Appendix 2.E

Phase transitions and fluctuations*)

[vortices] also ~~nuclei~~

~~domain wall~~

~~Time reversal~~ ~~Valtellina~~

Empty space, also with the quantal vacuum zero point fluctuations, is thought to be homogeneous and isotropic. Translational and rotational symmetry follows. But crystals, for example, of which all rocks are made, are neither homogeneous nor isotropic, displaying emergent properties like rigidity.

Not only a crystal occupies and defines a fixed position and a priviledged direction in space. Translational symmetry and isotropy is broken everywhere within it, in that the individual atoms all occupy fixed positions and varied groups of them define particular directions.

Lattice phonons are the corresponding fluctuations associated and restoring the broken symmetries by the individual atoms, while translation and rotation of the crystal is associated with symmetry restoration of the system as a whole.

Similarly, in a ferromagnetic crystal, where magnetization acquires

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*) See e.g. Anderson (1984) Book Basic Notions, Caianello (1964) and (1976) Gauge theories and Modern.

For a short overview see Brink and Broglia (2005) Ch. 1, p. 27. A more detailed account of the phenomenon is done in Ch. 6.

a value different from zero below the Curie temperature breaking rotational invariance. In this case, spin waves are associated with symmetry restoration. In other words, another emergent property of spontaneous symmetry breaking – namely the fact that many-body systems can have ground states which do not have the same symmetry as the Hamiltonian itself – aside from (generalized) rigidity, is the existence of long-wavelength collective motions of the order parameter (amplitude of density wave in a crystal, magnetization in a ferromagnet, etc), such as phonons and spin waves, which are the models of Goldstone and Higgs phenomena in field theories.

Superconductors break gauge symmetry, intimately related to charge and particle number conservation. A metallic superconductor has a rather perfect internal gauge phase order. Within this context, the BCS mechanism is most relevant to the mass problem because it introduces an energy (mass) gap for fermions, and the Goldstone – zeropoint motion of the total order parameter α_0 which is large and rapid ($\dot{\phi} = \lambda/\hbar$; pairing rotational bands in nuclei) – and Higgs (plasmon)*) modes.

Another major emergent property in broken symmetry systems is the appearance of singularities and textures of the order parameter like e.g. vortices in superfluid systems – namely, the possibility for a rotational invariant, spherical, quantal system to rotate (note van der Waals 1 – Cooper pair in ^{11}Li) – and of domain walls in ferromagnets.

Another feature of spontaneous symmetry breaking (SSB) is the

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*) For details see e.g. Shimizu et al (1989) Fig. 40

possibility of hierarchical*) SSB or "tumbling". Namely, SSB can be a cause for another SSB at a lower energy scale, an example being the chain crystal-phonon-superconductivity. Its Goldstone mode is the phonon which induces the Cooper pairing of electrons to cause superconductivity.

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*) See Broken Symmetry, Selected papers of Y. Nambu, Eds. T. Eguchi and K. Nishijima, World Scientific, Singapore, 1995.

Milano 11/10/17
QSE

reference Trieste P. W. Anderson + Aa. Bohr.

2.E.1 Pairing phase transition in small Particles

For bulk pure superconductors the large pair coherence length implies a very sharp, extremely narrow critical region as a function of the temperature (or of the magnetic field) so that, for example, the observed specific heat can be accurately described by the standard mean-field BCS approach. ~~For the nuclear system, the pairing interaction always leads to a coherence length which is large compared with~~

However, the size of the critical region becomes larger as the dimensions of the system decrease, below the coherence length. A limiting case corresponds to particles with dimensions smaller than the coherence length which form essentially zero-dimensional systems*).

*) See Perenboom et al (1981), P.W. Anderson et al (1959), Kubo (1962), Kubo (1968), Mühlschlegel et al (1972), Lauritzen et al (1993).

An interesting question one may pose is for which size of particles will superconductivity actually cease. It was conjectured*) that the usual Cooper instability will not exist any more and therefore superconductivity should disappear if the small superconducting particles are in the quantum-size-effect (QSE) regime when the energy difference δ between two discrete one-electron states is comparable to the energy gap of the superconducting state (Anderson criterion). This means that small superconductors with fewer than about 10^4 to 10^5 electrons should be affected by this effect.

Within this context, important information concerning pairing fluctuations is provided by the study of small Sn-particles at low temperatures**). To describe these fluctuations, use of techniques has been made that take into account large amplitude fluctuations

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*) Anderson (1959)

**) Perenboom et al (1981)

of the order parameter. In particular (7)
the static-path approximation*) (SPA)
with quadratic corrections**) which
mimic the RPA corrections.

The relevant parameter of this sort of calculations is the ratio of δ and kT_c, i.e.

$$\bar{\delta} = \frac{\delta}{kT_c} = \frac{2}{N(0)\,kT_c}, \quad (2.E.1)$$

where $N(0)$ is the single-spin energy density of states at the Fermi energy.

Fig. 1 Lauritzen

In Fig. 2.E.1 the results of the (SPA) +

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*) Mühlschlegel et al (1972)

**) Lauritzen et al (1993). It is of notice ~~that~~ in Mühlschlegel et al (1972), the basic result of the work of the same authors (Denton et al (1971)) on small normal-metal particles is mentioned to be the restriction to fixed electron number and the assertion is made that in the supercon-ducting case (where they use the grand canonical ensemble) the above restriction to fixed electron number is expected to be even more important.

quadratic correction model for specific heat and spin susceptibility as a function of temperature is shown for $\bar{\delta}$ ranging from $\delta = 0.01$ to $\bar{\delta} = 0.5$. The dashed curves show the static path results while the solid lines include the RPA-like corrections. For comparison, the BCS-results labelled by $\bar{\delta} = 0$ corresponding to the bulk system are also displayed.

For small values of $\bar{\delta}$ the system shows a sharp phase transition which can be well described by mean field theory. For $\bar{\delta} \approx 0.5$, corresponding to a small number of particles ($\sim 10^2$), the transition region has broaden so much as to blur the phase transition. RPA corrections are important only for relatively small particles.

Fig. 4 Lauritzen

2.E.2 Fine Sn particles

The electronic specific heat of small particles particles of Sn in a matrix with an average diameter ranging from 25 to 220 nm over a temperature range from $0.4 T_c$ to $1.5 T_c$ in zero magnetic field (Fig. 2.E.2) have been measured.*) The ~~small~~ particles were insulated from each other by oxide layers;*) In the same figure the results of the model discussed in the previous section is also shown.

There is a good quantitative agreement between the results of the model and the experimental findings**).

*) see Perenboom et al (1981) and references therein.

**) For details we refer to Lauritzen et al (1993) and Mühlschlegel et al (1972).

quadrupole pairing large σ excited 0^+ states

2.E.3 Time-reversal response function

To explain dirty superconductors a BCS type theory based on pairing each one-electron state with its exact time reverse, a generalization of the $\vec{k}$ up, $-k$ down pairing was developed*). It could explain the experimental observation that ~~while the pairing gap of a dirty superconductor is smaller than that of pure case, it is not so different from it. In fact~~

starting with a pure single crystal of a superconducting material, there is usually a rather sharp initial drop in the superconducting transition temperature as the first small percentage of chemical imperfection is added. If the impurities which are introduced are ordinary ones, the sharp drop stops rather soon and is replaced by a more gradual behavior. On the

*) Anderson (1959).

other hand, if the introduced impurities are magnetic ions rather than ordinary chemical impurities, the initial drop continues, and superconductivity is eventually destroyed.

In fact, it is clear that when time-reversal transformation cannot be made, that is, when the energy of the state $|\nu\rangle$ is not the same as the energy of $|\bar{\nu}\rangle$, a situation not found in in the case of ordinary impurities which do not affect Kramer's degeneracy, pair correlation weakens, and the transition temperature will continue to drop as the degree of magnetic scattering increases.

And within this scenario, one comes back again to the question of what size of particles and at what degree of (magnetic) scattering will superconductivity cease.

It is of notice that the nucleus

is itself a dirty superconductor in the sense that it can be viewed as a very fine particle — spatial quantization forcing it to be close to the QSE regime — with only a very small number of Fermi particles in it, in which Cooper pairs are associated with $|\nu\bar{\nu}\rangle$ correlations. Thus, rapid rotation of the nucleus as a whole which affects differently the different originally Kramers degenerate orbits, can be viewed as introducing an ever increasing amount of magnetic impurities. The associated eventual superfluid–normal phase transition taking place for values of the angular momentum*) $I_c \approx 20\hbar$. Physically this band crossing between the superfluid S (ground state based) and normal N (pair vibration two-quasiparticle based) bands, leads to the phenomenon known as backbending. Within the present scenario the S-N phase transition in nuclei can be studied in term of individual quantal states.

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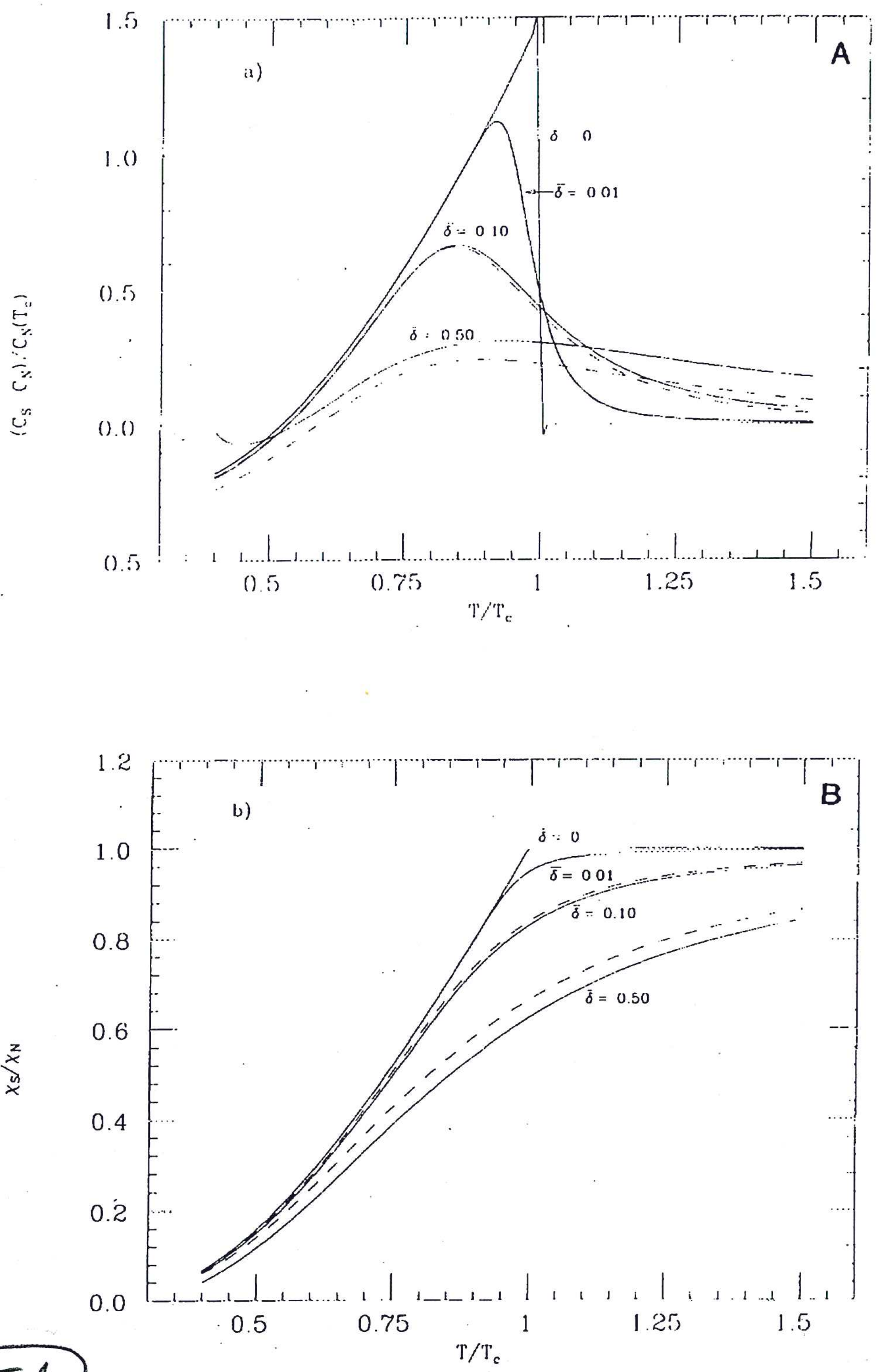
*) see e.g. Brink and Broglia Ch. 6, Fig. 6.3 and refs. therein.

Although not technically simple, two-nucleon transfer reactions induced by heavy ions can be employed in such studies*)

*) Broglia and Gallardo (1985), Shimizu et al (1989), Fig. 39.

14



2,E,1

FIG. 1. Specific heat and magnetic susceptibility in the static path approximation (dashed lines) and including quadratic corrections (solid lines). The curves labeled $\bar{\delta}=0$ show the finite temperature BCS results (after Lauritzen et al (1993)),

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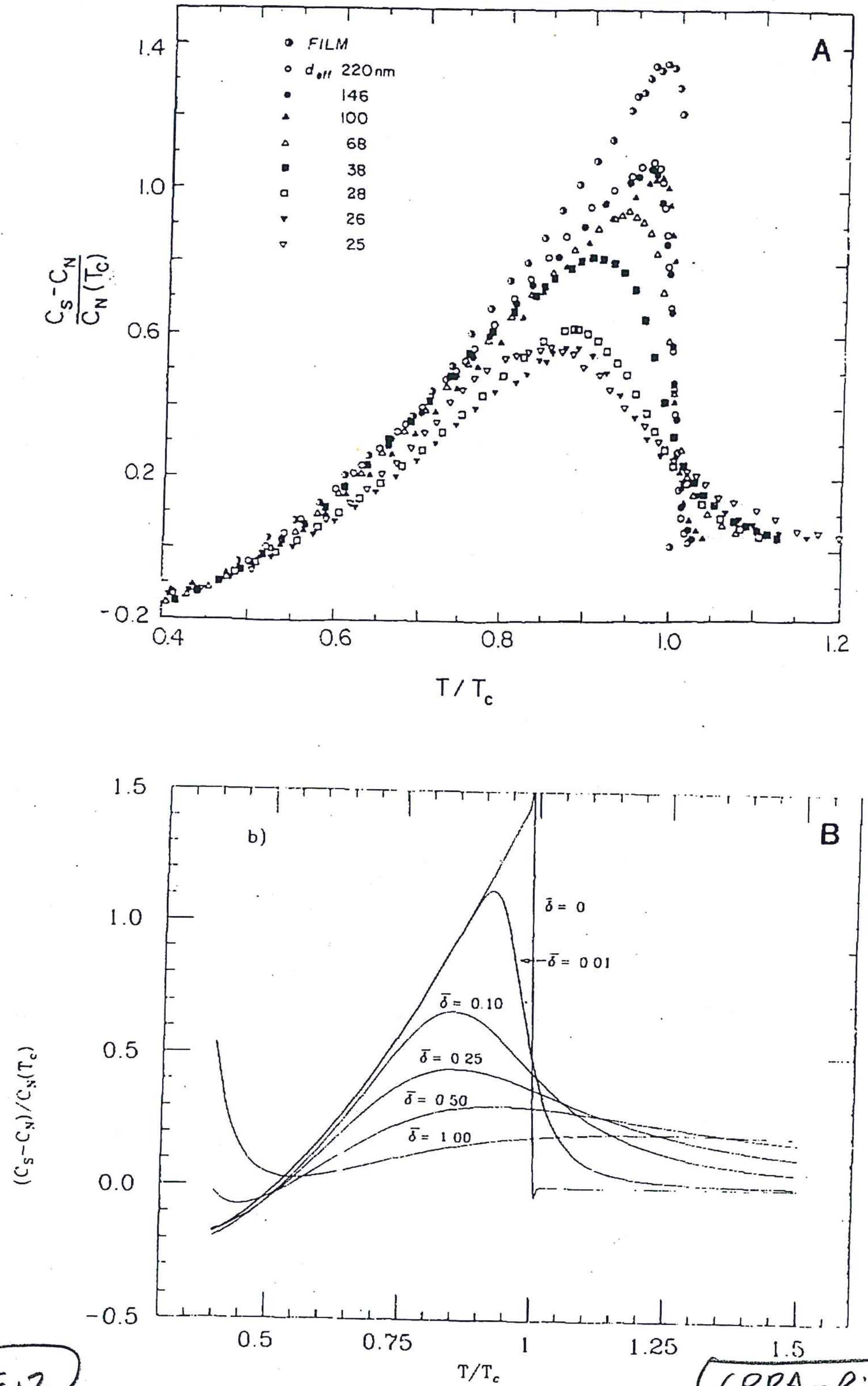

2.E.2

(RPA-like)

FIG. 4. (a) The measured specific heat is shown for a variety of Sn-particles. The figure is taken from ~~Ref. [16]~~ T. Tsuboi and T. Suzuki (1977). (b) The results of static path approximation with quadratic corrections are shown (after Lauritzen et al (1993)).

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