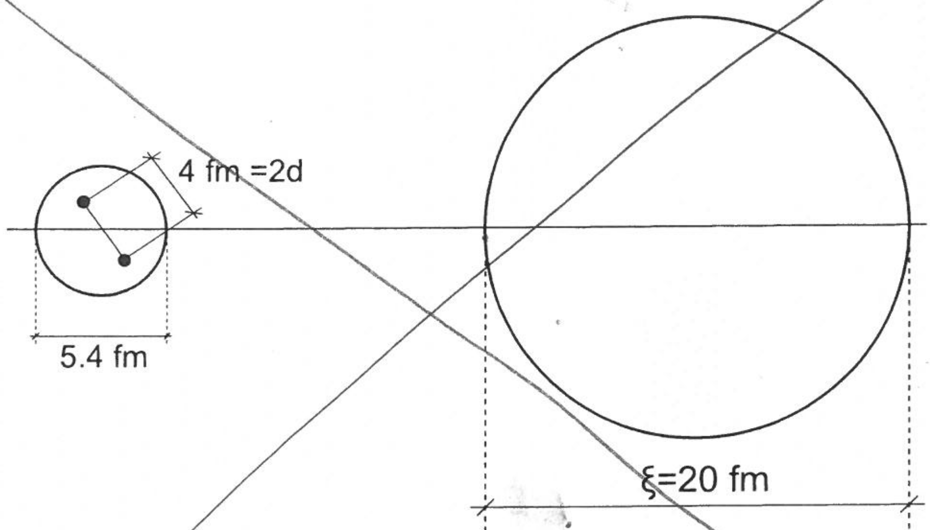


Figure 3.8.1: Schematic representation of the ^{11}Li nuclear radius and of the associated halo Cooper pair field. Twice the radius of ^{11}Li should be, according to systematics, given by $2R_0(^{11}\text{Li})=2\times(r_0 \times 11^{1/3}) = 5.4$ fm ($r_0 = 1.2$ fm). The average distance between nucleons is $2 \times d$, where d is the Wigner–Seitz radius $d = \left((4\pi/3)R_0^3/A\right)^{1/3} = (4\pi/3)^{1/3} r_0 \approx 2$ fm.

~~to p. 198~~

making use of the rules of nuclear field theory (NFT), the particle–vibration coupling (PVC) ~~Hamiltonian (mainly structure)~~, and the v_{np} (v : four point vertex), bare NN- interaction (mainly reaction). In this way one obtains quantities (energies, transition probabilities, absolute value of reaction cross sections) which can be directly compared with the experimental findings. Such a protocol can be carried out, in most cases, within the framework of ~~second order~~ perturbation theory. ~~For example, Rayleigh–Schrödinger or Brillouin–Wigner for structure and DWBA for reaction.~~

As a result, single-particle states ~~are clothed~~ move in a gas of vibrational quanta and become clothed. The quanta couple, in turn, to doorway states which renormalize their properties through self energy and vertex corrections. Similar couplings renormalize the bare NN–interaction in their the different channels. In particular in the $^1S_0$ (pairing) channel. Also as a result of the its interweaving, the variety of elementary modes of excitation ~~they can~~ may break in a number of states, eventually acquiring a lifetime and, within a coarse grain approximation, a damping width (imaginary component of the self energy). Moving into the continuum, as for example in the case of direct reactions, one such component is the imaginary part of the optical potential operating in the particular channel selected. It can be calculated microscopically using similar techniques and elements as e.g. those used in the calculation of the damping width of giant resonances. With the help of dispersion relations, the real part of the optical potential can be obtained from the knowledge of the energy dependence

For example, second order perturbation theory, in both reaction and structure, as exemplified in Fig. 1.9.3 displaying a NFT (r+s) graphical representation of contributions to the ^{11}Li(p,t)^{9}Li(gs) and ^{11}Li(p,t)^{9}Li($1/2^-$; 2.69 MeV) process (see also Fig. 6.1.3) intervening with and external $x_{\hat{F}}$ - field on

one obtain (b) and by time-ordering (c)

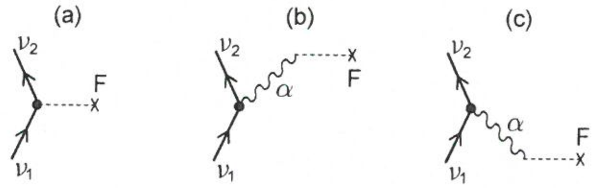


Figure 3.A.1: (a) F–moment of single–particle and (b,c) renormalization effects induced by the collective vibration α.

of the absorptive potential. In this way, the consistency circle structure–reaction based on elementary modes and codified by NFT could be closed. The rich variety of emergent properties found along the way eventually acquiring an important level of physical validation. In the case of halo exotic nuclei, in particular in the case of ^{11}Li (bootstrap, Van der Waals Cooper binding, halo pair addition mode (symbiosis of pairing vibration and pigmy) being few of the associated emergent properties) one is rather close to his goal. At that time it would be possible, arguably if there is one, to posit that the *ultima ratio* of structure and reactions, in any case that associated with pairing and Cooper pair transfer in nuclei, have been unveiled[7]

Effective moments

it is necessary to make reference to both of them simultaneously and in a unified way.

At the basis of the coupling between elementary modes of excitation, for example of single–particle motion and of collective vibrations, is the fact that, in describing the nuclear structure ~~nothing is won by describing the system in terms of solely one or the other degrees of freedom~~.

Within the harmonic approximation the above statement is economically embodied in e.g. the relation existing between the collective ($\hat{\alpha}$) and the single–particle ($\hat{F}$) representation[8] of the operator creating a $\beta = 0$ (β: transfer quantum number) excitation. That is (cf. Bohr, A. and Mottelson (1975), cf. also Brink, D. and

[7]In the above paragraph we allow ourselves to paraphrase Jacques Monod writing in connection with biology and life: L'*ultima ratio* de toutes les structures et performances téléonomiques des êtres vivants est donc enfermée dans les séquences des radicaux des fibres polypeptidiques "embryons" de ces démons de Maxwell biologiques que sont les protéines globulaires. En un sens, très réel, c'est à ce niveau d'organisation chimique qui gît, s'il y en a un, le secret de la vie. Et saurait–on non seullement décrire les séquences, mais énoncer la loi d'assemblage à laquelle obéissent, on pourrait dire que le secret est percé, l'ultima ratio découverte (J. Monod, Le hasard et la nécéssité, Editions du Seuil, Paris, 1970).

[8]

ki i.e. $\Gamma^{\dagger}_{ki}$

Broglia (2005) App. C),

$$
\begin{aligned}
F &= \left\{ \langle k|F|\tilde{i}\rangle \Gamma^{\dagger}_{@} + \langle \tilde{i}|F|k\rangle \Gamma_{ki} \right\} \\
&= \sum_{k,i,\alpha'} X^{\alpha'}_{ki} \Gamma^{\dagger}_{\alpha'} - Y^{\alpha'}_{ki} \Gamma_{\alpha'} \\
&= \sum_{\alpha'} \Lambda_{\alpha'} \sum_{ki} \frac{\left|\langle \tilde{i}|F|k\rangle\right|^2 2(\epsilon_i - \epsilon_k)}{(\epsilon_k - \epsilon_i)^2 - (\hbar\omega_{\alpha'})^2} \left(\Gamma^{\dagger}_{\alpha'} + \Gamma_{\alpha'}\right) \\
&= \frac{\Lambda_{\alpha'}}{K} \left(\Gamma^{\dagger}_{\alpha} + \Gamma_{\alpha}\right) = \sum_{\alpha'} \sqrt{\frac{\hbar\omega_{alpha}}{2C_{\alpha}}} \left(\Gamma^{\dagger}_{\alpha} + \Gamma_{\alpha}\right) = \hat{\alpha},
\end{aligned}
\qquad (3.A.1)
$$

kapp low κ case

it should read like

$$= \sum_{\alpha'} \frac{\Lambda_{\alpha'}}{\kappa} \left(\Gamma^{\dagger}_{\alpha'} + \Gamma_{\alpha'}\right) = \sum_{\alpha'} \sqrt{\frac{\hbar\omega_{\alpha'}}{2C_{\alpha'}}} \left(\Gamma^{\dagger}_{\alpha'} + \Gamma_{\alpha'}\right) = \hat{\alpha}$$

This is a consequence of the self consistent relation

$$\delta U(r) = \int d\mathbf{r}' \delta\rho(r) v(|\mathbf{r}-\mathbf{r}'|) \qquad (3.A.2)$$

existing between density (collective) and potential (single–particle) distortion, typical of normal modes of many–body systems.

Relation (3.A.1) implies that at the basis of these normal modes one finds the (attractive K < 0) separable interaction κ

$$H = \frac{K}{2} \hat{F}\hat{F} \qquad (3.A.3)$$

κ

but where now

$$\hat{F} = \sum_{\nu_1,\nu_2} \langle \nu_1|F|\nu_2\rangle a^{\dagger}_{\nu_1} a_{\nu_2} \qquad (3.A.4)$$

is a general single–particle operator, while $\hat{F}$ (see eq. (3.A.1)) is its harmonic representation acting in the particle (k)–hole (i) space, $\Gamma^{\dagger}_{ki}$ and Γ_{ki} being (quasi) bosons, i.e. respecting the commutation relation

$$\left[\Gamma_{ki}, \Gamma^{\dagger}_{k'i'}\right] = \delta(k,k')\delta(i,i'). \qquad (3.A.5)$$

In other words, the representation (3.A.1) which is at the basis of the RPA (as well as QRPA), does not allow for scattering vertices, processes which become operative by rewriting (3.A.3) in terms of the particle–vibration coupling Hamiltonian

κ

$$H_c = K\hat{\alpha}\hat{F} \qquad (3.A.6)$$

~~leading to the effective single–particle moments~~ (Fig. 3.A.1 (b)),

H $\langle \nu_2, n_\alpha = 1|H_c|\nu_1\rangle$

$$
\begin{aligned}
\langle \nu_2|\hat{F}|\nu_1\rangle_{(b)} &= \frac{\langle \nu_2|\hat{F}|\nu_2, n_\alpha = 1\rangle \langle \nu_2, n_\alpha = 1|\hat{F}|\nu_1\rangle}{(\epsilon_{\nu_1} - \epsilon_{\nu_2}) - \hbar\omega_\alpha} \\
&= \frac{\langle 0|\alpha|n_\alpha = 1\rangle K\alpha \langle \nu_2|F|\nu_1\rangle}{(\epsilon_{\nu_1} - \epsilon_{\nu_2}) - \hbar\omega_\alpha} \\
&= K\alpha^2 \frac{\langle \nu_2|F|\nu_1\rangle}{(\epsilon_{\nu_1} - \epsilon_{\nu_2}) - \hbar\omega_\alpha},
\end{aligned}
\qquad (3.A.7)
$$

κ — $\kappa\alpha$

κ

$\kappa\alpha^2$

It is of notice that κ is negative for an attractive field. Let us now calculate the effective single-particle moments

F

The sign of $\chi(0)$ is opposite to ~~that of~~ K, since the ~~static~~ polarization effect produced by an attractive coupling ($K<0$) is in phase with the single-particle moment, while a repulsive coupling ($K>0$) implies opposite phases for the polarization effect and the one-particle moment [8]

F

and (Fig. 3.A.1 (c))

$$\langle \nu_2|\hat{F}|\nu_1\rangle_{(c)} = \frac{\langle \nu_2|H_c|\nu_1, n_\alpha = 1\rangle\langle \nu_1, n_\alpha = 1|F|\nu_1\rangle}{\epsilon_{\nu_2} - (\epsilon_{\nu_1} + \hbar\omega_\alpha)}$$

$$= K\alpha^2\left(-\frac{\langle \nu_2|F|\nu_1\rangle}{(\epsilon_{\nu_1} - \epsilon_{\nu_2}) + \hbar\omega_\alpha}\right), \tag{3.A.8}$$

K

leading to

$$\langle \nu_2|\hat{F}|\nu_1\rangle_{(b)} + \langle \nu_2|\hat{F}|\nu_1\rangle_{(c)} = K\alpha^2\frac{2\hbar\omega_\alpha\langle \nu_2|F|\nu_1\rangle}{(epsilon_{\nu_1} - \epsilon_{\nu_2})^2 - (\hbar\omega_\alpha)^2}$$

$(\epsilon_{\nu_1} - \epsilon_{\nu_2})^2$ ϵ_{ν_1}

K

$$= \frac{K}{C_\alpha}\frac{(\hbar\omega_\alpha)^2\langle \nu_2|F|\nu_1\rangle}{(\epsilon_{\nu_1} - \epsilon_{\nu_2})^2 - (\hbar\omega_\alpha)^2} \tag{3.A.9}$$

This is in keeping with the fact that the ~~vibration zdro point fluctuation is~~ ZPF of the α-vibrational mode is

$$\alpha = \sqrt{\frac{\hbar\omega_\alpha}{2C_\alpha}}, \tag{3.A.10}$$

the particle–vibration coupling strength being

K

$$\Lambda_\alpha = K\alpha. \tag{3.A.11}$$

Together with $\langle \nu_2|\hat{F}|\nu_1\rangle_{(a)} = \langle \nu_2|F|\nu_1\rangle$ (see Fig. 3.A.1 (a)) one ~~can write~~ obtains

$\langle \nu_2|\hat{F}|\nu_1\rangle = (1+\chi(\omega))\langle \nu_2|F|\nu_1\rangle$

$$\langle \nu_2|\hat{F}|\nu_1\rangle = (1 + \chi_F(\omega))\langle \nu_2|F|\nu_1\rangle, \tag{3.A.12}$$

where

i.e. $\chi(\omega)$ ℓ

K

$$\chi(\omega) = \frac{K}{C_\alpha}\frac{\omega_\alpha^2}{\omega^2 - \omega_\alpha^2} \tag{3.A.13}$$

is the polarizability coefficient where

$\omega = |\epsilon_{\nu_1} - \epsilon_{\nu_2}|/\hbar.$

$$\boxed{\cancel{\hbar\omega = \epsilon_{\nu_1} - \epsilon_{\nu_2} > 0.}} \tag{3.A.14}$$

In the static limit, e.g. in the case in which α is a giant resonance and $\omega_\alpha \gg \omega$ one obtains

the whole equation should read $\chi(0) = -\frac{K}{C}$

K

$$\cancel{\lim_{\omega_\alpha \gg \omega}} \approx \chi_F(0) = -\frac{K}{C}. \tag{3.A.15}$$

ℓ

F – F

Let us now calculate the two–body pairing induced interaction arising from the exchange of collective vibrations[9] (symmetrizing between initial and final states)[10] cf. Brink, D. and Broglia (2005) p. 217) ℓ

ν

$$v^{ind}_{\nu\nu'}(a) + v^{ind}_{\nu\nu'}(b) = K^2\alpha^2\left|\langle \nu'|F|\nu\rangle\right|^2\left(\frac{1}{\epsilon_\nu - \epsilon_{nu'} - \hbar\omega_\alpha} + \frac{1}{\epsilon_{\nu'} - \epsilon_{nu} - \hbar\omega_\alpha}\right)$$

ν'

K

$$= K^2\alpha^2\left|\langle \nu'|F|\nu\rangle\right|^2\left(\frac{1}{(\epsilon_\nu - \epsilon_{nu'}) - \hbar\omega_\alpha} - \frac{1}{(\epsilon_\nu - \epsilon_{nu'}) - \hbar\omega_\alpha}\right)$$

ν'

$$= \Lambda_\alpha^2\left|\langle \nu'|F|\nu\rangle\right|^2\left(\frac{2\hbar\omega_\alpha}{(\epsilon_\nu - \epsilon_{nu'})^2 - (\hbar\omega_\alpha)^2}\right)$$

summing over the two time orderings and inside the parenthesis

$$= v^{ind}_{\nu\nu'}(c) + v^{ind}_{\nu\nu'}(d) \cancel{= 2M_{ind}}. \tag{3.A.16}$$

ℓ ν'

[8] Bohr and Mottelson (1975), Mottelson (1962).

[9] In the present discussion we do not consider spin modes. For details see e.g. Idini et. al (2015). See also Bortignon et al (1983)

[10]

(Bortignon, Broglia, Dasso NPA 398 (1983) 221

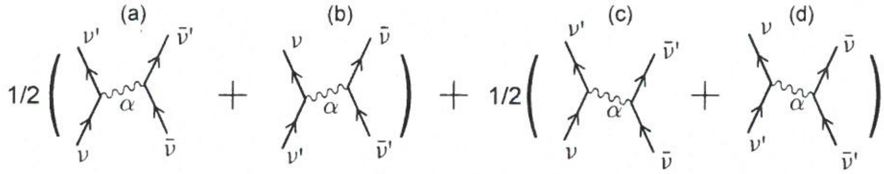
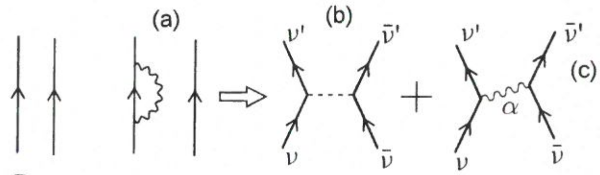
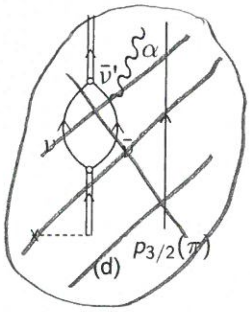


Figure 3.A.2: Diagrams associated with nuclear pairing induced interaction.





C - C →

from p. 213a handwritten

Kbhvn 24/4/16

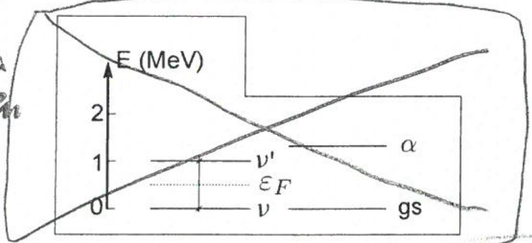


Figure 3.A.3: Schematic representation of ~~self energy (a) processes, and bare (b) and induced (c) pairing interaction. In (d) the process associated with the excitation of a particle–vibration state in Cooper pair transfer is shown. The inset provides a schematic picture of the .....~~

B - B →

p. 212a handwritten

Khvn 24/4/16

Summing to (3.A.~~8~~17) the matrix element of the bare interaction (3.A.3) K

$$v^{bare}_{\nu\nu'} = K\,|\langle\nu'|F|\nu\rangle|^2 \qquad (3.A.17)$$

13

and making use of (3.A.~~8~~13) one obtains for the total pairing matrix element 11 $v^{bare}_{\nu\nu'}(1+\chi(\omega))$

$$v_{\nu\nu'} = \cancel{v^{bare}_{\nu\nu'}\left(1+\chi(0)\frac{\omega_\alpha^2}{\omega^2-\omega_\alpha^2}\right)} = v^{bare}_{\nu\nu'}\left(1+v^{bare}_{\nu\nu'}\Pi_{\nu\nu'}(\omega,\omega_\alpha)\right), \qquad (3.A.18)$$

where, similarly to (3.A.14), we have introduced the frequency $\omega = |\varepsilon_\nu - \varepsilon_{\nu'}|/\hbar$.

where

$$\Pi_{\nu,\nu'} = \begin{cases} \left(C_\alpha\,|\langle\nu'|F|\nu\rangle|^2\right)^{-1}\frac{\omega_\alpha^2}{\omega^2-\omega_\alpha^2} \\ \left(D_\alpha\,|\langle\nu'|F|\nu\rangle|^2\right)^{-1}\frac{1}{\omega^2-\omega_\alpha^2} \end{cases} \qquad (3.A.19)$$

It is of notice that the second expression of $\Pi_{\nu\nu'}$ has the inertia of the phonon in the denominator, similar to the factor (Z/AM) appearing in ~~[32]~~ (3.A.75) below 11. Within the framework of (3.A.3) and of its role in (3.A.18) one finds, in the case of superconductivity in

footnote 11 →

(B) Thus

$$v^{ind}_{\nu\nu'} = \frac{1}{2}\left(v^{ind}_{\nu\nu'}(a) + v^{ind}_{\nu\nu'}(b)\right) + \frac{1}{2}\left(v^{ind}_{\nu\nu'}(c) + v^{ind}_{\nu\nu'}(d)\right)$$
$$= \Lambda_\alpha^2 |\langle \nu' | F | \nu \rangle|^2 \left(\frac{2\hbar\omega_\alpha}{(\epsilon_\nu - \epsilon_{\nu'})^2 - (\hbar\omega_\alpha)^2} \right), \quad (3.A.17)$$

The diagonal matrix element,

$$v^{ind}_{\nu\nu'} \equiv \langle \nu\bar{\nu} | v^{ind} | \nu\bar{\nu} \rangle$$
$$= -\frac{2\Lambda_\alpha^2 |\langle \nu | F | \nu \rangle|^2}{\hbar\omega_\alpha}, \quad (3.A.18)$$

testifies to the fact ~~that~~, for values of $\omega_\alpha \gtrsim \omega$, with

$$\omega = |\epsilon_\nu - \epsilon_{\nu'}|/\hbar \quad (3.A.19)$$

namely the frequency of the single-particle excitation energy, the induced pairing interaction is attractive. (B)

to p. 213

Kbhn 24/4/16

213a

© Fig. 3.A.3: Starting with two bare nucleons moving around a closed shell system N_0 in Hartree – Fock orbitals (arrowed lines far left), a graphical (NFT) representation of (a) self energy processes and of (b) bare and (c) induced pairing interactions are displayed. In (d) a two-nucleon transfer process which eventually forces to become real the exchanged phonon. ©

p. 213

Caption to Fig. 3.A.3

metals, to be discussed below ~~in Appendix 3.7~~, that the bare unscreened Coulomb interaction can be written as

$$U_c(r) = \frac{1}{2}\sum_{i,j}\frac{q_i q_j}{|\mathbf{r}_i - \mathbf{r}_j|}, \tag{3.A.20}$$

i, j running over all particles (nuclei and electrons) and $q_i = -e$ for electrons and Ze for nuclei.

end of footnote (91) p. 213

For nucleons moving in time reversal states close to the Fermi energy and for low–lying collective modes such that $\omega_\alpha \gtrsim \omega$ the induced interaction will lead to overall attraction regardless whether the bare interaction is attractive or repulsive, in keeping with the fact that the ω–dependent term can, in principle, overwhelm the constant term K. The situation is quite different in the static limit, in which case $\omega_\alpha \gg \omega$ and $\omega_\alpha^2/(\omega^2 - \omega_\alpha^2) \to -1$.

Summing up, the exchange of collective vibrations between particles moving in time reversal states lying close to the Fermi energy leads to an attractive pairing interaction. Two situations are of interest:

$$1) \qquad \omega = \omega_\alpha - \delta_\omega/2, \quad \delta\omega \ll \omega_\alpha \tag{3.A.21}$$

$$2) \qquad \omega_\alpha \gg \omega. \tag{3.A.22}$$

$\delta\omega$

That is

$$\lim \frac{\omega_\alpha^2}{\omega^2 - \omega_\alpha^2} = \begin{cases} -\frac{\omega_\alpha}{\delta\omega} \ll 1 & \text{plastic } \alpha \text{ modes} \\ -1 & \text{elastic } \alpha \text{ modes} \end{cases} \tag{3.A.23}$$

The first situation is typical of low–lying collective surface vibrations and of e.g. the pigmy resonance of ^{11}Li. The second of high lying giant resonances. While in this case one can parametrize the effect in terms of constants like effective moments and charges (see e.g. Bohr, A. and Mottelson (1975) pp. 421 and 432), the explicit treatment of the state (ω–dependence) of the first ones is unavoidable.

(G)-(G) p. 214a Kbhvn 25/4/16

Let us conclude this section by making a simple estimate of the contribution of the induced pairing interaction to the (empirical) nuclear pairing gap. For this purpose we introduce the quantity

$$\lambda = N(0)v_{\nu\nu'}^{ind} \tag{3.A.24}$$

where $N(0)$ is the density of levels of a single spin orientation at the Fermi energy. The above quantity is known as the nuclear mass enhancement factor. This is because of the role it plays in the nucleon ω–mass

$$m_\omega = (1+\lambda)m, \tag{3.A.25}$$

Systematic studies of this quantity, and of the related discontinuity occurring ~~by~~ in the single–particle occupation number at the Fermi energy, namely $Z_\omega = (m/m_\omega)$, testifies to the fact that $\lambda \approx 0.4$.

Ⓖ Let us now rewrite (3,A,18) as Kbhvn 25/4/16

$$v_{\nu\nu'} = v^{bare}_{\nu\nu'}\left(1 + |\chi(0)|\frac{\omega_\alpha^2}{\omega_\alpha^2 - \omega^2}\right), \quad (3.A.\alpha)$$

to discuss two particular situations of interest:

$$1) \qquad \omega = \omega_a - \delta\omega/2 \quad (\delta\omega \ll \omega_\alpha), \qquad (3.A.\beta)$$

$$2) \qquad \omega_\alpha \gg \omega. \qquad (3.A.\gamma)$$

That is (3,A,δ)

$$\lim_{\omega \to i} \frac{\omega_\alpha^2}{\omega_\alpha^2 - \omega^2} = \begin{cases} \frac{\omega_\alpha}{\delta\omega} \gg 1 & (i=1) \text{ plastic } \alpha\text{-modes} \\ 1 & (i=2) \text{ elastic } \alpha\text{-modes,} \end{cases} \qquad (3.A.\varepsilon)$$

The first situation is typical of low-lying collective surface vibrations. The second of high lying giant resonances. While in this case one can parametrize the effect in terms of constant effective moments [12], the explicit treatment of the state-(ω-) dependence of the first one is unavoidable.

p. 214 Ⓖ

[12] see e.g. Bohr and Mottelson (1975) pp. 421 and 432

The BCS expressions of the pairing gap in terms of λ are

$$\Delta = \begin{cases} 2\hbar\omega_D e^{-1/\lambda}, & \text{(weak coupling } \lambda \ll 1) \\ \hbar\omega_D\lambda, & (\lambda \geq 1) \end{cases} \tag{3.A.26}$$

where ω_D is the limiting frequency of the low–lying collective modes of nuclear excitation, typically of quadrupole and octupole vibrations. While for weak coupling one can use $\hbar\omega_D \approx 10$ MeV, for the strong coupling situation seem more proper $\hbar\omega_D \approx 2$ MeV.

Making use of $\lambda = 0.4$, intermediate between weak and strong coupling situation one obtains

$$\Delta \approx 1.6\,\text{MeV}, \tag{3.A.27}$$

and

$$\Delta \approx 0.8\,\text{MeV}, \tag{3.A.28}$$

to be compared with the empirical value

$$\Delta \approx 1.4\,\text{MeV} \tag{3.A.29}$$

of superfluid medium heavy mass nuclei like ^{120}Sn.

While the relations (3.A.26) can hardly be relied to provide a quantitative ~~number~~ estimate, they testify to the fact that induced pairing is expected to play an important role in nuclei. These expectations have been confirmed by detailed confrontation of theory and experiment. 5

Hindsight

Static polarization effects can be important in clothing induced single–particle states. ~~E.g.~~ For example, effective charges ~~associated with giant resonances)~~ and interactions (associated with moments induced by giant resonances 6. ~~(static deformations induced by an effective moment ($\alpha^2 = -KF_p/C$, Eq. (6-217) of Bohr, A. and Mottelson (1975) on a second nucleon, Eq. (6-228) of same reference).~~ However, retarded ω–dependent self–energy effects and induced interactions are essential in describing structure and reactions of many–body systems. Examples are provided by the bootstrap binding of the halo neutrons (pair addition mode) to ^{9}Li, leading to the fragile $|^{11}\text{Li}(gs)\rangle$, displaying a $S_{2n} \approx 0.380$ MeV as compared to typical values of $S_{2n} \approx 18$ MeV as far as structure goes, and by the $^1\text{H}(^{11}\text{Li},^9\text{Li}(1/2^-;2.69\,\text{MeV}))^3\text{H}$ population of the lowest member of the $(2^+ \times p_{3/2}(\pi))_J$ J- multiplet of ^{9}Li, as far as reaction goes. 7

If there was need for support coming from other fields of research, one can mention just two: van der Waals force and superconductivity.

It was recognized early in the study of dipole–dipole interaction in atomic systems that, of the variety of contributions to the van der Waals interaction, the retarded, fully quantal contribution, arising from (dipole) zero point fluctuations (ZPF) of the two interacting atoms or molecules, and only active also in the case of non–polar molecules[8], play an overwhelming role, static–induced interactions

[8] Within this context van der Waals and gravitation are two forces which are universally operative, acting among all bodies.

5 see e.g. Idini et al (2016) Phys. Rev. C RC

6 see e.g. Bohr and Mottelson (1975), Eqs. (6-217) and (6-228)

A consequence

being less important. ~~An exampl~~e of this result is the fact that the limiting size of globular proteins (≈ 50 Å) is controlled by the strong damping undergone by the retarded contribution to the amino acid interaction, when the frequency associated with the back and forth propagation of the force, ~~(C/2x distance between molecules)~~ matches the molecules electronic frequencies[9]. — e

Concerning superconductivity, the overscreening effect which binds weakly Cooper pairs stems from a delicate ω–dependent phenomenon leading, eventually, to one of the first macroscopic manifestations of quantum mechanics, as e.g. "permanent" magnetic field ~~and~~ supercurrents. associate with

T

The statement "Life at the edge of chaos"coined in connection with the study of emergent properties in biological molecules (e.g. protein evolution, folding and stability) reflects the idea, as expressed by de Gennes (1994), that truly important new properties and results can emerge in systems at the border between rigid order and randomness, as testified by the marginal stability and conspicuous fluctuations characterizing, for example, Cooper pairs at the dripline and in metals, and that of proteins of e.g. ~~lethal~~ viral. Let us conclude this short comment, ~~and eventually this short monograph~~ quoting again de Gennes but doing so with the hindsight twenty years of nucear research which have elapsed since "Les objets fragiles" was published. Chapter "Savoir s'arreter, savoir changer" starting at p. 180 starts "En ce moment, la physique nucléaire (la science des noyaux atomiques) est une science qui, à on avis, se trouve en fin de parcours... C'est une physique qui demande des moyens coûteux, et qui s'est constituée par ailleurs en un puissant lobby. Mais elle me semble naturellement ext 'enuée... je suis tenté de dire: "Arretons"... mais ce serait aussi absurde que de vouloir arreter un train a grande vitesse. Le mieux serait d'aiguiller ce train sur une autre voie, plus nouvelle et plus utile à la collectivité."

italics

in

nuclear

particles like the HIV-1- and HCV-proteases.

de Gennes remark, comunity

and

In a way, even without knowing ~~it~~, part of the nuclear physics ... have followed it, capitalizing on the novel embodiment that concepts like elementary modes of excitation, spontaneous symmetry breaking and phase transitions have had in this paradigm of finite many–body system the nucleus is, where fluctuations dominate, as a rule, over potential energy effects. The use of these concepts tainted by the FMB system effects as applied to proteins, in particular to the understanding of protein folding, arguably, may result in the design of drugs which do not create resistance[10]. — e

a quantity which is

Sect. 2.8.1

are

2.8.5

(correlation length)

physical picture of nuclear pairing. Even under the

[9]It is of notice that similar arguments (cf. ~~Appendix:~~) ~~is~~ are at the basis of the estimate (~~??~~) concerning the size of the halo nucleus ^{11}Li, influenced to a large extent by the maximum distance over which partners of a Cooper pair are solidly anchored to eaeh other (localized), and have to be seen as an (extended) bosonic entity and not as two fermions. ~~That one does not so in calculating e.g. the transfer//.~~ The fact that cooper pair transfer proceedes mainly in terms of successive transfer controlled by the single-particle mean field, reinforces the above the effect of extremely large, as compared to the pair correlation energy, external single-particle field, namely that of target and projectile, the Cooper pair field extends over the two nuclei, also through a tiny density overlaps.

permeating the whole summed nuclear volume

Broglia (2013) More is different: 50 Years of Nuclear BCS, in 50 Years of Nuclear BCS Eds. ... p. 643,

[10] See e.g. Broglia (2013) and refs therein

L.N. Cooper, Remembrance of superconductivity past, in BCS: 50 Years, eds. L.N. Cooper and D. Feldman, World Scientific, Singapore p. 12 (2012)

S. Weinberg, From BCS to LHC, in .... p. 559 (2012)

G.S. Stent, Ed. The double helix, Norton and Co, New York (1980)

3.B Absolute Cooper pair tunneling: quantitative novel physics at the edge between stability and chaos

to p. 222

In the study of many–body systems, in particular of finite many–body systems (FMBS) like the atomic nucleus, much can be learned from symmetries (group theory) as well as from the general phenomena of of spontaneous symmetry breaking. However, it is the texture of the associated emergent properties, concrete embodiment of symmetry breaking (potential energy) and of its restoration (fluctuations, collective modes), which provides insight into the eventual new physics. In fact, when one understands the many–body under study, in terms of the detailed motion of single–particles (nucleons) and collective motion, taking properly into account their couplings and associated zero point fluctuations, is that one can hope to have reached a solid, quantitative, understanding of the problem and of its solutions. Even more, that these solutions are likely transferable, at profit, to the study of other FMBS [12] like e.g. metal clusters, fullerenes [11] (cf. e.g. Broglia et al. (2004)), ~~quantum dots~~, and eventually proteins, let alone the fact that one can make ~~predictions.~~ Predictions which, in connection with the study of halo nuclei, in particular of pairing [13] (cf. e.g. Broglia, R. A. and Zelevinsky, V. (2013)) in such exotic, highly extended systems lying at the nucleon drip line, involve true novel physics [14] (cf. e.g. Barranco, F. et al. (2001); Tanihata, I. et al. (2008); Potel et al. (2010) and references therein). Within this context one can quote from Leon Cooper's contribution to the volume [15] BCS: 50 years: "It has become fashionable... to assert... that once gauge symmetry is broken the properties of superconductors follow... with no need to inquire into the mechanism by which the symmetry is broken[10]. This is not ...true, since broken gauge symmetry might led to molecule–like and a Bose–Einstein rather than BCS condensation... in 1957... the major problem was to show... how... an order parameter or condensation in momentum space could come about... to show how... gauge–invariant symmetry of the Lagrangian could be spontaneously broken due to interactions which were themselves gauge invariant".

I — In — italics — Broglia, Colò, Onida, Roman

Nuclear physics has brought this quest a step further. This time in connec-

[10] Detailed quoting (Weinberg (2012)): "...In consequence of this spontaneous symmetry breaking, products of any even number of electron fields have non–vanishing expectation values in a superconductor, though a single electron field does not. All of the dramatic exact properties of superconductors –zero electric resistance, the expelling of the magnetic fields from superconductors known as the Meissner effect, the quantization of magnetic flux through a thick superconducting ring, and the Josephson formula for the frequency of the ac current at a junction between two superconductors with different voltages– follow from the assumption the electromagnetic gauge invariance is broken in this way, with no need to inquire into the mechanism by which the symmetry is broken." ~~(Weinberg, S. in BCS: 50 years)~~. The above quotation is similar to saying that once the idea of a double DNA helix was thought, all about inheritance was solved and known, and that one could forget the X–ray plates of Rosalind Franklin, Maurice Wilkins and collaborators, let alone how DNA and protein interact with each other (cf. e.g. Stent (1980) and references therein). ~~Because a similar attitude was adopted once the human genome was sequenced, is that we are now, more than half a century after the paper of Crick and Watson (1953) using still primitive models of hard spheres to work out the working of inheritance (cf.e.g. ? and references therein).~~

11 13 14

12 E. Lipparini, Modern Many-Particle Physics: Atomic Gases, Quantum Dots and Quantum Fluids, World Scientific, Singapore (2003).

15 L.N. Cooper (2012)

being

tion with the "extension" of the study of BCS condensation to its origin, a single Cooper pair in the rarified atmosphere resulting from the strong radial (isotropic) deformation observed in light halo exotic nuclei in general, and in ^{11}Li in particular. During the last few years, the probing of this system in terms of absolute two–nucleon transfer (pick–up) reactions, has made this field a quantitative one, errors below the %10 limit ~~are~~ the rule. This achievement which has its basis on the remarkable experiments of Tanihata, I. et al. (2008), is also the result of the combined effort~~s~~ made in treating the structure and reaction aspects of the subject, two sides of the same physics, on equal footing. In particular regarding the ~~treatment~~ description of the continuum and of the fluctuations leading to both ~~to~~ single–particle and collective modes clothing, as well as present as ~~zero-point motion~~ ZPF of the ground state. New physics has been seen to emerge from situations in which these fluctuations diverge (like was also known to occur in the case of e.g. pairing rotational bands) or are on resonance, as in the case of the halo pair addition mode of ^{9}Li (i.e. ^{11}Li(gs)) and likely of ^{10}Be (i.e. Be(0^{+*}; 2.24 MeV)).

Saturation density, spill out and halo

to the chapter on bulk properties of nuclei

In the incipit of Bohr and Mottelson (1969) one reads: "The almost constant density of nuclear matter is associated with the finite range of nuclear forces; the range of the forces is r_0 (where r_0 enters the nuclear radius in the expression $R = r_0 A^{1/3}$) thus small compared to nuclear size. This "saturation" of nuclear matter is also reflected in the fact that the total binding energy of the nucleus is roughly proportional to A. In a minor way, thee features are modified by surface effects and long–range Coulomb forces acting between the protons".

Electron scattering experiments (see the figure 2-1 and the corresponding caption containing the references in p. 159 of the above reference) yield

$$\rho(0) = 0.17\,\text{fm}^{-3}. \tag{3.A.30}$$

Thus, one can posit that

$$\frac{4\pi}{3} R_0^3 \rho(0) = A, \tag{3.A.31}$$

leading to

$$r_0 = \left(\frac{3}{4\pi}\frac{1}{\rho(0)}\right)^{1/2}$$

$$r_0 (=) \left(\frac{3}{4\pi}\frac{1}{\rho(0)}\right)^{1/3} \approx 1.12\,\text{fm}. \tag{3.A.32}$$

Because the above relations imply a step function distribution, we have to add to (3.A.32) the nucleon spill out (16) $(a_0/R_0)\ln 2 \approx 0.07$ ($\approx (a_0/R_0)\ln 2 \approx (0.5/6) \times 0.69\,(A = 120)$) associated with the fact that a more realistic distribution is provided by a Fermi function of diffusivity $a_0 \approx 0.5$ fm. Thus $r_0 = (1.12 + 0.07)$ fm ≈ 1.2 fm. In the case of the nucleus ^{11}Li, observations indicate a mean square

(16) Bertsch and Broglia (2005)

$R(^{11}\mathrm{Li}) - R_0(^{9}\mathrm{Li}) = R_0(^{9}\mathrm{Li})\left(\frac{R(^{11}\mathrm{Li})}{R_0(^{9}\mathrm{Li})} - 1\right) = 0{,}83\, R_0(^{9}\mathrm{Li})$. In other words, ^{11}Li can be viewed as a "normal" ^{9}Li core and a two neutron skin extending over a radius of the order of that of the core.

17 radius $\langle r^2\rangle^{1/2} = 3.55 \pm 0.1$ fm (Kobayashi, T. et al. (1989)). Thus

To be noted that the actual mass number predicts

$$R(^{11}\mathrm{Li}) = \sqrt{\frac{5}{3}}\langle r^2\rangle^{1/2} \approx 4.58 \pm 1.3\,\mathrm{fm}. \qquad (3.A.33)$$

$(R_0(^{11}\mathrm{Li})$ 33

Making use of the relation $R_0 = 1.2A^{1/3}$ fm, the quantity (3.A.~~32~~) leads to $(4.58/1.2)^3 \approx$ 56, an effective mass number larger five times the actual value $A = 11$. ~~mass number connected with~~ a "systematic" value of the nuclear radius $R_0 \approx 2.7$ fm.

$= 1{,}2\,\mathrm{fm}\,(11)^{1/3}$

or halo region (skin),

The above results testifies to a very large "**radial deformation**", in keeping with the fact [11]. But even more that ~~this~~ deformation affects matter which is little complient to undergo compressions or, for that sake, ~~equally conspicuous~~ "depressions", without resulting in nuclear instability. In one case, through a mini supernova. In the second, by obliterating the effect of the short range strong force acting in the $^1S_0$ channel (pairing interaction).

( )] [,

either

the above mentioned

In fact, in the case of the halo Cooper pair of ^{11}Li, that is of the last two weakly bound neutrons, one is dealing with a rarefied nuclear atmosphere of density

$$\rho \approx \frac{2}{\frac{4\pi}{3}(R^3(^{11}\mathrm{Li}) - R_0^3(^{9}\mathrm{Li}))} \approx 0.6 \times 10^{-2}\,\mathrm{fm}^{-3} \qquad (3.A.34)$$

to typical,

where the value $R_0(^9\mathrm{Li}) \approx 2.5$ fm was used. That is, we are dealing with pairing in a nuclear system at a density which is only 4% of saturation density.

in which not only the line shape, but the main structure of the resonance depends on the state on which it is built, and to which it is deeply interweaved as to guarantee its stability 18.

The quest for the long range pairing mechanism which is at the basis of the binding of the halo Cooper pair of ^{11}Li to the ^{9}Li core ($S_{2n} \approx 0.380$ keV, to be compared ~~with~~ systematic values of $S_{2n} \approx 16$ MeV), has lead to the discovery of a novel nuclear mode of elementary excitation. The symbiotic halo pair addition mode, which has to carry its own source of binding (glue) like the hermit crab who gastropod shell to protect his body. A novel embodiment of the Axel–Brink scenario, ~~let alone~~ of the Bardeen–Frölich–Pines microscopic mechanism to break gauge invariance: through the exchange of ZPF which ~~restore~~ Galilean invariance to a nucleus displaying essentially a permanent dipole ~~mode~~, as a consequence of the almost degeneracy of the giant dipole pigmy resonance (centroid ~~≈ 0.6~~ MeV) with the ground state. To our knowledge, this is the first example of a van der Waals Cooper pair, atomic or nuclear.

It also provides a novel realization

carries a

quite large

ensures

moment

$\lesssim 1$

W

The NFT diagram shown in Fig. ~~??~~ 2.A.1 describing this binding seems quite involved and high order. Thus unlikely to be at the basis of a new elementary mode of nuclear excitation, if nothing else because of the apparent lack of "elementarity'. This is not the case and, in fact, the physics at the basis of the process depicted by the oyster–like and eagle–like networks displayed in (a) and (b) is ~~quite simple and~~ present throughout nuclear structure and reactions, ~~let alone~~ many–body theories and QED. In fact, it encompasses (see Fig. ~~??~~): (I,II) the changes in energy of single–particle levels as a function of quadrupole deformations (Nilsson model)

2.A.1

[11] ~~In fact~~ $R = R_0(1 + \alpha_{00}Y_{00}) = R_0(1 + \beta_0\frac{1}{\sqrt{4\pi}})$, ~~and~~ Thus $\beta_0 = \sqrt{4\pi}(\cancel{1 - \frac{R}{R_0}}) \approx 2.5$ which testifies to the extreme "exoticity" of the phenomenon.

$\left(\frac{R}{R_0} - 1\right)$

~~eventuall put text~~

17 Parametrizing the radius of ^{11}Li as (see Bohr and Mottelson (1975)),

18 Footnot 20 Physica Scripta with the two reference 51 and 79 to Axel and to Brink

see appendix

through the poor overlap between core and single-particle wavefunctions so as to be able to

19 Hamamoto and Susumu JPG 34, 2715 (2007)

(III) the interaction between particles through the exchange of (bosons) vibrations, (IV,V) Pauli principle, (VI,VII) the softening of collective modes due to ground state correlations ((ZPF)–components, QRPA) and eventually the permanent distortion of the system (phase transition), (VIII) the interaction between two non–polar systems through ZPF generated dipoles. Referring to general many–degree of freedom systems, (I,II) and (III) are at the basis of the fact that, in QED, the coupling between one and two photons is zero (Furry's theorem). It is also at the basis, through cancellation, of the small width displayed by giant quadrupole resonances as compared with single–particle widths at similar energies as well as .... inhomogeneous damping in NMR of molecules ~~as well as in e.g.~~ and in GDR ~~in nuclei~~ of atomic nuclei. Concerning (VIII), one can mention resonant interactions between fluctuating systems like e.g. two coupled harmonic oscillators. It is like to find a new particle. Either ~~you are~~ one is at the right energy (on resonance) or ~~you~~ one would not see it.

In the case of halo Cooper pair binding by GDPR in ^{11}Li, the system is essentially on resonance, in keeping with the fact $\epsilon_{p_{1/2}} - \epsilon_{s_{1/2}} \approx 0.3$ MeV, and that (see Fig. 3.A.4) independent particle motion emerges from the same properties of the force from which collective modes emerge. In other words the ^{10}Li inverted parity system is poised to acquire a permanent dipole moment or, almost equivalent, to display a large amplitude, dipole mode at very low energy as well as a collective $B(E1)$ to the halo ground state, of the order of a single-particle unit B_{sp}. This is the GDPR with centroid about 0.6–0.8 MeV, 8% of the EWSR and so screened from the GDR ~~that it can~~ retain essentially all of its B_{sp}, E1-strength, and could rightly be considered a new mode of excitation (see discussion after eq. (5.2.115)). In other words we are faced, already at the level of ~~simple~~ single particle single-particle spectrum, with the possibility of a plastic dipole mode, as it materializes in ^{11}Li. In this case, and making use of the relation

$$\frac{dn}{d\beta_L} = \frac{1}{4}\sqrt{\frac{2L+1}{3\pi}}A \tag{3.A.35}$$

defining the number of crossings n in terms of deformation (cf. Bertsch and Broglia (2005)), one obtains for $L = 0$ and $\beta_0 = 2.5$, $n \approx 2\cancel{2}$. i.e. $n \approx 2$.

It is of notice that all of these processes takes place inside the halo neutron pair addition vibrational mode of the closed shell system $^9_3$Li$_6$(gs), and thus in terms of virtual states. On the other hand ~~....~~ But let us now proceed ~~a~~ one step~~s~~ at a time. A very attractive, simple and economic picture of the giant dipole pygmy resonance was proposed ~~by ?~~ in 19. To explain parity inversion use is made of the fact that, for large prolate quadrupole deformations ($\beta_2 \approx 0.6 - 0.7$), the $m = 1/2$ member of the $1d_{5/2}$ and $1p_{1/2}$ orbitals, i.e. [220 1/2] and [101 1/2] in the Nilsson labeling of levels ([$Nn_3\Lambda\Omega$]), cross. This is in keeping with the fact that quadrupole distortion changes the energy of single–particle states; those having orbits lying in a plane containing the poles become, in the case of prolate deformations, lower ~~in~~ their energy, while those lying preferentially in a plane perpendicular to the symmetry axis, increase their energy. Now, this parity inversion is already observed between the resonant $1/2^-$ (≈ 0.5 MeV) and the virtual $1/2^+$ (≈ 0.2 MeV) states of ^{10}Li ($p_{1/2}$ and $2_{1/2}$ states). Thus, the energy difference of 0.3 MeV is not very different from the

footnote

intervening the processes depicted in Fig. 2.A.1 with external fields, e.g. those associated with one- and two-particle transfer processes, provides much of the physics which is at the basis of the exotic properties of ^{10}Li and ^{11}Li (see e.g. Fig 2.8.3(I), ~~and~~ 1.9.4 and 1.9.5. See also 6.1.3) that the above model predicts $R = R_0\left(1+\frac{\beta_2}{\sqrt{5}}\sqrt{\frac{5}{4\pi}}\right) = 2{,}7\,\mathrm{fm} \times 1{,}2 \approx 3{,}2\,\mathrm{fm}$ ($\beta_2 \approx 0{,}7$), in disagreement

(On the other hand,)

value of 0.6-0.7 MeV of the GDPR centroid. In any case, adjusting β_2 to the appropriate value this centroid energy is within reach. ~~Furthermore,~~ because the radius is ~~strongly~~ affected by deformation, one can posit ~~that $R = R_0(1 + 0.63\beta_2/\sqrt{5}) \approx 3.2$ fm, not in brilliant agreement~~ with the experimental finding.

Nonetheless, the ~~The~~ fact furthermore that the observed $\approx 8\%$ of the EWSR below ≈ 5 MeV for the GDPR corresponds to about $1B_{sp}(E1)$ for a single particle transition, provides another confirmation of the ~~soundness~~ attractiveness of the model. Now, static models (including also the group theoretical models like that provided by SU_3) imply that single–particle states are either occupied or empty. Experimentally, this does not seem the case in the reaction $^9\mathrm{Li}(d,p)^{10}\mathrm{Li}$, although one can argue that the situation is different in the case of the single–particle states in ^{11}Li.

Second, the $1/2^+$ ~~and~~ $1/2^-$ single–particle ~~states will be~~ transitions are also part of the GDR, which transition will essentially absorb ~~all~~ most of the $E1$ strength into the high energy mode. In fact, typical $E1$-low energy single particle transition display $\approx 10^{-4}B_{sp}(E1)$. Now, inhomogeneous damping ~~can bring~~ brings the dipole oscillations along the symmetry axis to an energy of

$$(\hbar\omega_D) \approx \frac{100\,\mathrm{MeV}}{\cancel{R_0(1+0.63\beta_2/\sqrt{5})}\ 3.2} \approx 30\,\mathrm{MeV} \qquad (3.A.36)$$

far away from the less than 1 MeV energy corresponding to the GDPR centroid.

Now, in order to calculate the giant dipole pygmy resonance based on the ground state of ^{11}Li ~~we~~ one needs to know the occupation factors of the $s_{1/2}$ and $p_{1/2}$ states. This has been done microscopically making use of the diagonalization of the NFT diagrams taking into account self–energy and induced interaction (vertex renormalization processes) leading to

$$|\tilde{0}\rangle = |0\rangle + 0.71|(p_{1/2}, s_{1/2})_{1^-} \otimes 1^-; 0\rangle + 0.1|(s_{1/2}, d_{5/2})_{2^+} \otimes 2^+; 0\rangle, \qquad (3.A.37)$$

and

$$|0\rangle = \cancel{0.75}\,0.45|s^2_{1/2}\rangle + 0.55|p^2_{1/2}\rangle + 0.04|d^2_{5/2}\rangle. \qquad (3.A.38)$$

In Eq. (3.A.37), the state $|1^-\rangle$ and $|2^+\rangle$ stand for the giant dipole pygmy resonance, and for the low–lying collective quadrupole vibration of ^{9}Li, respectively. As it emerges from (3.A.37) and (3.A.38), to calculate the microscopic structure of the state $|1^-\rangle$ (both wavefunction and transition density and consequently the particle–vibration coupling vertex) one needs to calculate $|0\rangle$. But to do ~~it~~. A similar calculation have been carried out for ^{12}Be(gs) and ^{12}Be(0^{+*}; 2.24 MeV). In the first case no pygmy is found, while in the second case ~~a~~ well developed GDPR is ~~observed with~~ predicted displaying a number of peaks ~~a~~ below 2 MeV and carrying a summed EWSR in the interval 0-5 of $\approx$ 6%. This result testifies to the fact that the symbiotic halo pair addition mode is a *bona fide* elementary mode of excitation. Its symbiotic GDPR allows to probe the state on which it is based, making the Axel–Brink mechanism a tool to ~~probe~~ study the structure of halo states. Within this context see Fig. 3.A.5.

so one needs to know the same $|1^-\rangle$ state, the vibrational mode which exchanged between the two neutrons of the halo provides most of its glue to the ^{9}Li core. From here, the symbiotic character of the 0^+ and 1^- (GDPR) entering the $|^{11}\mathrm{Li}(0^+_\nu \otimes p_{3/2}(\pi)); gs\rangle_{3/2}$ and $|^{11}\mathrm{Li}(1^- \otimes p_{3/2}(\pi))_{1/2,3/2,5/2}; \approx 0{,}8\,\mathrm{MeV}\rangle$ state.

In this figure an attempt is made to provide the (circumstantial) evidence, both experimental and theoretical which testify to the possibility to view the neutron halo pair addition mode and its symbiotic GDPR a new elementary mode of excitation which can be found in more than one nucleus and thus be used as a (composite) building block of the spectrum of exotic, light halo nuclei.

Fig. 3.A.5
Fig. A5 Physica Scripta p. 21 with caption and Refs.

N.W. Ashcroft and N.D. Mermin, Solid state physics, Holt, Rinehart and Wanston, Hong Kong (1987)

to p. 217

3.A.2 Metals

Plasmons and phonons (jellium model)

The expression of the electron plasmon frequency of the antenna–like oscillations of the free, conduction electrons against the positive charged background (jellium model) is

$$\omega_{ep}^2 = \frac{4\pi n_e e^2}{m_e} = \frac{3e^2}{m_e r_s^2}, \tag{3.A.39}$$

where

$$n_e = \frac{3}{4\pi}\frac{1}{r_s^3}, \tag{3.A.40}$$

are the number of electrons per unit volume, r_s being the radius of a sphere whose volume is equal to the volume per conduction electron,

$$r_s = \left(\frac{3}{4\pi n_e}\right)^{1/2}, \tag{3.A.41}$$

that is, the radius of the Wigner–Seitz cell.[20] Ashcroft and Mermin (1987)

For metallic Li (cf. page 5, table 1.1 of)

$$n_e = 4.70\frac{10^{22}}{\mathrm{cm}^3} = \frac{4\times 10^{-2}}{\mathring{A}^3} \tag{3.A.42}$$

while

$$r_s = \left(\frac{3\mathring{A}^3}{4\pi\times 4.7\times 10^{-2}}\right)^{1/3} = 1.72\mathring{A} \tag{3.A.43}$$

implying a value $(r/a_0) = 3.25$ in units of the Bohr radius ($a_0 = 0.529\mathring{A}$).

Making use of

$$\alpha = 7.2973 = \frac{e^2}{\hbar c} \tag{3.A.44}$$

and

$$e^2 = 14.4\,\mathrm{eV}\,\mathring{A}, \tag{3.A.45}$$

one obtains

$$\hbar c = \frac{14.4\,\mathrm{eV}\,\mathring{A}}{7.2973} = 1973.3\,\mathrm{eV}\,\mathring{A}. \tag{3.A.46}$$

Making use of the above values and of

$$m_e c^2 = 0.511\,\mathrm{MeV}, \tag{3.A.47}$$

one can write

$$\hbar^2\omega_{ep}^2 = \frac{(\hbar c)^2}{m_e c^2}\frac{3e^2}{r_s^3} = \frac{(1973.3\,\mathrm{eV}\,\mathring{A})}{0.511\times 10^6\,\mathrm{eV}}\frac{3\times 14.4\,\mathrm{eV}\,\mathring{A}}{(1.72\,\mathring{A})^3} = 64.69\,\mathrm{eV}^2 \tag{3.A.48}$$

[20]

C. Kittel, Itrodution to solid state physics, Wiley, NJ (1996)

leading to 21

1.94 × 10⁹ MHz

$$\hbar\omega_{ep} = 8.04\,\mathrm{eV} \approx \cancel{10^{15}\,\mathrm{sec}^{-1}} \tag{3.A.49}$$

~~(cf. Table 2 p. 262, ?)~~

~~One can now estimate the ion plasmon frequency, again~~ for metallic Li, ~~namely~~ one obtains

$$\hbar\omega_{ip} = \left(\frac{Zm_e}{AM}\right)^{1/2} \hbar\omega_{ep} = \left(\frac{3\times 0.5}{9\times 10^3}\right)^{1/2} \times 1.94 \times 10^{15}\,\mathrm{sec}^{-1}$$

$$\approx 2.5 \times \cancel{10^{-2}} \times 10^{15}\,\mathrm{sec}^{-1} \approx 10^{13}\,\mathrm{sec}^{-1} \approx 1.04 \times 10^2\ \cancel{40}\,\mathrm{meV}. \tag{3.A.53}$$

For the case of metal clusters of Li, the Mie resonance frequency is

$$\hbar\omega_M = \frac{\hbar\omega_{ep}}{\sqrt{3}} = 4.6\,\mathrm{eV}. \tag{3.A.50}$$

3.A.3 Elementary theory of phonon dispersion relation

~~Electron plasmon oscillation frequency (jellium model) can be written as (? p. 512)~~

(again, within the framework of the jellium model),

$$\boxed{\cancel{\omega_{ep}^2 = \frac{4\pi n_e e^2}{m_e}}} \tag{\cancel{3.A.52}}$$

~~Assuming an equally uniform negative background~~ one can estimate the long wavelength ionic plasma frequency introducing, in ~~the above equation~~ (3.A.39), the substitution $e \to Ze$, $m_e \to AM$ ($A = n + Z$, mass number, M nucleon mass), $n_e \to n_i = n_e/Z$,

$$\omega_{ip}^2 = \frac{4\pi n_i (Ze)^2}{AM} = \frac{Zm_e}{AM}\omega_{ep}^2. \tag{3.A.52}$$

Now, both ~~of~~ the ~~above~~ relations (3.A.39) and (3.A.52), although being quite useful, are wrong from a many–body point of view: ω_{ep} because electrons appear as bare electrons not dressed by the phonons, neither by the plasmons; ω_{ip} because the static negative background does not allow for an exchange of electron plasmons between ions, ~~thus~~ exchange eventually leading to a screened, short–range ionic Coulomb repulsive field. Namely ions interact in the approximation used above, in terms of the "bare" ion–ion Coulomb interaction. Being it infinite range it does not allow for a dispersion relation linear in k at long wavelengths (sound waves) but forces a finite "mass" also to the lattice phonons. Allowing for electron screening of the "bare" ion–ion Coulomb interaction, as embodied in the electron gas dielectric function $\epsilon(0, q) = q^2/(k_s^2 + q^2)$ one obtains the dressed phonon frequency

$$\omega_q^2 = \frac{\omega_{ip}^2}{\epsilon(q)} = \frac{Zm_e}{AM}\frac{\omega_{ep}^2}{q^2 + k_s^2}q^2. \tag{3.A.54}$$

The quantity k_s is the Thomas–Fermi screening wave vector, a quantity which is of the order of the Fermi momentum, the associated screening length being thus of the order of the Wigner-Seitz radius.

Thus,

$$\lim_{q\to 0} \omega(q) = c_s q, \tag{3.A.55}$$

21 Kittel (1996) Table 2, p. 278

where use has been made of

$$n_e = \frac{3}{4\pi}\frac{1}{r_s^3} = 4.7\times 10^{-2}\,\text{Å}^{-3} \quad (r_s = 1.72\,\text{Å, Li}),$$

and

$$\epsilon_F = \frac{50.1}{(r_s/a_0)} \approx 15.42\ \text{eV}) \quad (r_s/a_0 = 3.25,\ \text{Li})$$

where

$$c_s^2 = \frac{Zm_e}{AM}\frac{\omega_{ep}^2}{k_s^2}, \tag{3.A.56}$$

is the sound velocity. Making use of (22) (cf. ~~a~~ Mermin an Ashcroft (1987) p. 342)

$$k_s = \left(\frac{6\pi^2 n_e e^2}{\epsilon_F}\right)^{1/2} = \left(\frac{6\pi^2 Z n_i e^2}{\epsilon_F}\right) \approx 1.6\,\text{Å}^{-1} \tag{3.A.57}$$

and of (3.A.39),

$$\cancel{\boxed{\omega_{ep}^2 = \frac{4\pi n_e e^2}{m_e}}} \qquad \cancel{(3.A.58)}$$

one can write

$$c_s^2 = \frac{Zm_e}{AM}\frac{4\pi n_e e^2}{m_e}\frac{\epsilon_F}{6\pi^2 n_e e^2} = \frac{2Z}{3\pi AM}\epsilon_F = \frac{Zm_e}{3\pi AM}v_F^2, \tag{3.A.59}$$

With the help of

$$k_F = \frac{1.92}{r_s}, \tag{3.A.60}$$

and of the velocity of light,

$$c = 3\times 10^{10}\ \text{cm/sec}, \tag{3.A.61}$$

one obtains,

$$v_F = \left(\frac{\hbar}{m_e}\right)k_F = \left(\frac{\hbar c}{m_e c^2}\right)\times 3\times 10^{10}\frac{\text{cm}}{\text{sec}}\frac{1.92}{r_s} \approx 10^8\frac{\text{cm}}{\text{sec}} \tag{3.A.62}$$

(⁹Li) Li the associated quantality parameter is ....., see Table 2.3.1

Thus, for ~~a metal made out of $^9_3\text{Li}_6$~~

$$c_s^2 = \frac{1}{3}\frac{3m_e}{9M}v_F^2 \approx \cancel{10^{-3}}\ 4.2\times 10^{-3}\, v_F^2, \tag{3.A.63}$$

and

$$c_s \approx \cancel{10^{-3}}\ 0.42\times 10^{-2}\, v_F \approx \cancel{10^{5}}\ 0.54\times 10^{6}\frac{\text{cm}}{\text{sec}}. \tag{3.A.64}$$

That is, about a hundredth of the Fermi velocity, or of the order of 10^6 cm/sec, in overall agreement with order of magnitude experimental findings (Ashcroft and Mermin (1987), p. 514)

Let us now discuss the effective electron–electron interaction. Within the jellium model used above one can write it as

$$V(\mathbf{q},\omega) = \frac{U_c(q)}{\epsilon(\mathbf{q},\omega)}, \tag{3.A.65}$$

where the dielectric function

$$\epsilon(\mathbf{q},\omega) = \frac{\omega^2(q^2+k_s^2) - \omega_{ip}^2 q^2}{\omega^2 q^2} = \frac{(q^2+k_s^2)(\omega^2-\omega_q^2)}{\omega^2 q^2} \tag{3.A.66}$$

contains the effects due to both the ions and the background electrons, while

$$U_c(q) = \frac{4\pi n_e e^2}{q^2} \tag{3.A.67}$$

(22)

$$= \frac{2\times 10^3\,\text{Å eV}}{0.5\times 10^6\,\text{eV}}\times 3\times 10^{10}\frac{\text{cm}}{\text{sec}}\frac{1.92}{r_s} = \frac{2.3\times 10^{14}}{10^6}\frac{1}{(r_s)_{\text{Å}}}\frac{\text{cm}}{\text{sec}}$$

as compared to $U_c(r=5\text{Å}) \approx 2.9\,\text{eV}$,

is the Fourier transform of the bare Coulomb interaction

$$U_c(r) = \frac{e^2}{r}. \qquad (3.A.68)$$

For $\omega \gg \omega_{ip}$ one obtains the so called screened Coulomb field,

$$U_c^{scr}(q) = \frac{4\pi e^2 n_e}{q^2 + k_s^2}, \qquad (3.A.69)$$

its **r** space Fourier transform being

$$U_c^{scr}(r) = \frac{e^2}{r} e^{-k_s r}. \qquad (3.A.70)$$

a quantity that for ~~For~~ large values of r ~~it~~ falls off exponentially. Thus, in the high frequency limit, the electron–electron interaction, although strongly renormalized by the exchange of plasmons, as testified by the fact that (e.g. for Li),

$$U_c^{scr}(r = 5\,\text{Å}) \approx U_c(r = 5\,\text{Å}) e^{-1.6\times 5} \approx 1\text{meV}, \qquad (3.A.71)$$

1 meV

is still repulsive.

Let us now consider metallic frequencies $\omega \ll \omega_{ip}$ ~~but for~~ values of q of the order of a^{-1}, where a is the lattice constant ($a \approx 3-5$Å, $a^{-1} \approx 0.25$Å^{-1}) to be compared to $k_s \approx 1.6$Å^{-1} and $k_F \approx 1.12$Å^{-1} (~~case of~~ Li). In the case in which ~~If~~ $\omega_{ip}^2/\omega^2 > (q^2 + k_s^2)/q^2$ V ~~is~~ attractive (see Eqs. (3.A.65) and (3.A.66)). ~~and~~ this behavior explicitly involves the ions through ω_{ip} (electron–phonon coupling).

¶ The dispersion relation of the associated frequency collective modes follows from

$$\epsilon(\mathbf{q}, \omega) = 0. \qquad (3.A.72)$$

Making use of Eq. (3.A.66) one obtains the relation (3.A.54), as expected. One can now rewrite the reciprocal of the dielectric functions in terms of ω_q, that is,

$$\frac{1}{\epsilon(\mathbf{q}, \omega)} = \cancel{\frac{\omega^2 q^2}{(q^2 + k_s^2)}\left[\frac{(\omega_q^2 - \omega^2) + \omega_q^2}{\omega^2 - \omega_q^2}\right]} = \frac{q^2}{q^2 + k_s^2}\left[1 + \frac{\omega_q^2}{\omega^2 - \omega_q^2}\right]. \qquad (3.A.73)$$

$$\frac{\omega^2 q^2}{(q^2+k_s^2)(\omega^2-\omega_q^2)} =$$

. F

For $\omega \gg \omega_q$ one recovers the Thomas–Fermi dielectric function. For ω near, but smaller than ω_q the interaction is attractive.[23] The effective electron–electron interaction can then be written as

$$V(q, \omega) = \frac{4\pi n_e e^2}{q^2 + k_s^2} + \frac{4\pi n_e e^2}{q^2 + k_s^2} \frac{\omega_q^2}{\omega^2 - \omega_q^2},$$

$$= U_c^{scr}(q) + U_c^{scr}(q)\frac{\omega_q^2}{\omega^2 - \omega_q^2} = U^{scr}(q)\left(1 + U^{scr}(q)\Pi(q, \omega)\right) \qquad (3.A.74)$$

~~where~~ The quantity

$$\Pi(q, \omega) = \left(\frac{Z}{AM}\right) \frac{q^2}{\omega^2 - \omega_q^2} \qquad (3.A.75)$$

23 Schrieffer (1964), Fig. 6-11, p. 152

(24) J. Lindhard, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. ....., 28 No. 8 (1954)

title

Fig. 3.A.5 Schematic representation of the variety of contributions to the effective interaction in nuclei and in metals.

caption

is intimately connected with Lindhard's function.(24) ~~cf.~~ see also the close relation with the expression ~~(22)~~ 3.A.18 of the nuclear renormalized pairing interaction. The first term of $V(q,\omega)$ contains the screened Coulomb field arising from the exchange of ~~(electron)~~ plasmons between electrons (cf. Fig. ~~(??)~~ 3.A.5 bottom this page). The second term with the exchange of collective low frequency phonons calculated making use of the same screened interaction as emerges from (3.A.74).

Let us now introduce the ~~dimensional~~ dimensionless quantity

$$\lambda = \langle F|V|I\rangle = N(0)U_c^{scr}\left(1 + U_c^{scr}\Pi\right). \tag{3.A.76}$$

In the weak coupling limit ($\lambda^2 \ll \lambda$)

$$\Delta = 2\omega_D e^{-1/\lambda}, \tag{3.A.77}$$

where ω_D is the Debye energy.

Now, provided that we are in a situation in which ω is consistently different from ω_q,

$$\cancel{\boxed{U_c^{scr}\Pi \approx 0.2,}} \tag{3.A.78}$$

$$\frac{1}{\lambda} = \frac{1}{N(0)U_c^{scr}\left(1 + U_c^{scr}\Pi\right)} \approx \frac{1}{N(0)U_c^{scr}}\left(1 - U_c^{scr}\Pi\right), \tag{3.A.79}$$

~~an approximation which is correct within 4%~~. Thus

$$\frac{1}{\lambda} = \frac{1}{N(0)U_c^{scr}} - \frac{\Pi}{N(0)}, \tag{3.A.80}$$

Otherwise, the erro introduced through a resonant renormalization process, entering the expression of e.g. the pairing gap may be quite large.

and

$$\Delta = \left(2\omega_D e^{\frac{\Pi}{N(0)}}\right) e^{-\frac{1}{N(0)U_c^{scr}}}. \tag{3.A.81}$$

Thus, the renormalization effects ~~to~~ of the pairing gap associated with phonon exchange are independent on the approximation used to calculate U_c^{scr} (Thomas–Fermi in the above discussion), provided one has used the same "bare" (screened) Coulomb interaction to calculate ω_q^2.

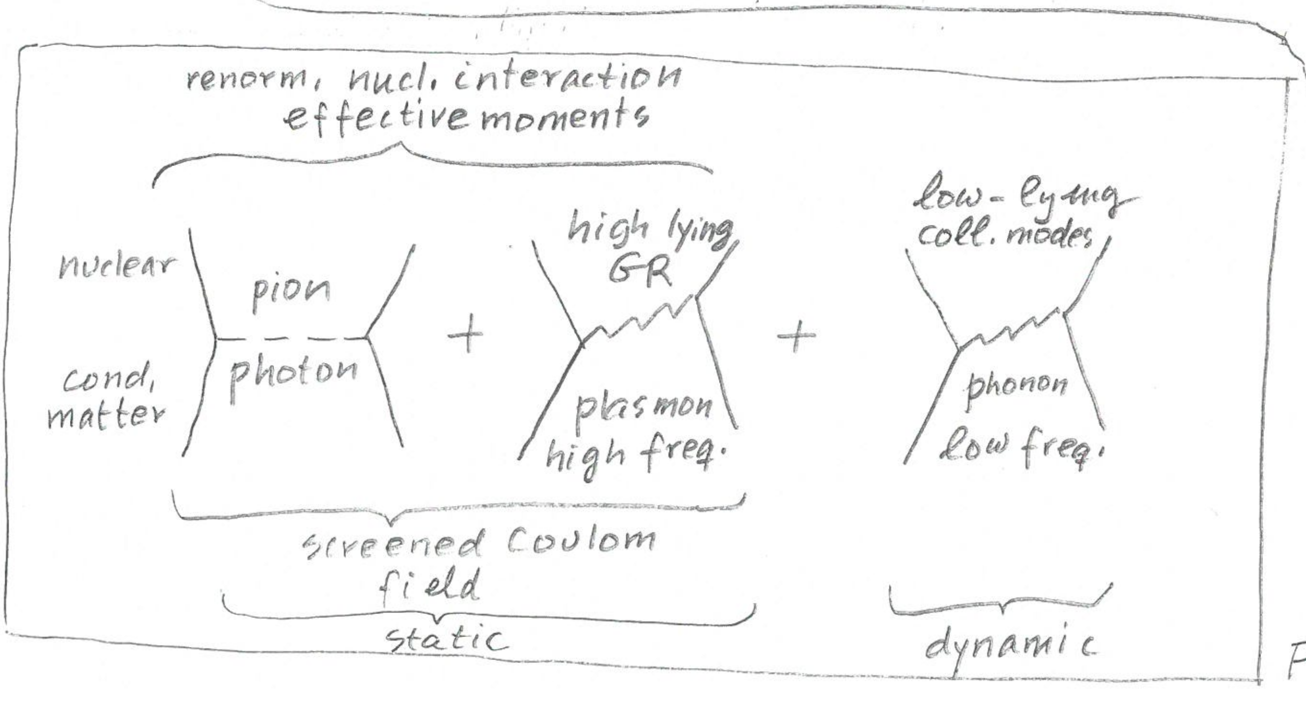


Fig. 3.A.5

Bibliography

V. Ambegaokar and A. Baratoff. Tunneling between superconductors. *Phys. Rev. Lett.*, 10:486, 1963.

P. W. Anderson. New method in the theory of superconductivity. *Phys. Rev.*, 110: 985, 1958.

P. W. Anderson. *The Many–Body Problem, Vol.2*. Academic Press, New York, 1964.

J. Bardeen. Tunnelling from a many-particle point of view. *Phys. Rev. Lett.*, 6:57, 1961.

J. Bardeen. Tunneling into superconductors. *Physical Review Letters*, 9:147, 1962.

Barranco, F., R. A. Broglia, and G. F. Bertsch. Exotic radioactivity as a superfluid tunneling phenomenon. *Phys. Rev. Lett.*, 60:507, 1988.

Barranco, F., G. Bertsch, R. A. Broglia, and E. Vigezzi. Large-amplitude motion in superfluid fermi droplets. *Nuclear Physics A*, 512:253, 1990.

Barranco, F., P. F. Bortignon, R. A. Broglia, G. Colò, and E. Vigezzi. The halo of the exotic nucleus ^{11}Li: a single Cooper pair. *Europ. Phys. J. A*, 11:385, 2001.

Bayman, B. F. and C. F. Clement. Sum rules for two–nucleon transfer reactions. *Phys. Rev. Lett.*, 29:1020, 1972.

G. Bertsch. Collective motion in Fermi droplets. In R. A. Broglia and J. R. Schrieffer, editors, *International School of Physics "Enrico Fermi" Collective motion in Fermi droplets*, page 41, Amsterdam, 1988. North Holland.

G. Bertsch and R. A. Broglia. *Oscillations in Finite Quantum Systems*. Cambridge University Press, Cambridge, 2005.

Bertsch, G. F., R. A. Broglia, and C. Riedel. Qualitative description of nuclear collectivity. *Nucl. Phys. A*, 91:123, 1967.

Bjerregaard, J. H., O. Hansen, Nathan, and S. Hinds. States of ^{208}Pb from double triton stripping. *Nucl. Phys.*, 89:337, 1966.

A. Bohr and B. R. Mottelson. *Nuclear Structure, Vol.I*. Benjamin, New York, 1969.

A. Bohr and O. Ulfbeck. Quantal structure of superconductivity gauge angle. In *First Topsøe summer School on Superconductivity and Workshop on Superconductors*, Roskilde, Denmark Riso/M/2756, 1988.

N. Bohr and J. A. Wheeler. The mechanism of nuclear fission. *Phys. Rev.*, 56:426, 1939.

Bohr, A. and B. R. Mottelson. *Nuclear Structure, Vol.II.* Benjamin, New York, 1975.

M. Born. Zur Quantenmechanik der Stoßvorgänge. *Zeitschr. f. Phys.*, 37:863, 1926.

Brink, D. and R. A. Broglia. *Nuclear Superfluidity.* Cambridge University Press, Cambridge, 2005.

2004 correct

R. A. Broglia and A. Winther. *Heavy Ion Reactions.* Westview Press, Boulder, CO., 2004.

~~R. Broglia and A. Winther. *Heavy Ion Reactions, 2nd ed.* Westview Press, Perseus Books, Boulder, 2005~~.

R. A. Broglia, G. Coló, G. Onida, and H. Roman. *Solid State Physics of Finite Systems: metal clusters, fullerenes, atomic wires.* Springer Verlag, Berlin, Heidelberg, 2004.

R. A. Broglia and A. Winther. On the pairing field in nuclei. *Physics Letters B*, 124:11, 1983.

Broglia, R. A., C. Riedel, and T. Udagawa. Sum rules and two-particle units in the analysis of two-neutron transfer reactions. *Nuclear Physics A*, 184:23, 1972.

Broglia, R. A. and Zelevinsky, V. . In R. A. Broglia and V. Zelevinsky, editors, *50 Years of Nuclear BCS.* World Scientific, Singapore, 2013.

Broglia, R.A., O. Hansen, and C. Riedel. Two–neutron transfer reactions and the pairing model. *Advances in Nuclear Physics*, 6:287, 1973. URL `www.mi.infn.it/?vigezzi/BHR/ BrogliaHansenRiedel.pdf`.

M. H. Cohen, L. M. Falicov, and J. C. Phillips. Superconductive tunneling. *Phys. Rev. Lett.*, 8:316, 1962.

P. G. de Gennes. *Les objets fragiles.* Plon, Paris, 1994.

Ferreira, L., R. Liotta, C. Dasso, R. A. Broglia, and A. Winther. Spatial correlations of pairing collective states. *Nuclear Physics A*, 426:276, 1984.

Gor'kov, L.P. Microscopic derivation of the Ginzburg–Landau eqautions in the theory of superconductivity. *Soviet Phys. JETP*, 9:1364, 1959.

Guazzoni, P., L. Zetta, A. Covello, A. Gargano, B. F. Bayman, G. Graw, R. Hertenberger, H.-F. Wirth, and M. Jaskola. Spectroscopy of ^{110}Sn via the high-resolution ^{112}Sn(p,t) ^{110}Sn reaction. *Phys. Rev. C*, 74:054605, 2006.

Guazzoni, P., L. Zetta, A. Covello, A. Gargano, B. F. Bayman, T. Faestermann, G. Graw, R. Hertenberger, H.-F. Wirth, and M. Jaskola. High-resolution measurement of the ^{118,124}Sn(p,t) ^{116,122} reactions: Shell-model and microscopic

Potel, G., A. Idini, F. Barranco, E. Vigezzi, and R. A. Broglia. Quantitative study of coherent pairing modes with two–neutron transfer: Sn isotopes. *Phys. Rev. C*, 87:054321, 2013b.

H. Rogalla and P. H. Kes. In H. Rogalla and P. H. Kes, editors, *100 years of superconductivity*. CRC Press, Taylor and Francis, FL, 2012.

J. J. Sakurai. *Modern Quantum Mechanics*. Addison-Wesley, Cambridge, MA, 1994.

Sargsyan, V. V., G. Scamps, G. G. Adamian, N. V. Antonenko, and D. Lacroix. Neutron pair transfer in sub–barrier capture processes. *arXiv:1311.4353v1*, 2013.

G. Satchler. *Introduction to Nuclear Reactions*. Mc Millan, New York, 1980.

Tanihata, I., M. Alcorta, D. Bandyopadhyay, R. Bieri, L. Buchmann, B. Davids, N. Galinski, D. Howell, W. Mills, S. Mythili, R. Openshaw, E. Padilla-Rodal, G. Ruprecht, G. Sheffer, A. C. Shotter, M. Trinczek, P. Walden, H. Savajols, T. Roger, M. Caamano, W. Mittig, P. Roussel-Chomaz, R. Kanungo, A. Gallant, M. Notani, G. Savard, and I. J. Thompson. Measurement of the two-halo neutron transfer reaction ^{1}H(^{11}Li,^{9}Li)^{3}H at 3A MeV. *Phys. Rev. Lett.*, 100:192502, 2008.

Thompson, I.J. Reaction mechanism of pair transfer. In R. A. Broglia and V. Zelevinsky, editors, *50 Years of Nuclear BCS*, page 455. World Scientific, Singapore, 2013.

W. von Oertzen and A. Vitturi. Pairing correlations of nucleons and multi–nucleon transfer between heavy nuclei. *Reports on Progress in Physics*, 64:1247, 2001.

von Oertzen, W. Enhanced two–nucleon transfer due to pairing correlations. In R. A. Broglia and V. Zelevinsky, editors, *50 Years of Nuclear BCS*, page 405. World Scientific, Singapore, 2013.