Appendix 3.C Absolute Cooper pair tunneling cross section: quantitative novel physics at the edge between stability and chaos

In the study of many–body systems, in particular of finite many–body systems (FMBS) like the atomic nucleus, much can be learned from symmetries (group theory) as well as from the general phenomena of of spontaneous symmetry breaking. However, it is the texture of the associated emergent properties, concrete embodiment of symmetry breaking (potential energy) and of its restoration (fluctuations, collective modes), which provides insight into the eventual new physics. in fact, when one understands the many–body under study, in terms of the detailed motion of single–particles (nucleons) and collective motion, taking properly into account their couplings and associated zero point fluctuations, is that one can hope to have reached a solid, quantitative, understanding of the problem and of its solutions. Even more, that these solutions are likely transferable, at profit, to the study of other FMBS like e.g. metal clusters, fullerenes[91], quantum dots[92], and eventually proteins*, let alone the fact that one can make predictions. Predictions which, in connection with the study of halo nuclei, in particular of pairing[93] in such exotic, highly extended systems lying at the nucleon drip line, involve true novel physics[94]. Within this context one can quote from Leon Cooper's contribution to the volume[95] BCS: 50 years: "It has become fashionable. . . to assert. . . that once gauge symmetry is broken the properties of superconductors follow. . . with no need to inquire into the mechanism by which the symmetry is broken[96]. This is not . . . true, since broken gauge symmetry might led to molecule–like and a Bose–Einstein rather than BCS condensation. . . in 1957. . . the major problem was to show. . . how. . . an order parameter or condensation in momentum space could come about. . . to show how. . . gauge–invariant symmetry of the Lagrangian could be spontaneously broken due to interactions which were themselves gauge invariant".

[91] Cf. e.g. Gunnarsson (2004), Broglia et al. (2004) and refs. therein.
[92] Lipparini (2003).
[93] Cf. e.g. Broglia, R. A. and Zelevinsky, V. (2013).
[94] Cf. e.g. Barranco, F. et al. (2001); Tanihata, I. et al. (2008); Potel et al. (2010) and references therein.
[95] Cooper (2011).
[96] Detailed quoting (Weinberg (2011)): ". . . In consequence of this spontaneous symmetry breaking, products of any even number of electron fields have non–vanishing expectation values in a superconductor, though a single electron field does not. All of the dramatic exact properties of superconductors –zero electric resistance, the expelling of the magnetic fields from superconductors known as the Meissner effect, the quantization of magnetic flux through a thick superconducting ring, and the Josephson formula for the frequency of the ac current at a junction between two superconductors with different voltages– follow from the assumption that electromagnetic gauge invariance is broken in this way, with no need to inquire into the mechanism by which the symmetry is broken." The above quotation is similar to saying that once the idea of a double DNA helix was thought, all about inheritance was solved and known, and that one could forget the X–ray plates of Rosalind Franklin, Maurice Wilkins and collaborators, let alone how DNA and proteins interact with each other (cf. e.g. G.S. Stent (1980) and references therein).

* Broglia (2013)

R. A. Broglia, More is different: 50 Years of Nuclear BCS in R. A. Broglia and V. Zelevinski, editors, 50 Years of Nuclear BCS, page 643. World Scientific, Singapore, 2013;

Nuclear physics has brought this quest a step further. This time in connection with the "extension" of the study of BCS condensation to its origin, a single Cooper pair in the rarified atmosphere resulting from the strong radial (isotropic) deformation observed in light halo, exotic nuclei in general, and in ^{11}Li in particular. During the last few years, the probing of this system in terms of absolute two–nucleon transfer (pick-up) reactions, has made this field a quantitative one, errors below the %10 limit being the rule. This achievement which has its basis on the remarkable experiments of Tanihata, I. et al. (2008), is also the result of the combined effort made in treating the structure and reaction aspects of the subject, two sides of the same physics, on equal footing. In particular regarding the description of the continuum and of the fluctuations leading to both single–particle and collective modes clothing, as well as present as ZPF of the ground state. New essentially physics has been seen to emerge from situations in which these fluctuations diverge (like was also known to occur in the case of e.g. pairing rotational bands) or are on (quasi) resonance, as in the case of the halo pair addition mode of ^{9}Li (i.e. ^{11}Li(gs)) and ~~likely~~ , arguably, of ^{10}Be (i.e. Be(0^{+*}; 2.24 MeV)).

3.C.1 Saturation density, spill out and halo

In the incipit to the Chapter on bulk properties of nuclei of Bohr and Mottelson*) ~~(1969) p. 139~~ one reads: "The almost constant density of nuclear matter is associated with the finite range of nuclear forces; the range of the forces is r_0 (where r_0 enters the nuclear radius in the expression $R = r_0 A^{1/3}$) thus small compared to nuclear size. This "saturation" of nuclear matter is also reflected in the fact that the total binding energy of the nucleus is roughly proportional to A. In a minor way, these features are modified by surface effects and long–range Coulomb forces acting between the protons".

Electron scattering experiments (see the figure 2-1, p. 159 of the above reference) yield

$$\rho(0) = 0.17\,\mathrm{fm}^{-3}. \tag{3.C.1}$$

Thus, one can posit that

$$\frac{4\pi}{3} R_0^3 \rho(0) = A, \tag{3.C.2}$$

leading to

$$r_0 = \left(\frac{3}{4\pi}\frac{1}{\rho(0)}\right)^{1/3} \approx 1.12\,\mathrm{fm}. \tag{3.C.3}$$

Because the above relations imply a step function distribution, we have to add to (3.C.3) the nucleon spill out[97] $(a_0/R_0)\ln 2$ ~~≈ 0.07 (≈ $(a_0/R_0)\ln 2$~~ (≈ (~~0.5~~ 0.6/6) ×

[97] Bertsch and Broglia (2005).

*) Bohr and Mottelson (1969) p. 139.

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0.07

~~0.59~~ $(A = 120)$) associated with the fact that a more realistic distribution is provided by a Fermi function of diffusivity $a_0 \approx 0.5$ fm. Thus $r_0 = (1.12 + 0.07)$ fm ≈ 1.2 fm. In the case of the nucleus ^{11}Li, observations indicate a mean square (gyration radius[98]) radius $\langle r^2 \rangle^{1/2} = 3.55 \pm 0.1$ fm[99]. Thus

$$R(^{11}\text{Li}) = \sqrt{\frac{5}{3}} \langle r^2 \rangle^{1/2} \approx 4.58 \pm 0.13 \,\text{fm}. \tag{3.C.4}$$

Making use of the relation $R(= R_0) \approx 1.2A^{1/3}$ fm, the quantity (3.C.4) leads to $(4.58/1.2)^3 \approx 56$, an effective mass number larger five times the actual value $A = 11$. To be noted that the actual mass number predicts a "systematic" value of the nuclear radius $R_0(^{11}\text{Li}) \approx 2.7$ fm.

The above results testifies to a very large "**isotropic radial deformation**", or halo region (skin), in keeping with the fact that[100] $R(^{11}\text{Li})$-$R_0(^9\text{Li})$=$R_0(^9\text{Li})(\frac{R(^{11}\text{Li})}{R_0(^9\text{Li})} - 1) = 0.83 R_0(^9\text{Li})$. In other words, ^{11}Li can be viewed as a normal ^{9}Li core and of a skin made out of two neutrons extending over a shell radius of the order of that of the core. But even more important, the above mentioned "deformation" affects matter which is little complient to undergo either compressions or, for that sake, "depressions", without resulting in nuclear instability. In one case, through a mini supernova. In the second, by obliterating the effect of the short range strong force acting in the $^1S_0$ channel (pairing interaction).

In fact, in the case of the halo Cooper pair of ^{11}Li, that is of the last two weakly bound neutrons, one is dealing with a rarefied nuclear atmosphere of density

what can be considered

$$\rho \approx \frac{2}{\frac{4\pi}{3}(R^3(^{11}\text{Li}) - R_0^3(^9\text{Li}))} \approx 0.6 \times 10^{-2} \,\text{fm}^{-3} \tag{3.C.5}$$

where the value $R_0(^9\text{Li}) \approx 2.5$ fm was used. That is, we are dealing with pairing in a nuclear system at a density which is only 4% of saturation density.

The quest for the long range pairing mechanism which is at the basis of the binding of the halo Cooper pair of ^{11}Li to the ^{9}Li core ($S_{2n} \approx 0.380$ keV, to be compared to typical systematic values of $S_{2n} \approx 16$ MeV), has lead to the discovery of ~~a~~ novel nuclear mode of elementary excitation. The symbiotic halo pair addition mode, which has to carry its own source of binding (glue) like the hermit crab who carries a gastropod shell to protect his body. A novel embodiment of the Axel–Brink scenario in which not only the line shape, but the main structure of the resonance depends on the state on which it is built, and to which it is deeply interweaved as to guarantee its stability[101]. It also provides a novel realization

and thus its own existenc

[98] The radius of gyration R_g is a measure of an object of arbitrary shape, R_g^2 being the second moment in 3D space. In the case of a sphere of radius R, $R_g^2 = 3R^2/5$.

[99] Kobayashi, T. et al. (1989).

[100] One ~~can~~ parametrize s the radius of ^{11}Li as (see Bohr, A. and Mottelson (1975)), $R = R_0(1 + \alpha_{00}Y_{00}) = R_0(1 + \beta_0 \frac{1}{\sqrt{4\pi}})$. Thus $\beta_0 = \sqrt{4\pi}(\frac{R}{R_0} - 1) \approx 2.5$ which testifies to the extreme "exoticity" of the phenomenon.

[101] Axel (1962); Brink (1955).

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of the Bardeen–Frölich–Pines microscopic mechanism to break gauge invariance: through the exchange of quite large ZPF which ensures Galilean invariance to a nucleus displaying essentially a permanent dipole moment, as a consequence of the almost degeneracy of the giant dipole pigmy resonance (**centroid $\lesssim$ 1 MeV**) with the ground state. To our knowledge, this is the first example of a van der Waals Cooper pair, atomic or nuclear (App. 2.A).

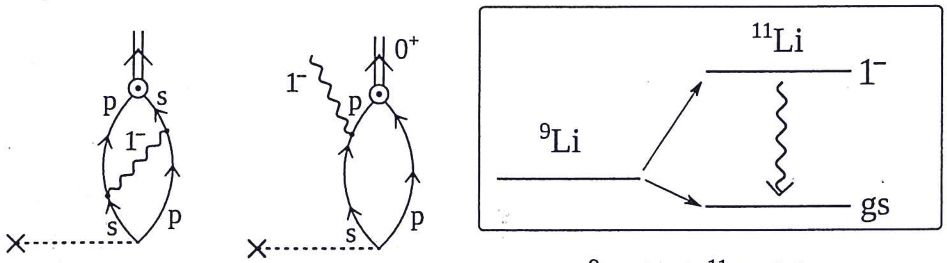
The NFT diagram shown in Fig. 2.A.1 describing this binding seems quite involved and high order. Thus unlikely to be at the basis of a new elementary mode of nuclear excitation, if nothing else because of the apparent lack of "elementarity'. This is not the case and, in fact, the physics at the basis of the process depicted by the oyster–like and eagle–like networks displayed in (a) and (b) is quite simple and ~~present~~ throughout nuclear structure and reactions, let alone many–body theories and QED. In fact, it encompasses (see Fig. 2.A.1): (I,II) the changes in energy of single–particle levels as a function of quadrupole deformations (Nilsson model) (III) the interaction between particles through the exchange of (bosons) vibrations, (IV,V) Pauli principle, (VI,VII) the softening of collective modes due to ground state correlations ((ZPF)–components, QRPA) and eventually the permanent distortion of the system (phase transition), (VIII) the interaction between two non–polar systems through ZPF generated dipoles. Referring to general many–degree of freedom systems, (I,II) and (III) are at the basis of the fact that, in QED, the coupling between one and two photons is zero (Furry's theorem). It is also at the basis, through cancellation, of the small width displayed by giant resonances as compared with single–particle widths at similar energies as well as quadrupole inhomogeneous damping in NMR of molecules and in GDR of atomic nuclei. Concerning (VIII), one can mention resonant interactions between fluctuating systems like e.g. two coupled harmonic oscillators. It is like to find a new particle. Either one is at the right energy (on shell ~~resonance~~) or one would not see it.

recurrent

In the case of halo Cooper pair binding by GDPR in ^{11}Li, the system is essentially on resonance, in keeping with the fact $\epsilon_{p_{1/2}} - \epsilon_{s_{1/2}} \approx 0.3$ MeV, and that independent particle motion emerges from the same properties of the force from which collective modes emerge. In other words the ^{10}Li inverted parity system is poised to acquire a permanent dipole moment or, almost equivalent, to display a large amplitude, dipole mode at very low energy as well as a collective $B(E1)$ to the halo ground state, of the order of a single–particle unit B_{sp}. This is the GDPR (see Fig. 3.C.1, see also Fig. 1.9.1) with centroid about ~~0.6–0.8~~ $\lesssim 1$ MeV, 8% of the EWSR and so screened from the GDR through the poor overlap between core and and halo single–particle wavefunctions so as to be able to retain essentially all of its B_{sp}, $E1$–strength which can rightly be considered a new mode of excitation (see discussion after Eq. (3.C.9)). In other words we are faced, already at the level of single–particle spectrum, with the possibility of a plastic dipole mode, as it materializes in ^{11}Li. In this case, and making ~~use of the relation~~ large amplitude

~~Cappuzzello~~

$$\frac{dn}{d\beta_L} = \frac{1}{4}\sqrt{\frac{2L+1}{3\pi}}A \tag{3.C.6}$$

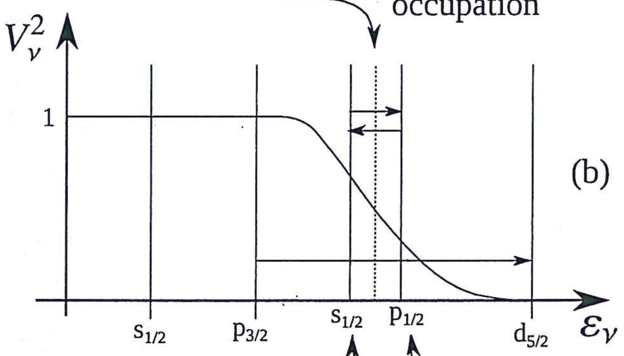


(c)

^{9}Li (t,p) ^{11}Li (1$^-$)
eventually γ -coincidence

$$|0\rangle_\nu = |0\rangle + 0.7|(p_{1/2}, s_{1/2})_{1^-} \otimes 1^-;0\rangle + 0.1|(s_{1/2}, d_{5/2})_{2^+} \otimes 2^+;0\rangle$$

partial
occupation



$$|0\rangle = 0.55|p_{1/2}^2\rangle + 0.45|s_{1/2}^2\rangle + 0.04|d_{5/2}^2\rangle$$

$$|1^-, \text{pigmy}\rangle = \alpha\,\Gamma_{\text{pigmy}}^{+}|\text{halo}\rangle + \beta\,\Gamma_{\text{GDR}}^{+}|\text{core}\rangle$$

(a) $\alpha^2 \gg \beta^2$

	$1p_{1/2}^{-1}2s_{1/2}$	$1p_{1/2}^{-1}3s_{1/2}$	$1p_{1/2}^{-1}4s_{1/2}$	$1p_{1/2}^{-1}1d_{3/2}$	$1p_{3/2}^{-1}5d_{5/2}$	$1p_{3/2}^{-1}6d_{5/2}$	$1p_{3/2}^{-1}7d_{5/2}$
X	0.847	-0.335	0.244	0.165	0.197	0.201	0.157
Y	0.088	0.060	0.088	0.008	0.165	0.173	0.138

8% EWSR $E_{1^-} \approx 0.7$ MeV

√ y

Figure 3.C.1: Schematic representation of (a) the QRPA calculation of the GDPR of ^{11}Li and associated results; namely X and Y amplitudes divided, for didactical purposes, into low–lying (pigmy) and high lying (GDPR) $p-h$ excitations. It is of notice that throughout the odd $p_{3/2}$ proton ~~state~~ is not shown being treated as a spectator, although the corresponding couplings are properly taken into account in the actual calculations (Barranco, F. et al. (2001)). (b) Schematic representation of the connection between occupation numbers and NFT wavefunction describing the two halo neutrons. (c) Gedanken eksperiment (^{9}Li$(t,p)^{11}$Li(GDPR)) to probe the GDPR wavefunction.

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defining the number of crossings n in terms of deformation *) ~~(cf. Bertsch and Broglia (2005))~~; one obtains for $L = 0$ and $\beta_0 = 2.5$, $n \approx 2$.

It is of notice that all of these processes takes place inside the halo neutron pair addition vibrational mode of the closed shell system ${}^9_3\text{Li}_6$(gs), and thus in terms of virtual states. On the other hand intervening the processes depicted in Fig. 2.A.1 with external fields, e.g. those associated with one–and two–particle transfer processes, provides much of the physics which is at the basis of the exotic properties of ^{10}Li and ^{11}Li (see e.g. Figs. 2.6.3 (I), 1.9.4 and 1.9.5. See also Fig. 6.1.3).

But let us now proceed one step at a time. A ~~very attractive,~~ simple and economic picture of the giant dipole pygmy resonance was proposed in 102. To explain parity inversion use is made of the fact that, for large prolate quadrupole deformations ($\beta_2 \approx 0.6 - 0.7$), the $m = 1/2$ member of the $1d_{5/2}$ and $1p_{1/2}$ orbitals, i.e. [220 1/2] and [101 1/2] in the Nilsson labeling of levels ([$Nn_3\Lambda\Omega$]), cross. This is in keeping with the fact that quadrupole distortion changes the energy of single–particle states; those having orbits lying in a plane containing the poles become, in the case of prolate deformations, lower in energy, while those lying preferentially in a plane perpendicular to the symmetry axis, increase their energy. Now, this parity inversion is already observed between the resonant $1/2^-$ (≈ 0.5 MeV) and the virtual $1/2^+$ (≈ 0.2 MeV) states of ^{10}Li ($p_{1/2}$ and $2_{1/2}$ states). Thus, the energy difference of 0.3 MeV is not very different from the value of 0.6-0.7 MeV of the GDPR centroid. In any case, adjusting β_2 to the appropriate value this centroid energy is within reach. On the other hand, because the radius is affected by deformation, one can posit that the above model predicts $R = R_0(1 + \frac{\beta_2}{\sqrt{5}}\sqrt{\frac{5}{4\pi}}) = 2.7\,\text{fm}\times 1.2\,\text{fm} \approx 3.2$ fm ($\beta_2 \approx 0.7$), in disagreement with the experimental finding.

Nonetheless, the fact furthermore that the observed $\approx 8\%$ of the EWSR below ≈ 5 MeV for the GDPR corresponds to about $1B_{sp}(E1)$ for a single particle transition, provides another confirmation of the attractiveness of the model. Now, static models (including also the group theoretical models like that provided by SU_3) imply that single–particle states are either occupied or empty. Experimentally, this does not seem the case in the reaction ^{9}Li$(d,p)^{10}$Li, although one can argue that the situation is different in the case of the single–particle states in ^{11}Li.

Second, the $1/2^+ \longleftrightarrow 1/2^-$ single–particle transitions are also part of the GDR transition, resonance which will essentially absorb most of the $E1$ strength into the high energy mode. In fact, typical $E1$–low energy single particle transition display $\approx 10^{-4}B_{sp}(E1)$. Inhomogeneous damping brings the dipole oscillations along the symmetry axis to an energy of

$$(\hbar\omega_D) \approx \frac{100\,\text{MeV}}{3.2} \approx 30\,\text{MeV} \tag{3.C.7}$$

far away from the less than 1 MeV energy corresponding to the GDPR centroid.

In order to calculate the giant dipole pygmy resonance based on the ground state of ^{11}Li one needs to know the occupation factors of the $s_{1/2}$ and $p_{1/2}$ states.

[102] Hamamoto and Shimoura (2007).

*) Brink D. and Broglia (2005), Eq. (7,35).

This has been done microscopically making use of ~~the diagonalization of the~~ NFT diagrams taking into account self–energy and induced interaction (vertex renormalization processes) leading to *)

$$|\tilde{0}\rangle = |0\rangle + 0.71|(p_{1/2}, s_{1/2})_{1^-} \otimes 1^-; 0\rangle + 0.1|(s_{1/2}, d_{5/2})_{2^+} \otimes 2^+; 0\rangle, \qquad (3.C.8)$$

and

$$|0\rangle = 0.45|s^2_{1/2}\rangle + 0.55|p^2_{1/2}\rangle + 0.04|d^2_{5/2}\rangle. \qquad (3.C.9)$$

In Eq. (3.C.8), the state $|1^-\rangle$ and $|2^+\rangle$ stand for the giant dipole pygmy resonance, and for the low–lying collective quadrupole vibration of ^{9}Li, respectively. As it emerges from (3.C.8) and (3.C.9), to calculate the microscopic structure of the state $|1^-\rangle$ (both wavefunction and transition density and consequently the particle–vibration coupling vertex) one needs to calculate $|0\rangle$. But to do so one needs to know the same $|1^-\rangle$ state, the vibrational mode which exchanged between the two neutrons of the halo provides most of its glue to the ^{9}Li core. From here, the symbiotic character of the 0^+ and 1^- (GDPR) entering the $|^{11}\text{Li}(0^+_\nu \otimes p_{3/2}(\pi))_{3/2^-}; gs\rangle$ and $|^{11}\text{Li}(1^-_\nu \otimes p_{3/2}(\pi))_{1/2,3/2,5/2^+}; \approx 0.8\,\text{MeV}\rangle$ states.

Similar calculations have been carried out for ^{12}Be(gs) and ^{12}Be(0^{+*}; 2.24 MeV). In the first case no pygmy is found, while in the second case a well developed GDPR is predicted displaying a number of peaks below 2 MeV and carrying a summed EWSR in the interval 0-5 of $\approx$ 6%. This result testifies to the fact that the symbiotic halo pair addition mode is a *bona fide* elementary mode of excitation. Its symbiotic GDPR allows to probe the state on which it is based, making the Axel–Brink mechanism a tool to study the structure of halo states. Within this context see Fig. 3.8.1.

Appendix 3.D pairing spatial correlation:simple estimate

Let us assume two equal nucleons above closed shell as the nuclear embodiment of Cooper's model. The two–particle wave function in configuration space can be written as,

$$\Psi(\mathbf{r}_1\sigma_1, \mathbf{r}_2\sigma_2) = \Psi_0(\mathbf{r}_1, \mathbf{r}_2)\chi_{S=0}(\sigma_1, \sigma_2) + [\Psi_1(\mathbf{r}_1, \mathbf{r}_2)\chi_{S=1}(\sigma_1\sigma_2)]_0, \qquad (3.D.1)$$

where $\chi_{S=0}$ and $\chi_{S=1}$ are the singlet and triplet spin wavefunctions, respectively.

In what follows we shall consider a pairing interaction acting on pairs of particles moving in time reversal states. Consequently we shall concentrate in the spin singlet radial component of (3.D.1). In the Tamm–Dancoff approximation (in keeping with Cooper ansatz) one can write

$$\Psi_0(\mathbf{r}_1, \mathbf{r}_2) = \sum_{nn'lj} X_{nn'lj} R_{nl}(r_1) R_{n'l}(r_2) \sqrt{\frac{2j+1}{2(2l+1)}} [Y_l(\hat{r}_1) Y_l(\hat{r}_2)]_0 . \qquad (3.D.2)$$

*) Barranco (2001)

where again (3.D.6) have been used. One can write

$$|\Psi_0(R_0, R_0, \theta)|^2 \sim |\sum_l P_l(\cos\theta)|^2. \qquad (3.D.11)$$

Assuming the closed shell nucleus to be ^{208}Pb and that N ~~implies only~~ the N single-particle levels are the neutron valence orbitals $2g_{9/2}, 1i_{11/2}, 1j_{15/2}, 3d_{5/2}, 4s_{1/2}, 2g_{7/2}$ and $3d_{3/2}$, one ~~would~~ obtains

$$\frac{|\Psi_0(R_0, R_0, \theta = 0°)|^2}{|\Psi_0(R_0, R_0, \theta = 180°)|^2} \approx \left(\frac{7}{5}\right)^2 \approx 2, \qquad (3.D.12)$$

in keeping with the fact that there is only a single state of opposite parity (intruder $j_{15/2}$).

Making use of an extended basis, containing a similar amount of positive and negative natural parity states (i.e. $\pi = (-1)^l$), that is taking into account a ~~large~~ number of major shells ($\pi = (-1)^N$, N principal quantum number), one can essentially bring to zero the value of *) $|\Psi_0(R_0, R_0, \theta = 180°)|^2$. ~~(for details see Ferreira, L. et al. (1984))~~. This of course materializes already within the basis of valence states in e.g. ^{11}Li, in keeping with the fact that in this case $s_{1/2}$ and $p_{1/2}$ play a similar role.

Summing up, the above results have something to do with the Cooper pair problem, but much more with the peculiarities of spatial quantization associated with the nuclear self–bound many–body system. That is, the fact that the nuclear Cooper pair phenomenon is to be expressed under the influence of a very strong external field which imposes not only confinement, but also spatial quantization with strong spin orbit effects resulting, among other things, in intruder states and thus parity mixing.

*) Ferreira et al (1984)

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cited in p. 234 footnote 86

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From BCS to the LHC