System		Δ_0		N_0		W_{con}		E_c	BE/A	$\frac{W_{con}}{E_c}$	$\frac{W_{con}}{BE}$
		meV	MeV	$\frac{\text{mev}^{-1}}{\text{atom}}$	MeV^{-1}	$\frac{\text{mev}}{\text{atom}}$	MeV	$\frac{\text{mev}}{\text{atom}}$	$\frac{\text{MeV}}{A}$		
Pb	^{120}Sn	1.4	1.5	276	4	3×10^{-4}	4.3	2030	8.5	10^{-7}	10^{-3}

Table 3.A.1: Summary of the quantities entering the calculation of the condensation energy seperconducting lead, and of the single open shell superfluid nucleus ^{120}Sn.

(Pb)

and

$$\Delta = \left(2\omega_D e^{\frac{\Pi}{N(0)}}\right) e^{-\frac{1}{N(0)U_c^{scr}}}. \qquad (3.A.73)$$

Consequently, the renormalization effects of the pairing gap associated with phonon exchange are independent of the approximation used to calculate U_c^{scr} (Thomas–Fermi in the above discussion), provided one has used the same "bare" (screened) Coulomb interaction to calculate ω_q^2. Otherwise, the error introduced through a resonant renormalization process entering the expression of e.g. the pairing gap may be quite large.

3.A.4 Pairing condensation (correlation) energy beyond level density

The condensation energy, namely the energy difference $W_N - W_S$ between the normal $N-$ and superfluid $S-$state is defined as (Eq. (2-35) of ref[63])

$$W_{con} = W_N - W_S = \frac{1}{2}N(0)\Delta_0^2, \qquad (3.A.74)$$

where $N(0)$ is the density of single–electron states of one–spin orientation evaluated at the Fermi surface (p. 31 of ref.[64]), and Δ_0 is the pairing gap at $T = 0$.

The correlation energy E_{corr} introduced in equation (6-618) of[64]

$$E_{corr} = -\frac{1}{2d}\Delta^2 \qquad (3.A.75)$$

to represent $W_S - W_N$ in the nuclear case, was calculated making use of a (single particle) spectrum of two–fold degenerate (Kramer degeneracy) equally spaced (spacing d) single–particle levels. Consequently, $2/d$ corresponds to the total level density, and $1/d = N(0)$. In keeping with the fact that a nucleus in the ground state (or in any single quantal state), is at zero temperature, (3.A.74) coincides with (3.A.75), taking into account the difference in sign in the definitions.

74 75

Nuclei

The empirical value of the level density parameter for both states $(\nu, \bar{\nu})$ (Kramers degeneracy, both spin orientations) is $a = A/8\ \text{MeV}^{-1}, A = N + Z$ being the mass

[63] Schrieffer (1964)
[64] Bohr, A. and Mottelson (1975)

number. Thus, for neutrons one can write $a_N = N/8$ and $N_N(0) = N/16$ MeV^{-1}. For $^{120}_{50}Sn_{70}$, $N_N(0) \approx 4$ MeV^{-1}. Because $\Delta = 1.46$ MeV, (Table 3.A.1)

$$W_{con} = \frac{1}{2} \times 4\ \text{MeV}^{-1} \times (1.46)^2\ \text{MeV}^2 \approx 4.3\ \text{MeV}. \tag{3.A.76}$$

The binding energy per nucleon is $BE/A = 8.504$ MeV. Thus $BE = 120 \times 8.504$ MeV $= 1.02 \times 10^3$ MeV, and

$$\frac{W_{con}}{BE} \approx 4.2 \times 10^{-3}. \tag{3.A.77}$$

Superconducting lead

Making use of the value[65]

$$N(0) = \frac{0.276\ \text{eV}^{-1}}{\text{atom}}, \tag{3.A.78}$$

and of $\Delta_0 = 1.4$ meV, one obtains

$$W_{con} = 0.27 \times 10^{-6}\text{eV/atom}. \tag{3.A.79}$$

In keeping with the fact that the cohesive energy of lead, namely the energy required to break all the bonds associated with one of its atoms is

$$E_c = 2.03 \frac{\text{eV}}{atom}, \tag{3.A.80}$$

one obtains

$$\frac{W_{con}}{E_c} \approx 1.3 \times 10^{-7}. \tag{3.A.81}$$

The different quantities are summarized in Table 3.A.1.

$\Pi(q,\omega)$

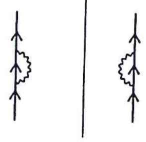
3.A.5 Hindsight

The function $\Pi(q + \omega)$ essentially at resonance ($\omega \lessapprox \omega_q$) and its nuclear analogue being $|\chi(0)|\frac{\omega_\alpha^2}{\omega_\alpha^2 - \omega^2}$ again close to resonance ($\omega \lessapprox \omega_\alpha$), are the sources of new physics eventually leading to observable emergent properties, provided one finds the proper embodiments. In the case of metals at low temperature there are permanent magnetic fields in a superconducting ring, the Josephson effect, etc. In the case of halo neutron drip line nuclei one finds (see Sect. 3.C in particular paragraph before Eq. (3.C.6)) symbiotic pair addition modes, essentially equality of the absolute one– and two–particle transfer cross section, etc.

In Fig. 3.A.5 we present a schematic parallel between the physical mechanisms at the basis of the origin of pairing in metals and in nuclei, and of some of the consequences associated with spontaneous breaking of gauge symmetry in these systems, in particular Cooper pair tunneling.

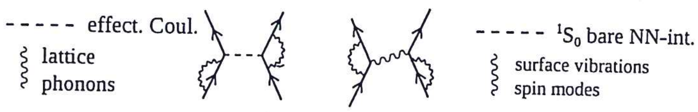
[65] Beck and Claus (1970).

Metals | Nuclei



dressed

electrons (m=0.5 MeV) | nucleons (m=10^3 MeV)



spontaneous breaking of gauge symmetry

$$\left(U'_\nu + V'_\nu e^{-2i\phi} a^\dagger_\nu a^\dagger_{\bar\nu}\right)|0\rangle$$

independent pair motion

$\xi\,(\xi/d)$

10^4 Å(10^4) | 20 fm (5)

overlapping pairs 10^6 | number of pairs 8

$\Delta(\Delta/\epsilon_F)$

$\approx$ 1 meV (10^{-4}) | $\approx$ 1 MeV (10^{-2})

generalized quantality parameter

$$q_\xi = \frac{\hbar^2}{2m\xi^2}\frac{1}{\Delta}$$

supercurrents

10^{-5} | 10^{-2}

pairing

probing of gauge deformation

$P_1=10^{-10}$

observation of ~~currents~~ supercurrents between two weakly coupled superconductors (barrier) which only allows ~~essentially~~ for normal single electron tunneling), ~~with~~ of 2e carriers (Josephson effect)

$P_1=10^{-3}$

Single Cooper pair tunneling mainly as successive transfer between member of a ~~piring~~ pairing rotational band fulfilling

$$\frac{\sigma(gs(N)\to gs(N+2))}{\sum_{exc}\sigma(gs(N)\to 0^+_{exc}(N+2))} \gg 1$$

$N = N_0, N_0+2, N_0+4\ldots N_0+14\ldots(N_0=10)$

$P_2=P_1$

Figure 3.A.5

Appendix 3.B Cooper pair: radial dependence

The fact that one is still trying to understand (BCS–like) pairing (abnormall density phenomena) in nuclei is, to a non negligible extent, due to the fact that, as a rule, pairing in these systems is constrained to manifest itself subject to a very strong "external" (mean, normal density) field[66]. Also, to same extent, due to the fact that the analysis of two–nucleon transfer data was made in terms of relative cross sections and not absolute cross sections as done now[67]. Within this context, Cooper pair transfer was viewed as simultaneous transfer, successive implying a breakup or, at least an anti–pairing disturbance of the pair. There exist a number of evidences which testify to the fact that the picture in which nucleon Cooper pairs are viewed as independent correlated entities over distances of the order of tens of fm (Fig. 3.2.1), contains a number of correct elements (see e.g. Fig. 3.B.1). In this Appendix an attempt at summarizing these evidences, already mentioned or partially discussed before is ~~attempted~~[68]. made

The problem that Cooper solved was that of a pair of electrons which interact above a quiescent Fermi sea ~~sphere~~ with an interaction of the kind that might be expected due to ~~the~~ exchange phonon and a screened Coulomb field[69]. What he showed approximating this retarded interaction by a non–local one, active on a thin energy shell near (above) the Fermi surface[70], was that the resulting spectrum has an eigenvalue $E = 2\epsilon_F - 2\Delta$, regardless how weak the interaction is, and consequently ~~as a consequence~~ the binding energy 2Δ of the pair, so long as the interaction is attractive. This result is a consequence of the Fermi statistic and of the existence of a Fermi sea background –the two electrons interact with each other but not with those in the sea, except via the Pauli principle– since it is well known that binding does not ordinarily occur in the two–body problem in three dimensions, until the strength of the attraction exceeds a finite threshold value.

sea

...phonon exchange and a screen

The wavefunction of the two electrons can be written as

$$\Psi(\mathbf{r}_1, \mathbf{r}_2) = \phi_q(\mathbf{r})e^{i\mathbf{q}\cdot\mathbf{R}}\chi(\sigma_1, \sigma_2) \tag{3.B.1}$$

where $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2$, $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$, and σ_1 and σ_2 denote the spins[71].

Let us consider the state with zero center of mass ($q = 0$) and relative momenta momentum, so that the two electrons carry equal and opposite momenta, aside of

---

[66] c.f. e.g. Matsuo, M. (2013).

[67] See Potel, G. et al. (2013a) and references therein.

[68] Of course such manifestation will be latent, expressing themselves indirectly. In other words, abnormal density can only be present when normal density, at ever so low values already is present. The pairing field does not have within this context an existence by itself uncoupled from the normal density. On the other hand this, in most cases latent (~~more than~~ virtual), and in only few cases factual, existence, has important consequences on nuclear properties. Within this context one can mention that the neutron halo normal density in ^{11}Li is not there before the associated abnormal density is operative. In fact ~~one~~ density requires the ~~other~~ to ~~exist~~ in this neutron dripline nucleus, and viceversa.

[69] Cooper (1956).

[70] States below the Fermi surface are frozen because of Pauli principle.

[71] In the limit $q \to 0$ the relative coordinate problem is spherically symmetric so that $\phi_0(\mathbf{r})$ (Schrieffer (1964)).

abnormal normal one developed

as a function of the cartesian coordinates of neutron 2, for fixed values of the ~~position~~ r_1 of particle 1 (for more details see Caption to Fig. 2.6.3)

resulting in

$$\phi_0(\mathbf{r}) \sim e^{-r/\xi} e^{ik_F r}. \tag{3.B.8}$$

Because we are dealing with a singlet state, and the total wavefunction has to be antisymmetric,

$$\phi_0(\mathbf{r}) \sim e^{-r/\xi} \cos k_F r, \tag{3.B.9}$$

A more proper solution of the Cooper pair problem leads to[73]

$$\phi_0(\mathbf{r}) \sim K_0(r/\pi\xi) \cos k_F r, \tag{3.B.10}$$

where K_0 id the zeroth–order modified Bessel function. For $x \gg 0$, $K_0(x) \sim (\pi/2x)^{1/2} \exp(-x)$, where $x = r/\pi\xi$.

A wavefunction which extends over distances much larger than the binding potential is a well–known phenomenon when the binding energy is small. For example, in the case of the deuteron. In any case, it is of notice that here we are discussing a rather subtle phenomenon, pairing or better Cooper pairing, which has to express itself in the presence of a very strong "external" field. Unless one does not relate the NN interaction binding the deuteron to proton–neutron pairing.

Be as it may, the large size of the Cooper pair wavefunction also explains why the electrostatic repulsion between electron pair does not appreciably influence the binding. The repulsion acts only over distances of the order of the correlation length.

Caption Fig 3.B.1 9 ^{9}Li

Synthesis of the spatial structure of ^{11}Li neutron halo Cooper pair calculated in NFT (Barranco, F. et al. (2001)). To make more direct the comparison between the simple estimates and the results of the above reference, it is assumed that $\xi = 7.5$ fm (dashed circle) instead of 10–11 fm as obtained from $\xi = \hbar v_F/(\pi|E_{corr}|)$ ($v_F/c \approx 0.08, E_{corr} \approx 0.4$ MeV). Diagrams (a) and (d) are the schematic representations of the modulus square $|\Psi_0(\mathbf{r}_1, \mathbf{r}_2)|^2 = |\langle \mathbf{r}_1, \mathbf{r}_2|0\rangle|^2$ describing the motion of the two halo neutrons of ^{11}Li, moving around the ^{11}Li core. Diagrams (b), (e) and (g) are the results of NFT (see also Fig. 2.6.3 (II) a) and b)). **(a)** The circles drawn with continuous lines correspond to the relative distance r at the radius of the ^{9}Li core and of ^{11}Li. The Cooper pair "intrinsic coordinate" r_{12} is also shown. Particle 1 of the Cooper pair is assumed to occupy the center of the nucleus ($r = 0$). **(b)** Result of NFT for a situation similar to the above. **(c)** Schematic representation of an uncorrelated pair in a potential weakly binding the pure configuration $p^2_{1/2}(0)(r = 0)$. **(d)** Same as (a) but for $r = 7.5$ fm. **(e)** Result of the NFT calculation for this configuration. **(f)** Schematic representation of a pure configuration $p^2_{1/2}(0)(r = 7$ fm), **(g)** The result of the microscopic calculation for a weakly bound

[73] Kadin (2007).

e

fermions

$p^2_{1/2}(0)$ configuration (r = 7.5 fm). **(h)** The variety of situations in (a) and (d) in comparison to each other in a single cartoon. **(i)** Schematic picture of the dynamics in the quantum state of the Cooper pair. It is a linear combination of motions away and towards one another. The ~~electrons~~ stay within a distance of the order ξ, root mean square radius of the Cooper pair (After Weisskopf (1981), see also Kadin (2007) and van Witsen (2014)).

Fig.

Fig.

Within the nuclear scenario, to interact at profit through long wavelength medium polarization pairing, pairs of nucleons have to have low momentum. To do so they have to reduce the effect of the strong external (mean) field by moving away from it, possible mechanisms being among other*s* halo (3.B.1), transfer processes (see e.g. 3.4.1), exotic decay[74] (see Fig. 3.B.3.) others

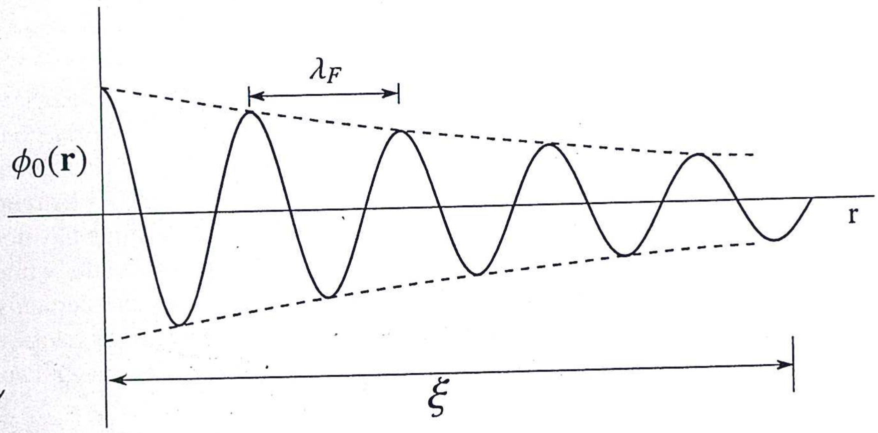


Figure 3.B.2: Schematic representation of the Cooper pair wavefunction. Indicated are the coherence length ξ and the Fermi wavelength $\lambda_F = h/p_F = 2\pi/k_F$. In the nuclear case $\lambda_F \approx 4.6$ fm and $\xi \approx \hbar v_F/2\Delta \approx 30$ fm ($v_F/c \approx 0.3, \Delta \approx 1$ MeV). Thus $\xi/\lambda_F \approx 7$ (after Weisskopf (1981)).

bulk

low temperature

l

3.B.1 Number of overlapping pairs

1,4

The coherence length for low–temperature superconductors is of the order of 10^4 Å. In fact, in the case of e.g. Pb, for which[75] $\Delta_0 = \cancel{0.62}$ meV and $v_F = 1.83 \times 10^8$ cm/s one obtains $\xi \approx 10^{-4}$ cm, where use of $c = 3 \times 10^{10}$ cm/s and $\hbar c \approx 2 \times 10^3$ Å eV has been made.

l

associate with

[74] In Fig. 3.B.3, a parallel is made between correlation lengths ~~between~~ pairing particle–particle or hole-hole modes and particle–hole vibrations, modes ~~which~~ also display a consistent spatial correlation (see e.g. Broglia et al. (1971)). These last

[75] ~~The standard quoted value is $\Delta_0 = \Delta(T = 0) = 7.19$ K. Making use of the conversion factor 1K = 8.6217×10^{-5} eV one obtains 0.62 meV.~~

Making use of the experimental values $T_c = 7{,}193$ K ($k_BT_c = 0{,}62$ meV) and $(2\Delta(0)/k_BT_c) = 4{,}38$ one obtains $\Delta = \Delta(0) = 1.4$ meV.

footnote 75

With the help of the approximate expression $\Omega \approx (2/3) A^{2/3}$ one obtains, for ^{120}Sn, $\alpha \approx 8$.

Since electrons in metals typically occupy a volume of the order of $(2Å)^3$ (Wigner–Seitz cell), there would be of the order of[76] $\xi^3/(2Å)^3 \approx 10^{11}$ other electrons within a "coherence volume". Eliminating the electrons deep within the Fermi sea as they behave essentially as the metal was in the normal phase, one gets[77] 10^6. In other words, about a million of other Cooper pairs have their center of mass falling inside the coherence volume of a pair. Thus, the isolated pair picture (Fig. 2.3.4) is not correct, but yes that displayed in Fig. 2.3.5.

In the nuclear case, the number of Cooper pairs participating in the condensate is

$$\alpha_0 = \langle BCS|P'^{\dagger}|BCS\rangle = \sum_j \frac{2j+1}{2} U'_j V'_j. \qquad (3.B.11)$$

A simple estimate of this number can be made with the help of the single j–shell model, in which case $V_j = (N/2\Omega)^{1/2}$ and $U_j = (1 - N/2\Omega)^{1/2}$, where $\Omega = (2j + 1)/2$. For a half–filled shell ($N = \Omega$) one obtains[78] $\alpha'_0 = \Omega/2$. ~~In the case of ^{120}Sn, $\alpha'_0 = 6 - 8$.~~

In keeping with the fact that $\xi > R_0$, in the nuclear case one has a complete overlap between all Cooper pairs participating in the condensate. This, together with the fact that the nuclear Cooper pairs press against the nuclear surface in an attempt to expand and are forced to bounce elastically off from it, receive strong circumstantial evidence from the following experimental results: **1)** while the moment of inertia of rotational bands is $\mathcal{J}_r/2$ it is $5\mathcal{J}_{irrot}$. In other words, while pairing in nuclei is important, its role is only partially exhausted, and certainly strongly distorted ~~(Bohr, A. and Mottelson (1975))~~; **2)** One– and two–nucleon transfer reactions in pairing correlated nuclei have the same order of magnitude. For example $\sigma(^{120}\mathrm{Sn}(p,d)^{119}\mathrm{Sn}(5/2^+; 1.09\,\mathrm{MeV})) = 5.35\mathrm{mb}$ ($2° < \theta_{cm} < 55°$), while $\sigma(^{120}\mathrm{Sn}(p,t)^{118}\mathrm{Sn}(gs)) = 2.25\mathrm{mb}$ ($7.6° < \theta_{cm} < 59.7°$). In this last reaction Cooper pair partners can be as far as 12–13 fm. In the case of a heavy ion reaction this distance becomes almost double (Fig. 3.4.1); **3)** The decay constant of the exotic decay $^{223}_{88}\mathrm{Ra}_{135} \rightarrow ^{14}_{6}\mathrm{C}_8 + ^{209}_{82}\mathrm{Pb}_{127}$ has been measured to be $\lambda_{exp} = 4.3 \times 10^{-16}\mathrm{sec}^{-1}$. For theoretical purposes it can be written as $\lambda = PfT$, product of the formation probability P of ^{14}C in ^{223}Ra (saddle configuration, see bottom Fig. 3.B.3), the knocking rate f and the tunneling probability T. These two last quantities hardly depend on pairing. On the other hand P changes from $\approx 2 \times 10^{-76}$ to 2.3×10^{-10}, and consequently the associated lifetimes from 10^{75}y to the observed value of 10^8y by allowing Cooper pairs to be correlated over distances which can be as large as 20 fm.

Within the above context, and as discussed in App. 3.C, exotic halo nuclei open new possibilities to understand the physics at the basis of pairing in nuclei. In particular, one may be to hide away in a (bare $^1S_0$ NN-potential the) $\approx 50\%$ contribution associated with the induced pairing interaction to the pairing gap in ^{120}Sn. Hardly, essentially all of the binding energy ($S_{2n} \approx 380$ keV) of the the neutron halo Cooper pair of ^{11}Li to the core ^{9}Li. ©-© p 228a handwritten

[76] Ketterson and Song (1999) p. 198.

[77] Schrieffer (1964) p. 43.

[78] Making use of the harmonic oscillator, one can write $\Omega = \frac{1}{2}(N+1)(N+2) \sim A^{2/3}$, where the proportionality constant has a value between 1/2 and 2/3.

(c) At the basis of the large magnitude of renormalization effects of elementary modes of excitation and of medium polarization contribution to the pairing interaction observed in light exotic halo nuclei like e.g. ^{11}Li, one finds a fundamental parameter of NFT, namely the effective degeneracy $\Omega\,(\approx 2/3\,A^{2/3})$ of the single-particle phase space, $1/\Omega$ being the small expansion parameter. In the case of ^{11}Li, Ω is rather smal (≈ 3) as compared with heavy nuclei like e.g. ^{210}Pb ($\Omega \approx 24$). Furthermore, in the case of ^{11}Li, the surface (S) to volume (V) ratio ($\approx aS/V$, a being the nuclear diffusivity) is much larger (≈ 0.74) than in the case of heavy nuclei lying along the stability valley (≈ 0.28 in the case of ^{210}Pb). (c)

to p. 228

3.B.2 Coherence length and quantality parameter for (ph) vibrations

Vibrations: correlated (ph) $(\alpha = 0)(pp)(\alpha = +2)$ and $(hh)(\alpha \doteq -2)$ modes, with energy $E_{corr}(< 0)$ and fulfilling the dispersion relation in nuclear matter,

$$|E_{corr}| = \frac{\hbar^2 k^2}{2m}.$$

$$\begin{aligned} \lambdabar &= \frac{1}{k} \approx \frac{\hbar^2 k_F}{2m}\frac{1}{|E_{corr}|} \\ &= \frac{\hbar v_F}{2|E_{corr}|} \end{aligned} \quad (3.B.12)$$

To be compared with

$$\xi = \frac{\hbar v_F}{2\Delta} \quad (3.B.13)$$

a similar

for superfluid nuclei. Thus, one can assume that both $\lambdabar$ and ξ describe ~~the same~~ physical phenomenon: correlation length of two fermions in normal $((ph), (pp), (hh))$ or in superfluid $((pp) + (hh))$ nuclei (see Fig. 3.B.3).

independent

✓**Caption Fig 3.B.3**

correspond to

Vibrations can be classified by the transfer quantum number α. Collective modes with $\alpha = 0$ correspond to correlated particle–hole (ph) excitation. For example low–lying quadrupole or octupole (surface and/or density) vibrations. Modes with $\alpha = \pm 2$ are ~~known as~~ correlated (pp) or (hh) modes, that is, pair addition and pair substraction modes. Thinking of these modes propagating in uniform nuclear matter, the reduced wavelength $\lambdabar = \lambda/2\pi = 1/k$ is estimated in terms of the correlation energy E_{corr}. The (generalized) quantality parameter, ratio of the quantal kinetic energy of localization and the correlation energy, gives a measure of the tendency to independent particle ($q_\xi \approx 1$) or pair ($q_\xi \ll 1$) motion, in keeping with the fact that potential energy is best profited by special arrangements between nu- (partially) cleons and thus lower symmetry than the original Hamiltonian, ~~while fluctuations~~ favor symmetry. Going from the infinite to the finite nuclear system, these modes change somewhat character, e.g. density turns into surface modes, becoming distorted by the mean field which acts as a strong external field (see also Fig. 3.B.1). A concrete example which testifies to the fact that (ph) excitations (large amplitude surface distortion) and independent (pp) motion (superfluidity) are correlated over dimensions larger than typical nuclear dimensions, is provided by e.g. fission and exotic decay, in particular $^{223}Ra \rightarrow {}^{14}C + {}^{209}Pb$. In keeping with the uncertainties affecting the above simple estimates (factor 2 or π in the denominator of ξ, $\langle r^2 \rangle^{1/2}_{Cooper}$ or $\sqrt{\frac{3}{5}}\langle r^2 \rangle^{1/2}_{Cooper}$, etc.), it seems fair to conclude that $10 \lesssim \xi \lesssim 20$. Thus, one is likely faced with an intermediate situation in which $1.3 \lesssim \xi/R \lesssim 2.6$.

Vibrations

(ph) | (pp), (hh)

correlated excitations (E_{corr})
with transfer quantum number

$\alpha=0$ | $\alpha = \pm 2$

waves on

the nuclear | the Fermi

surface

correlation length
infinite medium ($|E_{corr}| = \frac{\hbar^2 k^2}{2m}$)

$$\lambda\!\!\!^{-} = \frac{1}{k} \approx \frac{\langle k \rangle_F}{2m}\frac{1}{|E_{corr}|} = \frac{\hbar v_F}{\pi |E_{corr}|} = \xi$$

typical values (finite nuclei), E_{corr}=-2.0 MeV (-0.4 MeV, ^{11}Li), $v_F/c \approx 0.3$ ($\cancel{0.1}$ 0.14, ^{11}Li)

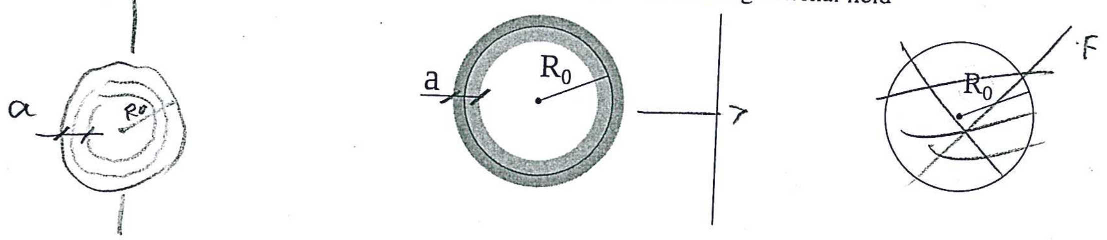
$\xi = 10$ fm (16 fm, ^{11}Li) ← $\xi \approx 10$ fm (22 fm, ^{11}Li)

genralized quantality parameter

$$q_\xi = \frac{\hbar^2}{2m(\xi)^2}\frac{1}{|E_{corr}|} \approx 0.1 \quad (\cancel{0.08}\ 0.1, {}^{11}\text{Li})$$

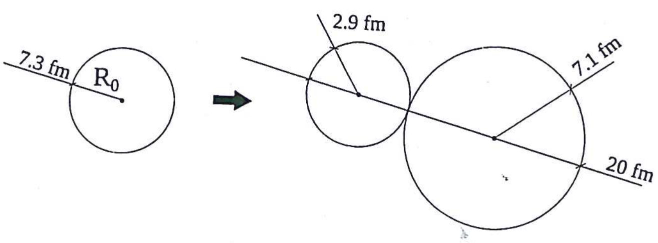
strongly correlated (cluster like $q_\xi \ll 1$), weakly bound ($|E_{corr}|/\epsilon_F \lesssim 0.06$)
very extended ($\xi/d \approx 5$, $d = \left(\frac{4\pi R^3}{3A}\right)^{1/3}$) objects $\gtrsim$

subject to a strong external field



example

^{223}Ra→^{14}C+^{209}Pb ($\lambda = PfT$)



$$P = \begin{cases} 10^{-76} & (\Delta = 0) \\ 10^{-10} & \Delta_{exp} \end{cases}$$

$$\langle r^2 \rangle^{1/2}_{Cooper} = \xi = \frac{\hbar v_F}{\pi \Delta} \quad (\approx 24\,\text{fm}; \Delta = 0.8\,\text{MeV})$$

√ Figure 3.B.3

Bringing the above arguments into reaction implies, as shown in Eq. (3.2.19), that $P_2 = P_1$.

U'_ν and V'_ν are real, and define the state,

$$
\begin{aligned}
|BCS(\phi)\rangle_{\mathcal{K}} &= \mathcal{G}(\phi)\prod_{\nu>0}(U_\nu + V_\nu a^\dagger_\nu a^\dagger_{\bar\nu})|0\rangle = e^{\frac{iN}{2}\phi}\prod_{\nu>0}(U'_\nu + V'_\nu e^{-2i\phi} a^\dagger_\nu a^\dagger_{\bar\nu})|0\rangle \\
&= e^{\frac{iN}{2}\phi}\prod_{\nu>0}(U'_\nu + V'_\nu a'^\dagger_\nu a'^\dagger_{\bar\nu})|0\rangle = |BCS(\phi)\rangle_{\mathcal{K}'} \qquad (3.B.17)
\end{aligned}
$$

where use has been made of the gauge transformation $a'^\dagger_\nu = \mathcal{G}(\phi)\, a^\dagger_\nu\, \mathcal{G}^{-1}(\phi)$, $\mathcal{G}(\phi) = e^{-iN\phi}$ inducing a rotation in gauge space. The labels $\mathcal{K}$ and $\mathcal{K}'$ indicate the laboratory and body–fixed reference frames respectively.

The state (3.B.17) displays off–diagonal–long–range–order (ODLRO) because each pair is in a state $(U'_\nu + V'_\nu e^{-2i\phi} a^\dagger_\nu a^\dagger_{\bar\nu})|0\rangle$ with the same phase as all the others. In fact, the wavefunction (3.B.17) leads to a two–particle density matrix with the property $\lim_{\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}_3,\mathbf{r}_4\to\infty}\phi(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}_3,\mathbf{r}_4) \neq 0$ under the assumption that $r_{12}, r_{34} < \xi$, $(\mathbf{r}_1,\mathbf{r}_2)$ and $(\mathbf{r}_3,\mathbf{r}_4)$ being the coordinates of a Cooper pair, r_{ij} the relative modulus of it and ξ the coherence length[79].

~~Let us bring this structure result into reaction. The fact that the wavefunction of the nucleons in the pair are phase–coherent ($(U'_\nu + V'_\nu e^{-2i\phi} a^\dagger_\nu a^\dagger_{\bar\nu})|0\rangle$) implies that to calculate the probability of two–nucleon transfer, one has to add the amplitudes of one–nucleon transfer before taking modulus squared, that is,~~

$$
\begin{aligned}
P_2 &= \lim_{\epsilon\to 0}\left|\frac{1}{\sqrt{2}}\left(e^{i\phi'}\sqrt{P_1} + e^{i\phi}\sqrt{P_1}\right)\right|^2 \qquad (\epsilon = \phi - \phi') \\
&= P_1 \lim_{\epsilon\to 0}(1+\cos\epsilon) = P_1. \qquad (3.B.18)
\end{aligned}
$$

Experiments observed values of maximum Josephson current of ≈ 1 mA, consistent with junctions resistances of $1\,\Omega$ per unit area (see Sect. 3.6. The 'importance of

In keeping with the parallel made with superconductors (see Fig. 3.A.5) one can mention that Josephson showed that at very low temperatures, the pair current is equal to the single–particle current at an equivalent voltage[80] $\pi\Delta/2e$. ~~How conclusive~~ this result ~~is~~ concerning the mechanism at the basis of Cooper pair transfer is connected with the fact that the probability of one–electron–tunneling across a typical dioxide layer giving rise to a weak $S-S$ coupling is 10^{-10}, Consequently, simultaneous pair transfer between two superconductors (S), with a probability $(10^{-10})^2$ cannot be observed[81]. (b) – (b)

One could argue that in the reaction ^{120}Sn$(p,t)^{118}$Sn(gs) one can hardly consider the triton as a pairing condensate. While this is correct one can hardly claim either that six–eight Cooper pairs (^{120}Sn) make a *bona fide* one. In any case, when one experimentally observes such unexpected ~~behaviour~~ phenomena ($\sigma_{2n} \approx \sigma_{1n}$) one is likely somewhat authorized at using similar concepts[82].

[79] See e.g. Ambegaokar (1969) and refs. therein.

[80] In the case of Pb $\Delta =$ ~~0.62~~ 1.4 meV (see footnote [75]) this voltage is $(\pi\times$ ~~0.62~~ 1.4$/2)\times 10^{-3}\times$eV/e$\approx$ ~~1~~ 2 mV (see e.g. McDonald (2001)). Co

[81] See e.g. McDonald (2001).

[82] Anderson (1972).

(b) Consequently, the Josephson current of carriers of charge 2e results from the tunneling of a Cooper pair partner at a time, equally pairing correlated when they are in the same superconductor (S) within the correlation length ξ, than when each of them is on a different of the two weakly coupled S. (b)

Aqui el signo de interrogación en el título es correcto.

$\frac{\hbar^2}{2m}\frac{1}{\xi^2}$ already discussed in connection with the generalized quantality parameter, through the relation $2|\alpha_0|T_\xi$ (for one type of nucleons), in keeping with the fact that (3.B.25) is expressed in term of single nucleon energies. Let us make a simple estimate which can help at providing a qualitative exmple of the above argument, and consider for the purpose the nucleus ^{223}Ra and $G \approx (22/A)$ MeV, $|\alpha_0| \approx 6$ and $\xi \approx 10$ fm: $T_\xi \approx 0.2$ MeV, $2\times(2\times|\alpha_0|\times T_\xi) = 4.8$ MeV, $2\times(-G|\alpha_0|^2) = -7.2$ MeV[84] (factors of 2,both protons and neutrons). The resulting pairing correlation energy thus being $E_{corr} = -2.6$ MeV. This number can be compared with a "realistic" estimate provided by the relation[85]

$$E_{corr} = -\frac{g\Delta^2}{4}, \tag{3.B.26}$$

where $g_n = N/16$ MeV^{-1} and $g_p = Z/16$ MeV^{-1}. Taking into account both types of particles $g = g_n + g_p = A/16$ MeV^{-1} and making use of $\Delta = 12/\sqrt{A}$ MeV, one obtains $E_{corr} = -\frac{144}{64}$ MeV =-2.25 MeV. With the help of E_{corr} and T_ξ, one can estimate the generalized quantality parameter, $q_\xi = T_\xi/|E_{corr}| = 0.2/2 \approx 0.08$, as well as make a consistency check on the value of ξ used, namely $\hbar v_F/(2|E_{corr}|) \approx$ 11.5 fm.

3.B.5 Possible (dream?) experiment[86]

The nuclear Cooper pair not only is forced to exist in a very strong external field, the HF mean field, of very reduced dimensions as compared to the correlation length. Because of spatial quantization, it is also forced to exist on selected orbitals of varied angular momentum and parity[87].

Correlations, in particular pairing correlations within such constrains will have opposite and apparently contradictory effects. As an example let's consider two neutrons moving around ^{9}Li in the $s^2_{1/2}(0)$ or in the $p^2_{1/2}(0)$ (pure) uncorrelated configurations. In such a situation, fixing one of the neutrons of the pair at a radius r_1, the other one will display equal ~~possibility~~ probability to be close or in the opposite side of the nucleus ($\theta_{12} = 0°$ or $\theta_{12} = 180°$), the average distance between neutrons

[84]This quantity, but divided by 2, i.e. -3.6 MeV can be compared with the effective pairing matrix element $v = \left(\frac{\Delta_\pi^2+\Delta_\nu^2}{4} \approx -2.9\right)$ MeV, operative at level crossing in the calculation of the inertia of the exotic decay ^{223}Ra$\to$^{14}Ca+^{209}Pb, cf. Brink, D. and Broglia (2005) p.159 and refs. therein.

[85]Brink, D. and Broglia (2005).

[86]The breaking of a prejudice: pairing plus long range force, i.e. pairing short range, many (high) relative angular momenta contributing (Kisslinger,L. S. and Sorensen, R. A. (1963); Soloviev (1965); Mottelson (1962, 1998).

[87]Some of them allowing for pure $j^2(0)$ configurations, with large (little) probability of $L = 0$ relative motion, which behaves as hot (cold) orbitals, their contribution to pairing correlations and to two–nucleon transfer reactions being very inhomogeneous, at variance of the situation found in solid state (see e.g. Broglia (2005)). This is also the reason why the second–order–like phase transition normal–superfluid taking place in nuclei as e.g. the number of pairs of nucleons moving outside closed shells, affected by strong pairing fluctuations, are conspicuously blurred as compared to the $N \to \infty$ case.

reaction ^{9}Li$(t,p)^{11}$Li(gs) as a function of the bombarding energy E_{CM}. For values of E_{CM}/A much larger than $\hbar\omega_{GDPR}(\lessapprox 1$ MeV), this mode will have hardly time to act, and the main component of halo pairing to become established, the associated cross section being expected to decrease as compared to the situation E_{CM}/A of few MeV[88]. Similar effects are expected in connection with the cross section associated with the process ^{9}Li$(t,p)^{11}$Li (1^-;GDPR), and the eventual observation of its γ decay. (see Fig. 3.C.1 ~~below~~).

γ-decay

3.B.28

2) Interference between positive ($(-1)^l = +1$) and negative ($(-1)^l = -1$) single–particle based $|(l,j)^2_0\rangle$ configurations, constructive at $\theta = 0°$ and destructive at $\theta = 180°$, $\theta = \hat{r_1 r_2}$ been the relative angle between the coordinates $\mathbf{r}_1$ and $\mathbf{r}_2$ of the Cooper pair partners. In other words the two nucleons will tend to be close to each other, in particular on the nuclear surface. As can be seen from (~~4~~), this effect is extreme in the case of the ground state of ^{11}Li. Now, such an effect has not much to do with pairing, BCS pairing at it and thus superconductivity[89], but with the properties of the nuclear mean field, result of spatial quantization which not only distorts the Cooper pair through isotropic confinement, but through admixtures of odd and even parity states controlled also by the very strong spin orbit term.

idea

Summing up, the difficulties of understanding pairing in nuclei as compared with condensed matter is (at least threefold): **a**) the bare interaction is attractive, a fact which lead to the ~~prejudice~~ that pairing force is short range and delayed the discovery of the other half of pairing, namely the retarded, medium polarization interaction, for a long time; **b**) particle number is small, thus pairing vibrations are important, and renormalize in a conspicuous way the variety of nuclear phenomena, in particular single–particle motion. The fact that such effects are still not being really considered is testified by the fact that a serious treatment of multipole pairing vibrations is still missing; **c**) spatial quantization leading to phenomena which by themselves can be very interesting[90], but which again has conditioned nuclear structure research, let alone reaction mechanism studies and the physics emerging from their interweaving.

~~point d) write better~~

[88] Tanihata, I. et al. (2008).

[89] Within this context it is of notice that in condensed matter literature Cooper pairs are viewed as fragile, extended di–electron entities, overlapping with a conspicuous number of other pairs, and displaying a delicate "rigid" quantal correlation between partners (generalized quantality parameter) and among Cooper pairs (off diagonal long range order). In fact, Weisskof's representaion of the radial (opposite) motion of electrons provides a useful picture of Cooper pir internal dynamics, In other words, approaching to or recessing from each other does not favour a particular anisotropic configuration , the two electrons being at the mean square radius of the Cooper pair, i.e. the coherence length ξ.

[90] Bertsch, G. F. et al. (1967); Ferreira, L. et al. (1984); Lotti et al. (1989); Matsuo (2006); Matsuo, M. (2013)