PREFACE (II) | 22/06/19

(9)

Because there is no restoring force associated with different orientations of $|N(\omega)\rangle_{\mathcal{K}'}$, fluctuations in the Euler angles diverge in just the right way to restore rotational invariance, leading to a rotational band whose members are

$$|IKM\rangle \sim \int d\omega\, \mathcal{D}^{I}_{MK}(\omega)\, |N(\omega)\rangle_{\mathcal{K}'}, \quad (0.1.47)$$

with energy

$$E_I = \frac{\hbar^2}{2\mathcal{J}} I(I+1). \quad (0.1.48)$$

The quantum numbers I, M, K are the total angular momentum I, and its third component M and K along the laboratory (z) and intrinsic (z') frame of reference respectively.

Rotational bands have been observed up to rather high angular momenta in terms of individual transitions. An example (^{152}Gd P. Twin) extending up to $I = 60\hbar$ is given in Fig ~~0.1.13~~ 0.4.2

0.4.2 Deformation in gauge space

Let us now turn to the pairing Hamiltonian. In the case in which $\hbar\omega_{\beta=+2} \approx \hbar\omega_{\beta=-2} \approx 0$, the system deforms, this time in gauge space. Calling $|BCS\rangle$ the eventual mean field solution, leads to a finite value

$$\alpha_0 = \langle BCS | P^{+} | BCS \rangle, \quad (0.1.49)$$

of the pair operator P^+ which can be viewed as the order parameter of the new (gauge deformed) phase of the system.

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~~Let us now turn to the pairing Hamiltonian. In the case in which $\hbar\omega_{+2} \approx \hbar\omega_{-2} \approx 0$, the system deforms, this time in gauge space. Calling $|BCS\rangle$ the eventual mean field solution, leads to a finite value in gauge space~~ (6)

top. (10)

$$\cancel{\alpha_0 = \langle BCS | P^\dagger | BCS \rangle} \quad \cancel{(0.1.39)}$$

~~of the pair operator $P^\dagger$ which can be viewed as the order parameter of this new (deformed) phase of the system. The total Hamiltonian can be~~ written as

$$H = H_{MF} + H_{fluct}, \qquad \cancel{(0.1.40)}\ (0.1.50)$$

where

$$H_{MF} = H_{sp} - \Delta\,(P^\dagger + P) + \frac{\Delta^2}{G} \qquad (0.1.51)\ \cancel{(0.1.41)}$$

and

$$H_{fluct} = -G\,(P^\dagger - \alpha_0)(P - \alpha_0) \qquad \cancel{(0.1.42)}\ (0.1.52)$$

The quantity

$$\Delta = G\alpha_0 \qquad \cancel{(0.1.45)}\ (0.1.53)$$

is the so called pairing gap, which measures the binding energy of Cooper pairs, α_0 counting the number of Cooper pairs.

~~$[[H,F],F]$ sum rules~~

~~$\hbar\omega_0 = \frac{41}{A^{1/3}}$ MeV G.R. GPV~~

København 11/08/18

~~(7)~~ (11)

The mean-field pairing Hamiltonian

$$H_{MF} = \sum_{\nu>0}(\varepsilon_\nu - \lambda)(a^+_\nu a_\nu + a^+_{\tilde\nu} a_{\tilde\nu}) - \Delta \sum_{\nu>0}(a^+_\nu a^+_{\tilde\nu} + a_{\tilde\nu} a_\nu) + \frac{\Delta^2}{G} \qquad (0.1.54)\ \cancel{(0.1.44)}$$

is a bilinear expression in the creation and annihilation operator, ν labeling the quantum numbers of the single-particle orbitals where nucleons are allowed to correlate e.g. $(nljm)$, while $\tilde\nu$ denotes the time reversal state which in this case is degenerate with ν and has quantum numbers $(nlj-m)$, $\nu>0$ implying ~~that~~ one sums over $m>0$. It is of notice that

$$\hat N = \sum_{\nu>0}(a^+_\nu a_\nu + a^+_{\tilde\nu} a_{\tilde\nu}), \qquad (0.1.55)$$

is the number operator, and $\lambda\hat N$ in Eq (0.1.54) acts as a the Coriolis force in the body-fixed frame of reference in gauge space.

One can diagonalize H_{MF} by a rotation in the (a^+, a)-space. This can be accomplished through the Bogoliubov-Valatin transformation

$$\alpha^+_\nu = U_\nu a^+_\nu - V_\nu a_{\tilde\nu}. \qquad (0.1.56)\ \cancel{(0.1.45)}$$

The BCS solution does not change the energies ε_ν (measured in (0.1.54) ~~(0.1.44)~~ from the Fermi energy λ) of the single-particle levels or associated wave functions $\varphi_\nu(\vec r)$, but the occupation probabilities for levels around the Fermi energy within an energy range 2Δ $(2\Delta/\lambda \approx 2\,\text{MeV}/36\,\text{MeV} \approx 0.06)$. The quasiparticle operator α^+_ν creates a particle in the single-particle ~~state~~ ν with probability U_ν^2, while it creates a hole (annihilates a particle) with probability V_ν^2. To be able to create a particle, the state ν should be empty, while to annihilate a particle it has to be filled, so U_ν^2 and V_ν^2 are the probabilities ~~that~~ the state ν is empty and is occupied respectively. Within this context, the one quasiparticle states

$$|\nu\rangle = \alpha^+_\nu |BCS\rangle \qquad (0.1.57)\ \cancel{(0.1.46)}$$

are orthonormal. In particular

(0.1.58) ~~(0.1.47)~~

$$\langle \nu | \nu \rangle = 1 = \langle BCS | \alpha_\nu \alpha_\nu^\dagger | BCS \rangle$$

$$= \langle BCS | \{\alpha_\nu, \alpha_\nu^\dagger\} | BCS \rangle = U_\nu^2 + V_\nu^2,$$

where the relations

(0.1.59) ~~(0.1.48)~~

$$\{a_\nu, a^\dagger_{\nu'}\} = \delta(\nu, \nu')$$

and

(0.1.60) ~~(0.1.49)~~

$$\{a_\nu, a_{\nu'}\} = \{a^\dagger_\nu, a^\dagger_{\nu'}\} = 0$$

have been used.

Note that the $|BCS\rangle$ state is the quasiparticle vacuum

(0.1.61) ~~(0.1.50)~~

$$\alpha_\nu |BCS\rangle = 0,$$

in a similar way in which $|0\rangle_F$ is the particle vacuum.

Inverting the quasiparticle transformation (0.1.56) ~~(0.1.45)~~ and its complex conjugate, i.e. expressing $a_\nu^\dagger$ and a_ν (and time reversals (tr)) in terms of $\alpha_\nu^\dagger$ and α_ν (and tr) one can rewrite ~~(0.1.44)~~ (0.1.54) in terms of quasiparticles. 2016/1/19

Minimizing the energy $E_0 = \langle BCS | H | BCS \rangle$ in terms of V_ν

(0.1.62) ~~(0.1.51)~~

$$\frac{\partial E_0}{\partial V_\nu} = 0$$

and making use of the expression for the average number of particles

(0.1.63) ~~(0.1.52)~~

$$N_0 = \langle BCS | \hat{N} | BCS \rangle = 2 \sum_{\nu > 0} V_\nu^2,$$

and of the number of Cooper pairs

(0.1.64) ~~(0.1.53)~~

$$\alpha_0 = \langle BCS | P^\dagger | BCS \rangle = \sum_{\nu > 0} U_\nu V_\nu$$

and thus of the pairing gap,

(0.1.65) ~~(0.1.54)~~

$$\Delta = G \sum_{\nu > 0} U_\nu V_\nu,$$

one obtains,

~~9~~ 13

$$H_{MF} = H_{11} + U \qquad (0.1.66) \; \cancel{(0.1.55)}$$

where

$$H_{11} = \sum_{\nu} E_\nu \, \alpha_\nu^\dagger \alpha_\nu \qquad (0.1.67) \; \cancel{(0.1.56)}$$

and

$$U = 2 \sum_{\nu>0} (\varepsilon_\nu - \lambda) V_\nu^2 - \frac{\Delta^2}{G}. \qquad (0.1.68) \; \cancel{(0.1.57)}$$

The quantity

$$E_\nu = \sqrt{(\varepsilon_\nu - \lambda)^2 + \Delta^2} \qquad (0.1.69) \; \cancel{(0.1.58)}$$

is the quasiparticle energy,
while the probability amplitudes are

$$V_\nu = \frac{1}{\sqrt{2}} \left(1 - \frac{\varepsilon_\nu - \lambda}{E_\nu}\right)^{1/2} \qquad (0.1.70) \; \cancel{(0.1.59)}$$

and

$$U_\nu = \frac{1}{\sqrt{2}} \left(1 + \frac{\varepsilon_\nu - \lambda}{E_\nu}\right)^{1/2} \qquad (0.1.71) \; \cancel{(0.1.60)}$$

From the relations (0.1.63) ~~(0.1.52)~~ and (0.1.65) ~~(0.1.54)~~
one obtains

$$N_0 = 2 \sum_{\nu>0} V_\nu^2 , \qquad \text{(number equation)} \quad 0.1.72 \; \cancel{(0.1.55)}$$

and

$$\frac{1}{G} = \sum_{\nu>0} \frac{1}{2E_\nu} , \qquad \text{(gap equation)} \quad (0.1.73) \; \cancel{(0.1.56)}$$

~~from which one can determine the parameter λ and Δ, and thus the occupation amplitudes~~ ~~10/6/19~~

These equations allow ~~us~~ one to calculate the parameters λ and Δ from the knowledge of G and ε_ν, parameters which completely determine ~~the~~ E_ν, ~~V_ν~~ V_ν ad U_ν and thus the BCS mean field solution (Fig. ~~0.1.14~~) Fig 1.9 Brink + Broglia

0.4.3

The validity of the BCS description of superfluid open shell nuclei have been confirmed throughout the mass table. We provide below recent examples.

The relation (~~0.1.50~~) (0.1.61), implies that

$$|BCS\rangle = \frac{1}{\text{Norm}} \prod_{\nu>0} \alpha_\nu \alpha_{\bar\nu} |0\rangle_F = \prod_{\nu>0} (U_\nu + V_\nu P^+_\nu) |0\rangle_F, \quad (\cancel{0.1.57})\ (0.1.74)$$

$$= \Big(\prod_{\nu>0} U_\nu\Big) \sum_{N\,\text{even}} \frac{\big(\sum_{\nu>0} c_\nu P^+_\nu\big)^{N/2}}{(N/2)!} |0\rangle_F, \quad (\cancel{0.4.58})\ (0.1.75)$$

where

$$P^+_\nu = a^+_\nu a^+_{\bar\nu} \quad \Big(P^+ = \sum_{\nu>0} P^+_\nu\Big), \quad c_\nu = V_\nu / U_\nu . \quad (\cancel{0.4.58})$$

In the above discussion of BCS we have treated in a rather cavalier fashion the fact ~~that~~ the amplitudes U_ν and V_ν are in fact complex quantities. A possible choice of phasing is*)

$$U_\nu = U'_\nu \quad ; \quad V_\nu = V'_\nu e^{-2i\phi}, \quad (0.1.76)\ (\cancel{0.4.59})$$

U'_ν and V'_ν being real quantities, while ϕ is the gauge angle, conjugate variable to the number of particles operator (0.1.55) (~~0.1.44~~).

Then**)

$$\hat\phi = i\,\partial/\partial N, \quad N \quad (0.1.76)\ (\cancel{0.1.60})$$

and (~~Appendix 0.A~~)

$$[\hat\phi, N] = i \quad (0.1.77)\ (\cancel{0.4.61})$$

where $N \equiv \hat N$ (Eq. ~~0.1.44~~ 0.1.55), gauge transfor-

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*) The same results as those which will be derived are obtained with the alternative choice $U_\nu = U'_\nu e^{i\phi}$, $V_\nu = V'_\nu e^{-i\phi}$.

**) See e.g. Brink and Broglia (2005) App. H and refs. therein.

See p. 20

app H Brink and Broglia

mations being induced by the operator

$$\mathcal{G}(\phi) = e^{-i\mathcal{N}\phi} \quad . \qquad (0.1.78) \; \cancel{(0.1.62)} \; (0.1.76)$$

Let us introduce the amplitudes ~~(0.1.59)~~ in (0.1.58),

$$|BCS\rangle = \left(\prod_{\nu>0} U'_\nu\right) \sum_{N\,even} e^{-iN\phi} |\Phi_{N'}\rangle = \left(\prod_{\nu>0} U'_\nu\right) \sum_{N\,even} e^{-i\mathcal{N}\phi} |\Phi_N\rangle \qquad \cancel{(0.1.63)} \; (0.1.79)$$

where

$$|\Phi_N\rangle = \frac{\left(\sum_{\nu>0} c'_\nu P^\dagger_\nu\right)^{N/2}}{(N/2)!} |0\rangle_F \qquad (0.1.80) \; \cancel{(0.1.64)}$$

with $c'_\nu = V'_\nu / U'_\nu$. It is of notice that

$$\sum_{\nu>0} c'_\nu P^\dagger_\nu |0\rangle_F \qquad (0.1.81)$$

is the single cooper pair state. As alread emerged from (0.1.39) and is explictely confirmed by the above expression, the state $|BCS\rangle$ does not have a definite number of particles but only in average ~~(0.1.52)~~ (0.1.63), being a wavepacket in N.

In fact, ~~(0.1.63)~~ (0.1.79) defines a priviledge direction in gauge space, being an eigenstate of $\hat{\Phi}$

$$\hat{\Phi}|BCS\rangle = i\frac{\partial}{\partial\mathcal{N}} \left(\prod_{\nu>0} U'_\nu\right) \sum_{N\,even} e^{-i\mathcal{N}\phi} |\Phi_N\rangle \qquad (0.1.82) \; \cancel{(0.1.65)}$$

$$= \phi\, |BCS\rangle$$

Expressing it differently ~~(0.1.63)~~ (0.1.79) can be viewed as an axially symmetric deformed system in gauge space, whose symmetry axis coincides with the z' component of the body-fixed frame of reference $\mathcal{K}'$, which makes an angle ϕ with the laboratory z-axis. (Fig ~~0.1.14~~)

Returning to the original, first line

~~0.1.15~~

Fig. 0.4.4

~~0.4.4~~

expression of (0.1.57) one can write,

$$|BCS(\phi=0)\rangle_{\mathcal{K}'} = \prod_{\nu>0} (U'_\nu + V'_\nu P'^{+}_\nu)|0\rangle_F , \quad (0.1.83) \; \cancel{(0.1.66)}$$

where use was made of the relations

$$\mathcal{G}(\phi)\, a^+_\nu \mathcal{G}^{-1}(\phi) = e^{-i\phi} a^+_\nu = a'^{+}_\nu , \quad (0.1.84) \; \cancel{(0.1.67)}$$

and

$$\mathcal{G}(\phi)\, P^+_\nu \mathcal{G}^{-1}(\phi) = e^{-2i\phi} P^+_\nu = P'^{+}_\nu , \quad (0.1.84) \; \cancel{(0.1.68)}$$

~~anti clockwise~~

It is to be noted that $\mathcal{G}$ induces an counter clockwise rotation, ~~(17pp.0A)~~

$$\mathcal{G}(\chi)\hat{\phi}\mathcal{G}^{-1}(\chi) = \hat{\phi} - \chi . \quad (0.1.85)$$

As a consequence, to rotate $|BCS(\phi=0)\rangle_{\mathcal{K}'}$ back into the laboratory system, use has to be made of the clockwise rotation of angle ϕ induced by $\mathcal{G}^{-1}(\phi)$,

$$\mathcal{G}^{-1}(\phi)|BCS(\phi=0)\rangle_{\mathcal{K}'} = \prod_{\nu>0} (U'_\nu + V'_\nu \mathcal{G}^{-1}(\phi) P'^{+}_\nu)|0\rangle_F$$

$$= \prod_{\nu>0} (U_\nu + e^{2i\phi} V_\nu P^+_\nu)|0\rangle_F \quad (0.1.86) \; \cancel{(0.1.69)}$$

~~20/6/14~~ $$= |BCS(\phi)\rangle_{\mathcal{K}} , \quad (0.1.84)$$

where use was made of ~~(0.1.68)~~

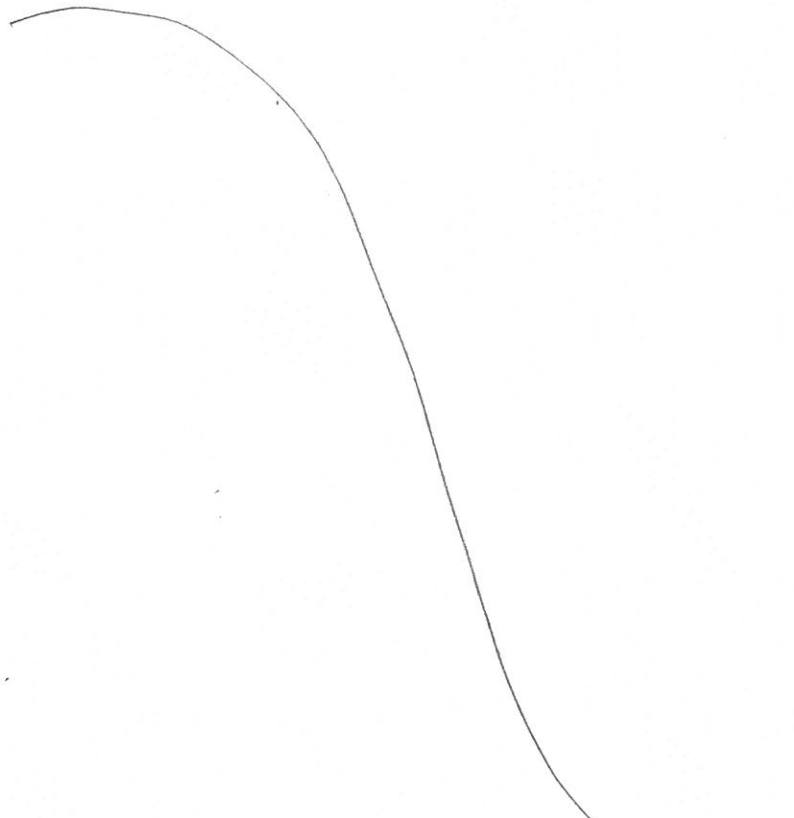
$$\mathcal{G}^{-1}(\phi)\left(\mathcal{G}(\phi) P^+_\nu \mathcal{G}^{-1}(\phi)\right)\mathcal{G}(\phi) = \mathcal{G}^{-1}(\phi) P'^{+}_\nu \mathcal{G}(\phi) \quad \cancel{(0.1.70)} \; (0.1.87)$$

We note furthermore

$$|BCS(\phi)\rangle_{\mathcal{K}'} = \prod_{\nu>0} (U'_\nu + V'_\nu P'^{+}_\nu)|0\rangle_F$$

$$= \prod_{\nu>0} (U_\nu + V_\nu P^+_\nu)|0\rangle_F \quad \cancel{(0.1.71)} \; (0.1.88)$$

Spontaneous broken symmetry in nuclei is, as a rule associated with the presence of rotational bands, as alread found in the case of quadrupole deformed nuclei. Consequently, one expects in nuclei with $\Delta \neq 0$ rotational bands in which particle number plays the role of angular momentum. That is pairing rotational bands. In what follow we will discuss the structure of H_{fluct} and single out the term responsible for restoring gauge invariance to the BCS mean field solution and thus, give rise to pairing rotation.



Spontaneously broken symmetry in nuclei is, as a rule associated with the presence of rotational bands, as already exemplified by the case of quadrupole deformed nuclei.

the different orientations (of $|BCS(\phi)\rangle_K$) in gauge space have the same energy, as no restoring force is associated with such a process. [~~Instead of proceeding as in the case of rotations in 3D-space~~] In what follows, we will extract ~~the component of H_{fluct}~~ responsible to generate rotation in gauge space. (discuss the structure of H_{fluct}, and ~~individuate~~ single out the ~~quad~~ term)

Consequently, one expects (in nuclei in which $\Delta \neq 0$) rotational bands in which particle number plays the role of angular momentum. That is pairing rotational bands.

In terms of quasiparticles, H_{fluct} can be expressed as

$$H_{fluct} = H'_p + H''_p + C \tag{0.1.89}$$

~~(0.1.72)~~

where

$$H'_p = -\frac{G}{4}\left(\sum_{\nu>0}(U_\nu^2 - V_\nu^2)(\Gamma_\nu^\dagger + \Gamma_\nu)\right)^2 \tag{0.1.90}$$

~~(0.1.73)~~

and

$$H''_p = \frac{G}{4}\left(\sum_{\nu>0}(\Gamma_\nu^\dagger - \Gamma_\nu)\right)^2, \tag{0.1.91}$$

~~(0.1.73)~~

with

$$\Gamma_\nu^\dagger = \alpha_\nu^\dagger \alpha_{\bar\nu}^\dagger . \tag{0.1.92}$$

~~(0.1.74)~~

The term C stands for constant terms, as well as for terms proportional to the number of quasiparticles, ~~and~~ which consequently vanish when acting on $|BCS\rangle$.

The term H_p' gives rise two-quasiparticle pairing vibrations with energies $\gtrsim 2\Delta$. It can be shown that it is the term H_p'' which restores gauge invariance*),

$$[H_{MF} + H_p'', \hat{N}] = 0 \qquad \begin{matrix}(0.1.93)\\ \cancel{(0.4.75)}\end{matrix}$$

We now diagonalize $H_{MF} + H_p''$ in the quasi-particle RPA (QRPA),

$$[H_{MF} + H_p'', \Gamma_n^+] = \hbar\omega_n \Gamma_n^+ , \qquad [\Gamma_n, \Gamma_{n'}^+] = \delta(n,n'), \qquad \begin{matrix}\cancel{(0.4.76)}\\ (0.1.94)\end{matrix}$$

where

$$\Gamma_n^+ = \sum_\nu (a_{n\nu}\Gamma_\nu^+ + b_{n\nu}\Gamma_\nu) , \qquad \Gamma_\nu^+ = \alpha_\nu^+ \alpha_{\bar\nu}^+ \qquad \begin{matrix}(0.1.95)\\ \cancel{(0.1.77)}\end{matrix}$$

is the creation operator of the nth vibrational mode. In the case of the $n=1$, lowest energy root, it can be written as

$$\Gamma_1^+ = \frac{\Lambda_1''}{2\Delta}(\hat{N} - N_0), \qquad \begin{matrix}(0.1.96)\\ \cancel{(0.4.78)}\end{matrix}$$

where $\hat{N}$ is the particle number operator written in terms of Γ_ν^+ and Γ_ν, and Λ_1'' is the strength of the quasiparticle-mode coupling. The prefactor is the zeropoint fluctuation (ZPF) of the mode, ~~that~~ is (Eq. 0.1.10)

$$\sqrt{\frac{\hbar\omega_1''}{2C_1''}} = \sqrt{\frac{\hbar^2}{2D_1''\hbar\omega_1''}} \qquad \left(\omega_1'' = \sqrt{\frac{C_1''}{D_1''}}\right) \qquad \begin{matrix}\cancel{(0.4.79)}\\ (0.1.97)\end{matrix}$$

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*) For details see Brink and Broglia (2005)

Because the frequency $\omega_1''=0$, the associated ZPF diverge ($\Lambda_1''\sim(\hbar\omega_1'')^{-1/2}$). It can be shown that this is because $C_1''\to 0$, while D_1'' remains finite. (15) (19)

In fact,

$$\frac{D_1''}{\hbar^2}=\frac{2\Delta^2}{\Lambda_1''^2\hbar\omega_1''}=4\sum_{\nu>0}\frac{U_\nu^2V_\nu^2}{2E_\nu}. \quad (0.1.98) \quad \cancel{(0.1.8)}$$

Because a rigid rotation in gauge space can be generated by a series of infinitesimal operations of type $\mathcal{G}(\delta\phi)=e^{i(\hat N-N_0)\delta\phi}$, the one phonon state $|1''\rangle=\Gamma_1^+|0''\rangle$, is obtained from rotations in gauge space of divergent amplitude. That is, fluctuations of ϕ over the whole $0-2\pi$ range.

By proper inclusion of these fluctuations one can restore gauge invariance violated by $|BCS\rangle_{\mathcal{K}'}$. The resulting state

$$|N_0\rangle\sim\int_0^{2\pi}d\phi\, e^{-iN_0\phi}|BCS(\phi)\rangle_{\mathcal{K}}\sim\Big(\sum_{\nu>0}c_\nu' P_\nu^+\Big)^{N/2}|0\rangle_F \quad (0.1.99)$$

have a definite number of particles and constitute the members of a pairing rotational band. ~~For example the that represented by the ground state of the ${}^{A}_{50}Sn_N$-isotopes, open shell, superfluid nuclei.~~

Making use of a simplified model (single j-shell[*)]) it can be shown that

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Brink and Broglia App. H

~~Conserving approx~~

the energy of these states can be written as, ~~(App. 0.A)~~ ← (App C Aage Bohr contrs. ~~(see App H Brink+Broglia)~~

$$E_N = \lambda(N-N_0) + \frac{G}{4}(N-N_0)^2, \quad (0.1.100)$$

where

$$\frac{G}{4} = \frac{\hbar^2}{2D_1''}. \quad (0.1.101)$$

An example of pairing rotational bands is provided by the ground state of the single open closed shell superfluid isotopes of the ${}^{A}_{50}Sn_N$ - isotopes (Fig. ~~0.1.15~~ 4.6 Fig. 9 paper PRC 87 Sn), $N_0 = 68$ having been used in the solution of the BCS number equation (0.1.72). Theory provides an overall account of the experimental findings. 0.4.5

Making use of the BCS pair transfer amplitudes,

$$\langle BCS | P^+_\nu | BCS \rangle = U_\nu V_\nu \quad (0.1.102)$$

in combination with a reaction software and of global optical parameters, one can account for the absolute value of the pair transfer ~~(specific probe)~~ differential cross section, within experimental errors ~~(Fig. 6.4.1)~~ ← (see Ch. 6).

The fact that projecting out the different Sn-isotopes from the intrinsic BCS state describing ^{118}Sn one obtains a quantitative description of observations carried out with the help of the specific probe of pairing correlations (Cooper pair transfer), testifies to the fact that pairing rotational bands can be considered elementary modes of nuclear excitation, emergent properties of spontaneous symmetry breaking of gauge invariance.

Furthermore, the fact that these results follow the use of QRPA*) in the calculation of the ZPF of the collective solutions of the pairing Hamiltonian indicates the importance of conserving approximations to describe many-body problems in general, and the finite size many-body problem (FSMB) of which the nuclear case represents a paradigmatic example.

(21)

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*) Using the Tamm-Dancoff approximation, i.e. setting $Y \equiv 0$ (and thus $\sum X^2 = 1$) in the QRPA approximation does not lead to particle number conservation, in keeping with the fact that the amplitudes Y are closely connected with ZPF.