2. Introduction

Globos tensión superficial ...

Nuclear Structure in a nutshell 24/12/13

The low-energy properties of quantal, many-body, Fermi systems ~~displaying a quantality parameter~~ displaying sizable values of zero-point-motion (kinetic energy) of localization compared to the strength of the NN-interaction and quantified by a quantality parameter $Q \gtrsim 0.15$, are determined by the laws which control independent particle motion close to the Fermi energy ε_F (on-the-energy shell), and by the correlations operating among them.

First of all, the Pauli principle, implying orbitals solidly anchored to the single-particle mean field, as testified by the Hartree-Fock ground state $|HF\rangle = \Pi_i a^+_i |0\rangle$, describing a step function separation in the probability of occupied $(\varepsilon_i \leq \varepsilon_F)$ and empty $(\varepsilon_k \geq \varepsilon_F)$ states (box 1).

Pairing acting on fermions moving in time reversal states lying close to ε_F alters this picture in a conspicuous way. In particular, in the case of $S=0$ configurations, in which case the radial component of the pair wavefunction does not display nodes. Within an energy range of the pair correlation energy E_{corr} ($\approx 2\Delta$ within BCS) centered around ε_F ($E_{corr}/\varepsilon_F \ll 1$), the system is now made out of pairs of fermions which

24/12/13 ②

flicker in and out of the correlated ($L=0, S=0$) configuration (Cooper pairs; box 2). For temperatures (intrinsic excitation energies) or stress (magnetic field in metals, Coriolis force in nuclei, etc) smaller than $\approx E_{corr}/2$ (critical value), Cooper pairs are present, in the ground ~~state~~ regime of the system, with a high probability. Because Cooper pairs respect Bose-Einstein statistics, the single-particle orbits on which they are correlated become dynamically dettached from the mean field, leading to a bosonic condensate and, at the same time, reducing in a conspicuous way the inertia of the system (e.g. the moment of inertia $\mathcal{J}$ of quadrupole rotational bands is much smaller than the rigid moment of inertia ($\mathcal{J} \approx \mathcal{J}_r/3$) expected from independent particle motion). Cooper pairs exist also in situation in which the environmental condition are above critical, e.g. in metals at room temperature or nuclei at high values of the angular momentum, although they break as soon as they are generated (pairing vibrations). While these pair addition and substraction fluctuations have little effect on condensed systems, they play an important role in mesoscopic systems, in particular in nuclei (box 3).

~~Coming back to below criticality values of the parameter determining cooper pair condensation in fermionic~~

Within the framework of the above picture, one can introduce at profit a collective coordinate α_0 (order parameter) which measures the number of Cooper pairs participating in the pairing condensate, and define a wavefunction for each pair $(U_\nu + V_\nu a^\dagger_\nu a^\dagger_{\tilde{\nu}})|0\rangle$ (independent pair motion, BCS approximation), adjusting the occupation parameters V_ν and U_ν (probability amplitudes that the two-fold (Kramer's-) degenerate pair state $(\nu, \tilde{\nu})$ is either occupied or empty), so as to minimize the energy of the system under the condition that the average number of nucleons is equal to N_0 (Coriolis force felt, in the intrinsic system, by the pairs, equal to $-\lambda N_0$). Thus, $|BCS\rangle = \Pi_{\nu>0}(U_\nu + V_\nu a^\dagger_\nu a^\dagger_{\tilde{\nu}})|0\rangle$ provides a valid description of the paired mean field ground state, and of the associated order parameter $\alpha_0 = \langle BCS|P^\dagger|BCS\rangle$, $P^\dagger = \Sigma_{\nu>0} a^\dagger_\nu a^\dagger_{\tilde{\nu}}$ being the pair creation operator (box 2).

It is then natural to posit that two-nucleon transfer reactions are specific to probe pairing correlations in many-body fermionic systems. Examples are provided by the Josephson effect in e.g. metallic superconductors, and (t,p) and (p,t) reactions in atomic nuclei.

Because away from the Fermi energy pair independent motion becomes independent particle motion, in particular in the nuclear case $|BCS\rangle \rightarrow |Nilsson\rangle$, one-particle transfer reactions like e.g. (d,p) and (p,d) can be used together with (t,p) and (p,t) processes, as a valid tool to cross check pair correlation predictions. In particular, to shed light on the origin of pairing in nuclei: in a nutshell, the relative importance of the bare NN-interaction and of the induced pairing interaction (box 4) ④

While the calculation of two-nucleon transfer spectroscopic amplitudes and differential cross sections are, a priori, more involved to be worked out than those associated with one-nucleon transfer reactions, the former are, as a rule, more intrinsically accurate than the later ones. This is because in the first case, the actual value of the variety of $\sqrt{j+1/2}\,U_\nu V_\nu$ quantities reflect coherence, and ~~thus~~ the averaging over many contributions which, inspite of the fact that each of them may be somewhat inaccurate, their overall sum leads to α_0 ($d\sigma(2n\text{-transfer})/d\Omega \sim |\alpha_0|^2$). On the other hand, $d\sigma(1n\text{-transfer})/d\Omega \sim U_\nu^4$ or $\sim V_\nu^2$, thus depending on the accuracy with which one is able to calculate the occupancy of a pure configuration (box 4).

reliable

inspite

(25/12/13) (5)

The above parlance is reflected in the calculation of the elements resulting from the encounter of structure and reaction, namely one- and two-nucleon modified transfer formfactors. While it is usually considered that these quantities carry all the structure information associated with the calculation of the corresponding cross sections, a consistent NFT calculation of structure and reaction will posit that equally much is contained in the distorted waves describing the relative motion of the colliding systems. This is because the optical potential (U+iW) which determines these scattering waves, emerges from the same modified formfactors, eventually including also inelastic processes (see comment on the optical potential Two-nucleon transfer in a nutshell p. 13). In other words, setting detectors in e.g. a definite two-particle transfer channel like $A+t \rightarrow B(=A+2)+p$, one needs to know what the single-particle states and collective modes of the systems $F(=A+1)$ and A and B are respectively, as well as their interweaving leading to dressed particle states (quasiparticles; fermions) and renormalized normal modes of excitation (bosons) are. But these are essentially all the elements needed to calculate the processes leading to the depopulation of the flux of the incoming channel ($A+t$, in the case under discussion). In particular, and assuming to work with spherical nuclei, so as to avoid strong inelastic processes, one-particle transfer is, as a rule (no particular Q-value closed channels) the main depopulation process, in keeping with the long range tail of the associated formfactor as compared to that of other processes, In keeping with this fact, and because U and W are connected by the Kramers-Krönig generalized dispersion relation (fluctuation-dissipation theorem), it is possible to calculate the nuclear dielectric function (optical potential) needed to describe the $A+t \to B+p$ process in question.

Concerning the modified formfactor associated with this process, we shall see in the next Chapter that it can be written as

$$\mathcal{U}_{LSJ}^{J_i J_f}(R) = \sum_{\substack{n_1, l_1, j_1 \\ n_2, l_2, j_2, n}} B(n_1 l_1 j_1, n_2 l_2 j_2; J J_i J_f)$$

$$\langle SLJ | j_1 j_2 J \rangle \langle n0, NL, L | n_1 l_1, n_2 l_2; L \rangle$$

$$\Omega_n \, R_{NL}(R),$$

where the overlaps

$$B(n_1 l_1 j_1, n_2 l_2 j_2; J J_i J_f)$$

$$= \langle \Psi^{J_f}(\xi_{A+2}) | [\Phi^J(n_1 l_1 j_1, n_2 l_2 j_2), \Psi^{J_i}(\xi_A)]^{J_f} \rangle$$

and

$$\Omega_n = \langle \phi_{n l m_l}(\vec{r}) | \phi_{000}(\vec{r}) \rangle$$

encodes for the physics of particle-particle (but also, to a large extent, particle-hole) correlations in nuclei, $\langle SLJ | j_1 j_2 J \rangle$ and $\langle n0, NL, L | n_1 l_1, n_2 l_2; L \rangle$ being LS-jj and Moshinsky transformation brackets, keeping track of symmetry and number of degrees conservation. In fact, the two-nucleon spectroscopic amplitude (B-coefficient) and the overlap Ω_n reflect the parentage in which the nucleus B can be written in terms of the system A and a Cooper pair,

$$\Psi_{exit} = \Psi^{J_f}_{M_f}(\xi_{A+2})\,\chi^{s_f}_{M_{s_f}}(\sigma_p),$$

where

$$\Psi^{J_f}_{M_f}(\xi_{A+2}) = \sum_{\substack{n_1 l_1 j_1,\\ n_2 l_2 j_2\\ J,\, J_i'}} B(n_1 l_1 j_1, n_2 l_2 j_2; J\, J_i'\, J_f)\left[\Phi^{J}(n_1 l_1 j_1, n_2 l_2 j_2)\,\Psi^{J_i'}(\xi_A)\right]^{J_f}_{M_f}$$

and

$$\Psi_{entrance} = \Psi^{J_i}_{M_i}(\xi_A)\,\Phi_t(\vec{r}_{n1}, \vec{r}_{n2}, r_p; \sigma_{n1}, \sigma_{n2}, \sigma_p)$$

with

$$\Phi_t = \left[\chi^{s}(\sigma_{n1}, \sigma_{n2})\,\chi^{s_f'}(\sigma_p)\right]^{s_i}_{M_{s_i}}\,\Phi_t^{L=0}\Big(\sum_{i>j}|\vec{r}_i - \vec{r}_j|\Big)$$

Assuming for simplicity a symmetric di-neutron radial wavefunction of the triton (i.e. neglecting the d-component of the corresponding wavefunction), for the relative and center of mass wavefunctions $\phi_{nlm}(\vec{r})$ and $\Phi_{N\Lambda M}(R)$ $(n=l=m=0,\ N=\Lambda=M=0)$, leads to Ω_n, a quantity which reflects both the non-orthogonality existing between the di-neutron wavefunctions into the final nucleus (Cooper pair) and in the triton. Another way to say the same thing, is that dineutron correlations in these two systems are different, a fact which underscores the limitations of light ion reactions to probe specifically pairing correlations in nuclei.

One can then conclude that, provided one makes use of a (sensible) complete single-particle basis (eventually including also the continuum), one can capture through $u^{J_iJ_f}_{LSJ}(R)$ most of the coherence of Cooper pair transfer, in keeping with the fact that major aspects of the associated di-neutron non-locality are taken care of by the n-summation weighted by the non-orthogonal overlaps Ω_n. This is in keeping with the ~~fact~~ ~~that~~, making use of more refined triton wavefunctions than employed above, the n-p (deuteron-like) correlations of this particle can be described with reasonable accuracy and thus the emergence of successive transfer. On the other hand, being the deuteron a bound system, this effective treatment of the associated resonances is not particular economic. Furthermore, zero-range approximation ($V(\rho)\phi_{000}(\rho)=D_0\,\delta(\vec{\rho})$) blocks such a posibility.

Nonetheless, the fact ~~that~~ one can still work out a detailed and consistent picture of two-nucleon transfer reactions in nuclei in terms of absolute cross sections with the help of a single parameter ($D_0^2 \approx (31.6 \pm 9.3)\,10^4$ MeV^2 fm^2) testifies to the fact ~~that~~ the above picture of Cooper pair transfer is a powerful picture, as it contains a large fraction of the physics which is at the basis of Cooper pair transfer in nuclei (Broglia et al 1973; Ch. 2). This is the reason why, treating explicitely the intermediate deuteron channel in terms of successive transfer, correcting both this and the simultaneous transfer channel for non-orthogonality contributions, makes the above picture the quantitative probe of Cooper pair correlations in nuclei (Potel et al, 20013; Ch. 4 and 5), as testified by Fig. all ang distrs. and Table all abs. cross sections

Within this context, we provide below two

footnote

~~and which~~ (*) Within the framework of (two-nucleon transfer) an almost charicature-like simplification, implies ~~that~~ one knows how to calculate the absolute value of the modified form-factor at around the nuclear radius $u^{J_iJ_f}_{LSJ}$ $(u^{J_iJ_f}_{LSJ}(R_0 \pm a/2))$.

Potel, G, .. Review paper 2013

Broglia et al, 1973; Broglia, R.A., Hansen, O. and Riedel, C. (1973) Two-neutron transfer and the pairing model, Adv. in Nucl. Phys. - - - -

Examples of B-coefficients. One for the case in which A and B(=A+2) are members of a pairing rotational, a second one, in the case in which they are members of a pairing vibrational band:

$$B(nlj,nlj;0,0,0) = \langle BCS(N+2) | [a^{\dagger}_{nlj} a^{\dagger}_{nlj}]^{0}_{0} | BCS(N) \rangle$$
$$= \sqrt{j+1/2}\; U_{nlj}(N)\, V_{nlj}(N+2)$$

and

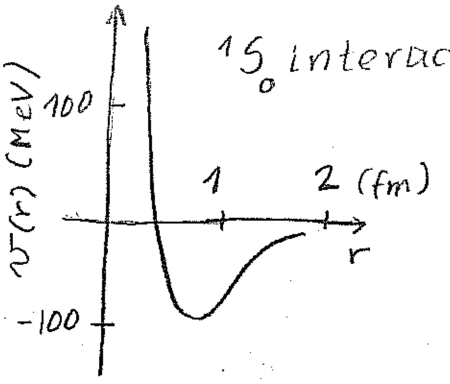
$$B(nlj,nlj;0,0,0) = \langle N_0+2(gs) | [a^{\dagger}_{nlj} a^{\dagger}_{nlj}]^{0}_{0} | N_0(gs) \rangle$$
$$= \begin{cases} \sqrt{j+1/2}\; X_a(n_k l_k j_k) & (\varepsilon_{j_k} > \varepsilon_F) \\ \sqrt{j+1/2}\; Y_a(n_i l_i j_i) & (\varepsilon_{j_i} \leq \varepsilon_F) \end{cases}$$

For actual numerical values see box 3 and Tables B-coeffs.

$\boxed{\text{Quantality parameter}}$ ratio of quantal kinetic energy of localization and potential energy; $\boxed{\text{box 1}}$

App. 2.A

$$Q = \frac{\hbar^2}{M_n a^2}\frac{1}{|v_0|}; \quad M_n = 0.939\ \text{GeV}/c^2 \text{ (neutron mass)}$$



$a \approx 1$ fm (range)

$v_0 \approx -100$ MeV (depth)

$^1S_0$: interaction between two nucleons in states of time reversal with $S=L=0$, and thus in a singlet state.

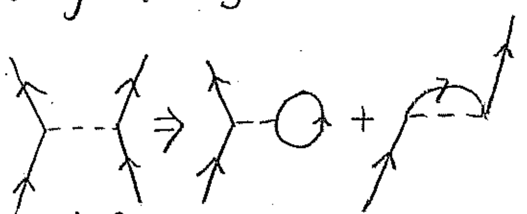
(QM: ZPF ($\Delta p_x \Delta x \gtrsim \hbar$) $\frac{\hbar^2}{Ma^2}$)

constituents	M/M_n	a (cm)	v_0 (eV)	Q	phase (T=0)	T_c
^3He	3	2.9(-8)	8.6(-4)	0.19	liquid a)	
^4He	4	2.9(-8)	8.6(-4)	0.14	liquid a)	
H_2	2	3.3(-8)	32(-4)	0.06	solid a)	
^{20}Ne	20	3.1(-8)	31(-4)	0.007	solid	
nucleons	1	9(-14)	100(+6)	0.5 c)	liquid a,b)	

a) liquid (condensed)
b) better, Non-Newtonian solid
c) Nucleus, paradigm of quantal many-body Fermi systems.

(B.R. Mottelson, Elementary features of nuclear structure, Les Houches, Session LXVI, Elsevier (1998))

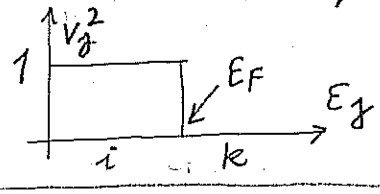
Fluctuations, quantal or classical, favor symmetry: gases and liquids are homogenous. Potential energy on the other hand prefers special arrangements: atoms like to be at specific distances from each other (spontaneous breaking of symmetry).



$$U(r) = \int d^3r' \rho(r') v(|r-r'|)$$

$$\rho(r) = \sum_i |\varphi_i(\vec r)|^2$$

$$U_x = -\sum_i \varphi_i^*(\vec r') v(|\vec r - \vec r'|) \varphi_i(\vec r)$$



For $Q \gtrsim 0.15$ independent-particle motion.

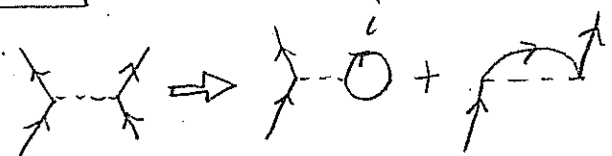
$\boxed{\text{Cooper pairs}}$ $\boxed{\cancel{\text{box2}}}$ app. 2.B

$\boxed{\text{HF}}$ $$H = \sum_{j_1 j_2} \langle j_1|T|j_2\rangle a^\dagger_{j_1} a_{j_2} + \frac{1}{4}\sum_{\substack{j_1 j_2 \\ j_3 j_4}} \langle j_1 j_2|v|j_3 j_4\rangle a^\dagger_{j_2} a^\dagger_{j_1} a_{j_3} a_{j_4}$$

$j(\equiv j, m)$

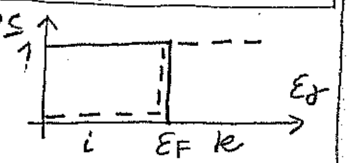
$\boxed{\text{independent particle motion } (Q=1/2)}$, $\boxed{\text{mean field}}$

$$a^\dagger_{j_2} a^\dagger_{j_1} a_{j_3} a_{j_4} \Longrightarrow a^\dagger_{j_2}\langle a^\dagger_{j_1} a_{j_3}\rangle a_{j_4} + \dots$$



$$\boxed{\begin{aligned} U(r) &= \int d^3r' \rho(r') v(|\vec r - \vec r'|) \\ U_x(r,r') &= -\sum_i \varphi^*_i(\vec r') v(|\vec r - \vec r'|)\varphi_i(\vec r) \\ \rho(r) &= \sum_i |\varphi_i(\vec r)|^2; \int d^3r \rho(r) = N \end{aligned}}$$

Hartree-Fock, complete separation between occupied($|i\rangle$) and empty($|k\rangle$) states



$(U_\nu^2 + V_\nu^2 = 1)$ $\varphi = \bar a^\dagger_\nu |0\rangle = (U_\nu + V_\nu a^\dagger_\nu)|0\rangle$; $V_\nu^2 = \begin{cases} 1 & \varepsilon_i \le \varepsilon_F \\ 0 & \varepsilon_i > \varepsilon_F \end{cases}$

$$|\text{Nilsson}(\Omega)\rangle_{\mathcal{K}} = \det(\varphi_\nu) = \prod \bar a^\dagger_\nu |0\rangle = \prod a^\dagger_i |0\rangle = \prod a^\dagger_i a^\dagger_{\tilde i} |0\rangle$$

$$|IKM\rangle \sim \int d\Omega\, \mathcal{D}^I_{MK}(\Omega)|\text{Nilsson}(\Omega)\rangle; \quad E_I = (\hbar^2/2\mathcal{J}) I(I+1); \quad \mathcal{J} = \mathcal{J}_{rig}$$

$\boxed{\text{independent pair motion}}$

constant m.els approx. $\langle j_1 j_2|v|j_3 j_4\rangle = -G$

$$\sum\langle a^\dagger_{j_2} a^\dagger_{j_1}\rangle a_{j_3} a_{j_4} + \sum a^\dagger_{j_2} a^\dagger_{j_1}\langle a_{j_3} a_{j_4}\rangle; \quad \varphi^{\text{COOPER}}_j = (U_j + V_j a^\dagger_{jm} a^\dagger_{j\tilde m})|0\rangle$$

$$|BCS\rangle = \prod_{jm>0}(U_j + V_j a^\dagger_{jm} a^\dagger_{j\tilde m})|0\rangle; \quad \alpha_0 = \langle BCS|\sum_{jm>0} a^\dagger_{jm} a^\dagger_{j\tilde m}|BCS\rangle$$

$$U_\nu = |U_\nu| = U'_\nu; \quad V_\nu = e^{-2i\phi} V'_\nu \; (V'_\nu \equiv |V_\nu|) \quad (\nu \equiv j, m)$$

$\mathcal{K}$: lab. system
$\mathcal{K}'$: intr. system

$$|BCS(\phi)\rangle_{\mathcal{K}} = \prod_{\nu>0}(U'_\nu + V'_\nu e^{-2i\phi} a^\dagger_\nu a^\dagger_{\tilde\nu})|0\rangle$$
$$= \prod_{\nu>0}(U'_\nu + V'_\nu a'^\dagger_\nu a'^\dagger_{\tilde\nu})|0\rangle = |BCS(\phi=0)\rangle_{\mathcal{K}'}$$

$$\alpha_0 = \alpha'_0 e^{-2i\phi}; \quad \alpha'_0 = \sum_{\nu>0} U'_\nu V'_\nu; \quad \left.\begin{matrix} V'_\nu \\ U'_\nu \end{matrix}\right\} = \frac{1}{\sqrt 2}\left(1 \mp \frac{1}{E_\nu}\right)^{1/2}$$

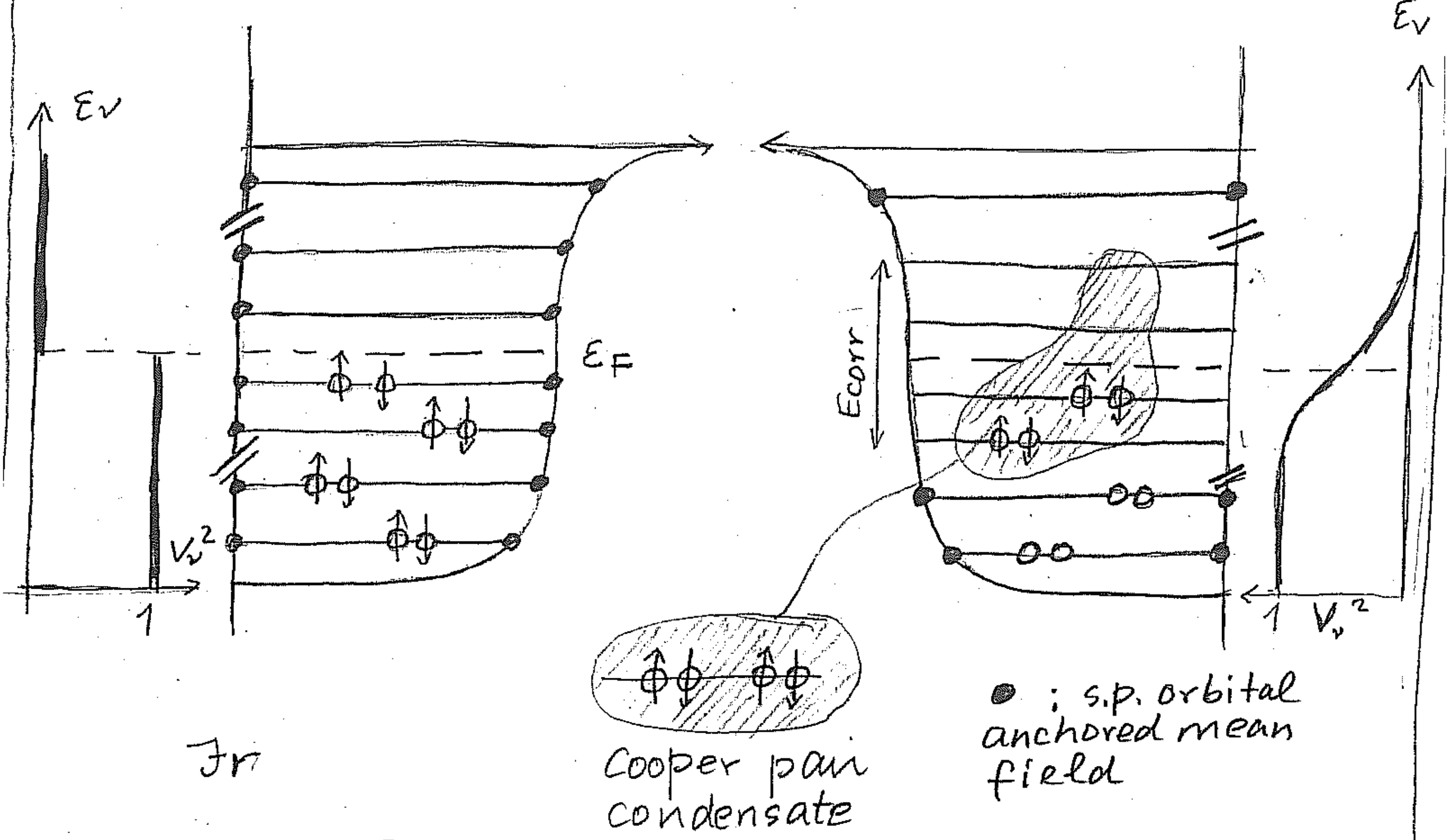
$$\Delta = G\alpha_0; \quad N_0 = 2\sum_{\nu>0} V_\nu^2; \quad \frac{1}{G} = \sum_{\nu>0}\frac{1}{2E_\nu}$$

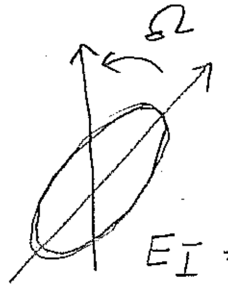
$E_\nu = ((\varepsilon_\nu - \lambda)^2 + \Delta^2)^{1/2}$

$$|N_0\rangle \sim \int_0^{2\pi} d\phi\, |BCS(\phi)\rangle_{\mathcal{K}} \sim \left(\sum_{\nu>0} c_\nu a^\dagger_\nu a^\dagger_{\tilde\nu}\right)^{N_0/2}|0\rangle; \quad E_N = (\hbar^2/2\mathcal{J}) N^2$$

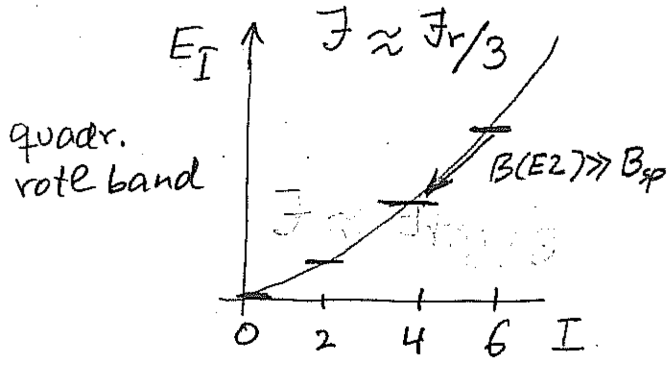
$\mathcal{J} \approx 2\hbar^2/G$

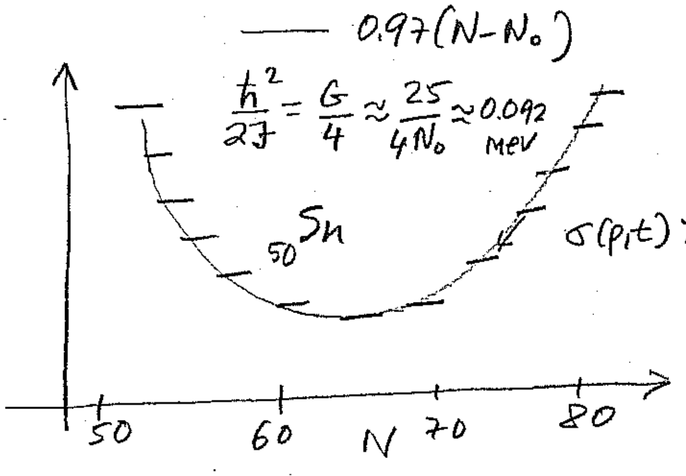
$\boxed{\text{box 2 contin.}}$





$$E_I = \frac{\hbar^2}{2\mathcal{J}} I(I+1)$$





paring rotl. band

pairing rotl. band

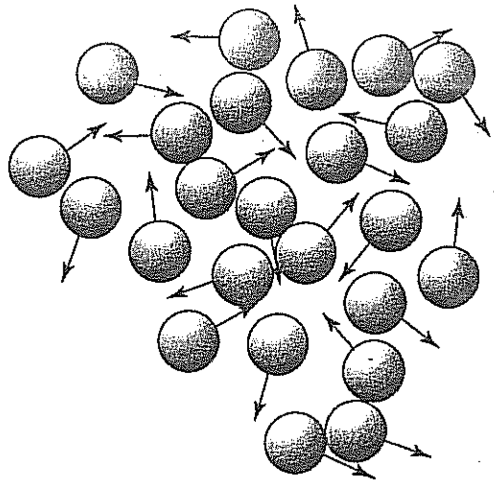


Figure 1.13. A system of independent Cooper pairs (Schafroth pairs). This situation corresponds to the incoherent solution of the many Cooper pair problem, the so called Fock state.

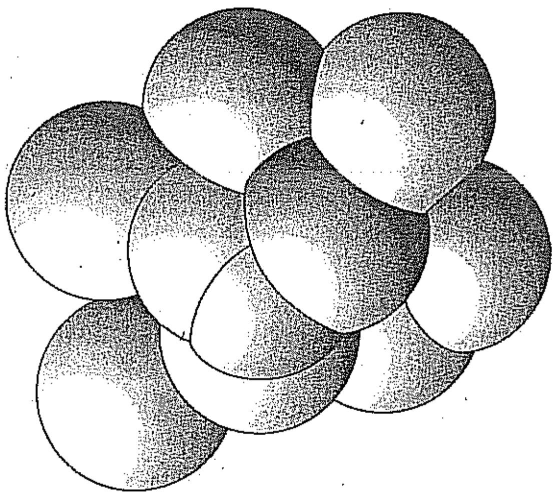


Figure 1.14. There are about 10^{18} Cooper pairs per cm^3 in a superconducting metal. A Cooper pair has a spatial extension of about 10^{-4} cm. Thus a given Cooper pair will overlap with 10^6 other Cooper pairs, leading to strong pair–pair correlation, as schematically shown. This solution corresponds to the coherent solution of the many Cooper pair problem (coherent state).