Ch. 7

All refs. should read as

Bayman, B.F. and Chen, J. (1982), One-step and two-step contributions to two-nucleon transfer reactions, Phys. Rev. C, 26: 1509.

Optical potential ^{11}Li, He ?

Description COOP p. 2 comento para gregory

Macro knock out

Put every thing format
Knock-out it reads well

collected ⓐ
cf. binder
ch. 7
①

In other words, because of its (intrinsic, virtual extension) Cooper pair transfer display equivalent pairing correlations both in simultaneous as in successive transfer *)

Version containing latest corrections sent by G in Sept 2013

Chapter 7

Two–particle transfer

go on and correct
3/10/13

Cooper pairs are the building blocks of pairing correlations in many-body ~~fermionic~~ systems. In particular in atomic nuclei. As a consequence, nuclear superfluidity can be specifically probed through Cooper pair tunneling. In the simultaneous transfer of two nucleons, one nucleon goes over from target to projectile, or viceversa, under the influence of the nuclear interaction responsible of the existence of a mean field potential, while the other follows suit by profiting of: 1) pairing correlations (simultaneous transfer); 2) the fact that the single-particle wavefunctions describing the motion of Cooper pair partners in both target and projectile are solutions of different single-particle potentials (non-orthogonality transfer). In the limit of independent particle motion, in which all of the nucleon-nucleon interaction is used up in generating a mean field, both contributions to the transfer process (simultaneous and non-orthogonality) cancel out exactly (cf. App. A).

7.C
App 7.A C

In keeping with the fact that nuclear Cooper pairs are weakly bound ($E_{corr} \ll \varepsilon_F$), this cancellation is, in actual nuclei, quite strong. Consequently, successive transfer, a process in which the ~~mean field~~ nuclear interaction acts twice is, as a rule, the main mechanism at the basis of Cooper pair transfer. Because of the same reason (weak binding), the correlation length of Cooper pairs ~~is larger~~ ($\xi = \hbar v_F/E_{corr} \gg R$) than nuclear dimensions, a fact which allows the two members of a Cooper pair to move between target and projectile, essentially as a whole, also in the case of successive transfer.

¶ In the present Chapter a number of Appendices

The present Chapter is structured in the following way. In section 7.1 we present a summary of two–nucleon transfer reaction theory. ~~These are all~~ It provides the elements needed to ~~calculate the absolute~~ two–nucleon transfer differential cross sections in second order DWBA, and thus to compare theory with experiment. ~~In this way~~ Within this context one can, after reading this section, ~~one can go~~ move directly to ~~the~~ Chapter 8 containing examples of ~~the~~ applications of this formalism (cf. App. 8A).

For the more theoretically oriented reader we provide in section 7.2 a detailed derivation of the equations presented in section 7.1 ~~and which are implemented~~ in the softwares used in the applications. These equations are implemented and made operative ~~Three appendixes~~ are provided. ~~One~~ In particular one in which the ~~cancellations existing between the different~~ variety of contributions to ~~the~~ two–nucleon transfer ~~spectroscopic~~ amplitudes (successive, simultaneous and non–orthogonality) are discussed in detail within the framework of the semi–classical approximation (App. 7.B), ~~Another~~ ~~one~~ Another one (App. 7.C in which simple estimates of the relative importance of successive and of simultaneous transfer are worked out, and other two (App. 7.D and 7.E). ~~Finally a~~ derivation of first order DWBA simultaneous transfer is worked out within a formalism tailored to focus the attention on the nuclear structure correlations aspects of the process leading to ~~a~~ effective two–nucleon transfer form factors.

e
e
e
e
to

*) footnote, see pp. ①a and ①b

*) Footnote p. 1 — 1a

In order for a nucleon to display independent-particle motion, all other nucleons must act coherently so as to leave the way free making feel their pullings and pushings only when the nucleon in question tries to leave the self-bound system, thus acting as a reflecting surface which inverts the momentum of the particle. It is then natural to consider the nuclear mean field, the most striking and fundamental collective feature in all nuclear phenomena. A close second is provided by the BCS mean field, resulting from the condensation of strongly overlapping (i.e. $\langle BCS|\sum_{\nu>0} a^\dagger_\nu a^\dagger_{\bar\nu}|BCS\rangle = \alpha_0 \neq 0$) Cooper pairs and leading to independent quasiparticle motion. It is a rather unfortunate perversity of popular terminology ~~that~~ regards these collective fields (HF and HFB) as well as successive transfer, as in some sense ~~opposite~~ and atithesis ~~of popular terminology~~ ~~that~~ ~~regards~~ to the nuclear collective ~~effects~~ mods ~~and to~~ (Mottelson 1962) and to simultaneous ~~transfer~~ ~~respectively~~ ~~respectively~~.

transfer respectively. Within this context 1 b it is of notice ~~that~~ the two-nucleon differential cross section ~~[illegible]~~ between ~~superfluid nuclei~~ the ground state of superfluid nuclei is proportional to α_0^2 and not to Δ^2. In fact, Cooper pair partners remain correlated even over regions in which $G=0$.

appendices 7.F, 7.G, 7.H and 7.I provide element to be used in the order of magnitude estimates mentioned above, while App. 7.A deals with nuclear structure processes renormalizing the properties of single particle and collective states and their relation with two-nucleon transfer processes

2

7.1 Summary of ~~2nd~~ second order DWBA

Let us illustrate the ~~calculation with the~~ theory of second order DWBA two–nucleon transfer reactions with the $A + t \rightarrow B(\equiv A+2) + p$ reaction, in which $A+2$ and A are even nuclei in their 0^+ ground state. The extension of the ~~following~~ expressions to the transfer of pairs coupled to arbitrary angular momentum is discussed in subsection 7.2.10.

¶ new line

The wavefunction of the nucleus $A+2$ can be written as

$$\Psi_{A+2}(\xi_A, \mathbf{r}_{A1}, \sigma_1, \mathbf{r}_{A2}, \sigma_2) = \psi_A(\xi_A) \sum_{l_i, j_i} [\phi^{A+2}_{l_i, j_i}(\mathbf{r}_{A1}, \sigma_1, \mathbf{r}_{A2}, \sigma_2)]^0_0, \tag{7.1.1}$$

where

$$\left[\phi^{A+2}_{l_i, j_i}(\mathbf{r}_{A1}, \sigma_1, \mathbf{r}_{A2}, \sigma_2\right]^0_0 = \sum_{nm} a_{nm} \left[\varphi^{A+2}_{n, l_i, j_i}(\mathbf{r}_{A1}, \sigma_1)\varphi^{A+2}_{m, l_i, j_i}(\mathbf{r}_{A2}, \sigma_2)\right]^0_0, \tag{7.1.2}$$

while the wavefunctions $\varphi^{A+2}_{n, l_i, j_i}(\mathbf{r})$ are eigenfunctions of a Saxon–Woods potential

$$U(r) = -\frac{V_0}{1 + \exp\left[\frac{r - R_0}{a}\right]}, \qquad R_0 = r_0 A^{1/3}, \tag{7.1.3}$$

together with a standard spin-orbit potential

of depth V_0 adjusted to reproduce the experimental single–particles energies. The radial dependence of the wavefunction of the two neutrons in the triton is written as $\phi_t(\mathbf{r}_{p1}, \mathbf{r}_{p2}) = \rho(r_{p1})\rho(r_{p2})\rho(r_{12})$, where r_{p1}, r_{p2}, r_{12} are the distances between neutron 1 and the proton, neutron 2 and the proton and between neutrons 1 and 2 respectively, ~~and~~ while $\rho(r)$ is the hard core ($r_{core} = 0.45$ fm) potential wavefunction depicted in Fig ~~7.1.1~~ 7.1.1.

The two–nucleon transfer differential cross section is written as

$$\frac{d\sigma}{d\Omega} = \frac{\mu_i \mu_f}{(4\pi\hbar^2)^2} \frac{k_f}{k_i} \left| T^{(1)} + T^{(2)}_{succ} - T^{(2)}_{NO} \right|^2, \tag{7.1.4}$$

where (see ~~also~~ e.g. Bayman and Chen (1982)),

Sabana Ben Bayman Appendix ?

$$T^{(1)} = 2 \sum_{l_i, j_i} \sum_{\sigma_1 \sigma_2} \int d\mathbf{r}_{tA} d\mathbf{r}_{p1} d\mathbf{r}_{A2} [\phi^{A+2}_{l_i, j_i}(\mathbf{r}_{A1}, \sigma_1, \mathbf{r}_{A2}, \sigma_2)]^{0*}_0 \chi^{(-)*}_{pB}(\mathbf{r}_{pB})$$

$$\times v(\mathbf{r}_{p1}) \phi_t(\mathbf{r}_{p1}, \mathbf{r}_{p2}) \chi^{(+)}_{tA}(\mathbf{r}_{tA}), \tag{7.1.5a}$$

$$T^{(2)}_{succ} = 2 \sum_{l_i, j_i} \sum_{l_f, j_f, m_f} \sum_{\substack{\sigma_1 \sigma_2 \\ \sigma_1' \sigma_2'}} \int d\mathbf{r}_{dF} d\mathbf{r}_{p1} d\mathbf{r}_{A2} [\phi^{A+2}_{l_i, j_i}(\mathbf{r}_{A1}, \sigma_1, \mathbf{r}_{A2}, \sigma_2)]^{0*}_0 \chi^{(-)*}_{pB}(\mathbf{r}_{pB}) v(\mathbf{r}_{p1})$$

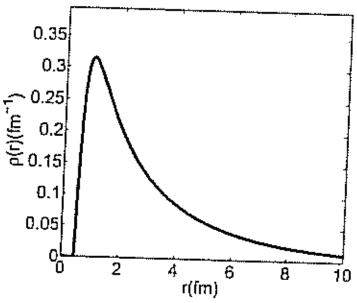
$$\times \phi_d(\mathbf{r}_{p1}) \varphi^{A+1}_{l_f, j_f, m_f}(\mathbf{r}_{A2}) \int d\mathbf{r}'_{dF} d\mathbf{r}'_{p1} d\mathbf{r}'_{A2} G(\mathbf{r}_{dF}, \mathbf{r}'_{dF})$$

$$\times \phi_d(\mathbf{r}'_{p1})^* \varphi^{A+1*}_{l_f, j_f, m_f}(\mathbf{r}'_{A2}) \frac{2\mu_{dF}}{\hbar^2} v(\mathbf{r}'_{p2}) \phi_d(\mathbf{r}'_{p1}) \phi_d(\mathbf{r}'_{p2}) \chi^{(+)}_{tA}(\mathbf{r}'_{tA}), \tag{7.1.5b}$$

Nota para Gregory

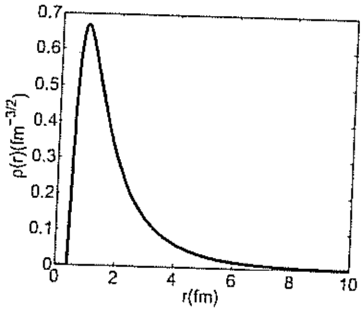
Los detalles de como ajustas V_0 y que potencial $\vec{l}\cdot\vec{s}$ usas (uno tipo para ^{11}Li (B+Mottelson) otro para Sn, etc) lo escribes en la descripción del software COOP.

3



Radial function $P(r)$ (hard core 0.45 fm) entering the

Figure 7.1.1: ~~T~~tritium wavefunction (cf. Tang and Herndon, 1965)



Radial wavefunction $P_d(r)$ (hard core 0.45 fm) entering the

Figure 7.1.2: ~~D~~deuteron wavefunction (cf. Tang and Herndon, 1965)

4

$$T^{(2)}_{NO} = 2\sum_{l_i,j_i}\sum_{l_f,j_f,m_f}\sum_{\substack{\sigma_1\sigma_2\\ \sigma_1'\sigma_2'}}\int d\mathbf{r}_{dF}d\mathbf{r}_{p1}d\mathbf{r}_{A2}[\phi^{A+2}_{l_i,j_i}(\mathbf{r}_{A1},\sigma_1,\mathbf{r}_{A2},\sigma_2)]^{0*}_0\chi^{(-)*}_{pB}(\mathbf{r}_{pB})v(\mathbf{r}_{p1})$$

$$\times\,\phi_d(\mathbf{r}_{p1})\varphi^{A+1}_{l_f,j_f,m_f}(\mathbf{r}_{A2})\int d\mathbf{r}'_{p1}d\mathbf{r}'_{A2}d\mathbf{r}'_{dF}$$

e

$$\times\,\phi_d(\mathbf{r}'_{p1})^*\varphi^{A+1*}_{l_f,j_f,m_f}(\mathbf{r}'_{A2})\phi_d(\mathbf{r}'_{p1})\phi_d(\mathbf{r}'_{p2})\chi^{(+)}_{tA}(\mathbf{r}'_{tA}). \qquad (7.1.5c)$$

The quantities $\mu_i, \mu_f(k_i, k_f)$ are the reduced masses (relative linear momenta) in both entrance (initial, i) and exit (final, f) channels, respectively. In the above expressions, $\varphi^{A+1}_{l_f,j_f,m_f}(\mathbf{r}_{A1})$ are the wavefunctions describing the intermediate states of the nucleus $F(\equiv(A+1))$ generated as solutions of a Woods–Saxon potential, ~~and~~ $\phi_d(\mathbf{r}_{p2})$ being the the deuteron bound wavefunction (see Fig. 7.1.2). Note that some or all of the single–particle states described by the wavefunctions $\varphi^{A+1}_{l_f,j_f,m_f}(\mathbf{r}_{A1})$ may lie in the continuum (case in which the nucleus F is loosely bound or unbound). Although there are a number of ways to exactly treat such states, discretization processes may be sufficiently accurate. They can be implemented by, for example, embedding ~~the Woods–Saxon~~ potential in a spherical box of sufficiently large radius. ~~In~~ actual calculations involving the halo nucleus ^{11}Li, and where $|F\rangle = |^{10}\mathrm{Li}\rangle$, one achieved convergence making use of ~~about~~ 20 continuum states and a box of 30 fm radius. Concerning the components of the triton wavefunction describing the relative motion of the dineutron, it was generated with the $p-n$ ~~Tang–Herndon~~ interaction of (Tang and Herndon, 1965)

$F(\equiv(A+1))$

of

approximately

$$v(r) = -v_0\exp(-k(r-r_c)) \quad r > r_c \qquad (7.1.6)$$

$$v(r) = \infty \quad r < r_c, \qquad (7.1.7)$$

where $k = 2.5$ fm^{-1} and $r_c = 0.45$ fm, the depth v_0 being adjusted to reproduce the experimental separation energies. The positive–energy wavefunctions $\chi^{(+)}_{tA}(\mathbf{r}_{tA})$ and $\chi^{(-)}_{pB}(\mathbf{r}_{pB})$ are the ingoing distorted wave in the initial channel and the outgoing distorted wave in the final channel respectively. They are continuum solutions of the Schrödinger equation associated with the corresponding optical potentials.

The transition potential responsible for the transfer of the pair is, in the *post* representation, (cf. Fig. 7.C.1)

$$V_\beta = v_{pB} - U_\beta, \qquad (7.1.8)$$

where v_{pB} is the interaction between the proton and nucleus B, and U_β is the optical potential in the final channel. We make the assumption that v_{pB} can be decomposed into a term containing the interaction between A and p and the potential describing the interaction between the proton and each of the transferred nucleons, namely

$$v_{pB} = v_{pA} + v_{p1} + v_{p2}, \qquad (7.1.9)$$

where v_{p1} and v_{p2} is the hard–core potential (~~7.2.116~~ 7.1.6). The transition potential is

$$V_\beta = v_{pA} + v_{p1} + v_{p2} - U_\beta. \qquad (7.1.10)$$

Assuming that $\langle\beta|v_{pA}|\alpha\rangle \approx \langle\beta|U_\beta|\alpha\rangle$ (i.e, assuming that the matrix element of the core–core interaction between the initial and final states is very similar to the matrix element of the real part of the optical potential), one obtains the final expression of the transfer potential in the *post* representation, namely,

$$V_\beta \simeq v_{p1} + v_{p2} = v(\mathbf{r}_{p1}) + v(\mathbf{r}_{p2}). \qquad (7.1.11)$$

5

We make the further approximation of using the same interaction potential in all (~~eg~~ i.e. initial, intermediate and final) the channels.

The extension to a heavy–ion reaction $A + a(\equiv b+2) \longrightarrow B(\equiv A+2) + b$ imply no essential modifications in the formalism. The deuteron and triton wavefunctions appearing in Eqs. (7.1.5a), (7.1.5b) and (7.1.5c) are to be substituted with the corresponding wavefunctions $\Psi_{b+2}(\xi_b, \mathbf{r}_{b1}, \sigma_1, \mathbf{r}_{b2}, \sigma_2)$, constructed in a similar way as ~~in (7.2.110,7.2.111)~~ those appearing in (7.1.1 and 7.1.2). The interaction potential used in Eqs. (7.1.5a), (7.1.5b) and (7.1.5c) will now be the Saxon–Woods used to define the initial (final) state in the post (prior) representation, instead of the proton–neutron interaction (~~7.2.116~~ 7.1.6).

The Green's function $G(\mathbf{r}_{dF}, \mathbf{r}'_{dF})$ appearing in (7.1.5b) propagates the intermediate channel d, F_0 ~~and~~ It can be expanded in partial waves as,

$$G(\mathbf{r}_{dF}, \mathbf{r}'_{dF}) = i \sum_l \sqrt{2l+1}\,\frac{f_l(k_{dF}, r_<)\, g_l(k_{dF}, r_>)}{k_{dF}\, r_{dF}\, r'_{dF}} \left[Y^l(\hat{r}_{dF}) Y^l(\hat{r}'_{dF})\right]^0_0 . \qquad (7.1.12)$$

The $f_l(k_{dF}, r)$ and $g_l(k_{dF}, r)$ are the regular and the irregular solutions of a Schrödinger equation for a suitable optical potential and an energy equal to the kinetic energy of the intermediate state. In most cases of interest, the result is hardly altered if we use the same energy of relative motion for all the intermediate states. This representative energy is calculated when both intermediate nuclei are in their corresponding ground states. It is of ~~note~~ notice that that the validity of this approximation can break down in some particular cases. If, for example, some relevant intermediate state become off shell, its contribution is significantly quenched. An interesting situation can arise when this happens to all possible intermediate states, so they can only be virtually ~~populated~~.

7.2 Detailed derivation of ~~2nd~~ second order DWBA

7.2.1 Simultaneous transfer: distorted waves

For a (t, p) reaction, the triton is represented by an incoming distorted wave. We make the assumption that the two neutrons are in an $S = L = 0$ state, and that the relative motion of the proton with respect to the dineutron is also $l = 0$. Consequently, the total spin of the triton is entirely due to the spin of the proton. We will explicitly treat it, as we will consider a spin–orbit term in the optical potential acting between the triton and the target. In what follows we ~~will use~~ follow the notation of Bayman (1971).

Following ~~(??)~~ (6.G.29) and (6.G.33), we can write the triton distorted wave as

$$\psi^{(+)}_{m_t}(\mathbf{R}, \mathbf{k}_i, \sigma_p) = \sum_{l_t} \exp\left(i\sigma^t_{l_t}\right) g_{l_t j_t}\, Y^{l_t}_0(\hat{\mathbf{R}}) \frac{\sqrt{4\pi(2l_t+1)}}{k_i R}\, \chi_{m_t}(\sigma_p), \qquad (7.2.1)$$

where use was made of $Y^{l_t}_0(\hat{\mathbf{k}}_i) = i^{l_t}\sqrt{\frac{2l_t+1}{4\pi}}\,\delta_{m_t,0}$, in keeping with the fact that $\mathbf{k}_i$ is oriented along the z–axis. Note the phase difference with eq. (7) of Bayman (1971), due to the use of time–reversal rather than Condon–Shortley phase convention. ~~If we write~~ Making use of the relation

$$Y^{l_t}_0(\hat{\mathbf{R}})\chi_{m_t}(\sigma_p) = \sum_{j_t} \langle l_t\ 0\ 1/2\ m_t | j_t\ m_t\rangle \left[Y^{l_t}(\hat{\mathbf{R}})\chi(\sigma_p)\right]^{j_t}_{m_t}, \qquad (7.2.2)$$

gregory check

we have

(6.G,35)

$$\psi_{m_t}^{(+)}(\mathbf{R}, \mathbf{k}_i, \sigma_p) = \sum_{l_t, j_t} \exp\left(i\sigma_{l_t}^t\right) \frac{\sqrt{4\pi(2l_t+1)}}{k_i R} g_{l_t j_t}(R) \times \langle l_t\ 0\ 1/2\ m_t | j_t\ m_t\rangle \left[Y^{l_t}(\hat{\mathbf{R}})\chi(\sigma_p)\right]^{j_t}_{m_t}. \tag{7.2.3}$$

We now turn our attention to the outgoing proton distorted wave, which, following ~~(27), we can write it as~~ can be written as,

$$\psi_{m_p}^{(-)}(\zeta, \mathbf{k}_f, \sigma_p) = \sum_{l_p j_p} \frac{4\pi}{k_f \zeta} i^{l_p} \exp\left(-i\sigma_{l_p}^p\right) f^*_{l_p j_p}(\zeta) \sum_m Y^{l_p}_m(\hat{\zeta}) Y^{l_p *}_m(\hat{\mathbf{k}}_f) \chi_{m_p}(\sigma_p). \tag{7.2.4}$$

Making use of the relation

$$\begin{aligned}
\sum_m Y^{l_p}_m(\hat{\zeta}) Y^{l_p *}_m(\hat{\mathbf{k}}_f) \chi_{m_p}(\sigma_p) &= \sum_{m, j_p} Y^{l_p *}_m(\hat{\mathbf{k}}_f) \langle l_p\ m\ 1/2\ m_p | j_p\ m+m_p\rangle \\
&\quad \times \left[Y^{l_p}(\hat{\zeta})\chi_{m_p}(\sigma_p)\right]^{j_p}_{m+m_p} \\
&= \sum_{m, j_p} Y^{l_p *}_{m-m_p}(\hat{\mathbf{k}}_f) \langle l_p\ m-m_p\ 1/2\ m_p | j_p\ m\rangle \left[Y^{l_p}(\hat{\zeta})\chi_{m_p}(\sigma_p)\right]^{j_p}_m,
\end{aligned} \tag{7.2.5}$$

one obtains

$$\psi_{m_p}^{(-)}(\zeta, \mathbf{k}_f, \sigma_p) = \frac{4\pi}{k_f \zeta} \sum_{l_p j_p, m} i^{l_p} \exp\left(-i\sigma_{l_p}^p\right) f^*_{l_p j_p}(\zeta) Y^{l_p *}_{m-m_p}(\hat{\mathbf{k}}_f) \times \langle l_p\ m-m_p\ 1/2\ m_p | j_p\ m\rangle \left[Y^{l_p}(\hat{\zeta})\chi(\sigma_p)\right]^{j_p}_m. \tag{7.2.6}$$

7.2.2 matrix element for the transition amplitude

We now turn our attention to the evaluation of

$$\begin{aligned}
\langle \Psi_f^{(-)}(\mathbf{k}_f)|V(r_{1p})|\Psi_i^{(+)}(k_i, \hat{\mathbf{z}})\rangle &= \frac{(4\pi)^{3/2}}{k_i k_f} \sum_{l_p l_t j_p j_t m} ((\lambda \tfrac{1}{2})_k (\lambda \tfrac{1}{2})_k | (\lambda\lambda)_0 (\tfrac{1}{2}\tfrac{1}{2})_0)_0 \sqrt{2l_t+1} \\
&\times \langle l_p\ m-m_p\ 1/2\ m_p | j_p\ m\rangle \langle l_t\ 0\ 1/2\ m_t | j_t\ m_t\rangle\, i^{-l_p} \exp[i(\sigma^p_{l_p} + \sigma^t_{l_t})] \\
&\times 2 Y^{l_p}_{m-m_p}(\hat{\mathbf{k}}_f) \sum_{\sigma_1 \sigma_2 \sigma_p} \int \frac{d\zeta d\mathbf{r} d\eta}{\zeta R} u_{\lambda k}(r_1) u_{\lambda k}(r_2) \left[Y^\lambda(\hat{\mathbf{r}}_1) Y^\lambda(\hat{\mathbf{r}}_2)\right]^{0*}_0 \\
&\times f_{l_p j_p}(\zeta) g_{l_t j_t}(R) \left[\chi(\sigma_1)\chi(\sigma_2)\right]^{0*}_0 \left[Y^{l_p}(\hat{\zeta})\chi(\sigma_p)\right]^{j_p *}_m V(r_{1p}) \\
&\times \theta^0_0(\mathbf{r}, \mathbf{s}) \left[\chi(\sigma_1)\chi(\sigma_2)\right]^0_0 \left[Y^{l_t}(\hat{\mathbf{R}})\chi(\sigma_p)\right]^{j_t}_{m_t},
\end{aligned} \tag{7.2.7}$$

where

$$\begin{aligned}
\mathbf{r} &= \mathbf{r}_2 - \mathbf{r}_1, \\
\mathbf{s} &= \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2) - \mathbf{r}_p, \\
\eta &= \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2), \\
\zeta &= \mathbf{r}_p - \frac{\mathbf{r}_1 + \mathbf{r}_2}{A+2}.
\end{aligned} \tag{7.2.8}$$

(cf. (6.6.17))

7

The sum over σ_1, σ_2 in (7.2.7) is found to be equal to 1. We will now simplify the term $\left[Y^{l_p}(\hat{\zeta})\chi(\sigma_p)\right]^{j_p*}_m \left[Y^{l_t}(\hat{\mathbf{R}})\chi(\sigma_p)\right]^{j_t}_{m_t}$, noting that

$$\left[Y^{l_p}(\hat{\zeta})\chi(\sigma_p)\right]^{j_p*}_m = (-1)^{1/2-\sigma_p+j_p-m}\left[Y^{l_p}(\hat{\zeta})\chi(-\sigma_p)\right]^{j_p}_{-m}. \qquad (7.2.9)$$

and that

$$\left[Y^{l_p}(\hat{\zeta})\,\chi(-\sigma_p)\right]^{j_p}_{-m}\left[Y^{l_t}(\hat{\mathbf{R}})\chi(\sigma_p)\right]^{j_t}_{m_t} = \sum_{JM}\langle j_p\ -m\ j_t\ m_t|J\ M\rangle \times\left\{\left[Y^{l_p}(\hat{\zeta})\chi(-\sigma_p)\right]^{j_p}\left[Y^{l_t}(\hat{\mathbf{R}})\chi(\sigma_p)\right]^{j_t}\right\}^J_M \qquad (7.2.10)$$

The only term which ~~do~~ does not vanish after the integration is performed is the one in which the angular and spin functions are coupled to $L=0, S=0, J=0$/~~thus~~. Thus,

$$\langle j_p\ -m\ j_t\ m_t|0\ 0\rangle\left\{\left[Y^{l_p}(\hat{\zeta})\chi(-\sigma_p)\right]^{j_p}\left[Y^{l_t}(\hat{\mathbf{R}})\chi(\sigma_p)\right]^{j_t}\right\}^0_0\delta_{l_pl_t}\delta_{j_pj_t}\delta_{mm_t}$$
$$= \frac{(-1)^{j_p+m_t}}{\sqrt{2j_p+1}}\left\{\left[Y^{l_p}(\hat{\zeta})\chi(-\sigma_p)\right]^{j_p}\left[Y^{l_t}(\hat{\mathbf{R}})\chi(\sigma_p)\right]^{j_t}\right\}^0_0\delta_{l_pl_t}\delta_{j_pj_t}\delta_{mm_t}. \qquad (7.2.11)$$

[illegible]

Coupling separately the spin and ~~spatial~~ the angular function, one obtains

$$\left\{\left[Y^{l}(\hat{\zeta})\chi(-\sigma_p)\right]^{j}\left[Y^{l}(\hat{\mathbf{R}})\chi(\sigma_p)\right]^{j}\right\}^0_0 = ((l\tfrac{1}{2})_j(l\tfrac{1}{2})_j|(ll)_0(\tfrac{1}{2}\tfrac{1}{2})_0)_0\left[\chi(-\sigma_p)\chi(\sigma_p)\right]^0_0\left[Y^l(\hat{\zeta})Y^l(\hat{\mathbf{R}})\right]^0_0. \qquad (7.2.12)$$

OK

We substitute (7.2.9),(7.2.30),(7.2.31) in (7.2.7) to obtain [illegible]

OK

$$\langle\Psi^{(-)}_f(\mathbf{k}_f)|V(r_{1p})|\Psi^{(+)}_i(k_i,\hat{\mathbf{z}})\rangle = -\frac{(4\pi)^{3/2}}{k_ik_f}\sum_{lj}((\lambda\tfrac{1}{2})_k(\lambda\tfrac{1}{2})_k|(\lambda\lambda)_0(\tfrac{1}{2}\tfrac{1}{2})_0)_0\sqrt{\frac{2l+1}{2j+1}}$$
$$\times\langle l\ m_t-m_p\ 1/2\ m_p|j\ m_t\rangle\langle l\ 0\ 1/2\ m_t|j\ m_t\rangle\, i^{-l}\exp[i(\sigma^p_l+\sigma^t_l)]$$
$$\times 2Y^l_{m_t-m_p}(\hat{\mathbf{k}}_f)\int\frac{d\zeta drd\eta}{\zeta R}u_{\lambda k}(r_1)u_{\lambda k}(r_2)\left[Y^\lambda(\hat{\mathbf{r}}_1)Y^\lambda(\hat{\mathbf{r}}_2)\right]^{0*}_0$$
$$\times f_{lj}(\zeta)g_{lj}(R)\left[Y^l(\hat{\zeta})Y^l(\hat{\mathbf{R}})\right]^0_0 V(r_{1p})\theta^0_0(\mathbf{r},\mathbf{s})$$
$$\times((l\tfrac{1}{2})_j(l\tfrac{1}{2})_j|(ll)_0(\tfrac{1}{2}\tfrac{1}{2})_0)_0\sum_{\sigma_p}(-1)^{1/2-\sigma_p}\left[\chi(-\sigma_p)\chi(\sigma_p)\right]^0_0. \qquad (7.2.13)$$

The last sum over σ_p leads to

$$\sum_{\sigma_p}(-1)^{1/2-\sigma_p}\left[\chi(-\sigma_p)\chi(\sigma_p)\right]^0_0 = \sum_{\sigma_pm}(-1)^{1/2-\sigma_p}\langle 1/2\ m\ 1/2\ -m|0\ 0\rangle \times\chi_m(-\sigma_p)\chi_{-m}(\sigma_p)$$
$$= \frac{1}{\sqrt{2}}\sum_{\sigma_pm}(-1)^{1/2-\sigma_p}(-1)^{1/2-m}\delta_{m,-\sigma_p}\delta_{-m,\sigma_p} = -\sqrt{2}. \qquad (7.2.14)$$

8

The $9j$–symbols can be evaluated to find

$$
\begin{aligned}
((\lambda\tfrac{1}{2})_k(\lambda\tfrac{1}{2})_k|(\lambda\lambda)_0(\tfrac{1}{2}\tfrac{1}{2})_0)_0 &= \sqrt{\frac{2k+1}{2(2\lambda+1)}} \\
((l\tfrac{1}{2})_j(l\tfrac{1}{2})_j|(ll)_0(\tfrac{1}{2}\tfrac{1}{2})_0)_0 &= \sqrt{\frac{2j+1}{2(2l+1)}},
\end{aligned}
\tag{7.2.15}
$$

and consequently,

$$
\begin{aligned}
\langle\Psi_f^{(-)}(\mathbf{k}_f)|V(r_{1p})|\Psi_i^{(+)}(k_i,\hat{\mathbf{z}})\rangle &= \frac{(4\pi)^{3/2}}{k_ik_f}\sum_{lj}\sqrt{\frac{2k+1}{2\lambda+1}} \\
&\times\langle l\; m_t-m_p\; 1/2\; m_p|j\; m_t\rangle\langle l\; 0\; 1/2\; m_t|j\; m_t\rangle\, i^{-l}\exp[i(\sigma_l^p+\sigma_l^t)] \\
&\times\sqrt{2}Y^l_{m_t-m_p}(\hat{\mathbf{k}}_f)\int\frac{d\zeta d\mathbf{r}d\eta}{\zeta R}u_{\lambda k}(r_1)u_{\lambda k}(r_2)\left[Y^\lambda(\hat{\mathbf{r}}_1)Y^\lambda(\hat{\mathbf{r}}_2)\right]_0^{0*} \\
&\times f_{lj}(\zeta)g_{lj}(R)\left[Y^l(\hat{\zeta})Y^l(\hat{\mathbf{R}})\right]_0^0 V(r_{1p})\theta_0^0(\mathbf{r},\mathbf{s}).
\end{aligned}
\tag{7.2.16}
$$

The ~~possible~~ values of the Clebsh–Gordan coefficients are, for $j=l-1/2$,

e

$$
\begin{aligned}
&\langle l\; m_t-m_p\; 1/2\; m_p|l-1/2\; m_t\rangle\langle l\; 0\; 1/2\; m_t|l-1/2\; m_t\rangle \\
&\qquad=\begin{cases}\dfrac{l}{2l+1} & \text{if } m_t=m_p \\[2ex] -\dfrac{\sqrt{l(l+1)}}{2l+1} & \text{if } m_t=-m_p\end{cases}
\end{aligned}
\tag{7.2.17}
$$

and, for $j=l+1/2$:

$$
\begin{aligned}
&\langle l\; m_t-m_p\; 1/2\; m_p|l+1/2\; m_t\rangle\langle l\; 0\; 1/2\; m_t|l+1/2\; m_t\rangle \\
&\qquad=\begin{cases}\dfrac{l+1}{2l+1} & \text{if } m_t=m_p \\[2ex] \dfrac{\sqrt{l(l+1)}}{2l+1} & \text{if } m_t=-m_p\end{cases}
\end{aligned}
\tag{7.2.18}
$$

One thus can write,

$$
\begin{aligned}
\langle\Psi_f^{(-)}(\mathbf{k}_f)|V(r_{1p})|\Psi_i^{(+)}(k_i,\hat{\mathbf{z}})\rangle &= \frac{(4\pi)^{3/2}}{k_ik_f}\sum_l\frac{1}{(2l+1)}\sqrt{\frac{(2k+1)}{(2\lambda+1)}}\exp[i(\sigma_l^p+\sigma_l^t)]i^{-l} \\
&\times\sqrt{2}Y^l_{m_t-m_p}(\hat{\mathbf{k}}_f)\int\frac{d\zeta d\mathbf{r}d\eta}{\zeta R}u_{\lambda k}(r_1)u_{\lambda k}(r_2)\left[Y^\lambda(\hat{\mathbf{r}}_1)Y^\lambda(\hat{\mathbf{r}}_2)\right]_0^{0*} \\
&\times V(r_{1p})\theta_0^0(\mathbf{r},\mathbf{s})\left[Y^l(\hat{\zeta})Y^l(\hat{\mathbf{R}})\right]_0^0 \\
&\times\Big[\big(f_{ll+1/2}(\zeta)g_{ll+1/2}(R)(l+1)+f_{ll-1/2}(\zeta)g_{ll-1/2}(R)l\big)\delta_{m_p,m_t} \\
&+\big(f_{ll+1/2}(\zeta)g_{ll+1/2}(R)\sqrt{l(l+1)}-f_{ll-1/2}(\zeta)g_{ll-1/2}(R)\sqrt{l(l+1)}\big)\delta_{m_p,-m_t}\Big].
\end{aligned}
\tag{7.2.19}
$$

9

We can further simplify this expression using

$$\begin{aligned}
\left[Y^{\lambda}(\hat{\mathbf{r}}_1)Y^{\lambda}(\hat{\mathbf{r}}_2)\right]_0^{0*} &= \left[Y^{\lambda}(\hat{\mathbf{r}}_1)Y^{\lambda}(\hat{\mathbf{r}}_2)\right]_0^{0} = \sum_m \langle \lambda\, m\, \lambda\, -m|0\ 0\rangle Y^{\lambda}_m(\hat{\mathbf{r}}_1)Y^{\lambda}_{-m}(\hat{\mathbf{r}}_2) \\
&= \sum_m (-1)^{\lambda-m}\langle \lambda\, m\, \lambda\, -m|0\ 0\rangle Y^{\lambda}_m(\hat{\mathbf{r}}_1)Y^{\lambda *}_m(\hat{\mathbf{r}}_2) \\
&= \frac{1}{\sqrt{2\lambda+1}}\sum_m Y^{\lambda}_m(\hat{\mathbf{r}}_1)Y^{\lambda *}_m(\hat{\mathbf{r}}_2) \\
&= \frac{\sqrt{(2\lambda+1)}}{4\pi}P_{\lambda}(\cos\theta_{12}).
\end{aligned} \tag{7.2.20}$$

Note that when using Condon–Shortley phases this last expression is to be multiplied by $(-1)^{\lambda}$, and that

$$\begin{aligned}
\left[Y^{l}(\hat{\zeta})Y^{l}(\hat{\mathbf{R}})\right]_0^{0} &= \sum_m \langle l\, m\, l\, -m|0\ 0\rangle Y^{l}_m(\hat{\zeta})Y^{l}_{-m}(\hat{\mathbf{R}}) \\
&= \frac{1}{\sqrt{(2l+1)}}\sum_m (-1)^{l+m} Y^{l}_m(\hat{\zeta})Y^{l}_{-m}(\hat{\mathbf{R}}).
\end{aligned} \tag{7.2.21}$$

Because the integral of the above expression is independent of m, one can eliminate the m–sum and multiply by $2l+1$ the $m=0$ term, leading to

$$\begin{aligned}
\left[Y^{l}(\hat{\zeta})Y^{l}(\hat{\mathbf{R}})\right]_0^{0} &\Rightarrow (-1)^l\sqrt{(2l+1)}\, Y^{l}_0(\hat{\zeta})_0 Y^{l}(\hat{\mathbf{R}}) \\
&= \sqrt{(2l+1)}Y^{l}_0(\hat{\zeta})Y^{l*}_0(\hat{\mathbf{R}}).
\end{aligned} \tag{7.2.22}$$

We now change the integration variables from $(\zeta, \mathbf{r}, \eta)$ to $(\mathbf{R}, \alpha, \beta, \gamma, r_{12}, r_{1p}, r_{2p})$, the quantity

$$\left|\frac{\partial(\mathbf{r}, \eta, \zeta)}{\partial(\mathbf{R}, \alpha, \beta, \gamma, r_{12}, r_{1p}, r_{2p})}\right| = r_{12}r_{1p}r_{2p}\sin\beta \tag{7.2.23}$$

being the Jacobian of the transformation. Finally,

$$\begin{aligned}
\langle \Psi_f^{(-)}(\mathbf{k}_f)|V(r_{1p})|\Psi_i^{(+)}(k_i, \hat{\mathbf{z}})\rangle &= \frac{\sqrt{8\pi}}{k_ik_f}\sum_l \sqrt{\frac{2k+1}{2l+1}}\exp[i(\sigma_l^p + \sigma_l^t)]i^{-l} \\
&\times Y^{l}_{m_t-m_p}(\hat{\mathbf{k}}_f)\int d\mathbf{R}\, Y^{l*}_0(\hat{\mathbf{R}})\int \frac{d\alpha\, d\beta\, d\gamma\, dr_{12}\, dr_{1p}\, dr_{2p}\ \sin\beta}{\zeta R}Y^{l}_0(\hat{\zeta}) \\
&\times u_{\lambda k}(r_1)u_{\lambda k}(r_2)V(r_{1p})\theta^0_0(\mathbf{r}, \mathbf{s})P_{\lambda}(\cos\theta_{12})r_{12}r_{1p}r_{2p} \\
&\times \Big[\big(f_{ll+1/2}(\zeta)g_{ll+1/2}(R)(l+1) + f_{ll-1/2}(\zeta)g_{ll-1/2}(R)l\big)\delta_{m_p,m_t} \\
&+\big(f_{ll+1/2}(\zeta)g_{ll+1/2}(R)\sqrt{l(l+1)} - f_{ll-1/2}(\zeta)g_{ll-1/2}(R)\sqrt{l(l+1)}\big)\delta_{m_p,-m_t}\Big].
\end{aligned} \tag{7.2.24}$$

~~It is~~ noted that the second integral is a function of solely **R** transforming under rotations as $Y^l_0(\hat{\mathbf{R}})$, in keeping with the fact that the full dependence on the orientation of **R** is contained in the spherical harmonic $Y^l_0(\hat{\zeta})$. The second integral can thus be cast into the form

$$\begin{aligned}
A(R)Y^l_0(\hat{\mathbf{R}}) = \int d\alpha\, d\beta\, d\gamma\, dr_{12}\, dr_{1p}\, dr_{2p}\ \sin\beta \\
\times F(\alpha, \beta, \gamma, r_{12}, r_{1p}, r_{2p}, R_x, R_y, R_z).
\end{aligned} \tag{7.2.25}$$

10

To evaluate $A(R)$, we set $\mathbf{R}$ along the z–axis

$$A(R) = 2\pi i^{-l}\sqrt{\frac{4\pi}{2l+1}}\int d\beta\, d\gamma\, dr_{12}\, dr_{1p}\, dr_{2p}\, \sin\beta \\ \times F(\alpha,\beta,\gamma,r_{12},r_{1p},r_{2p},0,0,R), \tag{7.2.26}$$

where a factor 2π results from the integration over α, the integrand not depending on α. Substituting (7.2.25) and (7.2.26) in (7.2.24) and, after ~~integrating~~ integration over the angular variables of $\mathbf{R}$, we obtain

$$\begin{aligned}\langle\Psi_f^{(-)}(\mathbf{k}_f)|V(r_{1p})|\Psi_i^{(+)}(k_i,\hat{\mathbf{z}})\rangle &= 2\frac{(2\pi)^{3/2}}{k_ik_f}\sum_l\sqrt{\frac{2k+1}{2l+1}}\exp[i(\sigma_l^p+\sigma_l^t)]i^{-l}\\ &\times Y^l_{m_t-m_p}(\hat{\mathbf{k}}_f)\int dR\, d\beta\, d\gamma\, dr_{12}\, dr_{1p}\, dr_{2p}\, R\sin\beta\, r_{12}r_{1p}r_{2p}\\ &\times u_{\lambda k}(r_1)u_{\lambda k}(r_2)V(r_{1p})\theta_0^0(\mathbf{r},\mathbf{s})P_\lambda(\cos\theta_{12})P_l(\cos\theta_\zeta)\\ &\times\Big[\Big(f_{ll+1/2}(\zeta)g_{ll+1/2}(R)(l+1)+f_{ll-1/2}(\zeta)g_{ll-1/2}(R)l\Big)\delta_{m_p,m_t}\\ &+\Big(f_{ll+1/2}(\zeta)g_{ll+1/2}(R)\sqrt{l(l+1)}-f_{ll-1/2}(\zeta)g_{ll-1/2}(R)\sqrt{l(l+1)}\Big)\delta_{m_p,-m_t}\Big]/\zeta,\end{aligned} \tag{7.2.27}$$

where use was made of the relation

$$Y_0^l(\hat{\zeta}) = i^l\sqrt{\frac{2l+1}{4\pi}}P_l(\cos\theta_\zeta). \tag{7.2.28}$$

The final expression of the differential cross section involves a sum over the spin orientations:

$$\frac{d\sigma}{d\Omega}(\hat{\mathbf{k}}_f) = \frac{k_f}{k_i}\frac{\mu_i\mu_f}{(2\pi\hbar^2)^2}\frac{1}{2}\sum_{m_t m_p}|\langle\Psi_f^{(-)}(\mathbf{k}_f)|V(r_{1p})|\Psi_i^{(+)}(k_i,\hat{\mathbf{z}})\rangle|^2. \tag{7.2.29}$$

When $m_p = 1/2, m_t = 1/2$ or $m_p = -1/2, m_t = -1/2$, the terms proportional to δ_{m_p,m_t} including the factor

$$|Y^l_{m_t-m_p}(\hat{\mathbf{k}}_f)\delta_{m_p,m_t}| = |Y_0^l(\hat{\mathbf{k}}_f)| = \left|i^l\sqrt{\frac{2l+1}{4\pi}}P_l^0(\cos\theta)\right|, \tag{7.2.30}$$

in the case in which $m_p = -1/2, m_t = 1/2$

$$|Y^l_{m_t-m_p}(\hat{\mathbf{k}}_f)\delta_{m_p,-m_t}| = |Y_1^l(\hat{\mathbf{k}}_f)| = \left|i^l\sqrt{\frac{2l+1}{4\pi}\frac{1}{l(l+1)}}P_l^1(\cos\theta)\right|, \tag{7.2.31}$$

and

$$|Y^l_{m_t-m_p}(\hat{\mathbf{k}}_f)\delta_{m_p,-m_t}| = |Y_{-1}^l(\hat{\mathbf{k}}_f)| = |Y_1^l(\hat{\mathbf{k}}_f)| = \left|i^l\sqrt{\frac{2l+1}{4\pi}\frac{1}{l(l+1)}}P_l^1(\cos\theta)\right|, \tag{7.2.32}$$

when $m_p = 1/2, m_t = -1/2$ Taking the squared modulus of (7.2.27), the sum over m_t and m_p yields a factor 2 multiplying each one of the 2 different terms of the sum

(11)

($m_t = m_p$ and $m_t = -m_p$). This is equivalent to multiply each amplitude by $\sqrt{2}$, so the final constant that multiply the amplitudes is

$$\frac{8\pi^{3/2}}{k_i k_f}. \tag{7.2.33}$$

Now, for the triton wavefunction we use

also used by

$$\theta_0^0(\mathbf{r}, \mathbf{s}) = \rho(r_{1p})\rho(r_{2p})\rho(r_{12}), \tag{7.2.34}$$

(1965)

$\rho(r)$ being a Tang–Herndon wave function as ~~done in~~ Bayman (1971). We obtain

$$\frac{d\sigma}{d\Omega}(\hat{\mathbf{k}}_f) = \frac{1}{2E_i^{3/2}E_f^{1/2}}\sqrt{\frac{\mu_f}{\mu_i}}\left(|I_{\lambda k}^{(0)}(\theta)|^2 + |I_{\lambda k}^{(1)}(\theta)|^2\right), \tag{7.2.35}$$

where

$$\begin{aligned} I_{\lambda k}^{(0)}(\theta) = &\sum_l P_l^0(\cos\theta)\sqrt{2k+1}\exp[i(\sigma_l^p + \sigma_l^t)] \\ &\times \int dR\, d\beta\, d\gamma\, dr_{12}\, dr_{1p}\, dr_{2p}\, R\sin\beta\, \rho(r_{1p})\rho(r_{2p})\rho(r_{12}) \\ &\times u_{\lambda k}(r_1)u_{\lambda k}(r_2)V(r_{1p})P_\lambda(\cos\theta_{12})P_l(\cos\theta_\zeta)r_{12}r_{1p}r_{2p} \\ &\times \left(f_{ll+1/2}(\zeta)g_{ll+1/2}(R)\,(l+1) + f_{ll-1/2}(\zeta)g_{ll-1/2}(R)\,l\right)/\zeta, \end{aligned} \tag{7.2.36}$$

and

$$\begin{aligned} I_{\lambda k}^{(1)}(\theta) = &\sum_l P_l^1(\cos\theta)\sqrt{2k+1}\exp[i(\sigma_l^p + \sigma_l^t)] \\ &\times \int dR\, d\beta\, d\gamma\, dr_{12}\, dr_{1p}\, dr_{2p}\, R\sin\beta\, \rho(r_{1p})\rho(r_{2p})\rho(r_{12}) \\ &\times u_{\lambda k}(r_1)u_{\lambda k}(r_2)V(r_{1p})P_\lambda(\cos\theta_{12})P_l(\cos\theta_\zeta)r_{12}r_{1p}r_{2p} \\ &\times \left(f_{ll+1/2}(\zeta)g_{ll+1/2}(R) - f_{ll-1/2}(\zeta)g_{ll-1/2}(R)\right)/\zeta. \end{aligned} \tag{7.2.37}$$

that

Note the absence of the $(-1)^\lambda$ factor with respect to what is found in Bayman (1971), is due to the use of time–reversed phases instead of Condon–Shortley phasing. This is compensated in the total result by a similar difference in the expression of the spectroscopic amplitudes. This ensures that, in either case, the contribution of all the single particle transitions tend to have the same phase for superfluid nuclei, adding coherently to enhance the transfer cross section.

In ~~If we are~~ dealing with a heavy ion reaction, $\theta_0^0(\mathbf{r}, \mathbf{s})$ ~~will be~~ are the spatial part of the wavefunction

$$\begin{aligned} \Psi(\mathbf{r}_{b1}, \mathbf{r}_{b2}, \sigma_1, \sigma_2) &= \left[\psi^{j_i}(\mathbf{r}_{b1}, \sigma_1)\psi^{j_i}(\mathbf{r}_{b2}, \sigma_2)\right]_0^0 \\ &= \theta_0^0(\mathbf{r}, \mathbf{s})\left[\chi(\sigma_1)\chi(\sigma_2)\right]_0^0, \end{aligned} \tag{7.2.38}$$

where $\mathbf{r}_{b1}, \mathbf{r}_{b2}$ are the positions of the two neutrons with respect to the b core. It can be shown to be

$$\boxed{\theta_0^0(\mathbf{r}, \mathbf{s}) = \frac{u_{l_i j_i}(r_{b1})u_{l_i j_i}(r_{b2})}{4\pi}\sqrt{\frac{2j_i+1}{2}}P_{l_i}(\cos\theta_i),} \tag{7.2.39}$$

(a) - (b)

Simultaneous light ions ~~Thus 3 different ways~~

~~Bayman light ions Applied Plenty~~

$T^{1step}_{2N}(\theta)$

(12)

where θ_i is the angle between $\mathbf{r}_{b1}$ and $\mathbf{r}_{b2}$. Neglecting the spin–orbit term in the optical potential, as is usually done for heavy ion reactions, one obtains

$T^{1step}_{2N}(\theta)$

$$\frac{d\sigma}{d\Omega}(\hat{\mathbf{k}}_f) = \frac{\mu_f \mu_i}{16\pi^2\hbar^4 k_i^3 k_f}|T_0^{j_i,j_f}(\theta)|^2, \tag{7.2.40}$$

where

$$\begin{aligned} T_l^{j_i,j_f}(\theta) = & \sum_l (2l+1) P_l(\cos\theta)\sqrt{(2j_i+1)(2j_f+1)}\exp[i(\sigma_l^p + \sigma_l^t)] \\ & \times \int dR\, d\beta\, d\gamma\, dr_{12}\, dr_{b1}\, dr_{b2}\, R\sin\beta\, u_{l_i j_i}(r_{b1}) u_{l_i j_i}(r_{b2}) \\ & \times u_{l_f j_f}(r_{A1}) u_{l_f j_f}(r_{A2}) V(r_{b1}) P_\lambda(\cos\theta_{12}) P_l(\cos\theta_\zeta) \\ & \times r_{12} r_{b1} r_{b2} P_{l_i}(\cos\theta_i)\frac{f_l(\zeta) g_l(R)}{\zeta}, \end{aligned} \tag{7.2.41}$$

(7.2.34)

obtained by using (7.2.39) in (7.2.7) instead of

$\mathbf{r}_{A1}, \mathbf{r}_{A2}$ being the coordinates of the two transferred of the two neutrons with respect to the A core.

~~alternating~~ ~~another derivation~~ ~~independent~~

to p. (11) ← (a) **Heavy–ion Reactions** (a)

~~The~~ A distorted waves for a reaction taking place between spinless nuclei, ~~are~~ namely,

$$\psi^{(+)}(\mathbf{r}_{Aa}, \mathbf{k}_{Aa}) = \sum_l \exp\left(i\sigma_l^i\right) g_l Y_0^l(\hat{\mathbf{r}}_{aA})\frac{\sqrt{4\pi(2l+1)}}{k_{aA} r_{aA}}, \tag{7.2.42}$$

and

$$\psi^{(-)}(\mathbf{r}_{bB}, \mathbf{k}_{bB}) = \frac{4\pi}{k_{bB} r_{bB}}\sum_l i^l \exp\left(-i\sigma_l^f\right) f_l^*(r_{bB})\sum_m Y_m^{l*}(\hat{\mathbf{k}}_{bB}) Y_m^l(\hat{\mathbf{r}}_{bB}). \tag{7.2.43}$$

~~Matrix element;~~ One can then write,

$$\begin{aligned} T^{1step}_{2N} = \langle\Psi_f^{(-)}(\mathbf{k}_{bB})|V(r_{1p})|\Psi_i^{(+)}(k_{aA},\hat{\mathbf{z}})\rangle = & \frac{(4\pi)^{3/2}}{k_{aA}k_{bB}}\sum_{llm}((l_f\tfrac{1}{2})_{j_f}(l_f\tfrac{1}{2})_{j_f}|(l_f l_f)_0(\tfrac{1}{2}\tfrac{1}{2})_0)_0 \\ & \times ((l_i\tfrac{1}{2})_{j_i}(l_i\tfrac{1}{2})_{j_i}|(l_i l_i)_0(\tfrac{1}{2}\tfrac{1}{2})_0)_0\,\sqrt{2l+1}\, i^{-l_p}\exp[i(\sigma_l^f + \sigma_l^i)] \\ & \times 2Y_m^l(\hat{\mathbf{k}}_{bB})\sum_{\sigma_1\sigma_2}\int\frac{dr_{bB}\,dr\,d\eta}{r_{bB} r_{aA}} u_{l_f j_f}(r_{A1}) u_{l_f j_f}(r_{A2}) u_{l_i j_i}(r_{b1}) u_{l_i j_i}(r_{b2}) \\ & \times\left[Y^{l_f}(\hat{\mathbf{r}}_{A1}) Y^{l_f}(\hat{\mathbf{r}}_{A2})\right]_0^{0*}\left[Y^{l_i}(\hat{\mathbf{r}}_{b1}) Y^{l_i}(\hat{\mathbf{r}}_{b2})\right]_0^0 \\ & \times f_l(r_{bB}) g_l(r_{aA})\left[\chi(\sigma_1)\chi(\sigma_2)\right]_0^{0*} Y_m^{l*}(\hat{\mathbf{r}}_{bB}) V(r_{1p}) \\ & \times\left[\chi(\sigma_1)\chi(\sigma_2)\right]_0^0 Y_0^l(\hat{\mathbf{r}}_{aA}). \end{aligned} \tag{7.2.44}$$

For control, in what follows we work out the same transition amplitude but starting from the

~~Simplifying a~~ which after a number of simplifications becomes

13

$$\begin{aligned}\langle \Psi_f^{(-)}(\mathbf{k}_{bB})|V(r_{1p})|\Psi_i^{(+)}(k_{aA},\hat{\mathbf{z}})\rangle &= \frac{(4\pi)^{3/2}}{k_{aA}k_{bB}}\sum_{l\bar{l}m}\sqrt{\frac{(2j_f+1)(2j_i+1)}{(2l_f+1)(2l_i+1)}}\\ &\times \sqrt{2\bar{l}+1}\,i^{-\bar{l}}\exp[i(\sigma_{\bar{l}}^f+\sigma_l^i)]\\ &\times Y_m^{\bar{l}}(\hat{\mathbf{k}}_{bB})\int\frac{d\mathbf{r}_{bB}d\mathbf{r}d\eta}{r_{bB}r_{aA}}u_{l_fj_f}(r_{A1})u_{l_fj_f}(r_{A2})u_{l_ij_i}(r_{b1})u_{l_ij_i}(r_{b2})\\ &\times\left[Y^{l_f}(\hat{\mathbf{r}}_{A1})Y^{l_f}(\hat{\mathbf{r}}_{A2})\right]_0^{0*}\left[Y^{l_i}(\hat{\mathbf{r}}_{b1})Y^{l_i}(\hat{\mathbf{r}}_{b2})\right]_0^{0}\\ &\times f_{\bar{l}}(r_{bB})g_l(r_{aA})Y_m^{\bar{l}*}(\hat{\mathbf{r}}_{bB})V(r_{1p})Y_0^l(\hat{\mathbf{r}}_{aA}),\end{aligned} \tag{7.2.45}$$

where ~~We clearly need~~ $l=\bar{l}$ and $m=0$. ~~We also introduce the~~ Making use of Legendre polynomials, leads to,

$$\begin{aligned}\langle \Psi_f^{(-)}(\mathbf{k}_{bB})|V(r_{1p})|\Psi_i^{(+)}(k_{aA},\hat{\mathbf{z}})\rangle &= \frac{(4\pi)^{-1/2}}{k_{aA}k_{bB}}\sum_{l}\sqrt{(2j_f+1)(2j_i+1)}\\ &\times \sqrt{2l+1}\,i^{-l}\exp[i(\sigma_l^f+\sigma_l^i)]Y_0^l(\hat{\mathbf{k}}_{bB})\\ &\times\int\frac{d\mathbf{r}_{bB}d\mathbf{r}d\eta}{r_{bB}r_{aA}}u_{l_fj_f}(r_{A1})u_{l_fj_f}(r_{A2})u_{l_ij_i}(r_{b1})u_{l_ij_i}(r_{b2})\\ &\times P_{l_f}(\cos\theta_A)P_{l_i}(\cos\theta_b)\\ &\times f_l(r_{bB})g_l(r_{aA})Y_0^{l*}(\hat{\mathbf{r}}_{bB})V(r_{1p})Y_0^l(\hat{\mathbf{r}}_{aA}),\end{aligned} \tag{7.2.46}$$

~~We~~ Changing ~~the~~ integration variables and ~~proceed~~ proceeding ~~as in~~ last section, ~~what involves mul-tiplying by~~ implying a multiplicative factor $2\pi\sqrt{\frac{4\pi}{2l+1}}$, the above expression becomes

$$\begin{aligned}\langle \Psi_f^{(-)}(\mathbf{k}_{bB})|V(r_{1p})|\Psi_i^{(+)}(k_{aA},\hat{\mathbf{z}})\rangle &= \frac{2\pi}{k_{aA}k_{bB}}\sum_{l}\sqrt{(2j_f+1)(2j_i+1)}\\ &\times i^{-l}\exp[i(\sigma_l^f+\sigma_l^i)]Y_0^l(\hat{\mathbf{k}}_{bB})\\ &\times\int dr_{aA}\,d\beta\,d\gamma\,dr_{12}\,dr_{b1}\,dr_{b2}\,r_{aA}\sin\beta\,r_{12}r_{b1}r_{b2}\\ &\times P_{l_f}(\cos\theta_A)P_{l_i}(\cos\theta_b)u_{l_fj_f}(r_{A1})u_{l_fj_f}(r_{A2})u_{l_ij_i}(r_{b1})u_{l_ij_i}(r_{b2})\\ &\times f_l(r_{bB})g_l(r_{aA})Y_0^{l*}(\hat{\mathbf{r}}_{bB})V(r_{1p})/r_{bB}.\end{aligned} \tag{7.2.47}$$

recasted, through the use of Legendre polynomials

~~We introduce some more polynomials,~~ which eventually can be in the form —

$$\begin{aligned}T_{2N}^{1\,step} = \langle \Psi_f^{(-)}(\mathbf{k}_{bB})|V(r_{1p})|\Psi_i^{(+)}(k_{aA},\hat{\mathbf{z}})\rangle &= \frac{1}{2k_{aA}k_{bB}}\sum_{l}\sqrt{(2j_f+1)(2j_i+1)}\\ &\times i^{-l}\exp[i(\sigma_l^f+\sigma_l^i)]P_l(\cos\theta)(2l+1)\\ &\times\int dr_{aA}\,d\beta\,d\gamma\,dr_{12}\,dr_{b1}\,dr_{b2}\,r_{aA}\sin\beta\,r_{12}r_{b1}r_{b2}\\ &\times P_{l_f}(\cos\theta_A)P_{l_i}(\cos\theta_b)u_{l_fj_f}(r_{A1})u_{l_fj_f}(r_{A2})V(r_{1p})\\ &\times u_{l_ij_i}(r_{b1})u_{l_ij_i}(r_{b2})f_l(r_{bB})g_l(r_{aA})P_l(\cos\theta_{if})/r_{bB}.\end{aligned} \tag{7.2.48}$$

expression which gives the same results as (7.2.41)

7.2.3 Coordinates for the calculation of simultaneous transfer ~~calculation~~

~~We refer to the notation used in~~ Making use of the notation of Bayman (1971), ~~We must~~ we find the expression of the variables appearing in the integral as functions of the integration variables $r_{1p}, r_{2p}, r_{12}, R, \beta, \gamma$

In what follows, we explicit the coordinates used in the calculation of the above equation.