previous T^{VV}_{2NT}

corresponding to $J = 0$ survives, which is indeed independent of M. We can thus omit the sum over M in (7.2.120) and multiply by $(2K + 1)$, obtaining

$T^{(2)}_{succ}$

$$
\begin{aligned}
T^{2step}_{2NT} = {} & \frac{64\mu_{Cc}(\pi)^{3/2} i}{\hbar^2 k_{Aa} k_{Bb} k_{Cc}} \frac{i^{-l}}{\sqrt{(2j_i+1)(2j_f+1)}} \\
& \times \sum_K (2K+1)((l_f\tfrac{1}{2})_{j_f}(l_i\tfrac{1}{2})_{j_i}|(l_f l_i)_K(\tfrac{1}{2}\tfrac{1}{2})_0)^2_K \\
& \times \sum_{l_c,l} \frac{e^{i(\sigma_i^l+\sigma_f^l)}}{\sqrt{(2l+1)}} Y^l_0(\hat{k}_{Bb}) S_{K,l,l_c},
\end{aligned}
\tag{7.2.128}
$$

where

$$
\begin{aligned}
S_{K,l,l_c} = {} & \int d^3 r_{Cc} d^3 r_{b1} v(r_{b1}) u_{l_f}(r_{A1}) u_{l_i}(r_{b1}) \frac{s_{K,l,l_c}(r_{Cc})}{r_{Cc}} \frac{F_l(r_{Bb})}{r_{Bb}} \\
& \times \left[Y^{l_f}(\hat{r}_{A1}) Y^{l_i}(\hat{r}_{b1})\right]^K_M \left[Y^{l_c}(\hat{r}_{Cc}) Y^l(\hat{r}_{Bb})\right]^{K*}_M ,
\end{aligned}
\tag{7.2.129}
$$

and

$$
\begin{aligned}
s_{K,l,l_c}(r_{Cc}) = {} & \int_{r_{Cc} fixed} d^3 r'_{Cc} d^3 r'_{A2} v(r'_{c2}) u_{l_f}(r'_{A2}) u_{l_i}(r'_{b2}) \frac{F_l(r'_{Aa})}{r'_{Aa}} \frac{f_{l_c}(k_{Cc}, r_<) P_{l_c}(k_{Cc}, r_>)}{r'_{Cc}} \\
& \times \left[Y^{l_f}(\hat{r}'_{A2}) Y^{l_i}(\hat{r}'_{b2})\right]^{K*}_M \left[Y^{l_c}(\hat{r}'_{Cc}) Y^l(\hat{r}'_{Aa})\right]^K_M .
\end{aligned}
\tag{7.2.130}
$$

It can be shown that the integrand in (7.2.129) is independent of M. Consequently, one can sum over M and divide by $(2K + 1)$, to get

$$
\begin{aligned}
& \frac{1}{2K+1} v(r_{b1}) u_{l_f}(r_{A1}) u_{l_i}(r_{b1}) \frac{s_{K,l,l_c}(r_{Cc})}{r_{Cc}} \frac{F_l(r_{Bb})}{r_{Bb}} \\
& \qquad \times \sum_M \left[Y^{l_f}(\hat{r}_{A1}) Y^{l_i}(\hat{r}_{b1})\right]^K_M \left[Y^{l_c}(\hat{r}_{Cc}) Y^l(\hat{r}_{Bb})\right]^{K*}_M .
\end{aligned}
\tag{7.2.131}
$$

This integrand is rotationally invariant (it is proportional to a T^L_M spherical tensor with $L = 0$, $M = 0$), so one can evaluate it in the "standard" configuration in which $\mathbf{r}_{Cc}$ is directed along the z–axis and multiply by $8\pi^2$ (see Bayman and Chen (1982)), obtaining the final expression for S_{K,l,l_c}:

$$
\begin{aligned}
S_{K,l,l_c} = {} & \frac{4\pi^{3/2}\sqrt{2l_c+1}}{2K+1} i^{-l_c} \\
& \times \int r^2_{Cc} dr_{Cc}\, r^2_{b1} dr_{b1} \sin\theta\, d\theta\, v(r_{b1}) u_{l_f}(r_{A1}) u_{l_i}(r_{b1}) \\
& \times \frac{s_{K,l,l_c}(r_{Cc})}{r_{Cc}} \frac{F_l(r_{Bb})}{r_{Bb}} \\
& \times \sum_M \langle l_c\, 0\, l\, M | K\, M\rangle \left[Y^{l_f}(\hat{r}_{A1}) Y^{l_i}(\theta+\pi, 0)\right]^K_M Y^{l*}_M(\hat{r}_{Bb}).
\end{aligned}
\tag{7.2.132}
$$

Similarly, one has

$$
\begin{aligned}
s_{K,l,l_c}(r_{Cc}) =& \frac{4\pi^{3/2}\sqrt{2l_c+1}}{2K+1} i^{l_c} \\
&\times \int r'^2_{Cc}\, dr'_{Cc}\, r'^2_{A2}\, dr'_{A2}\, \sin\theta'\, d\theta'\, v(r'_{c2}) u_{l_f}(r'_{A2}) u_{l_i}(r'_{b2}) \\
&\times \frac{F_l(r'_{Aa})}{r'_{Aa}} \frac{f_{l_c}(k_{Cc}, r_<) P_{l_c}(k_{Cc}, r_>)}{r'_{Cc}} \\
&\times \sum_M \langle l_c\, 0\, l\, M | K\, M\rangle \left[Y^{l_f}(\hat{r}'_{A2}) Y^{l_i}(\hat{r}'_{b2})\right]^{K*}_M Y^l_M(\hat{r}'_{Aa}).
\end{aligned}
\tag{7.2.133}
$$

Introducing the further approximations $\mathbf{r}_{A1} \approx \mathbf{r}_{C1}$ and $\mathbf{r}_{b2} \approx \mathbf{r}_{c2}$, one obtains the final expression

$$
\begin{aligned}
T^{2step}_{2NT} =& \frac{1024\mu_{Cc}\pi^{9/2} i}{\hbar^2 k_{Aa} k_{Bb} k_{Cc}} \frac{1}{\sqrt{(2j_i+1)(2j_f+1)}} \\
&\times \sum_K \frac{1}{2K+1} ((l_f \tfrac{1}{2})_{j_f} (l_i \tfrac{1}{2})_{j_i} | (l_f l_i)_K (\tfrac{1}{2} \tfrac{1}{2})_0)^2_K \\
&\times \sum_{l_c, l} e^{i(\sigma^l_i + \sigma^l_f)} \frac{(2l_c+1)}{\sqrt{2l+1}} Y^l_0(\hat{k}_{Bb}) S_{K,l,l_c},
\end{aligned}
\tag{7.2.134}
$$

$T^{(2)}_{succ}$

?

with

$$
\begin{aligned}
S_{K,l,l_c} =& \int r^2_{Cc}\, dr_{Cc}\, r^2_{b1}\, dr_{b1}\, \sin\theta\, d\theta\, v(r_{b1}) u_{l_f}(r_{C1}) u_{l_i}(r_{b1}) \\
&\times \frac{s_{K,l,l_c}(r_{Cc})}{r_{Cc}} \frac{F_l(r_{Bb})}{r_{Bb}} \\
&\times \sum_M \langle l_c\, 0\, l\, M | K\, M\rangle \left[Y^{l_f}(\hat{r}_{C1}) Y^{l_i}(\theta+\pi, 0)\right]^K_M Y^{l*}_M(\hat{r}_{Bb}),
\end{aligned}
\tag{7.2.135}
$$

and

$$
\begin{aligned}
s_{K,l,l_c}(r_{Cc}) =& \int r'^2_{Cc}\, dr'_{Cc}\, r'^2_{A2}\, dr'_{A2}\, \sin\theta'\, d\theta'\, v(r'_{c2}) u_{l_f}(r'_{A2}) u_{l_i}(r'_{c2}) \\
&\times \frac{F_l(r'_{Aa})}{r'_{Aa}} \frac{f_{l_c}(k_{Cc}, r_<) P_{l_c}(k_{Cc}, r_>)}{r'_{Cc}} \\
&\times \sum_M \langle l_c\, 0\, l\, M | K\, M\rangle \left[Y^{l_f}(\hat{r}'_{A2}) Y^{l_i}(\hat{r}'_{c2})\right]^{K*}_M Y^l_M(\hat{r}'_{Aa}).
\end{aligned}
\tag{7.2.136}
$$

draw coordinates ?

7.2.7 Coordinates for the successive transfer

In the standard configuration in which the integrals (7.2.135) and (7.2.136) are to be evaluated, we have

$$
\mathbf{r}_{Cc} = r_{Cc}\,\hat{\mathbf{z}}, \qquad \mathbf{r}_{b1} = r_{b1}(-\cos\theta\,\hat{\mathbf{z}} - \sin\theta\,\hat{\mathbf{x}}). \tag{7.2.137}
$$

where e.g. in the case of (7.2.145), one has

$$f(M) = \langle l_c\ 0\ l\ M|K\ M\rangle \left[Y^{l_f}(\hat{r}_{C1})Y^{l_i}(\theta+\pi,0)\right]^K_M Y^{l_*}_M(\hat{r}_{Bb}). \tag{7.2.148}$$

Note that all the vectors that come into play in the above expressions are in the (x,z)–plane. Consequently, the azimuthal angle ϕ is always equal to zero. Under these circumstances and for time–reversed phases, $(Y^{L*}_M(\theta,0) = (-1)^L Y^L_M(\theta,0))$ one has

$$f(-M) = (-1)^{l_c+l_f+l_i+l} f(M). \tag{7.2.149}$$

Consequently,

$$\begin{aligned}\sum_M \langle l_c\ 0\ l\ M|K\ M\rangle f(M) &= \langle l_c\ 0\ l\ 0|K\ 0\rangle f(0)\\ &+ \sum_{M>0}\langle l_c\ 0\ l\ M|K\ M\rangle f(M)\left(1+(-1)^{l_c+l+l_i+l_f}\right).\end{aligned} \tag{7.2.150}$$

Consequently, in the case in which $l_c + l + l_i + l_f$ is odd, we have only to evaluate the $M = 0$ contribution. This consideration is useful to restrict the number of numerical operations needed to calculate the transition amplitude.

7.2.9 non–orthogonality term

We write the non–orthogonality contribution to the transition amplitude (see Bayman and Chen (1982)):

$T^{(2)}_{NO}$

$$\begin{aligned}T^{NO}_{2NT} = 2\sum_{\substack{\sigma_1\sigma_2\\ \sigma'_1\sigma'_2\\ KM}} \int & d^3r_{Cc}d^3r_{b1}d^3r_{A2}d^3r'_{b1}d^3r'_{A2}\,\chi^{(-)*}(\mathbf{k}_{Bb},\mathbf{r}_{Bb})\\ &\times\left[\psi^{j_f}(\mathbf{r}_{A1},\sigma_1)\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\right]^{0*}_0 v(r_{b1})\left[\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\psi^{j_i}(\mathbf{r}_{b1},\sigma_1)\right]^K_M\\ &\times\left[\psi^{j_f}(\mathbf{r}'_{A2},\sigma'_2)\psi^{j_i}(\mathbf{r}'_{b1},\sigma'_1)\right]^{K*}_M\left[\psi^{j_i}(\mathbf{r}'_{b1},\sigma'_1)\psi^{j_i}(\mathbf{r}'_{b2},\sigma'_2)\right]^0_0\chi^{(+)}(\mathbf{r}'_{Aa}).\end{aligned} \tag{7.2.151}$$

This expression is equivalent to (7.2.110) if we make the replacement

$$\frac{2\mu_{Cc}}{\hbar^2}G(\mathbf{r}_{Cc},\mathbf{r}'_{Cc})v(r'_{A2}) \to \delta(\mathbf{r}_{Cc}-\mathbf{r}'_{Cc}). \tag{7.2.152}$$

Looking at the partial–wave expansions of $G(\mathbf{r}_{Cc},\mathbf{r}'_{Cc})$ and $\delta(\mathbf{r}_{Cc}-\mathbf{r}'_{Cc})$ (see Section ??), we find that we can use the above expressions for the successive transfer with the replacement

$$i\frac{2\mu_{Cc}}{\hbar^2}\frac{f_{l_c}(k_{Cc},r_<)P_{l_c}(k_{Cc},r_>)}{k_{Cc}} \to \delta(r_{Cc}-r'_{Cc}). \tag{7.2.153}$$

as an approximation for the incoming state. The first term of (7.2.158) gives rise to the simultaneous amplitude, while from second one leads to both the successive and the non-orthogonality contributions. To extract the amplitude $\mathcal{U}_{K,M}(\mathbf{r}_{cC})$, we define $f_{KM}(\mathbf{r}_{cC})$ as the scalar product

$$f_{KM}(\mathbf{r}_{cC}) = \left\langle \left[\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\psi^{j_{i1}}(\mathbf{r}_{b1},\sigma_1)\right]^K_M \middle| \Psi^{(+)}(\mathbf{r}_{aA},\mathbf{r}_{b1},\mathbf{r}_{b2},\sigma_1,\sigma_2)\right\rangle \quad (7.2.159)$$

for fixed $\mathbf{r}_{cC}$, which can be seen to obey the equation

$$\left(\frac{\hbar^2}{2\mu_{cC}}k^2_{cC} + \frac{\hbar^2}{2\mu_{cC}}\nabla^2_{r_{cC}} - U(r_{cC})\right) f_{KM}(\mathbf{r}_{cC}) = \left\langle \left[\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\psi^{j_{i1}}(\mathbf{r}_{b1},\sigma_1)\right]^K_M \middle| v(r_{c2}) \middle| \Psi^{(+)}(\mathbf{r}_{aA},\mathbf{r}_{b1},\mathbf{r}_{b2},\sigma_1,\sigma_2)\right\rangle. \quad (7.2.160)$$

The solution can be written in terms of the Green function $G(\mathbf{r}_{cC},\mathbf{r}'_{cC})$ defined by

$$\left(\frac{\hbar^2}{2\mu_{cC}}k^2_{cC} + \frac{\hbar^2}{2\mu_{cC}}\nabla^2_{r_{cC}} - U(r_{cC})\right) G(\mathbf{r}_{cC},\mathbf{r}'_{cC}) = \frac{\hbar^2}{2\mu_{cC}}\delta(\mathbf{r}_{cC}-\mathbf{r}'_{cC}). \quad (7.2.161)$$

Thus,

$$\begin{aligned} f_{KM}(\mathbf{r}_{cC}) &= \frac{2\mu_{cC}}{\hbar^2}\int d\mathbf{r}'_{cC} G(\mathbf{r}_{cC},\mathbf{r}'_{cC}) \left\langle \left[\psi^{j_f}(\mathbf{r}'_{A2},\sigma'_2)\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\right]^K_M \middle| v(r_{C2}) \middle| \Psi^{(+)}(\mathbf{r}'_{aA},\mathbf{r}'_{b1},\mathbf{r}'_{b2},\sigma'_1,\sigma'_2)\right\rangle \\ &\approx \frac{2\mu_{cC}}{\hbar^2}\sum_{\sigma'_1\sigma'_2}\int d\mathbf{r}'_{cC}d\mathbf{r}'_{A2}d\mathbf{r}'_{b1} G(\mathbf{r}_{cC},\mathbf{r}'_{cC}) \left[\psi^{j_f}(\mathbf{r}'_{A2},\sigma'_2)\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\right]^{K*}_M \\ &\quad \times v(r'_{c2})\chi^{(+)}(\mathbf{r}'_{aA})\left[\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\psi^{j_{i2}}(\mathbf{r}'_{b2},\sigma'_2)\right]^{\Lambda}_{\mu} = \mathcal{U}_{K,M}(\mathbf{r}_{cC}) \\ &\quad + \left\langle \left[\psi^{j_f}(\mathbf{r}'_{A2},\sigma_2)\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma_1)\right]^K_M \middle| \chi^{(+)}(\mathbf{r}'_{aA})\left[\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\psi^{j_{i2}}(\mathbf{r}'_{b2},\sigma'_2)\right]^{\Lambda}_{\mu}\right\rangle. \end{aligned} \quad (7.2.162)$$

Therefore

$$\begin{aligned} \mathcal{U}_{K,M}(\mathbf{r}_{cC}) &= \frac{2\mu_{cC}}{\hbar^2}\sum_{\sigma'_1\sigma'_2}\int d\mathbf{r}'_{cC}d\mathbf{r}'_{A2}d\mathbf{r}'_{b1} G(\mathbf{r}_{cC},\mathbf{r}'_{cC}) \left[\psi^{j_f}(\mathbf{r}'_{A2},\sigma'_2)\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\right]^{K*}_M \\ &\quad \times v(r'_{c2})\chi^{(+)}(\mathbf{r}'_{aA})\left[\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\psi^{j_{i2}}(\mathbf{r}'_{b2},\sigma'_2)\right]^{\Lambda}_{\mu} \\ &\quad - \left\langle \left[\psi^{j_f}(\mathbf{r}'_{A2},\sigma_2)\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma_1)\right]^K_M \middle| \chi^{(+)}(\mathbf{r}'_{aA})\left[\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\psi^{j_{i2}}(\mathbf{r}'_{b2},\sigma'_2)\right]^{\Lambda}_{\mu}\right\rangle. \end{aligned} \quad (7.2.163)$$

When we substitute $\mathcal{U}_{K,M}(\mathbf{r}_{cC})$ into (7.2.158) and (7.2.157), the first term gives rise to the successive amplitude for the two–particle transfer, while the second term is responsible for the non–orthogonal contribution.

$\left(T^{(2)}_{succ}\right)_\mu$ or $T^{(2)}_{succ}(\mu)$ likely better

7.2.11 Successive transfer contribution

We need to evaluate the integral

$$T_\mu^{succ} = \frac{4\mu_{cC}}{\hbar^2}\sum_{\sigma_1\sigma_2}\sum_{KM}\int d\mathbf{r}_{cC}d\mathbf{r}_{A2}d\mathbf{r}_{b1}d\mathbf{r}'_{cC}d\mathbf{r}'_{A2}d\mathbf{r}'_{b1}\left[\psi^{j_f}(\mathbf{r}_{A1},\sigma_1)\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\right]_0^{0*}$$
$$\times\chi^{(-)*}(\mathbf{r}_{bB})G(\mathbf{r}_{cC},\mathbf{r}'_{cC})\left[\psi^{j_f}(\mathbf{r}'_{A2},\sigma'_2)\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\right]_M^{K*}\chi^{(+)}(\mathbf{r}'_{aA})v(r'_{c2})v(r_{b1})$$
$$\times\left[\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\psi^{j_{i2}}(\mathbf{r}'_{b2},\sigma'_2)\right]_\mu^\Lambda\left[\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\psi^{j_{i1}}(\mathbf{r}_{b1},\sigma_1)\right]_M^K, \tag{7.2.164}$$

where we must substitute the Green function and the distorted waves by their partial wave expansions (see App. 7.K and App. 7.L). The integral over $\mathbf{r}'_{b1}$ is:

$$\sum_{\sigma'_1}\int d\mathbf{r}'_{b1}\left[\psi^{j_f}(\mathbf{r}'_{A2},\sigma'_2)\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\right]_M^{K*}\left[\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\psi^{j_{i2}}(\mathbf{r}'_{b2},\sigma'_2)\right]_\mu^\Lambda$$
$$=\sum_{\sigma'_1}\int d\mathbf{r}'_{b1}(-1)^{-M+j_f+j_{i1}-\sigma_1-\sigma_2}\left[\psi^{j_{i1}}(\mathbf{r}'_{b1},-\sigma'_1)\psi^{j_f}(\mathbf{r}'_{A2},-\sigma'_2)\right]_{-M}^K\left[\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\psi^{j_{i2}}(\mathbf{r}'_{b2},\sigma'_2)\right]_\mu^\Lambda$$
$$=\sum_{\sigma'_1}\int d\mathbf{r}'_{b1}(-1)^{-M+j_f+j_{i1}-\sigma_1-\sigma_2}\sum_P\langle K\ \Lambda\ -M\ \mu|P\ \mu-M\rangle((j_{i1}j_f)_K(j_{i1}j_{i2})_\Lambda|(j_{i1}j_{i1})_0(j_fj_{i2})_P)_P$$
$$\times\left[\psi^{j_{i1}}(\mathbf{r}'_{b1},-\sigma'_1)\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\right]_0^0\left[\psi^{j_f}(\mathbf{r}'_{A2},-\sigma'_2)\psi^{j_{i2}}(\mathbf{r}'_{b2},\sigma'_2)\right]_{\mu-M}^P$$
$$=(-1)^{-M+j_f+j_{i1}}\sqrt{2j_{i1}+1}\,u_{l_f}(r_{A2})u_{l_{i2}}(r'_{b2})\sum_P\langle K\ \Lambda\ -M\ \mu|P\ \mu-M\rangle$$
$$\times((j_{i1}j_f)_K(j_{i1}j_{i2})_\Lambda|(j_{i1}j_{i1})_0(j_fj_{i2})_P)_P((l_f\tfrac{1}{2})_{j_f}(l_{i2}\tfrac{1}{2})_{j_{i2}}|(l_fl_{i2})_P\ (\tfrac{1}{2}\tfrac{1}{2})_0)_P$$
$$\times\left[Y^{l_f}(\hat{\mathbf{r}}'_{A2})Y^{l_{i2}}(\hat{\mathbf{r}}'_{b2})\right]_{\mu-M}^P u_{l_f}(r_{A2})u_{l_{i2}}(r_{b2}). \tag{7.2.165}$$

Integrating over $\mathbf{r}_{A2}$ (see (7.2.117)) leads to,

$$\sum_{\sigma_2}\int d\mathbf{r}_{A2}\left[\psi^{j_f}(\mathbf{r}_{A1},\sigma_1)\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\right]_0^{0*}\left[\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\psi^{j_{i1}}(\mathbf{r}_{b1},\sigma_1)\right]_M^K$$
$$=-\sqrt{\frac{2}{2j_f+1}}((l_f\tfrac{1}{2})_{j_f}(l_{i1}\tfrac{1}{2})_{j_{i1}}|(l_fl_{i1})_K\ (\tfrac{1}{2}\tfrac{1}{2})_0)_K\left[Y^{l_f}(\hat{\mathbf{r}}_{A1})Y^{l_{i1}}(\hat{\mathbf{r}}_{b1})\right]_M^K u_{l_f}(r_{A1})u_{l_{i1}}(r_{b1}). \tag{7.2.166}$$

Let us examine the term

$$\sum_M(-1)^M\langle K\ \Lambda\ -M\ \mu|P\ \mu-M\rangle\left[Y^{l_f}(\hat{\mathbf{r}}_{A1})Y^{l_{i1}}(\hat{\mathbf{r}}_{b1})\right]_M^K\left[Y^{l_f}(\hat{\mathbf{r}}'_{A2})Y^{l_{i2}}(\hat{\mathbf{r}}'_{b2})\right]_{\mu-M}^P. \tag{7.2.167}$$

Making use of the relation

$$\langle l_1\ l_2\ m_1\ m_2|L\ M_L\rangle=(-1)^{l_2-m_2}\sqrt{\frac{2L+1}{2l_1+1}}\langle L\ l_2\ -M_L\ m_2|l_1\ -m_1\rangle, \tag{7.2.168}$$

We now couple this last term with the term $\left[Y^{l_c}(\hat{\mathbf{r}}'_{cC})Y^{l_c}(\hat{\mathbf{r}}_{cC})\right]^0_0$, arising from the partial wave expansion of the Green function. That is,

$$
\begin{aligned}
&\left[\left\{ \left[Y^{l_f}(\hat{\mathbf{r}}'_{A2})Y^{l_{i2}}(\hat{\mathbf{r}}'_{b2})\right]^P \left[Y^{l_f}(\hat{\mathbf{r}}_{A1})Y^{l_{i1}}(\hat{\mathbf{r}}_{b1})\right]^K\right\}^\Lambda \left[Y^{l_a}(\hat{\mathbf{r}}'_{aA})Y^{l_b}(\hat{\mathbf{r}}_{bB})\right]^\Lambda\right]^0_0 \left[Y^{l_c}(\hat{\mathbf{r}}'_{cC})Y^{l_c}(\hat{\mathbf{r}}_{cC})\right]^0 \\
&= ((l_a l_b)_\Lambda (l_c l_c)_0 | (l_a l_c)_P (l_b l_c)_K)_\Lambda \left[\left\{\left[Y^{l_f}(\hat{\mathbf{r}}'_{A2})Y^{l_{i2}}(\hat{\mathbf{r}}'_{b2})\right]^P \left[Y^{l_f}(\hat{\mathbf{r}}_{A1})Y^{l_{i1}}(\hat{\mathbf{r}}_{b1})\right]^K\right\}^\Lambda\right. \\
&\left.\left\{\left[Y^{l_a}(\hat{\mathbf{r}}'_{aA})Y^{l_c}(\hat{\mathbf{r}}'_{cC})\right]^P \left[Y^{l_b}(\hat{\mathbf{r}}_{bB})Y^{l_c}(\hat{\mathbf{r}}_{cC})\right]^K\right\}^\Lambda\right]^0_0 = ((l_a l_b)_\Lambda (l_c l_c)_0 | (l_a l_c)_P (l_b l_c)_K)_\Lambda \\
&\times ((PK)_\Lambda (PK)_\Lambda | (PP)_0 (KK)_0)_0 \left\{\left[Y^{l_f}(\hat{\mathbf{r}}'_{A2})Y^{l_{i2}}(\hat{\mathbf{r}}'_{b2})\right]^P \left[Y^{l_a}(\hat{\mathbf{r}}'_{aA})Y^{l_c}(\hat{\mathbf{r}}'_{cC})\right]^P\right\}^0_0 \\
&\times \left\{\left[Y^{l_f}(\hat{\mathbf{r}}_{A1})Y^{l_{i1}}(\hat{\mathbf{r}}_{b1})\right]^K \left[Y^{l_b}(\hat{\mathbf{r}}_{bB})Y^{l_c}(\hat{\mathbf{r}}_{cC})\right]^K\right\}^0_0 = ((l_a l_b)_\Lambda (l_c l_c)_0 | (l_a l_c)_P (l_b l_c)_K)_\Lambda \\
&\times \sqrt{\frac{2\Lambda+1}{(2K+1)(2P+1)}} \left\{\left[Y^{l_f}(\hat{\mathbf{r}}'_{A2})Y^{l_{i2}}(\hat{\mathbf{r}}'_{b2})\right]^P \left[Y^{l_a}(\hat{\mathbf{r}}'_{aA})Y^{l_c}(\hat{\mathbf{r}}'_{cC})\right]^P\right\}^0_0 \\
&\times \left\{\left[Y^{l_f}(\hat{\mathbf{r}}_{A1})Y^{l_{i1}}(\hat{\mathbf{r}}_{b1})\right]^K \left[Y^{l_b}(\hat{\mathbf{r}}_{bB})Y^{l_c}(\hat{\mathbf{r}}_{cC})\right]^K\right\}^0_0 .
\end{aligned}
\tag{7.2.175}
$$

Collecting all the contributions (including the constants and phases arising from the partial wave expansion of the distorted waves and the Green function), we get

$$
\begin{aligned}
T^{succ}_\mu =& (-1)^{j_f+j_{i1}} \frac{2048\pi^5 \mu_{Cc}}{\hbar^2 k_{Aa} k_{Bb} k_{Cc}} \sqrt{\frac{(2j_{i1}+1)}{(2\Lambda+1)(2j_f+1)}} \sum_{K,P} ((l_f \tfrac{1}{2})_{j_f} (l_{i2} \tfrac{1}{2})_{j_{i2}} | (l_f l_{i2})_P (\tfrac{1}{2}\tfrac{1}{2})_0)_P \\
&\times ((l_f \tfrac{1}{2})_{j_f} (l_{i1} \tfrac{1}{2})_{j_{i1}} | (l_f l_{i1})_K (\tfrac{1}{2}\tfrac{1}{2})_0)_K \; ((j_{i1} j_f)_K (j_{i1} j_{i2})_\Lambda | (j_{i1} j_{i1})_0 (j_f j_{i2})_P)_P \\
&\times \frac{(-1)^K}{(2K+1)\sqrt{2P+1}} \sum_{l_c, l_a, l_b} ((l_a l_b)_\Lambda (l_c l_c)_0 | (l_a l_c)_P (l_b l_c)_K)_\Lambda e^{i(\sigma_i^{l_a} + \sigma_f^{l_b})} i^{l_a - l_b} \\
&\times (2l_c+1)^{3/2} \left[Y^{l_a}(\hat{\mathbf{k}}_{aA}) Y^{l_b}(\hat{\mathbf{k}}_{bB})\right]^\Lambda_\mu S_{K,P,l_a,l_b,l_c},
\end{aligned}
\tag{7.2.176}
$$

$\left(T^{(2)}_{succ}\right)_\mu$

or

$T^{(2)}_{succ}(\mu)$

with (note that we have reduced the dimensionality of the integrals in the same fashion as for the L =0–angular momentum transfer calculation, see (7.2.132))

$$
\begin{aligned}
S_{K,P,l_a,l_b,l_c} =& \int r_{Cc}^2 \, dr_{Cc} \, r_{b1}^2 \, dr_{b1} \, \sin\theta \, d\theta \, v(r_{b1}) u_{l_f}(r_{C1}) u_{l_i}(r_{b1}) \\
&\times \frac{s_{P,l_a,l_c}(r_{Cc})}{r_{Cc}} \frac{F_{l_b}(r_{Bb})}{r_{Bb}} \\
&\times \sum_M \langle l_c \; 0 \; l_b \; M | K \; M \rangle \left[Y^{l_f}(\hat{r}_{C1}) Y^{l_{i1}}(\theta+\pi, 0)\right]^K_M Y^{l_b}_{-M}(\hat{r}_{Bb}),
\end{aligned}
\tag{7.2.177}
$$

and

$$s_{P,l_a,l_c}(r_{Cc}) = \int r'^2_{Cc}\, dr'_{Cc}\, r'^2_{A2}\, dr'_{A2}\, \sin\theta'\, d\theta'\, v(r'_{c2}) u_{l_f}(r'_{A2}) u_{l_i}(r'_{c2})$$
$$\times \frac{F_{l_a}(r'_{Aa})}{r'_{Aa}} \frac{f_{l_c}(k_{Cc}, r_<) P_{l_c}(k_{Cc}, r_>)}{r'_{Cc}} \tag{7.2.178}$$
$$\times \sum_M \langle l_c\ 0\ l_a\ M | P\ M\rangle \left[Y^{l_f}(\hat{r}'_{A2}) Y^{l_i}(\hat{r}'_{c2})\right]^P_M Y^{l_a}_{-M}(\hat{r}'_{Aa}).$$

We have evaluated the transition matrix element for a particular projection μ of the initial angular momentum of the two transferred nucleons. If they are coupled to a core of angular momentum J_f to total angular momentum J_i, M_i, the fraction of the initial wavefunction with projection μ is $\langle \Lambda\ \mu\ J_f\ M_i - \mu | J_i\ M_i\rangle$, and the cross section will be

?

$$\frac{d\sigma}{d\Omega}(\hat{\mathbf{k}}_{bB}) = \frac{k_{bB}}{k_{aA}} \frac{\mu_{aA}\mu_{bB}}{(2\pi\hbar^2)^2} \left| \sum_\mu \langle \Lambda\ \mu\ J_f\ M_i - \mu | J_i\ M_i\rangle T_\mu \right|^2. \tag{7.2.179}$$

For a non polarized incident beam,

?

$$\frac{d\sigma}{d\Omega}(\hat{\mathbf{k}}_{bB}) = \frac{k_{bB}}{k_{aA}} \frac{\mu_{aA}\mu_{bB}}{(2\pi\hbar^2)^2} \frac{1}{2J_i + 1} \sum_{M_i} \left| \sum_\mu \langle \Lambda\ \mu\ J_f\ M_i - \mu | J_i\ M_i\rangle T_\mu \right|^2. \tag{7.2.180}$$

This would be the differential cross section for a transition to a definite final state M_f. If we do not measure M_f we have to sum for all M_f,

$$\frac{d\sigma}{d\Omega}(\hat{\mathbf{k}}_{bB}) = \frac{k_{bB}}{k_{aA}} \frac{\mu_{aA}\mu_{bB}}{(2\pi\hbar^2)^2} \frac{1}{2J_i + 1} \sum_\mu |T_\mu|^2 \sum_{M_i, M_f} \left| \langle \Lambda\ \mu\ J_f\ M_f | J_i\ M_i\rangle \right|^2. \tag{7.2.181}$$

The sum over M_i, M_f of the Clebsh–Gordan coefficients gives $(2J_i + 1)/(2\Lambda + 1)$ (see Eq. (7.K.26)). One then gets,

? $\left|T^{(2)}_{succ}(\mu)\right|^2$

$$\frac{d\sigma}{d\Omega}(\hat{\mathbf{k}}_{bB}) = \frac{k_{bB}}{k_{aA}} \frac{\mu_{aA}\mu_{bB}}{(2\pi\hbar^2)^2} \frac{1}{(2\Lambda + 1)} \sum_\mu |T_\mu|^2, \tag{7.2.182}$$

where one can write

?

$$T_\mu = \sum_{l_a, l_b} C_{l_a, l_b} \left[Y^{l_a}(\hat{\mathbf{k}}_{aA}) Y^{l_b}(\hat{\mathbf{k}}_{bB})\right]^\Lambda_\mu$$
$$= \sum_{l_a, l_b} C_{l_a, l_b}\, i^{l_a} \sqrt{\frac{2l_a + 1}{4\pi}} \langle l_a\ l_b\ 0\ \mu | \Lambda\ \mu\rangle Y^{l_b}_\mu(\hat{\mathbf{k}}_{bB}). \tag{7.2.183}$$

Note that (7.2.182) takes into account only the spins of the heavy nucleus. In a (t, p) or (p, t) reaction, we have to sum over the spins of the proton and of the triton and divide by 2. If a spin–orbit term is present in the optical potential, the sum yields the combination of terms shown in Section (7.2.2),

$$\frac{d\sigma}{d\Omega}(\hat{\mathbf{k}}_{bB}) = \frac{k_{bB}}{k_{aA}} \frac{\mu_{aA}\mu_{bB}}{(2\pi\hbar^2)^2} \frac{1}{2(2\Lambda + 1)} \sum_\mu |A_\mu|^2 + |B_\mu|^2. \tag{7.2.184}$$

Appendix 7.A ZPF and Pauli principle at the basis of medium polarization effects: self–energy, vertex corrections and induced interaction

In keeping with a central objective of the formulation of quantum mechanics, namely that the basic concepts on which it is based relate directly to experiment[2] (Heisenberg (1925)), elementary modes of nuclear excitation (single–particle, collective vibrations and rotations), are solidly anchored on observation (inelastic and Coulomb excitation, one– and two–particle transfer reactions). Of all quantal phenomena, zero point fluctuations (ZPF), closely connected with virtual states, ~~are~~ likely to be most representative of the essential difference existing ~~between~~ quantum and classical mechanics. In fact, ZPF are intimately ~~connected~~ with the complementary principle (Bohr (1928)), and thus with ~~the~~ indeterminacy (Heisenberg (1927)) and non–commutative (Born and Jordan (1925), Born et al. (1925)) relations, and with the probabilistic interpretation (Born (1926)) of the (modulus squared) of the wavefunctions, solution of Schrödinger's or Dirac's equations (Schrödinger, E. (1925), Dirac (1926)).

e

Pauli principle (Pauli, 1925) brings about essential modifications of the virtual fluctuations of the many–body system, modifications which are instrumental in the dressing and interweaving of the elementary modes of excitation ~~(see Figs. 7.A.1 and 7.A.2)~~ (within the present context, see also Schrieffer (1964)).

In Fig. 7.A.1, NFT diagrams ⓒ–ⓒ from p. 126

here Figures 7.A.1 and 7.A.2

Appendix 7.B Coherence and effective formfactors

In what follows we shall work out a simplified derivation of the simultaneous two–nucleon transfer amplitude, within the framework of first order DWBA specially suited to discuss correlation aspects of pair transfer in general, and of the associated effective formfactors in particular.

We will concentrate on (t, p) reaction, namely reactions of the type $A(\alpha,\beta)B$ where $\alpha = \beta + 2$ and $B = A + 2$.

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[2] The abstract of this reference reads: "In this paper it will be attempted to secure foundations for a quantum theoretical mechanics which is exclusively based on relations between quantities which in principle are observables". Within the present context, namely that of the probing the nuclear structure (e.g. pairing correlations) with direct nuclear reactions, in particular Cooper pair transfer, one can hardly think of a better *incipit* for the introduction of elementary modes of excitation, modes which carry within them most of the correlations thus requiring for their theoretical treatment an effective field theory, like e.g. NFT to properly take into account the essential overcompleteness of the basis (non–orthogonality) as well as of Pauli violating processes.

where

Bring here Figure 7.B.1

$$\phi_{000}(\vec{r}) = R_{nl}(\nu^{1/2}r)Y_{lm}(\hat{r}). \tag{7.B.7}$$

The coordinate $\vec{\rho}$ is the radius vector which measures the distance between the center of mass of the dineutron and the proton, while the vector $\vec{r}$ is the dineutron relative coordinate (cf. Fig. 7.B.1).

To obtain the DWBA cross section we have to calculate the integral

$$T = \int d\xi_A\, d\vec{r}_1\, d\vec{r}_2\, d\vec{r}_p \chi_p^{(-)}(\vec{R}_2)\psi^*_\beta(\xi_{A+2},\sigma_\beta)V_\beta\psi_\alpha(\xi_A,\sigma_\alpha,\sigma_\beta)\psi_t^{(+)}(\vec{R}_1) \tag{7.B.8}$$

where the final state effective interaction $V_\beta(\rho)$ is assumed to depend only on the distance ρ between the center of mass of the di–neutron and of the proton. Instead of integrating over $\xi_A, \vec{r}_1, \vec{r}_2$ and $\vec{r}_p$ we would integrate over $\xi_A, \vec{r}', \vec{r}'$ and $\vec{r}_p$. The Jacobian of the transformation is equal to 1, i.e. $\partial(\vec{r}_1,\vec{r}_2)/\partial(\vec{r}',\vec{r}') = 1$.

To carry out the integral (7.B.8) we transform the wave function (7.B.4) into center of mass and relative coordinates. If we assume that both $\phi_{j_1}(\vec{r}_1)$ and $\phi_{j_2}(\vec{r}_2)$ are harmonic oscillator wave functions (used as a basis to expand the Saxon–Woods single–particle wavefunctions), this transformation can be carried with the aid of tha Moshinsky brackets. If $|n_1l_1, n_2l_2; \lambda\mu\rangle$ is a complete system of wave functions in the harmonic oscillator basis, depending on $\vec{r}_1$ and $\vec{r}_2$ and $|nl, NL; \lambda\mu\rangle$ is the corresponding one depending on $\vec{r}$ and $\vec{R}$, we can write

$$\begin{aligned} |n_1l_1, n_2l_2; \lambda\mu\rangle &= \sum_{nlNL} |nl, NL; \lambda\mu\rangle\langle nl, NL; \lambda\mu|)|n_1l_1, n_2l_2; \lambda\mu\rangle \\ &= \sum_{nlNL} |nl, NL; \lambda\mu\rangle\langle nl, NL; \lambda\mu|n_1l_1, n_2l_2; \lambda\rangle \end{aligned} \tag{7.B.9}$$

The labels n, l are the principal and angular momentum quantum numbers of the relative motion, ehile N, L are the corresponding ones corresponding to the center of mass motion of the two–neutron system. Because of energy and parity conservation we have

$$\begin{aligned} &2n_1 + l_1 + 2n_2 + l_2 = 2n + l + 2N + L \\ &(-1)^{l_1+l_2} = (-1)^{l+L}. \end{aligned} \tag{7.B.10}$$

The coefficients $\langle nl, NL, L|n_1l_1, n_2l_2, L\rangle$ are tabulated and were first discussed by (Moshinsky, 1959)

With the help of eq. (7.B.9) we can write the wave function $\psi^{J_f}_{M_f}(\xi_{A+2})$ as

$$T = D_0 \sum_L (LM_L J_i M_{J_i} | J_f M_{J_f}) \\ \times \int d\vec{R} \chi_p^{*(-)}(\frac{A}{A+2}\vec{R}) u_L^{j_i J_f}(R) Y^*_{LM_L}(\hat{R}) \chi_t^{(+)}(\vec{R}) \tag{7.B.21}$$

From Eq. (7.B.21) it is seen that the change in parity implied by the reaction is given by $\Delta\pi = (-1)^L$. Consequently, the selection rules for (t, p) and (p, t) reactions in zero–range approximation are,

$$\begin{aligned} \Delta S &= 0 \\ \Delta J &= \Delta L = L \\ \Delta\pi &= (-1)^L \end{aligned} \tag{7.B.22}$$

i.e. only normal parity states are excited.

The integral appearing in Eq. (7.B.21) has the same structure as the DWBA integral appearing in Eq. (6.F.16) which was derived for the case of one–nucleon transfer reactions.

The difference between the two processes manifests itself through the different structure of the two form factors. While $u_l(r)$ is a single–particle bound state wave function (cf. Eq. (6.F.1a)), $u_L^{J_i J_f}$ is a coherent summation over the center of mass states of motion of the two transferred neutrons (see Eq. (7.B.16)). In other words, an effective quantity (function). It is of notice that this difference essentially vanishes, when one considers dressed particles resulting from the coupling to collective motion, and leading, among other things, to ω–dependent effective masses. Examples of two–nucleon transfer form factors are given in Figs 7.B.2, 7.B.3 and 7.B.4

bring here figure

Appendix 7.C Relative importance of successive and simultaneous transfer and non-orthogonality corrections

In what follows we discuss the relative importance of successive and simultaneous two-neutron transfer and of non-orthogonality corrections associated with the reaction

$$\alpha \equiv a(= b + 2) + A \rightarrow b + B(= A + 2) \equiv \beta \tag{7.C.1}$$

in the limits of independent particles and of strongly correlated Cooper pairs, making use for simplicity of the semiclassical approximation (for details cf. Broglia and Winther (2004), Broglia (1975) and refs. therein), in which case the two-particle transfer differential cross section can be written as the reaction channel $f = (b+1) + F(= A+1)$ having been introduced, the quantity $S(n)$ being the one-neutron separation energy (see Fig. 7.C.3). The summation over $f(\equiv a'_1, a'_2)$ and $F(\equiv a_1, a_2)$ involves a restricted number of states, namely the valence shells in nuclei B and a.

The successive transfer amplitude $\tilde{a}^{(2)}_{\infty}$ written making use of the post-prior representation is equal to (see Fig. 7.C.2 (III))

$$\begin{aligned}\tilde{a}^{(2)}(\infty) = &\frac{1}{i\hbar}\int_{-\infty}^{\infty} dt (\psi^b\psi^B, (V_{bB} - < V_{bB} >)e^{i\sigma_1}\psi^f\psi^F)\\ &\times \exp[\frac{i}{\hbar}(E^{bB} - E^{fF})t + \gamma_1(t)]\\ &\times \frac{1}{i\hbar}\int_{-\infty}^{t} dt' (\psi^f\psi^F, (V_{fF} - < V_{fF} >)e^{i\sigma_2}\psi^a\psi^A >\\ &\times \exp[\frac{i}{\hbar}(E^{fF} - E^{aA})t' + \gamma_2(t)]. \end{aligned} \tag{7.C.7}$$

To gain insight into the relative importance of the three terms contributing to Eq. (7.C.3) we discuss two situations, namely, the independent-particle model and the strong-correlation limits.
Before doing so, let us describe in some detail the graphical description of the transfer amplitudes (7.C.4), (7.C.6) and (7.C.7) displayed in Figs. 7.C.1 and 7.C.2. It is of notice that the @-@

From p. 132 caption

7.C.1 Independent particle limit

In the independent particle limit, the two transferred particles do not interact among themselves but for antisymmetrization. Thus, the separation energies fulfill the relations (see Fig. 7.C.3)

$$S^B(2n) = 2S^B(n) = 2S^F(n), \tag{7.C.8}$$

and

$$S^a(2n) = 2S^a(n) = 2S^f(n). \tag{7.C.9}$$

In this case

$$\phi^{B(A)}(S^B(2n), \vec{r}_{1A}, \vec{r}_{2A}) = \sum_{a_1 a_2} \phi^{B(F)}_{a_1}(S^B(n), \vec{r}_{1A})\phi^{F(A)}_{a_2}(S^F(n), \vec{r}_{2a}), \tag{7.C.10}$$

and

$$\phi^{a(b)}(S^a(2n), \vec{r}_{1b}, \vec{r}_{2b}) = \sum_{a'_1 a'_2} \phi^{a(f)}_{a'_1}(S^a(n), \vec{r}_{2b})\phi^{f(b)}_{a'_2}(S^f(n), \vec{r}_{1b}), \tag{7.C.11}$$

where $(a_1, a_2) \equiv F$ and $(a'_1, a'_2) \equiv f$ span, as mentioned above, shells in nuclei B and a respectively.

Inserting Eqs. (7.C.8–7.C.11) in Eq. (7.C.4) one can show that

$$a^{(1)}(\infty) = a^{(NO)}(\infty). \tag{7.C.12}$$

terms of quasiparticle operators using (7.M.17)[3]:

$$
\begin{aligned}
P^\dagger(j_1, j_2, JM) = & \frac{1}{\sqrt{1+\delta_{j_1,j_2}}} \sum_{m_1,m_2} \langle j_1\ m_1\ j_2\ m_2 | J\ M \rangle \Big(U_{j_1} U_{j_2} \alpha^\dagger_{j_1 m_1} \alpha^\dagger_{j_2 m_2} \\
& + (-1)^{j_1+j_2-M} V_{j_1} V_{j_2} \alpha_{j_1 -m_1} \alpha_{j_2 -m_2} \\
& + (-1)^{j_2-m_2} U_{j_1} V_{j_2} \alpha^\dagger_{j_1 m_1} \alpha_{j_2 -m_2} \\
& - (-1)^{j_1-m_1} V_{j_1} U_{j_2} \alpha^\dagger_{j_2 m_2} \alpha_{j_1 -m_1} \\
& + (-1)^{j_1-m_1} V_{j_1} U_{j_2} \delta_{j_1 j_2} \delta_{-m_1 m_2} \Big).
\end{aligned}
\tag{7.N.8}
$$

If both nuclei are in zero–quasiparticle states, the only term that survives is the last one in the above expression, and (7.N.7) becomes

$$
\begin{aligned}
B(0,j,j) &= \frac{1}{\sqrt{2}} \sum_m \langle j\ m\ j\ -m | 0\ 0 \rangle (-1)^{j-m} V_j U_j \\
&= \frac{1}{\sqrt{2}} \sum_m \frac{(-1)^{j-m}}{\sqrt{(2j+1)}} (-1)^{j-m} V_j U_j \\
&= \frac{1}{\sqrt{2}} \sum_m \frac{1}{\sqrt{(2j+1)}} V_j U_j.
\end{aligned}
\tag{7.N.9}
$$

After doing the sum, we finally find

$$
B(0,j,j) = \sqrt{j+1/2}\ V_j U_j. \tag{7.N.10}
$$

Note that in this final expression V_j refers to the A nucleus, while U_j is related to the $A+2$ nucleus. In practice, it does not make a big difference to calculate both for the same nucleus.

Appendix 7.O Derivation of two–nucleon transfer transition amplitudes including recoil, non–orthogonality and successive transfer.

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In the present Appendix we reproduce what, arguably, was the first complete derivation (Bayman (1970)(unpublished)) of the different contributions needed to calculate absolute two–nucleon transfer cross sections in a systematic way (cf. Bayman (1971) and Bayman and Chen (1982)). Within this context we refer to Broglia et al. (1973) and Potel et al. (2013) in particular Fig. 10 of this reference.

[3] In what follows, we use the phase convention $\alpha_{j\tilde{m}=(-1)^{j-m}\alpha_{j-m}}$ instead of $\alpha_{j\tilde{m}=(-1)^{j+m}\alpha_{j-m}}$, consistent with (7.M.2). I don't know why, but it seems to be common practice... Had we stick to the definition (7.M.2), the amplitude $B(0,j,j)$ calculated below would have a minus sign, which would not have any physical consequence.

Reference

gasane

Gregory

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? what is all this

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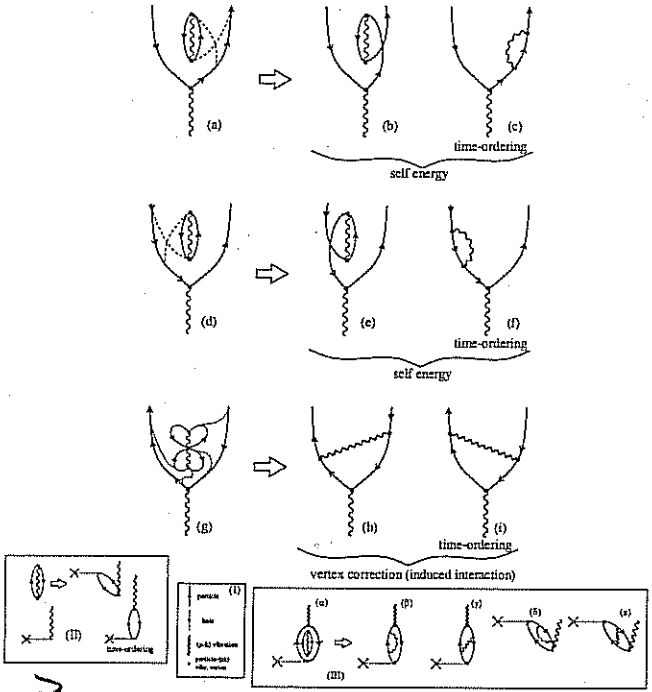
Fig. 7.A.1 NFT diagrams describing renormalization processes associate with ZPF



C ~~Figure 7.A.1: Nuclear field theory (NFT) diagrams~~ corresponding to the lowest order medium polarization effects renormalizing the properties of a particle–hole collective mode (wavy line), correlated particle–hole excitation, which in the shell model basis corresponds to a linear combination of particle–hole excitation ((up–going)–(down–going) arrowed lines) ~~excitations~~ calculated within the random phase approximation (RPA), ~~in of a bare interaction,~~ and leading to the particle–vibration coupling vertex (formfactor and strength, i.e. transition density (solid dot) see inset (I), bottom). The action of an external field on the zero point fluctuations (ZPF) of the vacuum (inset (II)), forces a virtual process to become real, leading to a collective vibration by annihilating a (virtual, spontaneous) particle–hole excitation (backwards RPA amplitude) or, in the time ordered process, by creating a particle–hole excitation which eventually, through the particle–vibration coupling vertex, correlate into the collective (coherent state; forwardsgoing amplitudes). Now, oyster–like diagrams associated with the vacuum ZPF can occur at any time (see inset (III)). Because the texture of the vacuum is permeated by symmetry rules (while one can violate energy conservation in a virtual state one cannot violate e.g. angular momentum conservation or the Pauli principle), the process shown in the inset III (α) leads, through Pauli principle correcting processes (exchange of fermionic arrowed lines) to self–energy (inset III (β), (δ)) and vertex corrections (induced p–h interaction; inset III (γ), (ε)) processes. The first ones are detailed in graphs (a)–(f), while the second ones in graphs (g)–(i). In keeping with the fact that the vibrational states can be viewed as a coherent state (cf. App. 7.J) exhausting a large fraction of the EWSR (e.g. a Giant Resonance) for which the associated uncertainty relations in momentum and coordinate fulfills the absolute minimum consistent with quantum mechanics ($\Delta\alpha_{\lambda\mu}\Delta\pi_{\lambda\mu} = \hbar/2$, $\alpha_{\lambda\mu} = (\hbar\omega_\mu/2C_\lambda^{1/2})(\Gamma^\dagger_{\lambda\mu} + \Gamma_{\lambda\mu})$ being the (harmonic) collective coordinate, $\pi_{\lambda\mu}$ being the conjugate momentum; cf. e.g. Glauber (1959)), there is a strong cancellation between the contribution of self–energy and vertex correction diagrams (Bortignon, 1981), implying small anharmonicities and long lifetimes ($\Gamma/E \ll 1$, where Γ is the width and E the centroid of the mode $|\lambda\mu\rangle = \Gamma^\dagger_{\lambda\mu}|0\rangle$, $(\hbar\omega_\lambda/2C_\lambda)^{1/2}$ being the ZPF amplitude (cf. e.g. Brink and Broglia (2005))).

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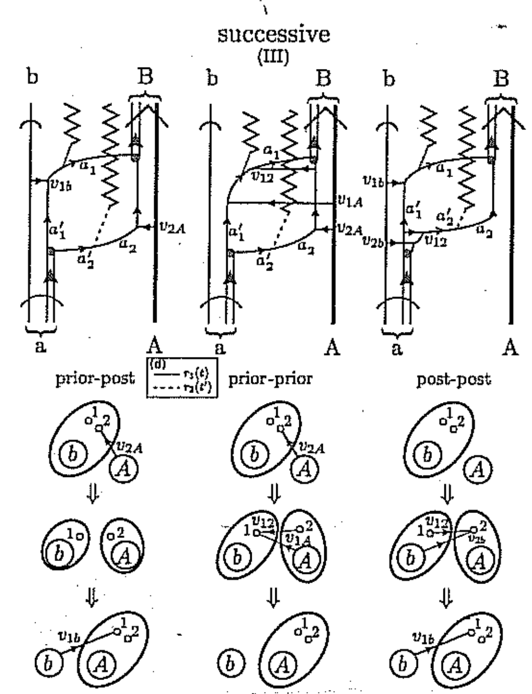

Caption

~~Fig 7.C.2~~. Graphical representation of the successive transfer of two nucleons

Figure 7.C.2: ~~Graphical representation of the lowest order ((I),(II) and (III) first and second-order in v respectively), two-nucleon transfer processes, which correctly converge to the strong-correlation (only simultaneous transfer), and to the independent-particle (only successive transfer) limits. The~~ time arrow is assumed to point upwards: (I) Simultaneous transfer, in which one particle is transferred by the nucleon-nucleon interaction (note that $U(r) = \int d^3r'\rho(r')v(|\vec{r} - \vec{r}'|)$) acting either in the entrance $\alpha \equiv a+A$ channel (prior) or in the final $\beta \equiv b+B$ channel (post), while the other particle follows suit making use of the particle-particle correlation (grey area) which binds the Cooper pair (see upper inset labelled (a)), represented by a solid arrow on a double line, to the projectile (curved arrowed lines) or to the target (opened arrowed lines). The above argument provides the explanation why when e.g. v_{1b} acts on one nucleon, the other nucleon also reacts instantaneously. In fact a Cooper pair displays generalized rigidy (emergent property in gauge space). A crossed open circle represents the particle-pair vibration coupling. The associated strength, together with an energy denominator, determines the amplitude $X_{a_1'a_2'}$ with which the pair mode (Cooper pair) is in the (time reversered) two particle configuration $a_1'a_2'$. In the transfer process, the ~~orbital~~ relative motion changes, the readjustement of the corresponding trajectory mismatch being operated by a Galilean operator $(\exp\{\vec{k}\cdot(\vec{r}_{1A}(t) + \vec{r}_{2A}(t))\})$. This phenomenon, known as recoil process, is represented by a jagged line which provides simultaneous information on the two transferred nucleons (single time appearing as argument of both single-particle coordinates r_1 and r_2; see inset labeled (b)). In other words, information on the coupling of structure and reaction modes. (II) Non-orthogonality contribution. While one of the nucleons of the Cooper pairs is transferred under the action of v, the other goes, uncorrelatedly over, profiting of the non-orthogonality of the associated single-particle wavefunctions (see inset (c)). In other words of the non-vanishing values of the overlaps, as shown in the inset. (III) Successive transfer. In this case, there are two time dependences associated with the acting of the nucleon-nucleon interaction twice (see inset (d)).

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