or

$$\chi_m^{1/2}(\sigma) = \delta_{m,\sigma}. \tag{7.K.11}$$

Thus, $\chi_m^{1/2*}(\sigma) = \chi_m^{1/2}(\sigma) = \delta_{m,\sigma}$, but we can also write

$$\chi_m^{1/2*}(\sigma) = (-1)^{1/2-m+1/2-\sigma}\chi_{-m}^{1/2}(-\sigma). \tag{7.K.12}$$

This trick enable us to write

$$\left[Y^l(\hat{r})\chi^{1/2}(\sigma)\right]_M^{J*} = (-1)^{1/2-\sigma+J-M}\left[Y^l(\hat{r})\chi^{1/2}(-\sigma)\right]_{-M}^{J}, \tag{7.K.13}$$

which can be derived in a similar way as (7.K.9). check Greg

7.K.4 angular momenta coupling

Relation between Clebsh–Gordan and $3j$ coefficients:

$$\langle j_1\ j_2\ m_1\ m_2|JM\rangle = (-1)^{j_1-j_2+M}\sqrt{2J+1}\begin{pmatrix} j_1 & j_2 & J \\ m_1 & m_2 & -M \end{pmatrix}. \tag{7.K.14}$$

Relation between Wigner and $9j$ coefficients:

$$\langle (j_1j_2)_{j_{12}}(j_3j_4)_{j_{34}}|(j_1j_3)_{j_{13}}(j_2j_4)_{j_{24}}\rangle_j =$$
$$\sqrt{(2j_{12}+1)(2j_{13}+1)(2j_{24}+1)(2j_{34}+1)}\begin{Bmatrix} j_1 & j_2 & j_{12} \\ j_3 & j_4 & j_{34} \\ j_{13} & j_{24} & j \end{Bmatrix}. \tag{7.K.15}$$

7.K.5 integrals

Let us now prove

$$\int d\Omega\left[Y^l(\hat{r})Y^l(\hat{r})\right]_M^I = \delta_{M,0}\delta_{I,0}\sqrt{2l+1}. \tag{7.K.16}$$

$$\begin{aligned}
\int d\Omega\left[Y^l(\hat{r})Y^l(\hat{r})\right]_M^I &= \sum_{\substack{m_1,m_2\\(m_1+m_2=M)}}\langle l\ l\ m_1\ m_2|IM\rangle\int d\Omega Y_{m_1}^l(\hat{r})Y_{m_2}^l(\hat{r}) \\
&= \sum_{\substack{m_1,m_2\\(m_1+m_2=M)}}(-1)^{l+m_1}\langle l\ l\ -m_1\ m_2|IM\rangle\int d\Omega Y_{m_1}^{l*}(\hat{r})Y_{m_2}^l(\hat{r}) \\
&= \delta_{M,0}\sum_m(-1)^{l+m}\langle l\ l\ -m\ m|I0\rangle \\
&= \delta_{M,0}\sqrt{2l+1}\sum_m\langle l\ l\ -m\ m|I0\rangle\langle l\ l\ -m\ m|00\rangle \\
&= \delta_{M,0}\delta_{I,0}\sqrt{2l+1},
\end{aligned} \tag{7.K.17}$$

where we have used

$$\langle l\ l\ -m\ m|0\ 0\rangle = \frac{(-1)^{l+m}}{\sqrt{2l+1}} \tag{7.K.18}$$

Let us now prove

$$\sum_{\sigma} \int d\Omega (-1)^{1/2-\sigma} \left[\Psi^j(\hat{r},-\sigma)\Psi^j(\hat{r},\sigma)\right]^I_M = -\delta_{M,0}\delta_{I,0}\sqrt{2j+1}. \tag{7.K.19}$$

$$\begin{aligned}
&\sum_{\sigma} \int d\Omega (-1)^{1/2-\sigma} \left[\Psi^j(\hat{r},-\sigma)\Psi^j(\hat{r},\sigma)\right]^I_M \\
&= \sum_{\substack{m_1,m_2 \\ (m_1+m_2=M)}} \langle j\ j\ m_1\ m_2|IM\rangle \sum_{\sigma} \int d\Omega \Psi^j_{m_1}(\hat{r},-\sigma)\Psi^j_{m_2}(\hat{r},\sigma) \\
&= \sum_{\substack{m_1,m_2 \\ (m_1+m_2=M)}} \langle j\ j\ m_1\ m_2|IM\rangle \sum_{\sigma} (-1)^{j+m_1} \int d\Omega \Psi^{j*}_{-m_1}(\hat{r},\sigma)\Psi^j_{m_2}(\hat{r},\sigma) \\
&= \sum_{\substack{m_1,m_2 \\ (m_1+m_2=M)}} \langle j\ j\ m_1\ m_2|IM\rangle (-1)^{j+m_1}\delta_{-m_1,m_2} \\
&= \delta_{M,0} \sum_{m} (-1)^{j+m} \langle j\ j\ m\ -m|I0\rangle \\
&= -\delta_{M,0}\sqrt{2j+1} \sum_{m} (-1)^{j+m} \langle j\ j\ m\ -m|I0\rangle \langle j\ j\ m\ -m|00\rangle \\
&= -\delta_{M,0}\delta_{I,0}\sqrt{2j+1}.
\end{aligned} \tag{7.K.20}$$

7.K.6 symmetry properties

Note also another useful property

$$\left[\Psi^{l_1}\Psi^{l_2}\right]^I_M = (-1)^{l_1+l_2-I}\left[\Psi^{l_2}\Psi^{l_1}\right]^I_M, \tag{7.K.21}$$

by virtue of the symmetry property of the Clebsh-Gordan coefficients

$$\langle l_1\ l_2\ m_1\ m_2|IM\rangle = (-1)^{l_1+l_2-I}\langle l_2\ l_1\ m_2\ m_1|IM\rangle. \tag{7.K.22}$$

Here's another symmetry property of the Clebsh-Gordan coefficients

$$\langle l_1\ l_2\ m_1\ m_2|IM\rangle = (-1)^{l_1+l_2-I}\langle l_1\ l_2\ -m_2\ -m_1|I-M\rangle. \tag{7.K.23}$$

Another one, which can be derived from the simpler properties of $3j$ symbols

$$\langle l_1\ l_2\ m_1\ m_2|IM\rangle = (-1)^{l_1-m_1}\sqrt{\frac{2I+1}{2l_2+1}}\langle l_1\ I\ m_1\ -M|l_2 m_2\rangle. \tag{7.K.24}$$

Let us use this last property to calculate sums of the type

$$\sum_{m_1,m_3} |\langle I_1\ I_2\ m_1\ m_2|I_3 m_3\rangle|^2 . \tag{7.K.25}$$

Using (7.K.24), we have

$$\sum_{m_1,m_3} |\langle I_1\ I_2\ m_1\ m_2|I_3 m_3\rangle|^2 =$$
$$\frac{2I_3+1}{2I_2+1}\sum_{m_1,m_3} |\langle I_1\ I_3\ m_1\ -m_3|I_2 m_2\rangle|^2 = \frac{2I_3+1}{2I_2+1}, \tag{7.K.26}$$

since

$$\sum_{m_1,m_3} |\langle I_1\ I_3\ m_1\ -m_3|I_2 m_2\rangle|^2 = \sum_{m_1,m_3} |\langle I_1\ I_3\ m_1\ m_3|I_2 m_2\rangle|^2 = 1. \tag{7.K.27}$$

Appendix 7.L distorted waves

Let us have a closer look at the partial wave expansion of the distorted waves

$$\chi^{(+)}(\mathbf{k},\mathbf{r}) = \sum_l \frac{4\pi}{kr} i^l e^{i\sigma^l} F_l \sum_m Y^l_m(\hat{r}) Y^{l*}_m(\hat{k}). \tag{7.L.1}$$

Of notice the very important fact that *this definition is independent of the phase convention*, since the l–dependent phase is multiplied by its complex conjugate.

$$\chi^{(-)}(\mathbf{k},\mathbf{r}) = \chi^{(+)*}(-\mathbf{k},\mathbf{r}) = \sum_l \frac{4\pi}{kr} i^{-l} e^{-i\sigma^l} F^*_l \sum_m Y^{l*}_m(\hat{r}) Y^l_m(-\hat{k}). \tag{7.L.2}$$

As $Y^l_m(-\hat{k}) = (-1)^l Y^l_m(\hat{k})$, we have

$$\chi^{(-)}(\mathbf{k},\mathbf{r}) = \sum_l \frac{4\pi}{kr} i^l e^{-i\sigma^l} F^*_l \sum_m Y^{l*}_m(\hat{r}) Y^l_m(\hat{k}), \tag{7.L.3}$$

which is also independent of the phase convention. With time–reversed phase convention

$$\chi^{(+)}(\mathbf{k},\mathbf{r}) = \sum_l \frac{4\pi}{kr} i^l \sqrt{2l+1} e^{i\sigma^l} F_l \left[Y^l(\hat{r}) Y^l(\hat{k})\right]^0_0 , \tag{7.L.4}$$

while with Condon–Shortley phase convention we get an extra $(-1)^l$ factor:

$$\chi^{(+)}(\mathbf{k},\mathbf{r}) = \sum_l \frac{4\pi}{kr} i^{-l} \sqrt{2l+1} e^{i\sigma^l} F_l \left[Y^l(\hat{r}) Y^l(\hat{k})\right]^0_0 . \tag{7.L.5}$$

The partial–wave expansion of the Green function $G(\mathbf{r},\mathbf{r}')$ is

$$G(\mathbf{r},\mathbf{r}') = i \sum_l \frac{f_l(k,r_<) P_l(k,r_>)}{krr'} \sum_m Y^l_m(\hat{r}) Y^{l*}_m(\hat{r}'), \tag{7.L.6}$$

where $f_l(k, r_<)$ and $P_l(k, r_>)$ are the regular and the irregular solutions of the homogeneous problem respectively. With *time–reversed* phase convention

$$G(\mathbf{r}, \mathbf{r}') = i \sum_l \sqrt{2l+1} \frac{f_l(k, r_<) P_l(k, r_>)}{krr'} \left[Y^l(\hat{r}) Y^l(\hat{r}') \right]_0^0 . \tag{7.L.7}$$

Appendix 7.M hole states and time reversal

Let us consider the state $|(jm)^{-1}\rangle$ obtained by removing a ψ_{jm} single–particle state from a $J = 0$ closed shell $|0\rangle$. The antisymmetrized product state

$$\sum_m \mathcal{A}\{\psi_{jm} |(jm)^{-1}\rangle\} \propto |0\rangle \tag{7.M.1}$$

is clearly proportional to $|0\rangle$. This gives us the transformation rules of $|(jm)^{-1}\rangle$ under rotations, which must be such that, when multiplied by a j, m spherical tensor and summed over m, yields a $j = 0$ tensor. It can be seen that these properties imply that $|(jm)^{-1}\rangle$ transforms like $(-1)^{j-m} T_{j-m}$, T_{j-m} being a spherical tensor. It also follows that the hole state $|(j\tilde{m})^{-1}\rangle$ transforms like a j, m spherical tensor if $\psi_{j\tilde{m}}$ is defined as the $\mathcal{R}$–conjugate to ψ_{jm} by the relation

$$\psi_{j\tilde{m}} \equiv (-1)^{j+m} \psi_{j-m}. \tag{7.M.2}$$

In other words, with the latter definition a *hole state* transforms under rotations with the right phase. We will now show that $\mathcal{R}$–conjugation is equivalent to a rotation of spin and spatial coordinates through an angle $-\pi$ about the y–axis:

$$e^{i\pi J_y} \psi_{jm} = (-1)^{j+m} \psi_{j-m} \equiv \psi_{j\tilde{m}}. \tag{7.M.3}$$

Let us begin by calculating $e^{i\pi L_y} Y_l^m$. The rotation matrix about the y–axis is

$$R_y(\theta) = \begin{pmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{pmatrix}, \tag{7.M.4}$$

so for $R_y(-\pi)$ we get

$$R_y(-\pi) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{7.M.5}$$

When applied to the generic direction $(\sin(\theta)\cos(\phi), \sin(\theta)\sin(\phi), \cos(\theta))$, we obtain $(-\sin(\theta)\cos(\phi), \sin(\theta)\sin(\phi), -\cos(\theta))$, which corresponds to making the substitutions

$$\theta \to \pi - \theta, \quad \phi \to \pi - \phi. \tag{7.M.6}$$

When we substitute these angular transformations in the spherical harmonic $Y_l^m(\theta, \phi)$, we obtain the rotated $Y_l^m(\theta, \phi)$:

$$e^{i\pi L_y} Y_l^m = (-1)^{l+m} Y_l^{-m}. \tag{7.M.7}$$

Let us now turn our attention to the spin coordinates rotation $e^{i\pi s_y}\chi_m$. The rotation matrix in spin space is

$$\begin{pmatrix}\cos(\theta/2) & -\sin(\theta/2)\\ \sin(\theta/2) & \cos(\theta/2)\end{pmatrix}, \tag{7.M.8}$$

which, for $\theta = -\pi$ is

$$\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}. \tag{7.M.9}$$

Applying it to the spinors, we find the rule

$$e^{i\pi s_y}\chi_m = (-1)^{1/2+m}\chi_{-m}, \tag{7.M.10}$$

so

$$\begin{aligned} e^{i\pi J_y}\psi_{jm} &= \sum_{m_l m_s}\langle l\ m_l\ 1/2\ m_s|j\ m\rangle\ e^{i\pi L_y}Y_l^{m_l}\ e^{i\pi s_y}\chi_{m_s}\\ &= \sum_{m_l m_s}(-1)^{1/2+m_s+l+m_l}\langle l\ m_l\ 1/2\ m_s|j\ m\rangle\ Y_l^{-m_l}\ \chi_{-m_s}\\ &= \sum_{m_l m_s}(-1)^{1+m-j+2l}\langle l\ -m_l\ 1/2\ -m_s|j\ -m\rangle\ Y_l^{-m_l}\ \chi_{-m_s}\\ &= (-1)^{m+j}\psi_{j-m} \equiv \psi_{j\tilde{m}}, \end{aligned} \tag{7.M.11}$$

where we have used $(-1)^{1+m-j+2l} = -(-1)^{m-j} = (-1)^{m+j}$, as j, m are always half–integers and l is always an integer.

We now turn our attention to the time reversal operation, which amounts to the transformations

$$\mathbf{r} \to \mathbf{r}, \quad \mathbf{p} \to -\mathbf{p}. \tag{7.M.12}$$

This is enough to define the operator of time reversal of a spinless particle (see Messiah). In the position representation, in which $\mathbf{r}$ is real and $\mathbf{p}$ pure imaginary, this (unitary antilinear) operator is the complex conjugation operator.

As angular momentum $\mathbf{l} = \mathbf{r}\times\mathbf{p}$ changes sign under time reversal, so does spin:

$$\mathbf{s} \to -\mathbf{s}, \tag{7.M.13}$$

which, along with (7.M.12), completes the set of rules that define the time reversal operation on a particle with spin. In the representation of eigenstates of s^2 and s_z, complex conjugation alone changes only the sign of s_y, so an additional rotation of $-\pi$ around the y–axis is necessary to change the sign of s_x, s_z and implement the transformation (7.M.13). If we call K the time–reversal operator, we have

$$K\psi_{jm} = e^{i\pi s_y}\psi^*_{jm}. \tag{7.M.14}$$

This is completely general and independent of the phase convention. It only depends on the fact that we have used the $\mathbf{r}$ representation for the spatial wave function and the representation of the eigenstates of s^2 and s_z for the spin part. *If we use time–reversal phases for the spherical harmonics* (see (7.K.2)),

$$Y_m^{l*} = (-1)^{l+m}Y_{-m}^l = e^{i\pi L_y}Y_m^l. \tag{7.M.15}$$

Check Gregory

So we can write

$$K\psi_{jm} = e^{i\pi J_y}\psi_{jm} = \psi_{j\bar{m}}. \tag{7.M.16}$$

Note again that this last expression is valid only if we use time–reversal phases for the spherical harmonics. Only in this case time–reversal coincides with $\mathcal{R}$–conjugation and hole states.

In BCS theory, the quasiparticles are defined in terms of linear combinations of particles and holes. With time–reversal phases, holes are equivalent to time–reversed states, and we get the definitions

$$\begin{aligned}
\alpha_\nu^\dagger &= u_\nu a_\nu^\dagger - v_\nu a_{\bar\nu} & a_\nu^\dagger &= u_\nu \alpha_\nu^\dagger + v_\nu \alpha_{\bar\nu} \\
\alpha_{\bar\nu}^\dagger &= u_\nu a_{\bar\nu}^\dagger + v_\nu a_\nu & a_{\bar\nu}^\dagger &= u_\nu \alpha_{\bar\nu}^\dagger - v_\nu \alpha_\nu \\
\alpha_\nu &= u_\nu a_\nu - v_\nu a_{\bar\nu}^\dagger & a_\nu &= u_\nu \alpha_\nu + v_\nu \alpha_{\bar\nu}^\dagger \\
\alpha_{\bar\nu} &= u_\nu a_{\bar\nu} + v_\nu a_\nu^\dagger & a_{\bar\nu} &= u_\nu \alpha_{\bar\nu} - v_\nu \alpha_\nu^\dagger
\end{aligned} \tag{7.M.17}$$

Appendix 7.N spectroscopic factors in the BCS approximation

The creation operator of a pair of fermions coupled to J, M can be expressed in second quantization as

$$P^\dagger(j_1, j_2, JM) = N \sum_m \langle j_1\ m\ j_2\ M-m | J\ M\rangle\, a^\dagger_{j_1 m} a^\dagger_{j_2 M-m}, \tag{7.N.1}$$

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where N is a normalization ~~constant.~~ To determine it, we write the wave function resulting from the action of (7.N.1) on the vacuum

$$\Psi = P^\dagger(j_1, j_2, JM)|0\rangle = \frac{N}{\sqrt{2}} \sum_m \langle j_1\ m\ j_2\ M-m | J\ M\rangle \times \Big(\phi_{j_1 m}(\mathbf{r}_1)\phi_{j_2 M-m}(\mathbf{r}_2) - \phi_{j_2 M-m}(\mathbf{r}_1)\phi_{j_1, m}(\mathbf{r}_2)\Big). \tag{7.N.2}$$

The norm is

$$\begin{aligned}
|\Psi|^2 = \frac{N^2}{2} \sum_{mm'} &\langle j_1\ m\ j_2\ M-m | J\ M\rangle \langle j_1\ m'\ j_2\ M-m' | J\ M\rangle \\
&\times \Big(\phi_{j_1 m}(\mathbf{r}_1)\phi_{j_2 M-m}(\mathbf{r}_2) - \phi_{j_2 M-m}(\mathbf{r}_1)\phi_{j_1, m}(\mathbf{r}_2)\Big) \\
&\times \Big(\phi_{j_1 m'}(\mathbf{r}_1)\phi_{j_2 M-m'}(\mathbf{r}_2) - \phi_{j_2 M-m'}(\mathbf{r}_1)\phi_{j_1, m'}(\mathbf{r}_2)\Big).
\end{aligned} \tag{7.N.3}$$

Integrating we get

$$
\begin{aligned}
1 &= \frac{N^2}{2}\sum_{mm'}\langle j_1\; m\; j_2\; M-m|J\; M\rangle\langle j_1\; m'\; j_2\; M-m'|J\; M\rangle \\
&\times\left(2\delta_{m,m'}-2\delta_{j_1,j_2}\delta_{m,M-m'}\right) \\
&= N^2\Bigg(\sum_m\langle j_1\; m\; j_2\; M-m|J\; M\rangle^2 \\
&-\delta_{j_1,j_2}\sum_m\langle j_1\; m\; j_2\; M-m|J\; M\rangle\langle j_1\; M-m\; j_2\; m|J\; M\rangle\Bigg) \\
&= N^2\left(1-\delta_{j_1,j_2}(-1)^{2j-J}\right),
\end{aligned}
\tag{7.N.4}
$$

where we have used the closure condition for Clebsh–Gordan coefficients and (7.K.22), and δ_{j_1,j_2} must be interpreted as a δ function regarding all the quantum numbers but the magnetic one. We see that two fermions with identical quantum numbers (but the magnetic one) *cannot couple to J odd.* If J is even, the normalization constant is

$$
N=\frac{1}{\sqrt{1+\delta_{j_1,j_2}}}. \tag{7.N.5}
$$

Check Gregory

To sum up,

$$
P^\dagger(j_1,j_2,JM)=\frac{1}{\sqrt{1+\delta_{j_1,j_2}}}\sum_m\langle j_1\; m\; j_2\; M-m|J\; M\rangle\, a^\dagger_{j_1m}a^\dagger_{j_2M-m}. \tag{7.N.6}
$$

The spectroscopic amplitude for finding in a $A+2$, J_f, M_f nucleus a couple of nucleons with quantum numbers j_1, j_2 coupled to J on top of a A, J_i nucleus is

$$
B(J,j_1,j_2)=\sum_{M,M_i}\langle J_i\; M_i\; JM|J_f\; M_f\rangle\langle\Psi_{J_fM_f}|P^\dagger(j_1,j_2,JM)|\Psi_{J_iM_i}\rangle. \tag{7.N.7}
$$

This is completely general. It depends on the structure model only through the way the $A+2$ and A nuclei are treated. We now want to turn our attention to the expression of $B(J,j_1,j_2)$ in the BCS approximation when both the $A+2$ and the A are 0^+, zero–quasiparticle ground states. In order to do this, we write (7.N.6) in terms of quasiparticle operators using (7.M.17)[3]:

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$$
\begin{aligned}
P^\dagger(j_1,j_2,JM) =&\frac{1}{\sqrt{1+\delta_{j_1,j_2}}}\sum_{m_1,m_2}\langle j_1\; m_1\; j_2\; m_2|J\; M\rangle\Big(U_{j_1}U_{j_2}\alpha^\dagger_{j_1m_1}\alpha^\dagger_{j_2m_2} \\
&+(-1)^{j_1+j_2-M}V_{j_1}V_{j_2}\alpha_{j_1-m_1}\alpha_{j_2-m_2} \\
&+(-1)^{j_2-m_2}U_{j_1}V_{j_2}\alpha^\dagger_{j_1m_1}\alpha_{j_2-m_2} \\
&-(-1)^{j_1-m_1}V_{j_1}U_{j_2}\alpha^\dagger_{j_2m_2}\alpha_{j_1-m_1} \\
&+(-1)^{j_1-m_1}V_{j_1}U_{j_2}\delta_{j_1j_2}\delta_{-m_1m_2}\Big).
\end{aligned}
\tag{7.N.8}
$$

[3] In what follows, we use the phase convention $\alpha_{j\tilde{m}=(-1)^{j-m}\alpha_{j-m}}$ instead of $\alpha_{j\tilde{m}=(-1)^{j+m}\alpha_{j-m}}$, consistent with (7.M.2). I don't know why, but it seems to be common practice... Had we stick to the definition (7.M.2) the amplitude $B(0,j,j)$ calculated below would have a minus sign, which would not have any physical consequence.

Check Gregory

If both nuclei are in zero–quasiparticle states, the only term that survives is the last one in the above expression, and (7.N.7) becomes

from p. (114) a next

$$\begin{aligned} B(0,j,j) &= \frac{1}{\sqrt{2}}\sum_m \langle j\ m\ j\ -m|0\ 0\rangle(-1)^{j-m}V_jU_j \\ &= \frac{1}{\sqrt{2}}\sum_m \frac{(-1)^{j-m}}{\sqrt{(2j+1)}}(-1)^{j-m}V_jU_j \\ &= \frac{1}{\sqrt{2}}\sum_m \frac{1}{\sqrt{(2j+1)}}V_jU_j. \end{aligned} \tag{7.N.9}$$

After doing the sum, we finally find

$$B(0,j,j) = \sqrt{j+1/2}\ V_jU_j. \tag{7.N.10}$$

Note that in this final expression V_j refers to the A nucleus, while U_j is related to the $A+2$ nucleus. In practice, it does not make a big difference to calculate both for the same nucleus.

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(a) Appendix O

Derivation of two-nucleon transfer transition amplitudes including recoil, non-orthogonaty and successive transfer

(Bayman 1975)

In the present [redacted] appendix we reproduce what, arguably, was the first complete derivation of the different contribution which allowed to calculated absolute two-nucleon transfer cross sections in a systematic way. Within this context we refer to Broglia, Hansen and Riedel (1973) and Potel et al (2013), in particular Fig. 10 of this reference. (a)

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Cf. Bayman (1971) and Bayman and Chen (1982)

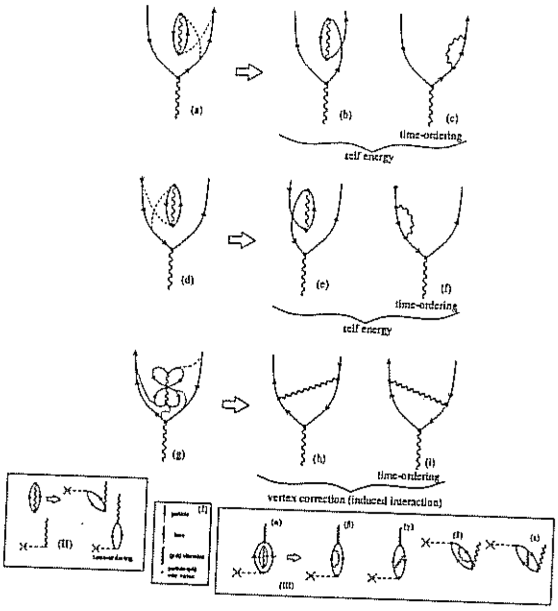

Figure 7.A.1: Nuclear field theory (NFT) diagrams corresponding to the lowest order medium polarization effects renormalizing the properties of a particle–hole collective mode (wavy line), correlated particle–hole excitation which in the shell model basis corresponds to a linear combination of particle–hole ((up–going)–(down–going) arrowed lines) excitations calculated within the random phase approximation (RPA) in of a bare interaction, and leading to the particle–vibration coupling vertex (formfactor and strength, i.e. transition density solid dot, see inset (I), bottom). The action of an external field on the zero point fluctuations (ZPF) of the vacuum (inset (II)), forces a virtual process to become real, leading to a collective vibration by annihilating a (virtual, spontaneous) particle–hole excitation (backwards RPA amplitude) or, in the time ordered process, by creating a particle–hole excitation which eventually, through the particle–vibration coupling vertex, correlate into the collective (coherent state; forwardsgoing amplitudes). Now, oyster–like diagrams associated with the vacuum ZPF can occur at any time (see inset (III)). Because the texture of the vacuum is permeated by symmetry rules (while one can violate energy conservation in a virtual state one cannot violate e.g. angular momentum conservation or the Pauli principle), the process shown in the inset III (α) leads, through Pauli principle correcting processes (exchange of fermionic arrowed lines) to self–energy (inset III (β), (δ)) and vertex corrections ~~(induced p–h interaction; inset III (γ), (ε)) processes~~. The first ones are detailed in graphs (a)–(f), while the second ones in graphs (g)–(i). In keeping with the fact that the vibrational states can be viewed as a coherent state (cf. App. 7F) exhausting a large fraction of the EWSR (e.g. a Giant Resonance) for which the associated uncertainty relations in momentum and coordinate fulfills the absolute minimum consistent with quantum mechanics ($\Delta\alpha_{\lambda\mu}\Delta\pi_{\lambda\mu} = \hbar/2, \alpha_{\lambda\mu} = (\hbar\omega_{\mu}/2C_{\lambda}^{1/2})(\Gamma^{\dagger}_{\lambda\mu} + \Gamma_{\lambda\mu})$ being the ~~(har-monic)~~ collective coordinate, $\pi_{\lambda\mu}$ being the conjugate momentum; cf. e.g. 3), there is a strong cancellation between the contribution of self–energy and vertex correction diagrams (6, implying small anharmonicities and long lifetimes ($\Gamma/E \ll 1$, where Γ is the width and E the centroid of the mode $|\lambda\mu\rangle = \Gamma^{\dagger}_{\lambda\mu}|0\rangle$, $(\hbar\omega_{\lambda}/2C_{\lambda})^{1/2}$ being the ZPF amplitude (cf. e.g. Brink and Broglia (2005))).

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Glauber (1969)

(P.F. Bortignon and R.A. Broglia, (1981)

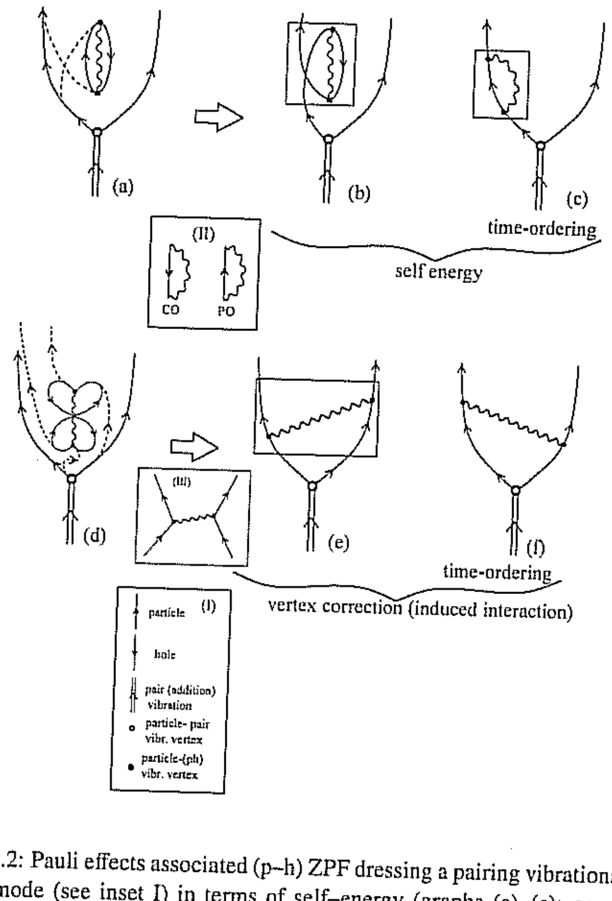

Figure 7.A.2: Pauli effects associated (p–h) ZPF dressing a pairing vibrational (pair addition) mode (see inset I) in terms of self–energy (graphs (a)–(c); correlation (CO) and polarization (PO) diagrams, inset II) and vertex correction (graphs (d)–(f); induced particle–particle (pairing) interaction,) processes (inset (III)).

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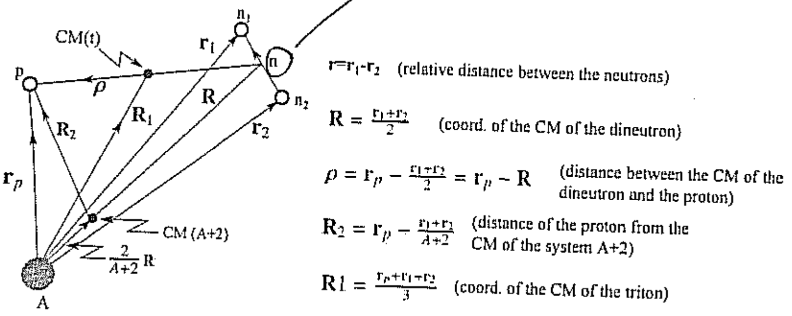
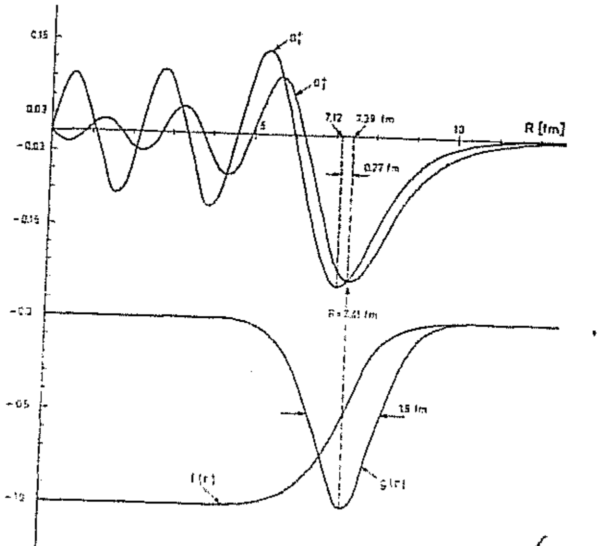


Figure 7.B.1: Coordinate system used in the calculation of the two-nucleon transfer amplitude.



($^{206}Pb(t,p)^{206}Pb$)

Figure 7.B.2: The upper part of the figure shows the modified form factor for the transition to the ground state (0^{1+}) and the pairing vibrational state (0^{2+}) at 4.87 MeV. Both curves are matched with appropriate Hankel functions. In the lower part the form factors of the real ($f(r)$) and the imaginary ($g(r)$) part of the optical potential are given in the same scale for the radius.

(use to calculate the differential cross sections (cf. Ch. 8).

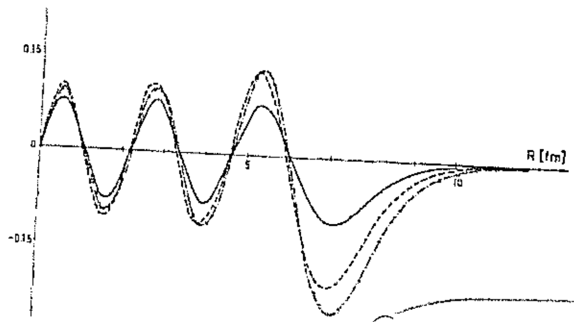


Figure 7.B.3: Modified form factor for the transition to the ground state $({}^{206}Pb(t,p)\,{}^{208}Pb(gs))$ calculated in different spectroscopic models (pure shell–model configuration ——, shell model plus pairing residual interaction – – –, including ground state correlations –o–o–). After Broglia and Riedel (1967)

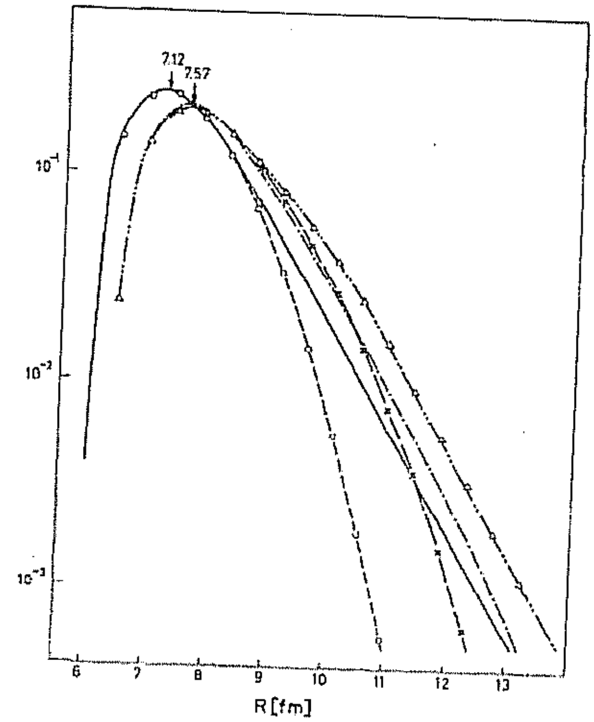


Figure 7.B.4:

Asymptotic behaviour of the $^{206}Pb(t,p)$ modified formfactor for the ground state transition for oscillator plus Hankel wavefunctions (——), oscillator wavefunctions alone (—o—o—), and Saxon-Woods wavefunctions with a variety of asymptotic matchings (—x—x—), (—Δ—Δ—), (—·—·—). After Broglia and Riedel 1967.

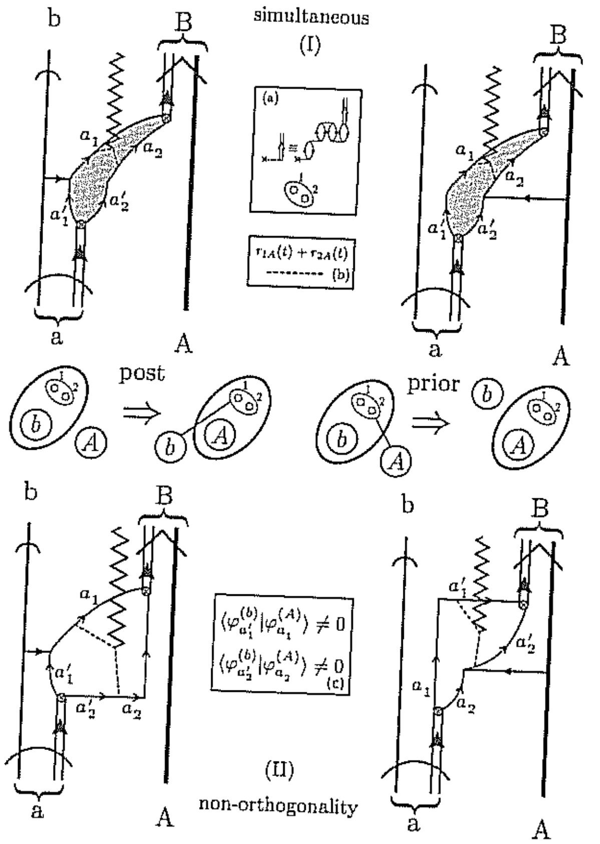


Figure 7.C.1: ~~See Fig. 7.C.2~~ Graphical representation of simultaneous and non-orthogonality transfer processes. For details cf. Caption to Fig. 7.C.2

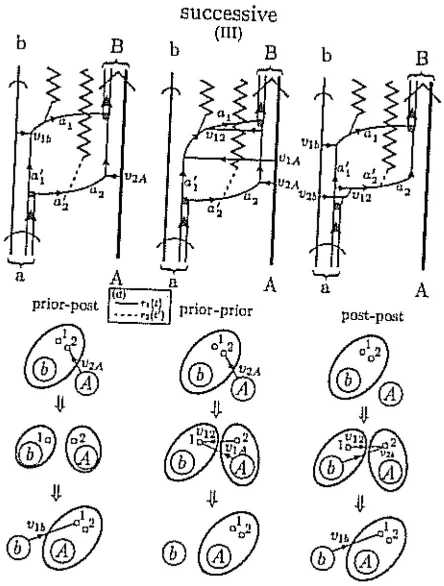
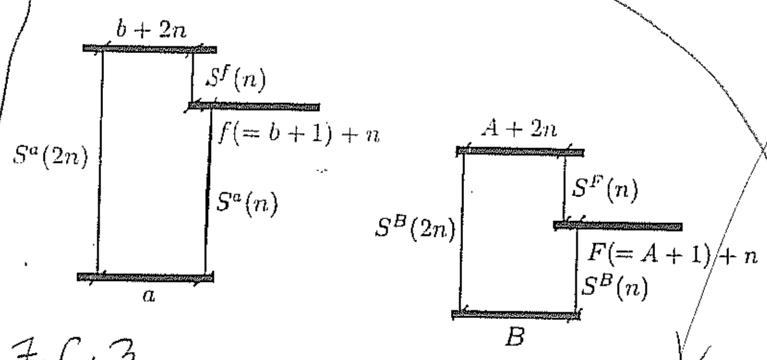


Figure 7.C.2: Graphical representation of the lowest order ((I),(II) and (III) first and second order in v respectively), two-nucleon transfer processes, which correctly converge to the strong-correlation (only simultaneous transfer), and to the independent-particle (only successive transfer) limits. The time arrow is assumed to point upwards: (I) Simultaneous transfer, in which one particle is transferred by the nucleon-nucleon interaction (note that $U(r) = \int d^3r' \rho(r') v(|\vec{r} - \vec{r}'|)$) acting either in the entrance $\alpha \equiv a+A$ channel (prior) or in the final $\beta \equiv b+B$ channel (post), while the other particle follows suit making use of the particle-particle correlation (grey area) which binds the Cooper pair (see upper inset labelled (a)), represented by a solid arrow on a double line, to the projectile (curved arrowed lines) or to the target (opened arrowed lines). The above argument provides the explanation why when e.g. v_{1b} acts on one nucleon, the other nucleon also reacts instantaneously. In fact a Cooper pair displays generalized rigidy (emergent property in gauge space). A crossed open circle represents the particle-pair vibration coupling. The associated strength, together with an energy denominator, determines the amplitude $X_{a_1'a_2'}$ (~~cf. Table I~~) with which the pair mode (Cooper pair) is in the (time reversered) two particle configuration $a_1'a_2'$. In the transfer process, the orbital of relative motion changes, the readjustement of the corresponding trajectory mismatch being operated by a Galilean operator $(\exp[\vec{k} \cdot (\vec{r}_{1A}(t) + \vec{r}_{2A}(t))])$. This phenomenon, known as recoil process, is represented by a jagged line which provides simultaneous information on the two transferred nucleons (single time appearing as argument of both single-particle coordinates r_1 and r_2; see inset labeled (b)). In other words, information on the coupling of structure and reaction modes. (II) Non-orthogonality contribution. While one of the nucleons of the Cooper pairs is transferred under the action of v, the other goes, uncorrelatedly over, profiting of the non-orthogonality of the associated single-particle wavefunctions (see inset (c)). In other words of the non-vanishing values of the overlaps, as shown in the inset. (III) Successive transfer. In this case, there are two time dependences associated with the acting of the nucleon-nucleon interaction twice (see inset (d)).

with Caption



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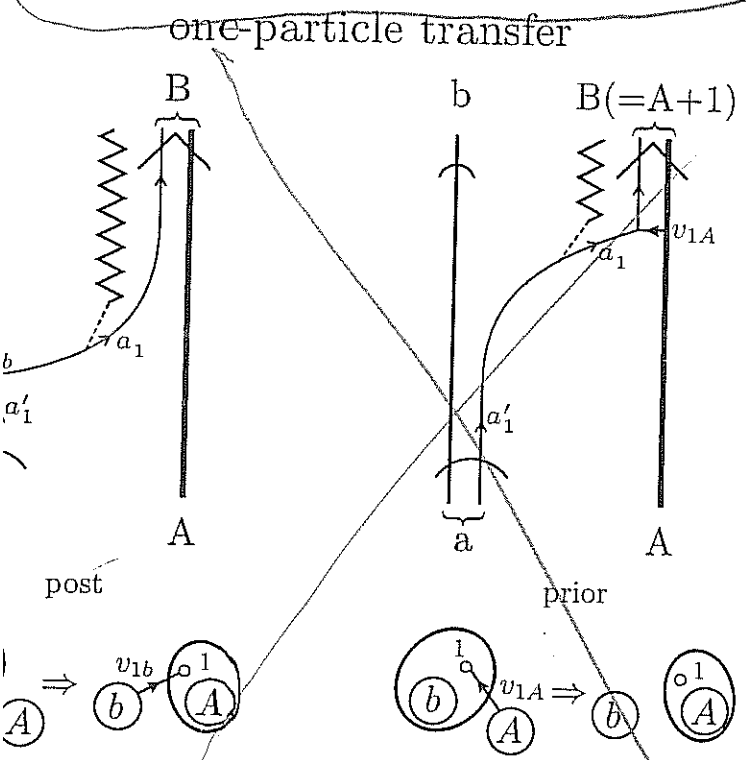
8. One- and two-neutron separation energies $S(n)$ and $S(2n)$ .ted with the channels $\alpha \equiv a(= b+2)+A \longrightarrow \gamma \equiv f(=$ + $F(= A+1) \longrightarrow \beta \equiv b + B(= A+2)$.



). One-particle transfer reactions $a(= b+1)+A \longrightarrow b$ + 1). The different symbols have been defined in the o figure 7.

ong correlation (cluster) limit

$$\times \frac{1}{i\hbar}\int_{-\infty}^{t} dt' (\psi^f\psi^F, (V_{aA} - \langle V_{aA}\rangle)\psi^a\psi^A)$$
$$\times \exp\left[\frac{i}{\hbar}(E^{fF} - E^{aA})t' + \gamma_2(t')\right].$$

The relations (A16), (A17) imply

$$E^{fF} - E^{aA} = S^a(n) - S^F(n) \gg \frac{\hbar}{\tau},$$

where τ is the collision time. Consequently the rea of $a^{(2)}(\infty)$ vanishes exponentially with the Q-value intermediate transition, while the imaginary part va inversely proportional to this energy. One can thus writ

$$\text{Re}\, a^{(2)}(\infty) \approx 0 \quad ($$

and

$$a^{(2)}(\infty) \approx \frac{1}{i\hbar}\frac{\tau}{\langle E^{fF}\rangle - E^{bB}}$$
$$\times \sum_{fF}(\psi^b\psi^B, (V_{bB} - \langle V_{bB}\rangle)\psi^f\psi^F)_{t=0}$$
$$\times(\psi^f\psi^F, (V_{aA} - \langle V_{aA}\rangle)\psi^a\psi^A)_{t=0}, \quad ($$

where one has made use of the fact that $E^{bB} \approx E^{aA}$. $v_{12} \to \infty$, $(\langle E^{fF}\rangle - E^{bB}) \to \infty$ and, consequently,

$$\lim_{v_{12}\to\infty} a^{(2)}(\infty) = 0. \quad ($$

Thus the complete two-nucleon transfer amplitu equal, in the strong coupling limit, to the amplitude $a^{(1)}($

Summing up, only when successive transfer and orthogonal corrections are included in the description o two-nucleon transfer process, does one obtain a consi description of the process, which correctly converges to the weak and the strong correlation limiting values.

Appendix B. Equivalence between NFT and F–G propagators

Chapter 8

Chapter 8

Appendix 8.A App

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