7.2.~~1~~7 coordinates for the successive transfer

In the standard configuration in which the integrals (7.135) and (7.136) are to be evaluated, we have

$$\mathbf{r}_{Cc} = r_{Cc}\,\hat{\mathbf{z}}, \qquad \mathbf{r}_{b1} = r_{b1}(-\cos\theta\,\hat{\mathbf{z}} - \sin\theta\,\hat{\mathbf{x}}). \tag{7.137}$$

Now,

$$\begin{aligned}\mathbf{r}_{C1} &= \mathbf{r}_{Cc} + \mathbf{r}_{c1} = \mathbf{r}_{Cc} + \frac{m_b}{m_b+1}\mathbf{r}_{b1}\\ &= \left(r_{Cc} - \frac{m_b}{m_b+1}r_{b1}\cos\theta\right)\hat{\mathbf{z}} - \frac{m_b}{m_b+1}r_{b1}\sin\theta\hat{\mathbf{x}},\end{aligned} \tag{7.138}$$

and

$$\mathbf{r}_{Bb} = \mathbf{r}_{BC} + \mathbf{r}_{Cb} = -\frac{1}{m_B}\mathbf{r}_{C1} + \mathbf{r}_{Cb}. \tag{7.139}$$

~~But~~ substituting the relation

$$\mathbf{r}_{Cb} = \mathbf{r}_{Cc} + \mathbf{r}_{cb} = \mathbf{r}_{Cc} - \frac{1}{m_b+1}\mathbf{r}_{b1}, \tag{7.140}$$

~~By substituting in~~ one (7.139) ~~we get~~ gets

$$\mathbf{r}_{Bb} = \left(\frac{m_B-1}{m_B}r_{Cc} + \frac{m_b+m_B}{m_B(m_b+1)}r_{b1}\cos\theta\right)\hat{\mathbf{z}} + \frac{m_b+m_B}{m_B(m_b+1)}r_{b1}\sin\theta\hat{\mathbf{x}}. \tag{7.141}$$

The primed variables are arranged in a similar fashion,

$$\mathbf{r}'_{Cc} = r'_{Cc}\,\hat{\mathbf{z}}, \qquad \mathbf{r}'_{A2} = r'_{A2}(-\cos\theta'\,\hat{\mathbf{z}} - \sin\theta'\,\hat{\mathbf{x}}). \tag{7.142}$$

~~And we get~~ Thus,

$$\mathbf{r}'_{c2} = \left(-r'_{Cc} - \frac{m_A}{m_A+1}r'_{A2}\cos\theta'\right)\hat{\mathbf{z}} - \frac{m_A}{m_A+1}r'_{A2}\sin\theta'\hat{\mathbf{x}}, \tag{7.143}$$

and

$$\mathbf{r}'_{Aa} = \left(\frac{m_a-1}{m_a}r'_{Cc} - \frac{m_A+m_a}{m_a(m_A+1)}r'_{A2}\cos\theta'\right)\hat{\mathbf{z}} - \frac{m_A+m_a}{m_a(m_A+1)}r'_{A2}\sin\theta'\hat{\mathbf{x}}. \tag{7.144}$$

7.2.~~2~~8 simplifying ~~simplification of~~ the vector coupling

We will now turn our attention to the vector–coupled quantities in (7.135) and (7.136),

$$\sum_M \langle l_c\;0\;l\;M|K\;M\rangle \left[Y^{l_f}(\hat{r}_{C1})Y^{l_i}(\theta+\pi,0)\right]^K_M Y^{l*}_M(\hat{r}_{Bb}), \tag{7.145}$$

and

$$\sum_M \langle l_c\;0\;l\;M|K\;M\rangle \left[Y^{l_f}(\hat{r}'_{A2})Y^{l_i}(\hat{r}'_{c2})\right]^{K*}_M Y^{l}_M(\hat{r}'_{Aa}). \tag{7.146}$$

~~We will simplify these expressions in order to ease the computational numerical evaluation.~~ We can express them both as

$$\sum_M f(M), \tag{7.147}$$

in the case of

Consequently,

where ~~(see e.g.~~ (7.145), we have

$$f(M) = \langle l_c\ 0\ l\ M|K\ M\rangle \left[Y^{l_f}(\hat{r}_{C1})Y^{l_i}(\theta+\pi,0)\right]^K_M Y^{l_c*}_M(\hat{r}_{Bb}). \tag{7.148}$$

Note that all the vectors that come into play in the above expressions are in the ~~xz~~ (x, z)–plane, ~~and thus~~ the azimuthal angle ϕ is always equal to zero. Under these circumstances and for time–reversed phases $(Y^{L*}_M(\theta,0) = (-1)^L Y^L_M(\theta,0))$ ~~so it is easy to verify that~~ one has

$$f(-M) = (-1)^{l_c+l_f+l_i+l} f(M). \tag{7.149}$$

~~From (7.149), we have~~ Consequently,

$$\sum_M \langle l_c\ 0\ l\ M|K\ M\rangle f(M) = \langle l_c\ 0\ l\ 0|K\ 0\rangle f(0) + \sum_{M>0} \langle l_c\ 0\ l\ M|K\ M\rangle f(M)\left(1+(-1)^{l_c+l+l_i+l_f}\right). \tag{7.150}$$

(Consequently, in the case in which ~~We see that when~~ $l_c + l + l_i + l_f$ is odd we only have to evaluate the $M = 0$ contribution. This consideration is useful to restrict the number of numerical operations needed to calculate the transition amplitude.

~~7.3~~ 2.9 non–orthogonality term

We write the non–orthogonality contribution to the transition amplitude (see [?]):

$$\begin{aligned}
T^{NO}_{2NT} = & 2\sum_{\substack{\sigma_1\sigma_2\\ \sigma'_1\sigma'_2\\ KM}} \int d^3r_{Cc}d^3r_{b1}d^3r_{A2}d^3r'_{b1}d^3r'_{A2}\chi^{(-)*}(\mathbf{k}_{Bb},\mathbf{r}_{Bb}) \\
& \times \left[\psi^{j_f}(\mathbf{r}_{A1},\sigma_1)\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\right]^{0*}_0 v(r_{b1}) \left[\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\psi^{j_i}(\mathbf{r}_{b1},\sigma_1)\right]^K_M \\
& \times \left[\psi^{j_f}(\mathbf{r}'_{A2},\sigma'_2)\psi^{j_i}(\mathbf{r}'_{b1},\sigma'_1)\right]^{K*}_M \left[\psi^{j_i}(\mathbf{r}'_{b1},\sigma'_1)\psi^{j_i}(\mathbf{r}'_{b2},\sigma'_2)\right]^0_0 \chi^{(+)}(\mathbf{r}'_{Aa}).
\end{aligned} \tag{7.151}$$

This expression is equivalent to (7.110) if we make the replacement

$$\frac{2\mu_{Cc}}{\hbar^2}G(\mathbf{r}_{Cc},\mathbf{r}'_{Cc})v(r'_{A2}) \to \delta(\mathbf{r}_{Cc}-\mathbf{r}'_{Cc}). \tag{7.152}$$

Looking at the partial–wave expansions of $G(\mathbf{r}_{Cc},\mathbf{r}'_{Cc})$ and $\delta(\mathbf{r}_{Cc}-\mathbf{r}'_{Cc})$ (see Section ??), we find that we can use the above expressions for the successive transfer with the replacement

$$i\frac{2\mu_{Cc}}{\hbar^2}\frac{f_{l_c}(k_{Cc},r_<)P_{l_c}(k_{Cc},r_>)}{k_{Cc}} \to \delta(r_{Cc}-r'_{Cc}). \tag{7.153}$$

We thus have

$$\begin{aligned}
T^{NO}_{2NT} = & \frac{512\pi^{9/2}}{k_{Aa}k_{Bb}}\frac{1}{\sqrt{(2j_i+1)(2j_f+1)}} \\
& \times \sum_K ((l_f\tfrac{1}{2})_{j_f}(l_i\tfrac{1}{2})_{j_i}|(l_fl_i)_K(\tfrac{1}{2}\tfrac{1}{2})_0)^2_K \\
& \times \sum_{l_c,l} e^{i(\sigma^l_i+\sigma^l_f)}\frac{(2l_c+1)}{\sqrt{2l+1}}Y^l_0(\hat{k}_{Bb})S_{K,l,l_c},
\end{aligned} \tag{7.154}$$

with

$$S_{K,l,l_c} = \int r_{Cc}^2\, dr_{Cc}\, r_{b1}^2\, dr_{b1}\, \sin\theta\, d\theta\, v(r_{b1}) u_{l_f}(r_{C1}) u_{l_i}(r_{b1}) \\ \times \frac{s_{K,l,l_c}(r_{Cc})}{r_{Cc}} \frac{F_l(r_{Bb})}{r_{Bb}} \\ \times \sum_M \langle l_c\; 0\; l\; M | K\; M\rangle \left[Y^{l_f}(\hat{r}_{C1}) Y^{l_i}(\theta+\pi, 0)\right]^K_M Y^{l*}_M(\hat{r}_{Bb}), \tag{7.155}$$

and

$$s_{K,l,l_c}(r_{Cc}) = r_{Cc} \int dr'_{A2}\, r'^2_{A2}\, \sin\theta'\, d\theta'\, u_{l_f}(r'_{A2}) u_{l_i}(r'_{c2}) \frac{F_l(r'_{Aa})}{r'_{Aa}} \\ \times \sum_M \langle l_c\; 0\; l\; M | K\; M\rangle \left[Y^{l_f}(\hat{r}'_{A2}) Y^{l_i}(\hat{r}'_{c2})\right]^{K*}_M Y^l_M(\hat{r}'_{Aa}). \tag{7.156}$$

2.10

7.~~4~~ Arbitrary orbital momentum transfer

the

We will now examine the case in which the two transferred nucleons carry an angular momentum Λ different from 0. Let us assume that two nucleons coupled to angular momentum Λ in the initial nucleus a are transferred into a final state of zero angular momentum in nucleus B. The transition amplitude is given by the integral

$$2 \sum_{\sigma_1 \sigma_2} \int d\mathbf{r}_{cC} d\mathbf{r}_{A2} d\mathbf{r}_{b1} \chi^{(-)*}(\mathbf{r}_{bB}) \left[\psi^{j_f}(\mathbf{r}_{A1}, \sigma_1) \psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)\right]^{0*}_0 \\ \times v(r_{b1}) \Psi^{(+)}(\mathbf{r}_{aA}, \mathbf{r}_{b1}, \mathbf{r}_{b2}, \sigma_1, \sigma_2). \tag{7.157}$$

?2

If we neglect core excitations, the above expression is exact as long as $\Psi^{(+)}(\mathbf{r}_{aA}, \mathbf{r}_{b1}, \mathbf{r}_{b2}, \sigma_1, \sigma_2)$ is the exact wavefunction. We can instead obtain an approximation for the transfer amplitude using

$$\Psi^{(+)}(\mathbf{r}_{aA}, \mathbf{r}_{b1}, \mathbf{r}_{b2}, \sigma_1, \sigma_2) \approx \chi^{(+)}(\mathbf{r}_{aA}) \left[\psi^{j_{i1}}(\mathbf{r}_{b1}, \sigma_1) \psi^{j_{i2}}(\mathbf{r}_{b2}, \sigma_2)\right]^\Lambda_\mu \\ + \sum_{K,M} \mathcal{U}_{K,M}(\mathbf{r}_{cC}) \left[\psi^{j_f}(\mathbf{r}_{A2}, \sigma_2) \psi^{j_{i1}}(\mathbf{r}_{b1}, \sigma_1)\right]^K_M \tag{7.158}$$

as an approximation for the incoming state. The first term of (7.158) gives rise to the simultaneous amplitude, while from second one we get the successive and the non-orthogonality contributions. To extract the amplitude $\mathcal{U}_{K,M}(\mathbf{r}_{cC})$, we define $f_{KM}(\mathbf{r}_{cC})$ as the scalar product

$$f_{KM}(\mathbf{r}_{cC}) = \left\langle \left[\psi^{j_f}(\mathbf{r}_{A2}, \sigma_2) \psi^{j_{i1}}(\mathbf{r}_{b1}, \sigma_1)\right]^K_M \middle| \Psi^{(+)}(\mathbf{r}_{aA}, \mathbf{r}_{b1}, \mathbf{r}_{b2}, \sigma_1, \sigma_2)\right\rangle \tag{7.159}$$

for fixed $\mathbf{r}_{cC}$, which can be seen to obey the equation

$$\left(\frac{\hbar^2}{2\mu_{cC}} k^2_{cC} + \frac{\hbar^2}{2\mu_{cC}} \nabla^2_{r_{cC}} - U(r_{cC})\right) f_{KM}(\mathbf{r}_{cC}) \\ = \left\langle \left[\psi^{j_f}(\mathbf{r}_{A2}, \sigma_2) \psi^{j_{i1}}(\mathbf{r}_{b1}, \sigma_1)\right]^K_M \middle| v(r_{c2}) \middle| \Psi^{(+)}(\mathbf{r}_{aA}, \mathbf{r}_{b1}, \mathbf{r}_{b2}, \sigma_1, \sigma_2)\right\rangle. \tag{7.160}$$

The solution can be written in terms of the Green function $G(\mathbf{r}_{cC},\mathbf{r}'_{cC})$ defined by

$$\left(\frac{\hbar^2}{2\mu_{cC}}k_{cC}^2+\frac{\hbar^2}{2\mu_{cC}}\nabla^2_{r_{cC}}-U(r_{cC})\right)G(\mathbf{r}_{cC},\mathbf{r}'_{cC})=\frac{\hbar^2}{2\mu_{cC}}\delta(\mathbf{r}_{cC}-\mathbf{r}'_{cC}). \tag{7.161}$$

Thus,

$$\begin{aligned}
f_{KM}(\mathbf{r}_{cC}) &= \frac{2\mu_{cC}}{\hbar^2}\int d\mathbf{r}'_{cC}G(\mathbf{r}_{cC},\mathbf{r}'_{cC})\left\langle\left[\psi^{j_f}(\mathbf{r}'_{A2},\sigma'_2)\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\right]^K_M\left|v(r_{C2})\right|\Psi^{(+)}(\mathbf{r}'_{aA},\mathbf{r}'_{b1},\mathbf{r}'_{b2},\sigma'_1,\sigma'_2)\right\rangle\\
&\approx \frac{2\mu_{cC}}{\hbar^2}\sum_{\sigma'_1\sigma'_2}\int d\mathbf{r}'_{cC}d\mathbf{r}'_{A2}d\mathbf{r}'_{b1}G(\mathbf{r}_{cC},\mathbf{r}'_{cC})\left[\psi^{j_f}(\mathbf{r}'_{A2},\sigma'_2)\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\right]^{K*}_M\\
&\times v(r'_{c2})\chi^{(+)}(\mathbf{r}'_{aA})\left[\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\psi^{j_{i2}}(\mathbf{r}'_{b2},\sigma'_2)\right]^\Lambda_\mu = \mathcal{U}_{K,M}(\mathbf{r}_{cC})\\
&+\left\langle\left[\psi^{j_f}(\mathbf{r}'_{A2},\sigma_2)\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma_1)\right]^K_M\middle|\chi^{(+)}(\mathbf{r}'_{aA})\left[\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\psi^{j_{i2}}(\mathbf{r}'_{b2},\sigma'_2)\right]^\Lambda_\mu\right\rangle.
\end{aligned} \tag{7.162}$$

Therefore

$$\begin{aligned}
\mathcal{U}_{K,M}(\mathbf{r}_{cC}) &= \frac{2\mu_{cC}}{\hbar^2}\sum_{\sigma'_1\sigma'_2}\int d\mathbf{r}'_{cC}d\mathbf{r}'_{A2}d\mathbf{r}'_{b1}G(\mathbf{r}_{cC},\mathbf{r}'_{cC})\left[\psi^{j_f}(\mathbf{r}'_{A2},\sigma'_2)\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\right]^{K*}_M\\
&\times v(r'_{c2})\chi^{(+)}(\mathbf{r}'_{aA})\left[\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\psi^{j_{i2}}(\mathbf{r}'_{b2},\sigma'_2)\right]^\Lambda_\mu\\
&-\left\langle\left[\psi^{j_f}(\mathbf{r}'_{A2},\sigma_2)\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma_1)\right]^K_M\middle|\chi^{(+)}(\mathbf{r}'_{aA})\left[\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\psi^{j_{i2}}(\mathbf{r}'_{b2},\sigma'_2)\right]^\Lambda_\mu\right\rangle.
\end{aligned} \tag{7.163}$$

When we substitute $\mathcal{U}_{K,M}(\mathbf{r}_{cC})$ into (7.158) and (7.157), the first term gives rise to the successive amplitude for the two–particle transfer, while the second term is responsible for the non–orthogonal contribution.

2.11

7.4.1 Successive contribution

We need to evaluate the integral

$$\begin{aligned}
T^{succ}_\mu = &\frac{4\mu_{cC}}{\hbar^2}\sum_{\sigma_1\sigma_2}\sum_{KM}\int d\mathbf{r}_{cC}d\mathbf{r}_{A2}d\mathbf{r}_{b1}d\mathbf{r}'_{cC}d\mathbf{r}'_{A2}d\mathbf{r}'_{b1}\left[\psi^{j_f}(\mathbf{r}_{A1},\sigma_1)\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\right]^{0*}_0\\
&\times\chi^{(-)*}(\mathbf{r}_{bB})G(\mathbf{r}_{cC},\mathbf{r}'_{cC})\left[\psi^{j_f}(\mathbf{r}'_{A2},\sigma'_2)\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\right]^{K*}_M\chi^{(+)}(\mathbf{r}'_{aA})v(r'_{c2})v(r_{b1})\\
&\times\left[\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\psi^{j_{i2}}(\mathbf{r}'_{b2},\sigma'_2)\right]^\Lambda_\mu\left[\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\psi^{j_{i1}}(\mathbf{r}_{b1},\sigma_1)\right]^K_M,
\end{aligned} \tag{7.164}$$

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where we must substitute the Green function and the distorted waves by their partial wave expansions (see Appendix). The integral over $\mathbf{r}'_{b1}$ is:

$$
\begin{aligned}
&\sum_{\sigma'_1}\int d\mathbf{r}'_{b1}\left[\psi^{j_f}(\mathbf{r}'_{A2},\sigma'_2)\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\right]^{K*}_{M}\left[\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\psi^{j_{i2}}(\mathbf{r}'_{b2},\sigma'_2)\right]^{\Lambda}_{\mu}\\
&=(-1)^{-M+j_f+j_{i1}-\sigma_1-\sigma_2}\left[\psi^{j_{i1}}(\mathbf{r}'_{b1},-\sigma'_1)\psi^{j_f}(\mathbf{r}'_{A2},-\sigma'_2)\right]^{K}_{-M}\left[\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\psi^{j_{i2}}(\mathbf{r}'_{b2},\sigma'_2)\right]^{\Lambda}_{\mu}\\
&=\sum_{\sigma'_1}\int(-1)^{-M+j_f+j_{i1}-\sigma_1-\sigma_2}\sum_P\langle K\,\Lambda\,-M\,\mu|P\,\mu-M\rangle((j_{i1}j_f)_K(j_{i1}j_{i2})_\Lambda|(j_{i1}j_{i1})_0(j_fj_{i2})_P)_P\\
&\qquad\times\left[\psi^{j_{i1}}(\mathbf{r}'_{b1},-\sigma'_1)\psi^{j_{i1}}(\mathbf{r}'_{b1},\sigma'_1)\right]^{0}_{0}\left[\psi^{j_f}(\mathbf{r}'_{A2},-\sigma'_2)\psi^{j_{i2}}(\mathbf{r}'_{b2},\sigma'_2)\right]^{P}_{\mu-M}\\
&\qquad=(-1)^{-M+j_f+j_{i1}}\sqrt{2j_{i1}+1}\,u_{l_f}(r_{A2})u_{l_{i2}}(r'_{b2})\sum_P\langle K\,\Lambda\,-M\,\mu|P\,\mu-M\rangle\\
&\qquad\times((j_{i1}j_f)_K(j_{i1}j_{i2})_\Lambda|(j_{i1}j_{i1})_0(j_fj_{i2})_P)_P((l_f\tfrac12)_{j_f}(l_{i2}\tfrac12)_{j_{i2}}|(l_fl_{i2})_P\,(\tfrac12\tfrac12)_0)_P\\
&\qquad\qquad\times\left[Y^{l_f}(\hat{\mathbf{r}}'_{A2})Y^{l_{i2}}(\hat{\mathbf{r}}'_{b2})\right]^{P}_{\mu-M}u_{l_f}(r_{A2})u_{l_{i2}}(r_{b2}).
\end{aligned}
\tag{7.165}
$$

integr. variabler

Integral over $\mathbf{r}_{A2}$ (see (7.117)):

$$
\begin{aligned}
&\sum_{\sigma_2}\int d\mathbf{r}_{A2}\left[\psi^{j_f}(\mathbf{r}_{A1},\sigma_1)\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\right]^{0*}_{0}\left[\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\psi^{j_{i1}}(\mathbf{r}_{b1},\sigma_1)\right]^{K}_{M}\\
&=-\sqrt{\frac{2}{2j_f+1}}((l_f\tfrac12)_{j_f}(l_{i1}\tfrac12)_{j_{i1}}|(l_fl_{i1})_K\,(\tfrac12\tfrac12)_0)_K\left[Y^{l_f}(\hat{\mathbf{r}}_{A1})Y^{l_{i1}}(\hat{\mathbf{r}}_{b1})\right]^{K}_{M}u_{l_f}(r_{A1})u_{l_{i1}}(r_{b1}).
\end{aligned}
\tag{7.166}
$$

Let us examine the term

$$
\sum_M(-1)^M\,\langle K\,\Lambda\,-M\,\mu|P\,\mu-M\rangle\left[Y^{l_f}(\hat{\mathbf{r}}_{A1})Y^{l_{i1}}(\hat{\mathbf{r}}_{b1})\right]^{K}_{M}\left[Y^{l_f}(\hat{\mathbf{r}}'_{A2})Y^{l_{i2}}(\hat{\mathbf{r}}'_{b2})\right]^{P}_{\mu-M}. \tag{7.167}
$$

~~By virtue of the property of Clebsh–Gordan coefficients~~ Making use of the relation

$$
\langle l_1\,l_2\,m_1\,m_2|L\,M_L\rangle=(-1)^{l_2-m_2}\sqrt{\frac{2L+1}{2l_1+1}}\langle L\,l_2\,-M_L\,m_2|l_1\,-m_1\rangle, \tag{7.168}
$$

2

the expression (7.167) is equivalent to

$$
(-1)^K\sqrt{\frac{2P+1}{2\Lambda+1}}\left\{\left[Y^{l_f}(\hat{\mathbf{r}}'_{A2})Y^{l_{i2}}(\hat{\mathbf{r}}'_{b2})\right]^{P}\left[Y^{l_f}(\hat{\mathbf{r}}_{A1})Y^{l_{i1}}(\hat{\mathbf{r}}_{b1})\right]^{K}\right\}^{\Lambda}_{\mu}. \tag{7.169}
$$

7 We now recouple

~~We re-couple~~ the foll~~owing~~ terms arising from the partial wave expansion of the incoming and outgoing distorted waves ~~to have~~ into the form

$$
\left[Y^{l_a}(\hat{\mathbf{r}}'_{aA})Y^{l_a}(\hat{\mathbf{k}}_{aA})\right]^{0}_{0}\left[Y^{l_b}(\hat{\mathbf{r}}_{bB})Y^{l_b}(\hat{\mathbf{k}}_{bB})\right]^{0}_{0} \tag{7.170}
$$

~~to have~~

$$
((l_al_a)_0(l_bl_b)_0|(l_al_b)_\Lambda(l_al_b)_\Lambda)_0\left\{\left[Y^{l_a}(\hat{\mathbf{r}}'_{aA})Y^{l_b}(\hat{\mathbf{r}}_{bB})\right]^{\Lambda}\left[Y^{l_a}(\hat{\mathbf{k}}_{aA})Y^{l_b}(\hat{\mathbf{k}}_{bB})\right]^{\Lambda}\right\}^{0}_{0}. \tag{7.171}
$$

The only term which does not vanish upon ~~that survives the~~ the integration is

$$\frac{(-1)^{\Lambda-\mu}}{\sqrt{(2l_a+1)(2l_b+1)}}\left[Y^{l_a}(\hat{\mathbf{r}}'_{aA})Y^{l_b}(\hat{\mathbf{r}}_{bB})\right]^{\Lambda}_{-\mu}\left[Y^{l_a}(\hat{\mathbf{k}}_{aA})Y^{l_b}(\hat{\mathbf{k}}_{bB})\right]^{\Lambda}_{\mu}. \tag{7.172}$$

Again, the only term surviving

$$\left\{\left[Y^{l_f}(\hat{\mathbf{r}}'_{A2})Y^{l_{i2}}(\hat{\mathbf{r}}'_{b2})\right]^{P}\left[Y^{l_f}(\hat{\mathbf{r}}_{A1})Y^{l_{i1}}(\hat{\mathbf{r}}_{b1})\right]^{K}\right\}^{\Lambda}_{\mu}\left[Y^{l_a}(\hat{\mathbf{r}}'_{aA})Y^{l_b}(\hat{\mathbf{r}}_{bB})\right]^{\Lambda}_{-\mu} \tag{7.173}$$

is

$$\frac{(-1)^{\Lambda+\mu}}{\sqrt{2\Lambda+1}}\left[\left\{\left[Y^{l_f}(\hat{\mathbf{r}}'_{A2})Y^{l_{i2}}(\hat{\mathbf{r}}'_{b2})\right]^{P}\right.\right.$$
$$\left.\left.\left[Y^{l_f}(\hat{\mathbf{r}}_{A1})Y^{l_{i1}}(\hat{\mathbf{r}}_{b1})\right]^{K}\right\}^{\Lambda}\left[Y^{l_a}(\hat{\mathbf{r}}'_{aA})Y^{l_b}(\hat{\mathbf{r}}_{bB})\right]^{\Lambda}\right]^{0}_{0}. \tag{7.174}$$

Now We couple this last term with the term $\left[Y^{l_c}(\hat{\mathbf{r}}'_{cC})Y^{l_c}(\hat{\mathbf{r}}_{cC})\right]^0_0$, ~~which arises~~ arising from the partial wave expansion of the Green function. That is,

$$\begin{aligned}
&\left[\left\{\left[Y^{l_f}(\hat{\mathbf{r}}'_{A2})Y^{l_{i2}}(\hat{\mathbf{r}}'_{b2})\right]^{P}\left[Y^{l_f}(\hat{\mathbf{r}}_{A1})Y^{l_{i1}}(\hat{\mathbf{r}}_{b1})\right]^{K}\right\}^{\Lambda}\left[Y^{l_a}(\hat{\mathbf{r}}'_{aA})Y^{l_b}(\hat{\mathbf{r}}_{bB})\right]^{\Lambda}\right]^{0}_{0}\left[Y^{l_c}(\hat{\mathbf{r}}'_{cC})Y^{l_c}(\hat{\mathbf{r}}_{cC})\right]^{0}\\
&= ((l_al_b)_\Lambda(l_cl_c)_0|(l_al_c)_P(l_bl_c)_K)_\Lambda\left[\left\{\left[Y^{l_f}(\hat{\mathbf{r}}'_{A2})Y^{l_{i2}}(\hat{\mathbf{r}}'_{b2})\right]^{P}\left[Y^{l_f}(\hat{\mathbf{r}}_{A1})Y^{l_{i1}}(\hat{\mathbf{r}}_{b1})\right]^{K}\right\}^{\Lambda}\right.\\
&\left.\left\{\left[Y^{l_a}(\hat{\mathbf{r}}'_{aA})Y^{l_c}(\hat{\mathbf{r}}'_{cC})\right]^{P}\left[Y^{l_b}(\hat{\mathbf{r}}_{bB})Y^{l_c}(\hat{\mathbf{r}}_{cC})\right]^{K}\right\}^{\Lambda}\right]^{0}_{0} = ((l_al_b)_\Lambda(l_cl_c)_0|(l_al_c)_P(l_bl_c)_K)_\Lambda\\
&\times((PK)_\Lambda(PK)_\Lambda|(PP)_0(KK)_0)_0\left\{\left[Y^{l_f}(\hat{\mathbf{r}}'_{A2})Y^{l_{i2}}(\hat{\mathbf{r}}'_{b2})\right]^{P}\left[Y^{l_a}(\hat{\mathbf{r}}'_{aA})Y^{l_c}(\hat{\mathbf{r}}'_{cC})\right]^{P}\right\}^{0}_{0}\\
&\times\left\{\left[Y^{l_f}(\hat{\mathbf{r}}_{A1})Y^{l_{i1}}(\hat{\mathbf{r}}_{b1})\right]^{K}\left[Y^{l_b}(\hat{\mathbf{r}}_{bB})Y^{l_c}(\hat{\mathbf{r}}_{cC})\right]^{K}\right\}^{0}_{0} = ((l_al_b)_\Lambda(l_cl_c)_0|(l_al_c)_P(l_bl_c)_K)_\Lambda\\
&\times\sqrt{\frac{2\Lambda+1}{(2K+1)(2P+1)}}\left\{\left[Y^{l_f}(\hat{\mathbf{r}}'_{A2})Y^{l_{i2}}(\hat{\mathbf{r}}'_{b2})\right]^{P}\left[Y^{l_a}(\hat{\mathbf{r}}'_{aA})Y^{l_c}(\hat{\mathbf{r}}'_{cC})\right]^{P}\right\}^{0}_{0}\\
&\times\left\{\left[Y^{l_f}(\hat{\mathbf{r}}_{A1})Y^{l_{i1}}(\hat{\mathbf{r}}_{b1})\right]^{K}\left[Y^{l_b}(\hat{\mathbf{r}}_{bB})Y^{l_c}(\hat{\mathbf{r}}_{cC})\right]^{K}\right\}^{0}_{0}.
\end{aligned} \tag{7.175}$$

Collecting ~~When we col~~lect all the contributions ~~pieces~~ (including the constants and phases ~~coming~~ arising from the partial wave expansion of the distorted waves and of the Green function), we ~~finally~~ get

$$\begin{aligned}
T^{succ}_{\mu} =&(-1)^{j_f+j_{i1}}\frac{2048\pi^5\mu_{Cc}}{\hbar^2k_{Aa}k_{Bb}k_{Cc}}\sqrt{\frac{(2j_{i1}+1)}{(2\Lambda+1)(2j_f+1)}}\sum_{K,P}((l_f\tfrac{1}{2})_{j_f}(l_{i2}\tfrac{1}{2})_{j_{i2}}|(l_fl_{i2})_P(\tfrac{1}{2}\tfrac{1}{2})_0)_P\\
&\times((l_f\tfrac{1}{2})_{j_f}(l_{i1}\tfrac{1}{2})_{j_{i1}}|(l_fl_{i1})_K(\tfrac{1}{2}\tfrac{1}{2})_0)_K\,((j_{i1}j_f)_K(j_{i1}j_{i2})_\Lambda|(j_{i1}j_{i1})_0(j_fj_{i2})_P)_P\\
&\times\frac{(-1)^K}{(2K+1)\sqrt{2P+1}}\sum_{l_c,l_a,l_b}((l_al_b)_\Lambda(l_cl_c)_0|(l_al_c)_P(l_bl_c)_K)_\Lambda e^{i(\sigma^{l_a}_i+\sigma^{l_b}_f)}i^{l_a-l_b}\\
&\times(2l_c+1)^{3/2}\left[Y^{l_a}(\hat{\mathbf{k}}_{aA})Y^{l_b}(\hat{\mathbf{k}}_{bB})\right]^{\Lambda}_{\mu}S_{K,P,l_a,l_b,l_c},
\end{aligned} \tag{7.176}$$

L=0

with (note that we have reduced the dimensionality of the integrals in the same fashion as for the ~~ℓ~~-angular momentum transfer calculation, see (7.132))

$$\begin{aligned} S_{K,P,l_a,l_b,l_c} = \int r_{Cc}^2\, dr_{Cc}\, r_{b1}^2\, dr_{b1}\, \sin\theta\, d\theta\, v(r_{b1}) u_{l_f}(r_{C1}) u_{l_i}(r_{b1}) \\ \times \frac{s_{P,l_a,l_c}(r_{Cc})}{r_{Cc}} \frac{F_{l_b}(r_{Bb})}{r_{Bb}} \\ \times \sum_M \langle l_c\ 0\ l_b\ M | K\ M\rangle \left[Y^{l_f}(\hat{r}_{C1}) Y^{l_i}(\theta+\pi, 0)\right]^K_M Y^{l_b}_{-M}(\hat{r}_{Bb}), \end{aligned} \tag{7.177}$$

and

$$\begin{aligned} s_{P,l_a,l_c}(r_{Cc}) = \int r'^2_{Cc}\, dr'_{Cc}\, r'^2_{A2}\, dr'_{A2}\, \sin\theta'\, d\theta'\, v(r'_{c2}) u_{l_f}(r'_{A2}) u_{l_i}(r'_{c2}) \\ \times \frac{F_{l_a}(r'_{Aa})}{r'_{Aa}} \frac{f_{l_c}(k_{Cc}, r_<) P_{l_c}(k_{Cc}, r_>)}{r'_{Cc}} \\ \times \sum_M \langle l_c\ 0\ l_a\ M | P\ M\rangle \left[Y^{l_f}(\hat{r}'_{A2}) Y^{l_i}(\hat{r}'_{c2})\right]^P_M Y^{l_a}_{-M}(\hat{r}'_{Aa}). \end{aligned} \tag{7.178}$$

~~Note that~~ We have evaluated the transition matrix element for a particular projection μ of the initial angular momentum of the two transferred nucleons. If they are coupled to a core of angular momentum J_f to total angular momentum J_i, M_i, the fraction of the initial wavefunction with projection μ is $\langle \Lambda\ \mu\ J_f\ M_i - \mu | J_i\ M_i\rangle$, and the cross section will be

$$\frac{d\sigma}{d\Omega}(\hat{\mathbf{k}}_{bB}) = \frac{k_{bB}}{k_{aA}} \frac{\mu_{aA}\mu_{bB}}{(2\pi\hbar^2)^2} \left| \sum_\mu \langle \Lambda\ \mu\ J_f\ M_i - \mu | J_i\ M_i\rangle T_\mu \right|^2 . \tag{7.179}$$

For a non polarized incident beam,

$$\frac{d\sigma}{d\Omega}(\hat{\mathbf{k}}_{bB}) = \frac{k_{bB}}{k_{aA}} \frac{\mu_{aA}\mu_{bB}}{(2\pi\hbar^2)^2} \frac{1}{2J_i+1} \sum_{M_i} \left| \sum_\mu \langle \Lambda\ \mu\ J_f\ M_i - \mu | J_i\ M_i\rangle T_\mu \right|^2 . \tag{7.180}$$

This would be the differential cross section for a transition to a definite final state M_f. If we ~~don't~~ do not measure M_f we have to sum for all M_f,

$$\frac{d\sigma}{d\Omega}(\hat{\mathbf{k}}_{bB}) = \frac{k_{bB}}{k_{aA}} \frac{\mu_{aA}\mu_{bB}}{(2\pi\hbar^2)^2} \frac{1}{2J_i+1} \sum_\mu |T_\mu|^2 \sum_{M_i, M_f} \left| \langle \Lambda\ \mu\ J_f\ M_f | J_i\ M_i\rangle \right|^2 . \tag{7.181}$$

The sum over M_i, M_f of the Clebsh–Gordan coefficients ~~is~~ give $(2J_i+1)/(2\Lambda+1)$ (see ??), ~~so we finally get~~ One then gets,

$$\frac{d\sigma}{d\Omega}(\hat{\mathbf{k}}_{bB}) = \frac{k_{bB}}{k_{aA}} \frac{\mu_{aA}\mu_{bB}}{(2\pi\hbar^2)^2} \frac{1}{(2\Lambda+1)} \sum_\mu |T_\mu|^2 . \tag{7.182}$$

where ~~we can~~ one can write

$$\begin{aligned} T_\mu &= \sum_{l_a, l_b} C_{l_a, l_b} \left[Y^{l_a}(\hat{\mathbf{k}}_{aA}) Y^{l_b}(\hat{\mathbf{k}}_{bB})\right]^\Lambda_\mu \\ &= \sum_{l_a, l_b} C_{l_a, l_b} i^{l_a} \sqrt{\frac{2l_a+1}{4\pi}} \langle l_a\ l_b\ 0\ \mu | \Lambda\ \mu\rangle Y^{l_b}_\mu(\hat{\mathbf{k}}_{bB}). \end{aligned} \tag{7.183}$$

appendix 7.B Coherence and effective formfactors

Note that (7.182) takes into account only the spins of the heavy nucleus. In a (t, p) or (p, t) reaction, we have to sum over the spins of the proton and of the triton and divide by 2. If a spin orbit term is present in the optical potential, the sum yields the combination of terms shown in section (7.1):

$$\frac{d\sigma}{d\Omega}(\mathbf{k}_{bB}) = \frac{k_{bB}}{k_{aA}}\frac{\mu_{aA}\mu_{bB}}{(2\pi\hbar^2)^2}\frac{1}{2(2\Lambda+1)}\sum_{\mu}|A_\mu|^2 + |B_\mu|^2. \tag{7.184}$$

~~7.5 Two-nucleon transfer reactions~~

~~Again we assume that the reaction is direct, and that it is adequately described by first-order distorted-wave Born approximation.~~

~~To be specific~~, ~~We~~ We will concentrate on (t, p) reaction, namely reactions of the type $A(\alpha,\beta)B$ where $\alpha = \beta + 2$ and $B = A + 2$.

The intrinsic wave functions are in this case

$$\begin{aligned}\psi_\alpha &= \psi^{J_i}_{M_i}(\xi_A)\sum_{ss'_f}\left[\chi^{s}(\sigma_\alpha)\chi^{s'_f}(\sigma_\beta)\right]^{s_i}_{M_{s_i}}\phi^{L=0}_t\Big(\sum_{i<j}|\vec r_i-\vec r_j|\Big)\\ &= \psi^{J_i}_{M_i}(\xi_A)\sum_{M_sM'_{s_f}}(sM_{s}s'_fM'_{s_f}|s_iM_{s_i})\chi^{s}_{M_s}(\sigma_\alpha)\chi^{s'_f}_{M'_{s_f}}(\sigma_\beta)\\ &\times\phi^{L=0}_t\Big(\sum_{i<j}|\vec r_i-\vec r_j|\Big)\end{aligned} \tag{7.185}$$

ℓ

while

$$\begin{aligned}\psi_\beta &= \psi^{J_f}_{M_f}(\xi_{A+2})\chi^{s_f}_{M_{s_f}}(\sigma_\beta)\\ &= \sum_{\substack{n_1l_1j_1\\n_2l_2j_2}}B(n_1l_1j_1,n_2l_2j_2);JJ'_iJ_f)\left[\phi^J(j_1j_2)\phi^{J'_i}(\xi_A)\right]^{J_f}_{M_f}\\ &\times\chi^{s_f}_{M_{s_f}}(\sigma_\beta)\end{aligned} \tag{7.186}$$

making use of the above equation one can define ~~But from eq. (7.186) is easy to see that~~ the spectroscopic amplitude B ~~is equal to~~ as

$$\begin{aligned}&B(n_1l_1j_1,n_2l_2j_2);JJ'_iJ_f)\\ &=\left\langle\psi^{J_f}_{M_f}(\xi_{A+2})\middle|\left[\phi^J(j_1j_2)\phi^{J'_i}(\xi_A)\right]^{J_f}_{M_f}\right\rangle\end{aligned} \tag{7.187}$$

where

$$\phi^J(j_1j_2) = \frac{\left[\phi_{j_1}(\vec r_1)\phi_{j_2}(\vec r_2)\right]^J - \left[\phi_{j_1}(\vec r_2)\phi_{j_2}(\vec r_1)\right]^J}{\sqrt{1+\delta(j_1,j_2)}} \tag{7.188}$$

is an antisymetrized, normalized wave function of the two transferred particles. The function $\chi^s_{M_s}(\sigma_\beta)$ appearing both in eq. (7.185) and (7.186) is the spin wave function of the proton while ~~$\chi^s(\sigma_\alpha)$ is equal to~~

$$\chi^s(\sigma_\alpha) = [\chi^{s_1}(\sigma_{n_1})\chi^{s_2}(\sigma_{n_2})]^s, \tag{7.189}$$

is the spin function of the two–neutron system.

discuss correlation aspects of pair transfer in general, and of the associated effective formfactors in particular. ~~[illegible]~~

In what follows we shall work out a simplified derivation of the simultaneous, two-nucleon transfer amplitude within the framework of first order DWBA specially suited to —

a convenient description of the intrinsic degrees of freedom of the triton is obtained by using a

~~The function $\phi_t^{L=0}$ describes the internal degree of freedom of the triton. A good description of this system is obtained by using~~ a wave function symmetric in the coordinates of all particles, i.e.

$$\phi_t^{L=0}(\sum_{i<j}|\vec{r}_i-\vec{r}_j|) = N_t\, e^{[(r_1-r_2)^2+(r_1-r_p)^2+(r_2-r_p)^2]}$$

$$= \phi_{000}(\vec{r})\phi_{000}(\vec{\rho}) \qquad (7.190)$$

where $\longrightarrow$ $\phi_{000}(\vec{r}) = R_{nl}(\nu^{1/2}r)Y_{lm}(\hat{r})$ while the

The coordinate $\vec{\rho}$ is the radius vector which measures the distance between the center of mass of the dineutron and the proton, while the ~~The~~ vector $\vec{r}$ is the dineutron relative coordinate (cf. Fig. 7B.1)

goes together with Fig 1

$$\vec{r} = \vec{r}_1 - \vec{r}_2 \quad \text{(relative distance between the neutrons)} \qquad (7.191a)$$

$$\vec{R} = \frac{\vec{r}_1+\vec{r}_2}{2} \quad \text{(coord. of the CM of the dineutron)} \qquad (7.191b)$$

$$\vec{\rho} = \vec{r}_p - \frac{\vec{r}_1+\vec{r}_2}{2} \quad \text{(distance between the CM of the dineutron and the proton)} \qquad (7.191c)$$

$$\vec{R}_2 = \vec{r}_p - \frac{\vec{r}_1+\vec{r}_2}{A+2} \quad \text{(distance of the proton from the CM of the system A+2)} \qquad (7.191d)$$

$$\vec{R}_1 = \frac{\vec{r}_p+\vec{r}_1+\vec{r}_2}{3} \quad \text{(coord. of the CM of the triton)} \qquad (7.191e)$$

To obtain the DWBA cross section we have to calculate the integral

$$T = \int d\xi_A\, d\vec{r}_1\, d\vec{r}_2\, d\vec{r}_p \chi_p^{(-)}(\vec{R}_2)\psi_\beta^*(\xi_{A+2},\sigma_\beta)V_\beta\psi_\alpha(\xi_A,\sigma_\alpha,\sigma_\beta)\psi_t^{(+)}(\vec{R}_1) \qquad (7.192)$$

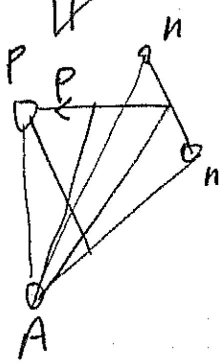


Fig. 7B.1

see next page (30) a

Instead of integrating over $\xi_A, \vec{r}_1, \vec{r}_2$ and $\vec{r}_p$ we would integrate over $\xi_A, \vec{r}, \vec{\rho}$ and $\vec{r}_p$. The Jacobian of the transformation is equal to 1, i.e. $\partial(\vec{r}_1,\vec{r}_2)/\partial(\vec{r},\vec{\rho}) = 1$.

To carry out the integral (7.192) we transform the wave function (7.188) into center of mass and relative coordinates. If we assume that both $\phi_{j_1}(\vec{r}_1)$ and $\phi_{j_2}(\vec{r}_2)$ are harmonic oscillator wave functions, this transformation can easily carried with the aid of tha Moshinsky brackets. If $|n_1l_1, n_2l_2; \lambda\mu\rangle$ is acomplete system of wave functions in the harmonic oscillator basis, depending on $\vec{r}_1$ and $\vec{r}_2$ and $|nl, NL; \lambda\mu\rangle$ is the corresponding one depending on $\vec{r}$ and $\vec{R}$, we can write

$$|n_1l_1, n_2l_2; \lambda\mu\rangle = (\sum_{nlNL}|nl, NL; \lambda\mu\rangle\langle nl, NL; \lambda\mu|)|n_1l_1, n_2l_2; \lambda\mu\rangle$$

$$= \sum_{nlNL}|nl, NL; \lambda\mu\rangle\langle nl, NL; \lambda\mu|n_1l_1, n_2l_2; \lambda\rangle \qquad (7.193)$$

The labels n, l are the principal and angular momentum quantum numbers of the relative motion, ehile N, L are the corresponding ones ~~corresponding to~~ associated with the center of mass motion of the two–neutron system. Because of energy and parity conservation we have

where the final state effective interaction $V_\beta(\rho)$ is assumed to depend only on the distance ρ between the center of mass of the di-neutron and of the proton. (cf. Eq. (6D.3) to trace a parallel between one- and two-particle transfer) where

$$\Phi^{J}(j_1 j_2) = \frac{[\phi_{j_1}(\vec{r}_1)\phi_{j_2}(\vec{r}_2)]^J - [\phi_{j_1}(\vec{r}_2)\phi_{j_2}(\vec{r}_1)]^J}{\sqrt{1+\delta(j_1,j_2)}} \quad (4)$$

is an antisymetrized, normalized wave function of the two trans-ferred particles. The function $\chi^{S}_{M_S}(\sigma_p)$ appearing both in eq(1) and (2) is the spin wave function of the proton while $\chi^{S}(\sigma_\alpha)$ is equal to

$$\chi^{S}(\sigma_\alpha) = [\chi^{S_1}(\sigma_{n_1})\,\chi^{S_1}(\sigma_{n_2})]^{S} \quad (5)$$

is the spin function of the two-neutron system.

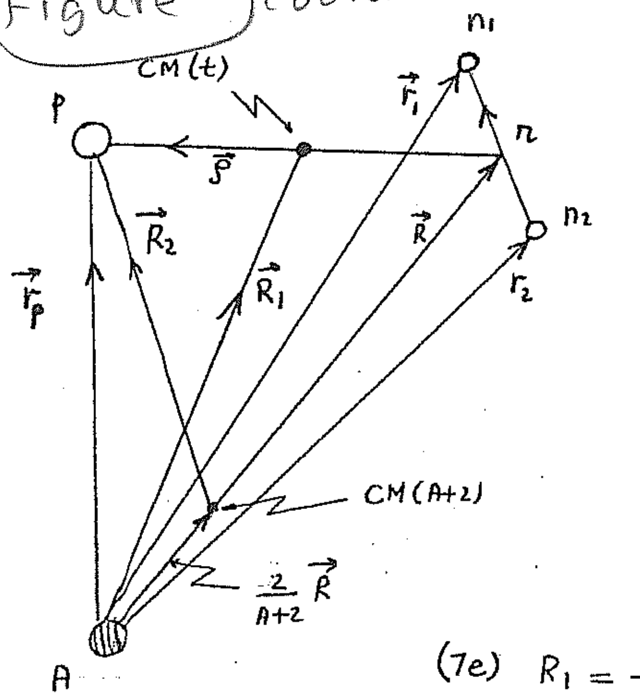
The function $\phi_t^{L=0}$ describes the internal degree of free-dom of the triton. A good description of this system is obtain-ed by using a wave function symmetric in the coordinates of all the particles, i.e.

$$\phi_t^{L=0}\Big(\sum_{i<j}|\vec{r}_i-\vec{r}_j|\Big) = N_t\, e^{-\eta^2[(r_1-r_2)^2+(r_1-r_p)^2+(r_2-r_p)^2]}$$
$$= \phi_{000}(\vec{r})\,\phi_{000}(\vec{\rho}) \quad (6)$$

$$\phi_{nlm}(\vec{r}) = R_{nl}(\nu^{1/2}r)\,Y_{lm}(\hat{r})$$

The coordinat $\vec{\rho}$ is the radius vector which measures the distance between the center of mass of the dineutron and the proton. The vector $\vec{r}$ is the di-neutron relative coordinate.

Figure coordinate



(7a) $\vec{r}=\vec{r}_1-\vec{r}_2$ (relative distance between the neutrons)

(7b) $\vec{R}=\frac{\vec{r}_1+\vec{r}_2}{2}$ (coord. of the CM of the dineutron)

(7c) $\vec{\rho} = \vec{r}_p - \frac{\vec{r}_1+\vec{r}_2}{2} = \vec{r}_p - \vec{R}$

(distance between the CM of the dineutron and the proton)

(7d) $R_2 = r_p - \frac{\vec{r}_1+\vec{r}_2}{A+2}$ (distance of the proton from the CM of the system A+2).

(7e) $R_1 = \frac{\vec{r}_p+\vec{r}_1+\vec{r}_2}{3}$ (coord. of the CM of the triton)

$$\begin{aligned} 2n_1 + l_1 + 2n_2 + l_2 &= 2n + l + 2N + L \\ (-1)^{l_1+l_2} &= (-1)^{l+L} \end{aligned} \tag{7.194}$$

The coefficients $\langle nl, NL, L|n_1l_1, n_2l_2, L\rangle$ are tabulated and were first discussed by M. Moshinsky in Nucl. Physics, 13 (1959) 104.

With the help of eq.(7.193) we can write the wave function $\psi^{J_f}_{M_f}(\xi_{A+2})$ as

$$\begin{aligned} \psi^{J_f}_{M_f}(\xi_{A+2}) &= \sum_{\substack{n_1l_1j_1\\ n_2l_2j_2\\ JJ'_i}} B(n_1l_1j_1, n_2l_2j_2; JJ'_iJ_f)\left[\phi^J(j_1j_2)\phi^{J'_i}(\xi_A)\right]^{J_f}_{M_f} \\ &= \sum_{\substack{n_1l_1j_1\\ n_2l_2j_2}} \sum_{JJ_i} B(n_1l_1j_1, n_2l_2j_2; JJ'_iJ_f) \\ &\times \sum_{M_JM'_{J_i}} \langle JM_JJ'_iM_{J_i}|J_fM_{J_f}\rangle \psi^{J'_i}_{M'_{J_i}}(\xi_A) \\ &\times \sum_{LS'} \langle S'LJ|j_1j_2J\rangle \sum_{M_LM'_S} \langle LM_LS'M'_S|JM_J\rangle \chi^{S'}_{M'_S}(\sigma_\alpha) \\ &\times \sum_{nlN\Lambda} \langle nl, N\Lambda, L|n_1l_1, n_2l_2, L\rangle \\ &\times \sum_{m_lM_\Lambda} \langle lm_l\Lambda M_\Lambda|LM_L\rangle \phi_{nlm_l}(\vec{r})\phi_{N\Lambda M_\Lambda}(\vec{R}) \end{aligned} \tag{7.195}$$

Integration over $\vec{r}$ gives

$$\langle \phi_{nlm_l}(\vec{r})|\phi_{000}(\vec{r})\rangle = \delta(l,0)\delta(m_l,0)\Omega_n \tag{7.196}$$

where

$$\Omega_n = \int R_{nl}(\nu_1^{1/2}r)R_{00}(\nu_2^{1/2}r)r^2\,dr \tag{7.197}$$

Note that there is no selection rule in the principal quantum number n, as the potential in which the two neutrons move in the triton has a frequency ν_2 which is different from the one that the two neutrons are subjected to, when moving in the system A.

Integration over ξ_A and multiplication of the spin functions gives

$$\begin{aligned} (\psi^{J_i}_{M_{J_i}}, (\overset{V'_\beta(\rho)}{\cancel{V(\vec{r}_1)+V(\vec{r}_2)+V(\vec{r}_p)-U}})\psi^{J'_i}_{M'_{J_i}}) &= \delta(J_i, J'_i)\delta(M_{J_i}, M_{J'_i})V_{\cancel{\mathrm{eff}}\,\beta}(\vec{\rho}) \qquad V_\beta(\rho) \\ (\chi^S_{M_S}(\sigma_\alpha), \chi^{S'}_{M_{S'}}(\sigma_\alpha)) &= \delta(S, S')\delta(M_S, M_{S'}) \\ (\chi^{S_f}_{M_{S_f}}(\sigma_\beta), \chi^{S'_f}_{M_{S'_f}}(\sigma_\beta)) &= \delta(S_f, S'_f)\delta(M_{S_f}, M_{S'_f}) \end{aligned} \tag{7.198}$$

The integral (7.192) can now be written as

$$
\begin{aligned}
T = &\sum_{\substack{n_1 l_1 j_1 \\ n_2 l_2 j_2}} \sum_{JM_J} \sum_{nN} \sum_{S} B(n_1 l_1 j_1, n_2 l_2 j_2; JJ_i' J_f) \\
&\times \langle JM_J J_i M_{J_i} | J_f M_{J_f} \rangle \langle SLJ | j_1 j_2 J \rangle \\
&\times \langle LM_L S M_S | JM_J \rangle \langle n0, NL, L | n_1 l_1, n_2 l_2, L \rangle \quad V_\beta \\
&\times \langle S M_S S_f M_{S_f} | S_i M_{S_i} \rangle \Omega_n \\
&\times \int d\vec{R}\, d\vec{r}_p \chi_i^{(+)*}(\vec{R}_1) \phi^*_{NLM_L}(\vec{R}) \cancel{V_{eff}}(\vec{\rho}) \phi_{000}(\vec{\rho}) \chi_i^{(+)}(\vec{R}_1)
\end{aligned}
\tag{7.199}
$$

~~where we have approximated V_β^l by an effective interaction V_{eff} depending~~ only on $\rho = |\vec{\rho}|$. It is important to point out that the two–~~body~~ interaction would act on the two–particle system at once, but the ~~single~~ particle potential would act on each particle independently. The ~~reason~~ why we can neglect the successive transfer of the nucleons (two–~~step process~~) is because the two neutrons in the triton are very strongly correlated and they build to a large extent a unity.

We now define the effective two–nucleon transfer form factor as

$$
\begin{aligned}
u^{j_i J_f}_{LSJ}(R) = &\sum_{n_1 l_1 j_1} B(n_1 l_1 j_1, n_2 l_2 j_2; JJ_i J_f) \langle SLJ | j_1 j_2 J \rangle \\
&\langle n0, NL, L | n_1 l_1, n_2 l_2; L \rangle \Omega_n R_{nL}(R)
\end{aligned}
\tag{7.200}
$$

We can now rewrite eq. (7.199) as

$$
\begin{aligned}
T = &\sum_J \sum_L \sum_S \langle JM_J J_i M_{J_i} | J_f M_{J_f} \rangle \langle S M_S S_f M_{S_f} | S_i M_{S_i} \rangle \langle LM_L S M_S | JM_J \rangle \\
\times &\int d\vec{R}\, d\vec{r}_p \chi_p^{*(-)}(\vec{R}_2) u^{j_i J_f}_{LSJ}(R) Y^*_{LM_L} V(\rho) \phi_{000}(\vec{\rho}) \chi_i^{(+)}(\vec{R}_1)
\end{aligned}
\tag{7.201}
$$

Because the di–neutron has $S = 0$, we have that

$$
\langle LM_L 00 | JM_J \rangle = \delta(J, L) \delta(M_L, M_J) \tag{7.202}
$$

and the summations over S and L disappear from eq. (7.201). Let us now

~~The integral to be carried out in eq. (7.201) is six–dimensional, and is a formidable task to calculate it exactly (one of these integrals takes ≈ 5 hs in a CDC 6600 computer).~~

~~One can~~ make also here the zero range approximation, , as done in app. for ~~the~~ one-particle transfer reactions, ~~is~~ that is,

$$
V(\rho) \phi_{000}(\vec{\rho}) = D_0 \delta(\vec{\rho}) \tag{7.203}
$$

This means that the proton interacts with the center of mass of the di–neutron, only when they are at the same point in space.

From eqs. (??) we obtain

$$
\begin{aligned}
\vec{R} &= \vec{R}_1 = \vec{r} \\
\vec{R}_2 &= \frac{A}{A+2} \vec{R}
\end{aligned}
\tag{7.204}
$$

Then eq. (7.199) can be written as