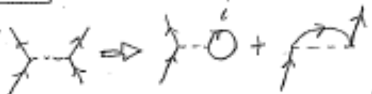
$\boxed{\text{Cooper pairs}}$ $\boxed{\text{box 2}}$ app. 2.B

$\boxed{\text{HF}}$ $$H=\sum_{j_1 j_2}\langle j_1|T|j_2\rangle a^{+}_{j_1}a_{j_2}+\frac{1}{4}\sum_{j_1 j_2 \ldots j_4}\langle j_1 j_2|v|j_3 j_4\rangle a^{+}_{j_2}a^{+}_{j_1}a_{j_3}a_{j_4}$$
$j(\equiv j,m)$

$\boxed{\text{independent particle motion } (Q=1/2)\text{, mean field}}$

$$a^{+}_{j_2}a^{+}_{j_1}a_{j_3}a_{j_4}\Rightarrow a^{+}_{j_2}\langle a^{+}_{j_1}a_{j_3}\rangle a_{j_4}+\ldots$$



$$U(r)=\int d^3r'\rho(r')v(|\vec r-\vec r'|)$$
$$U_x(r,r')=-\sum_i \varphi_i^*(\vec r')v(|\vec r-\vec r'|)\varphi_i(\vec r)$$
$$\rho(r)=\sum_i|\varphi_i(\vec r)|^2;\ \int d^3r\,\rho(r)=N$$

Hartree-Fock, complete separation between occupied ($|i\rangle$) and empty ($|k\rangle$) states

$(U_\nu^2+V_\nu^2=1)$ $\varphi=\bar a^{+}_{\nu}|0\rangle=(U_\nu+V_\nu a^{+}_{\nu})|0\rangle;\ V_\nu^2=\begin{cases}1 & \varepsilon_i\le\varepsilon_F\\ 0 & \varepsilon_i>\varepsilon_F\end{cases}$ (1; i, ε_F, k; ε_ν)

$$|\text{Nilsson}(\Omega)\rangle_{\mathcal K}=\det(\varphi_\nu)=\prod\bar a^{+}_{\nu}|0\rangle=\prod a^{+}_{i}|0\rangle=\prod a^{+}_{i}a^{+}_{\tilde i}|0\rangle$$

$$|IKM\rangle\sim\int d\Omega\,\mathcal D^{I}_{MK}(\Omega)|\text{Nilsson}(\Omega)\rangle;\ E_I=(\hbar^2/2\mathcal J)I(I+1);\ \mathcal J=\mathcal J_{rig}$$

$\boxed{\text{independent pair motion}}$

constant m.els approx. $\langle j_1 j_2|v|j_3 j_4\rangle=-G$

$$\sum\langle a^{+}_{j_2}a^{+}_{j_1}\rangle a_{j_3}a_{j_4}+\sum a^{+}_{j_2}a^{+}_{j_1}\langle a_{j_3}a_{j_4}\rangle;\ \varphi_j^{\text{COOPER}}=(U_j+V_j a^{+}_{jm}a^{+}_{j\tilde m})|0\rangle$$

$$|BCS\rangle=\prod_{jm>0}(U_j+V_j a^{+}_{jm}a^{+}_{j\tilde m})|0\rangle;\quad \alpha_0=\langle BCS|\sum_{jm>0}a^{+}_{jm}a^{+}_{j\tilde m}|BCS\rangle$$

$$U_\nu=|U_\nu|=U'_\nu;\quad V_\nu=e^{-2i\phi}V'_\nu\ (V'_\nu\equiv|V_\nu|)\quad(\nu\equiv j,m)$$

$$|BCS(\phi)\rangle_{\mathcal K}=\prod_{\nu>0}(U'_\nu+V'_\nu e^{-2i\phi}a^{+}_{\nu}a^{+}_{\tilde\nu})|0\rangle$$
$$=\prod_{\nu>0}(U'_\nu+V'_\nu a'^{+}_{\nu}a'^{+}_{\tilde\nu})|0\rangle=|BCS(\phi=0)\rangle_{\mathcal K'}$$

$\mathcal K$: lab. system
$\mathcal K'$: intr. system

$$\alpha_0=\alpha'_0 e^{-2i\phi};\quad \alpha'_0=\sum_{\nu>0}U'_\nu V'_\nu;\quad \left.\begin{matrix}V'_\nu\\U'_\nu\end{matrix}\right\}=\frac{1}{\sqrt2}\left(1\mp\frac{1}{E_\nu}\right)^{1/2}$$

$$\Delta=G\alpha_0;\quad N_0=2\sum_{\nu>0}V_\nu^2;\quad \frac{1}{G}=\sum_{\nu>0}\frac{1}{2E_\nu}\qquad E_\nu=((\varepsilon_\nu-\lambda)^2+\Delta^2)^{1/2}$$

$$|N_0\rangle\sim\int_0^{2\pi}d\phi\,|BCS(\phi)\rangle_{\mathcal K}\sim\Big(\sum_{\nu>0}c_\nu a^{+}_{\nu}a^{+}_{\tilde\nu}\Big)^{N_0/2}|0\rangle;\ E_N=(\hbar^2/2\mathcal J)N^2$$
$$\mathcal J\approx 2\hbar^2/G$$