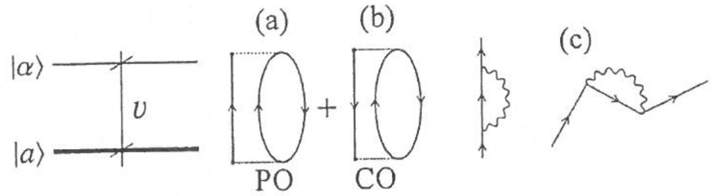

Figure 4.B.1: Two state schematic model describing the breaking of the strength of the pure single–particle state $|a\rangle$, through the coupling to collective vibrations (wavy line) associated with polarization (PO) and correlation (CO) processes. directly

the particle–vibration coupling parameter *) ~~cf. e.g.~~ Bohr, A. and Mottelson (1975); Brink, D. and Broglia (2005) and refs. therein). This is in keeping with the fact that the time the nucleon is coupled to the vibrations it cannot behave as a single–particle and can thus not contribute to e.g. the single–particle pickup cross section.

It is of notice that the selfconsistence requirements for the iterative solution of Eq. (4.A.1) (see Fig. 4.A.1 (d) and (d')) remind very much those associated with the solution of the Kohn–Sham equations in finite systems, S

$$H^{KS}\varphi_\gamma(\mathbf{r}) = \lambda_\gamma\varphi_\gamma(\mathbf{r}), \tag{4.A.6}$$

where

$$H^{KS} = -\frac{\hbar^2}{2m_e}\nabla^2 + U_H(\mathbf{r}) + V_{ext}(\mathbf{r}) + U_{xc}(\mathbf{r}), \tag{4.A.7}$$

H^{KS} being known as the Kohn–Sham Hamiltonian, $V_{ext}(\mathbf{r})$ being the field created by the ions and acting on the electrons. Both the Hartree and the exchange–correlation potentials $U_H(\mathbf{r})$ and $U_{xc}(\mathbf{r})$ depend on the (local) density, hence on the whole set of wavefunctions $\varphi_\gamma(\mathbf{r})$. Thus, the set of KS–equations must be solved selfconsistently **) ~~e.g.~~ (Broglia et al., 2004) and refs. therein).

4.A.1 Density of levels

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Appendix 4.B Model for single–particle strength function: Dyson equation

In the previous Appendix we schematically introduced arguments regarding the "impossibility" of defining a "bona fide" single–particle spectroscopic factor. It was done with the help of Feynman (NFT) diagrams. In what follows we essentially repeat the arguments, but this time in terms of Dyson's (Schwinger) language. For simplicity, we consider a two–level model where the pure single–particle state $|a\rangle$ couples to a more complicated (doorway) state $|\alpha\rangle$, made out of a fermion (particle or hole), coupled to a particle–hole excitation which, if iterated to all orders can give rise to a collective state (cf. Fig.4.B.1). The Hamiltonian describing the system is ***) (Bohr and Mottelson, 1969), l

$$H = H_0 + v, \tag{4.B.1}$$

*)

**) See e.g.

***)

4.A.1 Density of levels

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Making use of Eq.(4.A.2) with $\epsilon = \hbar\omega$, one can calculate $d\epsilon/dk$ (for a single nucleon and one spin orientation (Mahaux et al (198), p. 17). The inverse of this expression is (1)

$$\frac{dk}{d\epsilon} = \frac{m^*}{\hbar^2 k}, \quad (4.A.8)$$

which testifies to the fact that the energy spacing between levels, i.e. the density of levels (see below), changes as m^* does.

One can then calculate the average value over the Fermi distribution, obtaining

$$\left\langle \frac{dk}{d\epsilon} \right\rangle = \frac{m^*}{\hbar^2\, 2/3\, k_F} \,. \quad (4.A.9)$$

Let us now take into account all nucleons, both spin orientations and eliminate the unit length, i.e

$$\frac{2A}{k_F}\left\langle \frac{dk}{d} \right\rangle = 3A\,\frac{m^*}{\hbar^2 k_F^2} = 3A\,\frac{m^*}{2m\,\varepsilon_F} \quad (4.A.10)$$

Assuming $m^* = m_\omega m_k/m$ where m is the bare mass one obtains

$$\frac{3}{2}\,\frac{A}{\varepsilon_F}, \quad (4.A.11)$$

a value which coincides with the Fermi model estimate for g_0 (see e.g. Bohr+Mottelson (1969) Eq.(2-48)). Taking properly into account the geometry of the system, one obtains

$$a = \frac{\pi^2}{6}\,\frac{3}{2}\,\frac{A}{\varepsilon_F} \quad (4.A.12)$$

for the prefactor in the exponential of the Fermi expression of the total density of single particle levels.

Making use of $\varepsilon_F = 36$ MeV leads to,

$$a \approx \frac{A}{14}\,\text{MeV}^{-1}. \quad (4.A.13)$$

In keeping with the fact that one can interpret $d\epsilon/dk$ as the rate of change in energy when the momentum changes or, equivalently, when the number of nodes per unit length changes, and this can be used to label the single-particle states, (4.A.13) can be confronted at profit with the average degeneracy per unit energy of valence orbitals (see Table 4.A.1).

Within this context, it is of notice that an estimate of the quantity a based on the harmonic oscillator, leads to $a \approx \frac{\pi^2}{6}\frac{(N_{max}+3/2)^2}{\hbar\omega_0}$ i.e. an expression inversely proportional to the (constant) energy separation of levels (see Bohr and Mottelson (1969) p. 188, Eq. (2-125a))

Within this context, and in an attempt to bridge the gap between the nuclear matter expressions discussed above and finite nuclei, i.e. potential wells of finite range we consider

$$H = \frac{p^2}{2D} + \frac{C}{2}x^2, \qquad (4.A.13)$$

($p = D\dot{x}$) which leads to a constant level spacing,

$$\hbar\omega_0 = \hbar\sqrt{\frac{C}{D}}, \qquad (4.A.14)$$

and implies that the density of states is proportional to the square root of the particle inertia (mass). However, this result follows the assumption ~~that~~ the potential remains unchanged if the bare mass (in which case D is, for example, set equal to the HF- k-mass, m_k) is replaced by an effective mass m^* (e.g. $m_k m_\omega/m$, Eq. (4.A.3)). In this case the ground state wavefunction

$$\varphi_0 \sim \exp\left(-\frac{x^2}{2b^2}\right), \qquad (4.A.15)$$

with

$$b = \sqrt{\frac{\hbar}{m^*\omega_0}}, \qquad (4.A.16)$$

for a dressed nucleon of effective mass $m^* > D$ will shrink in space compared with the one of mass D and consequently the mean square radius of the system

$$\langle r^2 \rangle = \frac{\hbar}{m^* \omega_0}\left(N+\frac{3}{2}\right) = b^2\left(N+\frac{3}{2}\right)$$

(4.A.17)

will decrease. This is not correct, and one has to impose the condition b^2 = constant. A condition which implies that the energy difference between levels is inversely propor-tional to the effective mass of the nucleon, that is,

$$\hbar\omega_0 = \frac{\hbar^2}{m^* b^2}. \qquad (4.A.18)$$

Let us conclude this Appendix with a remark concerning the dimensions of the parameters D and C entering Eq. (4.A.13). Because the variable x has dimensions of length ($[x]$ = fm), the dimensions of the inertia and of the restoring force parameters are

$[C]$ = MeV fm^{-2} and $[D]$ = MeV×fm^{-2}×s^2.

Consequently the associated zero point fluctuations

$$\sqrt{\frac{\hbar\omega_0}{2C}} = \sqrt{\frac{\hbar^2}{2D}\frac{1}{\hbar\omega_0}} \qquad (4.A.18)$$

have dimensions of fm. It is of notice that in the case of the harmonic oscillator Hamilto-nian (1.B.10), the associated ZPF is is a c-number, in keeping with the fact that the (dynamic (1.B.18), static (1.B.26) deformation parameter in the variety of notations (α^L_M, β_L, b_L) is a dimensionless (collective) variable (CV).

	MeV^{-1}	
	empirical	a
${}^{208}_{82}Pb_{126}$	17 (10[a] + 7[b])	15 (9[a] + 6[b])
${}^{120}_{50}Sn_{70}$	4 [a]	5 [a]

a) neutrons
b) protons

Table 4.A.1

Comparison of the factor (4.A.13) ($a = n/14\ MeV^{-1}$, $n = A$) corresponding to ${}^{208}Pb$ for both $n = N$ and $N = Z$ and for ${}^{120}Sn$ for $n = N$, in comparison with the empirical value associated with the valence orbitals of these nuclei. That is ($h_{9/2}, f_{7/2}, i_{13/2}, p_{3/2}, f_{5/2}, p_{1/2}; g_{9/2}, i_{11/2}, d_{5/2}, j_{15/2}, s_{1/2}, g_{7/2}, d_{3/2}$) and ($g_{7/2}, d_{5/2}, h_{11/2}, d_{3/2}, s_{1/2}; h_{9/2}, f_{7/2}, i_{13/2}$) for neutrons and protons in ${}^{208}Pb$, leading to $\sum_j^N (2j+1)/\Delta E_N = 102/10\,MeV \approx 10\ MeV^{-1}$ and $\sum_j^Z (2j+1)/\Delta E_Z = 64/9\,MeV \approx 7\ MeV^{-1}$, where ΔE_i is the experimental energy interval over which the valence orbitals are distributed (see e.g. Bohr and Mottelson (1969) p. 325, Fig. 3-3). In the case of the neutrons of ${}^{120}Sn$, use is made of the dressed valence orbitals $d_{5/2}, g_{7/2}, s_{1/2}, d_{3/2}, h_{11/2}$ resulting from the renormalization

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of HF-SLy4 levels through the coupling of collective modes making use of nuclear field theory plus Nambu-Gorkov techniques ((NFT)+(NG); for details see Idini et al (2016) and Table I of Potel et al (2017)). The result, taking into account the breaking of the single-particle strength, in particular that of the $d_{5/2}$ orbital is

$$\sum_j^N (2j+1)/\Delta E_N \approx 32/8\,\mathrm{MeV} \approx 4\,\mathrm{MeV}^{-1}.$$

References

Bohr and Mottelson (1969), A. Bohr and B.R. Mottelson, Nuclear Structure, Vol I (1969).

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