$V(r_{ab}, \sigma_a, \sigma_b) = v(r_{ab})v_\sigma(\sigma_a, \sigma_b)$. Note that this rules out a spin–orbit term and terms proportional to $\mathbf{r} \cdot \sigma$, such as the tensor term! For the moment we will assume that the spin–dependent interaction is rotationally invariant (scalar with respect to rotations), such as, e.g., $v_\sigma(\sigma_a, \sigma_b) \propto \sigma_a \cdot \sigma_b$. Again, that excludes from our formalism tensor terms in the interaction. The transition amplitude is

$$T^{m'_a, m'_b}_{m_a, m_b} = \sum_{\sigma_a, \sigma_b} \int d\mathbf{r}_{aA} d\mathbf{r}_{bc} \chi^{(-)*}_{m'_a}(\mathbf{r}_{ac}, \sigma_a) \chi^{(-)*}_{m'_b}(\mathbf{r}_{bc}, \sigma_b)$$
$$\times v(r_{ab}) v_\sigma(\sigma_a, \sigma_b) \chi^{(+)}_{m_a}(\mathbf{r}_{aA}, \sigma_a) \psi^{l_b, j_b}_{m_b}(\mathbf{r}_{bc}, \sigma_b). \tag{6.F.28}$$

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**Distorted waves**

The distorted waves in (6.F.28) $\chi_m(\mathbf{r}, \sigma) = \chi(\mathbf{r})\phi^{1/2}_m(\sigma)$ have a spin dependence contained in the spinor $\phi^{1/2}_m(\sigma)$, where $\sigma$ is the spin degree of freedom and $m$ the projection of the spin along the quantization axis. The superscript $1/2$ reminds us that we are considering spin $1/2$ particles, which have important consequences when dealing with the spin–orbit term of the optical potentials. As for the spin–dependent term $v_\sigma(\sigma_a, \sigma_b)$, the value of the spin of the involved particles does not make much difference *as long as this term is rotationally invariant.* After (6.F.4)

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$$\chi^{(+)}(\mathbf{k}, \mathbf{r})\phi_m(\sigma) = \sum_{l,j} \frac{4\pi}{kr} i^l \sqrt{2l+1} e^{i\sigma^l} F_{l,j}(r) \left[Y^l(\hat{\mathbf{r}}) Y^l(\hat{\mathbf{k}})\right]^0_0 \phi^{1/2}_m(\sigma). \tag{6.F.29}$$

Note that now the sum is also over the total angular momentum $j$, because the radial functions $F_{l,j}(r)$ depend now on $j$ as well as on $l$, being solutions of an optical potential with a spin–orbit term proportional to $1/2\,(j(j+1) - l(l+1) - 3/4)$. We must then couple the radial and spin functions to total angular momentum $j$, noting that

$$\left[Y^l(\hat{\mathbf{r}})\ Y^l(\hat{\mathbf{k}})\right]^0_0 \phi^{1/2}_m(\sigma) = \sum_{m_l} \langle l\ m_l\ l\ -m_l | 0\ 0 \rangle Y^l_{m_l}(\hat{\mathbf{r}}) Y^l_{-m_l}(\hat{\mathbf{k}}) \phi^{1/2}_m(\sigma) =$$
$$\sum_{m_l} \frac{(-1)^{l-m_l}}{\sqrt{2l+1}} Y^l_{m_l}(\hat{\mathbf{r}}) Y^l_{-m_l}(\hat{\mathbf{k}}) \phi^{1/2}_m(\sigma), \tag{6.F.30}$$

and

$$Y^l_{m_l}(\hat{\mathbf{r}})\phi^{1/2}_m(\sigma) = \sum_j \langle l\ m_l\ 1/2\ m | j\ m_l + m \rangle \left[Y^l(\hat{\mathbf{r}})\phi^{1/2}(\sigma)\right]^j_{m_l+m}, \tag{6.F.31}$$

we can write

$$\left[Y^l(\hat{\mathbf{r}})\ Y^l(\hat{\mathbf{k}})\right]^0_0 \phi^{1/2}_m(\sigma) = \sum_{m_l, j} \frac{(-1)^{l+m_l}}{\sqrt{2l+1}} \langle l\ m_l\ 1/2\ m | j\ m_l + m \rangle$$
$$\times \left[Y^l(\hat{\mathbf{r}})\phi^{1/2}(\sigma)\right]^j_{m_l+m} Y^l_{-m_l}(\hat{\mathbf{k}}), \tag{6.F.32}$$

and the distorted waves in (6.F.28) are

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$$\chi^{(+)}_{m_a}(\mathbf{r}_{aA}, \mathbf{k}_a, \sigma_a) = \sum_{l_a, m_{l_a}, j_a} \frac{4\pi}{k_a r_{aA}} i^{l_a} (-1)^{l_a + m_{l_a}} e^{i\sigma^{l_a}} F_{l_a, j_a}(r_{aA})$$
$$\times \langle l_a\ m_{l_a}\ 1/2\ m_a | j_a\ m_{l_a} + m_a \rangle \left[Y^{l_a}(\hat{\mathbf{r}}_{aA})\phi^{1/2}(\sigma_a)\right]^{j_a}_{m_{l_a}+m_a} Y^{l_a}_{-m_{l_a}}(\hat{\mathbf{k}}_a), \tag{6.F.33}$$

$$\begin{aligned}\chi^{(-)*}_{m'_b}(\mathbf{r}_{bc},\mathbf{k}'_b,\sigma_b) = \sum_{l'_b,m_{l'_b},j'_b} \frac{4\pi}{k'_b r_{bc}} i^{-l'_b}(-1)^{l'_b+m_{l'_b}} e^{i\sigma^{l'_b}} F_{l'_b,j'_b}(r_{bc}) \\ \times \langle l'_b\ m_{l'_b}\ 1/2\ m'_b | j'_b\ m_{l'_b}+m'_b\rangle \left[Y^{l'_b}(\hat{\mathbf{r}}_{bc})\phi^{1/2}(\sigma_b)\right]^{j'_b *}_{m_{l'_b}+m'_b} Y^{l'_b *}_{-m_{l'_b}}(\hat{\mathbf{k}}'_b),\end{aligned} \tag{6.F.34}$$

$$\begin{aligned}\chi^{(-)*}_{m'_a}(\mathbf{r}_{ac},\mathbf{k}'_a,\sigma_a) = \sum_{l'_a,m_{l'_a},j'_a} \frac{4\pi}{k'_a r_{ac}} i^{-l'_a}(-1)^{l'_a+m_{l'_a}} e^{i\sigma^{l'_a}} F_{l'_a,j'_a}(r_{ac}) \\ \times \langle l'_a\ m_{l'_a}\ 1/2\ m'_a | j'_a\ m_{l'_a}+m'_a\rangle \left[Y^{l'_a}(\hat{\mathbf{r}}_{ac})\phi^{1/2}(\sigma_a)\right]^{j'_a *}_{m_{l'_a}+m'_a} Y^{l'_a *}_{-m_{l'_a}}(\hat{\mathbf{k}}'_a).\end{aligned} \tag{6.F.35}$$

The initial bounded wavefunction of particle $b$ is

$$\psi^{l_b,j_b}_{m_b}(\mathbf{r}_{bc},\sigma_b) = u_{l_b,j_b}(r_{bc})\left[Y^{l_b}(\hat{\mathbf{r}}_{bc})\phi^{1/2}(\sigma_b)\right]^{j_b}_{m_b}, \tag{6.F.36}$$

substituting in 6.F.28,

$$\begin{aligned}T^{m'_a,m'_b}_{m_a,m_b}(\mathbf{k}'_a,\mathbf{k}'_b) = \frac{64\pi^3}{k_a k'_a k'_b} \sum_{\sigma_a,\sigma_b} \sum_{l_a,m_{l_a},j_a} \sum_{l'_a,m_{l'_a},j'_a} \sum_{l'_b,m_{l'_b},j'_b} e^{i(\sigma^{l_a}+\sigma^{l'_a}+\sigma^{l'_b})} i^{l_a-l'_a-l'_b}(-1)^{l_a-m_{l_a}+l'_a-j'_a+l'_b-j'_b} \\ \times \langle l'_a\ m_{l'_a}\ 1/2\ m'_a|j'_a\ m_{l'_a}+m'_a\rangle\langle l_a\ m_{l_a}\ 1/2\ m_a|j_a\ m_{l_a}+m_a\rangle\langle l'_b\ m_{l'_b}\ 1/2\ m'_b|j'_b\ m_{l'_b}+m'_b\rangle \\ \times Y^{l_a}_{-m_{l_a}}(\hat{\mathbf{k}}_a)Y^{l'_b}_{-m_{l'_b}}(\hat{\mathbf{k}}'_b)Y^{l'_a}_{-m_{l'_a}}(\hat{\mathbf{k}}'_a)\int d\mathbf{r}_{aA}d\mathbf{r}_{bc}\left[Y^{l'_a}(\hat{\mathbf{r}}_{ac})\phi^{1/2}(\sigma_a)\right]^{j'_a}_{-m_{l'_a}-m'_a}\left[Y^{l'_b}(\hat{\mathbf{r}}_{bc})\phi^{1/2}(\sigma_b)\right]^{j'_b}_{-m_{l'_b}-m'_b} \\ \times \frac{F_{l_a,j_a}(r_{aA})F_{l'_a,j'_a}(r_{ac})F_{l'_b,j'_b}(r_{bc})}{r_{ac}r_{aA}r_{bc}} u_{l_b,j_b}(r_{bc})v(r_{ab})v_\sigma(\sigma_a,\sigma_b) \\ \times\left[Y^{l_a}(\hat{\mathbf{r}}_{aA})\phi^{1/2}(\sigma_a)\right]^{j_a}_{m_{l_a}+m_a}\left[Y^{l_b}(\hat{\mathbf{r}}_{bc})\phi^{1/2}(\sigma_b)\right]^{j_b}_{m_b},\end{aligned} \tag{6.F.37}$$

where we have used

$$\left[Y^{l}(\hat{\mathbf{r}})\phi^{1/2}(\sigma)\right]^{j*}_{m} = (-1)^{j-m}\left[Y^{l}(\hat{\mathbf{r}})\phi^{1/2}(\sigma)\right]^{j}_{-m}. \tag{6.F.38}$$

### Recoupling of angular momenta

Let us now separate spatial and spin coordinates, noting that the spin functions must be coupled to 0 (this is a consequence of the interaction $v_\sigma(\sigma_a,\sigma_b)$ being rotationally invariant). Starting with particle $a$,

$$\begin{aligned}\left[Y^{l'_a}(\hat{\mathbf{r}}_{ac})\phi^{1/2*}(\sigma_a)\right]^{j'_a}_{-m_{l'_a}-m'_a}\left[Y^{l_a}(\hat{\mathbf{r}}_{aA})\phi^{1/2}(\sigma_a)\right]^{j_a}_{m_{l_a}+m_a} = \\ \sum_K ((l'_a\tfrac{1}{2})_{j'_a}(l_a\tfrac{1}{2})_{j_a}|(l_a l'_a)_K(\tfrac{1}{2}\tfrac{1}{2})_0)_K \\ \times\left[Y^{l'_a}(\hat{\mathbf{r}}_{ac})Y^{l_a}(\hat{\mathbf{r}}_{aA})\right]^{K}_{-m_{l'_a}-m'_a+m_{l_a}+m_a}\left[\phi^{1/2*}(\sigma_a)\phi^{1/2}(\sigma_a)\right]^0_0.\end{aligned} \tag{6.F.39}$$

For particle $b$,

$$\begin{aligned}\left[Y^{l'_b}(\hat{\mathbf{r}}_{bc})\phi^{1/2*}(\sigma_b)\right]^{j'_b}_{-m_{l'_b}-m'_b}\left[Y^{l_b}(\hat{\mathbf{r}}_{bc})\phi^{1/2}(\sigma_b)\right]^{j_b}_{m_b} = \\ \sum_{K'} ((l'_b\tfrac{1}{2})_{j'_b}(l_b\tfrac{1}{2})_{j_b}|(l_b l'_b)_{K'}(\tfrac{1}{2}\tfrac{1}{2})_0)_{K'} \\ \times\left[Y^{l'_b}(\hat{\mathbf{r}}_{bc})Y^{l_b}(\hat{\mathbf{r}}_{bc})\right]^{K'}_{-m_{l'_b}-m'_b+m_b}\left[\phi^{1/2*}(\sigma_b)\phi^{1/2}(\sigma_b)\right]^0_0.\end{aligned} \tag{6.F.40}$$

The spin summation yields a constant factor,

$$\sum_{\sigma_a,\sigma_b}\left[\phi^{1/2^*}(\sigma_a)\phi^{1/2}(\sigma_a)\right]_0^0\left[\phi^{1/2^*}(\sigma_b)\phi^{1/2}(\sigma_b)\right]_0^0 v_\sigma(\sigma_a,\sigma_b)\equiv T_\sigma, \qquad (6.F.41)$$

and what we have yet to do is very similar to what we have done for spinless particles. First of all note that the necessity to couple all angular momenta to 0 imposes $K'=K$ and $m_{l_a}+m_a-m_{l'_a}-m'_a=m_{l'_b}+m'_b-m_b$ (see 6.F.39 and 6.F.40). If we set $M=m_{l_a}+m_a-m_{l'_a}-m'_a$ and take, as before, $\hat{\mathbf{k}}_a\equiv\hat{z}$

$$\begin{aligned}T^{m'_a,m'_b}_{m_a,m_b}(\mathbf{k}'_a,\mathbf{k}'_b)=&\frac{32\pi^{5/2}}{k_a k'_a k'_b}T_\sigma\sum_{l_a,j_a}\sum_{l'_a,j'_a}\sum_{l'_b,j'_b}\sum_{K,M}e^{i(\sigma^{l_a}+\sigma^{l'_a}+\sigma^{l'_b})}i^{l_a-l'_a-l'_b}(-1)^{l_a+l'_a+l'_b-j'_a-j'_b}\\
&\times\sqrt{2l_a+1}((l'_a\tfrac{1}{2})_{j'_a}(l_a\tfrac{1}{2})_{j_a}|(l_al'_a)_K(\tfrac{1}{2}\tfrac{1}{2})_0)_K((l'_b\tfrac{1}{2})_{j'_b}(l_b\tfrac{1}{2})_{j_b}|(l_bl'_b)_K(\tfrac{1}{2}\tfrac{1}{2})_0)_K\\
&\times\langle l'_a\ m_a-m'_a-M\ 1/2\ m'_a|j'_a\ m_a-M\rangle\langle l_a\ 0\ 1/2\ m_a|j_a\ m_a\rangle\langle l'_b\ m_b-m'_b+M\ 1/2\ m'_b|j'_b\ M+m_b\rangle\\
&\times Y^{l'_b}_{m'_b-m_b-M}(\hat{\mathbf{k}}'_b)Y^{l'_a}_{m'_a-m_a+M}(\hat{\mathbf{k}}'_a)\int d\mathbf{r}_{aA}d\mathbf{r}_{bc}\frac{F_{l_a,j_a}(r_{aA})F_{l'_a,j'_a}(r_{ac})F_{l'_b,j'_b}(r_{bc})}{r_{ac}r_{aA}r_{bc}}\\
&\times u_{l_b,j_b}(r_{bc})v(r_{ab})\left[Y^{l_a}(\hat{\mathbf{r}}_{aA})Y^{l'_a}(\hat{\mathbf{r}}_{ac})\right]^K_M\left[Y^{l_b}(\hat{\mathbf{r}}_{bc})Y^{l'_b}(\hat{\mathbf{r}}_{bc})\right]^K_{-M}. \qquad (6.F.42)\end{aligned}$$

The integral of the above expression is similar to the one in 6.F.18 so we obtain

$$\begin{aligned}T^{m'_a,m'_b}_{m_a,m_b}(\mathbf{k}'_a,\mathbf{k}'_b)=&\frac{128\pi^4}{k_ak'_ak'_b}T_\sigma\sum_{l_a,j_a}\sum_{l'_a,j'_a}\sum_{l'_b,j'_b}\sum_{K,M}e^{i(\sigma^{l_a}+\sigma^{l'_a}+\sigma^{l'_b})}i^{l_a-l'_a-l'_b}(-1)^{l_a+l'_a+l'_b-j'_a-j'_b}\\
&\times\frac{2l_a+1}{2K+1}((l'_a\tfrac{1}{2})_{j'_a}(l_a\tfrac{1}{2})_{j_a}|(l_al'_a)_K(\tfrac{1}{2}\tfrac{1}{2})_0)_K((l'_b\tfrac{1}{2})_{j'_b}(l_b\tfrac{1}{2})_{j_b}|(l_bl'_b)_K(\tfrac{1}{2}\tfrac{1}{2})_0)_K\\
&\times\langle l'_a\ m_a-m'_a-M\ 1/2\ m'_a|j'_a\ m_a-M\rangle\langle l'_b\ m_b-m'_b+M\ 1/2\ m'_b|j'_b\ M+m_b\rangle\\
&\times\langle l_a\ 0\ 1/2\ m_a|j_a\ m_a\rangle Y^{l'_b}_{m'_b-m_b-M}(\hat{\mathbf{k}}'_b)Y^{l'_a}_{m_a-m'_a+M}(\hat{\mathbf{k}}'_a)\mathcal{I}(l_a,l'_a,l'_b,j_a,j'_a,j'_b,K), \qquad (6.F.43)\end{aligned}$$

with

$$\begin{aligned}\mathcal{I}(l_a,l'_a,l'_b,j_a,j'_a,j'_b,K)=&\int dr_{aA}dr_{bc}d\theta r_{aA}r_{bc}\frac{\sin\theta}{r_{ac}}u_{l_b}(r_{bc})v(r_{ab})\\
&\times F_{l_a,j_a}(r_{aA})F_{l'_a,j'_a}(r_{ac})F_{l'_b,j'_b}(r_{bc})\\
&\times\sum_{M_K}\langle l_a\ 0\ l'_a\ M_K|K\ M_K\rangle\left[Y^{l_b}(\cos\theta,0)Y^{l'_b}(\cos\theta,0)\right]^K_{-M_K}Y^{l'_a}_{M_K}(\cos\theta_{ac},0). \qquad (6.F.44)\end{aligned}$$

Again, this is a 3–dimensional integral that can be evaluated with the method of Gaussian quadratures. The transition amplitude $T^{m'_a,m'_b}_{m_a,m_b}(\mathbf{k}'_a,\mathbf{k}'_b)$ depends explicitly on the initial $(m_a,m'_a)$ and final $(m'_a,m'_b)$ polarizations of $a,b$. If the particle $b$ is initially coupled to core $c$ to total angular momentum $J_A,M_A$, the amplitude to be considered is rather

$$T^{m'_a,m'_b}_{m_a}(\mathbf{k}'_a,\mathbf{k}'_b)=\sum_{m_b}\langle j_b\ m_b\ j_c\ M_A-m_b|J_A\ M_A\rangle T^{m'_a,m'_b}_{m_a,m_b}(\mathbf{k}'_a,\mathbf{k}'_b), \qquad (6.F.45)$$

and the multi–differential cross section for detecting particle $c$ (or $a$) is

$$\left.\frac{d\sigma}{d\mathbf{k}'_ad\mathbf{k}'_b}\right]^{m'_a,m'_b}_{m_a}=\frac{k'_a}{k_a}\frac{\mu_{aA}\mu_{ac}}{4\pi^2\hbar^4}\left|\sum_{m_b}\langle j_b\ m_b\ j_c\ M_A-m_b|J_A\ M_A\rangle T^{m'_a,m'_b}_{m_a,m_b}(\mathbf{k}'_a,\mathbf{k}'_b)\right|^2. \qquad (6.F.46)$$

All spin–polarization observables (analysing powers, etc.,) can be derived from this expression. But let us now work out the expression of the cross section for an unpolarized beam (sum over initial spin orientations divided by the number of such orientations) and when we do not detect the final polarizations (sum over final spin orientations),

$$\frac{d\sigma}{d\mathbf{k}_a' d\mathbf{k}_b'} = \frac{k_a'}{k_a}\frac{\mu_{aA}\mu_{ac}}{4\pi^2\hbar^4}\frac{1}{(2J_A+1)(2j_a+1)} \times \sum_{\substack{m_a,m_a'\\ M_A,m_b'}} \left|\sum_{m_b} \langle j_b\ m_b\ j_c\ M_A - m_b | J_A\ M_A\rangle\, T^{m_a',m_b'}_{m_a,m_b}(\mathbf{k}_a',\mathbf{k}_b')\right|^2. \tag{6.F.47}$$

The sum above can be simplified a bit. Let us consider a single particular value of $m_b$ in the sum over $m_b$,

$$\begin{aligned}\sum_{m_a,m_a',m_b'} \left|T^{m_a',m_b'}_{m_a,m_b}(\mathbf{k}_a',\mathbf{k}_b')\right|^2 \sum_{M_A}\left|\langle j_b\ m_b\ j_c\ M_A - m_b|J_A\ M_A\rangle\right|^2 = \\ \frac{2J_A+1}{2j_b+1}\sum_{m_a,m_a,m_b'}\left|T^{m_a',m_b'}_{m_a,m_b}(\mathbf{k}_a',\mathbf{k}_b')\right|^2 \\ \times\sum_{M_A}\left|\langle J_A\ -M_A\ j_c\ M_A - m_b|j_b\ m_b\rangle\right|^2,\end{aligned} \tag{6.F.48}$$

where we have used

$$\langle j_b\ m_b\ j_c\ M_A - m_b|J_A\ M_A\rangle = (-1)^{j_c - M_A + m_b}\sqrt{\frac{2J_A+1}{2j_b+1}}\langle J_A\ -M_A\ j_c\ M_A - m_b|j_b\ m_b\rangle. \tag{6.F.49}$$

As

$$\sum_{M_A}\left|\langle J_A\ -M_A\ j_c\ M_A - m_b|j_b\ m_b\rangle\right|^2 = 1, \tag{6.F.50}$$

we finally have

$$\frac{d\sigma}{d\mathbf{k}_a' d\mathbf{k}_b'} = \frac{k_a'}{k_a}\frac{\mu_{aA}\mu_{ac}}{4\pi^2\hbar^4}\frac{1}{(2j_b+1)(2j_a+1)}\sum_{m_a,m_a',m_b'}\left|\sum_{m_b} T^{m_a',m_b'}_{m_a,m_b}(\mathbf{k}_a',\mathbf{k}_b')\right|^2. \tag{6.F.51}$$

**Zero range approximation.**

The zero range approximation consists in taking $v(r_{ab}) = D_0\delta(r_{ab})$. Then, (see 6.F.2)

$$\begin{aligned}\mathbf{r}_{aA} &= \frac{c}{A}\mathbf{r}_{bc},\\ \mathbf{r}_{ac} &= \mathbf{r}_{bc}.\end{aligned} \tag{6.F.52}$$

The angular dependence of the integral can be readily evaluated. From 6.F.20, noting that $\hat{\mathbf{r}}_{aA} = \hat{\mathbf{r}}_{ac} = \hat{\mathbf{r}}_{bc} \equiv \hat{\mathbf{r}}$,

$$\begin{aligned}\left[Y^{l_a}(\hat{\mathbf{r}})Y^{l_a'}(\hat{\mathbf{r}})\right]^K_M \left[Y^{l_b}(\hat{\mathbf{r}})Y^{l_b'}(\hat{\mathbf{r}})\right]^K_{-M} = \\ \frac{(-1)^{K-M}}{\sqrt{2K+1}}\left\{\left[Y^{l_a}(\hat{\mathbf{r}})Y^{l_a'}(\hat{\mathbf{r}})\right]^K\left[Y^{l_b}(\hat{\mathbf{r}})Y^{l_b'}(\hat{\mathbf{r}})\right]^K\right\}^0_0.\end{aligned} \tag{6.F.53}$$

We can as before evaluate this expression in the configuration shown in Fig. 6.F.2 ($\hat{\mathbf{r}} = \hat{z}$), but now the multiplicative factor is $4\pi$. The corresponding contribution to the integral is

$$\frac{(-1)^K}{4\pi(2K+1)}\langle l_a\, 0\, l'_a\, 0|K\, 0\rangle\sqrt{(2l_a+1)(2l'_a+1)(2l_b+1)(2l'_b+1)}, \tag{6.F.54}$$

and

$$\begin{aligned}
T^{m'_a,m'_b}_{m_a,m_b}(\mathbf{k}'_a,\mathbf{k}'_b) &= \frac{16\pi^2}{k_a k'_a k'_b}\frac{c}{A}D_0 T_\sigma \sum_{l_a,j_a}\sum_{l'_a,j'_a}\sum_{l'_b,j'_b}\sum_{K,M} e^{i(\sigma^{l_a}+\sigma^{l'_a}+\sigma^{l'_b})} i^{l_a-l'_a-l'_b}(-1)^{l_a+l'_a+l'_b-j'_a-j'_b}\\
&\times\sqrt{(2l_a+1)(2l'_a+1)(2l_b+1)(2l'_b+1)}\,\langle l_a\, 0\, l'_a\, 0|K\, 0\rangle\\
&\times\frac{2l_a+1}{2K+1}((l'_a\tfrac{1}{2})_{j'_a}(l_a\tfrac{1}{2})_{j_a}|(l_a l'_a)_K(\tfrac{1}{2}\tfrac{1}{2})_0)_K\,((l'_b\tfrac{1}{2})_{j'_b}(l_b\tfrac{1}{2})_{j_b}|(l_b l'_b)_K(\tfrac{1}{2}\tfrac{1}{2})_0)_K\\
&\times\langle l'_a\, m_a-m'_a-M\; 1/2\; m'_a|j'_a\; m_a-M\rangle\langle l'_b\; m_b-m'_b+M\; 1/2\; m'_b|j'_b\; M+m_b\rangle\\
&\times\langle l\, 0\, 1/2\, m_a|j\, m_a\rangle Y^{l'_b}_{M+m_b+m'_b}(\hat{\mathbf{k}}'_b)Y^{l'_a}_{m_a+m'_a-M}(\hat{\mathbf{k}}'_a)\mathcal{I}_{ZR}(l_a,l'_a,l'_b,j_a,j'_a,j'_b),
\end{aligned} \tag{6.F.55}$$

where now the 1–dimensional integral to solve is

$$\mathcal{I}_{ZR}(l_a,l'_a,l'_b,j_a,j'_a,j'_b) = \int dr u_{l_b}(r)F_{l_a,j_a}(\tfrac{c}{A}r)F_{l'_a,j'_a}(r)F_{l'_b,j'_b}(r)/r. \tag{6.F.56}$$

## 6.F.3 One particle transfer

It may be interesting to state the expression for the one particle transfer reaction within the same context and using the same elements, in order to better compare these two type of experiments. In particle transfer, the final state of $b$ is a bounded state of the $B(=a+b)$ nucleus, and we can carry on in a similar way as done previously just by substituting the distorted wave (continuum) wave function (6.F.34) with

$$\psi^{l'_b,j'_b *}_{m'_b}(\mathbf{r}_{ab},\sigma_b) = u^*_{l'_b,j'_b}(r_{ab})\left[Y^{l'_b}(\hat{\mathbf{r}}_{ab})\phi^{1/2}(\sigma_b)\right]^{j'_b *}_{m'_b}, \tag{6.F.57}$$

so the transition amplitude is now

$$\begin{aligned}
T^{m'_a,m'_b}_{m_a,m_b}(\mathbf{k}'_a) &= \frac{8\pi^{3/2}}{k_a k'_a}\sum_{\sigma_a,\sigma_b}\sum_{l_a,j_a}\sum_{l'_a,m_{l'_a},j'_a} e^{i(\sigma^{l_a}+\sigma^{l'_a})} i^{l_a-l'_a}(-1)^{l_a+l'_a-j'_a-j'_b}\\
&\times\sqrt{2l_a+1}\,\langle l'_a\; m_{l'_a}\; 1/2\; m'_a|j'_a\; m_{l'_a}+m'_a\rangle\langle l_a\, 0\, 1/2\, m_a|j_a\, m_a\rangle\\
&\times Y^{l'_a}_{-m_{l'_a}}(\hat{\mathbf{k}}'_a)\int d\mathbf{r}_{aA}d\mathbf{r}_{bc}\left[Y^{l'_a}(\hat{\mathbf{r}}_{Bc})\phi^{1/2}(\sigma_a)\right]^{j'_a}_{-m_{l'_a}-m'_a}\left[Y^{l'_b}(\hat{\mathbf{r}}_{ab})\phi^{1/2}(\sigma_b)\right]^{j'_b}_{-m'_b}\\
&\times\frac{F_{l_a,j_a}(r_{aA})F_{l'_a,j'_a}(r_{Bc})}{r_{Bc}r_{aA}}u^*_{l'_b,j'_b}(r_{ab})u_{l_b,j_b}(r_{bc})v(r_{ab})v_\sigma(\sigma_a,\sigma_b)\\
&\times\left[Y^{l_a}(\hat{\mathbf{r}}_{aA})\phi^{1/2}(\sigma_a)\right]^{j_a}_{m_a}\left[Y^{l_b}(\hat{\mathbf{r}}_{bc})\phi^{1/2}(\sigma_b)\right]^{j_b}_{m_b}.
\end{aligned} \tag{6.F.58}$$

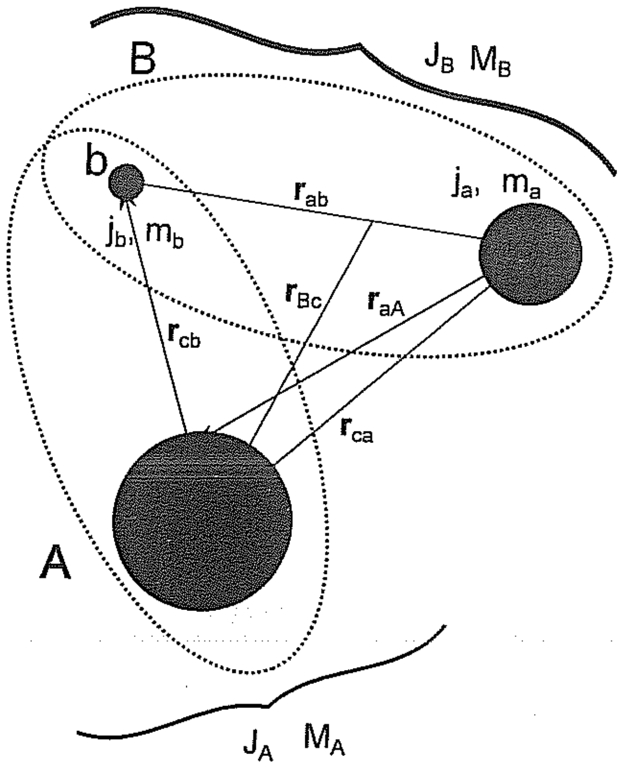


Figure 6.F.4: One particle transfer reaction $A(= c + b) + a \to B(= a + b) + c$.

Using (6.F.39), (6.F.40), (6.F.41), and setting $M = m_a - m'_a - m_{l'_a}$

$$T^{m'_a,m'_b}_{m_a,m_b}(\mathbf{k}'_a) = \frac{8\pi^{3/2}}{k_a k'_a} T_\sigma \sum_{l_a,j_a} \sum_{l'_a,j'_a} \sum_{K,M} e^{i(\sigma^{l_a}+\sigma^{l'_a})} i^{l_a - l'_a} (-1)^{l_a + l'_a - j'_a - j'_b}$$
$$\times ((l'_a \tfrac{1}{2})_{j'_a} (l_a \tfrac{1}{2})_{j_a} | (l_a l'_a)_K (\tfrac{1}{2}\tfrac{1}{2})_0)_K \, ((l'_b \tfrac{1}{2})_{j'_b} (l_b \tfrac{1}{2})_{j_b} | (l_b l'_b)_K (\tfrac{1}{2}\tfrac{1}{2})_0)_K$$
$$\times \sqrt{2l_a + 1} \langle l'_a \; m_a - m'_a - M \; 1/2 \; m'_a | j'_a \; m_a - M \rangle \langle l_a \; 0 \; 1/2 \; m_a | j_a \; m_a \rangle$$
$$\times Y^{l'_a}_{m_a - m'_a - M}(\hat{\mathbf{k}}'_a) \int d\mathbf{r}_{aA} d\mathbf{r}_{bc} \frac{F_{l_a,j_a}(r_{aA}) F_{l'_a,j'_a}(r_{Bc})}{r_{Bc} r_{aA}} u^*_{l'_b,j'_b}(r_{ab}) u_{l_b,j_b}(r_{bc}) v(r_{ab})$$
$$\times \left[ Y^{l_a}(\hat{\mathbf{r}}_{aA}) Y^{l'_a}(\hat{\mathbf{r}}_{Bc}) \right]^K_M \left[ Y^{l_b}(\hat{\mathbf{r}}_{bc}) Y^{l'_b}(\hat{\mathbf{r}}_{ab}) \right]^K_{-M}. \quad (6.F.59)$$

Aside from (6.F.2), we also need

$$\mathbf{r}_{Bc} = \frac{a+B}{B} \mathbf{r}_{aA} + \frac{b}{A} \mathbf{r}_{bc}. \quad (6.F.60)$$

From (6.F.20), (6.F.21), (6.F.22), (6.F.23), (6.F.24), we get

$$T^{m'_a,m'_b}_{m_a,m_b}(\mathbf{k}'_a) = \frac{32\pi^3}{k_a k'_a} T_\sigma \sum_{l_a,j_a} \sum_{l'_a,j'_a} \sum_{K,M} e^{i(\sigma^{l_a}+\sigma^{l'_a})} i^{l_a - l'_a} (-1)^{l_a + l'_a - j'_a - j'_b}$$
$$\times ((l'_a \tfrac{1}{2})_{j'_a} (l_a \tfrac{1}{2})_{j_a} | (l_a l'_a)_K (\tfrac{1}{2}\tfrac{1}{2})_0)_K \, ((l'_b \tfrac{1}{2})_{j'_b} (l_b \tfrac{1}{2})_{j_b} | (l_b l'_b)_K (\tfrac{1}{2}\tfrac{1}{2})_0)_K$$
$$\times \frac{2l_a + 1}{2K + 1} \langle l'_a \; m_a - m'_a - M \; 1/2 \; m'_a | j'_a \; m_a - M \rangle$$
$$\times \langle l_a \; 0 \; 1/2 \; m_a | j_a \; m_a \rangle Y^{l'_a}_{m_a - m'_a - M}(\hat{\mathbf{k}}'_a) \mathcal{I}(l_a, l'_a, j_a, j'_a, j'_b, K), \quad (6.F.61)$$

with

$$I(l_a, l'_a, j_a, j'_a, K) = \int dr_{aA} dr_{bc} d\theta \, r_{aA} r^2_{bc} \frac{\sin\theta}{r_{Bc}}$$
$$\times F_{l_a,j_a}(r_{aA}) F_{l'_a,j'_a}(r_{ac}) u^*_{l'_b,j'_b}(r_{ab}) u_{l_b,j_b}(r_{bc}) v(r_{ab})$$
$$\times \sum_{M_K} \langle l_a \; 0 \; l'_a \; M_K | K \; M_K \rangle \left[ Y^{l_b}(\cos\theta, 0) Y^{l'_b}(\cos\theta_{ab}, 0) \right]^K_{-M_K} Y^{l'_a}_{M_K}(\cos\theta_{Bc}, 0), \quad (6.F.62)$$

where (see (6.F.2), (6.F.60) and Fig. 6.F.2)

$$\cos\theta_{ab} = \frac{-r_{aA} - \frac{c}{A} r_{bc} \cos\theta}{\sqrt{\left(\frac{c}{A} r_{bc} \sin\theta\right)^2 + \left(r_{aA} + \frac{c}{A} r_{bc} \cos\theta\right)^2}}, \quad (6.F.63)$$

$$\cos\theta_{Bc} = \frac{\frac{a+B}{B} r_{aA} + \frac{b}{A} r_{bc} \cos\theta}{\sqrt{\left(\frac{b}{A} r_{bc} \sin\theta\right)^2 + \left(\frac{a+B}{B} r_{aA} + \frac{b}{A} r_{bc} \cos\theta\right)^2}}, \quad (6.F.64)$$

and

$$r_{Bc} = \sqrt{\left(\frac{b}{A} r_{bc} \sin\theta\right)^2 + \left(\frac{a+B}{B} r_{aA} + \frac{b}{A} r_{bc} \cos\theta\right)^2}. \quad (6.F.65)$$

Again, this is nothing new as many codes exist which deal with one particle transfer within the same DWBA formalism we have used here, but it may be useful to have our own code to better compare transfer and knock–out experiments. By the way, (6.F.61) can also be used when particle $b$ populates a resonant state in the continuum of nucleus $B$.

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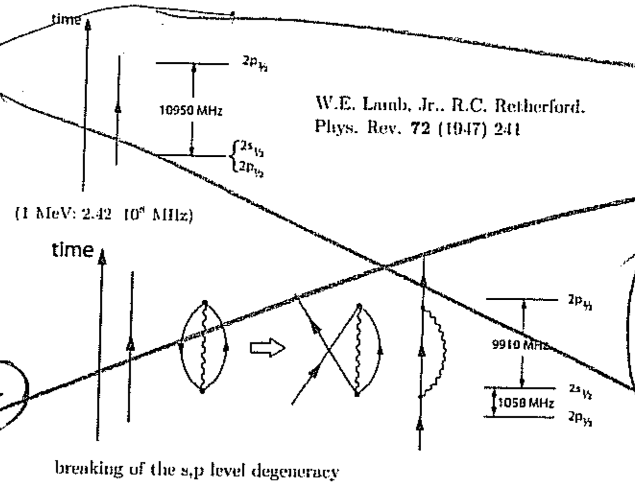


6.7

Figura de appendix 6.C The Lamb shift

Figure ~~6.14~~: Schematic representation of the processes associated with the Lamb shift.

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