properties
Symmetries
of empty space

~~Preface~~

~~The elementary modes of nuclear excitation are vibrations and rotations, are rotations of spatially deformed systems and vibration of both spherically and deformed systems, single-particle motion, and pairing vibrations and rotations.~~

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Preface

The elementary modes of nuclear excitation are vibrations and rotations, single-particle motion, and pairing vibrations and rotations. The specific reactions probing these modes are inelastic and Coulomb excitation, and single- and two-particle transfer processes respectively.

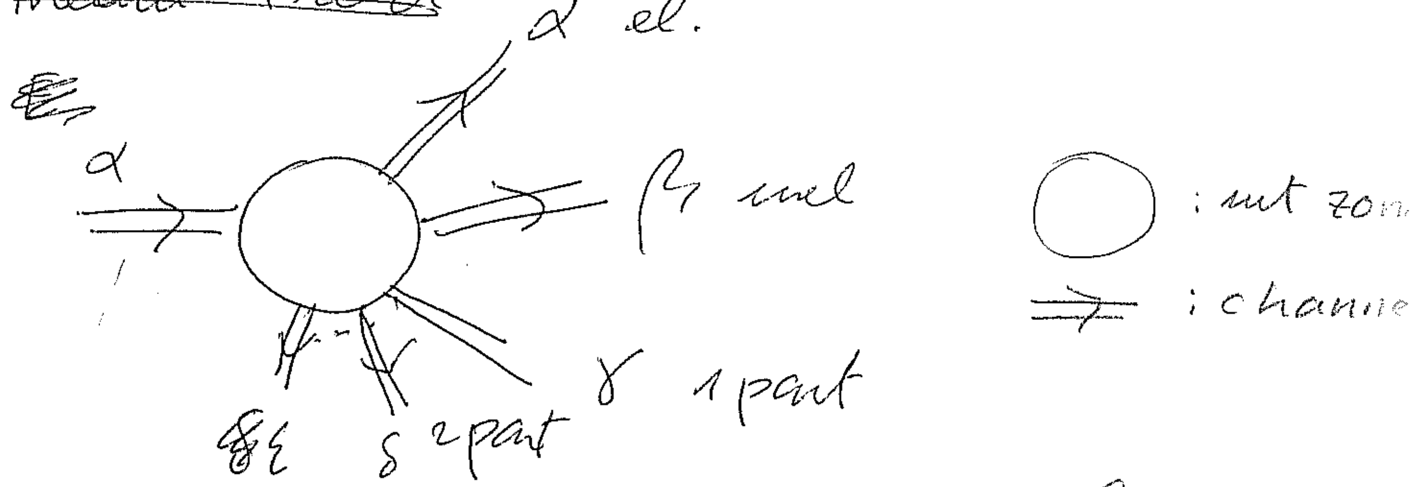
Pairing vibrations and rotations, closely connected with nuclear superfluidity are, arguably, a paradigm of quantal nuclear phenomena. They thus play a central role within the field of nuclear structure. It is only natural that two-nucleon, Cooper pair, transfer plays a similar role concerning direct nuclear reactions.

Book Gregory 9/04/2013

~~General~~ Background QM

General intr.

~~Mean Field~~



R: react
S: str.

Lecture Reactions

α: Elastic ~~react~~ R

mean field (HF) S

β: inel R

Coll. vibrs (TDHF) S

γ: 1-part. R

~~Dressed s.p. sta~~

beyond mean fiel β N S

δ: 2-part. R

Pairing correls. S

Light and Heavy Ions

succ. sim etc.

α': elastic revisited

absorptive potential

book ~~[illegible]~~ Gregory

$\boxed{\text{15 p. book}}$

1) Notes (small blue book)

* Pairing in Nuclei and neutron stars

2) Phys. Rep. version 1

3) " " " 2

4) Paper PRC Sn
~~with and [illegible]~~
without corrects referee

5) Paper PRC Sn
with corrs. referee

6) Pape ^{12}Be, ^{11}Li Belyaev

7) PRL ^{11}Li

8) PRL Sn

~~10~~

9) Review paper Heenen

~~10~~ 10) } blurbs on
11) } ~~paired~~ pairing vibr.
12) } halo
13)
:
:

$\boxed{^{132}Sn}$

anti pairing
effects
is OK ~~B~~ H + Mott

but "L (gs) exists

Thus
dynamical

Preface

The origin of these lectures attended by RAB and can be traced back to October 1965, date of the first lecture of the course on Nuclear Reactions held, during the fall semester by Ben Mottelson at the Universitets Institut for Teoretisk Physik (now The Niels Bohr Institute of the University of Copenhagen), in the unique setup of Aud. A, rich of tradition ~~but also of~~ as well as a beautiful look to Fælledparken. It was the fortune of RAB to start a collaboration with Claus Riedel on two-nucleon transfer reactions at that time, ~~to collaborate on~~ ~~the subject of~~ ~~to shed light~~ with which to probe the newly postulated pairing vibrations (and rotations), in particular in connection with the (t,p) ~~results~~ measurements ~~carried out~~ made at Harwell (UK) by the group of Ole Hansen and Ove Nathan with the collaboration of Stan Hinds. The Monday morning experimental groups meetings carried out at Aud. A (and moderated by Aage Bohr), in which essentially the raw data was confronted with ~~theoretical~~ speculations and numerical results, with ~~interventions~~ central contributions by Daniel Bes were instrumental ~~to shape~~ to shed important ~~light~~ ~~on nuclear~~ on pairing correlations in nuclei, as well as to make ~~it clear~~ operative that ~~direct~~ structure and reactions are but two aspects of the same physics, one referring mainly to bound

and to continuum states respectively.
The fact that this is not only a technical question, but at the basis of the observed nuclear properties, is solidly anchored on quantum mechanics, and the associated central role that quantal fluctuations and associated virtual states play in atomic nuclei, a fact that is at the basis of medium renormalization processes dressing the nuclear modes of elementary excitation, i.e. single-particle and collective vibrations and rotations, and leading to the actual value of the observables, i.e. effective mass, charge, energies, lifetimes, absolute cross sections, etc. This view led, under the guidance of Daniel Bes to the Nuclear Field Theory and of Aage Winther, to a corresponding formulation of Heavy Ion Reactions. The development of the consequences of these projects has taken many years to be ~~worked out~~ implemented. Regarding the structure aspects Francisco Barranco, George Bertsch, Pier Francesco Bortignon and Enrico Vigezzi have played an important role, the contribution of have been central both concerning the structure and (also

in lectures at the NBI story OUT

one from another

and have been rewritten

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Dept. of Physics

in lectures, at the NBI, Stony Brook, UT Knoxville

in one form or another

and have been

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During the last [redacted] few years new impetus [redacted] towards a quantitative description of the probing of pairing in nuclei have been pursued at Milan, in particular in connection with the probing of new mechanisms of cooper pair binding. The results of these efforts constitute~~s~~ an important part of the present monograph, [redacted] and also the basis for a ~~cour~~ series by the authors of lectures delivered [redacted] within the framework of the PhD program of the Department of Physics of the University of Milan.

RA
Gregory Potel

Ricardo A. Broglia

Preface

vibrations and rotations (two-particle transfer), will be discussed, ~~and the tools within the framework~~

The pairing interaction arizing from the exchange of mesons and empirically parametrized in terms of a nucleon-nucleon potential, like e.g. the Argonne potential, can be viewed as a contact interaction. A multipole expansion reveals that none of the multipole terms is more important than the other, and that one has to consider rather high multipoles to obtain convergence. These effects were already discussed in detail in the seminal papers of Beliaev [1], Mottelson [2] and Bayman [3]. In fact, one of the main subjects studied in these papers was that of the competition between pairing and deformation effects, the conclusion arrived was: single, low-multipoles control mean field (deformation), while many, high multipoles, glue pairs of nucleons together (Cooper pairs) giving rise to pairing correlations [4-6].

While this result is quite sound, it leaves out about half of the effects which control the nuclear structure. This is because it is based on a static view of the mean field (shell model), where nucleons move independently of each other feeling the pullings and pushings of all other nucleons when bouncing elastically off the surface. Single-particle levels are the solutions of a nucleon of mass equal to the bare mass moving in a static Saxon-Woods potential. This picture is, however, not quite correct, as the motion of the nucleons and their interactions are strongly renormalized by vibrations of the mean field (dynamical shell model [7-9]). This is not only the case for nuclei lying along the stability valley but it is especially true in the case of exotic nuclei in general and halo nuclei in particular [10,11].

In what follows (the first part of monography), ~~the case for~~ the dynamical shell model will be briefly reviewed contructed ~~one step at a time~~, starting from the study of the effective mass and of the lifetime of single-particle motion, resulting from the interweaving of nucleons with low-lying collective vibrations of the nuclear surface and arriving~~, in the second part of the monography~~, to the pairing induced interaction arizing from the exchange of these vibrations between pairs of nucleons. It will be concluded that ~~such a~~ the resulting description of the nuclear structure

the case for the renormalized picture of the nuclear structure in terms of elementary modes of nuclear excitation ~~will be briefly reviewed~~

single-particle motion (one-particle transfer) and pairing

In the second part, the direct reaction processes which specifically probe the variety degrees of freedom (mean field (elastic scattering), collective vibrations and rotations (inelastic scattering and Coulomb excitation), and reactions

of nuclear structure and reactions which reflect the bound and continuum aspects of the same physics,

in which the self-consistency existing in nature between density and potential and their fluctuations is correctly treated, provides a unified picture ~~of single-particle and collective motion~~, its natural mathematical framework being the nuclear field theory (NFT) [12-16]. ~~The study of the structure of atomic nuclei, paradigm of finite-many body systems provides, largely for free, the physical insight and the mathematical tools needed to understand other finite or compact systems like neutron stars, metal clusters [17], fullerenes [18], Bose Einstein condensates, etc. In particular, their behaviour at very low temperatures (superfluidity and superconductivity). This constitutes the last part of the present monography and, in a very real sense, is a gift that the practice of the discipline of nuclear structure provides to its practitioners.~~

(also the optical potential)

6/05/13

Chapter 2

Spectroscopy with direct reactions

2.1 Introduction

In a scattering experiment, a beam of particles is directed at a target containing the scattering material, and the energy and angular distributions of the outgoing particles is measured. In the case in which the scattering material (target) corresponds to unstable, short lived nuclei,like e.g. ^{132}Sn (39.7 sec, see [**?**]) (but not e.g. in the case of ^{210}Pb (22.3 y)), one can carry out the experiment by exchanging the roles of target and projectile, a method known as inverse kinematics.

There are two basic lengths governing the nuclear reactions, namely the radius of the nucleus ($\approx 10^{-13}$cm) and the distance from the target nucleus to the detector ($\approx 10^{2}$cm). Because of the difference of these two characteristic magnitudes, one can divide the scattering process in two separate parts, namely,

1. the analysis of the outgoing beam properties in terms of optical potentials and of single–particle strengths and spreading widths, (effective) deformation parameters and average value of the pair transfer operator, and
2. the relation between these parameters and the motion of nucleons in the nuclei.

In the first part of the monograph the motion of a particle displaying a large mean free path (quantality parameter of order unity)

2.2 Reaction channel

Let us consider the case in which ^{18}O is the target and the projectile is a proton. The following processes can take place, among others

$$\text{entrance channel}\left\{\begin{array}{ll} p+^{18}O\rightarrow & p+^{18}O\,(\text{gs})\;\;(Q=0)\;\;\text{Elastic scattering} \\ p+^{18}O\rightarrow & \left.\begin{array}{ll} p+^{18}O\,(6\text{MeV}) & +Q_1 \\ d+^{17}O\,(\text{gs}) & +Q_2 \\ t+^{16}O\,(\text{gs}) & +Q_3 \end{array}\right\}\;\text{reaction channels} \end{array}\right. \tag{2.1}$$

where

$$Q_1 = M_p + M(^{18}O) - (M_p +^{18} O\ (6\ \text{MeV})) = -6\text{MeV} \tag{2.2}$$

$$Q_2 = M_p + M(^{18}O) - (M_d +^{17} O\ (\text{gs})) \tag{2.3}$$

$$Q_3 = M_p + M(^{18}O) - (M_t +^{16} O\ (\text{gs})) \tag{2.4}$$

If $Q > 0$ the reaction can proceed at zero bombarding energy. For $Q < 0$ the reaction is not observed below the threshold E_t which is defined, for a general reaction $A(a, b)B$ as

$$E_{CM} = \frac{1}{2}\frac{M_a M_A}{M_a + M_A} V_a^2 = \frac{M_A}{M_a + M_A} E_{lab} = \frac{1}{1 + (M_a/M_A)} E_{lab} \tag{2.5}$$

($1/2 M_a V_a^2$ is the total energy of the system). If $E = |Q|$ we have

$$E_t^{lab} = \frac{M_a + M_A}{M_A}|Q| \to E_t^{lab} = \left(1 + \frac{M_a}{M_A}\right)|Q| \tag{2.6}$$

For the particular case

$$\begin{array}{l} p +^{18} O \to p +^{18} O \quad (6\ \text{MeV}) \\ E_t^{lab} = \frac{19}{18} \times 6\ \text{MeV} \end{array} \tag{2.7}$$

A complete specification of the type of two-particle breakup and of the internal states of the two particles, is called a channel and is specified by a product $\Psi_\alpha = \Psi_a \Psi_A$ of bound internal wave functions of the two nuclei. Here A and a denote the nuclei into which the system breaks up and also their state of excitation, their angular momenta and the projection of their angular momenta.

The same word channel is understood sometimes to include all the properties already mentioned, together with a definite value of the orbital angular momentum of the relative motion of the centers of mass of the two separating systems.

2.3 The reaction cross section

The initial situation can be described by a plane wave [1],

$$\Psi_{inc} = \Psi_\alpha e^{ik_\alpha z} \tag{2.8}$$

which represents a beam of particles of unit density, incident upon the scattering center in the z-direction. The price to pay for such a simplification is that momentum and energy are then absolutely defined, and it is no longer possible to follow the scattering process neither in space now in time (from $\Delta x \Delta p \geq \frac{\hbar}{2}$ we loose the localization in space and from $\Delta E \Delta t \geq \frac{\hbar}{2}$ the one in time).

This way of describing the incident beam is of course an idealization, and in most cases there is no problem of principle, as one can approach the situation described by Eq.(2.8) as closely as desired.

[1] This free particle wave function can be normalized in terms of the function or asking the function to obey periodic boundary conditions inside a box (see Appendix)

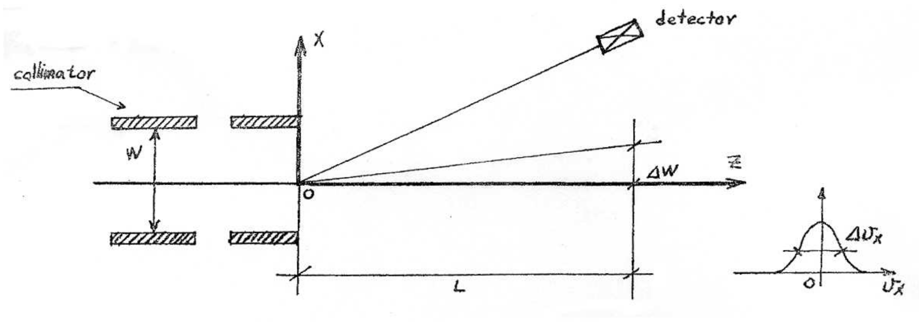


Figure 2.1:

One must be able, however, to place the detector outside the beam. It is important to verify that the corresponding spread of the wave packet does not produce any essential limitation. Let us call W the width of the beam (see fig. 2.1). The velocity perpendicular to the beam can be easily calculated

$$\begin{aligned} \Delta x \Delta p_x &\sim \hbar \quad , p_x = m v_x \\ \Delta p_x = m \Delta v_x & \quad , \Delta X = W \\ W(m \Delta v_x) &\sim \hbar \\ \Delta v_x &\sim \tfrac{\hbar}{MW} \end{aligned} \tag{2.9}$$

A particle leaving the collimator ($\langle v_x \rangle = 0$) has an uncertainty in v_x given by the equation (2.9). In the most unfavourable case $v_x = \Delta v_x$. After a time $\Delta t = t$ (we choose $t = 0$ when the particle leaves the collimator, namely for $z = 0$), the particle has travelled a distance L, and the spreading of the wave packet along x is equal to

$$\begin{aligned} \Delta W = v_x t & \approx \Delta v_x t \qquad t = \tfrac{L}{v} \\ \Delta W = \tfrac{\Delta v_x}{v} L & \approx \tfrac{\hbar L}{m W v} \end{aligned} \tag{2.10}$$

The relative increase of the wave packet is then equal to

$$\frac{\Delta W}{W} \approx \frac{\hbar L}{m W^2 v} \tag{2.11}$$

Let us put some numbers corresponding to the accelerator at Risø

$$\begin{aligned} L & \approx 10^2 \mathrm{cm} \\ v & \approx 10^9 \mathrm{cm/seg} \\ W & \approx 10^{-1} \mathrm{cm} \\ \tfrac{\Delta W}{W} & \approx 10^{-8} \end{aligned} \tag{2.12}$$

which is very small indeed.

The most dangerous case corresponds to reactions with slow neutrons, which usually are done also in large equipments (time of flight techniques). Let us put

$$\begin{aligned} L & \approx 10^4 \mathrm{cm} \\ v & \approx 2 \times 10^5 \mathrm{cm/seg} \\ W & \approx 10^{-1} \mathrm{cm} \\ \tfrac{\Delta W}{W} & \approx 10^{-2} \end{aligned} \tag{2.13}$$

The ratio is still small, but just on the limit of becoming important. Still if this ratio would be of order unit we could use the same concepts, but we should threat with more detail the problem of how the wave packet is constructed and the possible interference at the edge of the beam, with the outgoing wave packet.

The scattered wave (asymptotic region) must be the solution of the free field equation

$$\begin{aligned} H\Psi_{\text{scatt}} &= E\Psi_{\text{scatt}} \\ E &= \frac{\hbar^2 k_\alpha^2}{2m} \end{aligned} \tag{2.14}$$

where

$$H = \frac{p^2}{2m} = -\frac{\hbar^2}{2m}\nabla^2 = -\frac{\hbar^2}{2m}\left\{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\hat{L}^2\right\} \tag{2.15}$$

where $\hat{L}$ is the angular momentum operator, $m = \frac{M_B M_b}{M_b + M_B}$ the reduced mass, and k and r the relative momentum and coordinate of the nuclei b and B.
At large distances the angular momentum terms drops out as $\frac{1}{r^2}$ and it is easy to verify that the asymptotic solution is

$$\Psi_{\text{scatt}} = \frac{e^{ik_\alpha r}}{r} f_{\alpha\alpha}(E, \theta, \phi)\Psi_\alpha \tag{2.16}$$

Let us now calculate the incoming current and the scattered current of particles.

For a given wave function Ψ, (describing the motion of a particle of mass m), the associated current is equal to

$$\begin{aligned} \vec{I} &= \frac{\hbar}{2im}\left(\Psi^*\vec{\nabla}\Psi - \left(\vec{\nabla}\Psi^*\right)\Psi\right) \\ &= \frac{\hbar}{m}\mathcal{I}_m\left(\Psi^*\vec{\nabla}\Psi\right) \end{aligned} \tag{2.17}$$

The incident current is equal to (cf. eq. (9))

$$\begin{aligned} \vec{I}_{\text{inc}} &= \frac{\hbar}{m}\mathcal{I}_m e^{-ik_\alpha z}\frac{\mathrm{d}}{\mathrm{d}z}\left(e^{ik_\alpha z}\right)\hat{z} \\ &= \frac{\hbar k_\alpha}{m}\hat{z} = v_\infty\hat{z} \end{aligned} \tag{2.18}$$

where v_∞ is the velocity corresponding to the projectile incident energy.

The scattered current is equal to [2]

$$\vec{I}_{\text{scatt}} \approx \frac{|f(\theta,\phi)|^2}{r^2}\frac{\hbar k_\alpha}{m}\hat{r} \tag{2.19}$$

The differential cross section is defined as the flux of particles going into the solid angle dΩ at angle (θ, ϕ), divided by the incoming flux of incoming particles.

The flux of outgoing particles is given by the projection of I_{scatt} on the unit $\mathrm{d}\vec{s} = r^2\mathrm{d}\Omega\hat{r}$ of solid angle, namely

$$\vec{I}_{\text{scatt}}\mathrm{d}\vec{s} = |f(\theta,\phi)|^2\frac{\hbar k_\alpha}{m}\mathrm{d}\Omega \tag{2.20}$$

The incident flux is given by

$$\vec{I}_{\text{inc}}\hat{z} = \frac{\hbar k_\alpha}{m} \tag{2.21}$$

[2]In deriving this equation one assumes that $r \to \infty$ (asymptotic region).

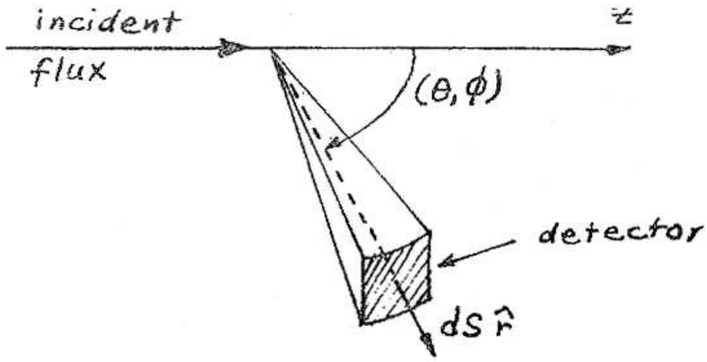


Figure 2.2:

The differential cross section is defined as the flux of particles into the solid angle dΩ at angle (θ, ϕ), divided by the incoming flux of incoming particles, namely

$$d\sigma = |f_{\alpha\alpha}(E, \theta, \phi)|^2 d\Omega \tag{2.22}$$

In other channels there will be no incident wave, and in general, the outgoing waves would have a different value of the wave number, i.e.

$$\Psi_{\text{scatt}} = \frac{1}{r} e^{ik_\beta r} f_{\alpha\beta}(E', \theta, \phi) \tag{2.23}$$

The symmetries of the problem can produce limitations in the form of $f_{\alpha\beta}(k_\beta, \theta, \phi)$. In general, using unpolarized particles, and not considering spin, $f_{\alpha\beta}(k_\beta, \theta, \phi)$ will not depend on the angle ϕ. This is a consequence of the fact that the incoming beam has zero projection of the angular momentum in the direction of the incident beam. Therefore, in the outgoing channel the angular momentum will mantain its zero projection and therefore the outgoing wave function cannot depend on ϕ.

2.4 Actual evaluation of the spreading of the wave packet

$$m_p = 1.7 \times 10^{-24} \text{gr} \tag{2.24}$$

$$1\text{MeV} = 1.6 \times 10^{-6} \text{erg} = 1.6 \times 10^{-6} \text{gr} \frac{\text{cm}^2}{\text{sec}^2} \tag{2.25}$$

$$\begin{aligned} v &= \sqrt{\tfrac{2E}{m}} = \sqrt{\tfrac{3.2E \times 10^{-6}}{1.7 \times 10^{-24}\text{gr}} \text{gr} \tfrac{\text{cm}^2}{\text{sec}^2}} \\ &= 10^9 \sqrt{1.88 E(\text{MeV})} \\ &\approx 1.4 \times 10^9 \sqrt{E(\text{MeV})} \tfrac{\text{cm}}{\text{sec}} \end{aligned} \tag{2.26}$$

For 20 MeV protons ($E = 20$ MeV)

$$v \approx 6 \times 10^9 \frac{\text{cm}}{\text{sec}} \tag{2.27}$$

$$\frac{\Delta W}{W} \approx \frac{\hbar L}{m W^2 v} \tag{2.28}$$

$$L \approx 10^2 \text{cm} \qquad\qquad W \approx 10^{-1} \text{cm} \tag{2.29}$$

$$\hbar = 1.054 \times 10^{-27} \text{erg sec} \tag{2.30}$$

$$\begin{aligned} \frac{\Delta W}{W} &= \frac{\left(1.054\times10^{-27}\text{gr}\frac{\text{cm}^2}{\text{sec}^2}\right)\left(\text{cm}\times10^2\right)}{1.7\times10^{-24}\text{gr}\times10^{-2}\text{cm}^2\times6\times10^9\frac{\text{cm}}{\text{sec}}} \\ &= \frac{1.054\times10^{-25}}{1.7\times6}\times10^{17} = \frac{1.054}{1.7\times6}10^{-8} \\ &\approx 10^{-9} \end{aligned} \tag{2.31}$$

2
Introduction

Spectroscopy with direct reactions

Ch. 2 $\boxed{C}$ pp 49-58

In a scattering experiment, a beam of particles is directed at a target containing the scattering material, and the energy and angular distributions of the outgoing particles is measured. In the case in which the scattering material (target) corresponds to unstable, short lived nuclei, like e.g. ^{132}Sn (39.7 sec, see [3]) (but not e.g. in the case of ^{210}Pb (22.3y)), one can carry out the experiment by exchanging the roles of target and projectile, a method known as inverse kinematics.

There are two basic lengths governing nuclear reactions, namely the radius of the nucleus ($\approx 10^{-13}$cm) and the distance from the target nucleus to the detector ($\approx 10^{2}$cm). Because of the difference of these two characteristic magnitudes, one can divide the scattering process in two separate parts, namely,

1. the analysis of the outgoing beam properties in terms of optical potentials and of single–particle strengths and spreading widths, (effective) deformation parameters and average value of the pair transfer operator, and

2. the relation between these parameters and the motion of nucleons in the nuclei.

In the first part of the monograph the motion of a particle displaying a large mean free path (quantality parameter of order of unity, see ??) in a medium, can be economically described in terms of a (complex) dielectric constant (function). In the nuclear case, this function is known as the optical potential for particles moving in the continuum (scattering states). The average, single–particle potential (Hartree–Fock mean field), corresponds to the real part of this function (see ??). The imaginary part describes the coupling between the entrance, elastic channel, and the different reaction channels (inelastic, transfer, compound, etc.), leading to the depopulation of the incoming beam. Particularly strong couplings cannot be treated this way (average imaginary function), and have to be included explicitly, as a rule, within the framework of a coupled channel formalism. For bound nucleons the dielectric function is known as the self–energy function. The real part is connected with the single–particle energy centroid and strength (quasiparticle pole and residue E_{qp}, S (see ??)). The imaginary part provides a compact measure of the range of energy over which the remaining strength is distributed (single–particle fractionation), as a result of the interweaving of single–particle and collective motion (quasiparticle lifetime $\hbar/\Gamma$). In the case in which the particle–vibration coupling strength becomes too strong ($\Gamma \gtrsim E_{qp}$), a full diagonalization is called for. As example one can refer to the self–energy function arizing from the coupling with collective quadrupole surface vibrations of closed shell nuclei. In this case the self–energy function can be calculated perturbatively. As nucleons are progressively added it may happen that $E_{2^+} \to 0$ and the nucleus acquires a permanent static deformation. The large breaking of the j–strength into the two–fold degenerate (Kramers degeneracy) calls for a change of basis and the used of a "deformed" average mean field (Nilsson model).

The real and imaginary parts of the dielectric functions describing the effects, on the nucleon motion, of the virtual quantal processes which do not conserve (off–the energy shell) conserve (on–the energy shell) energy, are not independent of each other, but must fulfill a dispersion relation known as the Kramers–Krönig relation. This has profound physical implications, as well as practical (computational) consequences. In fact, virtual processes, renormalizing the properties of a particle like mass, charge, etc., can lead to divergences, which forces one to introduce (energy–, momentum–, angular momentum–, etc.) cut–offs. Now, energy conserving contributions are free from such divergences. Consequently, calculating the imaginary part of the dielectric function and making use of the (so–called substracted) dispersion relations, can provide non–divergent real components of the dielectric function. This is a distinctive property of, so called, asymptotic free theories. In these theories one knows that something quite spectacular can happen in the infrared end of the spectrum (e.g. spontaneous breaking of rotational invariance associated with nuclear deformation), but that the consequences of such a phenomenon will not depend on contributions to observables from processes above a certain cut–off which can be simply defined introducing by just choosing a model (e.g. the Nilsson model in the case under discussion). In Part I of the monograph, the techniques necessary to deal with problem 1) will be worked out, as for as needed to deal with problem 2) which is the central subject of the present monograph and is treated in Part II. Appendices are given that provide the elements of nuclear structure needed for the calculation of the differential cross sections associated with the variety of reaction processes. In other words, the spectroscopic amplitudes associated with one– and two– nucleon transfer processes, and the effective deformation parameters and transition densities associated with anelastic processes. Each chapter introduces the subject in term of the definition of the quantities needed for the calculation of the differential cross sections. Approximations are then introduced (plane–wave, no–recoil, etc.) which allows to work out most of the technical aspects of the reaction machinery almost analytically. This is done to be able to explicit the nuclear structure (details on nucleonic motion in terms of single– and two–particle as well as (particle–hole)–wavefunctions), needed to calculate the differential cross section associated with the process, setting special emphasis in the nuclear structure information one can extract from the comparison with the experimental data. In the second part of each chapter, and eventually in appendices, the full details of spectroscopic amplitudes, formfactors and of the differential cross sections, without introducing but very generic approximations, are given eventually supplemented by numerical examples. The recent availability of low energy, light ion reaction data on exotic nuclei, but not only, requires the availability of the theoretical tools to extract the corresponding nuclear structure information. In particular concerning (dressed) single–particle and (medium renormalized) pairing degrees of freedom.

Many of these questions are dealt with in a unified fashion, within the framework of the applications which constitute thepart II of this monograph.

A CD with software which allows to apply some of the concepts, ideas and techniques developed in different chapters is also provided.

2.1 Reaction channel

Let us consider the case in which the nucleus ^{18}O is the target and the projectile is a proton. The following processes can take place, among others:

$$\text{entrance channel} \left\{ \begin{array}{ll} p+^{18}O \rightarrow & p+^{18}O\,(\text{gs}) \;\; (Q=0) \;\; \text{elastic scattering}\,, \\ & \left. \begin{array}{ll} p+^{18}O^{*}\,(6\text{MeV}) & +Q_1 \\ \boxed{\cancel{p+^{18}O \rightarrow}}\;\; d+^{17}O\,(\text{gs}) & +Q_2 \\ t+^{16}O\,(\text{gs}) & +Q_3 \end{array} \right\} \text{reaction channels,} \end{array} \right. \tag{2.1}$$

where, for example,

$$Q_1 = M_p + M(^{18}O) - (M_p +^{18} O\,(6\text{ MeV})) = -6\text{MeV}\,, \tag{2.2}$$

It is a very basic concept, and a necessarily loose one. It can be determined by specifying the products entering in the channel, the energy, spin, direction of outgoing particles, etc.

$$Q_2 = M_p + M(^{18}O) - (M_d + ^{17}O\ (\text{gs})) \tag{2.3}$$

$$Q_3 = M_p + M(^{18}O) - (M_t + ^{16}O\ (\text{gs})) \tag{2.4}$$

and where ^{18}O* labels the nucleus ^{18}O in an excited state. In general if $Q > 0$ the reaction can proceed at zero bombarding energy. For $Q < 0$ the reaction is not observed below the threshold E_t which is defined, for a general reaction $A(a, b)B$ as

$$E_{CM} = \frac{1}{2}\frac{M_a M_A}{M_a + M_A}V_a^2 = \frac{M_A}{M_a + M_A}E_{lab} = \frac{1}{1 + (M_a/M_A)}E_{lab} \tag{2.5}$$

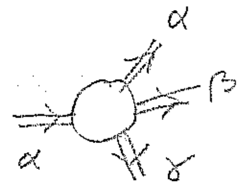
($1/2M_aV_a^2$ is the total energy of the system). If $E = |Q|$ we have

$$E_t^{lab} = \frac{M_a + M_A}{M_A}|Q| \rightarrow E_t^{lab} = \left(1 + \frac{M_a}{M_A}\right)|Q| \tag{2.6}$$

For the particular case

$$\begin{array}{l} p +^{18} O \rightarrow p +^{18} O^* \quad (6\ \text{MeV}) \\ E_t^{lab} = \frac{19}{18} \times 6\ \text{MeV} \end{array} \tag{2.7}$$

A complete specification of the type of two–particle breakup and of the internal states of the two particles, is called a channel and is specified by the product $\Psi_\alpha = \Psi_a\Psi_A$ of the (bound) internal wavefunctions of the two nuclei. Here A and a denote the nuclei into which the system breaks up and also their state of excitation, their angular momenta and the projection of their angular momenta. (see Fig.)



The same word channel is understood sometimes to include all the properties already mentioned, together with a definite value of the orbital angular momentum of the relative motion of the centers of mass of the two separating systems.

As all important concepts in physics, reaction channel is easy to apprehend but quite taxing to properly define. Let us profit of it without spending much time in reading it but better define intuitive

2.2 The reaction cross section

The initial situation can be described by a plane wave *,

$$\Psi_{inc} = \Psi_\alpha e^{ik_\alpha z} \tag{2.8}$$

which represents a beam of particles of unit density, incident upon the scattering center in the z-direction. The price to pay for such a simplification is that momentum and energy are then absolutely defined, and

* This free particle wavefunction can be normalized in a given ~~volume or requiring the~~ function to obey periodic boundary conditions inside a ~~box~~ (see Appendix)

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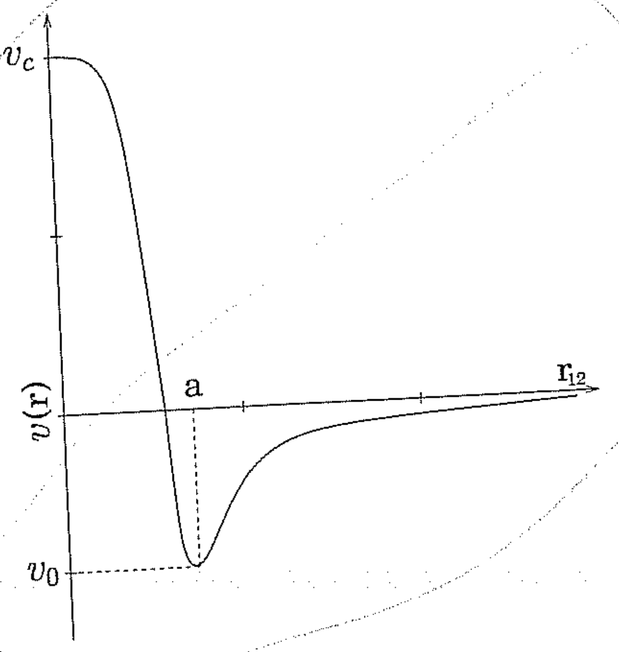

Fig. 2.1. Schematic representation of a reaction experiment

$$\begin{aligned} L &\approx 10^4\text{cm}, \\ v &\approx 2\times 10^5\text{cm/s}, \\ W &\approx 10^{-1}\text{cm}, \\ \tfrac{\Delta W}{W} &\approx 10^{-2}, \end{aligned} \tag{2.13}$$

The ratio is still small, but just on the limit of becoming important. Still if this ratio would be of order unit we could use the same concepts, but we should treat with more detail the problem of how the wave packet is constructed and the possible interference at the edge of the beam, with the outgoing wave packet.

The scattered wave (asymptotic region) must be the solution of the free field equation

$$\begin{aligned} H\Psi_{\text{scatt}} &= E\Psi_{\text{scatt}}, \\ E &= \tfrac{\hbar^2 k_\alpha^2}{2m}, \end{aligned} \tag{2.14}$$

with

$$H = \frac{p^2}{2m} = -\frac{\hbar^2}{2m}\nabla^2 = -\frac{\hbar^2}{2m}\left\{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\hat{L}^2\right\}, \tag{2.15}$$

where $\hat{L}$ is the angular momentum operator, $m = \frac{M_B M_b}{M_b+M_B}$ the reduced mass, and k and r the relative momentum and coordinate of the nuclei b and B.

At large distances the angular momentum terms drops out as $\frac{1}{r^2}$ and it is easy to verify that the asymptotic solution is

$$\Psi_{\text{scatt}} = \frac{e^{ik_\alpha r}}{r} f_{\alpha\alpha}(E,\theta,\phi)\Psi_\alpha, \tag{2.16}$$

where Ψ_α is the intrinsic channel wavefunction. Let us now calculate the incoming current and the scattered current of particles.

For a given wave function Ψ, (describing the motion of a particle of mass m), the associated current is equal to

$$\begin{aligned} \vec{I} &= \tfrac{\hbar}{2im}\left(\Psi^*\vec{\nabla}\Psi - \left(\vec{\nabla}\Psi^*\right)\Psi\right) \\ &= \tfrac{\hbar}{m}\mathcal{I}_m\left(\Psi^*\vec{\nabla}\Psi\right). \end{aligned} \tag{2.17}$$

The incident current is equal to (cf. eq. (9))

$$\begin{aligned} \vec{I}_{\text{inc}} &= \tfrac{\hbar}{m}\mathcal{I}_m e^{-ik_\alpha z}\tfrac{\mathrm{d}}{\mathrm{d}z}\left(e^{ik_\alpha z}\right)\hat{z} \\ &= \tfrac{\hbar k_\alpha}{m}\hat{z} = v_\infty\hat{z}. \end{aligned} \tag{2.18}$$

where v_∞ is the velocity corresponding to the projectile incident energy.

The scattered current is equal to †

$$\vec{I}_{\rm scatt} \approx \frac{|f(\theta,\phi)|^2}{r^2}\frac{\hbar k_\alpha}{m}\hat{r}. \tag{2.19}$$

The differential cross section is defined as the flux of particles going into the solid angle $\mathrm{d}\Omega$ at angle (θ,ϕ), divided by the incoming flux of incoming particles.

The flux of outgoing particles is given by the projection of $I_{\rm scatt}$ on the unit $\vec{ds} = r^2 \mathrm{d}\Omega \hat{r}$ of solid angle, namely

$$\vec{I}_{\rm scatt}\mathrm{d}\vec{s} = |f(\theta,\phi)|^2 \frac{\hbar k_\alpha}{m}\mathrm{d}\Omega. \tag{2.20}$$

The incident flux is given by

$$\vec{I}_{\rm inc}\hat{z} = \frac{\hbar k_\alpha}{m}. \tag{2.21}$$

The differential cross section is defined as the flux of particles into the solid angle $\mathrm{d}\Omega$ at angle (θ,ϕ), divided by the incoming flux of incoming particles, namely

$$\mathrm{d}\sigma = |f_{\alpha\alpha}(E,\theta,\phi)|^2 \mathrm{d}\Omega \tag{2.22}$$

In other channels there will be no incident wave, and in general, the outgoing waves would have a different value of the wave number, i.e.

$$\Psi_{\rm scatt} = \frac{1}{r}e^{ik_\beta r} f_{\alpha\beta}(E',\theta,\phi). \tag{2.23}$$

The symmetries of the problem can produce limitations in the form of $f_{\alpha\beta}(k_\beta,\theta,\phi)$. In general, using unpolarized particles, and not considering spin, $f_{\alpha\beta}(k_\beta,\theta,\phi)$ will not depend on the angle ϕ. This is a consequence of the fact that the incoming beam has zero projection of the angular momentum in the direction of the incident beam. Therefore, in the outgoing channel the angular momentum will mantain its zero projection and therefore the outgoing wave function cannot depend on ϕ.

2.3 Evaluation of the spreading of the wave packet

The proton mass is

$$m_p = 1.7 \times 10^{-24}\mathrm{gr}, \tag{2.24}$$

where

$$1\mathrm{MeV} = 1.6 \times 10^{-6}\mathrm{erg} = 1.6 \times 10^{-6}\mathrm{gr}\frac{\mathrm{cm}^2}{\mathrm{sec}^2} \tag{2.25}$$

† In deriving this equation one assumes that $r \to \infty$ (asymptotic region).

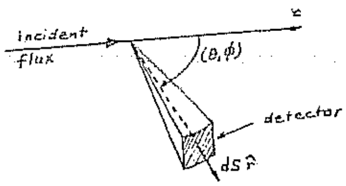

Fig. 2.2.

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Thus, the relation between velocity and energy for a proton can be written as

$$\begin{aligned} v &= \sqrt{\frac{2E}{m}} = \sqrt{\frac{3.2E\times10^{-6}}{1.7\times10^{-24}\mathrm{gr}}\mathrm{gr}\frac{\mathrm{cm}^2}{\mathrm{sec}^2}}, \\ &= 10^9\sqrt{1.88E(\mathrm{MeV})}, \\ &\approx 1.4\times10^9\sqrt{E(\mathrm{MeV})}\frac{\mathrm{cm}}{\mathrm{sec}}. \end{aligned} \tag{2.26}$$

For 20 MeV protons ($E = 20$ MeV) one obtains

$$v \approx 6\times10^9\frac{\mathrm{cm}}{\mathrm{sec}} \tag{2.27}$$

Thus

$$\frac{\Delta W}{W} \approx \frac{\hbar L}{mW^2 v}, \tag{2.28}$$

Typical values of L and W are

$$L \approx 10^2\mathrm{cm}, \qquad W \approx 10^{-1}\mathrm{cm}. \tag{2.29}$$

Using

$$\hbar = 1.054\times10^{-27}\,\mathrm{erg\ sec} \tag{2.30}$$

one obtains

$$\begin{aligned} \frac{\Delta W}{W} &= \frac{\left(1.054\times10^{-27}\mathrm{gr}\frac{\mathrm{cm}^2}{\mathrm{sec}^2}\right)\left(\mathrm{cm}\times10^2\right)}{1.7\times10^{-24}\mathrm{gr}\times10^{-2}\mathrm{cm}^2\times6\times10^9\frac{\mathrm{cm}}{\mathrm{sec}}} \\ &= \frac{1.054\times10^{-25}}{1.7\times6}\times10^{17} = \frac{1.054}{1.7\times6}10^{-8} \\ &\approx 10^{-9} \end{aligned} \tag{2.31}$$