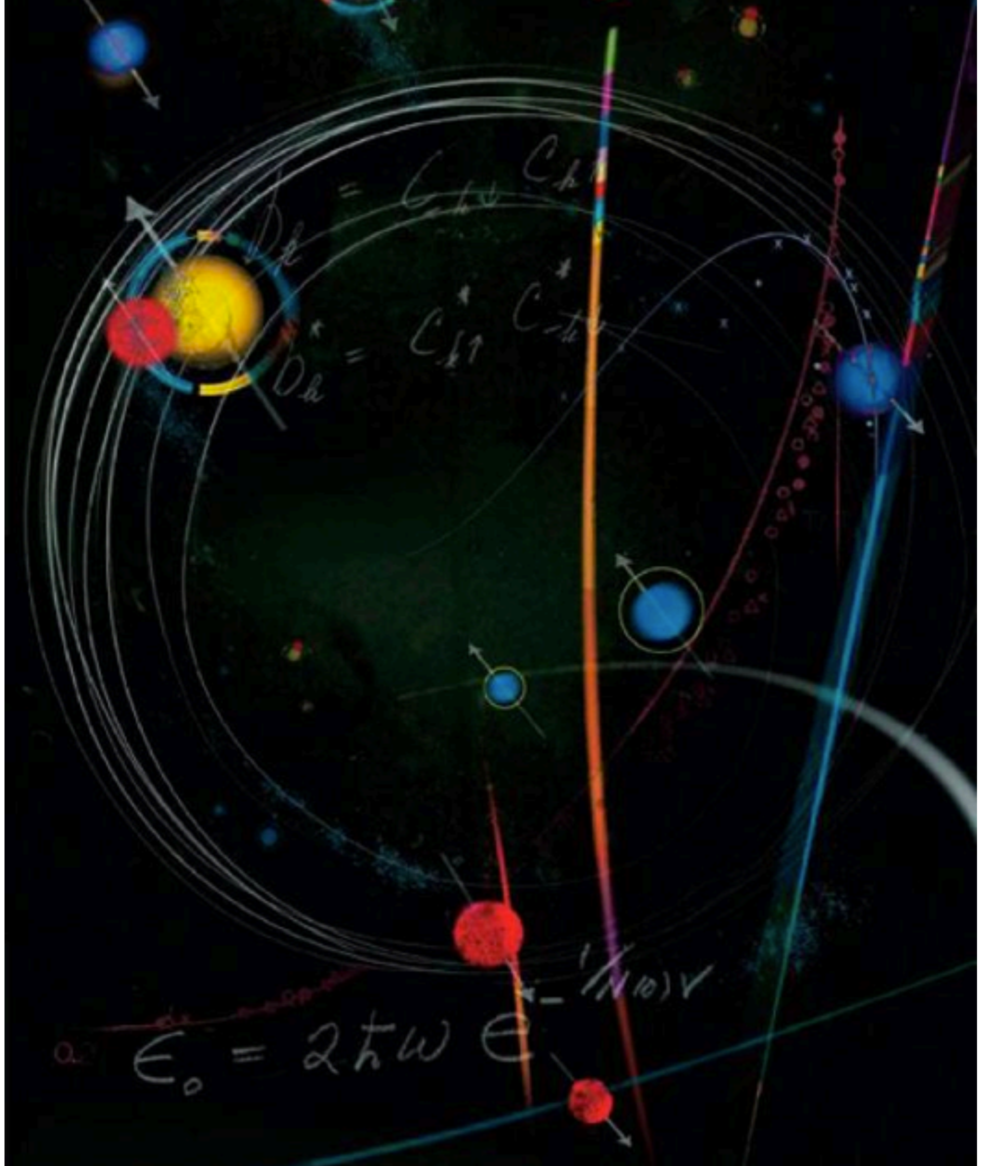
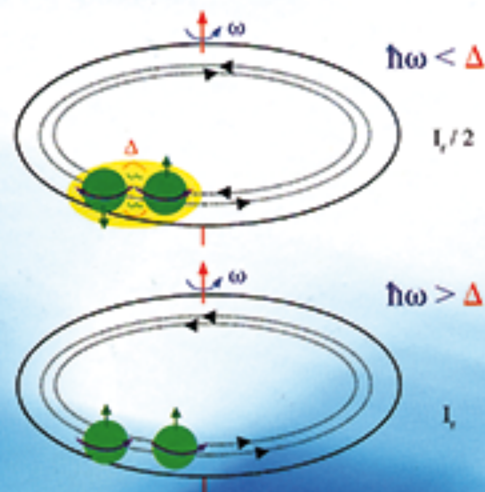
BCS: 50 Years



Edited by Leon N Cooper and Dmitri Feldman

Fifty Years of Nuclear BCS

Pairing in Finite Systems



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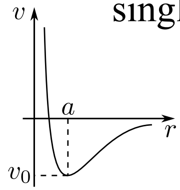
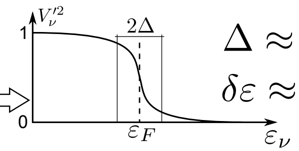
Classical localization and quantal ZPF

$\delta x \delta k \geq 1 \qquad \varepsilon = \frac{\hbar^2 k^2}{2M} \qquad \delta k = \frac{\delta\varepsilon}{\hbar v_F} \qquad (v_{F/c} \approx 0.3)$

$\boxed{\text{structure}}$

Independent motion of

single nucleons	pairs of nucleons
	
$a \approx 1$ fm	$\Delta \approx 1$ MeV
$v_0 = -100$ MeV	$\delta\varepsilon \approx 2\Delta; \delta x = \xi$
$\delta\varepsilon = \lvert v_0 \rvert; \delta x \approx a$	
$\delta x \delta k = \frac{a\lvert v_0\rvert}{\hbar v_F} \geq 1$	$\delta x \delta k = \frac{\xi 2\Delta}{\hbar v_F} \geq 1$
quantality parameter	correlation length
$q = \frac{\hbar v_F}{a\lvert v_0\rvert} \approx 0.6 \lesssim 1$	$\xi = \frac{\hbar v_F}{2\Delta} \approx 30 \text{ fm} \gg R$
delocalization	long range correlation

emergent property: generalized rigidy in

3D-space	gauge space

¿how does a short range force lead to

single-nucleon mean free paths	pairing correlations over distances

larger than nuclear dimension?

$2R \approx 20/k_F$

answer: quantal fluctuations

$\boxed{\text{reactions}}$

single particle transfer, e.g. *(p,d)*	Cooper pair transfer, e.g. *(p,t)*
$\frac{2R}{a} \approx 15$	$\frac{\xi}{a} \approx 30$
absolute cross section reflects the full nucleon probability amplitude distribution, and does not depend of the specific choice of v_{np}	Successive and simultaneous transfer amplitude contributions to the absolute cross section carry equally efficiently information concerning pair correlations

Classical localization and quantal ZPF

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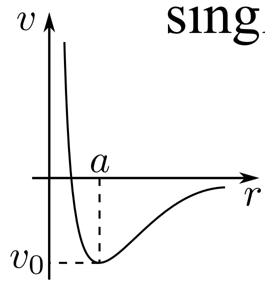	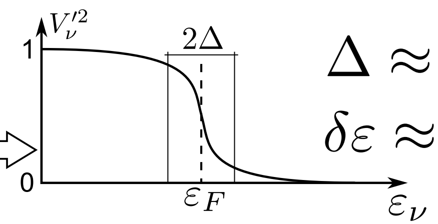
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Systematics (silent revolution)

$^{A+2}Sn(p,t)^{A}Sn$

A+2 = 112,114 116,118,120,122

Guazzoni et al, PRC
1999(122),2004(116),2006(112),
2008(120),2011(118,124),2012(114)

Major breakthrough

$^{1}H(^{11}Li,^{9}Li)^{3}H$

Tanihata et al, PRL 2008

Spectroscopic amplitudes

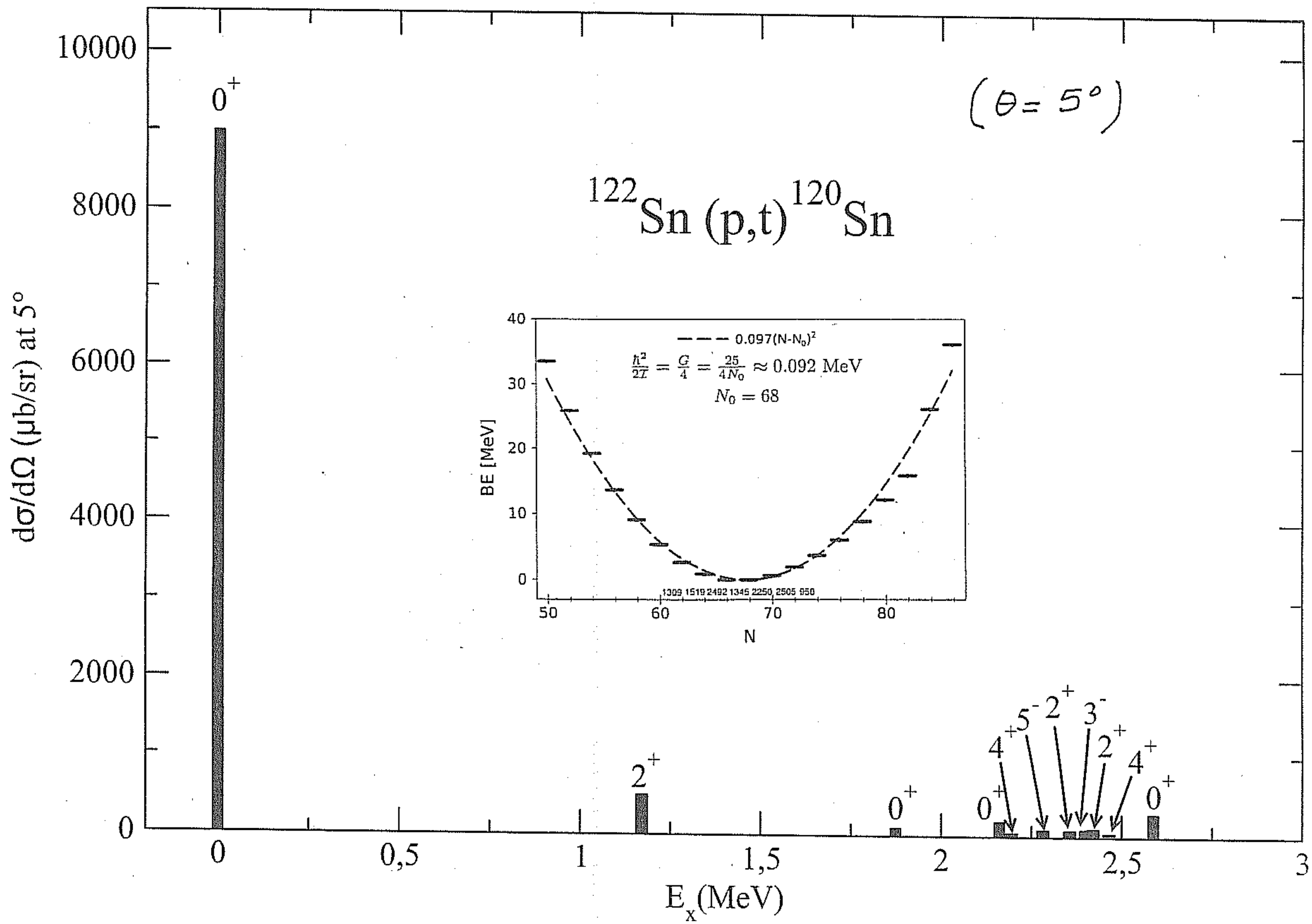
Pairing rotations $\quad {}^{A+2}Sn(p,t)^{A}Sn(gs)$

$U_\nu \; V_\nu$ (BCS)

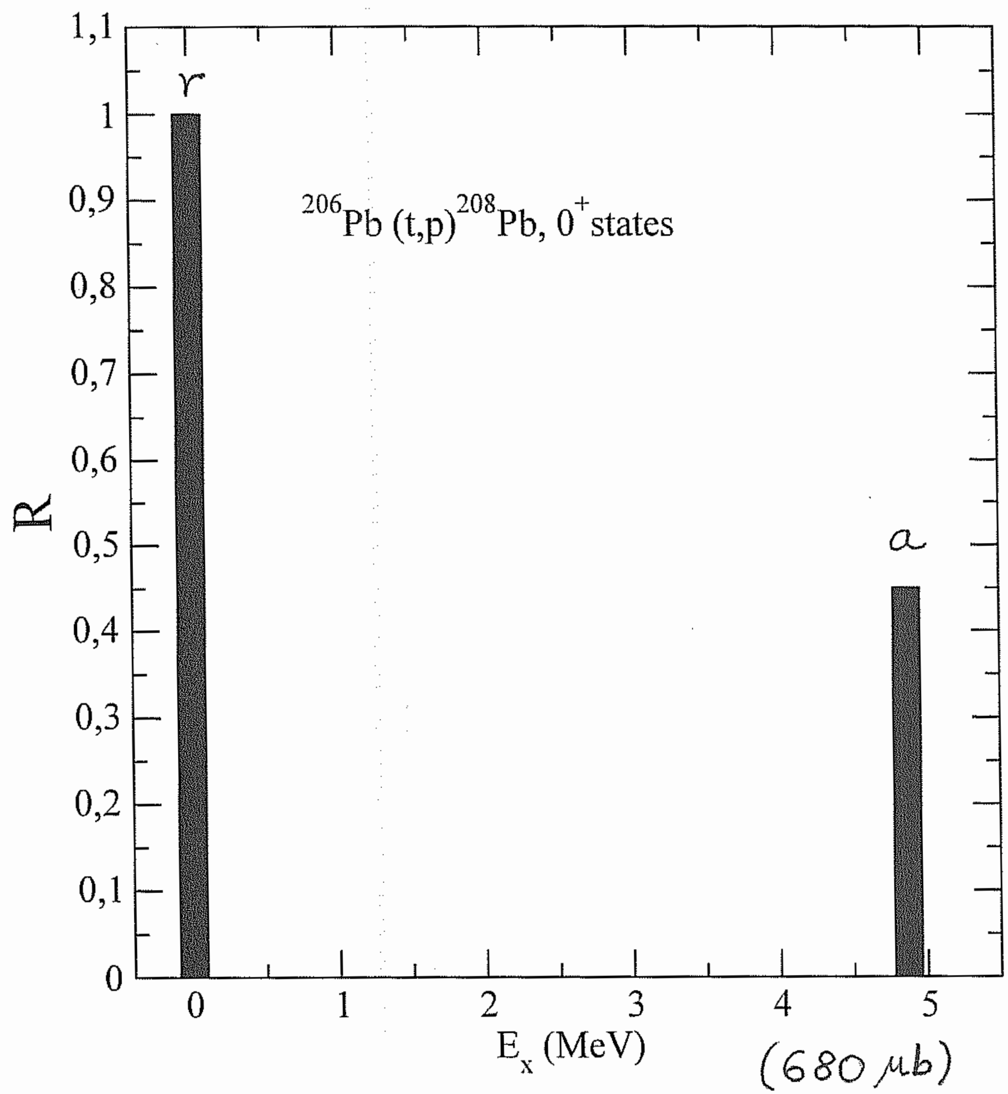
Pairing vibrations $\quad {}^{208}Pb(p,t)^{206}Pb(gs)$

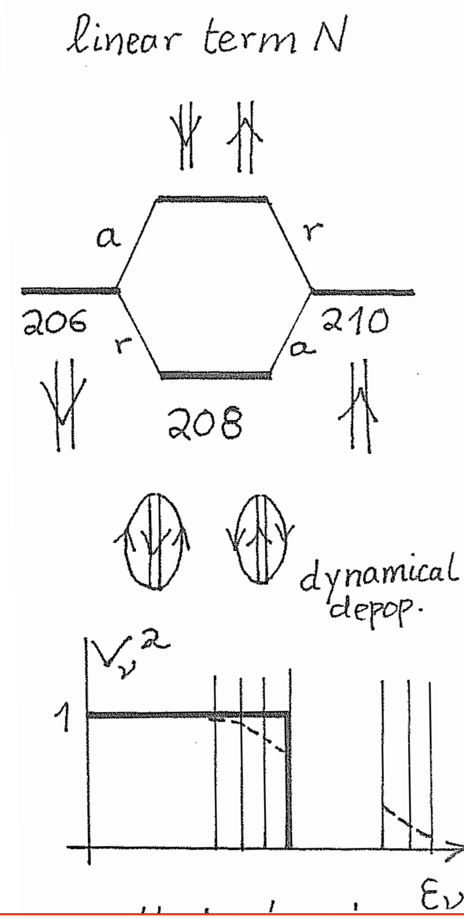
$X_r \; Y_r$ (RPA)

Coherent states: essentially exact



Potel et al., Rep. Prog. Phys. **76** (2013) 106301





Well developed vibrational nands, anharmonicities, cf. Bortignon et al., PL 76B(1978)153; Clark et al. PRL 96 (2006)032501)

$$P^\dagger = \sum_{\nu>0} a^\dagger_\nu a^\dagger_{\bar{\nu}}$$

$$x = \frac{2G\Omega}{D} = GN(0)$$

$x > 1$

$$\alpha_0 =< P^\dagger >= \frac{\Delta}{G} \approx 7$$

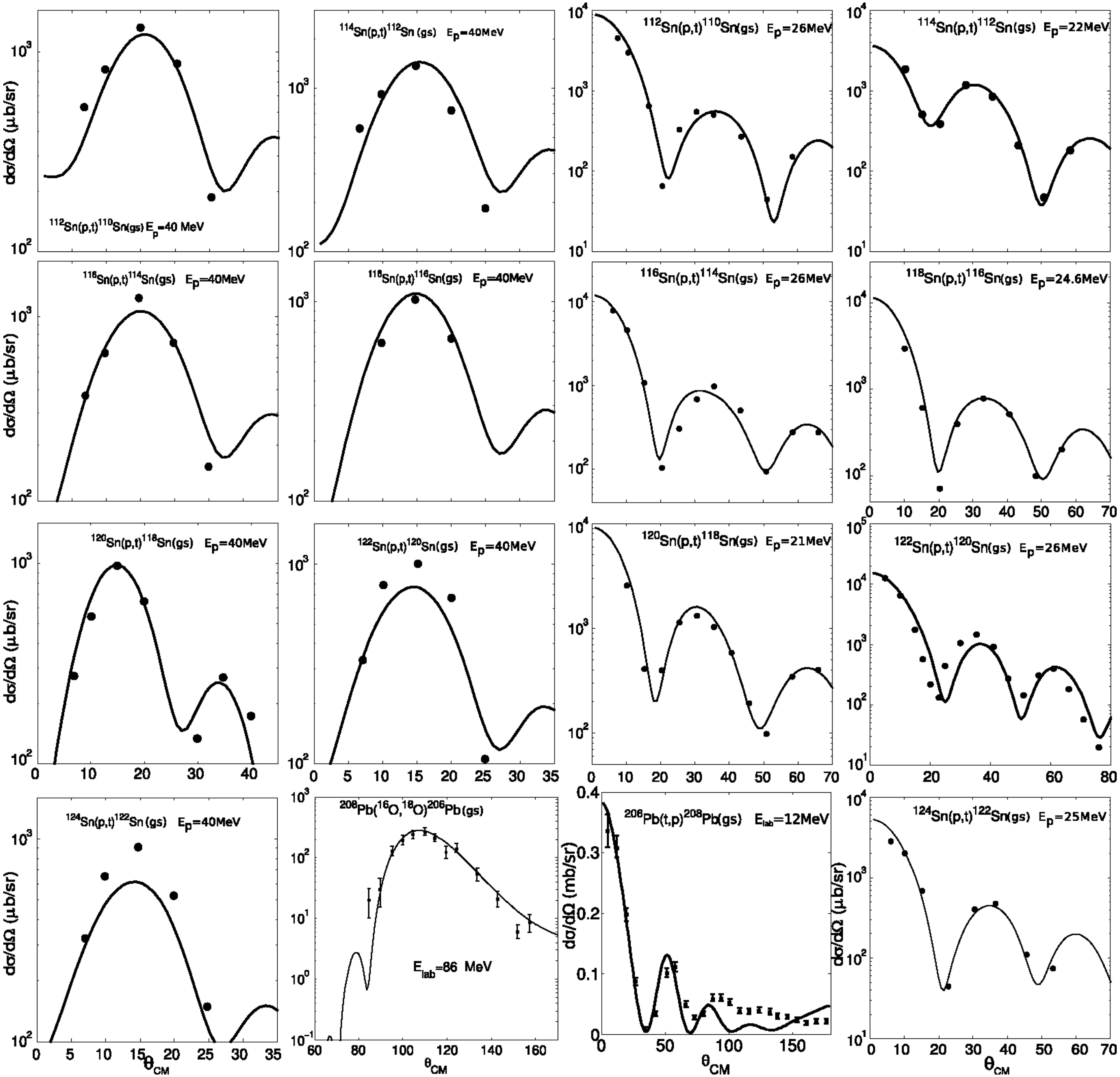
$x < 1$

$$\alpha_{dyn} = \frac{1}{G}\frac{<PP^\dagger>^{1/2}+<P^\dagger P>^{1/2}}{2}$$

$$\approx \quad \frac{1}{2}\left(\frac{E_{corr}(A+2)}{G} + \frac{E_{corr}(A-2)}{G}\right) \approx 10$$

$$\frac{\alpha_0}{\alpha_{dyn}} \approx 0.7$$

$$\frac{\beta_2}{(\beta_2)_{dyn}} \approx 3-6$$



112Sn(p,t)110Sn(gs) Ep=40 MeV
114Sn(p,t)112Sn (gs) Ep=40MeV
112Sn(p,t)110Sn(gs) Ep=26MeV
114Sn(p,t)112Sn(gs) Ep=22MeV
116Sn(p,t)114Sn(gs) Ep=40MeV
118Sn(p,t)116Sn(gs) Ep=40MeV
116Sn(p,t)114Sn(gs) Ep=26MeV
118Sn(p,t)116Sn(gs) Ep=24.6MeV
120Sn(p,t)118Sn(gs) Ep=40MeV
122Sn(p,t)120Sn(gs) Ep=40MeV
120Sn(p,t)118Sn(gs) Ep=21MeV
122Sn(p,t)120Sn(gs) Ep=26MeV
124Sn(p,t)122Sn (gs) Ep=40MeV
208Pb(16O,18O)206Pb(gs)
Elab=86 MeV
206Pb(t,p)208Pb(gs) Elab=12MeV
124Sn(p,t)122Sn(gs) Ep=25MeV
dσ/dΩ (μb/sr)
dσ/dΩ (mb/sr)
θCM

Evidence of the Giant Pairing Vibration in the ^{14}C and ^{15}C atomic nuclei

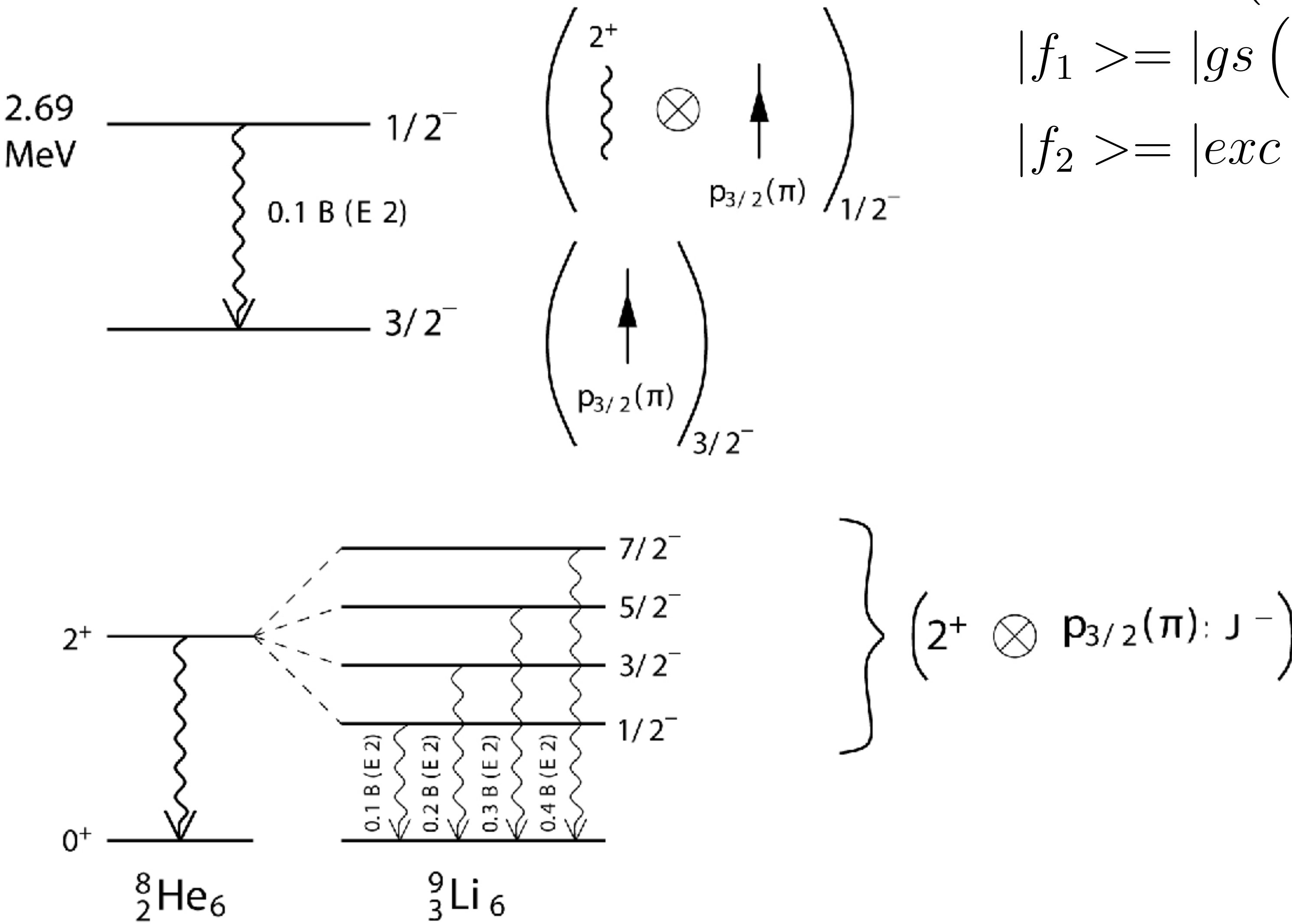
F. Cappuzzello[1,2*], D. Carbone[2], M. Cavallaro[2], M. Bondì[1,2], C. Agodi[2], F. Azaiez[3], A. Bonaccorso[4], A. Cunsolo[2], L. Fortunato[6,7], A. Foti[1,5], S. Franchoo[3], E. Khan[3], R. Linares[8], J. Lubian[8], J. A. Scarpaci[3], A. Vitturi[6,7]

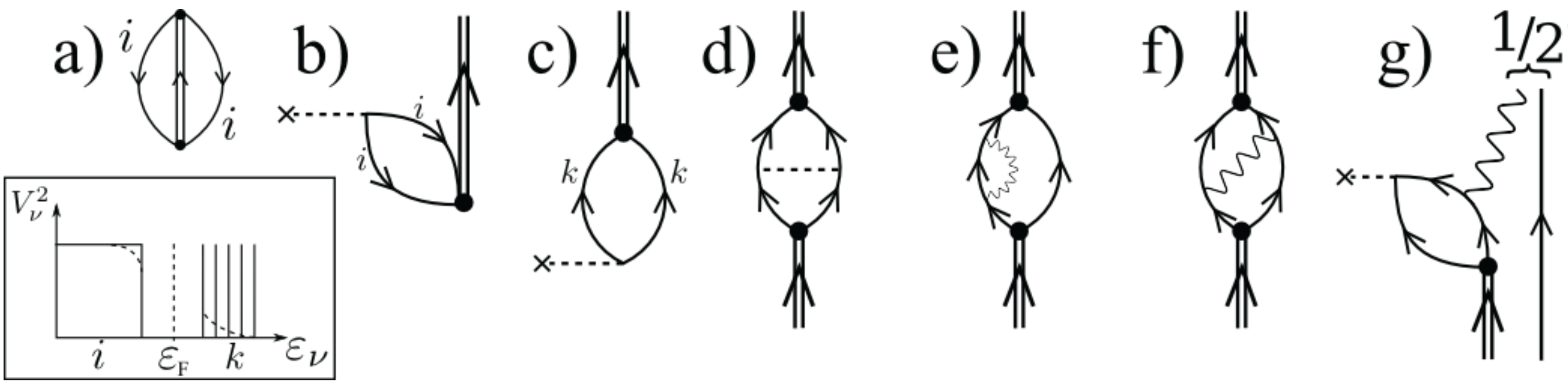
$$^{11}\mathrm{Li(i)}(^{1}\mathrm{H},^{3}\mathrm{H})^{9}\mathrm{Li(f)}$$

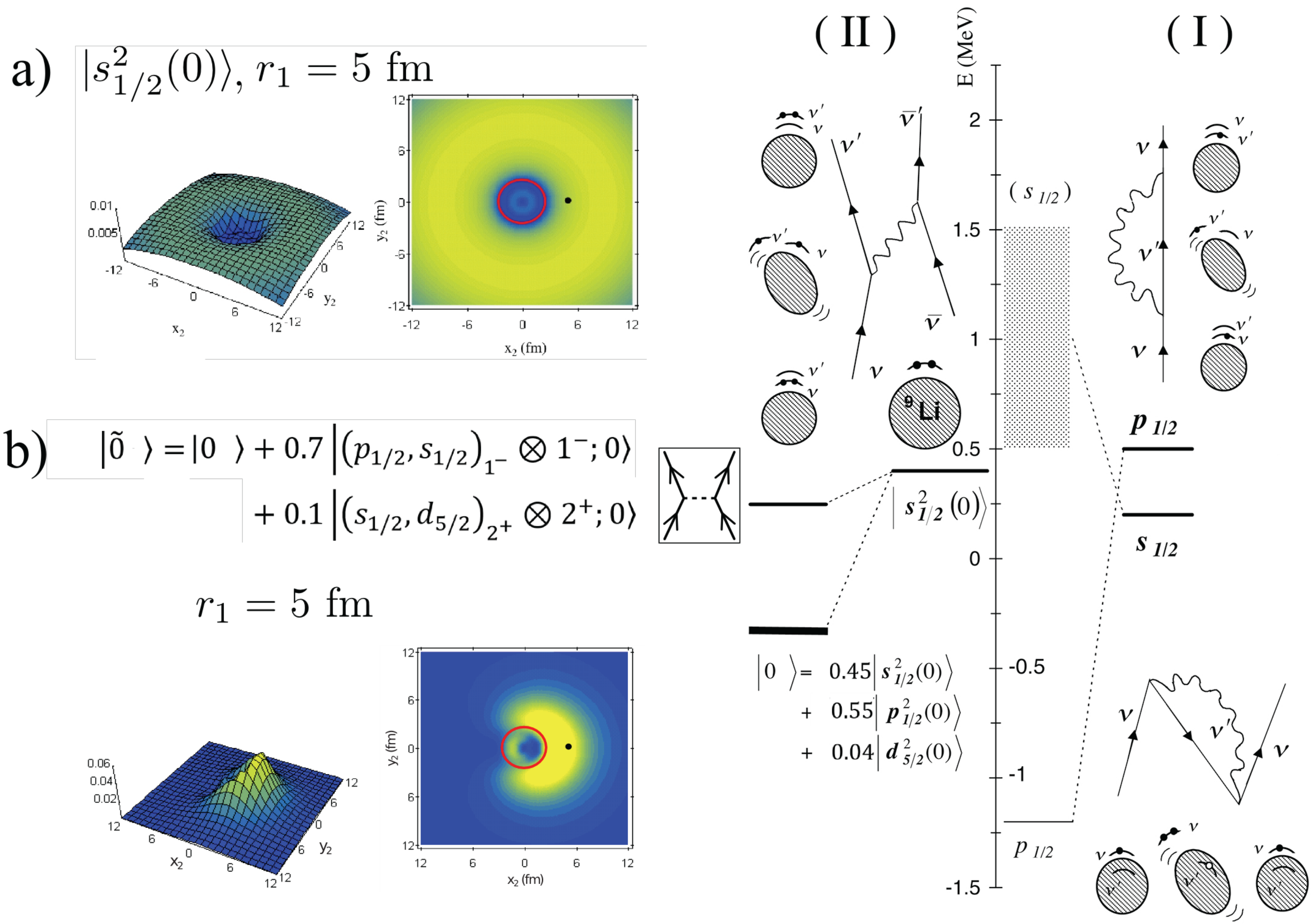
$$|i>=|gs\left(\frac{3}{2}\right)^{-}>$$

$$|f_1>=|gs\left(\frac{3}{2}\right)^{-}>$$

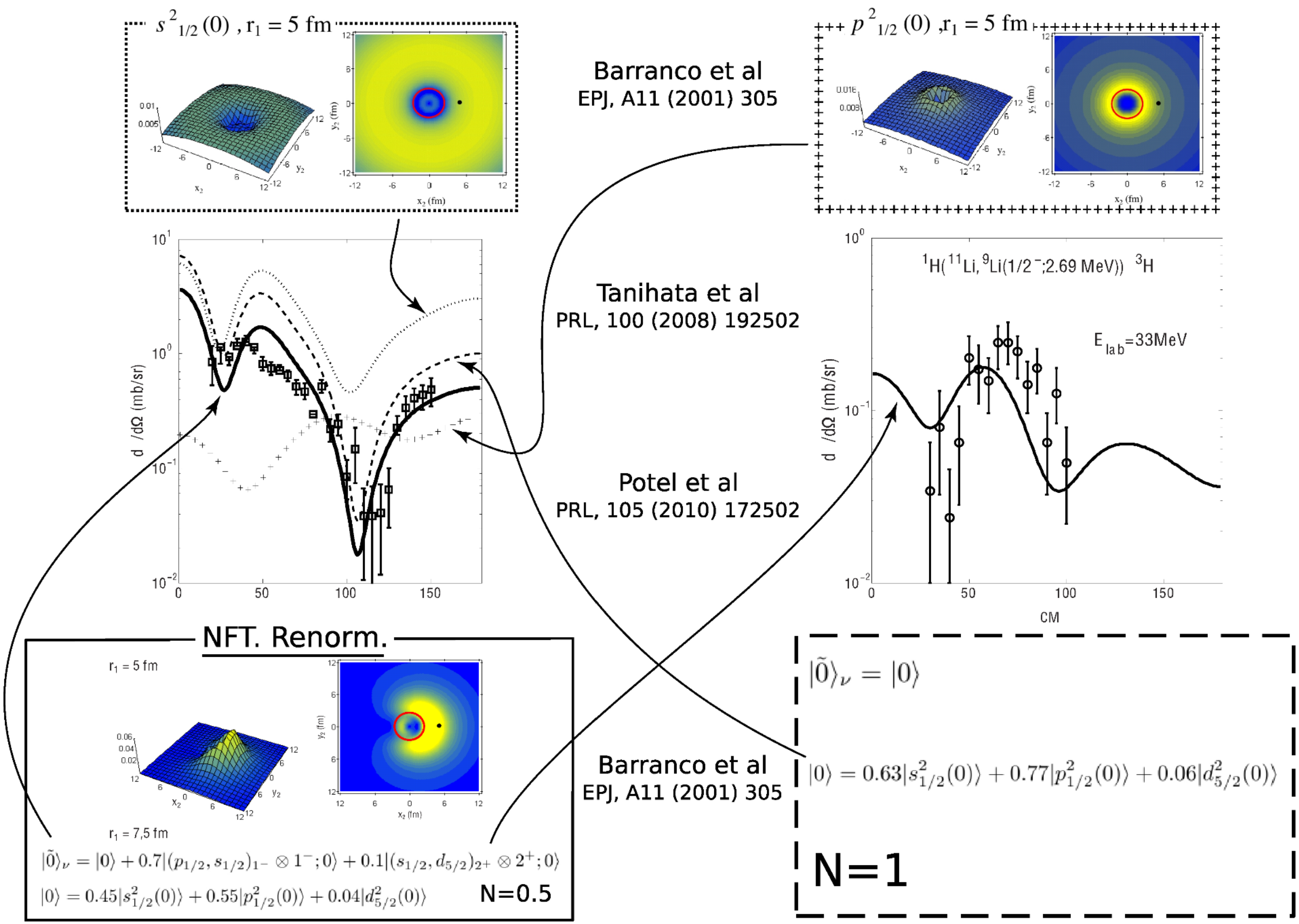
$$|f_2>=|exc\left(\frac{1}{2}\right)^{-}>$$




a)
i
i
b)
i
i
c)
k
k
d)
e)
f)
g)
1/2⁻
V_ν^2
i
ε_{F}
k
ε_ν

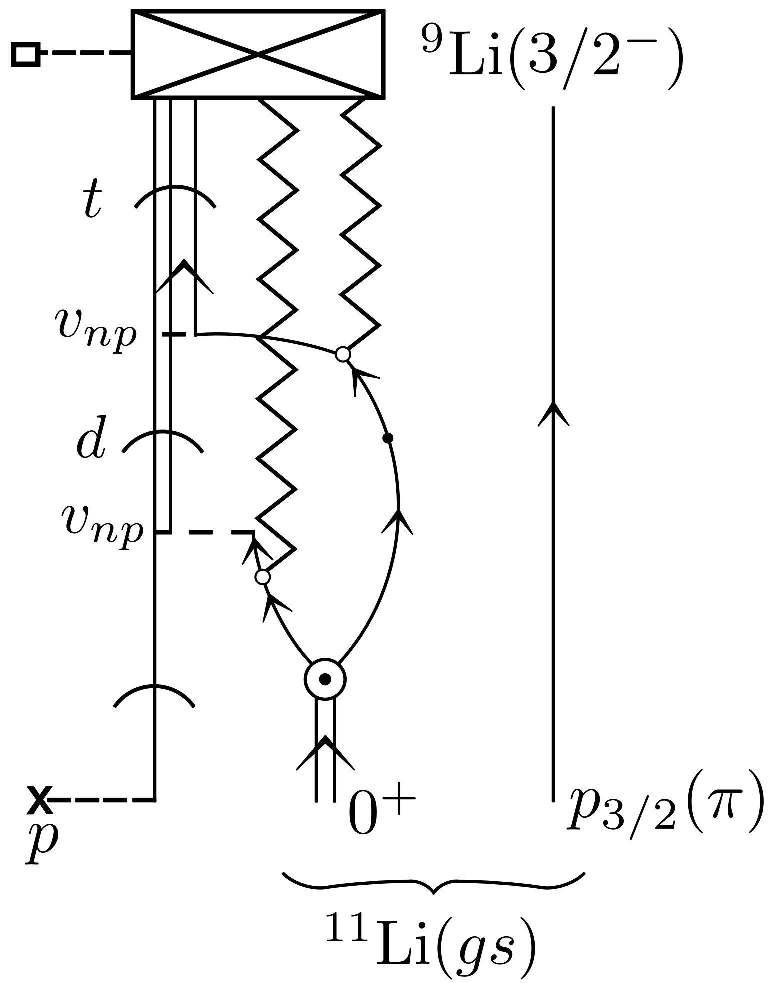


a) $|s^2_{1/2}(0)\rangle$, $r_1 = 5$ fm
b) $|\tilde{0}\rangle = |0\rangle + 0.7\left|(p_{1/2}, s_{1/2})_{1^-} \otimes 1^-; 0\right\rangle + 0.1\left|(s_{1/2}, d_{5/2})_{2^+} \otimes 2^+; 0\right\rangle$
$r_1 = 5$ fm
x2 (fm)
y2 (fm)
(II)
(I)
E (MeV)
^{9}Li
$(s_{1/2})$
$p_{1/2}$
$s_{1/2}$
$p_{1/2}$
$|s^2_{1/2}(0)\rangle$
$|0\rangle = 0.45|s^2_{1/2}(0)\rangle + 0.55|p^2_{1/2}(0)\rangle + 0.04|d^2_{5/2}(0)\rangle$



$s^2_{1/2}(0)$, $r_1 = 5$ fm
$p^2_{1/2}(0)$,$r_1 = 5$ fm
Barranco et al
EPJ, A11 (2001) 305
Tanihata et al
PRL, 100 (2008) 192502
Potel et al
PRL, 105 (2010) 172502
Barranco et al
EPJ, A11 (2001) 305
^{1}H(^{11}Li,^{9}Li(1/2$^-$;2.69 MeV)) ^{3}H
E$_{lab}$=33MeV
d /dΩ (mb/sr)
CM
NFT. Renorm.
$r_1 = 5$ fm
$r_1 = 7{,}5$ fm
$|\tilde{0}\rangle_\nu = |0\rangle + 0.7|(p_{1/2}, s_{1/2})_{1^-} \otimes 1^-; 0\rangle + 0.1|(s_{1/2}, d_{5/2})_{2^+} \otimes 2^+; 0\rangle$
$|0\rangle = 0.45|s^2_{1/2}(0)\rangle + 0.55|p^2_{1/2}(0)\rangle + 0.04|d^2_{5/2}(0)\rangle$
N=0.5
$|\tilde{0}\rangle_\nu = |0\rangle$
$|0\rangle = 0.63|s^2_{1/2}(0)\rangle + 0.77|p^2_{1/2}(0)\rangle + 0.06|d^2_{5/2}(0)\rangle$
N=1

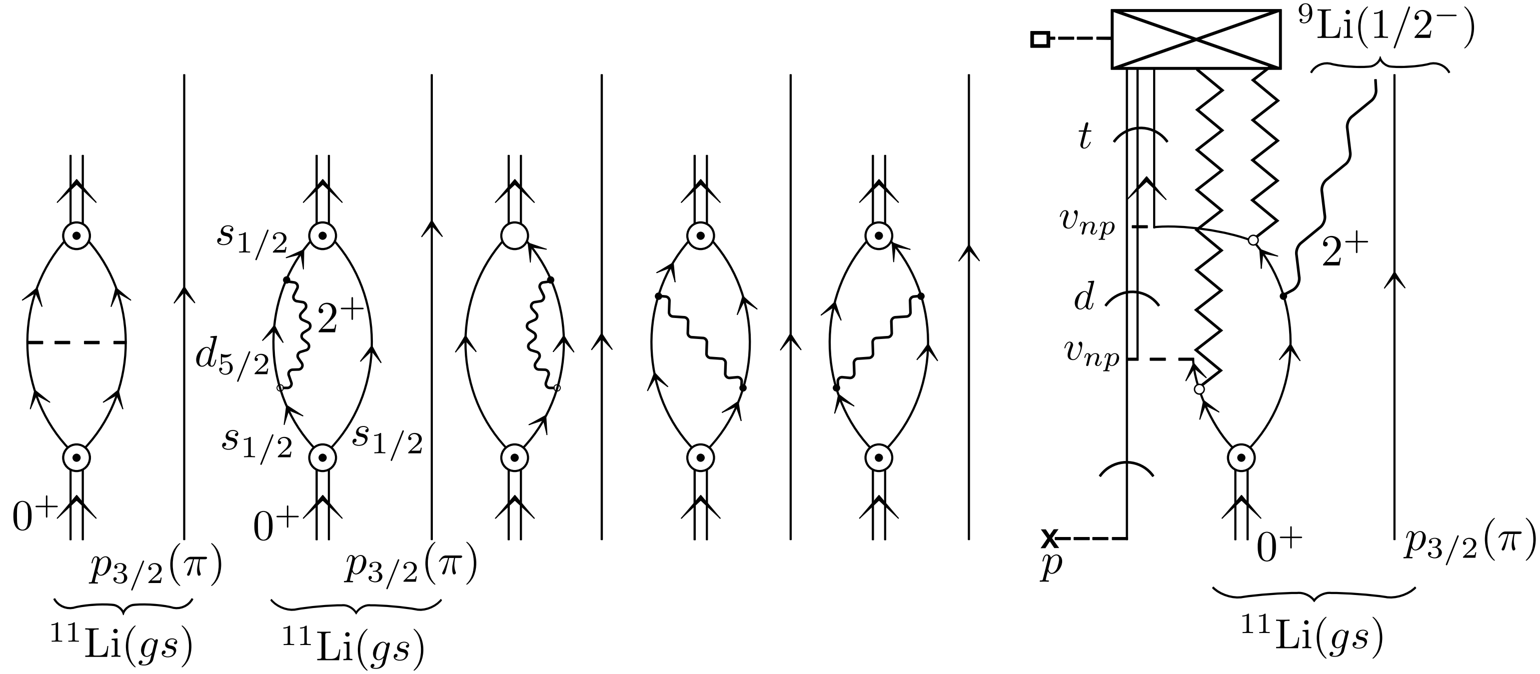
$^{1}\mathrm{H}(^{11}\mathrm{Li},^{9}\mathrm{Li}(gs))^{3}\mathrm{H}$



Variety of pv-coupling vertices (NFT)

- ⊙ pair
- • surface
- ∘ recoil

$^{1}\mathrm{H}(^{11}\mathrm{Li},^{9}\mathrm{Li}(1/2^{-}))^{3}\mathrm{H}$



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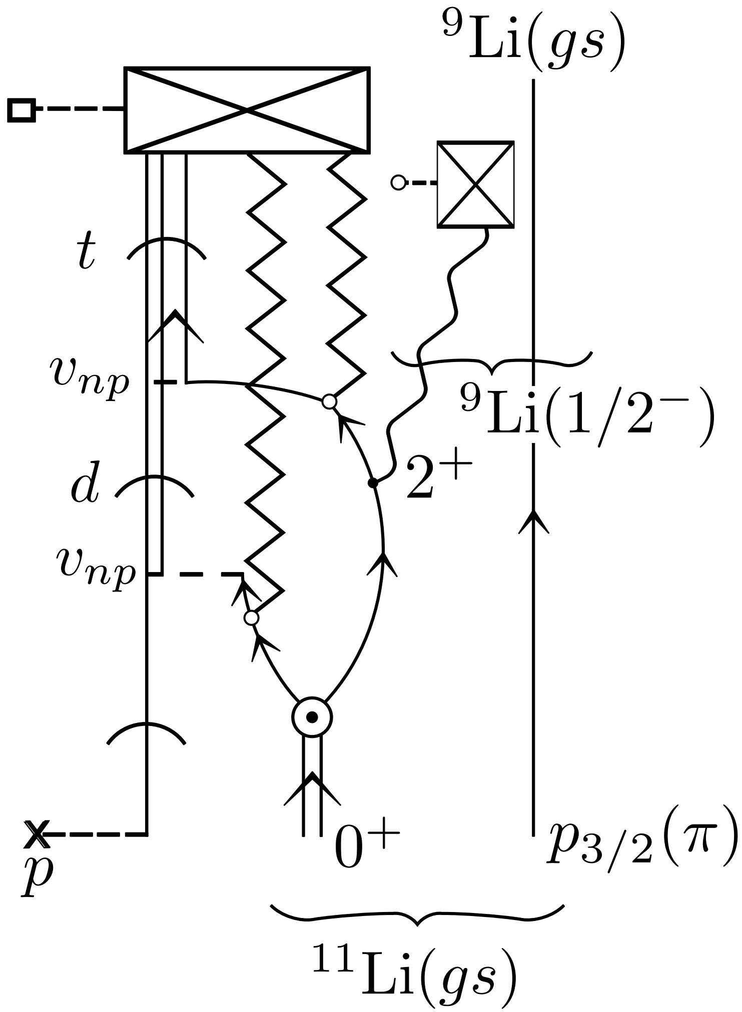


Variety of pv-coupling vertices (NFT)

⊙ pair
• surface
∘ recoil

$$V(r_{12}) = -4\pi V_0 \delta(|\vec{r}_1 - \vec{r}_2|)$$

$$\mathcal{R}_j = \sqrt{\tfrac{3}{R_0^3}}\Theta(r - R_0)$$

$$I(j) = \int_0^\infty dr \mathcal{R}_j^4 \, r^2 = \frac{3}{R_0^3}$$

$$M_j = < (j)^2_0 |V| (j)^2_0 > = - \tfrac{2j+1}{2} V_o I(j)$$

$$r = \frac{(M_j)_{halo}}{(M_j)_{syst}} = \left(\frac{R_0}{R}\right)^3$$

Halo anti-pairing effect
(cf. Bennaceur, Dobaczewski,
Hamamoto, Mottelson, Ploszajczak)

R_0 = 1.2 $A^{1/3}$ fm
(systematics)

R: halo radius

$$\Theta(r - R) = \begin{cases} 1 & r \leq R \\ 0 & r > R \end{cases}$$

core radius (systematics)

$$R_0 = 1.2 \times 9^{1/3} \approx 2.5 \text{ fm}$$

$$\mathcal{R} = \sqrt{\tfrac{3}{R_0^3}}\Theta(r - R_0)$$

$$\int_0^\infty dr\ r^2\mathcal{R}^2 \ = \frac{3}{R_0^3}\int_0^\infty \frac{dr^3}{3} = 1$$

Halo radius

$$R \approx \sqrt{\tfrac{5}{3}}\ (3.55\ \pm 0.1)\ \text{fm} \approx 4.6 \pm 1.3\ \text{fm}$$

$$\mathcal{R}_{halo} = \sqrt{\tfrac{3}{R^3}}\Theta(r - R)$$

Two-nucleon overlap probability

$$o = |<\mathcal{R}_{halo}|\mathcal{R}>|^2$$

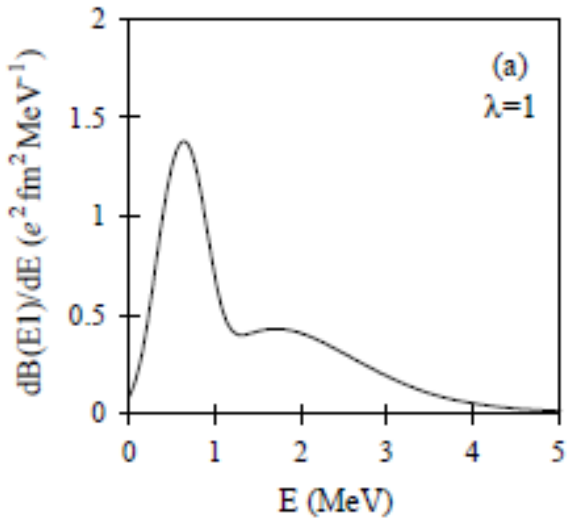
$$o = \left(\int_0^\infty\ dr\ r^2\mathcal{R}_{halo}\mathcal{R}\right)^2 = \left(\sqrt{\tfrac{3}{R^3}}\sqrt{\tfrac{3}{R_0^3}}\int_0^\infty \tfrac{dr^3}{3}\right)^2 = \left(\tfrac{R_0}{R}\right)^3 \approx 0.16$$

$$\Theta(r - R) = \begin{cases} 1 & r \le R \\ 0 & r > R \end{cases}$$

2^+ wavefunction

	$1p_{3/2}^{-1}1p_{1/2}$	$2s_{1/2}^{-1}5d_{3/2}$	$1p_{1/2}^{-1}6p_{3/2}$	$2s_{1/2}^{-1}3d_{5/2}$	$2s_{1/2}^{-1}5d_{5/2}$	$1p_{3/2}^{-1}1p_{1/2}$ (π)
$X_{\rm ph}$	0.824	0.404	0.151	0.125	0.126	0.16
$Y_{\rm ph}$	0.119	0.011	−0.002	−0.049	−0.011	0.07

B(E1) calculated with separable force; coupling constant tuned to reproduce experimental strength;
part of the strength comes from admixture of GDR

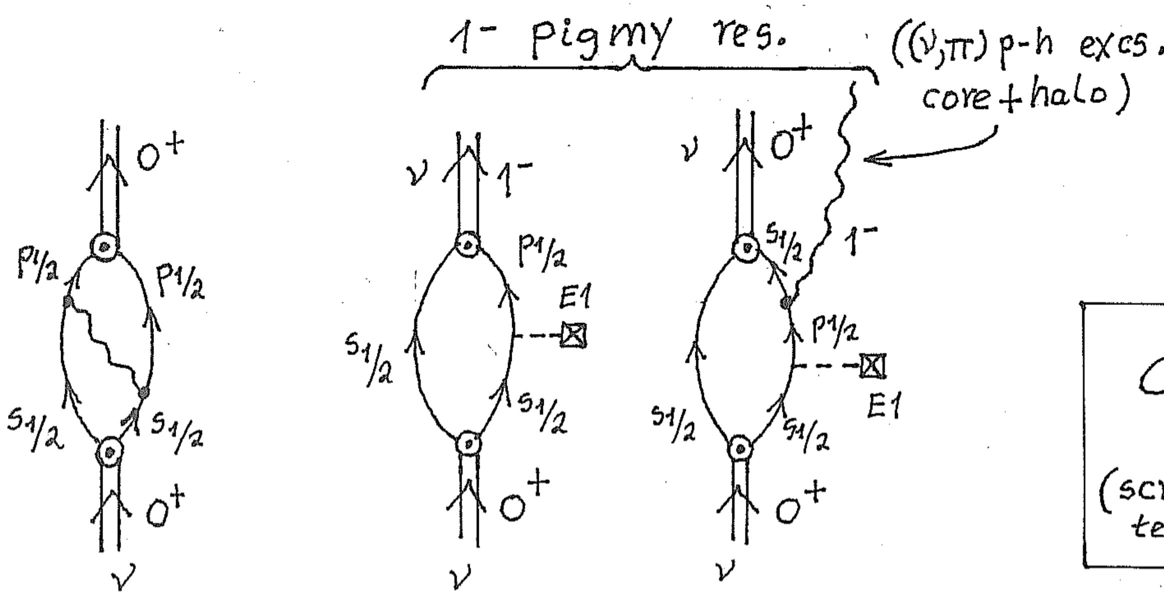


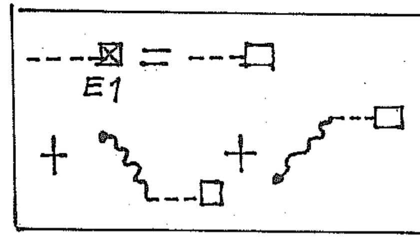
1^- wavefunction /low-lying strength

	$1p_{1/2}^{-1}2s_{1/2}$	$1p_{1/2}^{-1}3s_{1/2}$	$1p_{1/2}^{-1}4s_{1/2}$	$1p_{1/2}^{-1}1d_{3/2}$	$1p_{3/2}^{-1}5d_{5/2}$	$1p_{3/2}^{-1}6d_{5/2}$	$1p_{3/2}^{-1}7d_{5/2}$
$X_{\rm ph}$	0.847	−0.335	0.244	0.165	0.197	0.201	0.157
$Y_{\rm ph}$	0.088	0.060	0.088	0.008	0.165	0.173	0.138

Valence transitions

Transitions involving core states





$$\sigma \approx \left(\frac{R_0}{R}\right)^3 \approx 0.16$$

(screening of symmetry term)

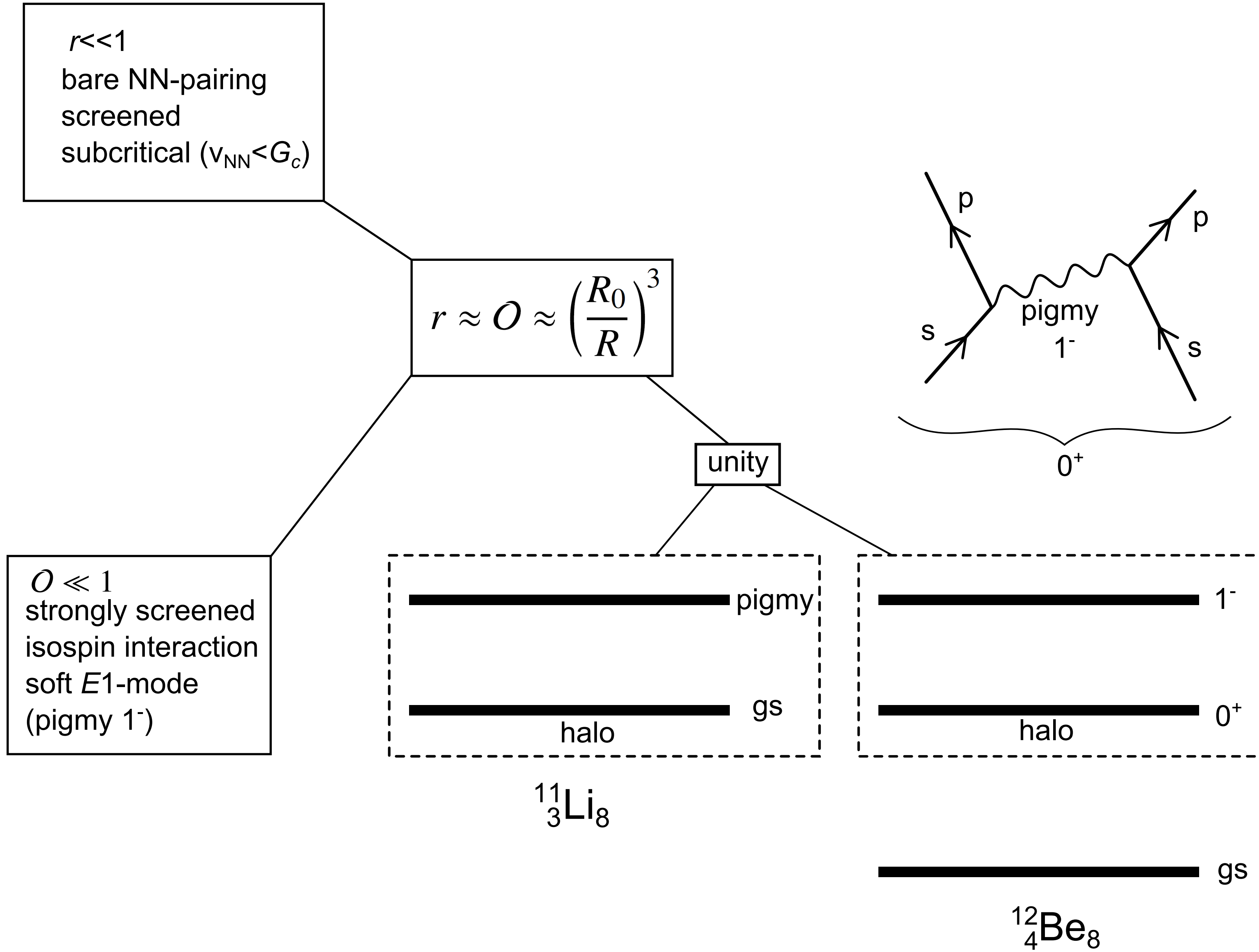
The ^{11}Li pigmy resonance can hardly be viewed but in symbiosis with the ^{9}Li halo neutron pair addition mode

anti – (halo-anti-pairing effect)

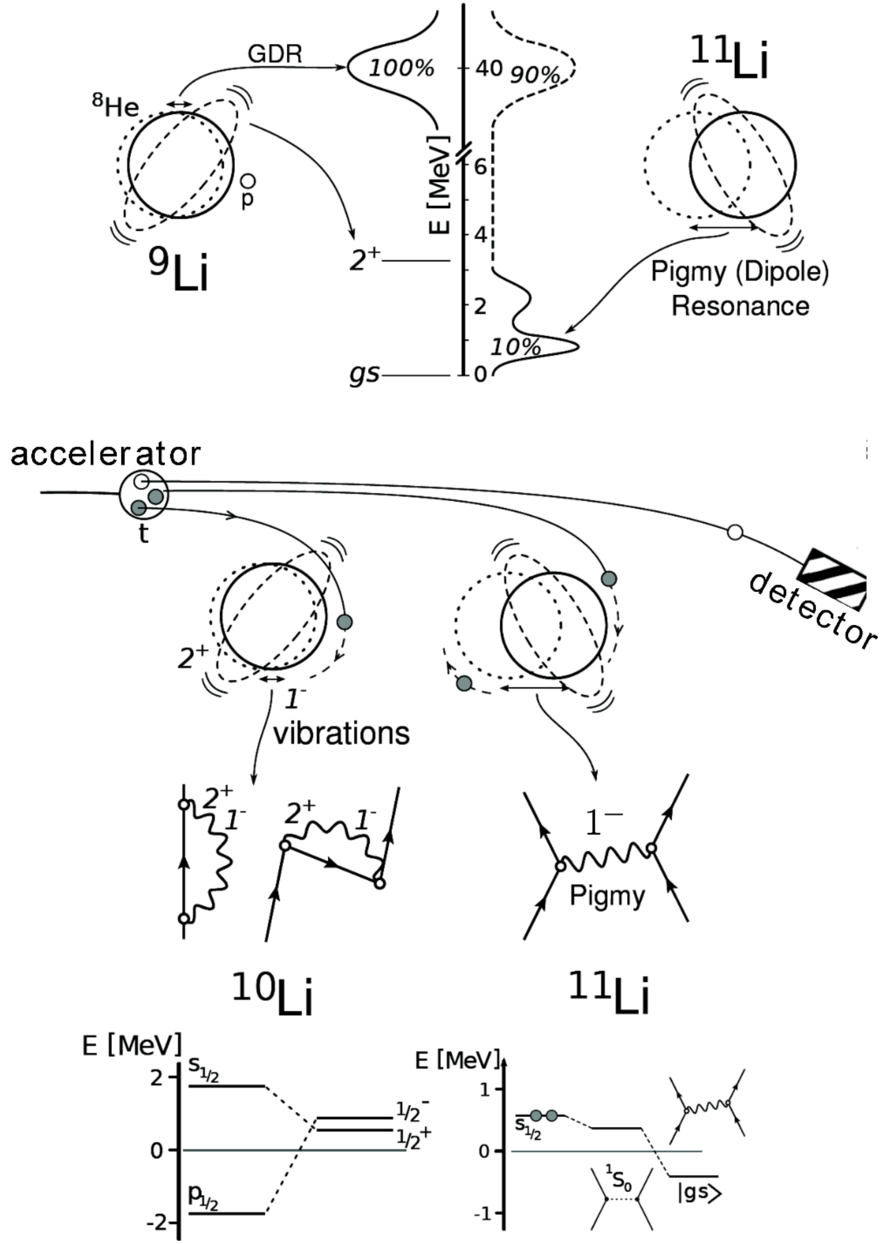
The pigmy resonance is built on a ground state with little overlap with the gs on which the GDR is built. It is thus a different (new) elementary mode of nuclear excitation

Extreme example of inhomogeneous damping (radial degree of freedom instead of quadrupole deformation)

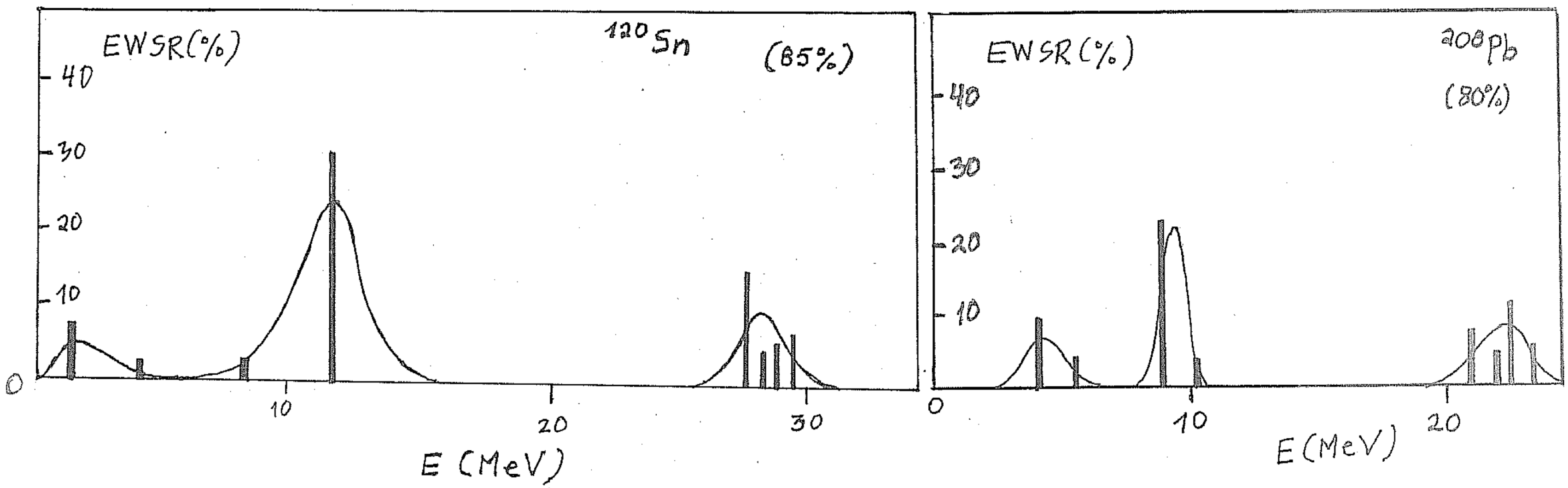
New physics



pair addition
halo mode



Bootstrap pairing correlations



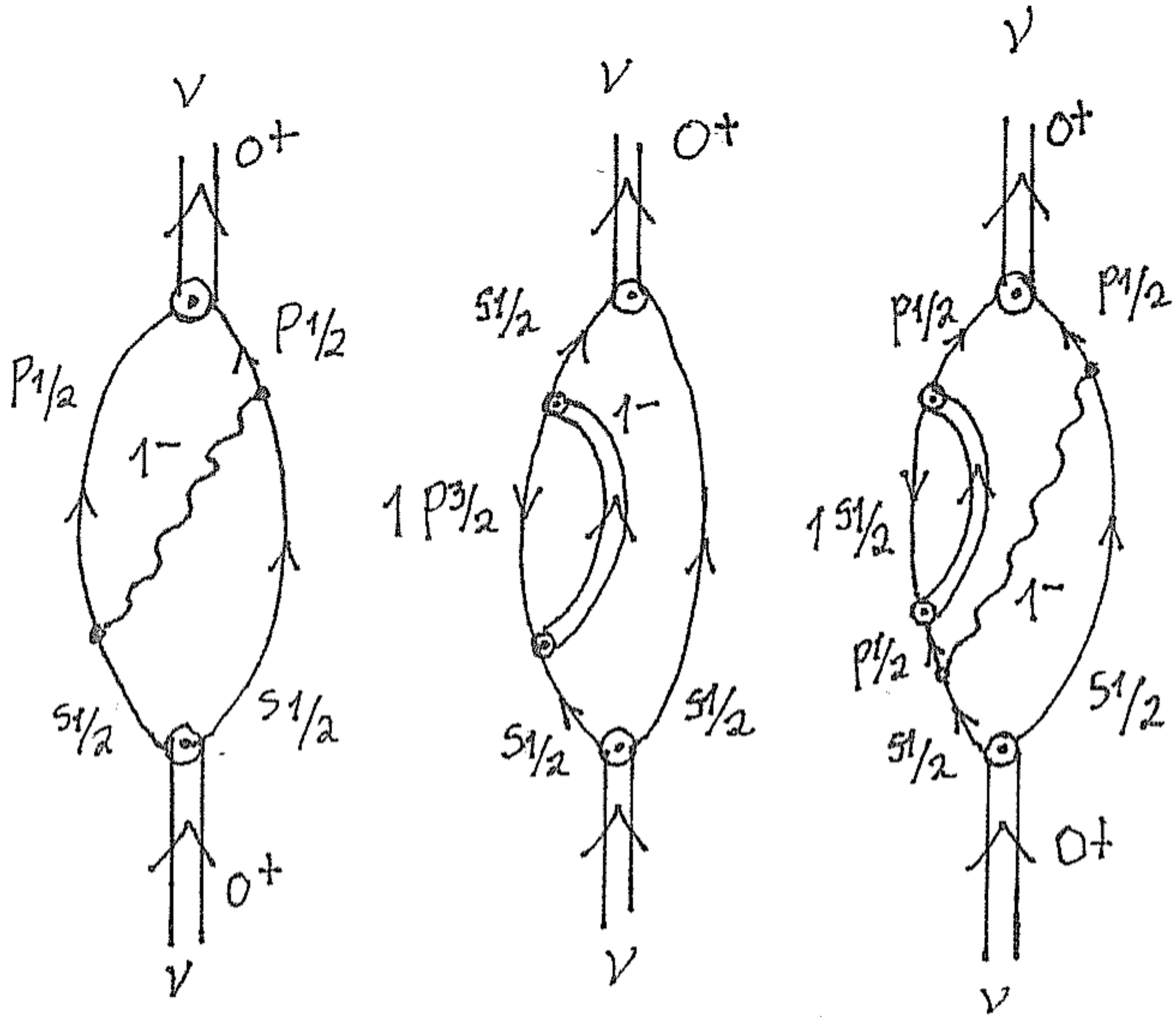
D.R. Bès, R.A. Broglia and B.S. Nilsson,
Microscopic description of isoscalar and
isovector giant quadrupole resonances,
Phys. Rep. 16 (1975) 1-56.

Dual origin of pairing in nuclei

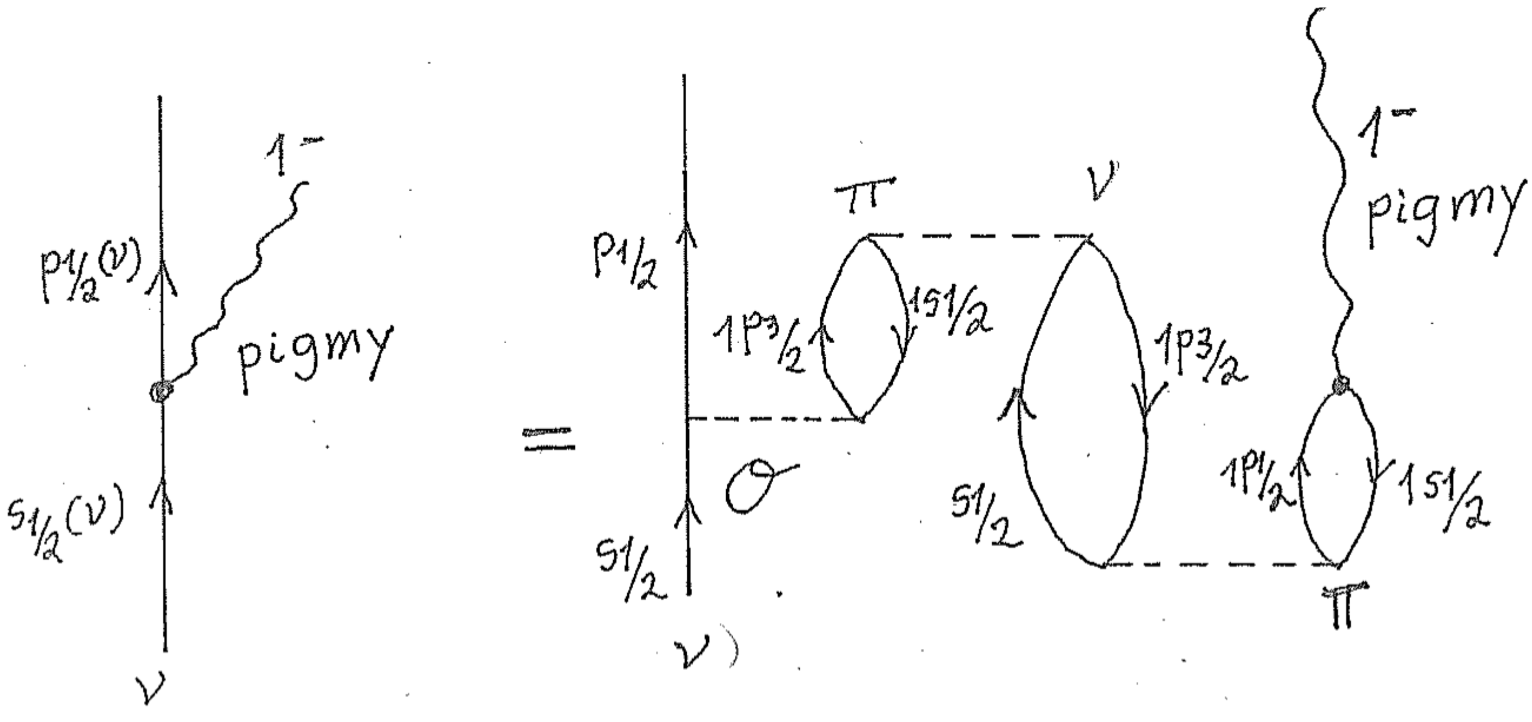
$v_{pair} = v_{bare}$ (N-N + 3N) $+ v_{ind}$

Direct and circumstantial evidence for v_{ind} in ^{11}Li

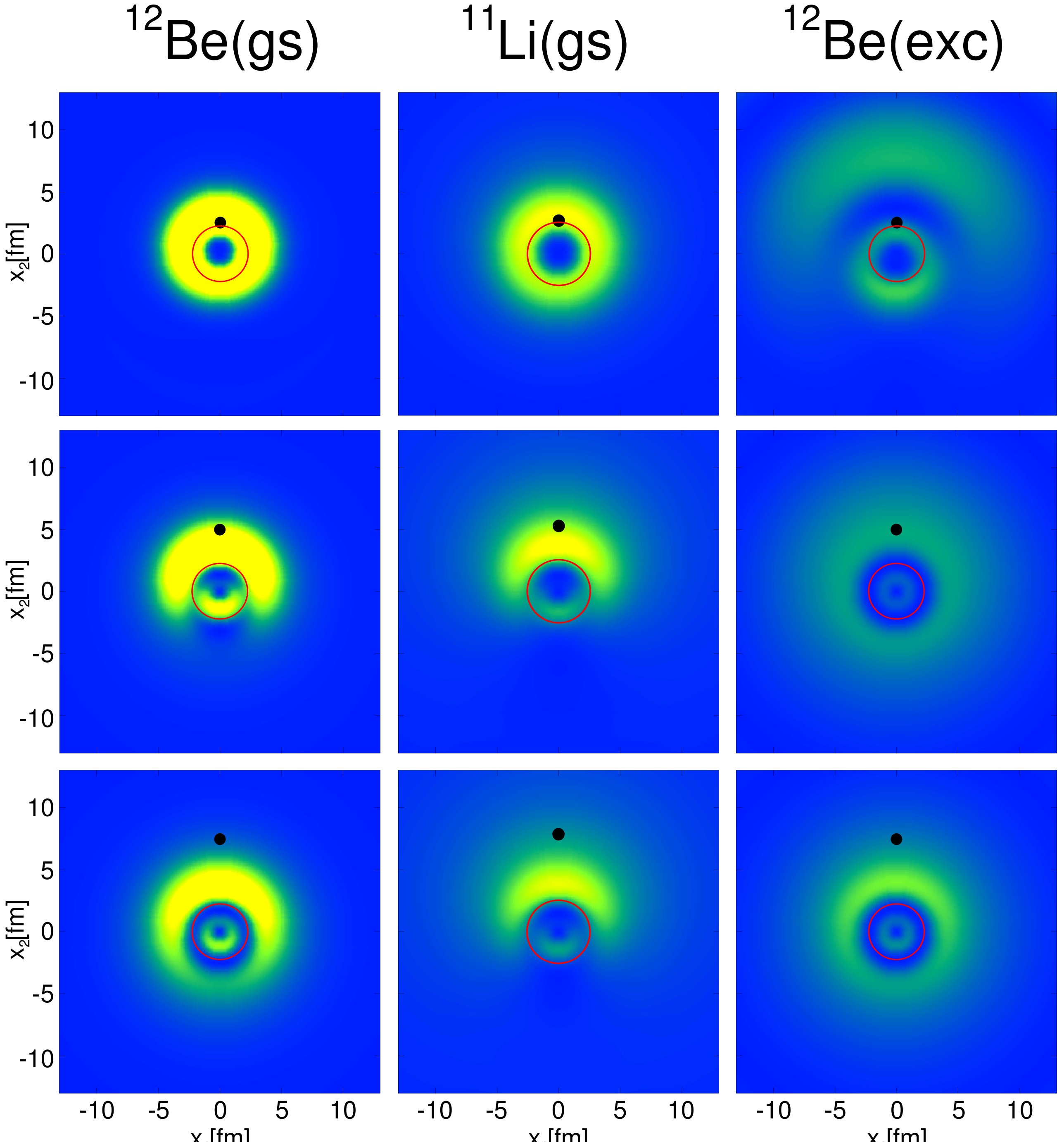
In nuclei along the stability valley, calculations estimate similar contributions from v_{pair} and v_{ind}
Open problem
(cf. A. Idini et al., nucl-th/1404.7365)



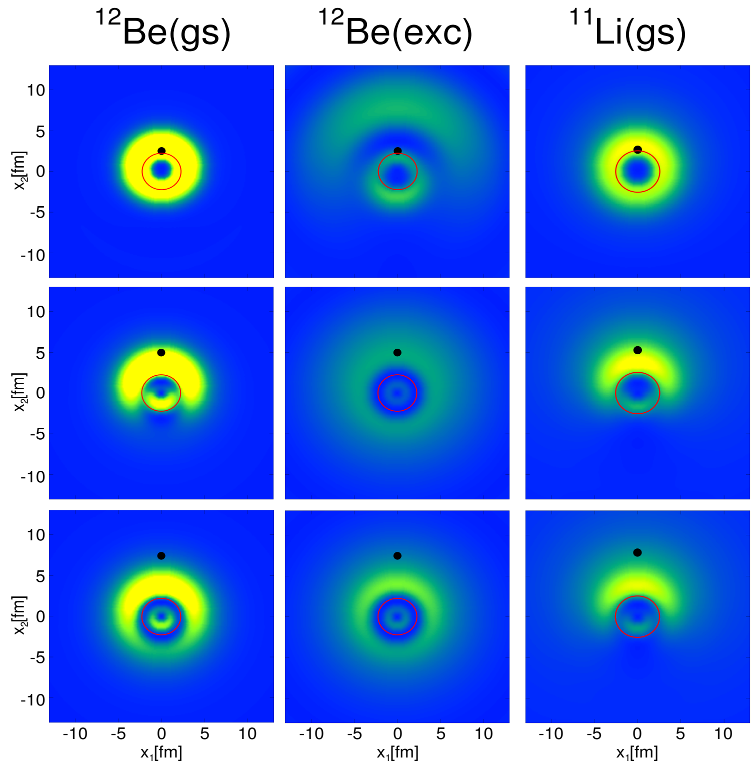
ν
0+
P1/2
P1/2
1−
s1/2
s1/2
0+
ν
ν
0+
s1/2
1−
1 p3/2
s1/2
s1/2
ν
ν
0+
p1/2
p1/2
1 s1/2
1−
p1/2
s1/2
s1/2
0+
ν



1⁻
pigmy
p1/2(ν)
s1/2(ν)
ν
=
p1/2
s1/2
ν
π
1p3/2
1s1/2
ν
1p3/2
s1/2
1⁻
pigmy
1p1/2
1s1/2
π



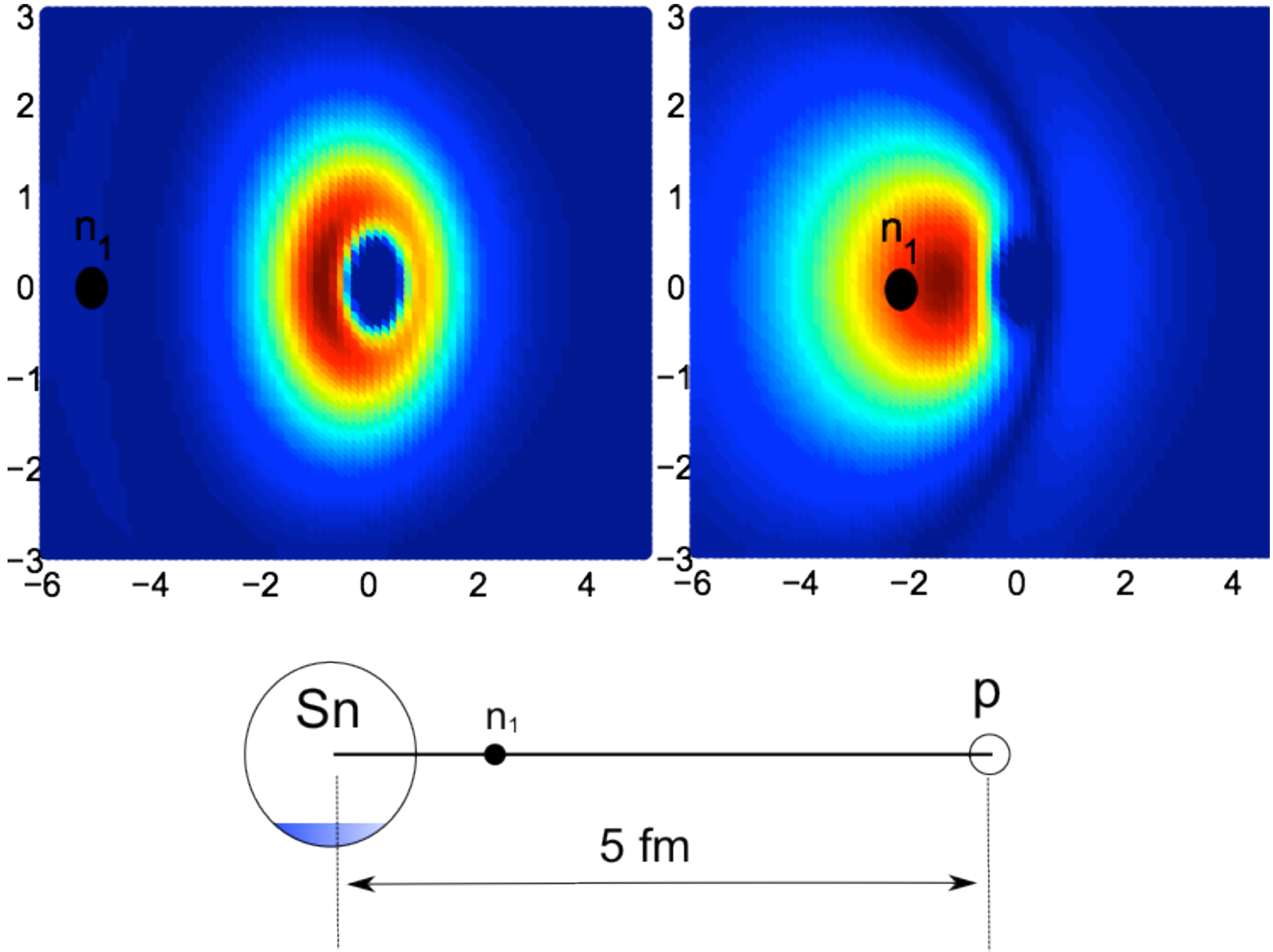
12Be(gs)
11Li(gs)
12Be(exc)
x2[fm]
x1[fm]
10
5
0
-5
-10



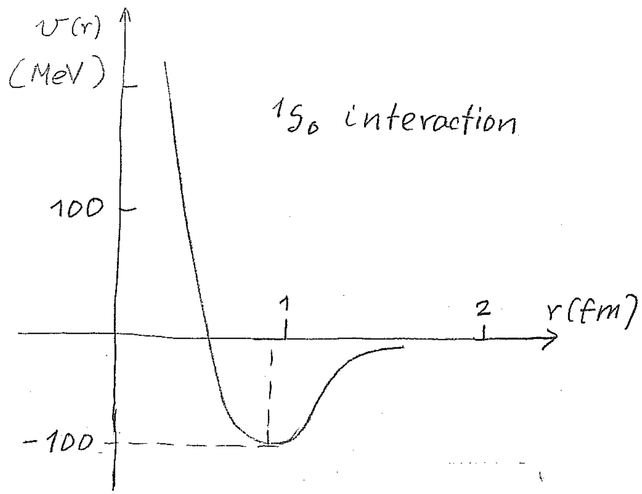
$$|0\rangle_\nu = |0\rangle + \alpha|(p,s)_{1^-} \otimes 1^-;0\rangle + \beta|(s,d)_{2^+} \otimes 2^+;0\rangle + \gamma|(p,d)_{3^-} \otimes 3^-;0\rangle$$

$$|0\rangle_\nu = a|s^2(0)\rangle + b|p^2(0)\rangle + c|d^2(0)\rangle$$

	^{11}Li(gs)	^{12}Be(gs)	^{12}Be(exc)
α	0.7	0.10	0.08
β	0.1	0.30	-0.39
γ	–	0.37	-0.1
a	0.45	0.37	0.89
b	0.55	0.50	0.17
c	0.04	0.60	0.19



n1
n1
Sn
n1
p
5 fm



Quantality parameter

$$q = \left(\frac{\hbar^2}{Ma^2}\right)\frac{1}{|v_0|}$$

$q << 1$ Crystalline structure (T=0)

$q \approx 1$ Quantum fluid (T=0)

$q \approx 0.4$ Nuclei

$$H = T + v = \underline{T + U + V_p} + (v - U - V_p)$$

MEAN FIELD

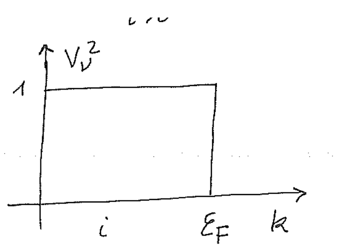
Diagon. $\alpha_\nu^\dagger = U_\nu a_\nu^\dagger - V_\nu a_{\bar{\nu}}$

g.s. $\alpha_\nu |\tilde{0}> = 0$

$$|\tilde{0}> = \Pi_{\nu>0}\, \alpha_\nu \alpha_{\bar{\nu}} |0> \sim \Pi_{\nu>0}(U_\nu + V_\nu a_\nu^\dagger a_{\bar{\nu}}^\dagger)|0>$$

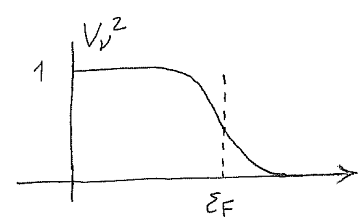
Ansatz 1 : sharp occupation distribution

$|\tilde{0}>$ = $|HF> = \Pi_{i>0}\, a_i^\dagger a_{\tilde{i}}^\dagger |0> = \Pi_i a_i^\dagger |0>$



Ansatz 2 : sigmoidal occupation distribution

$|\tilde{0}>$ = $|BCS> = \Pi_{\nu>0}\, (U_\nu + V_\nu a_\nu^\dagger a_{\bar{\nu}}^\dagger)|0>$



$$\xi = \frac{\hbar v_F}{E_{corr}} \approx 30\text{-}35\ fm$$

$$v_{np} \approx 0.4 fm$$

$$T^{(1)} = 2\sum_{l_i,j_i}\sum_{\sigma_1\sigma_2}\int d\mathbf{r}_{tA}d\mathbf{r}_{p1}d\mathbf{r}_{A2}\big[\phi^{A+2}_{l_i,j_i}(\mathbf{r}_{A1},\sigma_1,\mathbf{r}_{A2},\sigma_2)\big]^{0*}_0\chi^{(-)*}_{pB}(\mathbf{r}_{pB})v(r_{p1})\phi_t(r_{p1},\sigma_1,r_{p2},\sigma_2)\chi^{(+)}_{tA}(\mathbf{r}_{tA}), \tag{38a}$$

successive,

$$\begin{aligned} T^{(2)}_{\text{succ}} = 2\sum_{l_i,j_i}\sum_{l_f,j_f,m_f}\sum_{\substack{\sigma_1\sigma_2\\ \sigma_1'\sigma_2'}}\int d\mathbf{r}_{dF}d\mathbf{r}_{p1}d\mathbf{r}_{A2}\big[\phi^{A+2}_{l_i,j_i}(\mathbf{r}_{A1},\sigma_1,\mathbf{r}_{A2},\sigma_2)\big]^{0*}_0\chi^{(-)*}_{pB}(\mathbf{r}_{pB})v(r_{p1})\phi_d(r_{p1},\sigma_1)\varphi^{A+1}_{l_f,j_f,m_f}(\mathbf{r}_{\mathbf{A2}},\sigma_2)\\ \times\int d\mathbf{r}'_{dF}d\mathbf{r}'_{p1}d\mathbf{r}'_{A2}G(\mathbf{r}_{dF},\mathbf{r}'_{dF})\phi_d(r'_{p1},\sigma'_1)^*\varphi^{A+1*}_{l_f,j_f,m_f}(\mathbf{r}'_{\mathbf{A2}},\sigma'_2)\frac{2\mu_{dF}}{\hbar^2}v(r'_{p2})\phi_d(r'_{p1},\sigma'_1)\phi_d(r'_{p2},\sigma'_2)\chi^{(+)}_{tA}(\mathbf{r}'_{tA}), \end{aligned} \tag{38b}$$

and nonorthogonal,

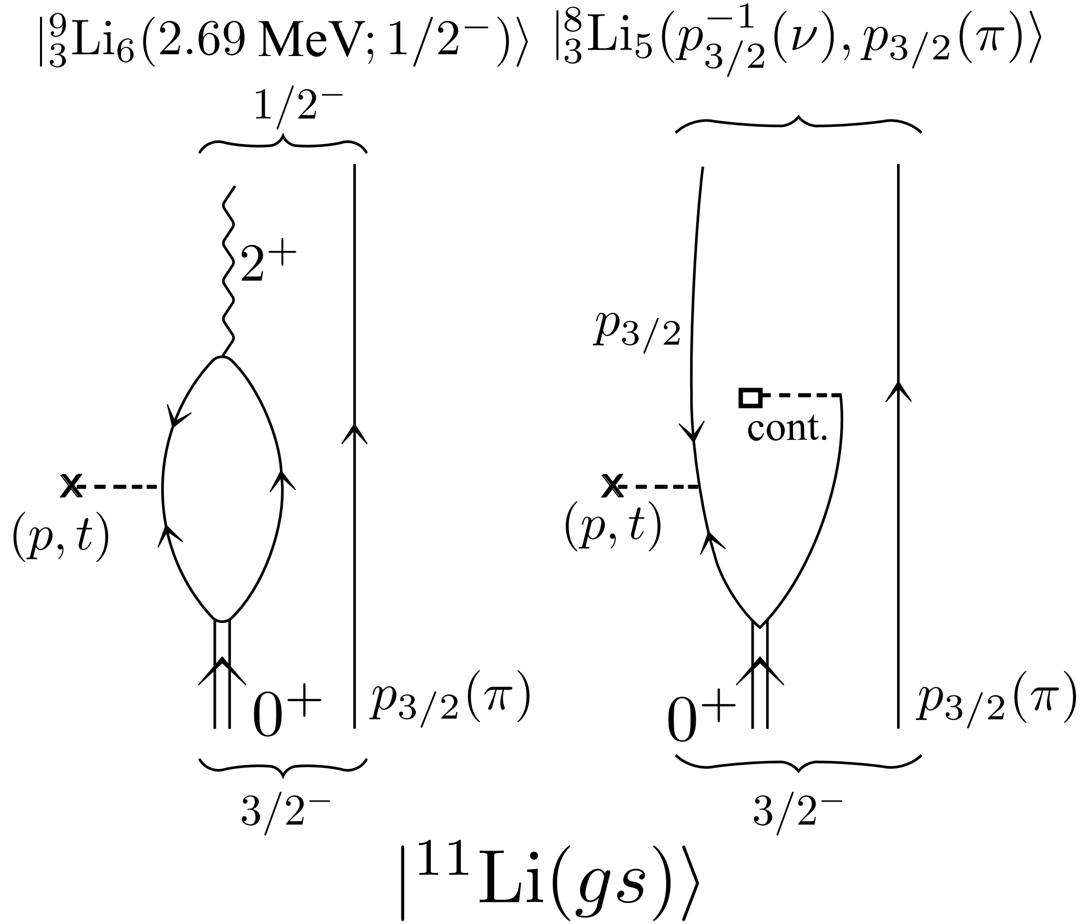
$$\begin{aligned} T^{(2)}_{\text{NO}} = 2\sum_{l_i,j_i}\sum_{l_f,j_f,m_f}\sum_{\substack{\sigma_1\sigma_2\\ \sigma_1'\sigma_2'}}\int d\mathbf{r}_{dF}d\mathbf{r}_{p1}d\mathbf{r}_{A2}\big[\phi^{A+2}_{l_i,j_i}(\mathbf{r}_{A1},\sigma_1,\mathbf{r}_{A2},\sigma_2)\big]^{0*}_0\chi^{(-)*}_{pB}(\mathbf{r}_{pB})v(r_{p1})\phi_d(r_{p1},\sigma_1)\varphi^{A+1}_{l_f,j_f,m_f}(r_{A2},\sigma_2)\\ \times\int d\mathbf{r}'_{p1}d\mathbf{r}'_{A2}d\mathbf{r}'_{dF}\phi_d(r'_{p1},\sigma'_1)^*\varphi^{A+1*}_{l_f,j_f,m_f}(\mathbf{r}'_{\mathbf{A2}},\sigma'_2)\phi_d(r'_{p1},\sigma'_1)\phi_d(r'_{p2},\sigma'_2)\chi^{(+)}_{tA}(\mathbf{r}'_{tA}), \end{aligned} \tag{38c}$$

New Technical Achievement

	σ(gs$\to$f)		
	f	Theory $^{a)\ b)\ f)}$	Experiment$^{f-m)}$
^{7}Li$(t,p)^9$Li	gs	14.3 $^{c)}$	14.7 ± 4.4 $^{c,i)}$ $[9.4^\circ < \theta < 108.7^\circ]$
^{1}H(^{11}Li,^{9}Li)^{3}H	gs	6.1 $^{c)}$	5.7 ± 0.9 $^{c,b)}$ $[20^\circ < \theta < 154.5^\circ]$
	$1/2^-$	0.7 $^{c)}$	1.0 ± 0.36 $^{c,b)}$ $[30^\circ < \theta < 100^\circ]$
^{10}Be$(t,p)^{12}$Be	gs	2.3 $^{c)}$	1.9 ± 0.57 $^{c,j)}$ $[4.4^\circ < \theta < 57.4^\circ]$
^{48}Ca$(t,p)^{50}$Ca	gs	0.55 $^{c)}$	0.56 ± 0.17 $^{c,m)}$ $[4.5^\circ < \theta < 174^\circ]$
^{112}Sn$(p,t)^{110}$Sn, $E_{CM} = 26$ MeV	gs	1301 $^{d)}$	$1309 \pm 200(\pm 14)$ $^{d,g)}$ $[6^\circ < \theta < 62.2^\circ]$
^{114}Sn$(p,t)^{112}$Sn, $E_{CM} = 22$ MeV	gs	1508 $^{d)}$	$1519 \pm 456(\pm 16.2)$ $^{d,g)}$ $[7.64^\circ < \theta < 62.24^\circ]$
^{116}Sn$(p,t)^{114}$Sn, $E_{CM} = 26$ MeV	gs	2078 $^{d)}$	$2492 \pm 374(\pm 32)$ $^{d,g)}$ $[4^\circ < \theta < 70^\circ]$
^{118}Sn$(p,t)^{116}$Sn, $E_{CM} = 24.4$ MeV	gs	1304 $^{d)}$	$1345 \pm 202(\pm 24)$ $^{d,g)}$ $[7.63^\circ < \theta < 59.6^\circ]$
^{120}Sn$(p,t)^{118}$Sn, $E_{CM} = 21$ MeV	gs	2190 $^{d)}$	$2250 \pm 338(\pm 14)$ $^{d,g)}$ $[7.6^\circ < \theta < 69.7^\circ]$
^{122}Sn$(p,t)^{120}$Sn, $E_{CM} = 26$ MeV	gs	2466 $^{d)}$	$2505 \pm 376(\pm 18)$ $^{d,g)}$ $[6^\circ < \theta < 62.2^\circ]$
^{124}Sn$(p,t)^{122}$Sn, $E_{CM} = 25$ MeV	gs	838 $^{d)}$	$958 \pm 144(\pm 15)$ $^{d,g)}$ $[4^\circ < \theta < 57^\circ]$
^{112}Sn$(p,t)^{110}$Sn, $E_p = 40$ MeV	gs	3349 $^{e)}$	3715 ± 1114 $^{e,h)}$
^{114}Sn$(p,t)^{112}$Sn, $E_p = 40$ MeV	gs	3790 $^{e)}$	3776 ± 1132 $^{e,h)}$
^{116}Sn$(p,t)^{114}$Sn, $E_p = 40$ MeV	gs	3085 $^{e)}$	3135 ± 940 $^{e,h)}$
^{118}Sn$(p,t)^{116}$Sn, $E_p = 40$ MeV	gs	2563 $^{e)}$	2294 ± 668 $^{e,h)}$
^{120}Sn$(p,t)^{118}$Sn, $E_p = 40$ MeV	gs	3224 $^{e)}$	3024 ± 907 $^{e,h)}$
^{122}Sn$(p,t)^{120}$Sn, $E_p = 40$ MeV	gs	2339 $^{e)}$	2907 ± 872 $^{e,h)}$
^{124}Sn$(p,t)^{122}$Sn, $E_p = 40$ MeV	gs	1954 $^{e)}$	2558 ± 767 $^{e,h)}$
^{206}Pb$(t,p)^{208}$Pb	gs	0.52 $^{c)}$	0.68 ± 0.21 $^{c,k)}$ $[4.5^\circ < \theta < 176.5^\circ]$
^{208}Pb(^{16}O,^{18}O)^{206}Pb	gs	0.80 $^{c)}$	0.76 ± 0.18 $^{c,f)}$ $[84.6^\circ < \theta < 157.3^\circ]$

Table 4:
It is of notice that the number in parenthesis (last column) corresponds to the statistical errors.

$^{a)}$ G. Potel et al., Phys. Rev. Lett. **107**, (2011) 092501.
$^{b)}$ G. Potel et al., Phys. Rev. Lett. **105**, (2010) 172502.
$^{c)}$ mb
$^{d)}$ μb
$^{e)}$ μb/sr ($\sum_{i=1}^{N}(d\sigma/d\Omega)$; differential cross section summed over the few, $N = 3-7$ experimental points).
$^{f)}$ B. Bayman and J. Chen, Phys. Rev. **C 26** (1982) 1509 and refs. therein.
$^{g)}$ P. Guazzoni, L. Zetta, et al., Phys. Rev. **C 60**, 054603 (1999).
P. Guazzoni, L. Zetta, et al., Phys. Rev. **C 69**, 024619 (2004).
P. Guazzoni, L. Zetta, et al., Phys. Rev. **C 74**, 054605 (2006).
P. Guazzoni, L. Zetta, et al., Phys. Rev. **C 83**, 044614 (2011).
P. Guazzoni, L. Zetta, et al., Phys. Rev. **C 78**, 064608 (2008).
P. Guazzoni, L. Zetta, et al., Phys. Rev. **C 85**, 054609 (2012).
$^{h)}$ G. Bassani et al. Phys. Rev. **139**, (1965)B830.
$^{i)}$ P.G. Young and R.H. Stokes, Phys. Rev. **C 4**, (1971) 1597.
$^{j)}$ H.T. Fortune, G.B. Liu and D.E. Alburger, Phys. Rev. **C 50**, (1994) 1355.
$^{k)}$ J.H. Bjerregaard et al., Nucl. Phys. **89**, (1966) 337.
$^{l)}$ J.H. Bjerregaard et al., Nucl. Phys. **A 113**, (1968) 484.
$^{m)}$ J. H. Bjerregaard et al., Nucl. Phys. **A 103**, (1967) 33.



$|^9_3\mathrm{Li}_6(2.69\,\mathrm{MeV};1/2^-)\rangle$
$|^8_3\mathrm{Li}_5(p^{-1}_{3/2}(\nu),p_{3/2}(\pi)\rangle$
$1/2^-$
2^+
(p,t)
0^+
$p_{3/2}(\pi)$
$3/2^-$
$p_{3/2}$
cont.
(p,t)
0^+
$p_{3/2}(\pi)$
$3/2^-$
$|^{11}\mathrm{Li}(gs)\rangle$