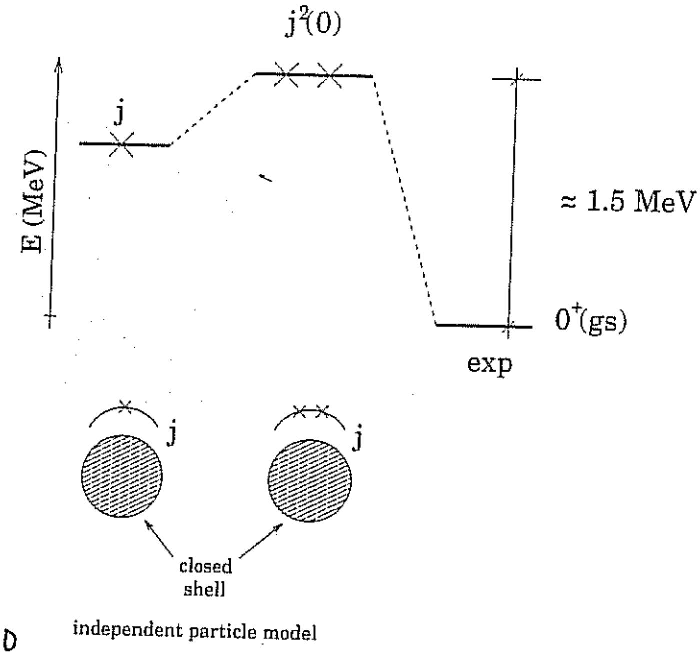

1.D.10

Figure ~~4.10~~: Schematic representation of the predictions of the independent particle model for one- and two-particles outside closed shell, in comparison with the experimental findings (e.g. for the case of ^{210}Pb, where $j = g_{9/2}$).

App. 1, E

Inelastic scattering

Old Chapter 5 ~~boo~~ book gregory
lectures
notes
copenhagen

App 1.E

~~Chapter~~ 5

Inelastic scattering

appendix: (how to extract values of the effective deformation parameter β_L from inelastic scattering absolute differential cross section)

In this ~~lecture~~ we discuss very briefly ~~some applications of the DW method,~~ in the most simple and straightforward way, ignoring all the complications associated with the spin carried by the particles, the spin orbit dependence of the optical model potential $\bar{U}(r_\beta)$ etc. ~~In following lectures we take again the problem in more detail.~~

5.1 Inelastic α-scattering

We start assuming that the interaction V'_β is equal to $V'_\beta = V'_\beta(\xi_\beta, r_\beta)$, which is usually called the stripping approximation.

We can then write eq. (7.2.126) in the DW approximation as

$$\frac{d\sigma}{d\Omega} = \frac{k_\beta}{k_\alpha}\frac{\mu_\alpha\mu_\beta}{(2\pi\hbar^2)}|(\psi_\beta(\xi_\beta)\chi^{(-)}(k_\beta,\vec{r}_\beta), V'_\beta(\xi_\beta, r_\beta)\psi_\alpha(\xi_\alpha)\chi^{(+)}(k_\alpha,\vec{r}_\alpha))|^2. \tag{5.1.1}$$

For the case of inelastic scattering $\xi_\alpha = \xi_\beta = \xi$, thus

$$\psi_\beta(\xi_\beta) = \psi^{I_\beta}_{M_{I\beta}}(\xi) \tag{5.1.2a}$$

$$\psi_\alpha(\xi_\alpha) = \psi^{I_\alpha}_{M_{I\alpha}}(\xi) \tag{5.1.2b}$$

$$\vec{r}_\alpha = \vec{r}_\beta \quad \mu_\alpha = \mu_\beta, \tag{5.1.2c}$$

i.e we are always in the entrance channel.

Equation (5.1.1) can now be rewritten as

$$\frac{d\sigma}{d\Omega} = \frac{k_\beta}{k_\alpha}\frac{m_\alpha^2}{(2\pi\hbar^2)^2}\frac{1}{2I_\alpha+1}\sum_{M_\alpha M_\beta}|(\chi^{(-)}(k_\beta,\vec{r}_\beta), V_{eff}(\vec{r})\chi^{(+)}(k_\alpha,\vec{r}_\alpha))|^2, \tag{5.1.3}$$

where

$$\begin{aligned} V_{eff} &= \int d\xi\, \psi^{I^*_\beta}_{M_{I\beta}}(\xi)V'_\beta(\xi,\vec{r})\psi^{I_\alpha}_{M_{I\alpha}}(\xi) \\ &= \int d\xi\, \psi^{I^*_\beta}_{M_{I\beta}}(\xi)V_\beta(\xi,\vec{r})\psi^{I_\alpha}_{M_{I\alpha}}(\xi) \end{aligned} \tag{5.1.4}$$

as ψ^{I_β} and ψ^{I_α} are orthogonal (remember $V'_\beta = V_\beta - \bar{U}(r)$). We expand the interaction in spherical harmonics, i.e.

$$\begin{aligned} V_\beta(\xi,\vec{r}) &= \sum_{LM} V^L_M(\xi,r)Y^L_M(\hat{r}) \\ &= \sum_{LM} V^L_M(\xi,\vec{r}). \end{aligned} \tag{5.1.5}$$

Defining

$$\int d\xi\, \psi^{I_\beta}_{M_{I\beta}}(\xi)[V^L_M(\xi,r)\psi^{I^\alpha}(\xi)]^{I_\beta}_{M_{I\beta}} = F_L(r), \tag{5.1.6}$$

we can write eq.(5.1.4) as

$$V_{eff}(\vec{r}) = \sum_{LM}(LMI_\alpha M_\alpha|I_\beta M_\beta)F_L(r)Y^L_M(\hat{r}). \tag{5.1.7}$$

Inserting (5.1.7) into (5.1.3) we obtain

$$\begin{aligned} \frac{d\sigma}{d\Omega} = \frac{k_\beta}{k_\alpha}\frac{m_\alpha^2}{(2\pi\hbar^2)^2}\frac{1}{2I_\alpha+1}\sum_{M_\alpha M_\beta}\Big|\sum_{LM}(LMI_\alpha M_\alpha|I_\beta M_\beta)\times \\ \int d\vec{r}\chi^{(-)*}(k_\beta,\vec{r}_\beta)F_L(r)Y^{L*}_M(\hat{r})\chi^{(+)}(k_\beta,\vec{r}_\beta)\Big|^2 = \\ \frac{k_\beta}{k_\alpha}\frac{m_\alpha^2}{(2\pi\hbar^2)^2}\frac{2I_\beta+1}{2I_\alpha+1}\times \\ \sum_{LM}\frac{1}{2L+1}\Big|\int d\vec{r}\chi^{(-)*}(k_\beta,\vec{r}_\beta)F_L(r)Y^{L*}_M(\hat{r})\chi^{(+)}(k_\beta,\vec{r}_\beta)\Big|^2, \end{aligned} \tag{5.1.8}$$

where we have used he orthogonality relation between Clebsch-Gordan coefficients

$$\begin{aligned} \sum_{M_\alpha M_\beta}(LMI_\alpha M_\alpha|I_\beta M_\beta)(L'MI_\alpha M_\alpha|I_\beta M_\beta) = \\ \sqrt{\frac{(2I_\beta+1)^2}{(2L+1)(2L'+1)}}\sum_{M_\alpha M_\beta}(I_\beta - M_\beta I_\alpha M_\alpha|L-M)\times \\ (I_\beta - M_\beta I_\alpha M_\alpha|L'-M) = \frac{2I_\beta+1}{2L+1}\delta_{LL'} \end{aligned} \tag{5.1.9}$$

(fixed M)

Let us now discuss the case of inelastic scattering of even spherical nuclei.

The macroscopic Hamiltonian describing the dynamics of the multipole surface vibrations in such nuclei can be written, in the harmonic approximation as

$$\mathrm{H} = \sum_{L,M}\left\{\frac{B_L}{2}|\dot{\alpha}^L_M|^2 + \frac{C_L}{2}|\alpha^L_M|^2\right\}, \tag{5.1.10}$$

where the collective coordinate α^L_M is defined through the equation of the radius

$$R(\hat{r}) = R_0\left[1 + \sum_{L,M} \alpha^L_M Y^{L*}_M(\hat{r})\right], \tag{5.1.11}$$

and where $R_0 = r_0 A^{1/3}$ fm.

The collective mode is generated from the interaction of the multipole field carried by each particle and the field of the rest of the particles. In turn this coupling modifies the single-particle motion. In particular the incoming prjectile would feel this coupling. The potential V'_β is equal to

$$\begin{aligned} V'_\beta(\xi, \vec{r}) &= U(r - R(\hat{r})) \\ &= U(r - R_0 - R_0 \sum_{L,M} \alpha^L_M Y^{L*}_M(\hat{r})) \\ &= U(r - R_0) - R_0 \sum_{L,M} \alpha^L_M Y^{L*}_M(\hat{r}) \frac{dU(r - R_0)}{dr} \\ &= V_\beta(\xi, r) - \bar{U}_\beta(r) \end{aligned} \tag{5.1.12}$$

$$\begin{aligned} \bar{U}_\beta(r) &= -U(r - R_0) \\ V_\beta(\xi, \vec{r}) &= R_0 \frac{d\bar{U}_\beta(r)}{dr} \sum_{L,M} \alpha^L_M Y^{L*}_M(\hat{r}) \end{aligned} \tag{5.1.13}$$

Comparing with eq. (5.1.5) we obtain

$$V^L_M(\alpha, r) = R_0 \frac{d\bar{U}_\beta(r)}{dr} \alpha^L_{+M} \tag{5.1.14}$$

Note that the Hamiltonian (5.1.10) is the Hamiltonian of a five-dimensional harmonic oscillator, and that α^L_M is a classical variable. One can quantize this Hamiltonian in the standard way (see for example Messiah "Quantum-Mechanics" Chapter 12)

$$\alpha^L_M = \sqrt{\frac{\hbar\omega_L}{2C_L}}(a^L_M - a^{+L}_{-M}) \tag{5.1.15}$$

where $\hbar\omega_L$ is the energy of the vibration, and a^{+L}_M is the creation operator of a phonon. For an even nucleus

$$\begin{aligned} &|\Psi^{I_\alpha}_{M_\alpha}\rangle = |0\rangle \quad (I_\alpha = M_\alpha = 0) \\ &|0\rangle : \quad \text{ground state} \end{aligned} \tag{5.1.16}$$

The one-phonon state is equal to

$$\begin{aligned} &|\Psi^{I_\alpha}_{M_\alpha}\rangle = |I; LM\rangle = a^{+L}_M|0\rangle \\ &(I_\beta = L; M_{I_\beta} = M) \end{aligned} \tag{5.1.17}$$

We can now calculate the matrix element of the operator (5.1.14), which connects states which differ in one phonon. Starting from the ground state we obtain

$$\begin{aligned}
&\langle I; LM|V_M^L(\alpha, r)|0\rangle = \\
&(-1)^{L-M} R_0 \frac{d\bar{U}_\beta(r)}{dr}\sqrt{\frac{\hbar\omega_L}{2C_L}}\langle 0|(a_M^L - a_{-M}^{+L})|0\rangle = \\
&R_0 \frac{d\bar{U}_\beta(r)}{dr}\sqrt{\frac{\hbar\omega_L}{2C_L}} = -\frac{R_0}{\sqrt{2L+1}}\frac{d\bar{U}_\beta(r)}{dr}\beta_L
\end{aligned} \tag{5.1.18}$$

where

$$\beta_L = \sqrt{\frac{(2L+1)\hbar\omega_L}{2C_L}} \tag{5.1.19}$$

Substituting (5.1.18) into eq. (5.1.8) and making use of eqs. (5.1.16) and (5.1.17) we get

$$\begin{aligned}
\frac{d\sigma}{d\Omega} &= \frac{k_\beta}{k_\alpha}\frac{\mu_\alpha^2}{(2\pi\hbar^2)^2}(\beta_L R_0)^2 \times \\
&\sum_M \frac{1}{2L+1}\left|\int d\vec{r}\chi^{(-)*}(k_\beta, \vec{r})\frac{dU(r)}{dr}Y_M^{L*}(\hat{r})\chi^{(+)}(k_\alpha, \vec{r}_\beta)\right|^2
\end{aligned} \tag{5.1.20}$$

Suppose now that the nucleus has a permanent axially-symmetric deformation. For a $K = 0$ band, the nuclear wave function has the form

$$\Psi_{IMK=0} = \sqrt{\frac{2I+1}{8\pi^2}}\mathcal{D}_{M0}^I(\omega)\chi_{K=0} \quad \text{(intrinsic)} \tag{5.1.21}$$

where we have used $(\omega) = (\theta, \phi, \psi)$ to label the Eulerian angles which serve as orientation parameters.

In the intrinsic frame (which we take to coincide with the space-fixed axis when $\theta = \phi = \psi = 0$) the nuclear surface has the shape

$$R(\hat{r}) = R_0\left[1 + \sum_L b_L Y_0^L(\hat{r})\right] \tag{5.1.22}$$

where the b_L introduced here is α_0^L in the intrinsic frame. When the nucleus has orientation ω, this shape is rotated into

$$\hat{P}_\omega R(\hat{r}) = R_0\left[1 + \sum_L b_L \mathcal{D}_{M0}^L(\omega) Y_0^L(\hat{r})\right] \tag{5.1.23}$$

we then have that

$$W(r - R(\hat{r})) = W(r - R_0) - R_0\frac{dW(r-R_0)}{dr}\sum_L b_L \mathcal{D}_{M0}^L(\omega) Y_0^L(\hat{r}) \tag{5.1.24}$$

which is the equivalent to eq. (5.1.12) for the case of deformed nuclei. Then

$$V_M^L(b,r;\omega) = -\frac{d\bar{U}_\beta(r-R_0)}{dr} b_L \mathcal{D}^L_{M0}(\omega) \qquad (5.1.25)$$

The effective interaction is now equal to

$$\begin{aligned}
&\langle \Psi_{IMK=0}, V_M^L(b,r;\omega)\Psi_{000}\rangle = \\
&-R_0\frac{d\bar{U}(r-R_0)}{dr} b_L \sqrt{\frac{(2L+1)^2}{8\pi^2}} \int d\omega \mathcal{D}^{L*}_{M0}(\omega)\mathcal{D}^L_{M0}(\omega) = \\
&-R_0\frac{d\bar{U}(r-R_0)}{dr} b_L = -\frac{R_0}{\sqrt{(2L+1)}}\frac{d\bar{U}(r-R_0)}{dr}\beta_L = F_L(r) \\
&(\beta_L = \sqrt{(2L+1)}b_L)
\end{aligned} \qquad (5.1.26)$$

in complete analogy to (5.1.18). Thus the same formfactor is used for both types of collective excitation.

The normalization factor $(\beta_L R_0)^2$ which is the only free parameter what is obtained from the comparison of the experimental and theoretical (DWBA) differential cross section. The quantity β_L is known as the multipole deformation (dynamic or static) parameter, and gives a direct measure of the coupling of the projectile to the vibrational field.

The value of β_L can also be obtained from the $B(E_L)$ reduced transition probability, in which case one has a measure of the electric moment, instead of the mass moment.

References

Introduction

R1

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