Big Letters

Nuclear Structure and Reactions
paring in nuclei with Cooper pair transfer

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January 13, 2014

Chapter ~~1~~ 0

Preface

O eliminar el número del Chapter (dejar simplemente Preface) o llamarlo Ch. 0

The elementary modes of nuclear excitation are vibrations and rotations, single–particle (quasiparticle) motion, and pairing vibrations and rotations. The specific reactions probing these modes are inelastic, single– and two– particle transfer processes respectively. Within this context one can posit that nuclear structure (bound) and reactions (continuum) are but two aspects of the same physics. This is the reason why they can be treated on equal footing in terms of elementary modes of excitation, within the framework of nuclear field theory (NFT). This theory provides the rules to diagonalize in a compact and economic way the nuclear Hamiltonian for both bound and continuum states correcting for overcompletness of the basis (particle–vibration coupling (structure), non–orthogonality (reaction)), and for Pauli principle violation.

Pairing vibrations and rotations, closely connected with nuclear superfluidity are, arguably, paradigms of quantal nuclear phenomena. They thus play an important role within the field of nuclear structure. It is only natural that two–nucleon transfer plays a similar role concerning direct nuclear reactions. In fact, this is the central subject of the present monograph.

At the basis of fermionic pairing phenomena one finds Cooper pairs, weakly bound, extended, strongly overlapping (quasi–) bosonic entities, made out of pairs of nucleons dressed by collective vibrations and interacting through the exchange of these vibrations as well as through the bare NN–interaction, eventually corrected by $3N$ contributions. Cooper pairs not only change the statistics of the nuclear stuff around the Fermi surface and, condensing, the properties of nuclei close to their ground state. They also display a rather remarkable mechanism of tunnelling between target and projectile in direct two–nucleon transfer reaction. In fact, being weakly bound ($\ll \epsilon_F$, Fermi energy) they display correlations over distances (correlation length) much larger than nuclear dimensions ($\gg R$, nuclear radius). On the other hand, Cooper pairs are forced to be confined within such dimensions by the action of the average potential, which can be viewed as an external field as far as these pairs are concerned.

The correlation length paradigm comes into evidence, for example, when two

It is of notice the relation existing between K and Lindemann's classical parameter (App. 1.A)

app. 1.A

1

Chapter ~~2~~

Pair Structure and ~~reactions~~ transfer in a nutshell

~~Appendix 1.A~~

~~2~~1.1 Nuclear Structure in a nutshell

the fermion K

The low–energy properties of quantal, many–body, Fermi systems displaying sizable values of zero–point–motion (kinetic energy) of localization compared to the strength of the NN–interaction and quantified by a quantality parameter $Q \gtrsim 0.15$, are determined by the laws which control independent particle motion close to the Fermi energy ϵ_F (on–the–energy shell), and by the correlations operating among them. relations

Fig. 1.1.1

First of all, the Pauli principle, implying orbitals solidly ~~anchored to the~~ single–particle mean field, as testified by the Hartree–Fock ~~ground~~ state $|HF\rangle = \Pi_i a_i^\dagger|0\rangle$, describing a step function separation in the probability of occupied ($\epsilon_i \leq \epsilon_F$) and empty ($\epsilon_k \geq \epsilon_F$) states ~~(box 1)~~ ([redacted] box 1).

Pairing acting on fermions moving in time reversal states lying close to ϵ_F alters this picture in a conspicuous way. In particular, in the case of $S = 0$ configurations, in which case the radial component of the pair wavefunction does not display nodes. Within an energy range of the pair correlation energy $E_{corr}(\approx 2\Delta$ within BCS) centered around $\epsilon_F(E_{corr}/\epsilon_F \ll 1)$ the system is now made out of pairs of fermions which flicker in and out of the correlated ($L = 0, S = 0$) configuration (Cooper pairs, ~~box 2~~). For temperatures (intrinsic excitation energies) or stress regimes (magnetic field in metals, Coriolis force in nuclei, etc.) smaller than $\approx E_{corr}/2$ (critical value), Cooper pairs respect Bose–Einstein statistics, the single–particle orbits on which they are correlated become dynamically detached from the mean field, leading to a bosonic condensate and, at the same time, reducing in a conspicuous way the inertia of the system (e.g. the moment of inertia I of quadrupole rotational bands is much smaller than the rigid moment of inertia I_r ($I \approx I_r/3$) expected from independent particle motion). Cooper pairs exist also in situations in which the environmental condition are above critical, e.g. in metals at room temperature or nuclei at high values of the angular momentum, although they

Fig. 1.1.2

[redacted]

should not be part of box 2

should be a Figure of the text

Fig. 1.1.3

[redacted]

lying along the stability valley

Schmidt. app. 1.B

Appendix 1.B

(but at $T \approx T$ (Schmidt 1968, Schmid 1966, 196

break as soon as they are generates (pairing vibrations). While these pair addition and substraction fluctuations have little effect on condensed systems, they play an important role in mesoscopic systems, in particular in nuclei (box 3).

Within the rframework of the above picture, one can introduce at profit a collective coordinate α_0 (order parameter) which measures the number of Cooper pairs participating in the pairing condensate, and define a wavefunction for each pair $\left(U_\nu + V_\nu a^\dagger_\nu a^\dagger_{\bar{\nu}}\right)|0\rangle$ (independent pair motion, BCS approximation), adjusting the occupation parameters V_ν and U_ν (probability amplitudes that the two–fold (Kramer's–)degenerate pair state $(\nu, \bar{\nu})$ is either occupied or empty), so as to minimize the energy of the system under the condition that the average number of nucleons is equal to N_0 (Coriolis force felt, in the inrinsic system, by the pairs, equal to $-\lambda N_0$). Thus, $|BCS\rangle = \Pi_{\nu>0}\left(U_\nu + V_\nu a^\dagger_\nu a^\dagger_{\bar{\nu}}\right)|0\rangle$ provides a valid description of the paired mean field ground state, and of the associated order parameter $\alpha_0 = \langle BCS|P^\dagger|BCS\rangle$, $P^\dagger = \sum_{\nu>0} a^\dagger_\nu a^\dagger_{\bar{\nu}}$ being the pair creation operator (box 2).

but in particular in the case of light exotic halo nuclei (App. 1.C)

It is then natural to posit that two–nucleon transfer reactions are specific to probe pairing correlations in many–body fermionic systems. Examples are provided by the Josephson effect in e.g. metallic superconductors, and (t,p) and (p,t) reactions in atomic nuclei.

Due to the fact that

~~Because~~, away from the Fermi energy pair independent motion becomes independent particle motion, in particular in the nuclear case $|BCS\rangle \to |\text{Nilsson}\rangle$, one–particle transfer reactions like e.g. (d,p) and (p,d) can be used together with (t,p) and (p,t) processes as a valid tool to cross check pair correlation predictions. In particular, to shed light on the origin of pairing in nuclei: in a nutshell, the relative importance of the bare NN–interaction and the induced pairing interaction (box 4).

(within this context see also app. 1.D)

app. 1.C

While the calculation of two–nucleon transfer spectroscopic amplitudes and differential cross sections are, a priori, more involved to be worked out than those associated with one–nucleon transfer reactions, the former are, as a rule, more intrinsically "accurate" than the latter ones. ~~This is because in the first case, the actual value of the variety of quantities reflect coherence, and thus the averaging sum over many contributions~~ $\cancel{\sqrt{j_\nu+1/2}\,U_\nu V_\nu}$ ~~thus the averaging which, in spite of the fact that each of them may be somewhat inaccurate, they overall sum leads to~~ $\cancel{\alpha_0 (d\sigma(2n\text{–transfer})/d\Omega \sim |\alpha_0|^2)}$. On the other hand, $d\sigma(1n\text{–transfer})/d\Omega \sim |U_\nu|^4$ ($\sim |V_\nu|^4$) ~~then~~ depending on the accuracy with which one is able to calculate the occupancy of a pure configuration itself (box 4).

soundness of the

(app. 1.B)

accuracy which goes beyond ~~that of~~ the individual contributions.

The above parlance is reflected in the calculation of the elements resulting from the encounter of structure and reaction, namely one– and two–nucleon modified transfer formfactors. While it is usually considered that these quantities carry all the structure information associated with the calculation of the corresponding cross sections, a consistent NFT calculation of structure and reaction will posit that equally much is contained in the distorted waves describing the relative motion of the colliding systems. This is because the optical potential $(U + iW)$ which determines the scattering waves, emerges from the same modified formfactors, eventually including also inelastic processes. In other words, setting detectors in

(cf. app. 1.E)

(order parameter)

This is ~~because~~, in the case of two-nucleon ~~transfer reactions~~, the quantity which expresses the ~~collectivity of the~~ members of a pairing rotational band reflect the properties of a coherent state ($|BCS\rangle$). In other words, it results from the sum over many contribution ($\sqrt{j_\nu+1/2}\,U_\nu V_\nu$) all of them having the same phase. Consequently, errors are averaged out in the summed value $|\alpha_0|$, conferring the two nucleon transfer crossection $d\sigma(2n\text{-transfer})/d\Omega \sim |\alpha_0|^2$ a quantitative, for

e.g. a ~~definite~~ given two–particle transfer channel like $A + t \rightarrow B(= A + 2) + p$, one needs to know what the single–particle states and collective modes of the systems A, $F(= A + 1)$ ~~and~~ and B are, respectively, ~~as well as their~~ interweaving ~~leading to~~ dressed particle states (quasiparticles) fermions) ~~are~~ renormalized normal modes of excitation (bosons) ~~etc~~. But these are essentially all the elements needed to calculate the processes leading to the depopulation of the flux of the incoming channel ($A + t$ in the case under discussion). In particular, and assuming to work with spherical nuclei, so as to avoid strong inelastic processes, one–particle transfer is, as a rule (in particular Q–value closed channels), the main depopulation process, in keeping with the long range tail of the associated formfactor as compared to that of other processes, e.g. inelastic processes.

e.g. Furthermore, one needs to to take into account the of these modes of excitation which result in and vibrational e.g.

In keeping with this fact, and because U and W are connected by the Kramers–Krönig generalized dispersion relation (fluctuation dissipation theorem), it is possible to calculate the nuclear dielectric function (optical potential) ~~needed to describe the $A + a \rightarrow B + p$ process in question.~~ associated with the elastic channels under discussion, (i.e. (A, t) and (B, p) in the present case)

Concerning the modified formfactor associated with ~~this~~ the (t, p) process, we shall see in ~~the next~~ Chapter 6, that it can be written as

$$
\begin{aligned}
u_{LSJ}^{J_iJ_f}(R) = \sum_{\substack{n_1l_1j_1 \\ n_2l_2j_2, n}} & B(n_1l_1j_1, n_2l_2j_2; JJ_iJ_f) \\
& \langle SLJ|j_1j_2J\rangle\langle \cancel{n0}\, nD, NL, L|n_1l_1, n_2l_2; L\rangle \\
& \Omega_n R_{NL}(R)\,),
\end{aligned}
$$

where the overlaps

$$
\begin{aligned}
& B(n_1l_1j_1, n_2l_2j_2; JJ_iJ_f) \\
& = \langle \Psi^{J_f}(\xi_{A+2})| \left[\phi^J(n_1l_1j_1, n_2l_2j_2), \Psi^{J_i}(\xi_A)\right]^{J_f}\rangle
\end{aligned}
$$

and

$$
\Omega_n = \langle \phi_{nlm_l}(\mathbf{r})|\phi_{000}(\mathbf{r})\rangle
$$

zero

lower case

~~encodes~~ encode for the physics of particle–particle (but also, to a large extent, particle–hole) correlations in nuclei, $\langle SLT|j_1j_2J\rangle$ and $\langle \cancel{n0}\, n0, NL, L|n_1l_1, n_2l_2; L\rangle$ being $LS - jj$ and Moshinsky transformation brackets, keeping track of symmetry and number of degrees conservation. In fact, the two–nucleon spectroscopic amplitude (B–coefficient) and the overlap Ω_n reflect the parentage ~~in~~ with which the nucleus B can be written in terms of the system A and a Cooper pair,

$$
\Psi_{exit} = \Psi_{M_f}^{J_f}(\xi_{A+2})\chi_{M_{s_f}}^{S_f}(\sigma_p),
$$

where

$$
\begin{aligned}
\Psi_{M_f}^{J_f}(\xi_{A+2}) &= \sum_{\substack{n_1l_1j_1 \\ n_2l_2j_2 \\ J, J_i'}} B(n_1l_1j_1, n_2l_2j_2; JJ_iJ_f) \\
&= \left[\phi^J(n_1l_1j_1, n_2l_2j_2)\Psi^{J_i'}(\xi_A)\right]_{M_f}^{J_f}
\end{aligned}
$$

and

$$\Psi_{entrance} = \Psi^{J_i}_{M_i}(\xi_A)\phi_t(\mathbf{r}_{n1}, \mathbf{r}_{n2}, r_p; \sigma_{n1}, \sigma_{n2}, \sigma_p)$$

with

$$\phi_t = \left[\chi^S(\sigma_{n1}, \sigma_{n2})\chi^{S'}{}_t(\sigma_p)\right]^{S_t}_{M_t i} \phi_t^{L=0}\Big(\sum_{i>j} |\mathbf{r}_i - \mathbf{r}_j|\Big)$$

Assuming for simplicity a symmetric di–neutron ~~radial~~ wavefunction of ~~the~~ triton, i.e. neglecting the d–component of ~~the corresponding~~ wavefunction, ~~for~~ the relative and center of mass wavefunctions $\phi_{nlm}(\mathbf{r})$ and $\Phi_{N\Lambda M}(R)$ ($n = l = m = 0, N = \Lambda = M = 0$), leads to Ω_n, a quantity ~~that~~ reflects both the non–orthogonality existing between the di–neutron wavefunctions in the final nucleus (Cooper pair) and in the triton. Another way to say the same thing is that dineutron correlations in these two systems are different, a fact which underscores the limitations of ~~the~~ light ion reactions to probe specifically pairing correlations in nuclei.

ρ regarding which

One can then conclude that, provided one makes use of a (sensible) complete single–particle basis (eventually including also the continuum), one can capture through $\mathcal{U}^{J_iJ_f}_{LSJ}(R)$ most of the coherence of Cooper pair transfer, ~~in keeping with the fact that~~ major ~~aspects~~ fraction of the associated di–neutron non–locality ~~are~~ is taken care of by the n–summation, weighted by the non–orthogonal ity overlaps Ω_n. This is in keeping with the fact that, making use of a more refined triton wavefunction than employed above, the $n-p$ (deuteron–like) correlations of this particle can be described with reasonable accuracy and thus the emergence of successive transfer. On the other hand, being the deuteron a bound system, this effective treatment of the associated resonances is not particular economic. Furthermore, zero–range approximation ($V(\rho)\phi_{000}(\rho) = D_0\delta(\vec{\rho})$) blocks such a possibility.

as a; the U that (see app. 1, D) appearing in the expression of U, (In other words,

Nonetheless, the fact that one can still work out a detailed ~~and consistent~~ picture of two–nucleon transfer reactions in nuclei in terms of ~~absolute~~ cross sections with the help of a single parameter ($D_0^2 \approx (31.6 \pm 9.3) 10^4 \text{MeV}^2\text{fm}^2$) testifies to the fact that the above picture of Cooper pair transfer is a powerful picture, as it contains a large fraction of the physics which ~~is~~ at the basis of Cooper pair transfer in nuclei (?; Ch. 2). ~~This is the reason~~ why, treating explicitly the intermediate deuteron channel in terms of successive transfer, correcting both this and the simultaneous transfer channel for non–orthogonality contributions, makes the above picture the quantitative probe of Cooper pair correlations in nuclei (?; Ch. 4 and 5), as testified by Fig. ?? and Table ??. Within ~~this~~ context, we provide below two examples of B–coefficients. One ~~for~~ the case in which A and $B(= A + 2)$ are members of a pairing rotational band. A second one, in the case in which they are members of a pairing vibrational band: That is,

the above

1) $$B(nlj, nlj; 000) = \langle BCS(N+2)|[a^\dagger_{nlj}a^\dagger_{nlj}]^0_0|BCS(N)\rangle = \sqrt{j+1/2}\,U_{nlj}(N)V_{nlj}(N+2),$$

and

2) $$B(nlj, nlj; 000) = \langle N_0 + 2(gs)|[a^\dagger_{nlj}a^\dagger_{nlj}]^0_0|N_0(gs)\rangle = \begin{cases} \sqrt{j+1/2}X_a(n_kl_kj_k) & (\epsilon_{j_k} > \epsilon_F), \\ \sqrt{j+1/2}Y_a(n_il_ij_i) & (\epsilon_{j_k} \le \epsilon_F). \end{cases}$$

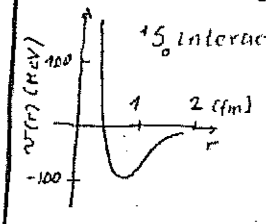
This is in keeping with the fact ~~that~~ the Cooper pair correlation length is much larger than nuclear dimensions and, consequently, simultaneous and successive transfer, ~~reflect the~~ feel the same ~~[illegible]~~ pairing correlation (see Sec. 1.2)

For actual numerical values see ~~[illegible] Tables 1.A.1 and 1.A.2~~

app. 2.B (Tables 2.B.1 (case 1)) and app 1.E Tables 1.E.1 and 1.E.2 (case 2))

$\boxed{\text{Quantality parameter}}$ ratio of quantal kinetic energy of localization and potential energy; $\boxed{\text{box 1}}$ App. 2.A

$$Q = \frac{\hbar^2}{M a^2}\frac{1}{|v_0|};\ M_n = 0.939\ \mathrm{GeV}/c^2$$ (neutron mass)



$a \approx 1$ fm (range)

$v_0 \approx -100$ MeV (depth)

$^1S_0$: interaction between two nucleons in states of time reversal with $S = L = 0$, and thus in a singlet state.

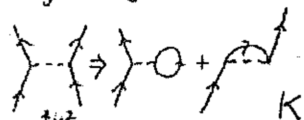
(QM : ZPF ($\Delta p_r \Delta r \geq \hbar$) $\frac{\hbar^2}{Ma^2}$)

constituents	M/M_n	a (cm)	v_0 (eV)	Q	phase ($T=0$)	T_c
^3He	3	2.9(-8)	8.6(-4)	0.19	liquid [a)]	
^4He	4	2.9(-8)	8.6(-4)	0.14	liquid [a)]	
H_2	2	3.3(-8)	32(-4)	0.06	solid [a)]	
^{20}Ne	20	3.1(-8)	31(-4)	0.007	solid	
nucleons	1	9(-14)	100(+6)	0.5 [c)]	liquid [a,b)]	

a) liquid (condensed)
b) better, Non-Newtonian solid.
c) Nucleus, paradigm of quantal many-body Fermi systems.

(B.R. Mottelson, Elementary features of nuclear structure, Les Houches, Session LXVI, Elsevier (1998))

Fluctuations, quantal or classical, favor symmetry: gases and liquids are homogenous. Potential energy on the other hand prefers special arrangements: atoms like to be at specific distances from each other (spontaneous breaking of symmetry).

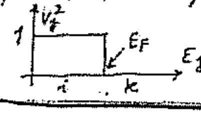


$$U(r) = \int d^3r'\rho(r')\,v(|r-r'|)$$

$$\rho(r) = \sum_i |\varphi_i(\vec r)|^2$$

K

$$U_x = -\sum_i \varphi_i^*(\vec r\,')\,v(|\vec r-\vec r\,'|)\,\varphi_i(\vec r)$$



For $Q \gtrsim 0.15$ independent-particle motion.

Figure ~~2.1.1~~ 1.1.1 : Quantality parameter and independent particle motion

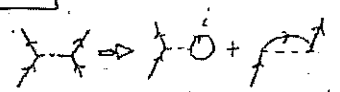
$\boxed{\text{Cooper pairs}}$ $\boxed{\text{Box 2}}$ ~~app.~~

$$H = \sum_{j_1 j_2} \langle j_1 | T | j_2 \rangle a^\dagger_{j_1} a_{j_2} + \frac{1}{4} \sum_{j_1 j_2 j_3 j_4} \langle j_1 j_2 | v | j_3 j_4 \rangle a^\dagger_{j_2} a^\dagger_{j_1} a_{j_3} a_{j_4}$$

$j (\equiv j, m)$

$\boxed{\text{independent particle motion } (\hbar = 1/2), \text{ mean field}}$

$\boxed{\text{HF}}$ $a^\dagger_{j_2} a^\dagger_{j_1} a_{j_3} a_{j_4} \Rightarrow a^\dagger_{j_2} \langle a^\dagger_{j_1} a_{j_3} \rangle a_{j_4} + \ldots$



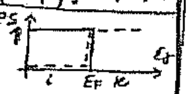
$U(r) = \int d^3r' \rho(r') v(|\vec{r} - \vec{r}'|)$

$U_x(r, r') = -\sum_i \varphi^*_i(\vec{r}') v(|\vec{r} - \vec{r}'|) \varphi_i(\vec{r})$

$\rho(r) = \sum_i |\varphi_i(\vec{r})|^2; \int d^3r \rho(r) = N$

Hartree-Fock, complete separation between occupied ($|i\rangle$) and empty ($|k\rangle$) states

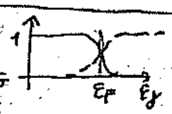
$(U_\nu^2 + V_\nu^2 = 1)$ $\varphi = \tilde{a}^\dagger_\nu |0\rangle = (U_\nu + V_\nu a^\dagger_\nu)|0\rangle; \; V_\nu^2 = \begin{cases} 1 & \varepsilon_i \le \varepsilon_F \\ 0 & \varepsilon_i > \varepsilon_F \end{cases}$



$|\text{Nilsson}(\Omega)\rangle_{\mathcal{K}} = \det(\varphi_\nu) = \prod \tilde{a}^\dagger_\nu |0\rangle = \prod a^\dagger_i |0\rangle = \prod a^\dagger_i a^\dagger_{\tilde{i}} |0\rangle$

$|IKM\rangle \sim \int d\Omega \mathcal{D}^I_{MK}(\Omega) |\text{Nilsson}(\Omega)\rangle; \; E_I = (\hbar^2/2\mathcal{J}) I(I+1); \; \mathcal{J} = \mathcal{J}_{r}$ $\mathcal{J}_r$

$\boxed{\text{BCS}}$ $\boxed{\text{independent pair motion}}$



constant m.els approx. $\langle j_1 j_2 | v | j_3 j_4 \rangle = -G$

$\sum \langle a^\dagger_{j_2} a^\dagger_{j_1} \rangle a_{j_3} a_{j_4} + \sum a^\dagger_{j_2} a^\dagger_{j_1} \langle a_{j_3} a_{j_4} \rangle; \; \varphi^{\text{cooper}}_j = (U_j + V_j a^\dagger_{jm} a^\dagger_{j\tilde{m}})|0\rangle$

$|BCS\rangle = \prod_{jm>0} (U_j + V_j a^\dagger_{jm} a^\dagger_{j\tilde{m}})|0\rangle; \quad \alpha_0 = \langle BCS | \sum_{jm>0} a^\dagger_{jm} a^\dagger_{j\tilde{m}} | BCS \rangle$

$U_\nu = |U_\nu| = U'_\nu; \quad V_\nu = e^{-2i\phi} V'_\nu \; (V'_\nu \equiv |V_\nu|) \quad (\nu \equiv j, m)$

$|BCS(\phi)\rangle_{\mathcal{K}} = \prod_{\nu>0} (U'_\nu + V'_\nu e^{-2i\phi} a^\dagger_\nu a^\dagger_{\tilde{\nu}})|0\rangle$ $\quad \mathcal{K}$: lab. system; $\mathcal{K}'$: intr. system

$= \prod_{\nu>0} (U'_\nu + V'_\nu a'^\dagger_\nu a'^\dagger_{\tilde{\nu}})|0\rangle = |BCS(\phi = 0)\rangle_{\mathcal{K}'}$

$\alpha_0 = \alpha'_0 e^{-2i\phi}; \quad \alpha'_0 = \sum_{\nu>0} U'_\nu V'_\nu; \quad \left.\begin{matrix} V'_\nu \\ U'_\nu \end{matrix}\right\} = \frac{1}{\sqrt{2}} \left(1 \mp \frac{1}{E_\nu}\right)^{1/2}$

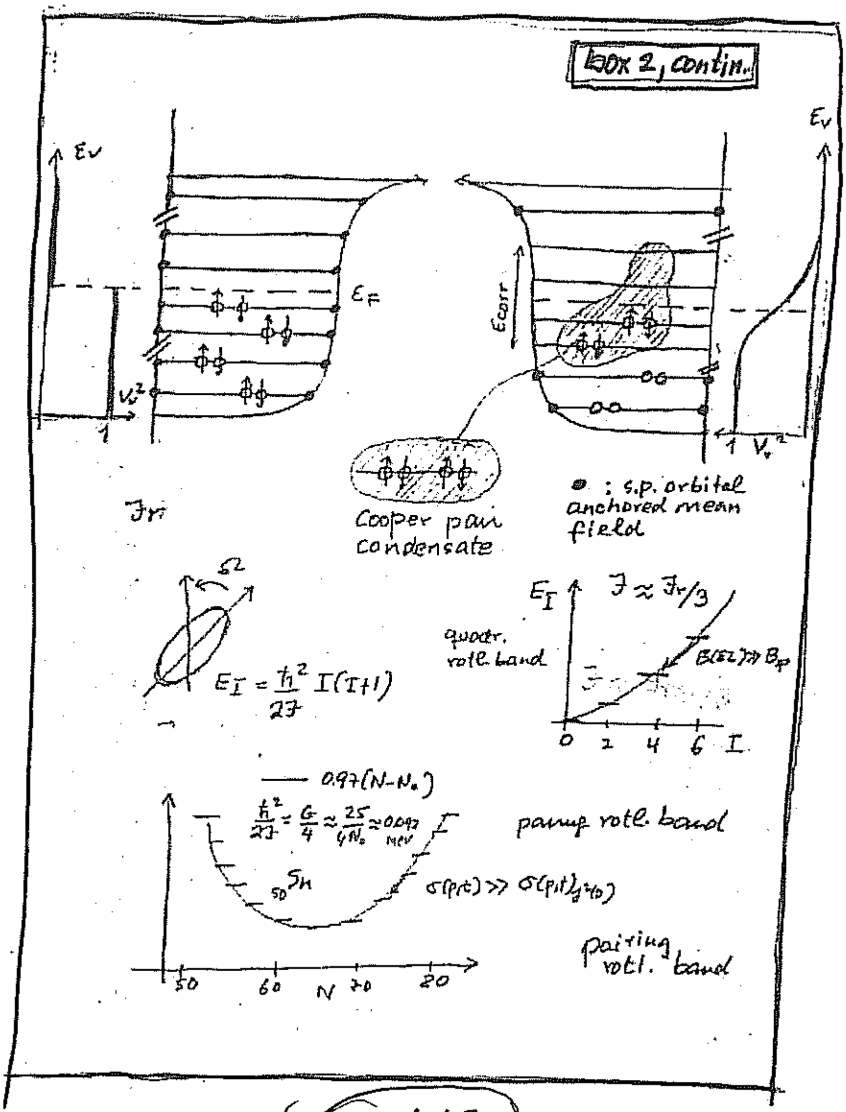
$\Delta = G\alpha_0; \quad N_0 = 2\sum_{\nu>0} V_\nu^2; \quad \frac{1}{G} = \sum_{\nu>0} \frac{1}{2E_\nu}$ $\quad E_\nu = ((\varepsilon_\nu - \lambda)^2 + \Delta^2)^{1/2}$

$|N_0\rangle \sim \int_0^{2\pi} d\phi \, |BCS(\phi)\rangle_{\mathcal{K}} \sim \left(\sum_{\nu>0} c_\nu a^\dagger_\nu a^\dagger_{\tilde{\nu}}\right)^{N_0/2} |0\rangle; \; E_N = (\hbar^2/2\mathcal{J}) N^2$

$\mathcal{J} \approx 2\hbar^2/G$

V Fig. 1.1.2 Co

Independent pair motion



1.1.3

Figure ~~2.1.2.~~

Appendix 1.B

Two-nucleon spectroscopic amplitudes associated with pairing vibrational modes in closed-shell systems $\boxed{\text{box 3}}$ I App. 2C

The solution of the pairing Hamiltonian

$$H = H_{sp} + H_p ,$$

where

$$H_{sp} = \sum_\nu \varepsilon_\nu a_\nu^\dagger a_\nu$$

and

$$H_p = -G P^\dagger P ,$$

with

$$P^\dagger = \sum_{\nu>0} a_\nu^\dagger a_{\bar\nu}^\dagger ,$$

in the Harmonic approximation (RPA) leads to pair addition (a) / a pair removal (r) two-particle, two-hole correlated mode, the associated creation and annihilation operators being

$$\Gamma_a^\dagger(n) = \sum_k X_n^a(k)\,\Gamma_k^\dagger + \sum_i Y_n^a(i)\,\Gamma_i$$

and

$$\Gamma_r^\dagger(n) = \sum_i X_n^r(i)\,\Gamma_i^\dagger + \sum_k Y_n^r(k)\,\Gamma_k ,$$

with

$$\sum X^2 - \sum Y^2 = 1$$

and

$$\Gamma_k^\dagger = a_k^\dagger a_{\bar k}^\dagger , \qquad (\varepsilon_k > \varepsilon_F),$$

$$\Gamma_i^\dagger = a_{\bar i} a_i . \qquad (\varepsilon_k \le \varepsilon_F).$$

The relations

$$[H, \Gamma_a^\dagger(n)] = \hbar W_n (\beta = +2)$$

(and

box 3 cont'd (II)

$$[H, \Gamma_\nu^\dagger(n)] = \hbar W_n(\beta = -2),$$

where β is the transfer quantum, while n labels the roots of the corresponding dispersion relations

$$\frac{1}{G(\pm 2)} = \sum_k \frac{(\Omega_k/2)}{2\varepsilon_k \mp W_n(\pm 2)} + \sum_i \frac{(\Omega_i/2)}{2E_i \pm W_n(\pm 2)}$$

in increasing order of energy (centroid).

For the case of the pair addition and pair substraction modes of ^{208}Pb the above equation can be graphically solved (cf. Fig. 1), the minimum of the dispersion relation coincides with the Fermi energy.

One then obtain

manca 17
spostate ~~di~~ immediatamente dietro p. 30 e prima di p. 31

Table 1.B.1

$$X_1^r(i) = \frac{\frac{1}{2}\Omega_i^{1/2}\Lambda_1(-2)}{2(|\epsilon_i| - |\epsilon_{p_{1/2}}|) + E_{corr}(-2)} \; ; \; Y_1^r(k) = \frac{\frac{1}{2}\Omega_k^{1/2}\Lambda_1(-2)}{2(|\epsilon_{g_{9/2}}| - |\epsilon_k|) + 2(|\epsilon_{p_{1/2}}| - |\epsilon_{g_{9/2}}|) - E_{corr}(-2)}$$ (1)OX3 (3)

$$E_{corr}(-2) = 0.5\,\text{MeV (cf. Fig. 1)} \qquad \Omega = 1+1/2$$

$$2(|\epsilon_{p_{1/2}}| - |\epsilon_{g_{9/2}}|) = 6.82\,\text{MeV} \qquad 2(|\epsilon_{p_{1/2}}| - |\epsilon_{g_{9/2}}|) - E_{corr} = (6.82 - 0.5)\,\text{MeV} = 6.32\,\text{MeV}$$

Thus

$$\begin{cases} X_1^r(i) = \dfrac{\frac{1}{2}\Omega_i^{1/2}\Lambda_1(-2)}{2(|\epsilon_i| - |\epsilon_{p_{1/2}}|) + 0.5\,\text{MeV}} ; \\ Y_1^r(k) = -\dfrac{\frac{1}{2}\Omega_k^{1/2}\Lambda_1(-2)}{2(|\epsilon_{g_{9/2}}| - |\epsilon_k|) + 6.32\,\text{MeV}} \end{cases}$$

Table ~~2.C.1~~ 1.B.1

units		MeV	MeV^{-1}		TDA
nlj	Ω_i	$\lvert\epsilon_i\rvert - \lvert\epsilon_{p_{1/2}}\rvert$	$A(i) = \frac{\frac{1}{2}\Omega_i^{1/2}}{2(\lvert\epsilon_i\rvert - \lvert\epsilon_{p_{1/2}}\rvert) + 0.5\,\text{MeV}}$	$X_1^r(i)$	$X_1^r(i)$
$2p_{1/2}$	1	0	1	0.83	0.80
$1f_{5/2}$	3	0.57	0.528	0.44	0.42
$2p_{3/2}$	2	0.90	0.307	0.25	0.25
$0i_{13/2}$	7	1.64	0.350	0.29	0.28
$1f_{7/2}$	4	2.35	0.192	0.16	0.15
$0h_{9/2}$	5	3.47	0.150	0.12	0.12

$\sum A^2(i) = 1.5549$ Very similar to column labeled 208Pb Table III ... $\frac{1}{\sqrt{1.5549}} = 0.802 \leftarrow \sum(X_1^r(i))^{TD} = 1$

units		$\lvert E\rvert$ (MeV)	MeV^{-1}	
nlj	Ω_k	$\lvert\epsilon_{g_{9/2}}\rvert - \lvert\epsilon_k\rvert$	$B(k) = \frac{\frac{1}{2}\Omega_k^{1/2}\Lambda_1(-2)}{2(\lvert\epsilon_{g_{9/2}}\rvert - \lvert\epsilon_k\rvert) + 6.32\,\text{MeV}}$	$Y_1^r(k)$
$1g_{9/2}$	5	0	0.179	−0.15
$0i_{11/2}$	6	0.77	0.158	−0.13
$0j_{15/2}$	8	1.41	0.156	−0.13
$2d_{5/2}$	3	1.56	0.093	−0.08
$3s_{1/2}$	1	2.03	0.046	−0.04
$1g_{7/2}$	4	2.47	0.090	−0.07
$2d_{3/2}$	2	2.51	0.063	−0.05

$\sum B^2(k) = 0.10418$.

$\Lambda_1(-2) = 0.83025$ MeV $\quad \Lambda_1^2(-2)\left(\sum_i A^2(i) - \sum_k B^2(k)\right) = \Lambda_1^2(-2)\,(1.5549 - 0.10418) = \Lambda_1^2(-2)\; 1.45073 = 1$

⑤

$E_{corr}(+2) = 1.5\ \text{MeV}$ (conf. Fig. 1) Box 3

$\Delta\varepsilon_{sp} = 2\left(|\varepsilon_{p_{1/2}}| - |\varepsilon_{g_{9/2}}|\right) = 6.82\ \text{MeV}$

Thus $2\Delta\varepsilon_{sp} - E_{corr} = (6.82 - 1.5)\ \text{MeV} = 5.32\ \text{MeV}$

$$\begin{cases} X_1^a(k) = \dfrac{\frac{1}{2}\Omega_k^{1/2}\,\Lambda_1(+2)}{2(|\varepsilon_{g_{9/2}}| - |\varepsilon_k|) + 1.5\ \text{MeV}} \\ Y_1^a(i) = -\dfrac{\frac{1}{2}\Omega_i^{1/2}\,\Lambda_1(+2)}{2(|\varepsilon_i| - |\varepsilon_{p_{1/2}}|) + 5.32\ \text{MeV}} \end{cases} \qquad \Omega_j = \frac{2j+1}{2}$$

B

1.~~A~~.2

Table ~~2.6.2~~

Units		MeV	MeV⁻¹	
$n l_{j_k}$	Ω_k	$\lvert\varepsilon_{g_{9/2}}\rvert - \lvert\varepsilon_k\rvert$	$C(k) = \frac{\frac{1}{2}\Omega_k^{1/2}}{2(\lvert\varepsilon_{g_{9/2}}\rvert - \lvert\varepsilon_k\rvert) + 1.5\ \text{MeV}}$	$X_1^a(k)$
$1g_{9/2}$	5	0	0.745	0.82
$0i_{11/2}$	6	0.77	0.403	0.44
$0j_{15/2}$	8	1.41	0.327	0.36
$2d_{5/2}$	3	1.56	0.187	0.21
$3s_{1/2}$	1	2.03	0.090	0.10
$1g_{7/2}$	4	2.47	0.155	0.17
$2d_{3/2}$	2	2.51	0.108	0.12

$\sum_k C^2(k) = 0.903$

coincides exactly with column (X) of Table XVI Adv. in N.P

Units		(MeV)	MeV⁻¹	
$n l_{j_i}$	Ω_i	$\lvert\varepsilon_i\rvert - \lvert\varepsilon_{p_{1/2}}\rvert$	$D(i) = -\frac{\frac{1}{2}\Omega_i^{1/2}}{2(\lvert\varepsilon_i\rvert - \lvert\varepsilon_{p_{1/2}}\rvert) + 5.32\ \text{MeV}}$	$Y_1^a(i)$
$2p_{1/2}$	1	0	−0.094	−0.10
$1f_{5/2}$	3	0.57	−0.134	−0.15
$2p_{3/2}$	2	0.90	−0.099	−0.11
$0i_{13/2}$	7	1.64	−0.154	−0.17
$1f_{7/2}$	4	2.35	−0.100	−0.11
$0h_{9/2}$	5	3.47	−0.091	−0.10

$\sum_i D^2(i) = 0.079$

$\Lambda_1^2(+2)\left(\sum_k C^2(k) - \sum_i D^2(i)\right) = (0.903 - 0.079)\ \text{MeV}^{-2} = 0.824\ \text{MeV}^{-2}$; $\Lambda_1(+2) = \frac{\text{MeV}}{\sqrt{0.824}} = 1.102\ \text{MeV}$

$\Lambda_1(+2) = 1.102\ \text{MeV}$

1.B.1

Figure 2.~~1.3~~:

appendix 1.C

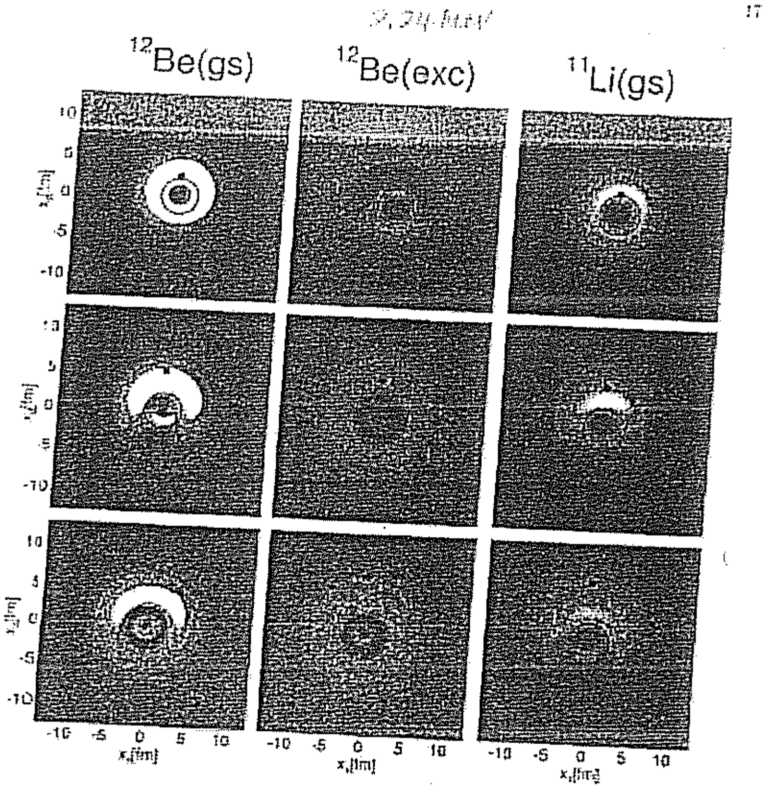
Microscopic mechanism to break gauge invariance

Box 4 (1)

App. 2D

Pairing is intimately connected with particle number violation and thus spontaneous breaking of gauge invariance, as testified by the order parameter $\langle BCS|P^{\dagger}|BCS\rangle = \alpha_0$. Now, in the nuclear case and at variance with condensed matter, dynamical breaking of gauge symmetry is equally important (pairing vibrations around closed shell nuclei, cf. Fig. 2 box 3). The fact that the average single-particle field acts as an external potential (like e.g. magnetic field in metallic superconductors) is at the basis of of the existence of a critical value of the pairing strength G to bind Cooper pairs in nuclei. In fact, spatial quantization in finite systems at large and in nuclei in particular, intimately connecte with the paramount role the surface has in these systems, is at the basis of the existence of a critical G value. Also of the fact that in nuclei an important fraction (30-50%) of Cooper pair binding is due to the exchange of collective vibrations between the partners of the pair, the rest being associated with the bare NN interaction in the $^1S_0$ channel (cf. [illegible])

Now, there are situations in which spatial quantization screens, essentially completely, the NN-interaction. This happens in the case in which the nuclear valence orbitals are s,p-states at threshold (pairing anti-halo effect). Example of situations of this



$$|0\rangle_v = |0\rangle + \alpha|(p,s)_{1^-} \otimes 1^-;0\rangle + \beta|(s,d)_{2^+} \otimes 2^+;0\rangle + \gamma|(p,d)_{3^-} \otimes 3^-;0\rangle$$

$$|0\rangle_v = a|s^2(0)\rangle + b|p^2(0)\rangle + c|d^2(0)\rangle$$

exotic decay

	^{11}Li(gs)	^{12}Be(gs)	^{12}Be(exc)
α	0.7	0.18	0.08
β	0.7	0.30	-0.30
γ	–	0.37	-0.3
a	0.45	0.37	0.82
b	0.55	0.59	0.17
c	0.04	0.60	0.19

← pigmy resonance, in Pb

$s^2(0)$ at threshold

Figure ~~2.1.4:~~ 1.B.1

C