with refs ~~Schmidt~~

Contact Osvaldo

Corrected 17/07/2015

Nature but also Nuclear Physics News

Elastic response of the atomic nucleus in gauge space: Giant Pairing Vibrations

R. A. Broglia,
Department of Physics,
University of Milan, Milan, Italy
and
The Niels Bohr Institute
University of Copenhagen,
Copenhagen, Denmark

July 3, 2015

of

Systems displaying many degrees of freedom can be described at profit in terms of field theories of fermions and of bosons and their interweaving (Weinberg, 1996). Examples are provided by Quantum Electro Dynamics (QED) (Feynman, 1961) and by Nuclear Field Theory (NFT) (Bès et al., 1974; Bortignon, P. F. et al., 1977). In QED, electrons and positrons are the fermions, photons the bosons. In NFT, taylored after Feynman's version of QED in order to describe the nuclear structure in general, and in particular that around closed shells, the nucleons, namely particles (p) and holes (h) are the fermions, while correlated particle–hole (ph), (pp) and (hh) collective vibrations are the (composite) bosons.

In QED the photon field and the electron field are in interaction (fine structure constant). As a consequence, the identification of each field by these names is only an approximate one. What one calls physically an electron is only partially to be associated with the electron field alone. It is also partially to be associated with the photon field. Physically, an electron can sometimes radiate a photon and, at a later time reabsorb it. Conversely, what one calls a photon, propagating through empty space can occasionally materialize itself in space and become replaced by an electron and a positron (particle (e^-)–hole (e^+) pair). Each of these fermions can radiate and reabsorb a photon (self energy) or exchange it (vertex correction), and then, in the course of time, recombine to reform a photon. In other occasions,
e
before the electron reabsorbs ~~the~~ the radiated photon it can annihilate with the positron ~~with the production of~~ a photon. This process measures the interaction between one– and two– photon states.
a
states
producing

In NFT, the nucleon field and the vibrational fields are in interaction through the particle–vibration coupling vertices. A physical nucleon propagating in the nuclear medium can change orbital by bouncing inelastically off the nuclear surface and setting it into (ph) vibration, eventually reabsorbing it at a later time (self–energy process). Conversely, a correlated particle–hole can decay into one of its (ph)–components and eventually couple to $2p$–$2h$ containing a (ph) collective vibration. This vibration can either be reabsorbed by the same fermion which virtually excited it (self energy) or be exchanged with the other fermion (vertex correction) before the particle (p) falls into the hole (h) and reconstitutes the collective vibration (Bertsch et al., 1983). It can also propagate asymptotically and be joined by a second (ph)–collective vibration produced by the annihilation of the p by the h, a process giving a measure of the interaction between one– and two– phonon vibrational states. It is at this point that the analogy between NFT and QED ends up.

In QED the coupling between one– and two– photon states is exactly zero (Furry theorem), and there is a complete cancellation between self–energy and vertex corrections, the photon being an elementary particle with infinite lifetime. The above results are a direct consequence of the exact symmetry existing between particle (electron) and hole (positron) states.
e

If this was also the case in nuclei, no (ph) collective vibrational states, in particular the Giant Dipole Resonance (GDR), will display a damping width (finite lifetime). Furthermore, all multiphonon spectra will be harmonic, let alone the fact that no vibrational state will display a finite value of the quadrupole moment. Applied to low–lying collective 2^+ states, the above results imply that no shape phase transition could take place and, as a consequence, no quadrupole rotational bands should be observed in nuclei.

The above expectations are clearly contradicted by the experimental findings (Bohr, A. and Mottelson, 1975; Soloviev, 1992; Bortignon, P.F. et al., 1998). Giant resonances (the elastic response of nuclei to sudden solicitations) in general, and GDR in particular damp out after few periods of oscillations. Similarly, low–lying (ph) collective vibrations (the plastic response of the nucleus ~~under~~ to long solicitations) in general, and 2^+ modes in particular leads to sizable reorientation effects, and multiphonon states show conspicuous anharmonicities. Finally, quadrupole rotational bands have been observed up to the critical value of spin leading to fission (Twin et al., 1986).

Summing up, in nuclei there is no symmetry between particle and hole states. Nonetheless, the physical importance of learning about Giant Pairing (pp, hh) Vibrations (GPV) (Broglia and Bes, 1977; Cappuzzello, F. et al., 2015) is comparable to that which is at the basis of studies of Giant Dipole (Pigmy) Resonances ~~(2)~~ (Savran et al, 2013).

A large number of excited 0^+ states are known in the low–energy nuclear spectrum. Several mechanisms may produce collective states of this spin and parity. The best studied ones correspond to oscillations in the shape or in the size of the nucleus ($\lambda^\pi = 2^+$ (ph) vibrations in quadrupole deformed nuclei, and (ph) monopole vibrations in both spherical and shape deformed nuclei). These modes are associated with changes in the binding field of each particle, i.e. a field which conserves the number of particles. A special case of this type of modes is provided by the so called coexistence states in $N = Z$ nuclei, in particular in ^{16}O. The 0^+ state observed at 6.05 MeV contains a large component of $4p - 4h$ admixture of deformed configurations (Brown, 1964; Engeland, 1965),

In addition to the previous modes, nuclei display vibrations based on fields which create or annihilate two particles. Namely, vibrations in gauge space (pair addition and pair substraction modes) based on pairing fields associated with the pairing interaction and corresponding to two–particle (pp) (two–hole (hh)) correlated modes. Because $\boxed{\text{of}}$ $\boxed{\text{all}}$ the associated configurations contribute in phase to the two–nucleon transfer formfactors, these reactions are the specific tools to probe pairing vibrations. Suggested by Bohr in terms of the baryon (transfer) quantum number (Bohr, 1964), studied in terms of a simple model (Högaasen-Feldman, 1961), implicitly included in spectroscopic studies of single–closed nuclei (Arvieu et al., 1963) and of β–vibrations in deformed nuclei (Bès, 1963), ~~was~~ collective modes in gauge space were eventually formulated in detail in terms of pairing rotational and pairing vibrational bands (Bès,D. R. and Broglia, 1966). The predicted 4.95 MeV two–phonon state of ^{208}Pb, product of the monopole pair removal and pair addition modes ($|^{206}\text{Pb(gs)}\rangle \otimes |^{210}\text{Pb(gs)}\rangle$) was observed in the ^{206}Pb(t, p) reaction, and the expected properties confirmed (Bjerregaard, J. H. et al., 1966; Broglia and Riedel, 1967) (cf. also (Mottelson, 1976)).

Within this context, it is to be noted ~~that~~ the low-lying 0^+ (coexistence) state of ^{16}O mentioned above, is opposite to a multi-phonon pairing vibrational state, in keeping with ~~the~~ fact ~~that~~ deformation (low level density) opposes pairing (Bohr and Mottelson 1975; p. 386 and 641; Mottelson 1976).

The low-lying collective pairing vibrations around closed shells, i.e. ~~(pp) and~~ pairs of particles (pp) ~~(H. Schmidt 1964)~~

new / watch out / eliminate refs?

, thus expected in all nuclei, disregarding whether they are normal or superfluid, spherical or deformed,

et al.,

excitations

and of holes

(hh), ~~states~~ moving and correlating in the valence orbitals, have been studied in detail, and states made up to three pairing vibrational ~~states~~ observed (Flynn, E. R. et al., 1972; Broglia, R.A. et al., 1973). These vibrations also dress the valence nucleons, mixing particle with hole states, and giving rise to retarded contributions to the state dependent effective mass throughout (~~Flynn, E. R. et al., 1972~~; Broglia, R.A. et al., 1973; Bès and Broglia, 1971a; Flynn, ~~E.R. and Igo, G. and Barnes, P.D. and Kovar, D. and Bès, D. R. and Broglia, R.A.,~~ 1971; Bès and Broglia, 1971b; Broglia et al., 1974), and to dealignments in deformed, rotating nuclei (Barranco et al., 1987; Shimizu, Y. R. et al., 1989).

Now, because of spatial quantization, single–particle levels in nuclei are bunched in major shells separated by an average energy $\hbar\omega_0 \approx \frac{41}{A^{1/3}}\text{MeV} \approx \frac{50\,\text{MeV fm}}{R}$, where $R = 1.2A^{1/3}$ fm is the nuclear radius. This is the origin of Giant Pairing Vibrations (GPV), that is (elastic) vibrations adding (removing) two nucleons and based on $2p$ $(2h)$ across major shells, ~~expected~~ at an excitation energy of $\hbar\omega_{GPV} \approx 1.7\hbar\omega_0$, and carrying a two–nucleon transfer cross section of the order of that associated with the low–lying (plastic) pairing vibrations. Predicted almost four decades ago (Broglia and Bes, 1977), serious experimental candidate to the role of pair addition GPV have been found in a recently reported experiment (Cappuzzello, F. et al., 2015) (within this context, see also (Crawley et al., 1977, 1980; Mouginot et al., 2011)), as the result of an experimental *tour de force*, backed by a systematic and less than straightforward theoretical calculation of the background. The importance of this work is that it ushers the experimental probing of the elastic response of the atomic nucleus in gauge space to state of the art levels, providing information about the associated elastic modulus, as well as concerning effective two–nucleon transfer amplitudes (cf. Fig. 1 ref (Broglia and Bes, 1977)), parallel to the nucleon effective charges associated with (ph) giant resonances, in particular with the GDR (cf. e.g. p. 486 of ref. (Bohr, A. and Mottelson, 1975)).

Cooper

The possibility to carry out similar studies is denied to pair transfer between metallic superconductors (~~Josephson effect~~ (Josephson, 1962)), although, arguably, possible in the case of quantum dots. Furthermore, the possibility to carry out systematic studies of GPV are expected to be instrumental in the test of two–nucleon transfer reaction mechanisms. In particular, to get further insight concerning the relative role successive and simultaneous transfer play in the calculation of the absolute value of two–nucleon transfer differential cross sections. Among other things, the relation to the spatial correlation of the Cooper pair partners (Bertsch, G. F. et al., 1967; ~~Tonozuka and Arima, 1979;~~ Ferreira, L. et al., 1984; Herzog et al., 1985; Lotti et al., 1989), correlations which can also be estimated in terms of the pair quantality parameter (Q_{pair}, cf. also (Mottelson, 1998)). That is, the ratio of the kinetic energy of localization ~~with~~ within the correlation length (ξ) ~~localization~~ and the correlation energy, $Q_{pair} = \frac{\hbar^2}{2m\xi^2}\frac{1}{2E_{corr}} \approx 0.02$ ($\xi = \hbar v_F/2E_{corr} \approx 20\,\text{fm}$, $E_{corr} \approx 1.5\,\text{MeV}$, $v_F/c \approx 0.3$). The above value implies a strong localization of one nucleon partner of the Cooper pair with respect to the second one. All these effects are rather subtle (Bardeen, 1962; Pippard, 2012; Anderson, 1964; Cohen et al.,

, Brink and Broglia 2005

1962) and, at the same time, fundamental subjects needed to understand ~~BCS~~ superconductivity and superfluidity in fermionic systems at large and ~~in~~ atomic nuclei in particular.

It is of notice ~~that~~ these

The extension of monopole ($J^{\pi} = 0^{+}$) GPV to other multipolarities and parities ($J^{\pi} = 1^{-}, 2^{+}, 3^{-}, 4^{+}$, etc.) may likely be of importance in connection with the background of the monopole GPV reported in (Cappuzzello, F. et al., 2015), and will likely match that carried out in connection with the low–lying multipole pairing vibrations (cf. (Brink, D. and Broglia, 2005) p. 108). ~~These~~ (plastic) vibrations may play a central role in double charge exchange reactions like ^{40}Ca(^{18}O,^{18}Ne)^{40}Ar, of interest in the quest to determine the value of the matrix element involved in the neutrinoless double β–decay, an important test of the standard model (Rodin and Faessler, 2009).

renormalized by the GPV,

Frekers et al 2013, Guess et al 2011,

two

The existence of major shells with alternating parity and separated by energies of the order of 8–10 MeV, is also at the basis of (ph) giant resonances, in particular of the GDR, namely the sloshing back and forth of neutrons against protons in an antenna–like motion with which an atomic nucleus absorbs energy from a γ–beam, and observed in essentially all nuclei throughout the mass table. Because one has to pay a conspicuous energetic prize to separate protons from neutrons (symmetry potential), the energy centroid of the GDR lies at high energy in the nuclear spectrum, estimated to be $\hbar\omega_D \approx 2\hbar\omega_0$ ($\approx \frac{100\,\mathrm{MeV\,fm}}{R}$).Its inverse proportionality to the nuclear radius testifies to the elastic character of the GDR.

It is interesting to note that one of todays growing points in nuclear research regards the study of low–energy E_1 strength found in ~~nuclei with large neutron~~ excess, Dipole Pigmy Resonances. That is, the ~~study~~ of the plastic response of the atomic nucleus to a dipole external field (?), allowing for new and unexpected roles regarding the low energy fraction of the Thomas–Reiche–Kuhn sum rule (Reiche and Thomas, 1925; Kuhn, 1925) (i.e. for the giant dipole pigmy resonance (GDPR)). For example, that of acting as intermediate boson in gluing the halo neutron Cooper pair of ^{11}Li to the core ^{9}Li (Barranco, F. et al., 2001; Potel et al., 2010), let alone to ~~give a~~ new possibilities to test the Axel–Brink hypothesis (stating that all nuclear states have a dipole mode on top of it (Brink, 1955; Axel et al., 1967)), important not only in the study of the nuclear structure, but also of the compound nucleus decay (Bertsch, G. F. and Broglia, 1986; Bortignon, P.F. et al., 1998).

(Savran et al 2013)

)

open

e

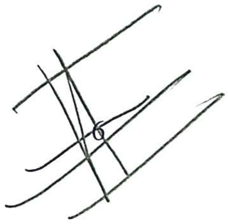
)

Last, but not least, the fact that the GDR of ^{11}Li can be viewed as a correlated Cooper pair with quantum numbers $J^{\pi} = 1^{-}$, namely those associated with a quantal nuclear vortex (Bertsch et al., 1988; Avogadro et al., 2007) (dipole pairing vibrational mode), and thus amenable to studies with two–nucleon transfer reactions (e.g. ^{9}Li(t,p)^{11}Li (1^{-};0.4 MeV)). ~~All these results, testifying~~ to the surprising and unexpected connection and physical unity of the studies of giant (pairing (pigmy) resonances lying at the forefront of today nuclear research. That is, the mapping in gauge ~~and~~ (isospin) space of the elastic ~~and/~~plastic properties of this ever surprising drop of non Newtonian fuid, namely the atomic nucleus.

e

summing up, the above results testify

Discussions with Francesco Cappuzzello, Titti Agodi, Manuela Cavallaro, Diana Carbone and Gregory Potel are gratefully acknowledged.



elminore

References

p. 113,

~~Bibliography~~

Gregory por favor todos los refs. tienen que tener pagina y año !!

P. W. Anderson. *The Many–Body Problem, Vol.2*. Academic Press, New York, 1964.

R. Arvieu, E. Baranger, M. Veneroni, M. Baranger, and V. Gillet. Description des noyaux $Z = 50$ par un modele de quasi–particules en interaction. *Physics Letters*, 4:119, 1963.

P. Avogadro, F. Barranco, R. A. Broglia, and E. Vigezzi. Quantum calculation of vortices in the inner crust of neutron stars. *Phys. Rev. C*, 75:012805, 2007.

P. Axel, D. M. Drake, S. Whetstone, and S. S. Hanna. Evidence for the isobaric splitting of the giant resonance from the reaction $^{89}\mathrm{Y}(p,\gamma_0)^{90}\mathrm{Zr}$. *Phys. Rev. Lett.*, 19:1343, 1967.

J. Bardeen. Tunneling into superconductors. *Physical Review Letters*, 9:147, 1962.

F. Barranco, M. Gallardo, and R. A. Broglia. Nuclear field theory of spin dealignment in strongly rotating nuclei and the vacuum polarization induced by pairing vibrations. *Phys. Lett. B*, 198:19, 1987.

Barranco, F., P. F. Bortignon, R. A. Broglia, G. Colò, and E. Vigezzi. The halo of the exotic nucleus $^{11}\mathrm{Li}$: a single Cooper pair. *Europ. Phys. J. A*, 11:385, 2001.

G. F. Bertsch, P. F. Bortignon, and R. A. Broglia. Damping of nuclear excitations. *Rev. Mod. Phys.*, 55:287, 1983.

G. F. Bertsch, R. A. Broglia, and S. R. J.R. Schrieffer, Rotating superfluidity in nuclei. *Nuovo Cimento*, 100:283, 1988.

Bertsch, G. F. and R. A. Broglia. Giant resonances in hot nuclei. *Physics Today*, 39 (8):44, 1986.

Bertsch, G. F., R. A. Broglia, and C. Riedel. Qualitative description of nuclear collectivity. *Nucl. Phys. A*, 91:123, 1967.

D. R. Bès. Beta–vibrations in even nuclei. *Nuclear Physics*, 49:544 – 565, 1963.

D. R. Bès and R. A. Broglia. Effect of the multipole pairing and particle-hole fields in the particle-vibration coupling of $^{209}\mathrm{Pb}$. I. *Physical Review C*, 3:2349, 1971a.

Elementary modes of excitations and their coupling

D. R. Bès and R. A. Broglia. Effective operators in the analysis of single–nucleon tansfer reactions on closed shell nuclei. *Physical Review C*, 3:2389, 1971b.

D. R. Bès, G. G. Dussel, R. Broglia, R. Liotta, and B. R. Mottelson. Nuclear field theory as a method of treating the problem of overcompleteness in descriptions involving elementary modes of both quasi–particles and collective type. *Phys. Lett. B*, 52:253, 1974.

Bès,D. R. and R. A. Broglia. Pairing vibrations. *Nucl. Phys.*, 80:289, 1966.

Bjerregaard, J. H., O. Hansen, Nathan, and S. Hinds. States of ^{208}Pb from double triton stripping. *Nucl. Phys.*, 89:337, 1966.

A. Bohr. In *Comptes Rendus du Congrès International de Physique Nucléaire*, volume 1, page 487. Centre National de la Recherche Scientifique, 1964.

Bohr, A. and B. R. Mottelson. *Nuclear Structure, Vol.II*. Benjamin, New York, 1975.

Bortignon, P. F., R. A. Broglia, D. R. Bès, and R. Liotta. Nuclear field theory. *Physics Reports*, 30:305, 1977.

Bortignon, P.F., A. Bracco, and R. Broglia. *Giant Resonances*. Harwood Academic Publishers, Amsterdam, 1998.

D. M. Brink. *PhD Thesis*. Oxford University, 1955.

Brink, D. and R. A. Broglia. *Nuclear Superfluidity*. Cambridge University Press, Cambridge, 2005.

R. Broglia and D. Bes. High-lying pairing resonances. *Physics Letters B*, 69:129, 1977.

R. A. Broglia and C. Riedel. Pairing vibration and particle-hole states excited in the reaction ^{206}Pb(t, p)^{208}Pb. *Nucl. Phys.*, 92:145, 1967.

R. A. Broglia, D. R. Bès, and B. S. Nilsson. Strength of the multipole pairing coupling constant. *Physics Letters B*, 50:213, 1974.

Broglia, R.A., O. Hansen, and C. Riedel. Two–neutron transfer reactions and the pairing model. *Advances in Nuclear Physics*, 6:287, 1973. URL `www.mi.infn.it/?vigezzi/BHR/ BrogliaHansenRiedel.pdf`.

G. E. Brown. In *Comptes Rendus du Congrès International de Physique Nucléaire*, volume 1, page 129. Centre National de la Recherche Scientifique, 1964.

Cappuzzello, F. et al. *Nature Communications*, 6:6743, 2015.

M. H. Cohen, L. M. Falicov, and J. C. Phillips. Superconductive tunneling. *Phys. Rev. Lett.*, 8:316, 1962.

Signatures of the Giant Pairing vibration in the ^{14}C and ^{15}C atomic nuclei

D. Frekers, P. Puppe, J. H. Thies and H. Ejiri, Gamow-Teller strength excitation from (^{3}He,t) reactions, Nucl. Phys A916, 219 (2013)

C. J. Guess et al The ^{150}Nd(^{3}He,t) and ^{150}Sm(t,^{3}He) reactions with applications to ββ decay of ^{150}Nd, Phys. Rev. C83, 064318 (2011)

G. M. Crawley, W. Benenson, D. Weber, and B. Zwieglinski. Observation of hole states at high excitation in (p,t) reactions. *Phys. Rev. Lett.*, 39:1451, 1977.

G. M. Crawley, S. Gales, D. Weber, B. Zwieglinski, W. Benenson, D. Friesel, A. Bacher, and B. M. Spicer. Deep hole states observed in (p,t) reactions. *Phys. Rev. C*, 22:316, 1980.

T. Engeland. Core excitation in ^{18}O. *Nuclear Physics*, 72:68, 1965.

Ferreira, L., R. Liotta, C. Dasso, R. A. Broglia, and A. Winther. Spatial correlations of pairing collective states. *Nuclear Physics A*, 426:276, 1984.

R. P. → R. P. Feynman. *Quantum Electrodynamics*. Benjamin, Reading, Mass., 1961.

Flynn, E. R., G. J. Igo, and R. A. Broglia. Three–phonon monopole and qudrupole pairing vibrational states in ^{206}Pb. *Phys. Lett. B*, 41:397, 1972.

Flynn, E.R. and Igo, G. and Barnes, P.D. and Kovar, D. and Bès, D. R. and Broglia, R.A. Effect of multipole pairing particle–hole fields in the particle–vibration couplig of ^{208}Pb (II). The ^{207}Pb$(t,p)^{209}$Pb reaction at 20 MeV. *Phys. Rev. C*, 3: 2371, 1971.

M. W. Herzog, R. J. Liotta, and L. J. Sibanda. Pair clustering and giant pairing resonances. *Phys. Rev. C*, 31:259, 1985.

J. Högaasen-Feldman. A study of some approximations of the pairing force. *Nuclear Physics*, 28:258, 1961.

B. D. Josephson. Possible new effects in superconductive tunnelling. *Phys. Lett.*, 1:251, 1962.

W. Kuhn. Über die Gesamtstärke der von einem Zustande ausgehenden Absorptionslinien. *Z. Phys.*, 33:408, 1925.

P. Lotti, F. Cazzola, P. F. Bortignon, R. A. Broglia, and A. Vitturi. Spatial correlation of pairing modes in nuclei at finite temperature. *Phys. Rev. C*, 40:1791, 1989.

B. Mottelson. Elementary features of nuclear structure. In Niefnecker, Blaizot, Bertsch, Weise, and David, editors, *Trends in Nuclear Physics, 100 years later, Les Houches, Session LXVI*, page 25, Amsterdam, 1998. Elsevier, 1998

B. R. Mottelson. *Elementary Modes of Excitation in Nuclei, Le Prix Nobel en 1975.* Imprimerie Royale Norstedts Tryckeri, Stockholm, 1976. p. 80.

B. Mouginot, E. Khan, R. Neveling, F. Azaiez, E. Z. Buthelezi, S. V. Förtsch, S. Franchoo, H. Fujita, J. Mabiala, J. P. Mira, P. Papka, A. Ramus, J. A. Scarpaci, F. D. Smit, I. Stefan, J. A. Swartz, and I. Usman. Search for the giant pairing vibration through (*p*,*t*) reactions around 50 and 60 mev. *Phys. Rev. C*, 83:037302, 2011.

A. B. Pippard. The historical context of Josephson discovery. In H. Rogalla and P. H. Kes, editors, *100 years of superconductivity*, page 30. CRC Press, Taylor and Francis, FL, 2012.

G. Potel, F. Barranco, E. Vigezzi, and R. A. Broglia. Evidence for phonon mediated pairing interaction in the halo of the nucleus ^{11}Li. *Phys. Rev. Lett.*, 105:172502, 2010.

F. Reiche and W. Thomas. Über die Zahl der Dispersionselektronen, die einem stationären Zustand zugeordnet sind. *Z. Phys.*, 34:510, 1925.

V. Rodin and A. Faessler. Can one measure nuclear matrix elements of neutrinoless double β decay? *Phys. Rev. C*, 80:041302, 2009.

Shimizu, Y. R., J. D. Garrett, R. A. Broglia, M. Gallardo, and E. Vigezzi. Pairing fluctuations in rapidly rotating nuclei. *Reviews of Modern Physics*, 61:131, 1989.

V. G. Soloviev. *The theory of atomic nuclei*. Institute of Physics Publishing, London, 1992.

~~I. Tonozuka and A. Arima. Surface α-clustering and α-decays of ^{212}Po. *Nuclear Physics A*, 323:45, 1979.~~

P. J. Twin, B. M. Nyakó, A. H. Nelson, J. Simpson, M. A. Bentley, H. W. Cranmer-Gordon, P. D. Forsyth, D. Howe, A. R. Mokhtar, J. D. Morrison, J. F. Sharpey-Schafer, and G. Sletten. Observation of a discrete-line superdeformed band up to $60\hbar$ in ^{152}Dy. *Phys. Rev. Lett.*, 57:811, 1986.

S. Weinberg. *The Quantum Theory of Fields*, ~~volume 2~~ Vols. 1 and 2. Cambridge University Press, Cambridge, 1996.

D. Savran, T. Aumann and A. Zilges, Experimental studies of the pygmy dipole resonance, Prog. Part. Nucl. Phys. 70, 210 (2013)

~~H. Schmidt, Monopole vibrations of closed shell nuclei, Zeits. Phys., 181, 532 (1964)~~