App. 6.F Simple estimates revisited (Ch. 6) 1/12/14 (1)

The interest of the simple estimates carried out in App. 2F, Sect. 2.F.1 is not so much because of the actual numerical results. For that we refer to the full, NFT microscopic calculation (Barranco et al 2001), and for the experimental test of the associated predictions carried out at TRIUMF (Tanihata et al (2008)) and eventually analized, within ~~[illegible]~~ ~~framework scenario~~, the framework of NFT in 2009 and published[1] in Potel et al (2010). The interest of 2.F.1 lies on the simplicity of the estimates, which allows the role of the different ~~mechanisms in~~ elementary modes of excitation participating ~~and the mechanism~~ in the halo binding process and the mechanism at the basis of it, to ~~[illegible]~~ explicit themselves. Within this scenario in what follows we test the stability of the results ~~which~~

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(8) of the type $|(p_{1/2}, s_{1/2})_{1^-} \otimes 1^-(^9Li)\rangle$. This is true, but with the proviso ~~that~~ the associated α-amplitude is expected in this case to be $\approx 10^{-2}$, and ~~that~~ the final state is likely to be found at an energy of ≈ 10 MeV ~~in the best of case~~, within the most favorable scenario. (9)

(6) as one could superficially expect from the largest corresponding component of (6.1.1) ($\alpha = 0.7$). In fact, the ~~[illegible]~~ associated 1^- state exists only in ^{11}Li in symbiosis with the ground state, and "evaporates" when ~~the~~ this state is ~~"removed" in the~~ annihilated in the (p,t) process. One could argue ~~that~~ (6.1.1) should contain another α-like component (7)

$((0.45)^2 + (0.55)^2 + (0.04)^2 \approx 1/2)$. This positive, albeit indirect experimental result, finds strong (circumstantial) support in the "non observation" of ~~the~~ a state $1/2^+$ $(1^- \otimes p_{3/2}(\pi))_{1/2^+}$, (5) (Tanihata et al 2008)

) is only 1/2 that expected from the two-particle strength associated with the phase space $|s^2_{1/2}(0)\rangle$, $|p^2_{1/2}(0)\rangle$, as predicted by the amplitude $\alpha = 0.7$ ($\alpha^2 \approx 1/2$), which reflects the normalization of $|0\rangle$ (Eq. (6.1.3)) (3) as an extrabonus

1) It is of notice that it is not only the observation of the population of the first excited state of ^9Li in the reaction $^{11}Li(p,t)^9Li(1/2^-; 2.69\,MeV)$ through the component $\beta = 0.1$ of the halo pair addition mode wavefunction given in Eqs (6.1.1)–(6.1.3) which confirms the predictions of the model, providing the first direct observation of phonon mediated pairing in nuclei, but, ~~also~~, equally important, the confirmation that the absolute ground state cross section associated with the ~~ground state~~ transition $^{11}Li(p,t)^9Li(gs)$ in

with respect to some of the inputs. (Results of More recent studies than those available to Bortignon et al (1998) concerning the symmetry energy indicate $V_1 = 33\,\mathrm{MeV}$ rather than 25 MeV. Furthermore, let us use 6% for the dipole EWSR (8%/e^2). Thus

$$2 \times 0.06 \frac{\hbar^2 A}{2M} = 0.12 \times 20\,\mathrm{MeV\,fm}^2 \times 11 \approx 26.4\,\mathrm{MeV\,fm}^2$$

and

$$K_1 = -0.022 \left(\frac{33}{25}\right) \mathrm{MeV\,fm}^{-2} \approx -0.029\,\mathrm{MeV\,fm}^{-2};$$

leading to

$$-K_1 \times 2 \times 6\%\,\mathrm{EWSR} \approx -(-0.029\,\mathrm{MeV\,fm}^{-2}) \times 26.4\,\mathrm{MeV\,fm}^2$$
$$\approx 0.77\,\mathrm{MeV}^2.$$

Consequently

$$(\hbar\omega_{pigmy})^2 = (0.3\,\mathrm{MeV})^2 + 0.77\,\mathrm{MeV}^2$$
$$\approx 0.86\,\mathrm{MeV}^2,$$

and

$$\hbar\omega_{pigmy} \approx 0.92\,\mathrm{MeV}.$$

Similarly

$$\Lambda^2 = \left(\frac{2 \times 0.92\,\mathrm{MeV} \times 26.4\,\mathrm{MeV\,fm}^2/(4.83)^2\,\mathrm{fm}^2}{[(0.3\,\mathrm{MeV})^2 - (0.92\,\mathrm{MeV})^2]^2}\right)^{-1},$$

$$= \left(\frac{2.1}{0.76}\right)^{-1} \mathrm{MeV}^2 = 0.36\,\mathrm{MeV}^2; \quad (\Lambda = 0.6\,\mathrm{MeV}).$$

One can then write

$$M_{ind} = -\frac{2\Lambda^2}{\hbar\omega_{pigmy}} \approx -\frac{0.72\,\mathrm{MeV}^2}{0.92\,\mathrm{MeV}} = -0.8\,\mathrm{MeV}$$

and

$$E_{corr} = |0.4\,\mathrm{MeV} - 0.1\,\mathrm{MeV} - 0.8\,\mathrm{MeV}| \approx 0.5\,\mathrm{MeV}$$

Let us now carry a second test.
Let us use the ~~above~~ ~~center of~~ the ~~TRK~~ EWSR ~~[illegible]~~ ~~taken~~ making explicit use of the fact that the nucleus has both protons and neutrons. That is replace $\hbar^2 A/2M$ by (cf. Bohr and Mottelson (1975) p. 403, cf. also Bortignon et al (1998) p. 60)

$$14.8\,\frac{NZ}{A}\,\text{fm}^2\,\text{MeV} = 14.8 \times \frac{8\times 3}{11}\,\text{fm}^2\,\text{MeV}$$

$$\approx 32.3\,\text{MeV fm}^2$$

Let us also use $V_1 = 33$ MeV instead of 25 MeV. Thus

$$K_1^0 = -0.49 \times \frac{33}{25}\,\text{fm}^{-2}\,\text{MeV} \approx -0.65\,\text{fm}^{-2}\,\text{MeV}.$$

Then, and in keeping with the fact of the explicit use of of Z (core) we introduce also the overlap $\mathcal{O}$,

$$-\mathcal{O}(K_1^0) \times 2 \times 6\% \times 32.3\,\text{MeV fm}^2$$

$$\approx 0.17 \times 0.65\,\text{fm}^{-2}\,\text{MeV} \times 0.12 \times 32.3\,\text{MeV fm}^2$$

$$\approx 0.43\,\text{MeV}^2,$$

and

$$(\hbar\omega_{pigmy})^2 = (0.3)\,\text{MeV} + 0.43\,\text{MeV}^2$$

$$\approx 0.52\,\text{MeV}^2,$$

leading to

$$\hbar\omega_{pigmy} \approx 0.72\,\text{MeV}.$$

Then

$$\Lambda^2 = \left(\frac{2\times 0.72\,\text{MeV} \times 2\times 0.06 \times 32.3\,\text{MeV fm}^2/(4.83)^2\,\text{fm}^2}{[(0.3)^2 - (0.72)^2]^2}\right)^{-1}$$

$$\approx \left(\frac{0.8}{0.4}\right)^{-1}\text{MeV}^2 \approx 0.5\,\text{MeV}^2, \quad (\Lambda \approx 0.7\,\text{MeV}),$$

Finally

$$M_{ind} \simeq -\frac{2\times 0.5\,\mathrm{MeV}^2}{0.72\,\mathrm{MeV}} \approx -1.4\,\mathrm{MeV}$$

and

$$E_{corr} \approx |0.4\,\mathrm{MeV} - 0.1\,\mathrm{MeV} - 1.4\,\mathrm{MeV}|$$
$$\approx 1\,\mathrm{MeV}$$

The above estimates together with those of Section 2.F.1 of App. 2.F imply

1
0.72
0.92
1.64
1.00
2.64

$$\hbar\omega_{pigmy} \approx 0.9 \pm 0.2\ \mathrm{MeV},$$

$$\Lambda \approx 0.6 \pm 0.1\ \mathrm{MeV},$$

and

0.6
0.7
0.6
1.9

$$E_{corr} \approx 0.6 \pm 0.4\ \mathrm{MeV}.$$

Summing, the different versions ~~Ecorr~~ of the above estimates aside from providing errors to the corresponding values, ~~[illegible]~~ tells two essential things. First, each property of a many-body system is interweaved with many others. If one changes values ~~of one at one~~ of one of them you are likely to recognize that you have to change other because of consistency. Second, ~~Calculations~~ calculations and predictions carry errors, not so much because one ~~is not able~~ does not know how to calculate, but more likely because one does not have yet a full grasp of the physics at the basis of the phenomena one wants to describe.

App. 6.F
Simple estimates revisited (Ch. 6)

1/12/14

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The interest of the simple estimates carried out in App. 2F, Sect. 2.F.1 is not so much because of the actual numerical results. For that we refer to the full, NFT microscopic calculation (Barranco et al 2001), and for the experimental test of the associated predictions carried out at TRIUMF (Tanihata et al (2008)) and eventually analized, within ~~[illegible]~~ ~~framework scenario~~, the framework of NFT in 2009 and published[1)] in Potel et al (2010). The interest of 2.F.1 lies on the simplicity of the estimates, which allows the role of the different ~~[illegible]~~ elementary modes of excitation participating ~~[illegible]~~ in the halo binding process and the mechanism at the basis of it, to ~~[illegible]~~ explicit themselves. Within this scenario in what follows we test the stability of the results ~~which~~

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(Results of More recent studies than those available to Bortignon et al (1998) concerning the symmetry energy indicate $V_1 = 33\,\text{MeV}$ rather than 25 MeV. Furthermore, let us use 6% for the dipole EWSR ($8\%/e^2$). Thus

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