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Nuclear Structure and Reactions
paring in nuclei with Cooper pair transfer

G. Potel and R. A. Broglia

June 6, 2014

which is
abnormal
is the normal
state

{ conclude

pair structure
and direct
reaction

and didatics
 simply given in the Appendix
 Text tough.

Chapter 1

Pair Structure and pair transfer in a nutshell

1.1 Nuclear Structure in a nutshell

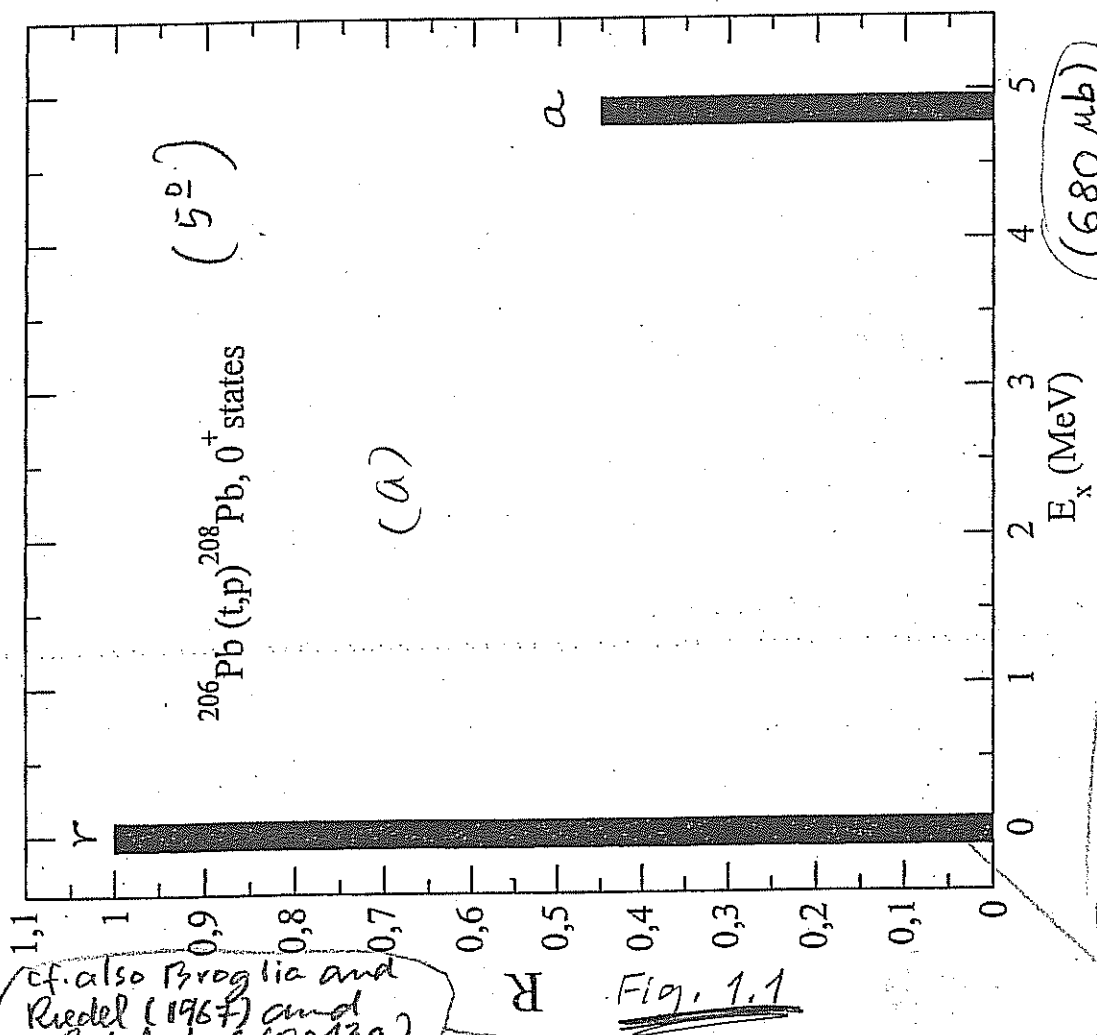
The low-energy properties of quantal, many-body, Fermi systems displaying sizable values of zero-point-motion (kinetic energy) of localization compared to the strength of the NN -interaction and quantified by the quantality parameter $q \gtrsim 0.15$ (see App. 1.A, Fig. 1.A.1 and Table 1.A.1), are determined by the laws which control independent-particle fermion motion close to the Fermi energy ϵ_F (on the energy shell) and by the relations/correlations operating among them. First of all, the Pauli principle, implying orbitals solidly anchored to the single-particle mean field, as testified by the Hartree-Fock ground state $|HF\rangle = \Pi_i a_i^\dagger |0\rangle$ (Figs. 1.A.2), describing a step function separation in the probability of occupied ($\epsilon_i \leq \epsilon_F$) and empty ($\epsilon_i \geq \epsilon_F$) states (see Fig. 1.A.3). It is of notice the relation existing between q and Lindemann's classical parameter (see App. 1.B).

Pairing acting on fermions moving in time reversal states lying close to ϵ_F alters this picture in a conspicuous way. In particular, in the case of $S = 0$ configurations, in which case the radial component of the pair wavefunction does not display nodes. Within an energy range of the pair correlation energy $E_{corr} (\approx 2\Delta$ within BCS) centered around ϵ_F ($E_{corr}/\epsilon_F \ll 1$) the system is now made out of pairs of fermions which flicker in and out of the correlated ($L = 0, S = 0$) configuration (Cooper pairs, App. 1.D). For temperatures (intrinsic excitation energies) or stress regimes (magnetic field in metals, Coriolis force in nuclei, etc.) smaller than $\approx E_{corr}/2$ (critical value), Cooper pairs respect Bose-Einstein statistics, the single-particle orbits on which they are correlated become dynamically detached from the mean field, leading to a bosonic condensate and, at the same time, reducing in a conspicuous way the inertia of the system (e.g. the moment of inertia I of quadrupole rotational bands is much smaller than the rigid moment of inertia ($I \approx I_r/3$) expected from independent particle motion see Fig. 1.A.3). [Cooper pairs exist also in situations in which the environmental conditions are above crit-

of superfluid nuclei
 with open shells of
 both protons and neutrons

end anharmonic
sect 1.E.1

(Fig. 1.1) p. 4

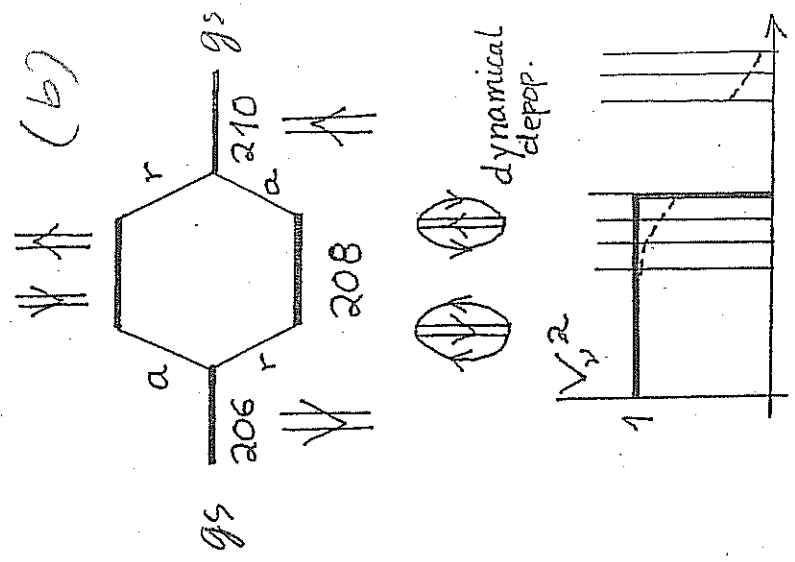


cf. also Broglia and Riedel (1967) and Potel et al (2013a)

Fig. 1.1

- (a) Ratio of the absolute $L=0$ differential cross sections $d\sigma(E_x, \theta)/d\sigma(g.s., \theta)$ for the reaction $^{206}\text{Pb}(t,p)^{208}\text{Pb}$ at $\theta=5^\circ$ (Bjerregaard et al 1966, ...)
- (b) Schematic representation of the pairing vibrational spectrum around ^{208}Pb . Concerning the anharmonicities

linear term N



Well developed vib. bands (very anharmonic at the edge of phase transition, thus very collective cf. Clark et al. PRL 96 (2006) 032501)

Why not to put everything in abs. cross sections

→ of these modes cf. last paragraph sect 1.E.1, App. E.

5

Bjerregaard, J.H., Hansen, O., Nathan, D. and
Hinds, S. (1966) States of ^{208}Pb from
double ^{triton}stripping, Nucl. Phys. 89, 337

(6)

Broglia, R.A. and Reidel, C. (1967),
Pairing vibration and particle-hole
states excited in the reaction $^{206}\text{Pb}(\text{t}, \text{p})^{208}\text{Pb}$
Nucl. Phys. A92, 145-174.

cf. also references Fig 1.3

Schmid, 1966, Schmidt, 1968, Abraham & Woo, 1968
Schmid, 1969

(Fig. 1.1, App. 1E)

Fig. 1.3 Padova

around closed shells, especially

In

4 CHAPTER 1. STRUCTURE AND PAIR TRANSFER IN A NUTSHELL

(7)

ical, e.g. in metals at room temperature, in closed shell nuclei as well as in deformed open shell ones at high values of the angular momentum, although they break as soon as they are generated (pairing vibrations). While these pair addition and subtraction fluctuations have little effect in condensed matter systems with the exception than at $T \approx T_c$ (referencias see Schmidt 68, 66, 69), they play an important role in mesoscopic systems, in particular in nuclei, in particular in the case of light, exotic halo nuclei (see App. 1.F). ~~(E) - (E) handwritten~~

, highly polarisable,

Within the framework of the above picture, one can introduce at profit a collective coordinate α_0 (order parameter) which measures the number of Cooper pairs participating in the pairing condensate, and define a wavefunction for each pair $(U_\nu + V_\nu a_\nu^\dagger a_\nu^\dagger)|0\rangle$ (independent pair motion, BCS approximation), adjusting the occupation parameters V_ν and U_ν (probability amplitudes that the two-fold (Kramer's-)degenerate pair state $(\nu, \bar{\nu})$ is either occupied or empty), so as to minimize the energy of the system under the condition that the average number of nucleons is equal to N_0 (Coriolis-like force felt, in the intrinsic system in gauge space, by the pairs, being equal to $-\lambda N_0$). Thus, $|BCS\rangle = \prod_{\nu>0} (U_\nu + V_\nu a_\nu^\dagger a_\nu^\dagger)|0\rangle$ provides a valid description of the paired mean field ground state, and of the associated order parameter $\alpha_0 = \langle BCS | P^\dagger | BCS \rangle$, $P^\dagger = \sum_{\nu>0} a_\nu^\dagger a_\nu^\dagger$ being the pair creation operator. It is then natural to posit that two-nucleon transfer reactions are specific to probe pairing correlations in many-body fermionic systems. Examples are provided by the Josephson effect in e.g. metallic superconductors, and (t, p) and (p, t) reactions in atomic nuclei (Fig. 1.3 and Fig. 1.4).

Cooper

e

$$= \sum_{\nu>0} U_\nu V_\nu$$

Due to the fact that, away from the Fermi energy pair independent motion becomes independent particle motion, in particular in the nuclear case $|BCS\rangle \rightarrow |\text{Nilsson}\rangle$, one-particle transfer reactions like e.g. (d, p) and (p, d) can be used together with (t, p) and (p, t) processes as a valid tool to cross check pair correlation predictions (within this context see Capitulo transfer in a nutshell). In particular, to shed light on the origin of pairing in nuclei: in a nutshell, the relative importance of the bare NN -interaction and the induced pairing interaction (see App. 1.F).

While the calculation of two-nucleon transfer spectroscopic amplitudes and differential cross sections are, a priori, more involved to be worked out than those associated with one-nucleon transfer reactions, the former are, as a rule, more "intrinsically" accurate than the latter ones. This is because, in the case of two nucleon transfer reactions, the quantity (order parameter α_0) which expresses the collectivity of the members of a pairing rotational band reflect the properties of a coherent state $|BCS\rangle$. In other words, it results from the sum over many contributions $(U_\nu V_\nu)$, all of them having the same phase. Consequently, errors are averaged out in the summed value $|\alpha_0|$, conferring the two nucleon transfer cross section $d\sigma(2N \text{ transfer})/d\Omega \sim |\alpha_0|^2$, a quantitative accuracy which goes beyond that of the individual contributions.

On the other hand, $d\sigma(1n\text{-transfer})/d\Omega \sim |U_\nu|^4 \sim |V_\nu|^2$ depends on the accuracy with which one is able to calculate the occupancy of a single pure configuration, (see App. 1.F).

The soundness of the above parlance reflects itself in the calculation of the el-

$$((j_0 + 1/2)^{1/2} U_\nu V_\nu, \text{ cf. App. D})$$

(of notice within this context of the accuracy with which one can calculate the nuclear density and that associate with a single orbital which can be probed in (e, e') experiments)

relative

Fig. 1.1
Fig. 1.3
Padova
Trento

Fig. 1.2
Padova
Trento
Fig. 1.3

Fig. 1.4
Fig. 1.0
PRC

Fig. 1.2
Fig. 6.24
Bryant
and
Prossler

(E)

(8)

(4)_a

From this vantage point of view one can posit that it is not so much, or at least not only, the superfluid state which is abnormal in the nuclear case, but the normal state of ~~many~~ closed shell systems.

It is of notice nonetheless, the role pairing vibrations play in the phase transition between superfluid and normal nuclear phases (cf. Fig. 1, 2) as a function of the rotational frequency (angular momentum) as emerged from the experimental studies of high spin states carried out by

Garrett and collaborators (~~cf. e.g.~~, Brink and Broglie, 2005, Ch. 6 and refs. therein)

p. 4 ~~Voronov~~ 6/7/14

cf. Shimizu et al 1989; cf. also

B+B
Fig
24

(Shimizu et al 1989, Shimizu and Broglia, 1990, Shimizu 2013)

(4)

From Fig. 1.2 it is seen that while the dynamic pairing ^{gap} associated with pairing vibrations leads to a $\approx 20\%$ ^{increase} of the static pairing gap for low rotational frequencies, it becomes the overwhelming contribution for angular momenta above critical. In any case, the central role played by pairing vibrations within the presence circumstances, that to restore particle-number conservation. Within this context, there are a number of methods which allow to go beyond mean-field approximation (HFB). Generally referred to as number projection method (NP), they make use of a variety of techniques (Generator Coordinate Method, pfaffians, etc) as well as protocols (variation after projection, gradient method, etc); (Ring and Schuck, 1980, Egido, 2013, Robledo and Bertsch, 2013; cf. also Frauendorf 2013, Ring 2013, Heenen et al 2013, and refs. therein). The advantages of NP method over the RPA is to lead to smooth functions for both the correlation energy and the pairing gap at the pairing phase transition. The above ^{results} underscores the fact that at the basis of an operative coarse grain approximation to the nuclear many-body problem, ^{one} finds a ^{judicious} choice of the collective coordinate/es. In other words, pairing vibrations are the elementary modes of excitation containing the right physics to restore gauge invariance through their interweaving with the quasiparticle states.

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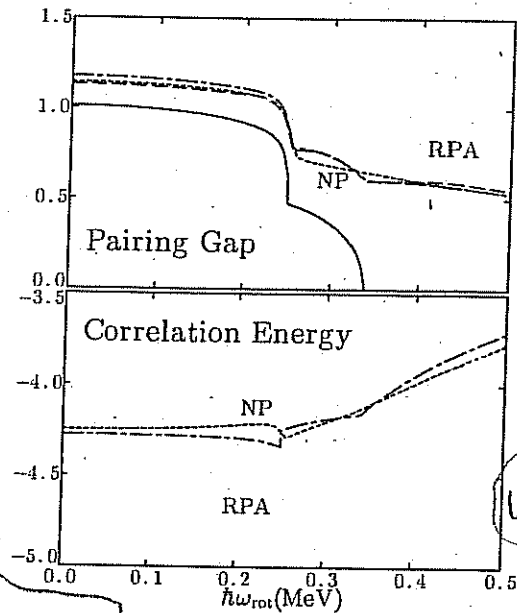
D. M. Brink and Broglia, R. A. (2005), Nuclear Superfluidity, Cambridge University Press, Cambridge.

weighted sum rule $S_0(\text{RPA})$ describes the contribution of pairing fluctuations associated with the monopole pair-transfer operator, $P^\dagger = \sum_{i>0} a_i^\dagger a_i^\dagger$, to the effective (RPA) gap. Note that $\sum_{n \neq \text{AGN}}$ means that the divergent contribution from the zero energy mode (pairing rotation) is to be excluded, in keeping the fact that its contribution to equation (6.61) is included through the static (BCS) pairing gap Δ . In Shimizu *et al.* (1989), $S_0(\text{RPA})$ was calculated making use of the expression

$$S_0(\text{RPA}) \approx \frac{1}{\pi} \int_{\omega_{\text{cut}}}^{\infty} \text{Im Tr}[\mathcal{R}(\omega + i\varepsilon)] d\omega, \quad (6)$$

where $\mathcal{R}(\omega) \equiv \mathcal{R}^{(\lambda=1)}(\omega)$ is the RPA response function, whose dimension corresponds to $Q_1 = (P^\dagger + P)/\sqrt{2}$ and $Q_2 = i(P^\dagger - P)/\sqrt{2}$. A finite value of ε and a low-energy cutoff ω_{cut} are used to get rid of the AGN mode contribution numerically. This is the same approximation as that used in calculating the pairing correlation energy, and can then be avoided using the path shown in Fig. 6.23. In this way one avoids the singularity associated with an eventual zero mode

number projection
p.v.b.s.
corrected
of exc.



✓ Fig. 1.2

Figure 6.24: RPA pairing gap (upper panel) and RPA correlation energy (lower panel) for neutrons in ^{164}Er as a function of the rotational frequency. Both quantities are in MeV. The dash-dotted curves denote the results of calculations with $\varepsilon = 200 \text{ keV}$ and $\hbar\omega_{\text{cut}} = 400 \text{ keV}$. The value of the static (mean-field) pairing gap Δ , which vanishes at $\hbar\omega_{\text{rot}} = 0.34 \text{ MeV}$, is also displayed in the upper panel (continuous curve). The results of the number projection (NP) calculations are shown as dotted curves.

Pairing gap taking into account pairing vibrations in the RPA approximation
 $(\Delta = (\Delta_{\text{BCS}}^2 + \frac{1}{2} G^2 S_0(\text{RPA}))^{1/2})$

(the correlation associated with)

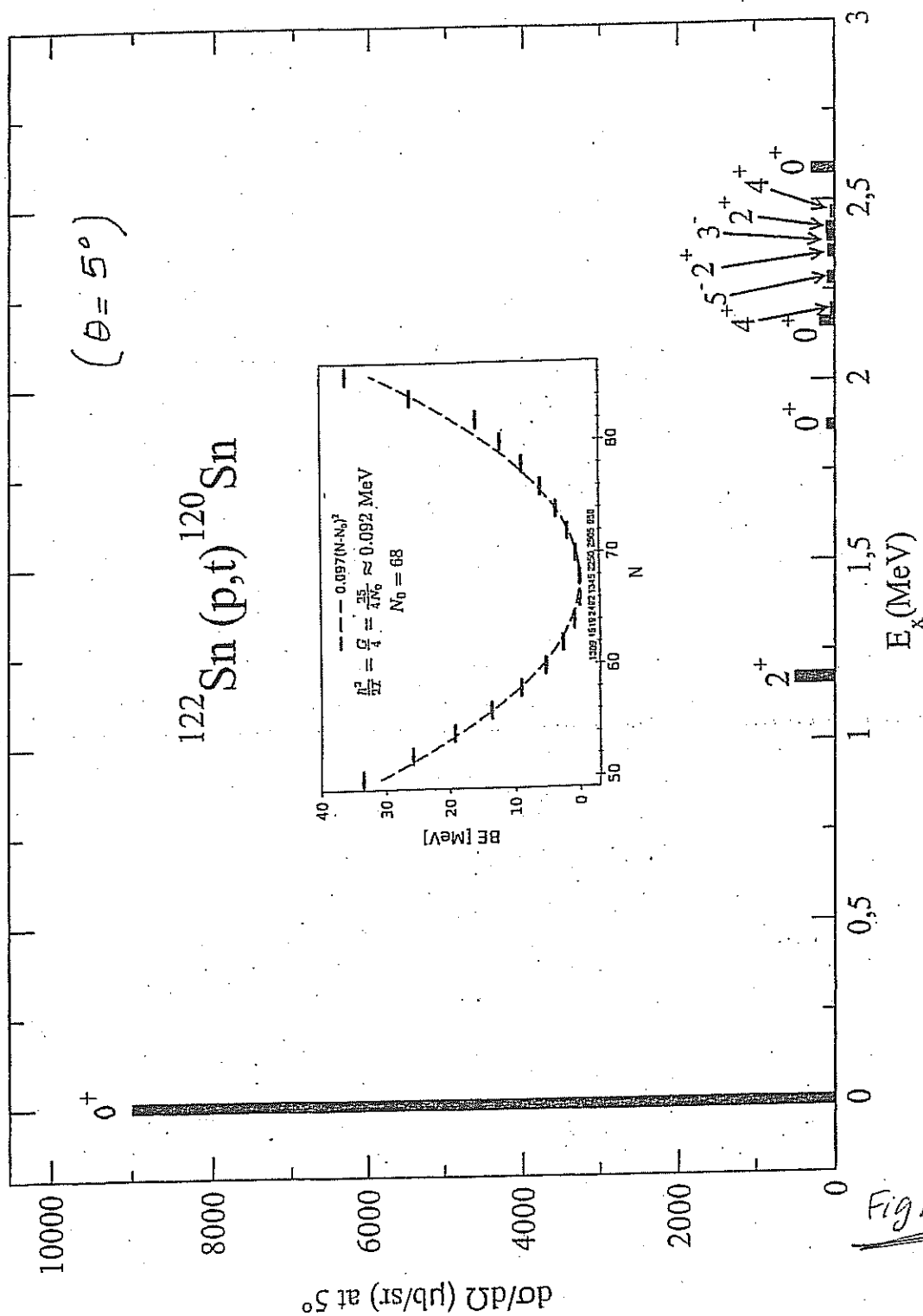


Fig. 1.3

Potel et al.. Rep. Prog. Phys. 76 (2013) 106301

Caption to Fig. 1.3

(14)

Excitation function associated with the reaction $^{122}\text{Sn}(p,t)^{120}\text{Sn}(J^\pi)$. The absolute experimental value (Guazzoni et al, 1999) of $d\sigma(J^\pi)/d\Omega|_{50}$ is given as a function of the excitation energy. ^(Potel et al 2013)

In the inset is displayed the pairing rotational band associated with the ~~gs~~ ground states of the Sn-isotopes [Potel et al 2013b, ~~Potel~~ and Broglia 2013]. ~~The numbers given in the abscissa are the absolute value of the experimental gs \rightarrow gs cross section (Guazzoni et al, 1999, 2004, 2008, 2011, 2012). The estimates of $\hbar^2/2I$ were obtained using the simple 7-shell model (cf. Brink and Broglia, 2005, app. H). For more details see~~

Guazzoni, et al 1999 \rightarrow [110]
 2004 \rightarrow [111]
 2008 \rightarrow [112]
 2011 \rightarrow [113]
 2012 \rightarrow [114]

Refs. Rep. Prog. Phys
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Pairing Correlations ~~with~~ with single
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Nuclear BCS, eds. ~~R. A. Broglia~~ R. A. Broglia
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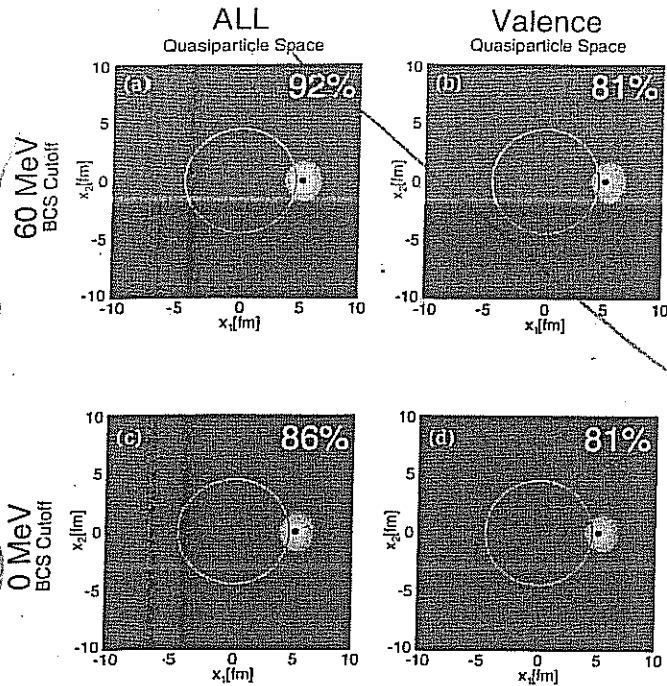


FIG. 8. (Color online) Spatial structure of a two-neutron Cooper pair of ^{120}Sn [see Eqs. (B16), (B17), and (B23)]. The modulus squared wave function $|\Psi_0(\vec{r}_1, \vec{r}_2)|^2 = |\langle \vec{0} | \vec{r}_1, \vec{r}_2 \rangle|^2$ (see Tables I and II), multiplied by $16\pi^2 r_1^2 r_2^2$ and normalized to unity, is displayed as a function of the Cartesian coordinates $x_1 = r_2 \cos \theta_{12}$ and $x_2 = r_2 \sin \theta_{12}$ of particle 2, for a fixed value of $r_1 = x_1 = 5$ fm (black dot) of particle 1, close to the surface of the nucleus (red circle). The numerical percentages correspond to the two-nucleon integrated density in a spherical box of radius 4 fm centered at the coordinates of the fixed particle.

94 μb . Thus, the discrepancies between theory and experiment are bound in the interval $0 \leq |\sigma_{\text{exp}}(i \rightarrow f) - \sigma_{\text{th}}(i \rightarrow f)| / \sigma_{\text{exp}}(i \rightarrow f) \leq 0.09$, the average discrepancy being 5%.

In Fig. 10 the excited, pairing rotational band associated with the average value of the 0^+ pairing vibrational states with energy ≤ 3 MeV, is displayed together with the best parabolic fit. Also given is the relative (p, t) integrated cross section normalized with respect to the $gs \rightarrow gs$ transitions, a value which is in all cases $\leq 8\%$, in overall agreement with the single j -shell estimate (see Ref. [10], Appendix H), given in the inset to the figure. The result testifies to the weak cross talk between pairing rotational bands and thus of the robust off-diagonal, long-range order coherence of these modes.

B. Pairing vibrational band in closed-shell nuclei

The expected pairing vibrational spectrum (harmonic approximation, see Refs. [10–12] and references therein) associated with the closed-shell exotic nucleus ^{132}Sn [2,3], up to two-phonon states has been published in Fig. 3 of Ref. [45]. Within this approximation, the one-phonon states are the pair addition $|a\rangle = |gs(^{134}\text{Sn})\rangle$ and pair removal $|r\rangle = |gs(^{130}\text{Sn})\rangle$ modes. The two-phonon 0^+ ($|pv(^{132}\text{Sn})\rangle = |r\rangle \otimes |a\rangle = |0^+(^{132}\text{Sn}; 6.5 \text{ MeV})\rangle$) pairing vibrational [(2p-2h)-like] state of ^{132}Sn , is predicted at an excitation energy of 6.5 MeV (see Fig. 3). The absolute two-particle transfer differential

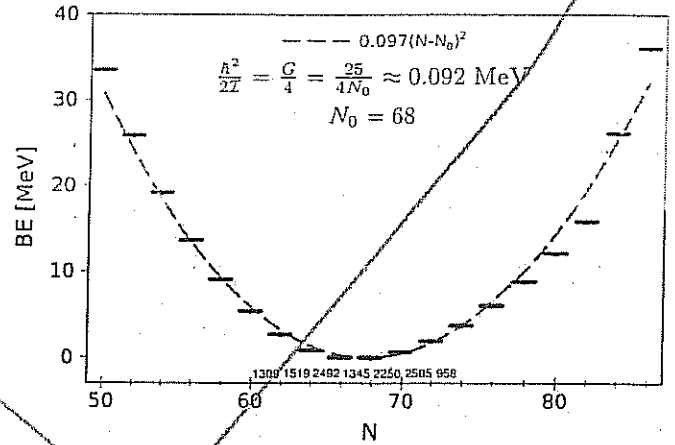


FIG. 9. (Color online) Pairing rotational band along the tin isotopes. The lines represent the energies calculated according to the expression $BE = B(^{50+N}\text{Sn}_N) - 8.124N + 46.33$ [10], subtracting the contribution of the single nucleon addition to the nuclear binding energy obtained by a linear fitting of the binding energies of the whole Sn chain. The estimate of $\hbar^2/2I$ was obtained using the single j -shell model (see, e.g., Ref. [10], Appendix H). The numbers given on the abscissa are the absolute value of the experimental $gs \rightarrow gs$ cross section (in units of μb ; see Table IV).

cross sections associated with $|a\rangle$ and $|r\rangle$, namely,

$$^{134}\text{Sn}(p, t)^{132}\text{Sn}(gs), \quad (E_{\text{CM}} = 20 \text{ MeV}), \quad (41)$$

$$^{132}\text{Sn}(p, t)^{130}\text{Sn}(gs), \quad (E_{\text{CM}} = 26 \text{ MeV}), \quad (42)$$

have been reported in the insets. Using detailed balance the reactions

$$^{130}\text{Sn}(t, p)^{132}\text{Sn}(0^+; 6.5 \text{ MeV}), \quad (E_{\text{CM}} = 20 \text{ MeV}), \quad (43)$$

$$^{134}\text{Sn}(p, t)^{132}\text{Sn}(0^+; 6.5 \text{ MeV}), \quad (E_{\text{CM}} = 26 \text{ MeV}), \quad (44)$$

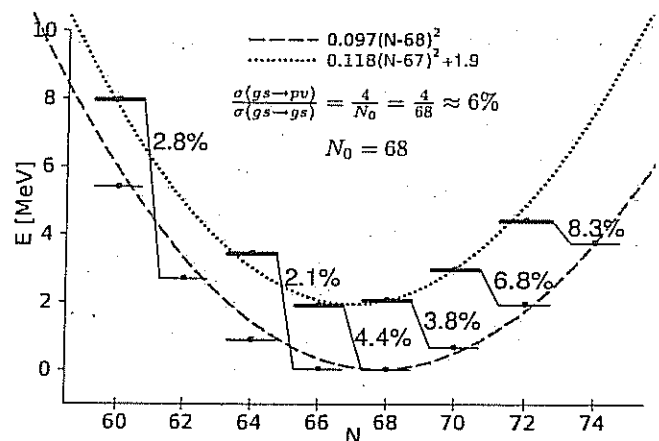


FIG. 10. (Color online) The weighted average energies ($E_{\text{exc}} = \sum_i E_i \sigma_i / \sum_i \sigma_i$) of the excited 0^+ states below 3 MeV in the Sn isotopic chain are shown on top of the pairing rotational band, already displayed in Fig. 9. Also indicated is the percentage of cross section for two-neutron transfer to excited states, normalized to the cross sections populating the ground states. The estimate of the ratio of cross sections displayed on top of the figure was obtained making use of the single j -shell model (see, e.g., Ref. [10], Appendix H).

(17)

to describe
the reaction

5

1.1. NUCLEAR STRUCTURE IN A NUTSHELL

elements resulting from the encounter of structure and reaction, namely one- and two-nucleon modified transfer formfactors. While it is usually considered that these quantities carry all the structure information associated with the calculation of the corresponding cross sections, a consistent NFT treatment of structure and reaction will posit that equally much is contained in the distorted waves describing the relative motion of the colliding systems (potential optical potential transfer in a nutshell). This is because the optical potential ($U + iW$) which determines the scattering waves, emerges from the same modified formfactors, eventually including also inelastic processes. In other words, setting detectors in e.g. a given two-particle transfer channel like $A + t \rightarrow B(= A + 2) + p$, one needs to know what the single-particle states and collective modes of the systems A ($F(= A + 1)$) and B are, respectively. Furthermore one needs to take into account the interweaving of these modes of excitation which results in dressed particle states (quasiparticles; fermions) and renormalized normal vibrational modes of excitation (bosons). But these are essentially all the elements needed to calculate the processes leading to the depopulation of e.g. the flux in the incoming channel ($A + t$ in the case under discussion). In particular, and assuming to work with spherical nuclei, one-particle transfer is, as a rule (in particular for well matched Q -value channels), the main depopulation process, in keeping with the long range tail of the associated formfactors as compared to that of other processes, e.g. inelastic processes (mirar figura 1B3 apendice 2B sobre optical potential capitulo 2).

are, equally
well than those
of nuclei A

In keeping with this fact, and because U and W are connected by the Kramers-Krönig generalized dispersion relation (fluctuation-dissipation theorem), it is possible to calculate the nuclear dielectric function (optical potential) associated with the elastic channels under discussion (i.e. (A, t) and (B, p) in the present case) making use of the above described elements.

for Ch. 1+2
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Concerning the modified formfactor associated with the (t, p) process, we shall see in the (chapter 7, 2pt) that it can be written as

$$u_{LSJ}^{J_i J_f}(R) = \sum_{\substack{n_1 l_1 j_1 \\ n_2 l_2 j_2, n}} B(n_1 l_1 j_1, n_2 l_2 j_2; J J_i J_f) \\ \langle S L J | j_1 j_2 J \rangle \langle n 0, N L, L | n_1 l_1, n_2 l_2; L \rangle \\ \Omega_n R_{NL}(R),$$

where the overlaps

$$B(n_1 l_1 j_1, n_2 l_2 j_2; J J_i J_f) \\ = \langle \Psi^{J_f}(\xi_{A+2}) | [\phi^J(n_1 l_1 j_1, n_2 l_2 j_2), \Psi^{J_i}(\xi_A)]^{J_f} \rangle$$

and

$$\Omega_n = \langle \phi_{nlm_l}(\mathbf{r}) | \phi_{000}(\mathbf{r}) \rangle$$

encode for the physics of particle-particle (but also, to a large extent, particle-hole) correlations in nuclei, $\langle S L J | j_1 j_2 J \rangle$ and $\langle n 0, N L, L | n_1 l_1, n_2 l_2; L \rangle$ being $LS - jj$

6 CHAPTER 1. STRUCTURE AND PAIR TRANSFER IN A NUTSHELL

and Moshinsky transformation brackets, keeping track of symmetry and number of degrees conservation. In fact, the two-nucleon spectroscopic amplitude (B-coefficient) and the overlap Ω_n reflect the parentage with which the nucleus B can be written in terms of the system A and a Cooper pair,

$$\Psi_{exit} = \Psi_{M_f}^{J_f}(\xi_{A+2}) \chi_{M_f}^{S_f}(\sigma_p),$$

where

$$\Psi_{M_f}^{J_f}(\xi_{A+2}) = \sum_{\substack{n_1 l_1 j_1 \\ n_2 l_2 j_2 \\ J, J_1}} B(n_1 l_1 j_1, n_2 l_2 j_2; J J_1 J_f)$$

$$\otimes [\phi^J(n_1 l_1 j_1, n_2 l_2 j_2) \Psi_{M_f}^{J_1}(\xi_A)]_{M_f}^{J_f},$$

and

$$\Psi_{entrance} = \Psi_{M_i}^{J_i}(\xi_A) \phi_i(\mathbf{r}_{n1}, \mathbf{r}_{n2}, \mathbf{r}_p; \sigma_{n1}, \sigma_{n2}, \sigma_p),$$

with

$$\phi_i = [\chi^S(\sigma_{n1}, \sigma_{n2}) \chi^{S'}(\sigma_p)]_{M_{\phi}}^{S_i} \phi_i^{L=0} \left(\sum_{i>j} |\mathbf{r}_i - \mathbf{r}_j| \right).$$

M_{S_i}

Assuming for simplicity a symmetric di-neutron radial wavefunction of the triton (i.e. neglecting the d -component of the corresponding wavefunction) regarding the relative and center of mass wavefunctions $\phi_{nlm}(\mathbf{r})$ and $\phi_{N\Lambda M}(R)$ ($n = l = m = 0, N = \Lambda = M = 0$), leads to Ω_n , a quantity which reflects both the non-orthogonality existing between the di-neutron wavefunctions in the final nucleus (Cooper pair) and in the triton. Another way to say the same thing is that dineutron correlations in these two systems are different, a fact which underscores the limitations of light ion reactions to probe specifically pairing correlations in nuclei.

One can then conclude that, provided one makes use of a (sensible) complete single-particle basis (eventually including also the continuum), one can capture through $u_{LSJ}^{J_f}(R)$ most of the coherence of Cooper pair transfer, as a major fraction of the associated di-neutron non-locality is taken care of by the n -summation appearing in the expression of u , weighted by the non-orthogonality overlaps Ω_n . This is in keeping with the fact that, making use of a more refined triton wavefunction than that employed above, the $n-p$ (deuteron-like) correlations of this particle can be described with reasonable accuracy and thus, the emergence of successive transfer (ver capitulo transfer in a nutshell). On the other hand, being the deuteron a bound system, this effective treatment of the associated resonances is not particularly economic. Furthermore, zero-range approximation ($V(\rho)\phi_{000}(\rho) = D_0\delta(\rho)$) blocks such a possibility.

Nonetheless, the fact that one can still work out a detailed and consistent picture of two-nucleon transfer reactions in nuclei in terms of absolute cross sections with the help of a single parameter ($D_0^2 \approx (31.6 \pm 9.3)10^4 \text{ MeV}^2 \text{ fm}^2$) testifies to the fact that the above picture of Cooper pair transfer is a powerful one, as it contains

appearing

eliminates

Anyhow

within this context
see Broglia and Wauther
Phys. Lett. \rightarrow $S_n + S_n$ + von Oertzen
Maggiore Vittori

in average, errors cancel against each other, an ansatz found at the basis of all collective approximation, e.g. RPA. On the other hand this does not guarantee that each contribution is correctly calculated but, alas, the opposite. It will thus not be surprising that if one can ~~make~~ each, or at least a number, of the ~~total~~ virtual individual contributions to become real in a ~~quantum~~ experiment, one would find a variety of degrees of agreement.

(19)

1.A. QUANTALITY PARAMETER

a large fraction of the physics which is at the basis of Cooper pair transfer in nuclei (Broglia et al. (1973)). This is in keeping with the fact that the Cooper pair correlation length is much larger than nuclear dimensions and, consequently, simultaneous and successive transfer feel the same pairing correlations (ver transfer in a nutshell). In other words, treating explicitly the intermediate deuteron channel in terms of successive transfer, correcting both this and the simultaneous transfer channel for non-orthogonality contributions, makes the above picture the quantitative probe of Cooper pair correlations in nuclei (Potel et al. (2013)). as testified by (figura con distribuciones angulares y tabla con valores absolutos).

within this context one may find through mean field approximation a good description of the energy of the valence orbitals but for a specific level (example d5/2 level of 19,124 Sn). It is not said that observing the correlation, such cases constitute a sobering experience concerning the problem.

the intricacies of the many-body problem in general, and the nuclear one (finite many-body system, FMS) in particular.

Within the above context, we provide below two examples of B-coefficients. One for the case in which A and B(= A + 2) are members of a pairing rotational band. A second one, in the case in which they are members of a pairing vibrational band. That is,

associated with coherent states. Namely,

$$1), B(nlj, nlj; 000) = \langle BCS(N+2) | [a_{nlj}^\dagger a_{nlj}^\dagger]_0^0 | BCS(N) \rangle = \sqrt{j+1/2} U_{nlj}(N) V_{nlj}(N+2),$$

and

$$2), B(nlj, nlj; 000) = \langle N_0 + 2(gs) | [a_{nlj}^\dagger a_{nlj}^\dagger]_0^0 | N_0(gs) \rangle = \begin{cases} \sqrt{j+1/2} X_a(n_k l_k j_k) & (\epsilon_{jk} > \epsilon_F) \\ \sqrt{j+1/2} Y_a(n_l l_l j_l) & (\epsilon_{jk} \leq \epsilon_F). \end{cases}$$

correct

For actual numerical values see box 3 (App. 1-E y apendice de los B-coeficientes gs-gs) and Tables

App. D, Table 1.D.1 and App. E, Tables 1.E.2 - 1.E.5

from $(\alpha_1 - \alpha_6)$

Appendix 1.A Quantality Parameter

Ratio of quantal kinetic energy of localization and potential energy, (cf. Fig. 1.A.1 and Table 1.A.1). Fluctuations, quantal or classical, favor symmetry: gases and liquids are homogeneous. Potential energy on the other hand prefers special arrangements: atoms like to be at specific distances from each other (spontaneous breaking of translational symmetry reflecting the homogeneity of empty space). When q is small, quantal effects are small and the lower state for $T < T_c$ will have a crystalline structure, while for sufficiently large q (> 0.15) the system will display particle delocalization and, very likely, will be amenable, in first approximation, to a mean field description (Fig. 1.A.2).

values of

space q (> 0.15)

at least

In fact, the step (delocalization → mean field) is certainly

not automatic, neither guaranteed. In any case, not for all properties, neither for all levels of the system. In fact, it is, arguably, true that independent particle motion can be viewed as the most collective nuclear property, reflecting the effect of all nucleons on a given one, and thus leading to a macroscopic effect (confinement). Consequently, it should be easy to calculate as the sum of many contributions whose relative

Let us elaborate on these points.

two more in Colombia

(a little bit)

(n) In other words, one is dealing with a self-confined strongly interacting, finite many-body system generated from collisions resulting and associated with a variety of astrophysical events and thus of the coupling and interweaving of different scattering channels and resonances, as e.g. the Hoyle monopole one ($\alpha + \alpha + \alpha \rightarrow {}^{12}\text{C}$). (7)
(20)
~~such phenomena more to allow organic matter and eventually life in the universe~~

(grand design)

Within the antropomorphic scenario such phenomena are found in the evolution of the Universe to eventually allow for the presence of organic matter and, arguably, life more likely than to make mean field approximation a valid description of nuclear structure and reactions.

(n)

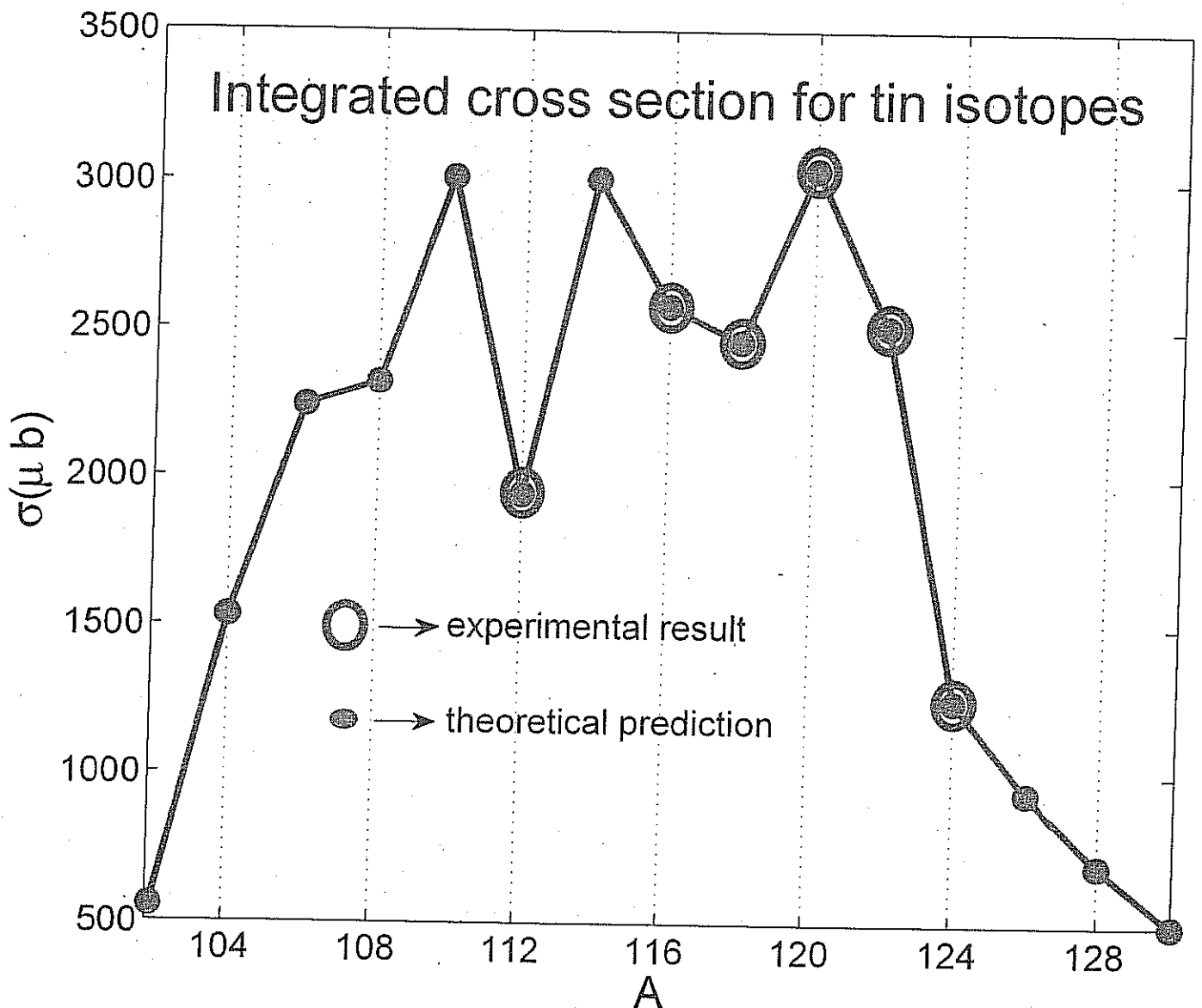


Fig. 1.5 Absolute value of the calculate two-nucleon transfer cross section $A+2 \text{ Sn}(p, t) A \text{ Sn}(g.s.)$ ($A=112, 116, 118, 120, 122, 124$; cf. Potel et al 2013a, 2013b, Potel and Broglia 2013) in comparison with the experimental data (Guazzoni et al 1999, 2004, 2008, 2011, 2012).

(d₁) - (d₆) p. 7

(d₁)

We conclude this section by remarking that, in spite ^(of the fact) that one is dealing with the connection ⁽²²⁾ between structure and direct transfer reactions, no mention has been made of spectroscopic factors in relation with one-particle transfer processes, let alone when discussing two-particle transfer. In fact, one ^{meets when} will be using throughout the present monograph ^(exception explicitly mentioned), absolute cross sections as the solely link between spectroscopic amplitudes and experimental observations.

Let us elaborate on this question in connection with one-particle transfer reactions [cf. Dickhoff and Van Neck (2005), Jennings (2011), Kramer et al (2001), Barbieri 2009, Schiffer et al 2012, ^(top) ^(d₄) Duguet and Hagen 2012, Furnstahl and Schwenk (2010)] [Mahaux et al (1985), Brink and Broglia (2005) ^{and refs. therein}]. Elementary modes of nuclear excitation, namely single-particle motion, vibrations and rotations, being tailored to economically describe the nuclear response to external probes, contain a large fraction of the many-body correlations. ^(consequently) ^(to each other) their wavefunctions are non-orthogonal, ^(in keeping with the fact that all the degrees of freedom of the nucleus) The resulting overlaps constitute a measure of the strength of ^(with which the different modes couple to each other) their coupling. Within the framework of mean field, ^(for structure calculations) arguably the most convenient of basis, the corresponding vertices are the particle-vibration

are exhausted by those of the nucleus

coupling vertices (and particle-rotor vertices) (d. to be treated according to BRST techniques cf. Bes and Kurchan, 1990, or approximately in terms of large amplitude, plastic-like vibrations, cf. Apps. F Mar and G) (23)

coherence and collectivity
large amplit. vibs.

to be diagonalized within the framework of nuclear field theory (NFT, cf. Bortignon et al (1977), (1978))

it is not hard to make local

Box 4 app. minimal

→ As a result of the interweaving of single-particle and collective motion, the nucleons acquire a state dependent self-energy $\Delta E_s(\omega)$ which, for level far away from the Fermi energy, can become complex. ~~As a result~~ ^{consequently}, the single-particle potential which was already non-local in space (exchange potential, due to Pauli principle), becomes also non-local in time (retardation effects). There are a number of techniques to make it local. In particular the Local Density Approximation (LDA) and the effective mass approximation. In this last case one can describe the single-particle motion in terms of local (complex) potential of real part $U(r) = (m/m^*) U(r)$, where m is the nucleon mass, $m^* = m_R m_\omega / m$

mass, and $m_w = m(1+\lambda)$, $\lambda = -\partial \Delta E(\omega) / \partial (hw)$, ^{the}

so called mass enhancement factor. It reflects the ability with which vibrations dress single particles. In other words, the probability with which ~~the~~ a nucleon moving at $t = -\infty$ in the "pure" orbital j can be found in a $2p-1h$ -like (doorway state)

$|j' \otimes L; j\rangle$, L being the multipolarity of the vibrational state.

Within this context, the discontinuity taking place at the Fermi energy in the ~~fermionic dressed~~ dressed particle picture is $Z_w = (m/m_w)$, closely connected with the ^{single-particle} occupancy probability. In keeping with the fact that $m_k \approx 0.6-0.7m$

m_k being the so called k -mass (non-locality in space in keeping with $\Delta x \Delta k_x \geq 1$),

and that $m^* \approx m$, as testified by the ^(good) fitting standard Saxon-Woods potentials provide to the valence orbitals of nucleon of mass m around closed shells, one obtains $m_w \approx 1.4-1.7m$. Thus $Z_w \approx 0.6-0.7$. It is still an open question how much of the observed single-particle degradation is due to hard core effects, shifting

the strength to very high moment levels (hundreds of MeV). Arguably, an estimate of such an effect of about 15-20%, effect which do not quantitative change the long-wavelength estimates of ZW given above. A much larger effect of the deprotonation of the hard core does not seem compatible with medium polarization effects.

(a)-(a)

from π/d_3

(25)

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sum rules
does work

(26)

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50% - 70% BSWF
IPSM

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0.55
quenching
factor

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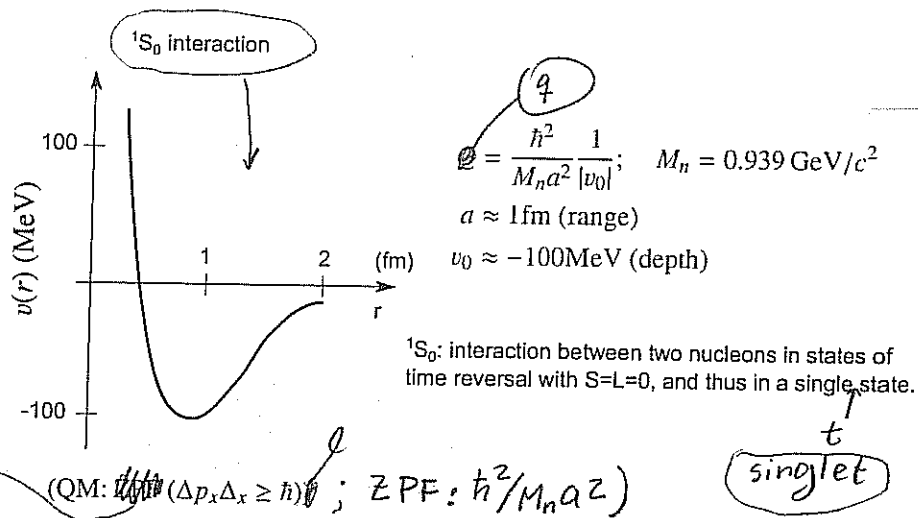


Figure 1.A.1: Schematic representation of the bare NN -interaction acting among nucleons and used to estimate the quantality parameter q , ratio of the zero point fluctuations (ZPF) of confinement and the potential energy.

q

constituents	M/M_n	$a(\text{cm})$	$v_0(\text{eV})$	q	phase($T = 0$)	T
^3He	3	2.9(-8)	8.6(-4)	0.19	liquid ^{a)}	3, 32
^4He	4	2.9(-8)	8.6(-4)	0.14	liquid ^{a)}	5, 18
H_2	2	3.3(-8)	32(-4)	0.06	solid ^{a)}	3, 32
^{20}Ne	20	3.1(-8)	31(-4)	0.007	solid	3, 32
nucleons	1	9(-14)	100(+6)	0.5 ^{c)}	liquid ^{a), b)}	3, 32

Table 1.A.1: a) Delocalized (condensed), b) Non-Newtonian solid, that is, systems which react elastically to sudden sollicitations and plastically under prolonged strain. c) Paradigm of quantal strongly fluctuating, many-body finite systems. While delocalization or less does not seem to depend much on whether one is dealing with fermions or bosons (Mott Les Houches 1998), the detailed properties of the corresponding single-particle motion are strongly dependent on the statistics obeyed by the associated particle (cf. App. 1.C).

Nucleus,

(29)

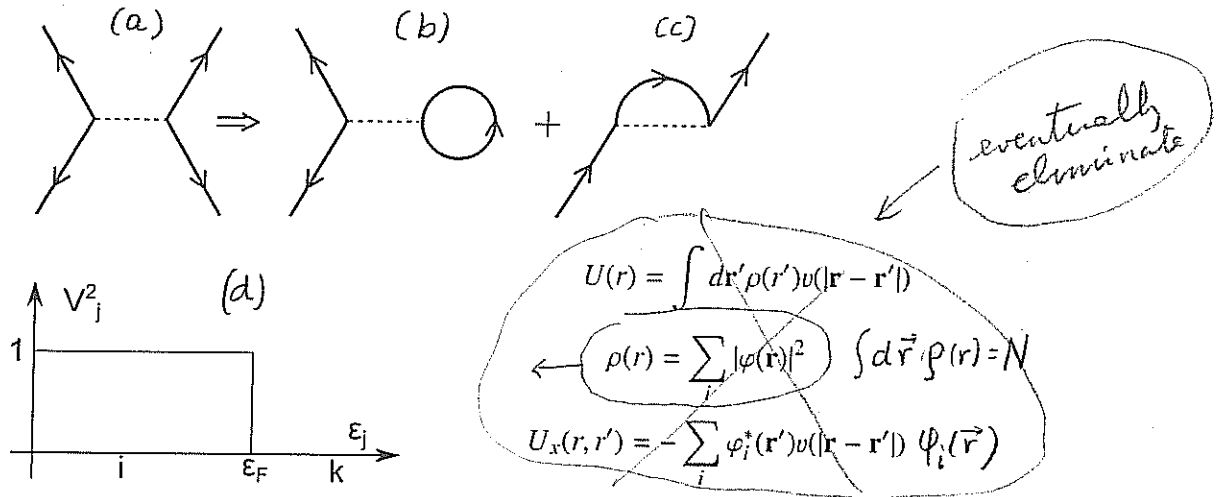
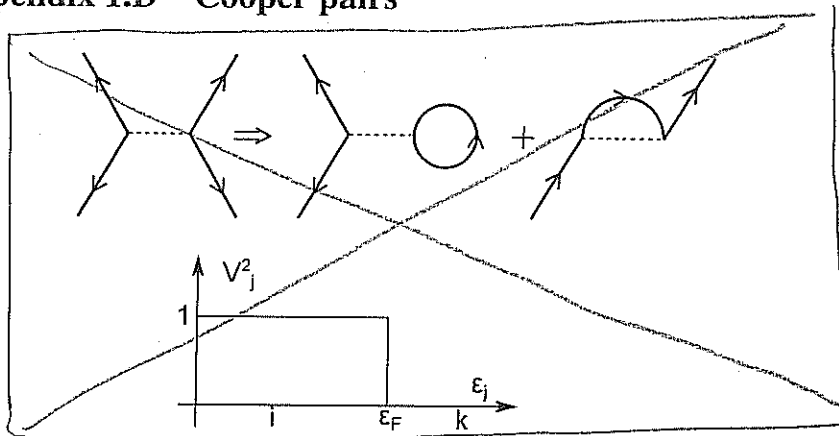


Figure 1.A.2: Schematic representation of (a) nucleon-nucleon scattering through the bare NN -interaction, (b) the associated contribution to the Hartree potential $U(r)$ and, (c) to the Fock (exchange) potential, $\rho(r)$ being the nucleon density. The Hartree-Fock solution leads to a sharp discontinuity at the Fermi energy ϵ_F . That is, single-particle levels with energy $\epsilon_i \leq \epsilon_F$ are fully occupied. Those with $\epsilon_k \geq \epsilon_F$ empty.

✓✓ Appendix 1.B Lindemann

✓ Appendix 1.C Answer to Ricci

✓✓ Appendix 1.D Cooper pairs

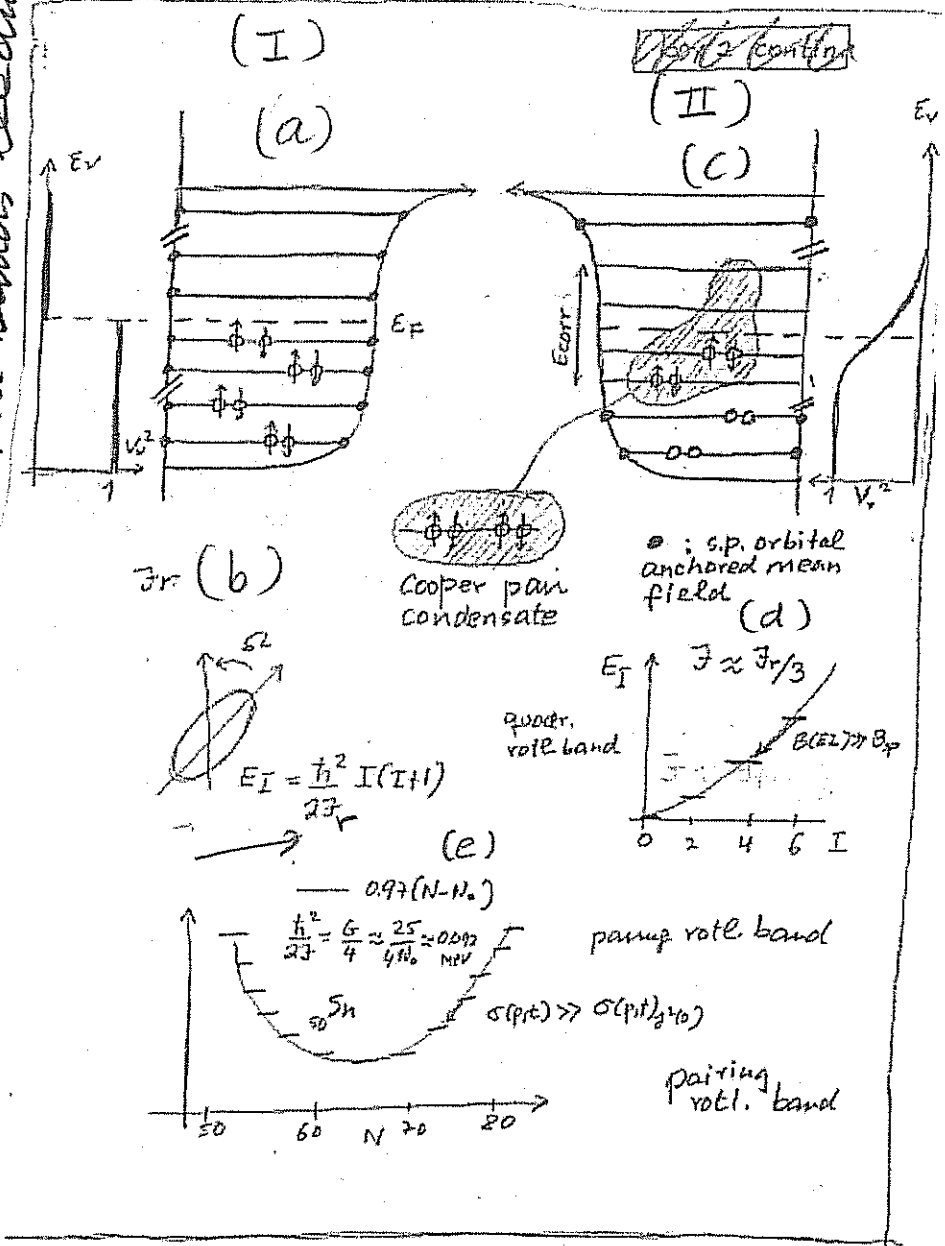


$$H = \sum_{j_1 j_2} \langle j_1 | T | j_2 \rangle a_{j_1}^\dagger a_{j_2} + \frac{1}{4} \sum_{\substack{j_1 j_2 \\ j_3 j_4}} \langle j_1 j_2 | v | j_3 j_4 \rangle a_{j_2}^\dagger a_{j_1}^\dagger a_{j_3} a_{j_4}$$

Let us assume ^{that the motion of} nucleons is described by the Hamiltonian

(30)

representation of independent nucleon Cooper pair motion in which few (of the order of $5-8$) pairs lead to a sigmoidal occupation transition, ~~the~~ having uncoupled themselves from the mean field being now ~~the~~ (quasi) bosons they do not essentially contribute to (a) the moment of inertia of ~~the~~ ~~rotational~~ bands leading to $I \approx I_r/3$.



(a) Pairing rotational band in ground space, in which the mean field has been provided by the ground state of the superfluid Sn-150 isotope.

Figure 1.A.3:

(I) Schematic representation of normal independent particle motion of nucleons in (a) orbits and (b) a deformed rotating nucleus to a rigid moment of inertia I_r . (II) Schematic representation of pairing rotational band in ground space, in which the mean field has been provided by the ground state of the superfluid Sn-150 isotope.