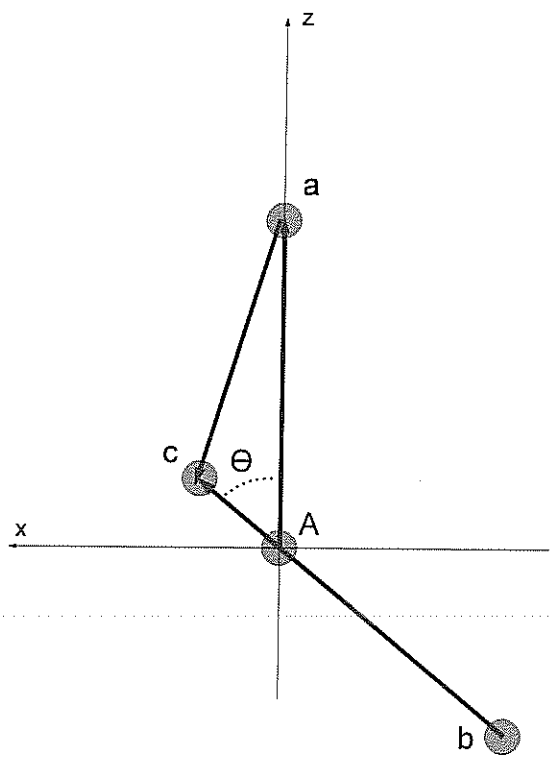

Figure 6.F.2: Coordinates in the "standard" configuration.

Then

$$\begin{aligned}
&\left\{\left[Y^{l_a}(\hat{\mathbf{r}}_{aA})Y^{l'_a}(\hat{\mathbf{r}}_{ac})\right]^K \left[Y^{l_b}(\hat{\mathbf{r}}_{bc})Y^{l'_b}(\hat{\mathbf{r}}_{bc})\right]^K\right\}_0^0 = \\
&\sum_{M_K}\langle K\; M_K\; K\; -M_K|0\;0\rangle \left[Y^{l_a}(\hat{\mathbf{r}}_{aA})Y^{l'_a}(\hat{\mathbf{r}}_{ac})\right]^K_{M_K} \left[Y^{l_b}(\hat{\mathbf{r}}_{bc})Y^{l'_b}(\hat{\mathbf{r}}_{bc})\right]^K_{-M_K} = \\
&\sqrt{\frac{2l_a+1}{4\pi}}\sum_{M_K}\frac{(-1)^{K+M_K}}{\sqrt{2K+1}}\langle l_a\; 0\; l'_a\; M_K|K\; M_K\rangle \\
&\times\left[Y^{l_b}(\hat{\mathbf{r}}_{bc})Y^{l'_b}(\hat{\mathbf{r}}_{bc})\right]^K_{-M_K} Y^{l'_a}_{M_K}(\hat{\mathbf{r}}_{ac}).
\end{aligned} \tag{6.F.23}$$

Remembering the $8\pi^2$ factor, the term arising from (6.F.20) to be considered in the integral is

$$\begin{aligned}
&4\pi^{3/2}\frac{\sqrt{2l_a+1}}{2K+1}(-1)^K\sum_{M_K}(-1)^{M_K}\langle l_a\; 0\; l'_a\; M_K|K\; M_K\rangle \\
&\qquad\times\left[Y^{l_b}(\cos\theta,0)Y^{l'_b}(\cos\theta,0)\right]^K_{-M_K} Y^{l'_a}_{M_K}(\cos\theta_{ac},0),
\end{aligned} \tag{6.F.24}$$

with

$$\cos\theta_{ac} = \frac{\frac{b}{A}r_{bc}\cos\theta - r_{aA}}{\sqrt{\left(\frac{b}{A}r_{bc}\sin\theta\right)^2 + \left(\frac{b}{A}r_{bc}\cos\theta - r_{aA}\right)^2}}, \tag{6.F.25}$$

(see (6.F.21)). The final expression of the transition amplitude is

$$\begin{aligned}
T_{m_b}(\mathbf{k}'_a,\mathbf{k}'_b) = &\frac{128\pi^{7/2}}{k_a k'_a k'_b}\sum_{KM}\frac{(-1)^{l'_b+m_b}}{2K+1}\langle l_a\; 0\; l'_a\; -M|K\; -M\rangle\langle l_b\; m_b\; l'_b\; -M-m_b|K\; -M\rangle \\
&\times\sum_{l_a,l'_a,l'_b}(2l_a+1)e^{i(\sigma^{l_a}+\sigma^{l'_a}+\sigma^{l'_b})}Y^{l'_b}_{-M-m_b}(\hat{\mathbf{k}}'_b)Y^{l'_a}_{-M}(\hat{\mathbf{k}}'_a)\,\mathcal{I}(l_a,l'_a,l'_b,K),
\end{aligned} \tag{6.F.26}$$

where

$$\begin{aligned}
\mathcal{I}(l_a,l'_a,l'_b,K) = &\int dr_{aA}dr_{bc}d\theta r_{aA}r_{bc}\frac{\sin\theta}{r_{ac}}u_{l_b}(r_{bc})v(r_{ab})F_{l_a}(r_{aA})F_{l'_a}(r_{ac})F_{l'_b}(r_{bc}) \\
&\times\sum_{M_K}(-1)^{M_K}\langle l_a\; 0\; l'_a\; M_K|K\; M_K\rangle\left[Y^{l_b}(\cos\theta,0)Y^{l'_b}(\cos\theta,0)\right]^K_{-M_K} Y^{l'_a}_{M_K}(\cos\theta_{ac},0)
\end{aligned} \tag{6.F.27}$$

is a 3–dimensional integral that can be numerically evaluated with, e.g., Gaussian integration.

6.F.2 Particles with spin

We will now turn to the case in which the clusters have a definite spin (see Fig. 6.F.3), and the optical potentials $U(r_{aA})$, $U(r_{cb})$, $U(r_{ac})$ are now central potentials with a spin–orbit term proportional to the usual product $\mathbf{l}\cdot\mathbf{s} = 1/2(j(j+1) - l(l+1) - 3/4)$ for particles with spin 1/2. In addition, the interaction $V(r_{ab},\sigma_a,\sigma_b)$ between a and b is taken to be a separable function of the distance r_{ab} and of the spin orientations,

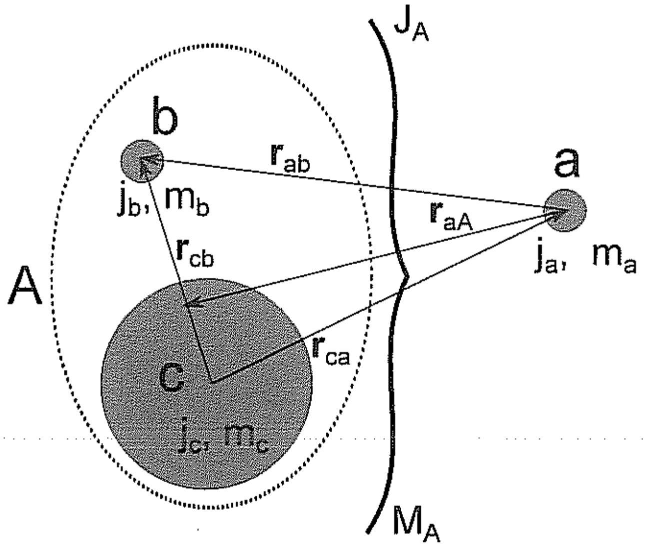

Figure 6.F.3: Now all three clusters a, b, c have definite spins and projections. The nucleus A is coupled to total spin J_A, M_A.

$V(r_{ab}, \sigma_a, \sigma_b) = v(r_{ab})v_\sigma(\sigma_a, \sigma_b)$. Note that this rules out a spin–orbit term and terms proportional to $\mathbf{r} \cdot \boldsymbol{\sigma}$, such as the tensor term! For the moment we will assume that the spin–dependent interaction is rotationally invariant (scalar with respect to rotations), such as, e.g., $v_\sigma(\sigma_a, \sigma_b) \propto \boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_b$. Again, that excludes from our formalism tensor terms in the interaction. The transition amplitude is

$$T^{m'_a, m'_b}_{m_a, m_b} = \sum_{\sigma_a, \sigma_b} \int d\mathbf{r}_{aA} d\mathbf{r}_{bc} \chi^{(-)*}_{m'_a}(\mathbf{r}_{ac}, \sigma_a) \chi^{(-)*}_{m'_b}(\mathbf{r}_{bc}, \sigma_b) \\ \times v(r_{ab}) v_\sigma(\sigma_a, \sigma_b) \chi^{(+)}_{m_a}(\mathbf{r}_{aA}, \sigma_a) \psi^{l_b, j_b}_{m_b}(\mathbf{r}_{bc}, \sigma_b). \tag{6.F.28}$$

( )

Distorted waves

The distorted waves in (6.F.28) $\chi_m(\mathbf{r}, \sigma) = \chi(\mathbf{r})\phi^{1/2}_m(\sigma)$ have a spin dependence contained in the spinor $\phi^{1/2}_m(\sigma)$, where σ is the spin degree of freedom and m the projection of the spin along the quantization axis. The superscript 1/2 reminds us that we are considering spin 1/2 particles, which have important consequences when dealing with the spin–orbit term of the optical potentials. As for the spin–dependent term $v_\sigma(\sigma_a, \sigma_b)$, the value of the spin of the involved particles does not make much difference *as long as this term is rotationally invariant.* After (6.F.4)

( )

eliminate italics

$$\chi^{(+)}(\mathbf{k}, \mathbf{r})\phi_m(\sigma) = \sum_{l,j} \frac{4\pi}{kr} i^l \sqrt{2l+1} e^{i\sigma^l} F_{l,j}(r) \left[Y^l(\hat{\mathbf{r}}) Y^l(\hat{\mathbf{k}})\right]^0_0 \phi^{1/2}_m(\sigma). \tag{6.F.29}$$

Note that now the sum is also over the total angular momentum j, because the radial functions $F_{l,j}(r)$ depend now on j as well as on l, being solutions of an optical potential with a spin–orbit term proportional to $1/2\,(j(j+1) - l(l+1) - 3/4)$. We must then couple the radial and spin functions to total angular momentum j, noting that

$$\left[Y^l(\hat{\mathbf{r}})\ Y^l(\hat{\mathbf{k}})\right]^0_0 \phi^{1/2}_m(\sigma) = \sum_{m_l} \langle l\ m_l\ l\ -m_l | 0\ 0\rangle Y^l_{m_l}(\hat{\mathbf{r}}) Y^l_{-m_l}(\hat{\mathbf{k}}) \phi^{1/2}_m(\sigma) = \\ \sum_{m_l} \frac{(-1)^{l-m_l}}{\sqrt{2l+1}} Y^l_{m_l}(\hat{\mathbf{r}}) Y^l_{-m_l}(\hat{\mathbf{k}}) \phi^{1/2}_m(\sigma), \tag{6.F.30}$$

and

$$Y^l_{m_l}(\hat{\mathbf{r}})\phi^{1/2}_m(\sigma) = \sum_j \langle l\ m_l\ 1/2\ m | j\ m_l + m\rangle \left[Y^l(\hat{\mathbf{r}})\phi^{1/2}(\sigma)\right]^j_{m_l+m}, \tag{6.F.31}$$

we can write

$$\left[Y^l(\hat{\mathbf{r}})\ Y^l(\hat{\mathbf{k}})\right]^0_0 \phi^{1/2}_m(\sigma) = \sum_{m_l, j} \frac{(-1)^{l+m_l}}{\sqrt{2l+1}} \langle l\ m_l\ 1/2\ m | j\ m_l + m\rangle \\ \times \left[Y^l(\hat{\mathbf{r}})\phi^{1/2}(\sigma)\right]^j_{m_l+m} Y^l_{-m_l}(\hat{\mathbf{k}}), \tag{6.F.32}$$

( )

and the distorted waves in (6.F.28) are

$$\chi^{(+)}_{m_a}(\mathbf{r}_{aA}, \mathbf{k}_a, \sigma_a) = \sum_{l_a, m_{l_a}, j_a} \frac{4\pi}{k_a r_{aA}} i^{l_a} (-1)^{l_a + m_{l_a}} e^{i\sigma^{l_a}} F_{l_a, j_a}(r_{aA}) \\ \times \langle l_a\ m_{l_a}\ 1/2\ m_a | j_a\ m_{l_a} + m_a\rangle \left[Y^{l_a}(\hat{\mathbf{r}}_{aA})\phi^{1/2}(\sigma_a)\right]^{j_a}_{m_{l_a}+m_a} Y^{l_a}_{-m_{l_a}}(\hat{\mathbf{k}}_a), \tag{6.F.33}$$

$$\chi^{(-)*}_{m'_b}(\mathbf{r}_{bc},\mathbf{k}'_b,\sigma_b) = \sum_{l'_b,m_{l'_b},j'_b} \frac{4\pi}{k'_b r_{bc}} i^{-l'_b}(-1)^{l'_b+m_{l'_b}} e^{i\sigma^{l'_b}} F_{l'_b,j'_b}(r_{bc})$$
$$\times \langle l'_b\ m_{l'_b}\ 1/2\ m'_b|j'_b\ m_{l'_b}+m'_b\rangle \left[Y^{l'_b}(\hat{\mathbf{r}}_{bc})\phi^{1/2}(\sigma_b)\right]^{j'_b*}_{m_{l'_b}+m'_b} Y^{l'_b*}_{-m_{l'_b}}(\hat{\mathbf{k}}'_b), \tag{6.F.34}$$

$$\chi^{(-)*}_{m'_a}(\mathbf{r}_{ac},\mathbf{k}'_a,\sigma_a) = \sum_{l'_a,m_{l'_a},j'_a} \frac{4\pi}{k'_a r_{ac}} i^{-l'_a}(-1)^{l'_a+m_{l'_a}} e^{i\sigma^{l'_a}} F_{l'_a,j'_a}(r_{ac})$$
$$\times \langle l'_a\ m_{l'_a}\ 1/2\ m'_a|j'_a\ m_{l'_a}+m'_a\rangle \left[Y^{l'_a}(\hat{\mathbf{r}}_{ac})\phi^{1/2}(\sigma_a)\right]^{j'_a*}_{m_{l'_a}+m'_a} Y^{l'_a*}_{-m_{l'_a}}(\hat{\mathbf{k}}'_a). \tag{6.F.35}$$

The initial bounded wavefunction of particle b is

$$\psi^{l_b,j_b}_{m_b}(\mathbf{r}_{bc},\sigma_b) = u_{l_b,j_b}(r_{bc})\left[Y^{l_b}(\hat{\mathbf{r}}_{bc})\phi^{1/2}(\sigma_b)\right]^{j_b}_{m_b}, \tag{6.F.36}$$

substituting in (6.F.28),

$$T^{m'_a,m'_b}_{m_a,m_b}(\mathbf{k}'_a,\mathbf{k}'_b) = \frac{64\pi^3}{k_a k'_a k'_b} \sum_{\sigma_a,\sigma_b}\sum_{l_a,m_{l_a},j_a}\sum_{l'_a,m_{l'_a},j'_a}\sum_{l'_b,m_{l'_b},j'_b} e^{i(\sigma^{l_a}+\sigma^{l'_a}+\sigma^{l'_b})} i^{l_a-l'_a-l'_b}(-1)^{l_a-m_{l_a}+l'_a-j'_a+l'_b-j'_b}$$
$$\times \langle l'_a\ m_{l'_a}\ 1/2\ m'_a|j'_a\ m_{l'_a}+m'_a\rangle\langle l_a\ m_{l_a}\ 1/2\ m_a|j_a\ m_{l_a}+m_a\rangle\langle l'_b\ m_{l'_b}\ 1/2\ m'_b|j'_b\ m_{l'_b}+m'_b\rangle$$
$$\times Y^{l_a}_{-m_{l_a}}(\hat{\mathbf{k}}_a)Y^{l'_b}_{-m_{l'_b}}(\hat{\mathbf{k}}'_b)Y^{l'_a}_{-m_{l'_a}}(\hat{\mathbf{k}}'_a)\int d\mathbf{r}_{aA}d\mathbf{r}_{bc}\left[Y^{l'_a}(\hat{\mathbf{r}}_{ac})\phi^{1/2}(\sigma_a)\right]^{j'_a}_{-m_{l'_a}-m'_a}\left[Y^{l'_b}(\hat{\mathbf{r}}_{bc})\phi^{1/2}(\sigma_b)\right]^{j'_b}_{-m_{l'_b}-m'_b}$$
$$\times \frac{F_{l_a,j_a}(r_{aA})F_{l'_a,j'_a}(r_{ac})F_{l'_b,j'_b}(r_{bc})}{r_{ac}r_{aA}r_{bc}} u_{l_b,j_b}(r_{bc})v(r_{ab})v_\sigma(\sigma_a,\sigma_b)$$
$$\times\left[Y^{l_a}(\hat{\mathbf{r}}_{aA})\phi^{1/2}(\sigma_a)\right]^{j_a}_{m_{l_a}+m_a}\left[Y^{l_b}(\hat{\mathbf{r}}_{bc})\phi^{1/2}(\sigma_b)\right]^{j_b}_{m_b}, \tag{6.F.37}$$

where we have used

$$\left[Y^{l}(\hat{\mathbf{r}})\phi^{1/2}(\sigma)\right]^{j*}_{m} = (-1)^{j-m}\left[Y^{l}(\hat{\mathbf{r}})\phi^{1/2}(\sigma)\right]^{j}_{-m}. \tag{6.F.38}$$

Recoupling of angular momenta

Let us now separate spatial and spin coordinates, noting that the spin functions must be coupled to 0 (this is a consequence of the interaction $v_\sigma(\sigma_a,\sigma_b)$ being rotationally invariant). Starting with particle a,

$$\left[Y^{l'_a}(\hat{\mathbf{r}}_{ac})\phi^{1/2*}(\sigma_a)\right]^{j'_a}_{-m_{l'_a}-m'_a}\left[Y^{l_a}(\hat{\mathbf{r}}_{aA})\phi^{1/2}(\sigma_a)\right]^{j_a}_{m_{l_a}+m_a} =$$
$$\sum_K ((l'_a\tfrac{1}{2})_{j'_a}(l_a\tfrac{1}{2})_{j_a}|(l_a l'_a)_K(\tfrac{1}{2}\tfrac{1}{2})_0)_K$$
$$\times\left[Y^{l'_a}(\hat{\mathbf{r}}_{ac})Y^{l_a}(\hat{\mathbf{r}}_{aA})\right]^{K}_{-m_{l'_a}-m'_a+m_{l_a}+m_a}\left[\phi^{1/2*}(\sigma_a)\phi^{1/2}(\sigma_a)\right]^0_0. \tag{6.F.39}$$

For particle b,

$$\left[Y^{l'_b}(\hat{\mathbf{r}}_{bc})\phi^{1/2*}(\sigma_b)\right]^{j'_b}_{-m_{l'_b}-m'_b}\left[Y^{l_b}(\hat{\mathbf{r}}_{bc})\phi^{1/2}(\sigma_b)\right]^{j_b}_{m_b} =$$
$$\sum_{K'} ((l'_b\tfrac{1}{2})_{j'_b}(l_b\tfrac{1}{2})_{j_b}|(l_b l'_b)_{K'}(\tfrac{1}{2}\tfrac{1}{2})_0)_{K'}$$
$$\times\left[Y^{l'_b}(\hat{\mathbf{r}}_{bc})Y^{l_b}(\hat{\mathbf{r}}_{bc})\right]^{K'}_{-m_{l'_b}-m'_b+m_b}\left[\phi^{1/2*}(\sigma_b)\phi^{1/2}(\sigma_b)\right]^0_0. \tag{6.F.40}$$

The spin summation yields a constant factor,

$$\sum_{\sigma_a,\sigma_b}\left[\phi^{1/2^*}(\sigma_a)\phi^{1/2}(\sigma_a)\right]_0^0\left[\phi^{1/2^*}(\sigma_b)\phi^{1/2}(\sigma_b)\right]_0^0 v_\sigma(\sigma_a,\sigma_b)\equiv T_\sigma, \qquad (6.F.41)$$

and what we have yet to do is very similar to what we have done for spinless particles. First of all note that the necessity to couple all angular momenta to 0 imposes $K'=K$ and $m_{l_a}+m_a-m_{l'_a}-m'_a=m_{l'_b}+m'_b-m_b$ (see (6.F.39) and (6.F.40)). If we set $M=m_{l_a}+m_a-m_{l'_a}-m'_a$ and take, as before, $\hat{\mathbf{k}}_a\equiv\hat{z}$

$$\begin{aligned}T^{m'_a,m'_b}_{m_a,m_b}(\mathbf{k}'_a,\mathbf{k}'_b)=&\frac{32\pi^{5/2}}{k_ak'_ak'_b}T_\sigma\sum_{l_a,j_a}\sum_{l'_a,j'_a}\sum_{l'_b,j'_b}\sum_{K,M}e^{i(\sigma^{l_a}+\sigma^{l'_a}+\sigma^{l'_b})}i^{l_a-l'_a-l'_b}(-1)^{l_a+l'_a+l'_b-j'_a-j'_b}\\&\times\sqrt{2l_a+1}((l'_a\tfrac12)_{j'_a}(l_a\tfrac12)_{j_a}|(l_al'_a)_K(\tfrac12\tfrac12)_0)_K\,((l'_b\tfrac12)_{j'_b}(l_b\tfrac12)_{j_b}|(l_bl'_b)_K(\tfrac12\tfrac12)_0)_K\\&\times\langle l'_a\ m_a-m'_a-M\ 1/2\ m'_a|j'_a\ m_a-M\rangle\langle l_a\ 0\ 1/2\ m_a|j_a\ m_a\rangle\langle l'_b\ m_b-m'_b+M\ 1/2\ m'_b|j'_b\ M+m_b\rangle\\&\times Y^{l'_b}_{m'_b-m_b-M}(\hat{\mathbf{k}}'_b)Y^{l'_a}_{m'_a-m_a+M}(\hat{\mathbf{k}}'_a)\int d\mathbf{r}_{aA}d\mathbf{r}_{bc}\frac{F_{l_a,j_a}(r_{aA})F_{l'_a,j'_a}(r_{ac})F_{l'_b,j'_b}(r_{bc})}{r_{ac}r_{aA}r_{bc}}\\&\times u_{l_b,j_b}(r_{bc})v(r_{ab})\left[Y^{l_a}(\hat{\mathbf{r}}_{aA})Y^{l'_a}(\hat{\mathbf{r}}_{ac})\right]^K_M\left[Y^{l_b}(\hat{\mathbf{r}}_{bc})Y^{l'_b}(\hat{\mathbf{r}}_{bc})\right]^K_{-M}. \qquad (6.F.42)\end{aligned}$$

The integral of the above expression is similar to the one in (6.F.18) so we obtain

$$\begin{aligned}T^{m'_a,m'_b}_{m_a,m_b}(\mathbf{k}'_a,\mathbf{k}'_b)=&\frac{128\pi^4}{k_ak'_ak'_b}T_\sigma\sum_{l_a,j_a}\sum_{l'_a,j'_a}\sum_{l'_b,j'_b}\sum_{K,M}e^{i(\sigma^{l_a}+\sigma^{l'_a}+\sigma^{l'_b})}i^{l_a-l'_a-l'_b}(-1)^{l_a+l'_a+l'_b-j'_a-j'_b}\\&\times\frac{2l_a+1}{2K+1}((l'_a\tfrac12)_{j'_a}(l_a\tfrac12)_{j_a}|(l_al'_a)_K(\tfrac12\tfrac12)_0)_K\,((l'_b\tfrac12)_{j'_b}(l_b\tfrac12)_{j_b}|(l_bl'_b)_K(\tfrac12\tfrac12)_0)_K\\&\times\langle l'_a\ m_a-m'_a-M\ 1/2\ m'_a|j'_a\ m_a-M\rangle\langle l'_b\ m_b-m'_b+M\ 1/2\ m'_b|j'_b\ M+m_b\rangle\\&\times\langle l_a\ 0\ 1/2\ m_a|j_a\ m_a\rangle Y^{l'_b}_{m'_b-m_b-M}(\hat{\mathbf{k}}'_b)Y^{l'_a}_{m_a-m'_a+M}(\hat{\mathbf{k}}'_a)\mathcal{I}(l_a,l'_a,l'_b,j_a,j'_a,j'_b,K), \qquad (6.F.43)\end{aligned}$$

with

$$\begin{aligned}\mathcal{I}(l_a,l'_a,l'_b,j_a,j'_a,j'_b,K)=&\int dr_{aA}dr_{bc}d\theta r_{aA}r_{bc}\frac{\sin\theta}{r_{ac}}u_{l_b}(r_{bc})v(r_{ab})\\&\times F_{l_a,j_a}(r_{aA})F_{l'_a,j'_a}(r_{ac})F_{l'_b,j'_b}(r_{bc})\\&\times\sum_{M_K}\langle l_a\ 0\ l'_a\ M_K|K\ M_K\rangle\left[Y^{l_b}(\cos\theta,0)Y^{l'_b}(\cos\theta,0)\right]^K_{-M_K}Y^{l'_a}_{M_K}(\cos\theta_{ac},0). \qquad (6.F.44)\end{aligned}$$

Again, this is a 3–dimensional integral that can be evaluated with the method of Gaussian quadratures. The transition amplitude $T^{m'_a,m'_b}_{m_a,m_b}(\mathbf{k}'_a,\mathbf{k}'_b)$ depends explicitly on the initial (m_a,m'_a) and final (m'_a,m'_b) polarizations of a,b. If the particle b is initially coupled to core c to total angular momentum J_A,M_A, the amplitude to be considered is rather

$$T^{m'_a,m'_b}_{m_a}(\mathbf{k}'_a,\mathbf{k}'_b)=\sum_{m_b}\langle j_b\ m_b\ j_c\ M_A-m_b|J_A\ M_A\rangle\,T^{m'_a,m'_b}_{m_a,m_b}(\mathbf{k}'_a,\mathbf{k}'_b), \qquad (6.F.45)$$

and the multi–differential cross section for detecting particle c (or a) is

$$\left.\frac{d\sigma}{d\mathbf{k}'_ad\mathbf{k}'_b}\right]^{m'_a,m'_b}_{m_a}=\frac{k'_a}{k_a}\frac{\mu_{aA}\mu_{ac}}{4\pi^2\hbar^4}\left|\sum_{m_b}\langle j_b\ m_b\ j_c\ M_A-m_b|J_A\ M_A\rangle\,T^{m'_a,m'_b}_{m_a,m_b}(\mathbf{k}'_a,\mathbf{k}'_b)\right|^2. \qquad (6.F.46)$$

All spin–polarization observables (analysing powers, etc.,) can be derived from this expression. But let us now work out the expression of the cross section for an unpolarized beam (sum over initial spin orientations divided by the number of such orientations) and when we do not detect the final polarizations (sum over final spin orientations),

$$\begin{aligned}\frac{d\sigma}{d\mathbf{k}'_a d\mathbf{k}'_b} &= \frac{k'_a}{k_a}\frac{\mu_{aA}\mu_{ac}}{4\pi^2\hbar^4}\frac{1}{(2J_A+1)(2j_a+1)}\\ &\times \sum_{\substack{m_a,m'_a\\ M_A,m'_b}}\left|\sum_{m_b}\langle j_b\ m_b\ j_c\ M_A-m_b|J_A\ M_A\rangle\, T^{m'_a,m'_b}_{m_a,m_b}(\mathbf{k}'_a,\mathbf{k}'_b)\right|^2 .\end{aligned} \tag{6.F.47}$$

The sum above can be simplified a bit. Let us consider a single particular value of m_b in the sum over m_b,

$$\begin{aligned}\sum_{m_a,m'_a,m'_b}&\left|T^{m'_a,m'_b}_{m_a,m_b}(\mathbf{k}'_a,\mathbf{k}'_b)\right|^2\sum_{M_A}\left|\langle j_b\ m_b\ j_c\ M_A-m_b|J_A\ M_A\rangle\right|^2 =\\ &\frac{2J_A+1}{2j_b+1}\sum_{m_a,m_a,m'_b}\left|T^{m'_a,m'_b}_{m_a,m_b}(\mathbf{k}'_a,\mathbf{k}'_b)\right|^2\\ &\times\sum_{M_A}\left|\langle J_A\ -M_A\ j_c\ M_A-m_b|j_b\ m_b\rangle\right|^2,\end{aligned} \tag{6.F.48}$$

where we have used

$$\langle j_b\ m_b\ j_c\ M_A-m_b|J_A\ M_A\rangle = (-1)^{j_c-M_A+m_b}\sqrt{\frac{2J_A+1}{2j_b+1}}\langle J_A\ -M_A\ j_c\ M_A-m_b|j_b\ m_b\rangle. \tag{6.F.49}$$

As

$$\sum_{M_A}\left|\langle J_A\ -M_A\ j_c\ M_A-m_b|j_b\ m_b\rangle\right|^2 = 1, \tag{6.F.50}$$

we finally have

$$\frac{d\sigma}{d\mathbf{k}'_a d\mathbf{k}'_b} = \frac{k'_a}{k_a}\frac{\mu_{aA}\mu_{ac}}{4\pi^2\hbar^4}\frac{1}{(2j_b+1)(2j_a+1)}\sum_{m_a,m'_a,m'_b}\left|\sum_{m_b}T^{m'_a,m'_b}_{m_a,m_b}(\mathbf{k}'_a,\mathbf{k}'_b)\right|^2 . \tag{6.F.51}$$

Zero range approximation.

The zero range approximation consists in taking $v(r_{ab}) = D_0\delta(r_{ab})$. Then, (see 6.F.2)

$$\begin{aligned}\mathbf{r}_{aA} &= \frac{c}{A}\mathbf{r}_{bc},\\ \mathbf{r}_{ac} &= \mathbf{r}_{bc}.\end{aligned} \tag{6.F.52}$$

The angular dependence of the integral can be readily evaluated. From 6.F.20, noting that $\hat{\mathbf{r}}_{aA} = \hat{\mathbf{r}}_{ac} = \hat{\mathbf{r}}_{bc} \equiv \hat{\mathbf{r}}$,

$$\begin{aligned}&\left[Y^{l_a}(\hat{\mathbf{r}})Y^{l'_a}(\hat{\mathbf{r}})\right]^K_M\left[Y^{l_b}(\hat{\mathbf{r}})Y^{l'_b}(\hat{\mathbf{r}})\right]^K_{-M} =\\ &\qquad\frac{(-1)^{K-M}}{\sqrt{2K+1}}\left\{\left[Y^{l_a}(\hat{\mathbf{r}})Y^{l'_a}(\hat{\mathbf{r}})\right]^K\left[Y^{l_b}(\hat{\mathbf{r}})Y^{l'_b}(\hat{\mathbf{r}})\right]^K\right\}^0_0 .\end{aligned} \tag{6.F.53}$$

We can as before evaluate this expression in the configuration shown in Fig. 6.F.2 ($\hat{\mathbf{r}} = \hat{z}$), but now the multiplicative factor is 4π. The corresponding contribution to the integral is

$$\frac{(-1)^K}{4\pi(2K+1)}\langle l_a\, 0\, l'_a\, 0|K\, 0\rangle \sqrt{(2l_a+1)(2l'_a+1)(2l_b+1)(2l'_b+1)}, \tag{6.F.54}$$

and

$$\begin{aligned} T^{m'_a,m'_b}_{m_a,m_b}(\mathbf{k}'_a,\mathbf{k}'_b) = \frac{16\pi^2}{k_a k'_a k'_b}\frac{c}{A} D_0 T_\sigma \sum_{l_a,j_a}\sum_{l'_a,j'_a}\sum_{l'_b,j'_b}\sum_{K,M} e^{i(\sigma^{l_a}+\sigma^{l'_a}+\sigma^{l'_b})} i^{l_a-l'_a-l'_b}(-1)^{l_a+l'_a+l'_b-j'_a-j'_b} \\ \times \sqrt{(2l_a+1)(2l'_a+1)(2l_b+1)(2l'_b+1)}\,\langle l_a\, 0\, l'_a\, 0|K\, 0\rangle \\ \times \frac{2l_a+1}{2K+1}((l'_a\tfrac{1}{2})_{j'_a}(l_a\tfrac{1}{2})_{j_a}|(l_a l'_a)_K(\tfrac{1}{2}\tfrac{1}{2})_0)_K\,((l'_b\tfrac{1}{2})_{j'_b}(l_b\tfrac{1}{2})_{j_b}|(l_b l'_b)_K(\tfrac{1}{2}\tfrac{1}{2})_0)_K \\ \times \langle l'_a\, m_a-m'_a-M\ 1/2\, m'_a|j'_a\, m_a-M\rangle\langle l'_b\, m_b-m'_b+M\ 1/2\, m'_b|j'_b\, M+m_b\rangle \\ \times \langle l\, 0\, 1/2\, m_a|j\, m_a\rangle Y^{l'_b}_{M+m_b+m'_b}(\hat{\mathbf{k}}'_b)Y^{l'_a}_{m_a+m'_a-M}(\hat{\mathbf{k}}'_a)\mathcal{I}_{ZR}(l_a,l'_a,l'_b,j_a,j'_a,j'_b), \end{aligned} \tag{6.F.55}$$

where now the 1–dimensional integral to solve is

$$\mathcal{I}_{ZR}(l_a,l'_a,l'_b,j_a,j'_a,j'_b) = \int dr u_{l_b}(r)F_{l_a,j_a}(\tfrac{c}{A}r)F_{l'_a,j'_a}(r)F_{l'_b,j'_b}(r)/r. \tag{6.F.56}$$

6.F.3 One-particle transfer

It may be interesting to state the expression for the one particle transfer reaction within the same context and using the same elements, in order to better compare these two type of experiments. In particle transfer, the final state of b is a bounded state of the $B(= a + b)$ nucleus, and we can carry on in a similar way as done previously just by substituting the distorted wave (continuum) wave function (6.F.34) with

$$\psi^{l'_b,j'_b*}_{m'_b}(\mathbf{r}_{ab},\sigma_b) = u^*_{l'_b,j'_b}(r_{ab})\left[Y^{l'_b}(\hat{\mathbf{r}}_{ab})\phi^{1/2}(\sigma_b)\right]^{j'_b*}_{m'_b}, \tag{6.F.57}$$

so the transition amplitude is now

$$\begin{aligned} T^{m'_a,m'_b}_{m_a,m_b}(\mathbf{k}'_a) = \frac{8\pi^{3/2}}{k_a k'_a}\sum_{\sigma_a,\sigma_b}\sum_{l_a,j_a}\sum_{l'_a,m_{l'_a},j'_a} e^{i(\sigma^{l_a}+\sigma^{l'_a})} i^{l_a-l'_a}(-1)^{l_a+l'_a-j'_a-j'_b} \\ \times \sqrt{2l_a+1}\,\langle l'_a\, m_{l'_a}\ 1/2\, m'_a|j'_a\, m_{l'_a}+m'_a\rangle\langle l_a\, 0\, 1/2\, m_a|j_a\, m_a\rangle \\ \times Y^{l'_a}_{-m_{l'_a}}(\hat{\mathbf{k}}'_a)\int d\mathbf{r}_{aA}d\mathbf{r}_{bc}\left[Y^{l'_a}(\hat{\mathbf{r}}_{Bc})\phi^{1/2}(\sigma_a)\right]^{j'_a}_{-m_{l'_a}-m'_a}\left[Y^{l'_b}(\hat{\mathbf{r}}_{ab})\phi^{1/2}(\sigma_b)\right]^{j'_b}_{-m'_b} \\ \times \frac{F_{l_a,j_a}(r_{aA})F_{l'_a,j'_a}(r_{Bc})}{r_{Bc}r_{aA}} u^*_{l'_b,j'_b}(r_{ab})u_{l_b,j_b}(r_{bc})v(r_{ab})v_\sigma(\sigma_a,\sigma_b) \\ \times \left[Y^{l_a}(\hat{\mathbf{r}}_{aA})\phi^{1/2}(\sigma_a)\right]^{j_a}_{m_a}\left[Y^{l_b}(\hat{\mathbf{r}}_{bc})\phi^{1/2}(\sigma_b)\right]^{j_b}_{m_b}. \end{aligned} \tag{6.F.58}$$

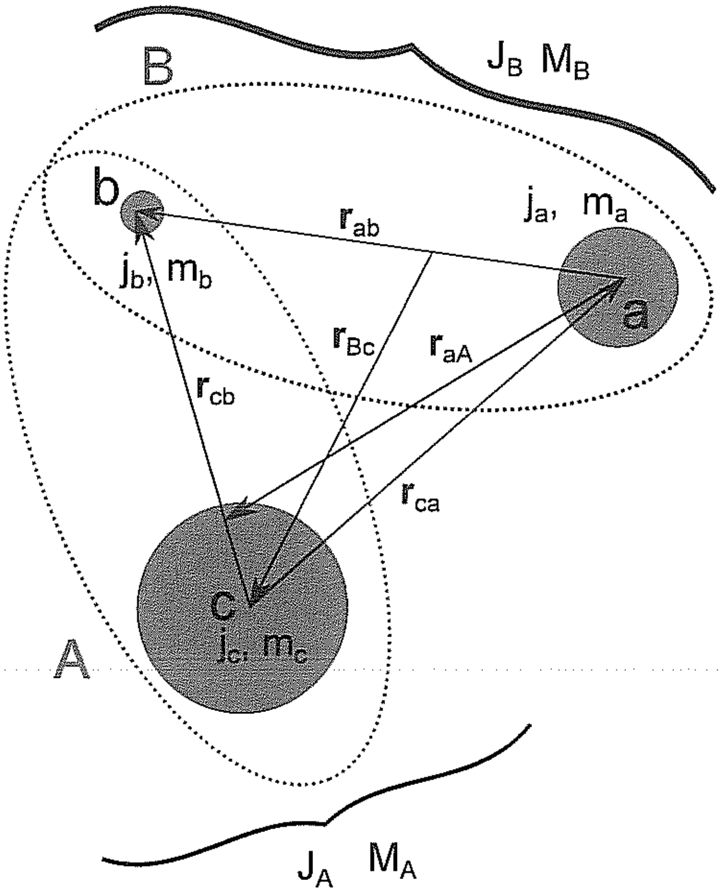

Figure 6.F.4: One particle transfer reaction $A(= c + b) + a \rightarrow B(= a + b) + c$.

Using (6.F.39), (6.F.40), (6.F.41), and setting $M = m_a - m'_a - m_{l'_a}$

$$
\begin{aligned}
T^{m'_a,m'_b}_{m_a,m_b}(\mathbf{k}'_a) = \frac{8\pi^{3/2}}{k_a k'_a} T_\sigma \sum_{l_a,j_a} \sum_{l'_a,j'_a} \sum_{K,M} e^{i(\sigma^{l_a}+\sigma^{l'_a})} i^{l_a-l'_a} (-1)^{l_a+l'_a-j_a-j'_b} \\
\times ((l'_a \tfrac{1}{2})_{j'_a}(l_a \tfrac{1}{2})_{j_a} | (l_a l'_a)_K (\tfrac{1}{2}\tfrac{1}{2})_0)_K \, ((l'_b \tfrac{1}{2})_{j'_b}(l_b \tfrac{1}{2})_{j_b} | (l_b l'_b)_K (\tfrac{1}{2}\tfrac{1}{2})_0)_K \\
\times \sqrt{2l_a+1} \, \langle l'_a \; m_a - m'_a - M \; 1/2 \; m'_a | j'_a \; m_a - M \rangle \langle l_a \; 0 \; 1/2 \; m_a | j_a \; m_a \rangle \\
\times Y^{l'_a}_{m_a-m'_a-M}(\hat{\mathbf{k}}'_a) \int d\mathbf{r}_{aA} d\mathbf{r}_{bc} \frac{F_{l_a,j_a}(r_{aA}) F_{l'_a,j'_a}(r_{Bc})}{r_{Bc} r_{aA}} u^*_{l'_b,j'_b}(r_{ab}) u_{l_b,j_b}(r_{bc}) v(r_{ab}) \\
\times \left[Y^{l_a}(\hat{\mathbf{r}}_{aA}) Y^{l'_a}(\hat{\mathbf{r}}_{Bc}) \right]^K_M \left[Y^{l_b}(\hat{\mathbf{r}}_{bc}) Y^{l'_b}(\hat{\mathbf{r}}_{ab}) \right]^K_{-M}. \quad (6.F.59)
\end{aligned}
$$

Aside from (6.F.2), we also need

$$
\mathbf{r}_{Bc} = \frac{a+B}{B} \mathbf{r}_{aA} + \frac{b}{A} \mathbf{r}_{bc}. \quad (6.F.60)
$$

From (6.F.20), (6.F.21), (6.F.22), (6.F.23), (6.F.24), we get

$$
\begin{aligned}
T^{m'_a,m'_b}_{m_a,m_b}(\mathbf{k}'_a) = \frac{32\pi^3}{k_a k'_a} T_\sigma \sum_{l_a,j_a} \sum_{l'_a,j'_a} \sum_{K,M} e^{i(\sigma^{l_a}+\sigma^{l'_a})} i^{l_a-l'_a} (-1)^{l_a+l'_a-j_a-j'_b} \\
\times ((l'_a \tfrac{1}{2})_{j'_a}(l_a \tfrac{1}{2})_{j_a} | (l_a l'_a)_K (\tfrac{1}{2}\tfrac{1}{2})_0)_K \, ((l'_b \tfrac{1}{2})_{j'_b}(l_b \tfrac{1}{2})_{j_b} | (l_b l'_b)_K (\tfrac{1}{2}\tfrac{1}{2})_0)_K \\
\times \frac{2l_a+1}{2K+1} \langle l'_a \; m_a - m'_a - M \; 1/2 \; m'_a | j'_a \; m_a - M \rangle \\
\times \langle l_a \; 0 \; 1/2 \; m_a | j_a \; m_a \rangle Y^{l'_a}_{m_a-m'_a-M}(\hat{\mathbf{k}}'_a) \mathcal{I}(l_a, l'_a, j_a, j'_a, j'_b, K), \quad (6.F.61)
\end{aligned}
$$

with

$$
\begin{aligned}
I(l_a, l'_a, j_a, j'_a, K) = \int dr_{aA} dr_{bc} d\theta \, r_{aA} r^2_{bc} \frac{\sin\theta}{r_{Bc}} \\
\times F_{l_a,j_a}(r_{aA}) F_{l'_a,j'_a}(r_{ac}) u^*_{l'_b,j'_b}(r_{ab}) u_{l_b,j_b}(r_{bc}) v(r_{ab}) \\
\times \sum_{M_K} \langle l_a \; 0 \; l'_a \; M_K | K \; M_K \rangle \left[Y^{l_b}(\cos\theta, 0) Y^{l'_b}(\cos\theta_{ab}, 0) \right]^K_{-M_K} Y^{l'_a}_{M_K}(\cos\theta_{Bc}, 0), \quad (6.F.62)
\end{aligned}
$$

where (see (6.F.2), (6.F.60) and Fig. 6.F.2)

$$
\cos\theta_{ab} = \frac{-r_{aA} - \frac{c}{A} r_{bc}\cos\theta}{\sqrt{\left(\frac{c}{A} r_{bc}\sin\theta\right)^2 + \left(r_{aA} + \frac{c}{A} r_{bc}\cos\theta\right)^2}}, \quad (6.F.63)
$$

$$
\cos\theta_{Bc} = \frac{\frac{a+B}{B} r_{aA} + \frac{b}{A} r_{bc}\cos\theta}{\sqrt{\left(\frac{b}{A} r_{bc}\sin\theta\right)^2 + \left(\frac{a+B}{B} r_{aA} + \frac{b}{A} r_{bc}\cos\theta\right)^2}}, \quad (6.F.64)
$$

and

$$
r_{Bc} = \sqrt{\left(\frac{b}{A} r_{bc}\sin\theta\right)^2 + \left(\frac{a+B}{B} r_{aA} + \frac{b}{A} r_{bc}\cos\theta\right)^2}. \quad (6.F.65)
$$

Again, this is nothing new as many codes exist which deal with one particle transfer within the same DWBA formalism we have used here, but it may be useful to have our own code to better compare transfer and knock–out experiments. By the way, (6.F.61) can also be used when particle b populates a resonant state in the continuum of nucleus B.

Bibliography

D. Bès and R. Broglia. In A. Bohr and R. Broglia, editors, *International School of Physics "Enrico Fermi" Course LXIX, Elementary Modes of Excitation in Nuclei*, page 55, Amsterdam, 1977. North Holland.

A. Bohr and B. R. Mottelson. *Nuclear Structure, Vol.II*. Benjamin, New York, 1975.

R. Broglia and A. Winther. *Heavy Ion Reactions*. Westview Press, Cambridge, MA., 2004.

R. Broglia, O. Hansen, and C. Riedel. Two–neutron transfer reactions and the pairing model. *Advances in Nuclear Physics*, 6:287, 1973. URL `http://merlino.mi.infn.it/repository/BrogliaHansenRiedel.pdf`.

R. Broglia, G. Coló, G. Onida, and H. Roman. *Solid State Physics of Finite Systems: metal clusters, fullerenes, atomic wires*. Springer Verlag, Berlin, Heidelberg, 2004.

C. Mahaux et al. Dynamics of the shell model. *Physics Reports*, 120:1–274, 1985.

R. Feynman. *Theory of fundamental processes*. Benjamin, Reading, Mass., 1975.

G. Satchler. *Introduction to Nuclear Reactions*. Mc Millan, New York, 1980.

Broglia, R.A., Hansen, O. and Riedel, C. (1973) Two-nucleon transfer reactions and the pairing model, Adv. Nucl. Phys., 6: 287-457, http://www.mi.ifn.it/-vigezzi/BHR/BrogliaHansenRiedel.pdf

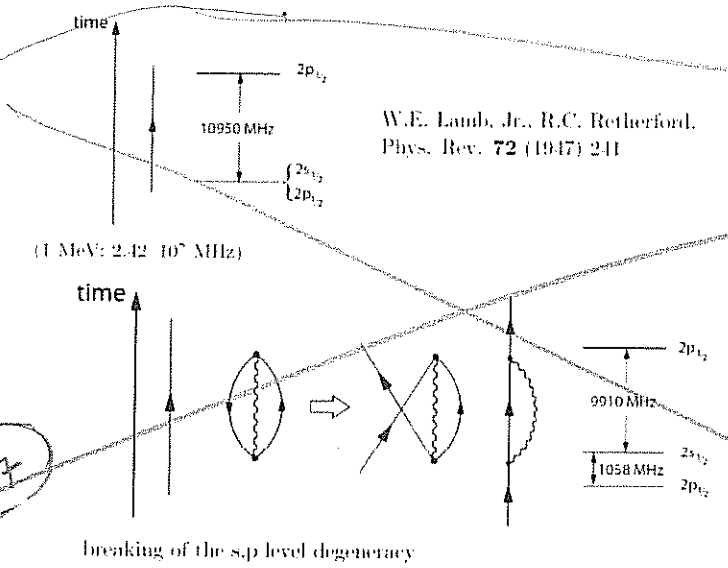

Figura de appendix 6.C The Lamb Shift

Figure ~~6.14~~ 6.7: Schematic representation of the processes associated with the Lamb shift.

Bibliography

(1975) D. Bès and R. Broglia. In A. Bohr ~~and~~ R. Broglia, editors, *International School of Physics "Enrico Fermi" Course LXIX, Elementary Modes of Excitation in Nuclei*, page 55, Amsterdam, 1977. North Holland.

A. Bohr and B. R. Mottelson. *Nuclear Structure, Vol.II.* Benjamin, New York, ~~1975.~~

~~R.~~ R.A. Broglia ~~and~~ A. Winther. *Heavy Ion Reactions*, (1991) ~~Westview Press, Cambridge, MA., 2004.~~ Addison-Wesley, Redwood City, California

R. Broglia, O. Hansen, and C. Riedel. Two–neutron transfer reactions and the pairing model. *Advances in Nuclear Physics*, 6:287, 1973. URL http://merlino.mi.infn.it/repository/BrogliaHansenRiedel.pdf.

~~R.~~ R.A. Broglia, ~~G.~~ G. Colò, ~~G.~~ E. Onida, and ~~H.~~ H.E. Roman. (2004) *Solid State Physics of Finite Systems: metal clusters, fullerenes, atomic wires.* Springer Verlag, Berlin, Heidelberg, ~~2004.~~

bold

C. Mahaux ~~et al.~~ Dynamics of the shell model. *Physics Reports*, 120:1–274, ~~1985.~~

R. Feynman. *Theory of fundamental processes.* Benjamin, Reading, Mass. (1975.)

G. Satchler. *Introduction to Nuclear Reactions.* Mc Millan, New York (1980.)

Bortignon, P.F., Broglia, R.A., and Dasso, C.H. (1985)

Bes, D.R. and Broglia, R.A. (1977), Nuclear superfluidity and Field Theory of Elementary Excitations, in Procs. of the International School of Physics "Enrico Fermi" Course LXIX on "Elementary Modes of Excitation in Nuclei", Eds. A. Bohr and R.A. Broglia, North Holland, Amsterdam, pp. 55–134.

Potel, G. Barranco, F., Vigezzi, E. and Broglia, R.A. (2010), Evidence of phonon mediated pairing interaction in the halo of the nucleus ^{11}Li, Phys. Rev. Lett. 105, 172502.

Tanihata, I., Alcorta, M., ---- Thompson, I.J. (2008) Measurement of the two-halo neutron transfer reaction $^1H(^{11}Li, ^9Li)^3H$ at 3 A MeV, Phys. Rev. Lett. 100, 192502.

Brink, D.M. and Broglia, R.A. (2005) Nuclear Superfluidity, Cambridge University Press, Cambridge.

Anderson, P.W. (1984) Basic Notions of Condensed Matter Physics, Benjamin, Menlo Park, California.

Migdal, A. (1958) Interaction between electrons and lattice vibrations in normal metals, Soviet Physics JETP, 7: 996-1001.

Bardeen, J. and Pines, D. (1955) Electron-phonon interaction in metals, Phys. Rev 99, 1189-1190.

Cobb W.R. and Guth, D.B. (1957) ---- title, Phys. Rev. 107, 181.