Chapter 5

Two–particle transfer

Cooper pairs are the building blocks of pairing correlations in many-body fermionic systems. In particular in atomic nuclei. As a consequence, nuclear superfluidity can be specifically probed through Cooper pair tunneling. In the simultaneous transfer of two nucleons, one nucleon goes over from target to projectile, or vicev-ersa, under the influence of the nuclear interaction responsible of the existence of a mean field potential, while the other follows suit by profiting of: 1) pairing cor-relations (simultaneous transfer); 2) the fact that the single-particle wavefunctions describing the motion of Cooper pair partners in both target and projectile are so-lutions of different single-particle potentials (non–orthogonality term). In the limit of independent particle motion, in which all of the nucleon-nucleon interaction is used up in generating a mean field, both contributions to the transfer process (si-multaneous and non-orthogonality) cancel out exactly (~~cf.~~ App. 5.C)

(mean field in the post- post- representation)

In keeping with the fact that nuclear Cooper pairs are weakly bound ($E_{corr} \ll \epsilon_F$), this cancellation is, in actual nuclei, quite strong. Consequently, successive transfer, a process in which the nuclear interaction acts twice is, as a rule, the main mechanism at the basis of Cooper pair transfer. Because of the same reason (weak ~~binding), the~~ correlation length of Cooper pairs is larger than nuclear dimensions ($\xi = \hbar v_F/(2E_{corr}) \gg R$), a fact which allows the two members of a Cooper pair to move between target and projectile, essentially as a whole, also in the case of successive transfer. In other words, because of its (intrinsic, virtual extension) Cooper pair transfer display equivalent pairing correlations both in simultaneous ~~as~~ than in successive transfer. [1]

[1] In order for a nucleon to display independent particle motion, all other nucleons must act co-herently so as to leave the way free making feel their pullings and pushings only when the nucleon in question tries to leave the self–bound system,thus acting as a reflecting surface which inverts the momentum of the particle. It is then natural to consider the nuclear mean field the most striking and fundamental collective feature in all nuclear phenomena (Mottelson (1962)). A close second is provided by the BCS mean field, resulting from the condensation of a number of strongly overlap-ping Cooper pairs ($\approx \langle BCS|\sum_{\nu>0} a^\dagger_\nu a^\dagger_{\bar\nu}|BCS\rangle = \alpha_0 \neq 0$) and leading to independent pair motion. It is a rather unfortunate perversity of popular terminology that regards these collective fields (HF and HFB) as well as successive transfer, as in some sense an antithesis to the nuclear collective modes and to simultaneous transfer respectively. Within this context it is of notice that the differential cross

The present Chapter is structured in the following way. In section 5.1 we present a summary of two–nucleon transfer reaction theory. It provides, together with Section 3.1 the elements needed to calculate the absolute two–nucleon transfer differential cross sections in second order DWBA, and thus to compare theory with experiment. Within this context one can, after reading this section, move directly to Chapter 6 containing examples of applications of this formalism. For the practitioner in search of details and clarification we present in section 5.2 a derivation of the equations presented in section 5.1. These equations are implemented and made operative in the software COOPER used in the applications (cf. App. 6.D).

A number of Appendices are provided. Appendix 5.A briefly reminds the quantal basis for the dressing of elementary modes of excitation and of pairing interaction. In App. 5.B the derivation of first order DWBA simultaneous transfer is worked out within a formalism tailored to focus the attention on the nuclear structure correlations aspects of the process leading to effective two–nucleon transfer form factors. In App. 5.C the variety of contributions to two–nucleon transfer amplitudes (successive, simultaneous and non–orthogonality) are discussed in detail within the framework of the semi–classical approximation which provides a rather intuitive vision of the different processes. Appendices 5.D–5.G contain relations used in Sect. 5.2 as well as in the derivation of two–nucleon transfer spectroscopic amplitudes. Finally Appendix 5.H provides a glimpse of original material due to Ben Bayman (Bayman and Kallio (1967), Bayman (1970), Bayman (1971), Bayman and Chen (1982)) which was instrumental to render quantitative, ~~studies~~ of two–nucleon transfer, studies which can now be carried out in terms of absolute cross sections and not relative ones as done previously.

5.1 Summary of second order DWBA

Let us illustrate the theory of second order DWBA two–nucleon transfer reactions with the $A + t \to B(\equiv A + 2) + p$ reaction, in which $A + 2$ and A are even nuclei in their 0^+ ground state. The extension of the expressions to the transfer of pairs coupled to arbitrary angular momentum is discussed in subsection 5.2.10.

The wavefunction of the nucleus $A + 2$ can be written as

$$\Psi_{A+2}(\xi_A, \mathbf{r}_{A1}, \sigma_1, \mathbf{r}_{A2}, \sigma_2) = \psi_A(\xi_A) \sum_{l_i, j_i} [\phi^{A+2}_{l_i,j_i}(\mathbf{r}_{A1}, \sigma_1, \mathbf{r}_{A2}, \sigma_2)]^0_0, \tag{5.1.1}$$

where

$$[\phi^{A+2}_{l_i,j_i}(\mathbf{r}_{A1}, \sigma_1, \mathbf{r}_{A2}, \sigma_2)]^0_0 = \sum_{nm} a_{nm} \left[\varphi^{A+2}_{n,l_i,j_i}(\mathbf{r}_{A1}, \sigma_1)\varphi^{A+2}_{m,l_i,j_i}(\mathbf{r}_{A2}, \sigma_2)\right]^0_0, \tag{5.1.2}$$

section associated with the two–nucleon transfer transitions between the ground state of superfluid nuclei is proportional to α_0^2 and not to Δ^2. In fact, Cooper pairs partners remain correlated even over regions in which $G = 0$.

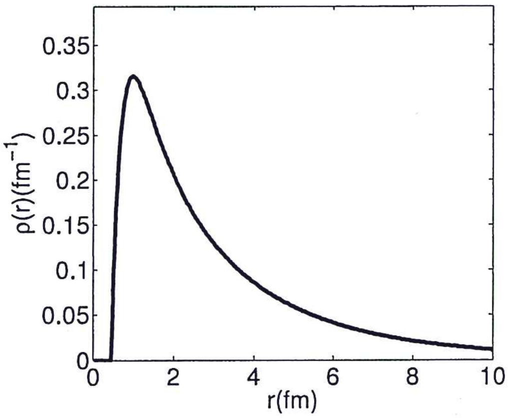


Figure 5.1.1: Radial function $\rho(r)$ (hard core 0.45 fm) entering the tritium wave-function (cf. Tang and Herndon (1965)).

while the wavefunctions $\varphi^{A+2}_{n,l_i,j_i}(\mathbf{r})$ are eigenfunctions of a Saxon–Woods potential

$$U(r) = -\frac{V_0}{1+\exp\left[\frac{r-R_0}{a}\right]}, \qquad R_0 \doteq r_0 A^{1/3}, \tag{5.1.3}$$

of depth V_0 adjusted to reproduce the experimental single–particles energies, together with a standard spin–orbit potential. The radial dependence of the wavefunction of the two neutrons in the triton is written as $\phi_t(\mathbf{r}_{p1}, \mathbf{r}_{p2}) = \rho(r_{p1})\rho(r_{p2})\rho(r_{12})$, where r_{p1}, r_{p2}, r_{12} are the distances between neutron 1 and the proton, neutron 2 and the proton and between neutrons 1 and 2 respectively, while $\rho(r)$ is the hard core (r_{core} = 0.45 fm) potential wavefunction depicted in Fig 5.1.1.

The two–nucleon transfer differential cross section is written as

$$\frac{d\sigma}{d\Omega} = \frac{\mu_i\mu_f}{(4\pi\hbar^2)^2}\frac{k_f}{k_i}\left|T^{(1)}(\theta) + T^{(2)}_{succ}(\theta) - T^{(2)}_{NO}(\theta)\right|^2, \tag{5.1.4}$$

*) where, (see ~~e.g.~~ Bayman and Chen (1982) and App. 5.H)

$$T^{(1)}(\theta) = 2\sum_{l_i,j_i}\sum_{\sigma_1\sigma_2}\int d\mathbf{r}_{tA}d\mathbf{r}_{p1}d\mathbf{r}_{A2}[\phi^{A+2}_{l_i,j_i}(\mathbf{r}_{A1},\sigma_1,\mathbf{r}_{A2},\sigma_2)]^{0*}_0\chi^{(-)*}_{pB}(\mathbf{r}_{pB})$$
$$\times\, v(\mathbf{r}_{p1})\phi_t(\mathbf{r}_{p1},\mathbf{r}_{p2})\chi^{(+)}_{tA}(\mathbf{r}_{tA}), \tag{5.1.5a}$$

$$T^{(2)}_{succ}(\theta) = 2\sum_{l_i,j_i}\sum_{l_f,j_f,m_f}\sum_{\substack{\sigma_1\sigma_2\\ \sigma'_1\sigma'_2}}\int d\mathbf{r}_{dF}d\mathbf{r}_{p1}d\mathbf{r}_{A2}[\phi^{A+2}_{l_i,j_i}(\mathbf{r}_{A1},\sigma_1,\mathbf{r}_{A2},\sigma_2)]^{0*}_0\chi^{(-)*}_{pB}(\mathbf{r}_{pB})v(\mathbf{r}_{p1})$$
$$\times\,\phi_d(\mathbf{r}_{p1})\varphi^{A+1}_{l_f,j_f,m_f}(\mathbf{r}_{A2})\int d\mathbf{r}'_{dF}d\mathbf{r}'_{p1}d\mathbf{r}'_{A2}G(\mathbf{r}_{dF},\mathbf{r}'_{dF})$$
$$\times\,\phi_d(\mathbf{r}'_{p1})^*\varphi^{A+1*}_{l_f,j_f,m_f}(\mathbf{r}'_{A2})\frac{2\mu_{dF}}{\hbar^2}v(\mathbf{r}'_{p2})\phi_d(\mathbf{r}'_{p1})\phi_d(\mathbf{r}'_{p2})\chi^{(+)}_{tA}(\mathbf{r}'_{tA}), \tag{5.1.5b}$$

*)

representative energy is calculated when both intermediate nuclei are in their corresponding ground states. It is of notice that the validity of this approximation can break down in some particular cases. If, for example, some relevant intermediate state become off shell, its contribution is significantly quenched. An interesting situation can arise when this happens to all possible intermediate states, so they can only be virtually populated.

5.2 Detailed derivation of second order DWBA

5.2.1 Simultaneous transfer: distorted waves

For a (t, p) reaction, the triton is represented by an incoming distorted wave. We make the assumption that the two neutrons are in an $S = L = 0$ state, and that the relative motion of the proton with respect to the dineutron is also $l = 0$. Consequently, the total spin of the triton is entirely due to the spin of the proton. We will explicitly treat it, as we will consider a spin–orbit term in the optical potential acting between the triton and the target. *) In what follows we will use the notation of Bayman (1971) (cf. also App. 5.H).

Following (5.E.1), we can write the triton distorted wave as

$$\psi^{(+)}_{m_t}(\mathbf{R}, \mathbf{k}_i, \sigma_p) = \sum_{l_t} \exp\left(i\sigma^{t}_{l_t}\right) g_{l_t j_t} Y^{l_t}_0(\hat{\mathbf{R}}) \frac{\sqrt{4\pi(2l_t+1)}}{k_i R} \chi_{m_t}(\sigma_p), \tag{5.2.1}$$

where use was made of $Y^{l_t}_0(\hat{\mathbf{k}}_i) = i^{l_t}\sqrt{\frac{2l_t+1}{4\pi}}\delta_{m_t,0}$, in keeping with the fact that $\mathbf{k}_i$ is oriented along the z–axis. Note the phase difference with eq. (7) of Bayman (1971), due to the use of time–reversal rather than Condon–Shortley phase convention. Making use of the relation

$$Y^{l_t}_0(\hat{\mathbf{R}})\chi_{m_t}(\sigma_p) = \sum_{j_t} \langle l_t\ 0\ 1/2\ m_t | j_t\ m_t\rangle \left[Y^{l_t}(\hat{\mathbf{R}})\chi(\sigma_p)\right]^{j_t}_{m_t}, \tag{5.2.2}$$

we have

$$\begin{aligned}\psi^{(+)}_{m_t}(\mathbf{R}, \mathbf{k}_i, \sigma_p) = \sum_{l_t, j_t} &\exp\left(i\sigma^{t}_{l_t}\right) \frac{\sqrt{4\pi(2l_t+1)}}{k_i R} g_{l_t j_t}(R) \\ &\times \langle l_t\ 0\ 1/2\ m_t | j_t\ m_t\rangle \left[Y^{l_t}(\hat{\mathbf{R}})\chi(\sigma_p)\right]^{j_t}_{m_t}.\end{aligned} \tag{5.2.3}$$

We now turn our attention to the outgoing proton distorted wave, which, following (5.E.3) can be written as

$$\psi^{(-)}_{m_p}(\zeta, \mathbf{k}_f, \sigma_p) = \sum_{l_p j_p} \frac{4\pi}{k_f \zeta} i^{l_p} \exp\left(-i\sigma^{p}_{l_p}\right) f^*_{l_p j_p}(\zeta) \sum_m Y^{l_p}_m(\hat{\zeta}) Y^{l_p *}_m(\hat{\mathbf{k}}_f)\chi_{m_p}(\sigma_p). \tag{5.2.4}$$

*)

and

$$|Y^l_{m_t-m_p}(\hat{\mathbf{k}}_f)\delta_{m_p,-m_t}| = |Y^l_{-1}(\hat{\mathbf{k}}_f)| = |Y^l_1(\hat{\mathbf{k}}_f)| = \left| i^l \sqrt{\frac{2l+1}{4\pi}\frac{1}{l(l+1)}} P^1_l(\cos\theta) \right|, \tag{5.2.32}$$

when $m_p = 1/2, m_t = -1/2$ Taking the squared modulus of (5.2.27), the sum over m_t and m_p yields a factor 2 multiplying each one of the 2 different terms of the sum ($m_t = m_p$ and $m_t = -m_p$). This is equivalent to multiply each amplitude by $\sqrt{2}$, so the final constant that multiply the amplitudes is

$$\frac{8\pi^{3/2}}{k_i k_f}. \tag{5.2.33}$$

Now, for the triton wavefunction we use

$$\theta^0_0(\mathbf{r}, \mathbf{s}) = \rho(r_{1p})\rho(r_{2p})\rho(r_{12}), \tag{5.2.34}$$

$\rho(r)$ being a Tang–Herndon (1965) wave function also used by Bayman (1971). We obtain

$$\frac{d\sigma}{d\Omega}(\hat{\mathbf{k}}_f) = \frac{1}{2E_i^{3/2}E_f^{1/2}}\sqrt{\frac{\mu_f}{\mu_i}}\left(|I^{(0)}_{\lambda k}(\theta)|^2 + |I^{(1)}_{\lambda k}(\theta)|^2\right), \tag{5.2.35}$$

where

$$\begin{aligned} I^{(0)}_{\lambda k}(\theta) = \sum_l & P^0_l(\cos\theta)\sqrt{2k+1}\exp[i(\sigma^p_l + \sigma^t_l)] \\ & \times \int dR\, d\beta\, d\gamma\, dr_{12}\, dr_{1p}\, dr_{2p}\, R\sin\beta\, \rho(r_{1p})\rho(r_{2p})\rho(r_{12}) \\ & \times u_{\lambda k}(r_1)u_{\lambda k}(r_2)V(r_{1p})P_\lambda(\cos\theta_{12})P_l(\cos\theta_\zeta)r_{12}r_{1p}r_{2p} \\ & \times \left(f_{ll+1/2}(\zeta)g_{ll+1/2}(R)\,(l+1) + f_{ll-1/2}(\zeta)g_{ll-1/2}(R)\,l\right)/\zeta, \end{aligned} \tag{5.2.36}$$

and

$$\begin{aligned} I^{(1)}_{\lambda k}(\theta) = \sum_l & P^1_l(\cos\theta)\sqrt{2k+1}\exp[i(\sigma^p_l + \sigma^t_l)] \\ & \times \int dR\, d\beta\, d\gamma\, dr_{12}\, dr_{1p}\, dr_{2p}\, R\sin\beta\, \rho(r_{1p})\rho(r_{2p})\rho(r_{12}) \\ & \times u_{\lambda k}(r_1)u_{\lambda k}(r_2)V(r_{1p})P_\lambda(\cos\theta_{12})P_l(\cos\theta_\zeta)r_{12}r_{1p}r_{2p} \\ & \times \left(f_{ll+1/2}(\zeta)g_{ll+1/2}(R) - f_{ll-1/2}(\zeta)g_{ll-1/2}(R)\right)/\zeta. \end{aligned} \tag{5.2.37}$$

Note that the absence of the $(-1)^\lambda$ factor with respect to what is found in the work of Bayman *), ~~(1971),~~ is due to the use of time–reversed phases instead of Condon–Shortley phasing. This is compensated in the total result by a similar difference in the expression of the spectroscopic amplitudes. This ensures that, in either case, the contribution of all the single particle transitions tend to have the same phase for superfluid nuclei, adding coherently to enhance the transfer cross section.

*) Bayman (1970)

The rest of the formulae are identical to the (t, p) ones. We list them for convenience,

$$\mathbf{r}_{A1} = \begin{bmatrix} d_1 \sin(\beta) \\ 0 \\ R + d_1 \cos(\beta) \end{bmatrix}, \tag{5.2.75}$$

$$\mathbf{r}_{A2} = \begin{bmatrix} d_1 \sin(\beta) + r_{12} \cos(\beta)\cos(\gamma)\sin(\alpha) - r_{12}\sin(\beta)\cos(\alpha) \\ r_{12}\sin(\gamma)\sin(\alpha) \\ R + d_1\cos(\beta) - r_{12}\sin(\beta)\cos(\gamma)\sin(\alpha) - r_{12}\cos(\alpha)\cos(\beta) \end{bmatrix}. \tag{5.2.76}$$

We we also find

$$\mathbf{r}_{b1} = \frac{1}{m_b}(\mathbf{r}_{A2} + (m_b + 1)\mathbf{r}_{A1} - m_a\mathbf{R}), \tag{5.2.77}$$

and

$$\mathbf{r}_{b2} = \frac{1}{m_b}(\mathbf{r}_{A1} + (m_b + 1)\mathbf{r}_{A2} - m_a\mathbf{R}). \tag{5.2.78}$$

One can readily obtain

$$\cos\theta_{12} = \frac{r_{A1}^2 + r_{A2}^2 - r_{12}^2}{2r_{A1}r_{A2}}, \tag{5.2.79}$$

and

$$\cos\theta_i = \frac{r_{b1}^2 + r_{b2}^2 - r_{12}^2}{2r_{b1}r_{b2}}. \tag{5.2.80}$$

5.2.4 Matrix element for the transition amplitude (alternative derivation)

that is

In what follows we work out ~~an~~ alternative ~~derivation of~~ T_{2N}^{1step}, more closely related to heavy ion reactions *), ~~Following~~ *) Bayman and Chen (1982) ~~it can be written as~~

$$\begin{aligned}
T^{(1)}(\theta) =& 2\frac{(4\pi)^{3/2}}{k_{Aa}k_{Bb}} \sum_{l_p j_p m l_t j_p} i^{-l_p} \exp[i(\sigma_{l_p}^p + \sigma_{l_t}^t)]\sqrt{2l_t + 1} \\
&\times \langle l_p\; m - m_p\; 1/2\; m_p | j_p\; m\rangle\langle l_t\; 0\; 1/2\; m_t | j_t\; m_t\rangle Y^{l_p}_{m-m_p}(\hat{\mathbf{k}}_{Bb}) \\
&\times \sum_{\sigma_1\sigma_2\sigma_p} \int d\mathbf{r}_{Cc} d\mathbf{r}_{b1} d\mathbf{r}_{A2} \left[\psi^{j_f}(\mathbf{r}_{A1}, \sigma_1)\psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)\right]_0^{0*} \\
&\times v(r_{b1})\left[\psi^{j_i}(\mathbf{r}_{b1}, \sigma_1)\psi^{j_i}(\mathbf{r}_{b2}, \sigma_2)\right]_0^0 \frac{g_{l_t j_t}(r_{Aa}) f_{l_p j_p}(r_{Bb})}{r_{Aa} r_{Bb}} \\
&\times \left[Y^{l_t}(\hat{\mathbf{r}}_{Aa})\chi(\sigma_p)\right]_{m_t}^{j_t} \left[Y^{l_p}(\hat{\mathbf{r}}_{Bb})\chi(\sigma_p)\right]_m^{j_p *}.
\end{aligned} \tag{5.2.81}$$

*)

and divide by 2. If a spin–orbit term is present in the optical potential, the sum yields the combination of terms shown in Section (5.2.2),

$$\frac{d\sigma}{d\Omega}(\hat{\mathbf{k}}_{bB}) = \frac{k_{bB}}{k_{aA}} \frac{\mu_{aA}\mu_{bB}}{(2\pi\hbar^2)^2} \frac{1}{2(2\Lambda+1)} \sum_{\mu} |A_\mu|^2 + |B_\mu|^2. \tag{5.2.184}$$

Appendix 5.A ZPF and Pauli principle at the basis of medium polarization effects: self–energy, vertex corrections and induced interaction

In keeping with a central objective of the formulation of quantum mechanics, namely that the basic concepts on which it is based relate directly to experiment[2], ~~(Heisenberg (1925))~~, elementary modes of nuclear excitation (single–particle, collective vibrations and rotations), are solidly anchored on observation (inelastic and Coulomb excitation, one– and two–particle transfer reactions). Of all quantal phenomena, zero point fluctuations (ZPF), closely connected with virtual states, are likely to be most representative of the essential difference existing between quantum and classical mechanics. In fact, ZPF are intimately connected with the complementary principle a) ~~(Bohr (1928~~)), and thus with indeterminacy b) ~~(Heisenberg (1927))~~ and non–commutative c) ~~(Born and Jordan (1925), Born et al. (1926))~~ relations, and with the probabilistic interpretation d) ~~(Born, 1926~~) of the (modulus squared) of the wavefunctions, solution of Schrödinger's or Dirac's equations e) (~~Schrödinger, E. (1926), Dirac (1930~~)). Pauli principle f) ~~(Pauli, 1925)~~ brings about essential modifications of the virtual fluctuations of the many–body system, modifications which are instrumental in the dressing and interweaving of the elementary modes of excitation g) (Within the present context, see also Schrieffer (1964)).

In Fig. 5.A.1, NFT diagrams are given which correspond to the lowest order medium polarization effects renormalizing the properties of a particle–hole collective mode (wavy line), correlated particle–hole excitation which in the shell model basis corresponds to a linear combination of particle–hole excitations ((up-going)–(down–going) arrowed lines) calculated within the random phase approximation (RPA), and leading to the particle–vibration coupling vertex (formfactor and strength, i.e. transition density (solid dot), see inset (I), bottom). The action of an external field on the zero point fluctuations (ZPF) of the vacuum (inset (II)), forces a virtual process to become real, leading to a collective vibration by annihilating a (virtual, spontaneous) particle–hole excitation (backwards RPA amplitude)

g)

f) Pauli (1925)

[2] The abstract of this reference reads: "In this paper it will be attempted to secure foundations for a quantum theoretical mechanics which is exclusively based on relations between quantities which in principle are observables". Within the present context, namely that of the probing the nuclear structure (e.g. pairing correlations) with direct nuclear reactions, in particular Cooper pair transfer, one can hardly think of a better *incipit* for the introduction of elementary modes of excitation, modes which carry within them most of the correlations thus requiring for their theoretical treatment an effective field theory, like e.g. NFT to properly take into account the essential overcompleteness of the basis (non–orthogonality) as well as of Pauli violating processes.

e) Schrödinger, E. (1926), Dirac (1930)

(Heisenberg (1925)

a) Bohr (1928)
b) Heisenberg (1927)
c) Born and Jordan (1925), Born et al (1926)
d) Born (1926)

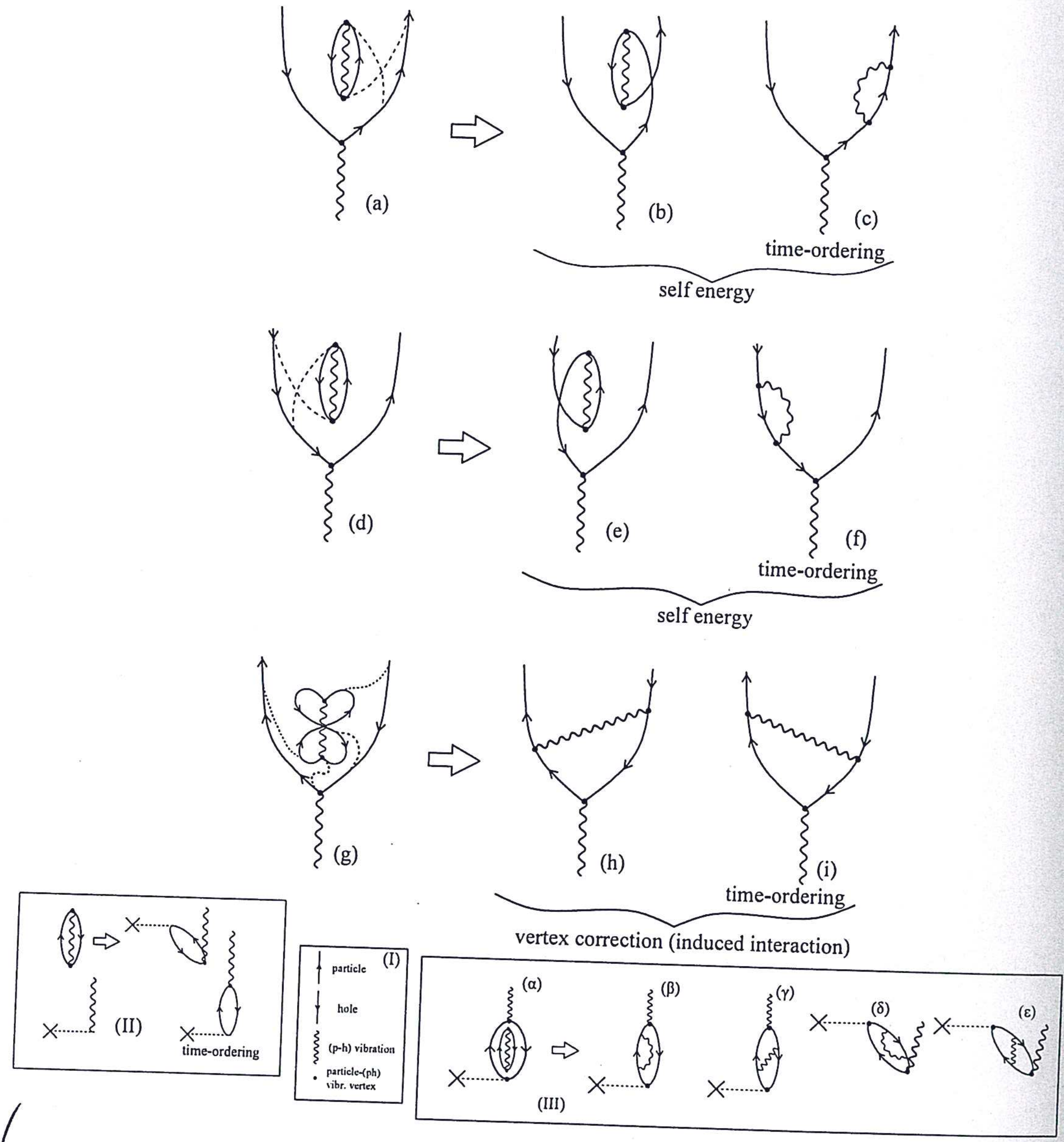


Figure 5.A.1: Nuclear field theory (NFT) diagrams describing renormalization processes associated with ZPF. For details cf. caption to Fig. 5.A.2.

of Fig. 5.A.1.

or, in the time ordered process, by creating a particle–hole excitation which eventually, through the particle–vibration coupling vertex, correlate into the collective (coherent) state (forwardsgoing amplitudes). Now, oyster–like diagrams associated with the vacuum ZPF can occur at any time (see inset (III)). Because the texture of the vacuum is permeated by symmetry rules (while one can violate energy conser-

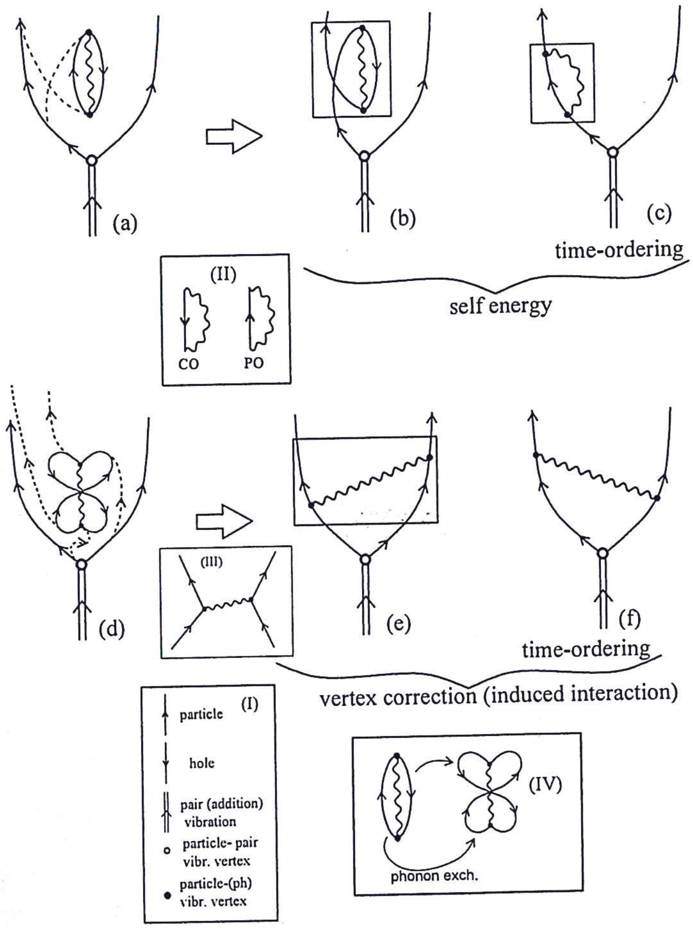


Figure 5.A.2: Pauli effects associated (p–h) ZPF dressing a pairing vibrational (pair addition) mode (see inset I) in terms of self–energy (graphs (a)–(c); correlation (CO) and polarization (PO) diagrams, inset II) and vertex correction (graphs (d)–(f); induced particle–particle (pairing) interaction,) processes (inset (III)), associated with phonon exchange (inset (IV)).

collective T diagrams modes

vation in a virtual state one cannot violate e.g. angular momentum conservation ~~or~~ the Pauli principle). The process shown in the inset III (α) leads, through Pauli principle correcting ~~processes~~ (exchange of fermionic arrowed lines) to self–energy (inset III (β), (δ)) and vertex corrections ~~(induced~~ p–h interaction; inset III (γ), (ε)). processes (phonon exchange, cf. inset (IV) of Fig. 5.A.2). ~~The first ones are detailed in graphs (a)–(f), while the second ones in graphs (g)–(i).~~ In keeping with the fact that the vibrational ~~states~~ can be viewed as ~~a~~ coherent states exhausting a consistent fraction of the EWSR (e.g. a Giant Resonance) for which the associated uncertainty relations in momentum and coordinate fulfills the absolute minimum con-

Similar processes are found in the interplay between ZPF and pair addition modes as shown in Fig 5.A.2. Note the parallel between diagrams 5.A.1 (g)–(i) and 5.A.2 (d)–(f)

(the indeterminacy relations)

sistent with ~~quantum mechanics~~ ($\Delta\alpha_{\lambda\mu}\Delta\pi_{\lambda\mu} = \hbar/2, \alpha_{\lambda\mu} = (\hbar\omega_\mu/2C_\lambda^{1/2})(\Gamma^\dagger_{\lambda\mu} + \Gamma_{\lambda\mu})$ being the (harmonic) collective coordinate, $\pi_{\lambda\mu}$ being the conjugate momentum, *)) ~~cf. e.g. Glauber (1969)),~~ there is a strong cancellation between the contribution of self–energy and vertex correction diagrams **) ~~(Bortignon and Broglia, 1981~~), implying small anharmonicities and long lifetimes ($\Gamma/E \ll 1$, where Γ is the width and E the centroid of the mode $|\lambda\mu\rangle = \Gamma^\dagger_{\lambda\mu}|0\rangle$, $(\hbar\omega_\lambda/2C_\lambda)^{1/2}$ being the ZPF amplitude ***). ~~(cf. e.g. Brink, D. and Broglia (2005)))~~.

Appendix 5.B Coherence and effective formfactors

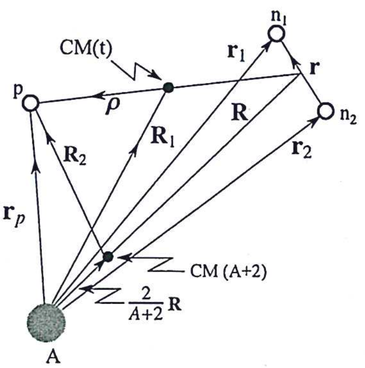
In what follows we shall work out a simplified derivation of the simultaneous two–nucleon transfer amplitude, within the framework of first order DWBA specially suited to discuss correlation aspects of pair transfer in general, and of the associated effective formfactors in particular. ****)

We will concentrate on (t, p) reaction, namely reactions of the type $A(\alpha,\beta)B$ where $\alpha = \beta + 2$ and $B = A + 2$.

The intrinsic wave functions are in this case

$$\begin{aligned}\psi_\alpha &= \psi^{J_i}_{M_i}(\xi_A)\sum_{ss'_f}\left[\chi^s(\sigma_\alpha)\chi^{s'_f}(\sigma_\beta)\right]^{s_i}_{M_{s_i}}\phi_t^{L=0}(\sum_{i<j}|\vec{r}_i-\vec{r}_j|)\\ &= \psi^{J_i}_{M_i}(\xi_A)\sum_{M_sM'_{s_f}}(sM'_s s'_f M'_{s_f}|s_iM_{s_i})\chi^s_{M'_s}(\sigma_\alpha)\chi^{s'_f}_{M'_{s_f}}(\sigma_\beta)\\ &\quad\times\phi_t^{L=0}(\sum_{i<j}|\vec{r}_i-\vec{r}_j|)\end{aligned} \tag{5.B.1}$$

while



$\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$ (relative distance between the neutrons)

$\mathbf{R} = \frac{\mathbf{r}_1+\mathbf{r}_2}{2}$ (coord. of the CM of the dineutron)

$\rho = \mathbf{r}_p - \frac{\mathbf{r}_1+\mathbf{r}_2}{2} = \mathbf{r}_p - \mathbf{R}$ (distance between the CM of the dineutron and the proton)

$\mathbf{R}_2 = \mathbf{r}_p - \frac{\mathbf{r}_1+\mathbf{r}_2}{A+2}$ (distance of the proton from the CM of the system A+2)

$\mathbf{R}1 = \frac{\mathbf{r}_p+\mathbf{r}_1+\mathbf{r}_2}{3}$ (coord. of the CM of the triton)

√ Figure 5.B.1: Coordinate system used in the calculation of the two–nucleon transfer amplitude.

****) Glendenning (1965)

*) see e.g. Glauber (1969)

**) Bortignon and Broglia (1981)

***) Brink, D. and Broglia (2005))

of integrating over $\xi_A, \vec{r}_1, \vec{r}_2$ and $\vec{r}_p$ we would integrate over $\xi_A, \vec{r}', \vec{r}'$ and $\vec{r}_p$. The Jacobian of the transformation is equal to 1, i.e. $\partial(\vec{r}_1, \vec{r}_2)/\partial(\vec{r}', \vec{r}') = 1$.

To carry out the integral (5.B.8) we transform the wave function (5.B.4) into center of mass and relative coordinates. If we assume that both $\phi_{j_1}(\vec{r}_1)$ and $\phi_{j_2}(\vec{r}_2)$ are harmonic oscillator wave functions (used as a basis to expand the Saxon–Woods single–particle wavefunctions), this transformation can be carried with the aid of tha Moshinsky brackets. If $|n_1 l_1, n_2 l_2; \lambda\mu\rangle$ is a complete system of wave functions in the harmonic oscillator basis, depending on $\vec{r}_1$ and $\vec{r}_2$ and $|nl, NL; \lambda\mu\rangle$ is the corresponding one depending on $\vec{r}$ and $\vec{R}$, we can write

$$
\begin{aligned}
|n_1 l_1, n_2 l_2; \lambda\mu\rangle &= \sum_{nlNL} |nl, NL; \lambda\mu\rangle\langle nl, NL; \lambda\mu|)|n_1 l_1, n_2 l_2; \lambda\mu\rangle \\
&= \sum_{nlNL} |nl, NL; \lambda\mu\rangle\langle nl, NL; \lambda\mu|n_1 l_1, n_2 l_2; \lambda\rangle
\end{aligned} \tag{5.B.9}
$$

The labels n, l are the principal and angular momentum quantum numbers of the relative motion, ehile N, L are the corresponding ones corresponding to the center of mass motion of the two–neutron system. Because of energy and parity conservation we have

$$
\begin{aligned}
2n_1 + l_1 + 2n_2 + l_2 &= 2n + l + 2N + L \\
(-1)^{l_1+l_2} &= (-1)^{l+L}.
\end{aligned} \tag{5.B.10}
$$

The coefficients $\langle nl, NL, L|n_1 l_1, n_2 l_2, L\rangle$ are tabulated and were first discussed by (Moshinsky, ~~1959~~)*)

With the help of eq. (5.B.9) we can write the wave function $\psi^{J_f}_{M_f}(\xi_{A+2})$ as

$$
\begin{aligned}
\psi^{J_f}_{M_f}(\xi_{A+2}) &= \sum_{\substack{n_1 l_1 j_1 \\ n_2 l_2 j_2 \\ J J_i}} B(n_1 l_1 j_1, n_2 l_2 j_2; J J_i'J_f)\left[\phi^J(j_1 j_2)\phi^{J_i'}(\xi_A)\right]^{J_f}_{M_f} \\
&= \sum_{\substack{n_1 l_1 j_1 \\ n_2 l_2 j_2}} \sum_{J J_i} B(n_1 l_1 j_1, n_2 l_2 j_2; J J_i' J_f) \\
&\times \sum_{M_J M'_{J_i}} \langle J M_J J_i' M_{J_i}|J_f M_{J_f}\rangle \psi^{J_i'}_{M'_{J_i}}(\xi_A) \\
&\times \sum_{LS'} \langle S' L J|j_1 j_2 J\rangle \sum_{M_L M'_S} \langle L M_L S' M'_S|J M_J\rangle \chi^{S'}_{M'_S}(\sigma_\alpha) \\
&\times \sum_{nlN\Lambda} \langle nl, N\Lambda, L|n_1 l_1, n_2 l_2, L\rangle \\
&\times \sum_{m_l M_\Lambda} \langle l m_l \Lambda M_\Lambda|L M_L\rangle \phi_{nlm_l}(\vec{r})\phi_{N\Lambda M_\Lambda}(\vec{R})
\end{aligned} \tag{5.B.11}
$$

*) Moshinsky (1959)

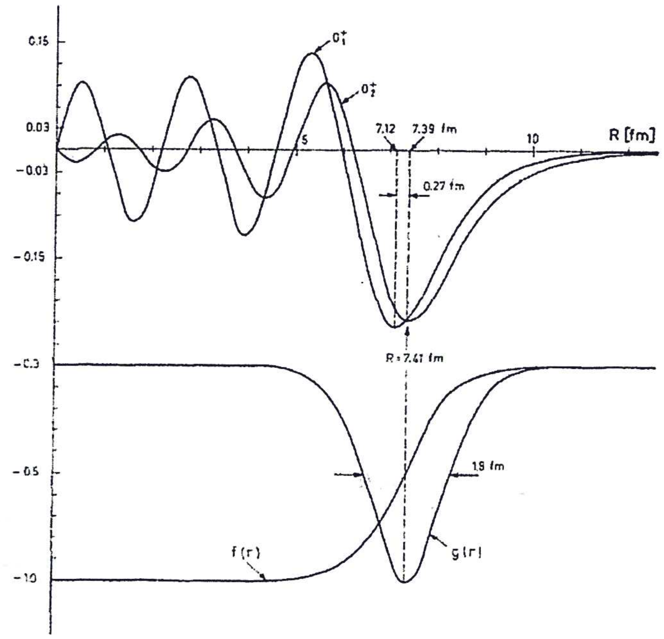


Figure 5.B.2: The upper part of the figure shows the modified formfactor for the ^{206}Pb(t,p)^{208}Pb transition to the ground state (0_1^+) and the pairing vibrational state (0_2^+) at 4.87 MeV. Both curves are matched with appropriate Hankel functions. In the lower part the form factors of the real ($f(r)$) and the imaginary ($g(r)$) part of the optical potential used to calculate the differential cross sections (cf. Fig. 3.4.4), are given in the same scale for the radius. After Broglia and Riedel (1967).

Appendix 5.C Relative importance of successive and simultaneous transfer and non-orthogonality corrections

In what follows we discuss the relative importance of successive and simultaneous two-neutron transfer and of non-orthogonality corrections associated with the reaction

$$\alpha \equiv a(= b + 2) + A \to b + B(= A + 2) \equiv \beta \tag{5.C.1}$$

in the limits of independent particles and of strongly correlated Cooper pairs, making use for simplicity of the semiclassical approximation *) (for details cf. Broglia and Winther (2004), ~~Broglia and refs. therein)~~, in which case the two-particle transfer differential cross section can be written as

*)

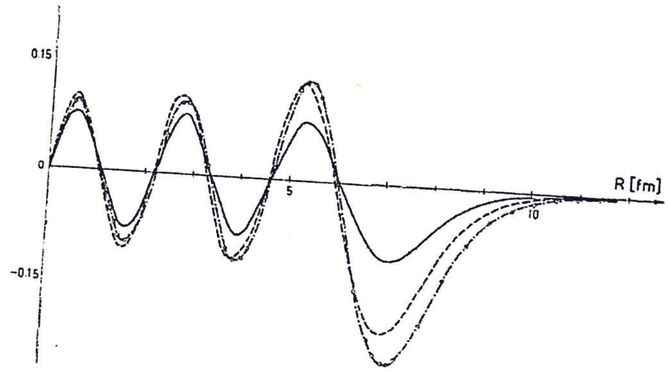


Figure 5.B.3: Modified formfactor for the transition to the ground state (^{206}Pb(t,p)^{208}Pb(gs); see Fig. 3.4.4 (a)) calculated in different spectroscopic models (pure shell–model configuration ——, shell model plus pairing residual interaction – – –, including ground state correlations –o–o–). After Broglia and Riedel (1967).

$$\frac{d\sigma_{\alpha\to\beta}}{d\Omega} = P_{\alpha\to\beta}(t=+\infty)\sqrt{\left(\frac{d\sigma_\alpha}{d\Omega}\right)_{el}}\sqrt{\left(\frac{d\sigma_\beta}{d\Omega}\right)_{el}}, \tag{5.C.2}$$

where P is the absolute value squared of a quantum mechanical transition amplitude. It gives the probability that the system at $t = +\infty$ is found in the final channel. The quantities $(d\sigma/d\Omega)_{el}$ are the classical elastic cross sections in the center of mass system, calculated in terms of the deflection function, namely the functional relating the impact parameter and the scattering angle.

The transfer amplitude can be written as

$$a(t=+\infty) = a^{(1)}(\infty) - a^{(NO)}(\infty) + \tilde{a}^{(2)}(\infty), \tag{5.C.3}$$

where $\tilde{a}^{(2)}(\infty)$ at $t = +\infty$ labels the successive transfer amplitude expressed in the post-prior representation (see below). The simultaneous transfer amplitude is given by (see Fig. 5.C.1 (I))

$$\begin{aligned} a^{(1)}(\infty) &= \frac{1}{i\hbar}\int_{-\infty}^{\infty} dt(\psi^b\psi^B, (V_{bB} - < V_{bB} >)\psi^a\psi^A) \times \exp[\frac{i}{\hbar}(E^{bB} - E^{aA})t] \\ &\approx \frac{2}{i\hbar}\int_{-\infty}^{\infty} dt\left(\phi^{B(A)}(S^B_{(2n)}; \vec{r}_{1A}, \vec{r}_{2A}), U(r_{1b})e^{i(\sigma_1+\sigma_2)}\phi^{a(b)}(S^a_{(2n)}; \vec{r}_{1b}, \vec{r}_{2b})\right) \\ &\times \exp[\frac{i}{\hbar}(E^{bB} - E^{aA})t + \gamma(t)] \end{aligned} \tag{5.C.4}$$

shell–model potential U. This is done by including the effects of the collective field generated by a small displacement of the nucleus, giving rise to a coupling Hamiltonian proportional to the gradient of U. The spectrum of normal modes generated by such coupling contains a zero frequency mode, orthogonal to the additional normal modes which represent 1^- states, displaying a divergent ZPF but a finite inertia equal to AM, result which testifies to translational invariance restoration*) ~~(cf. Bohr, A. and Mottelson (1975) p. 445)~~.

5.C.1 Independent particle limit

In the independent particle limit, the two transferred particles do not interact among themselves but for antisymmetrization. Thus, the separation energies fulfill the relations (see Fig. 5.C.3)

$$S^B(2n) = 2S^B(n) = 2S^F(n), \tag{5.C.8}$$

and

$$S^a(2n) = 2S^a(n) = 2S^f(n). \tag{5.C.9}$$

In this case

$$\phi^{B(A)}(S^B(2n), \vec{r}_{1A}, \vec{r}_{2A}) = \sum_{a_1a_2} \phi^{B(F)}_{a_1}(S^B(n), \vec{r}_{1A})\phi^{F(A)}_{a_2}(S^F(n), \vec{r}_{2a}), \tag{5.C.10}$$

and

$$\phi^{a(b)}(S^a(2n), \vec{r}_{1b}, \vec{r}_{2b}) = \sum_{a'_1a'_2} \phi^{a(f)}_{a'_1}(S^a(n), \vec{r}_{2b})\phi^{f(b)}_{a'_2}(S^f(n), \vec{r}_{1b}), \tag{5.C.11}$$

where $(a_1, a_2) \equiv F$ and $(a'_1, a'_2) \equiv f$ span, as mentioned above, shells in nuclei B and a respectively.

Inserting Eqs. (5.C.8–5.C.11) in Eq. (5.C.4) one can show that

$$a^{(1)}(\infty) = a^{(NO)}(\infty). \tag{5.C.12}$$

It can be further demonstrated that within the present approximation, $Im\ \tilde{a}^{(2)} = 0$, and that

$$\begin{aligned}\tilde{a}^{(2)}(\infty) = \frac{1}{i\hbar}\int_{-\infty}^{\infty} & dt(\psi^b\psi^B, (V_{bB} - < V_{bB} >)e^{i\sigma_1}\psi^f\psi^F > \\ & \times\exp[\frac{i}{\hbar}(E^{bB} - E^{fF})t + \gamma_1(t)] \\ \times\frac{1}{i\hbar}\int_{-\infty}^{\infty} & dt'(\psi^f\psi^F, (V_{fF} - < V_{fF} >)e^{i\sigma_2}\psi^a\psi^A) \\ & \times\exp[\frac{i}{\hbar}(E^{fF} - E^{aA})t' + \gamma_2(t)].\end{aligned} \tag{5.C.13}$$

*) Bohr and Mottelson (1975) p. 445.

If both nuclei are in zero–quasiparticle states, the only term that survives is the last one in the above expression, and (5.G.7) becomes (see also Sect. 2.4.2 and equation 2.1.8).

$$
\begin{aligned}
B_j = B(j^2(0)) &= \frac{1}{\sqrt{2}} \sum_m \langle j\, m\; j\; -m|0\;0\rangle (-1)^{j-m} V_j U_j \\
&= \frac{1}{\sqrt{2}} \sum_m \frac{(-1)^{j-m}}{\sqrt{(2j+1)}} (-1)^{j-m} V_j U_j \\
&= \frac{1}{\sqrt{2}} \sum_m \frac{1}{\sqrt{(2j+1)}} V_j U_j.
\end{aligned}
\tag{5.G.9}
$$

After carrying out the summation one finds,

$$
B_j = B(j^2(0)) = \sqrt{j + 1/2}\; V_j U_j. \tag{5.G.10}
$$

Note that in this final expression V_j refers to the A nucleus, while U_j is related to the $A+2$ nucleus. In practice, it does not make a big difference to calculate both for the same nucleus.

Appendix 5.H Derivation of two–nucleon transfer transition amplitudes including recoil, non–orthogonality and successive transfer.

In the present Appendix we reproduce with the permission of the author the first (manuscript) page (cf. Fig. 5.H.1) of what, arguably, was the first complete derivation* (~~Bayman (1970)(unpublished)~~) of the different contributions needed to calculate absolute two–nucleon transfer cross sections in a systematic way** (~~cf.~~ Bayman (1971) and Bayman and Chen (1982)). Within this context we refer to Broglia R.A. et al. (1973) and Potel, G. et al. (2013) in particular Fig. 10 of this reference.

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*) Bayman (1970) (unpublished)

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