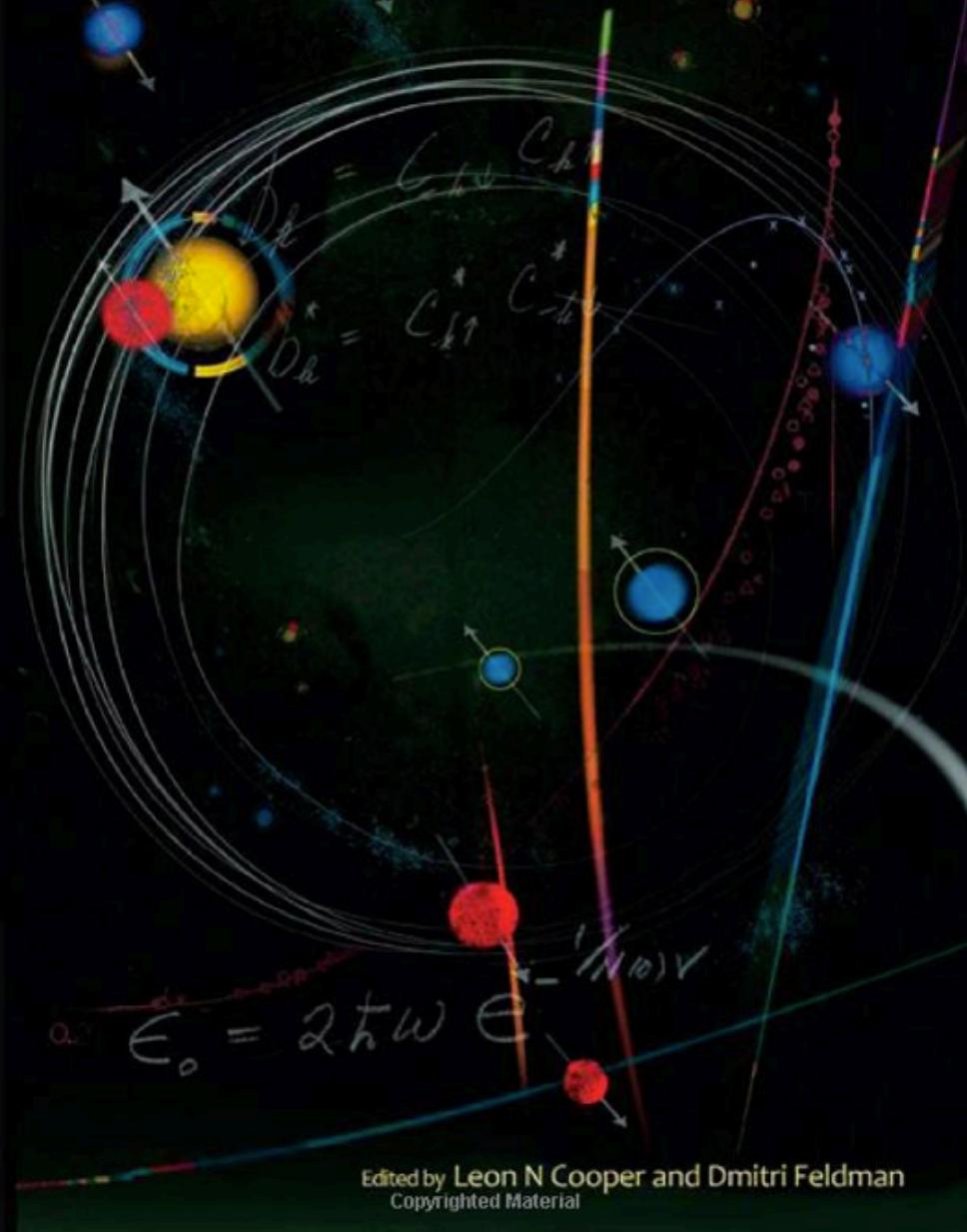
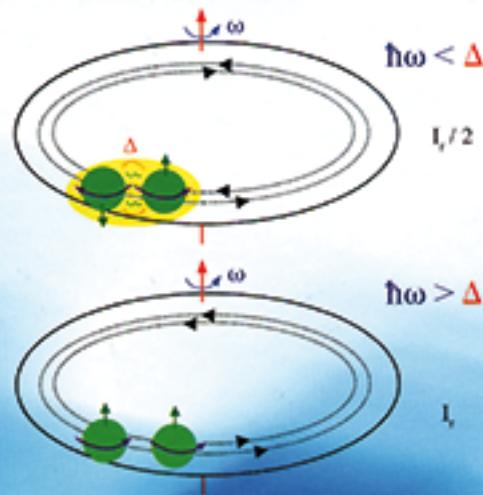


BCS: 50 Years



Fifty Years of Nuclear BCS

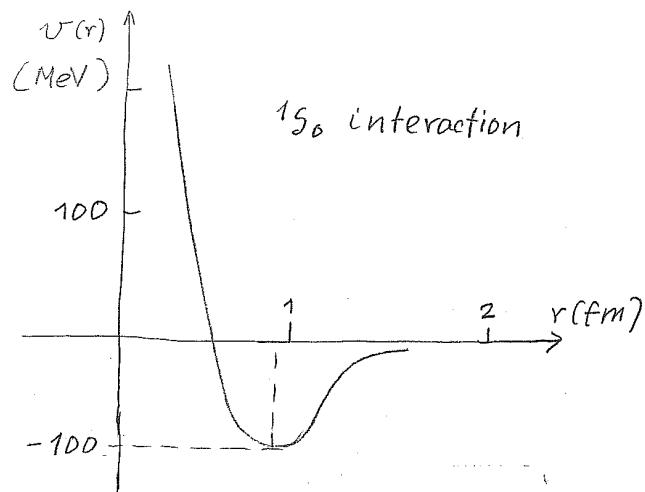
Pairing in Finite Systems



Ricardo A Broglia
Vladimir Zelevinsky

editors

 World Scientific



Quantity parameter

$q \ll 1$ Crystalline structure ($T=0$)

$$q = \left(\frac{\hbar^2}{Ma^2} \right) \frac{1}{|v_0|}$$

$q \approx 1$ Quantum fluid ($T=0$)

$$q \approx 0.4$$

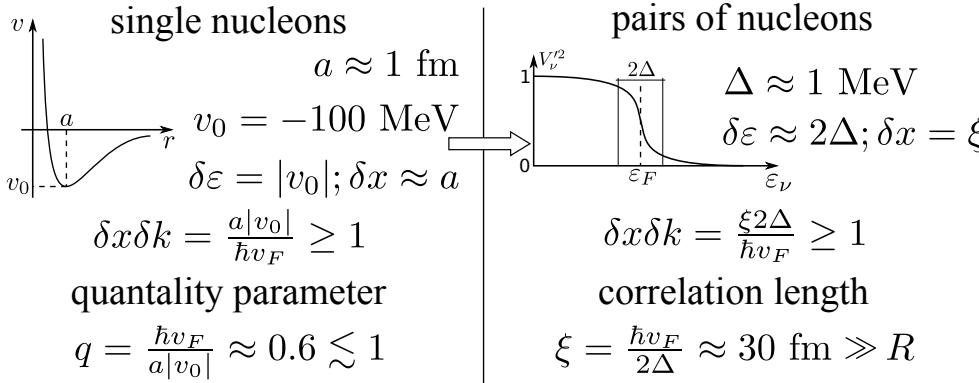
Nuclei

Classical localization and quantal ZPF

$$\delta x \delta k \geq 1 \quad \varepsilon = \frac{\hbar^2 k^2}{2M} \quad \delta k = \frac{\delta \varepsilon}{\hbar v_F} \quad (v_F/c \approx 0.3)$$

structure

Independent motion of



emergent property: generalized rigidy in
3D-space

↳ how does a short range force lead to

single-nucleon mean free paths
larger than nuclear dimension?

$2R \approx 20/k_F$
answer: quantal fluctuations
reactions

single particle transfer, e.g. (p,d)

$$\frac{2R}{a} \approx 15$$

absolute cross section reflects
the full nucleon probability
amplitude distribution, and does
not depend of the specific choice
of v_{np}

Cooper pair transfer, e.g. (p,t)

$$\frac{\xi}{a} \approx 30$$

Successive and simultaneous
transfer amplitude contributions to
the absolute cross section carry
equally efficiently information
concerning pair correlations

$$H = T + v = \underline{T + U + V_p} + (v - U - V_p)$$

MEAN FIELD

Diagon. $\alpha_\nu^\dagger = U_\nu a_\nu^\dagger - V_\nu a_{\bar{\nu}}^\dagger$

g.s. $\alpha_\nu |\tilde{0}\rangle = 0$

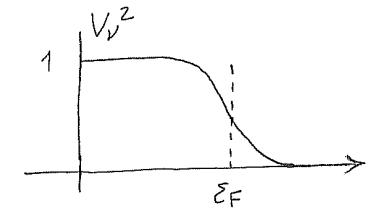
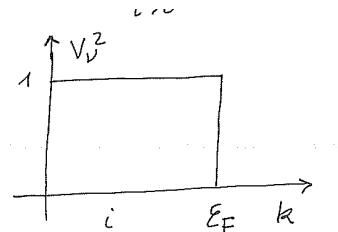
$$|\tilde{0}\rangle = \prod_{\nu>0} \alpha_\nu \alpha_{\bar{\nu}} |0\rangle \approx \prod_{\nu>0} (U_\nu + V_\nu a_\nu^\dagger a_{\bar{\nu}}^\dagger) |0\rangle$$

Ansatz 1 : sharp occupation distribution

$$|\tilde{0}\rangle = |HF\rangle = \prod_{i>0} a_i^\dagger a_{\tilde{i}}^\dagger |0\rangle = \prod_i a_i^\dagger |0\rangle$$

Ansatz 2 : sigmoidal occupation distribution

$$|\tilde{0}\rangle = |BCS\rangle = \prod_{\nu>0} (U_\nu + V_\nu a_\nu^\dagger a_{\bar{\nu}}^\dagger) |0\rangle$$



$$P^\dagger = \textstyle\sum_{\nu>0} a_\nu^\dagger a_\nu^\dagger$$

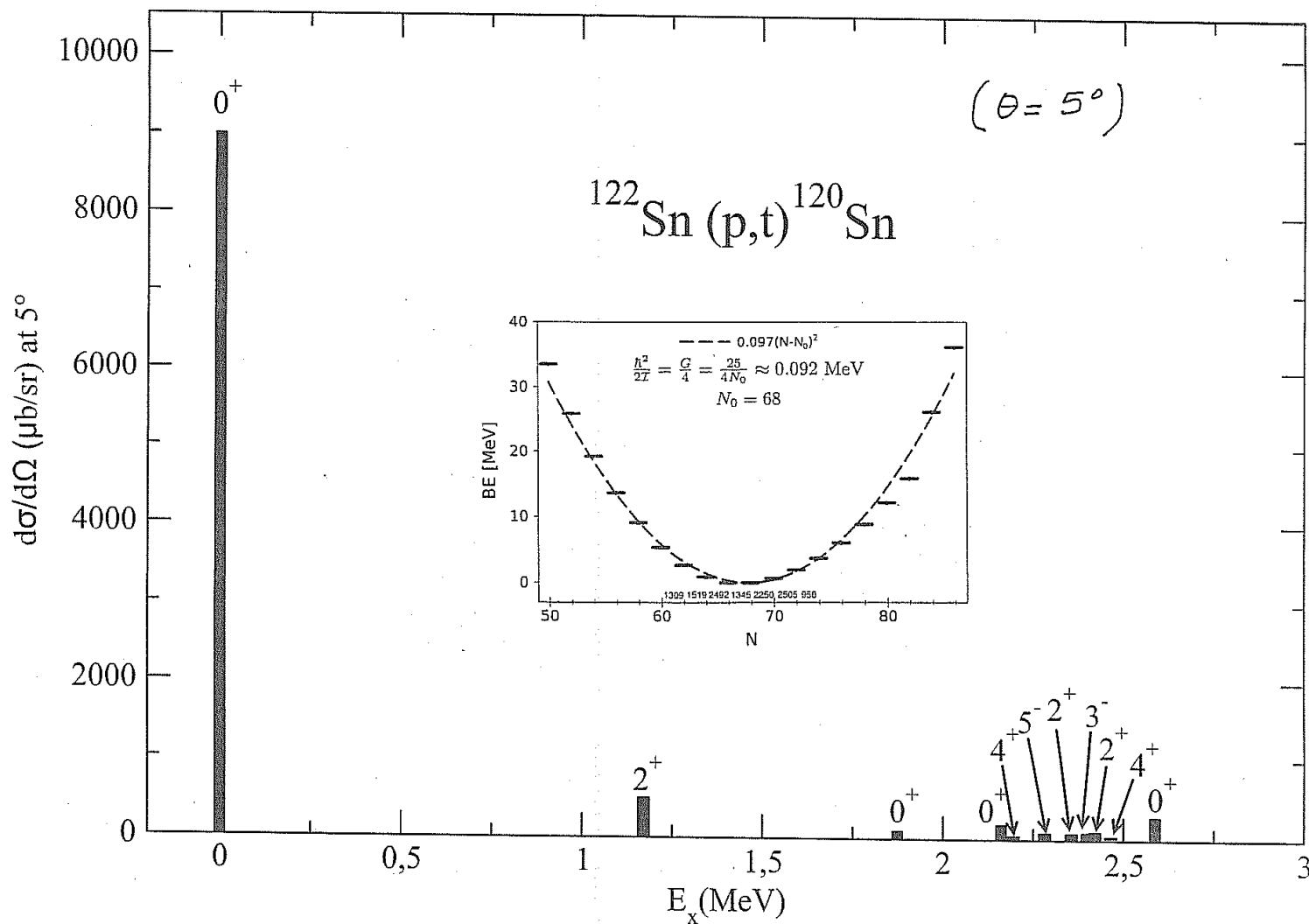
$$x=\tfrac{2G\Omega}{D}=GN(0)$$

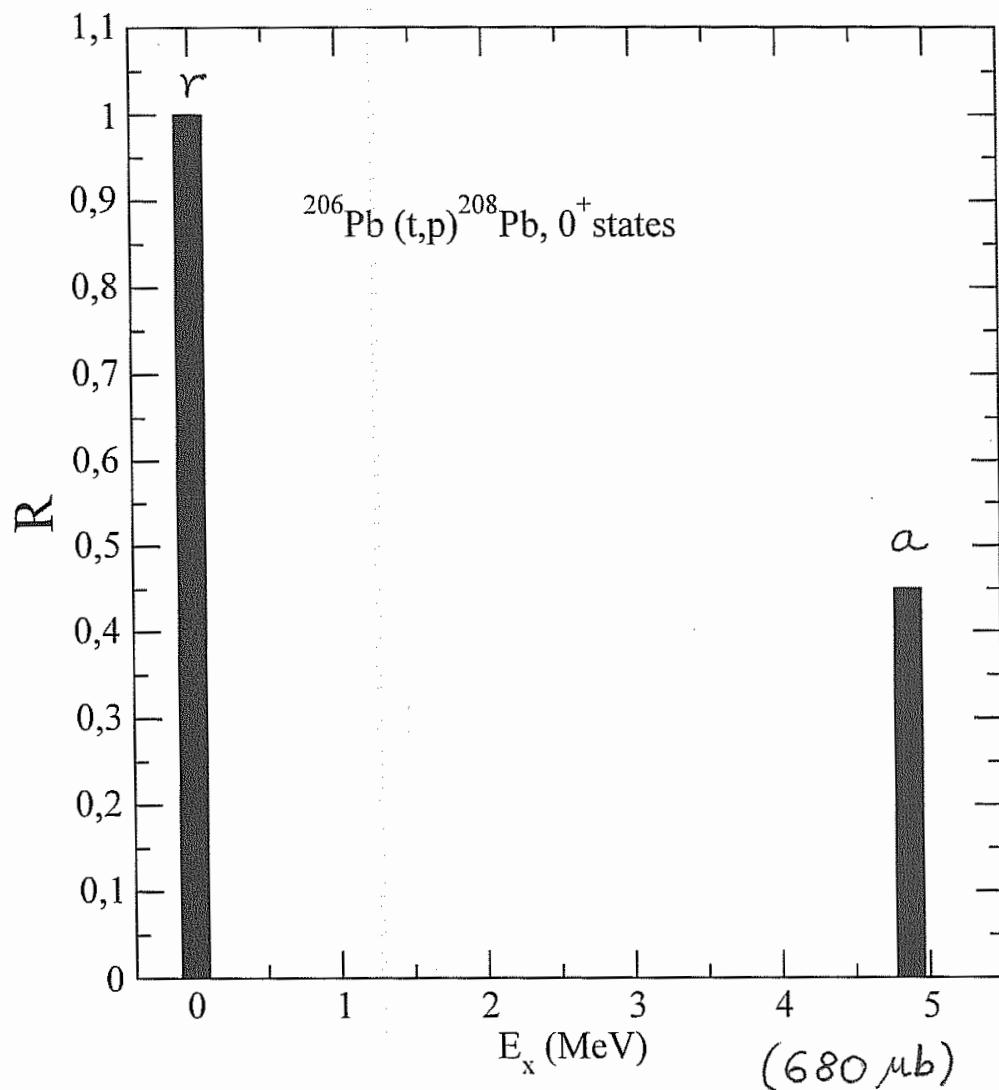
$$\begin{array}{ccc} x > 1 & & x < 1 \\ \hline \end{array}$$

$$\begin{array}{ccc} \alpha_0=< P^\dagger >=\frac{\Delta}{G}\approx 7 & \left| \right. & \alpha_{dyn}=\frac{1}{G}\frac{<PP^\dagger>^{1/2}+<P^\dagger P>^{1/2}}{2}\\ & & \approx \quad \frac{1}{2}\left(\frac{E_{corr}(A+2)}{G}+\frac{E_{corr}(A-2)}{G}\right)\approx 10 \end{array}$$

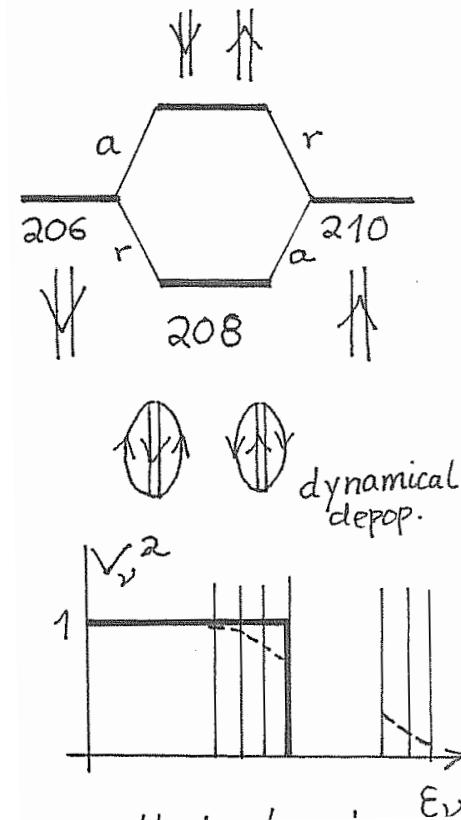
$$\frac{\alpha_0}{\alpha_{dyn}}\approx 0.7$$

$$\frac{\beta_2}{(\beta_2)_{dyn}}\approx 3-6$$





linear term N



Well developed vibr. bands
(very anaharmonic at the edge of
phase transition, thus very collective
cf. Clark et al. PRL 96 (2006)032501)

Spectroscopic amplitudes

Pairing rotations $A+2\text{Sn}(p,t)A\text{Sn(gs)}$

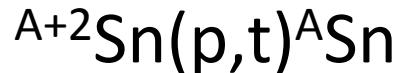
$U_v \ V_v$ (BCS)

Pairing vibrations $^{208}\text{Pb}(p,t)^{206}\text{Pb(gs)}$

$X_r \ Y_r$ (RPA)

Coherent states: essentially exact

Systematics (silent revolution)



$$A+2 = 112, 114, 116, 118, 120, 122$$

Guazzoni et al, PRC
1999(122), 2004(116), 2006(112),
2008(120), 2011(118, 124), 2012(114)

Major breakthrough



Tanihata et al, PRL 2008

$$\xi = \frac{\hbar v_F}{E_{corr}} \approx 30\text{-}35\,fm$$

$$v_{np}\approx 0.4\,fm$$

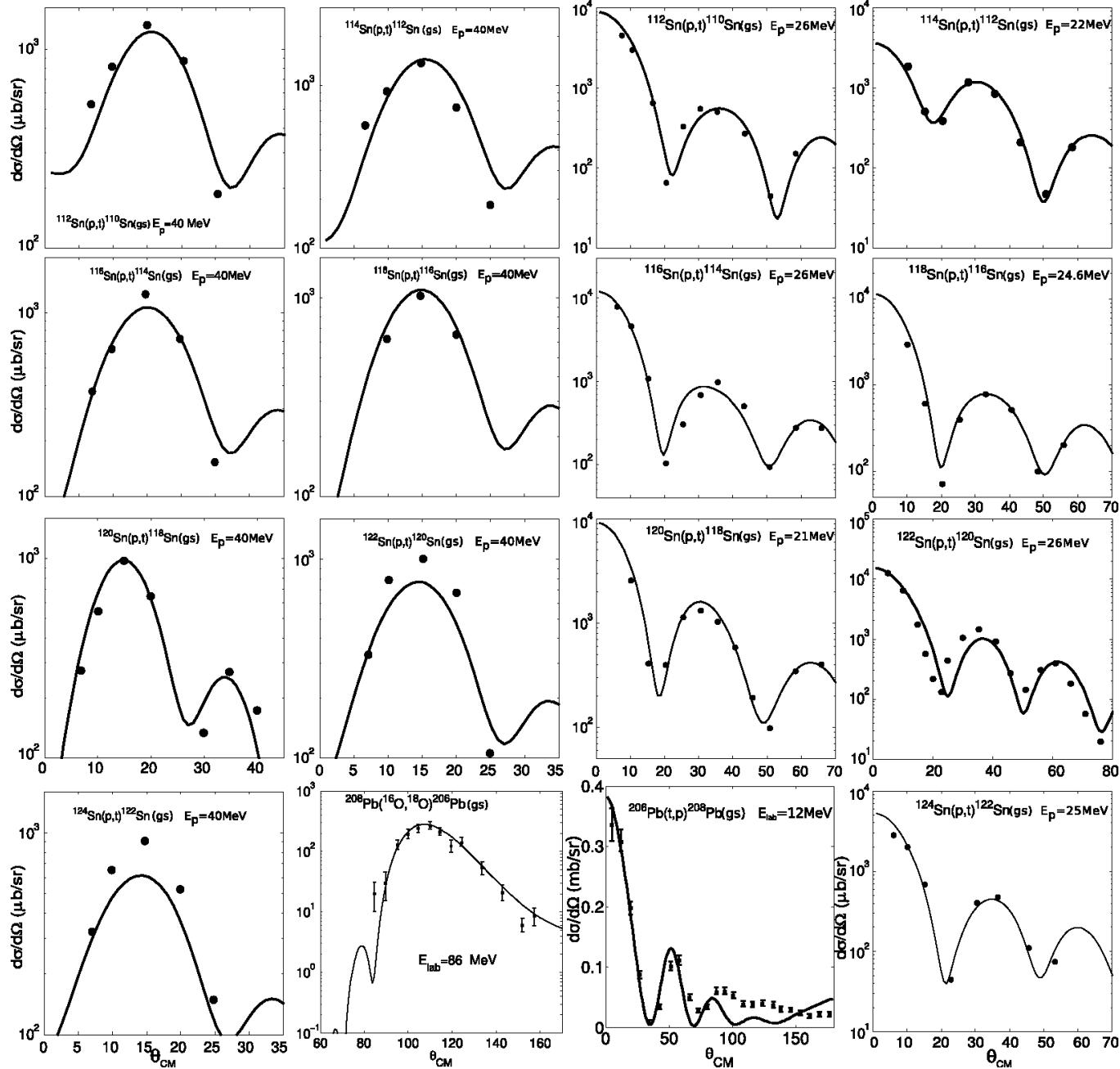
$$T^{(1)} = 2 \sum_{l_i, j_i} \sum_{\sigma_1 \sigma_2} \int d\mathbf{r}_{tA} d\mathbf{r}_{p1} d\mathbf{r}_{A2} [\phi_{l_i, j_i}^{A+2}(\mathbf{r}_{A1}, \sigma_1, \mathbf{r}_{A2}, \sigma_2)]_0^{0*} \chi_{pB}^{(-)*}(\mathbf{r}_{pB}) v(r_{p1}) \phi_t(r_{p1}, \sigma_1, r_{p2}, \sigma_2) \chi_{tA}^{(+)}(\mathbf{r}_{tA}), \quad (38a)$$

successive,

$$\begin{aligned} T_{\text{succ}}^{(2)} &= 2 \sum_{l_i, j_i} \sum_{l_f, j_f, m_f} \sum_{\substack{\sigma_1 \sigma_2 \\ \sigma'_1 \sigma'_2}} \int d\mathbf{r}_{dF} d\mathbf{r}_{p1} d\mathbf{r}_{A2} [\phi_{l_i, j_i}^{A+2}(\mathbf{r}_{A1}, \sigma_1, \mathbf{r}_{A2}, \sigma_2)]_0^{0*} \chi_{pB}^{(-)*}(\mathbf{r}_{pB}) v(r_{p1}) \phi_d(r_{p1}, \sigma_1) \varphi_{l_f, j_f, m_f}^{A+1}(\mathbf{r}_{A2}, \sigma_2) \\ &\times \int d\mathbf{r}'_{dF} d\mathbf{r}'_{p1} d\mathbf{r}'_{A2} G(\mathbf{r}_{dF}, \mathbf{r}'_{dF}) \phi_d(r'_{p1}, \sigma'_1)^* \varphi_{l_f, j_f, m_f}^{A+1*}(\mathbf{r}'_{A2}, \sigma'_2) \frac{2\mu_{dF}}{\hbar^2} v(r'_{p2}) \phi_d(r'_{p1}, \sigma'_1) \phi_d(r'_{p2}, \sigma'_2) \chi_{tA}^{(+)}(\mathbf{r}'_{tA}), \end{aligned} \quad (38b)$$

and nonorthogonal,

$$\begin{aligned} T_{\text{NO}}^{(2)} &= 2 \sum_{l_i, j_i} \sum_{l_f, j_f, m_f} \sum_{\substack{\sigma_1 \sigma_2 \\ \sigma'_1 \sigma'_2}} \int d\mathbf{r}_{dF} d\mathbf{r}_{p1} d\mathbf{r}_{A2} [\phi_{l_i, j_i}^{A+2}(\mathbf{r}_{A1}, \sigma_1, \mathbf{r}_{A2}, \sigma_2)]_0^{0*} \chi_{pB}^{(-)*}(\mathbf{r}_{pB}) v(r_{p1}) \phi_d(r_{p1}, \sigma_1) \varphi_{l_f, j_f, m_f}^{A+1}(\mathbf{r}_{A2}, \sigma_2) \\ &\times \int d\mathbf{r}'_{p1} d\mathbf{r}'_{A2} d\mathbf{r}'_{dF} \phi_d(r'_{p1}, \sigma'_1)^* \varphi_{l_f, j_f, m_f}^{A+1*}(\mathbf{r}'_{A2}, \sigma'_2) \phi_d(r'_{p1}, \sigma'_1) \phi_d(r'_{p2}, \sigma'_2) \chi_{tA}^{(+)}(\mathbf{r}'_{tA}), \end{aligned} \quad (38c)$$



New Technical Achievement

			$\sigma(g\text{-}s \rightarrow f)$
	f	Theory ^{a) b) f)}	Experiment ^{f-m)}
$^7\text{Li}(t, p)^9\text{Li}$	gs	14.3 ^{c)}	14.7 ± 4.4 ^{c,i)} $[9.4^\circ < \theta < 108.7^\circ]$
$^1\text{H}(^{11}\text{Li}, ^9\text{Li})^3\text{H}$	gs	6.1 ^{c)}	5.7 ± 0.9 ^{c,b)} $[20^\circ < \theta < 154.5^\circ]$
	1/2 ⁻	0.7 ^{c)}	1.0 ± 0.36 ^{c,b)} $[30^\circ < \theta < 100^\circ]$
$^{10}\text{Be}(t, p)^{12}\text{Be}$	gs	2.3 ^{c)}	1.9 ± 0.57 ^{c,j)} $[4.4^\circ < \theta < 57.4^\circ]$
$^{48}\text{Ca}(t, p)^{50}\text{Ca}$	gs	0.55 ^{c)}	0.56 ± 0.17 ^{c,m)} $[4.5^\circ < \theta < 174^\circ]$
$^{112}\text{Sn}(p, t)^{110}\text{Sn}$, $E_{CM} = 26$ MeV	gs	1301 ^{d)}	1309 ± 200 (± 14) ^{d,g)} $[6^\circ < \theta < 62.2^\circ]$
$^{114}\text{Sn}(p, t)^{112}\text{Sn}$, $E_{CM} = 22$ MeV	gs	1508 ^{d)}	1519 ± 456 (± 16.2) ^{d,g)} $[7.64^\circ < \theta < 62.24^\circ]$
$^{116}\text{Sn}(p, t)^{114}\text{Sn}$, $E_{CM} = 26$ MeV	gs	2078 ^{d)}	2492 ± 374 (± 32) ^{d,g)} $[4^\circ < \theta < 70^\circ]$
$^{118}\text{Sn}(p, t)^{116}\text{Sn}$, $E_{CM} = 24.4$ MeV	gs	1304 ^{d)}	1345 ± 202 (± 24) ^{d,g)} $[7.63^\circ < \theta < 59.6^\circ]$
$^{120}\text{Sn}(p, t)^{118}\text{Sn}$, $E_{CM} = 21$ MeV	gs	2190 ^{d)}	2250 ± 338 (± 14) ^{d,g)} $[7.6^\circ < \theta < 69.7^\circ]$
$^{122}\text{Sn}(p, t)^{120}\text{Sn}$, $E_{CM} = 26$ MeV	gs	2466 ^{d)}	2505 ± 376 (± 18) ^{d,g)} $[6^\circ < \theta < 62.2^\circ]$
$^{124}\text{Sn}(p, t)^{122}\text{Sn}$, $E_{CM} = 25$ MeV	gs	838 ^{d)}	958 ± 144 (± 15) ^{d,g)} $[4^\circ < \theta < 57^\circ]$
$^{112}\text{Sn}(p, t)^{110}\text{Sn}$, $E_p = 40$ MeV	gs	3349 ^{e)}	3715 ± 1114 ^{e,h)}
$^{114}\text{Sn}(p, t)^{112}\text{Sn}$, $E_p = 40$ MeV	gs	3790 ^{e)}	3776 ± 1132 ^{e,h)}
$^{116}\text{Sn}(p, t)^{114}\text{Sn}$, $E_p = 40$ MeV	gs	3085 ^{e)}	3135 ± 940 ^{e,h)}
$^{118}\text{Sn}(p, t)^{116}\text{Sn}$, $E_p = 40$ MeV	gs	2563 ^{e)}	2294 ± 668 ^{e,h)}
$^{120}\text{Sn}(p, t)^{118}\text{Sn}$, $E_p = 40$ MeV	gs	3224 ^{e)}	3024 ± 907 ^{e,h)}
$^{122}\text{Sn}(p, t)^{120}\text{Sn}$, $E_p = 40$ MeV	gs	2339 ^{e)}	2907 ± 872 ^{e,h)}
$^{124}\text{Sn}(p, t)^{122}\text{Sn}$, $E_p = 40$ MeV	gs	1954 ^{e)}	2558 ± 767 ^{e,h)}
$^{206}\text{Pb}(t, p)^{208}\text{Pb}$	gs	0.52 ^{c)}	0.68 ± 0.21 ^{c,k)} $[4.5^\circ < \theta < 176.5^\circ]$
$^{208}\text{Pb}(^{16}\text{O}, ^{18}\text{O})^{206}\text{Pb}$	gs	0.80 ^{c)}	0.76 ± 0.18 ^{c,f)} $[84.6^\circ < \theta < 157.3^\circ]$

Table 4:

It is of notice that the number in parenthesis (last column) corresponds to the statistical errors.

^{a)} G. Potel et al., Phys. Rev. Lett. **107**, (2011) 092501.

^{b)} G. Potel et al., Phys. Rev. Lett. **105**, (2010) 172502.

^{c)} mb

^{d)} μb

^{e)} $\mu\text{b}/\text{sr}$ ($\sum_{i=1}^N (d\sigma/d\Omega)$; differential cross section summed over the few, $N = 3 - 7$ experimental points).

^{f)} B. Bayman and J. Chen, Phys. Rev. **C 26** (1982) 1509 and refs. therein.

^{g)} P. Guazzoni, L. Zetta, et al., Phys. Rev. **C 60**, 054603 (1999).

P. Guazzoni, L. Zetta, et al., Phys. Rev. **C 69**, 024619 (2004).

P. Guazzoni, L. Zetta, et al., Phys. Rev. **C 74**, 054605 (2006).

P. Guazzoni, L. Zetta, et al., Phys. Rev. **C 83**, 044614 (2011).

P. Guazzoni, L. Zetta, et al., Phys. Rev. **C 78**, 064608 (2008).

P. Guazzoni, L. Zetta, et al., Phys. Rev. **C 85**, 054609 (2012).

^{h)} G. Bassani et al. Phys. Rev. **139**, (1965) B830.

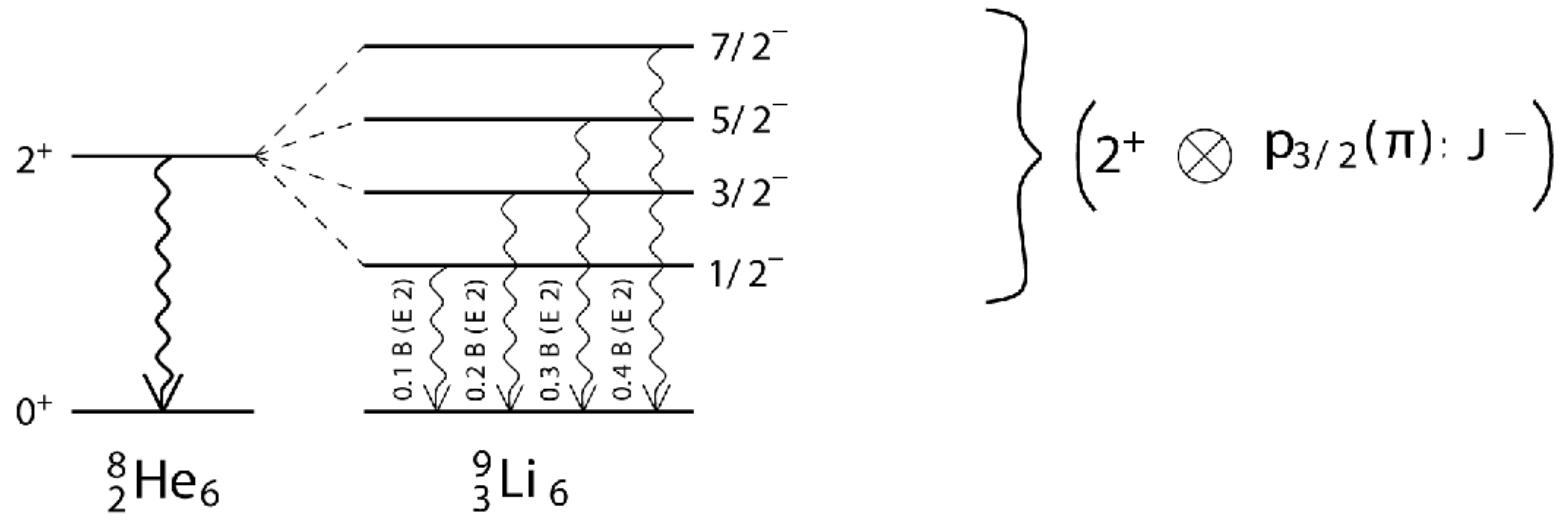
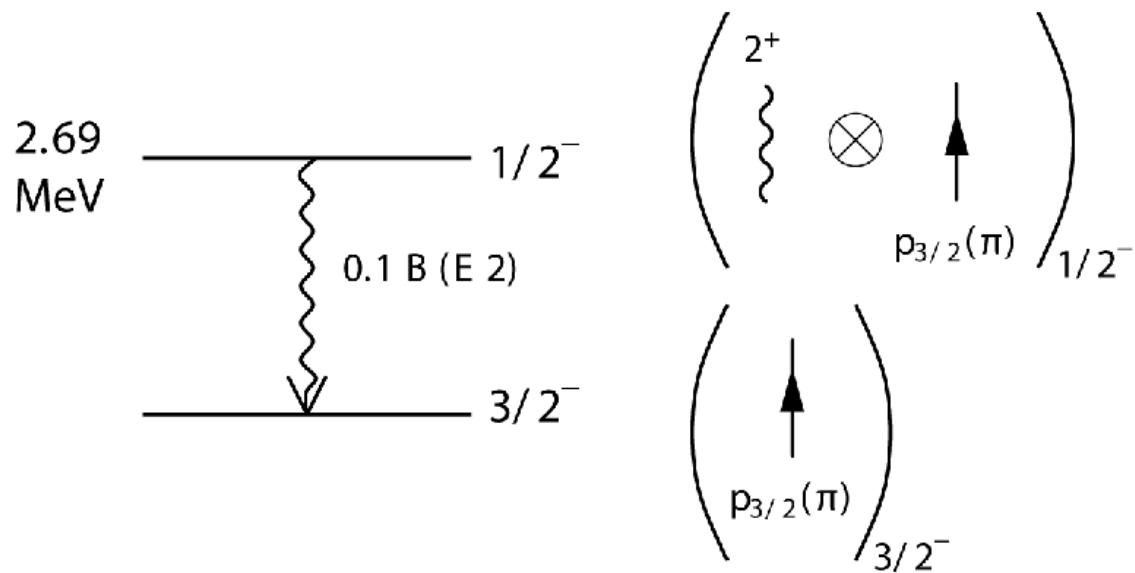
ⁱ⁾ P.G. Young and R.H. Stokes, Phys. Rev. **C 4**, (1971) 1597.

^{j)} H.T. Fortune, G.B. Liu and D.E. Alburger, Phys. Rev. **C 50**, (1994) 1355.

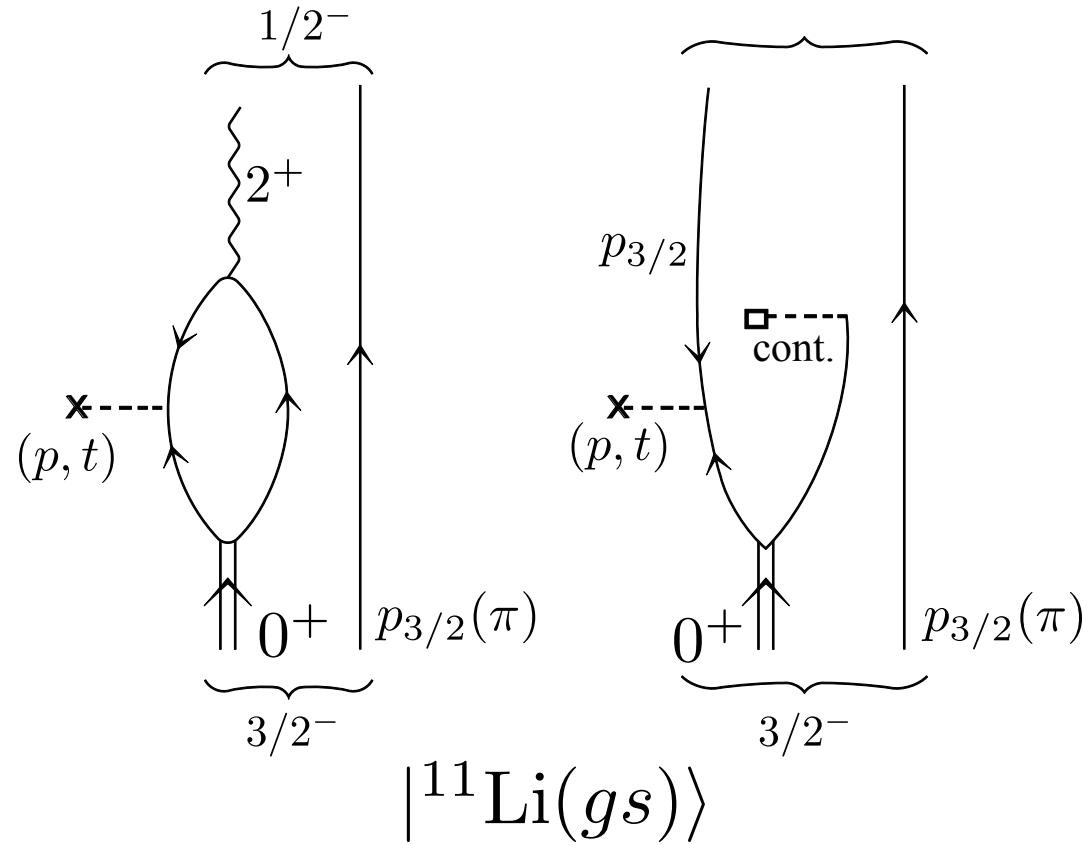
^{k)} J.H. Bjerregaard et al., Nucl. Phys. **89**, (1966) 337.

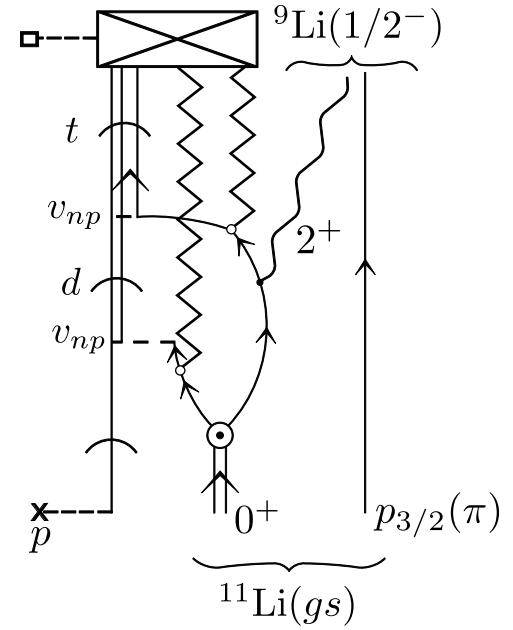
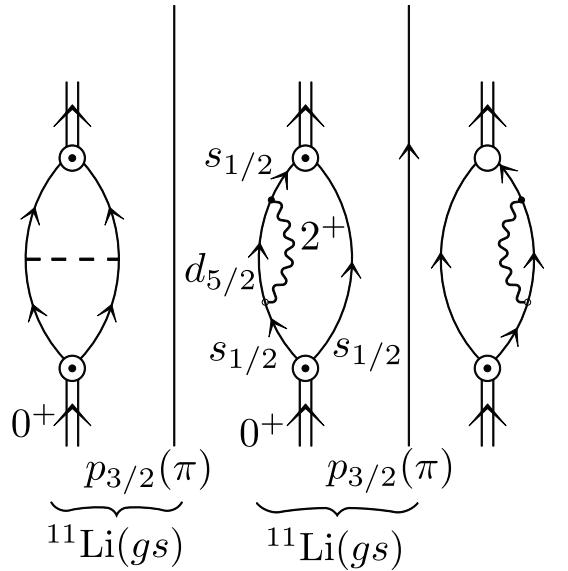
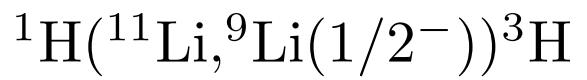
^{l)} J.H. Bjerregaard et al., Nucl. Phys. **A 113**, (1968) 484.

^{m)} J. H. Bjerregaard et al., Nucl. Phys. **A 103**, (1967) 33.



$$|{}^9_3\text{Li}_6(2.69 \text{ MeV}; 1/2^-)\rangle \ |{}^8_3\text{Li}_5(p_{3/2}^{-1}(\nu), p_{3/2}(\pi))\rangle$$



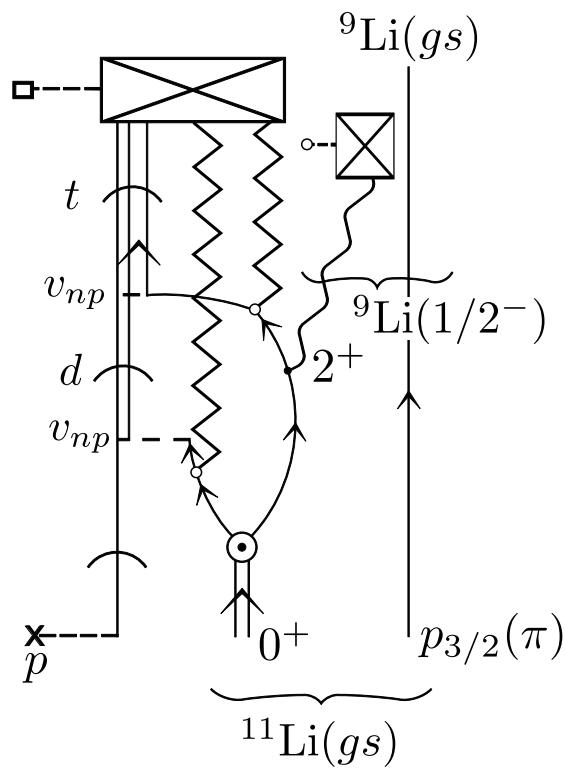


Variety of pv-coupling vertices
(NFT)

\times

\circ

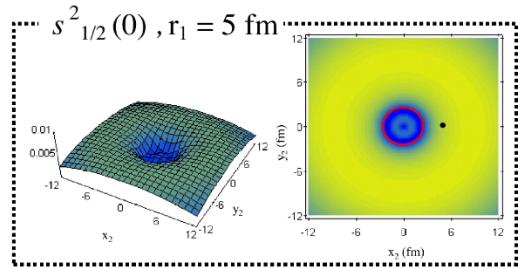
pair
surface
recoil



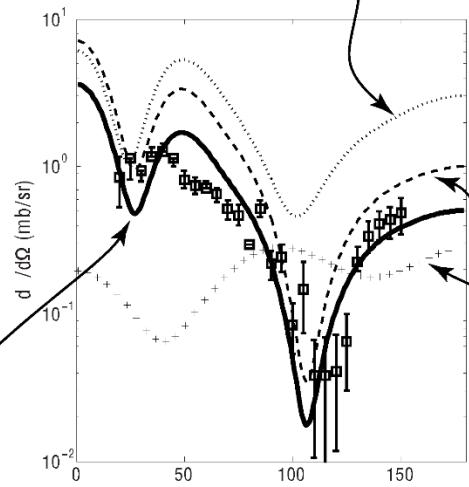
Variety of pv-coupling vertices

(NFT)

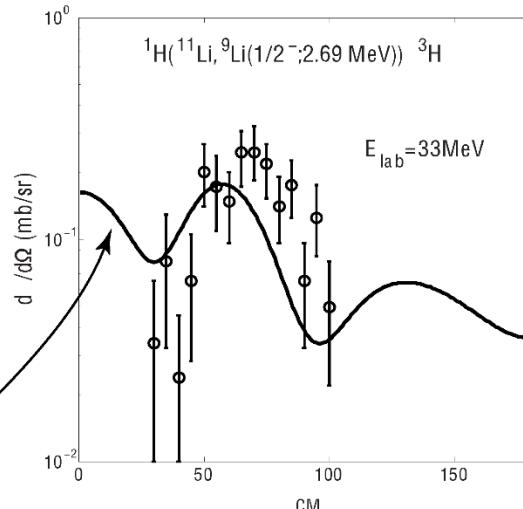
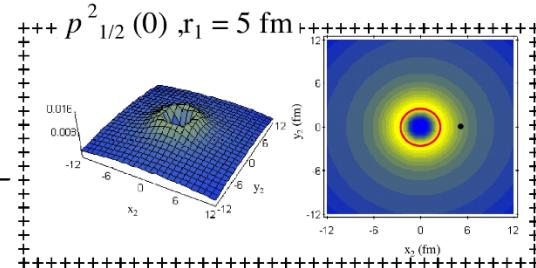
- pair
- surface
- recoil



Barranco et al
EPJ, A11 (2001) 305

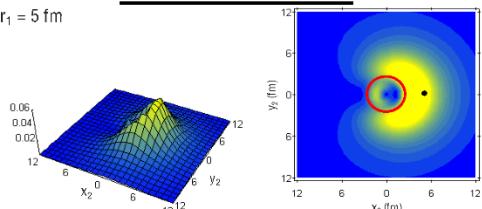


Tanikata et al
PRL, 100 (2008) 192502



NFT. Renorm.

$r_1 = 5$ fm



$r_1 = 7.5$ fm

$$|\tilde{0}\rangle_\nu = |0\rangle + 0.7|(p_{1/2}, s_{1/2})_{1^-} \otimes 1^-; 0\rangle + 0.1|(s_{1/2}, d_{5/2})_{2^+} \otimes 2^+; 0\rangle$$

$$|0\rangle = 0.45|s^2_{1/2}(0)\rangle + 0.55|p^2_{1/2}(0)\rangle + 0.04|d^2_{5/2}(0)\rangle$$

N=0.5

Barranco et al
EPJ, A11 (2001) 305

$|\tilde{0}\rangle_\nu = |0\rangle$

$|0\rangle = 0.63|s^2_{1/2}(0)\rangle + 0.77|p^2_{1/2}(0)\rangle + 0.06|d^2_{5/2}(0)\rangle$

N=1

core radius (systematics)

$$R_0 = 1.2 \times 9^{1/3} \approx 2.5 \text{ fm}$$

$$\mathcal{R} = \sqrt{\frac{3}{R_0^3}} \Theta(r - R_0)$$

$$\int_0^{R_0} dr \ r^2 \mathcal{R}^2 = \frac{3}{R_0^3} \int_0^{R_0} \frac{dr^3}{3} = 1$$

Halo radius

$$R \approx 3.5 \text{ fm}$$

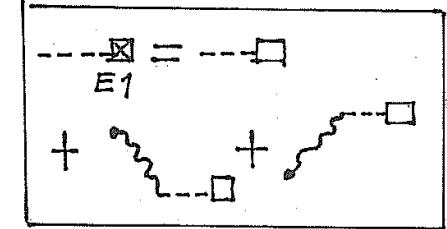
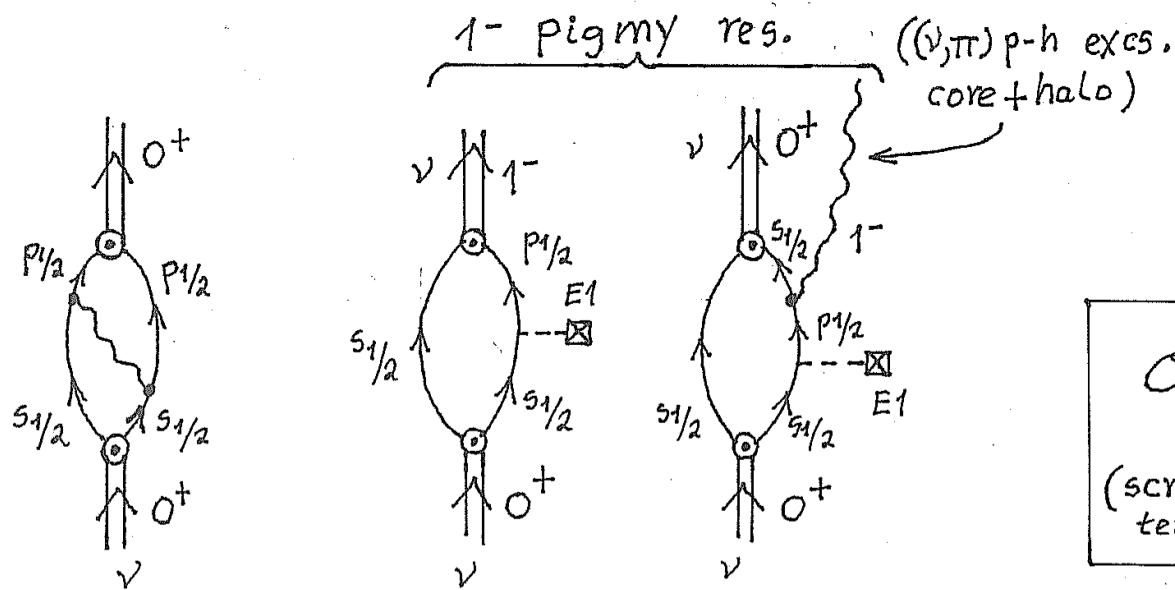
$$\mathcal{R}_{halo} = \sqrt{\frac{3}{R^3}} \Theta(r - R)$$

Two-nucleon overlap

$$o = | < \mathcal{R}_{halo} | \mathcal{R} > |^2$$

$$o = \left(\int_0^{R_0} dr r^2 \mathcal{R}_{halo} \mathcal{R} \right)^2 = \left(\sqrt{\frac{3}{R^3}} \sqrt{\frac{3}{R_0^3}} \int_0^{R_0} \frac{dr^3}{3} \right)^2 = \left(\frac{R_0}{R} \right)^3$$

$$\Theta(r - R) = \begin{cases} 1 & r \leq R \\ 0 & r > R \end{cases}$$



$$\sigma \approx \left(\frac{R_o}{R}\right)^3 \approx 0.16$$

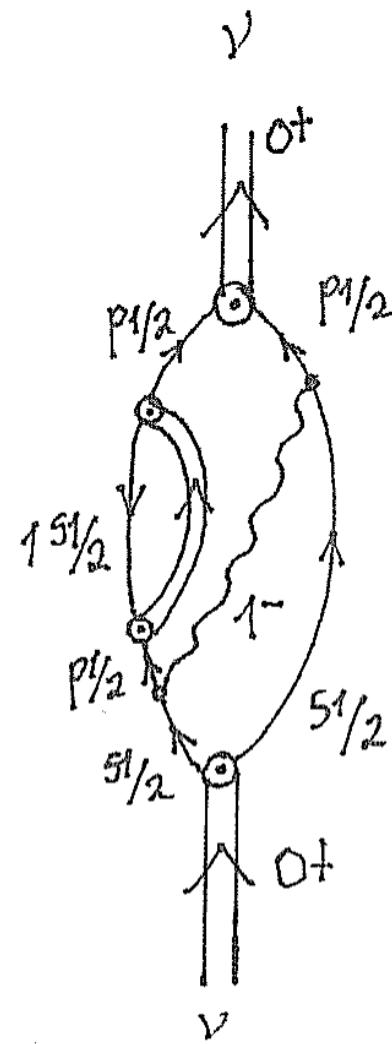
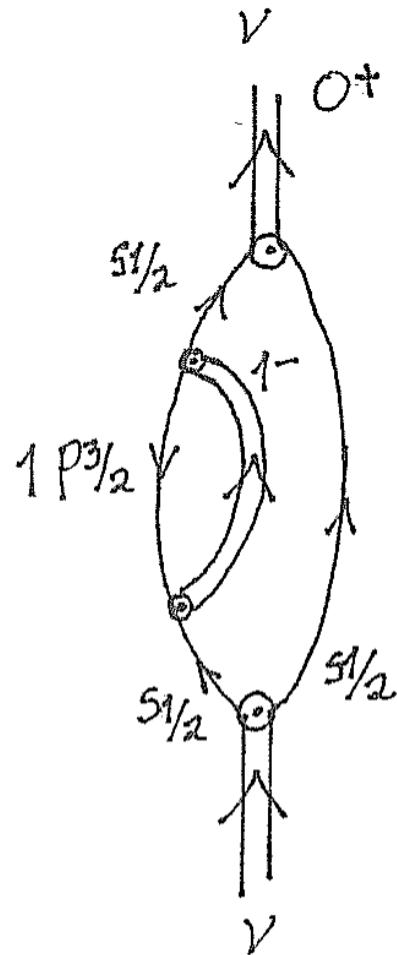
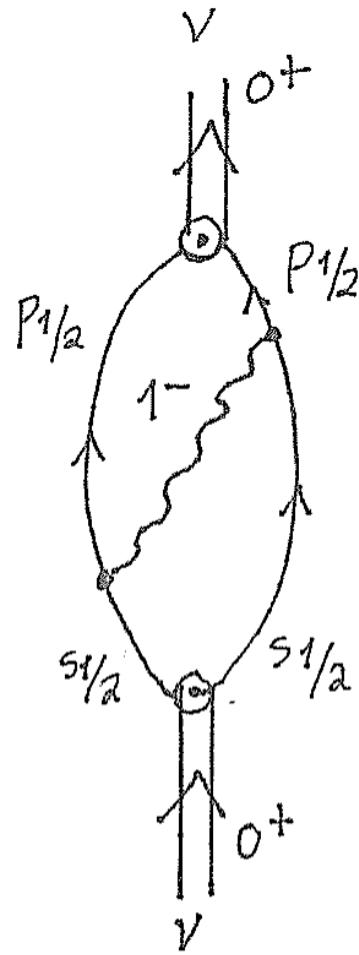
(screening of symmetry term)

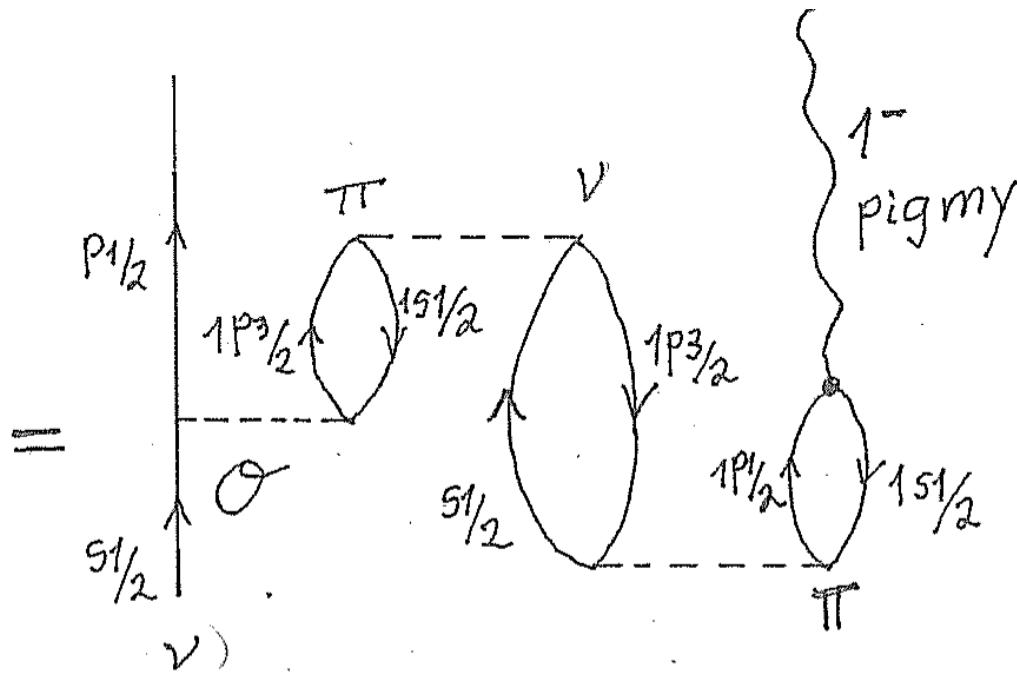
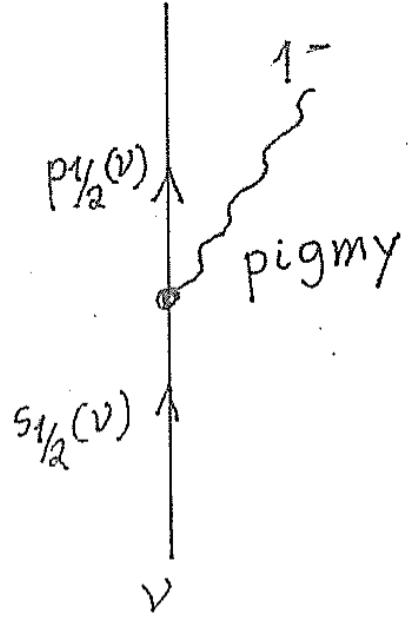
The ^{11}Li pigmy resonance can hardly be viewed but in symbiosis with the ^9Li halo neutron pair addition mode

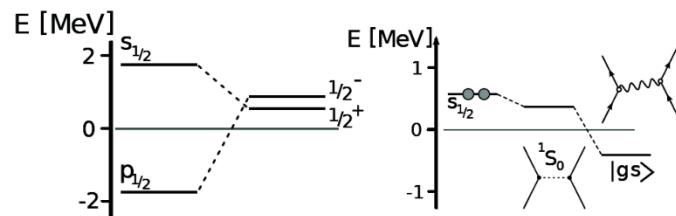
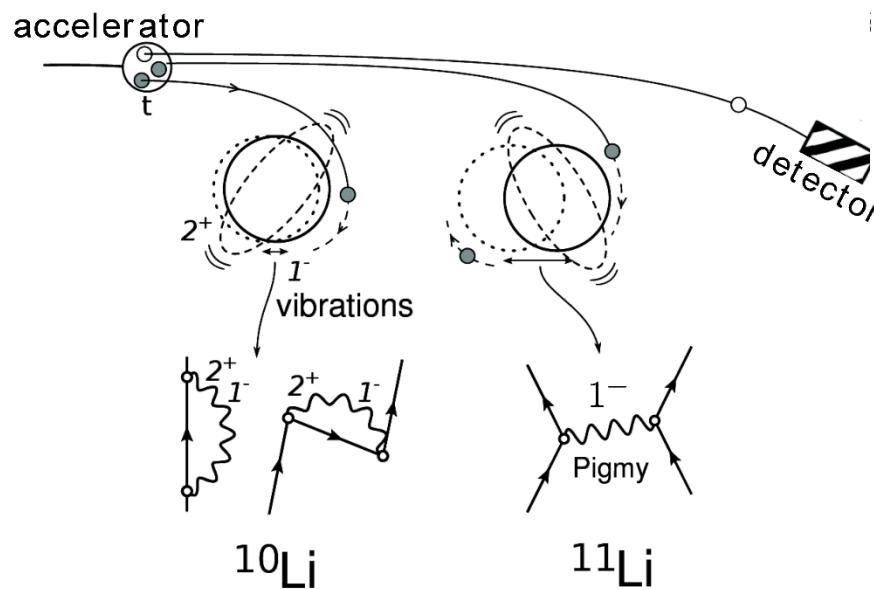
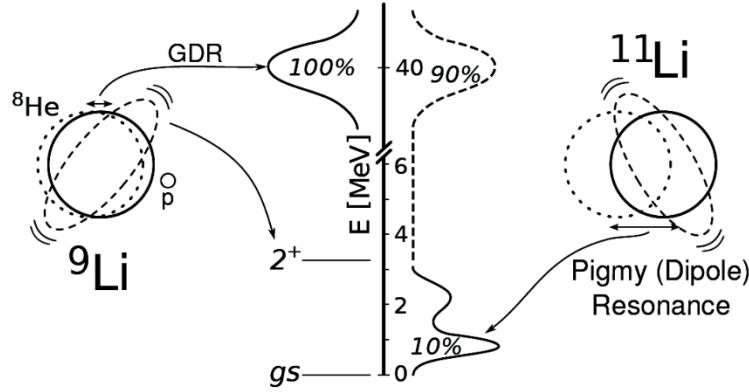
anti-(halo-anti-pairing effect)

The pigmy resonance is built on a ground state with little overlap with the gs on which the GDR is built. It is thus a different (new) elementary mode of nuclear excitation

Extreme example of inhomogeneous damping (radial degree of freedom instead of quadrupole deformation)







Bootstrap pairing correlations

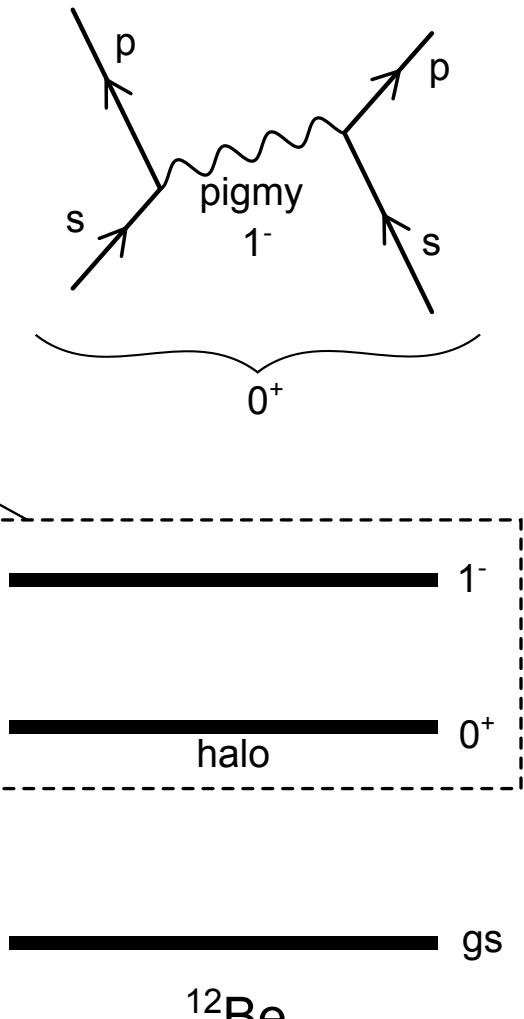
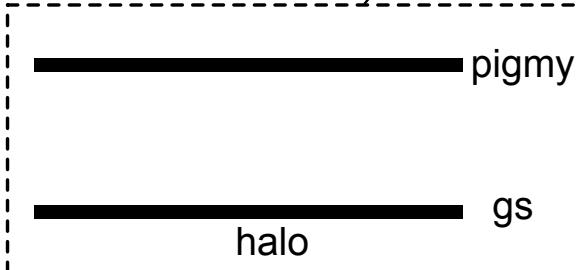
New physics

$r \ll 1$
 bare NN-pairing
 screened
 subcritical ($v_{NN} < G_c$)

$$r \approx O \approx \left(\frac{R_0}{R}\right)^3$$

unity

$O \ll 1$
 strongly screened
 isospin interaction
 soft $E1$ -mode
 (pigmy 1^-)



pair addition
 halo mode

$$V(r_{12}) = -4\pi V_0 \delta(|\vec{r}_1 - \vec{r}_2|)$$

$$\mathcal{R}_j = \sqrt{\frac{3}{R_0^3}} \Theta(r - R_0)$$

$$I(j) = \int_0^{R_0} dr \mathcal{R}_j^4 r^2 = \frac{3}{R_0^3}$$

$$M_j = <(j)_0^2 |V|(j)_0^2> = \frac{2j+1}{2} V_o I(j)$$

$$r = \frac{(M_j)_{halo}}{(M_j)_{syst}} = \left(\frac{R_0}{R}\right)^3$$

Halo anti-pairing effect
 (cf. Bennaceur, Dobaczewski,
 Hamamoto, Mottelson, Ploszajczak)

$R_0 = 1.2 A^{1/3}$ fm R: halo radius
 (systematics)

Dual origin of pairing in nuclei

$$v_{\text{pair}} = v_{\text{bare}} \cdot (N-N + 3N) + v_{\text{ind}}$$

Direct and circumstantial
evidence for v_{ind} in ^{11}Li

In nuclei along the stability valley,
calculations estimate similar
contributions from v_{pair} and v_{ind}

Open problem

(cf. A. Idini et al., nucl-th/1404.7365)

G. Potel (Livermore)

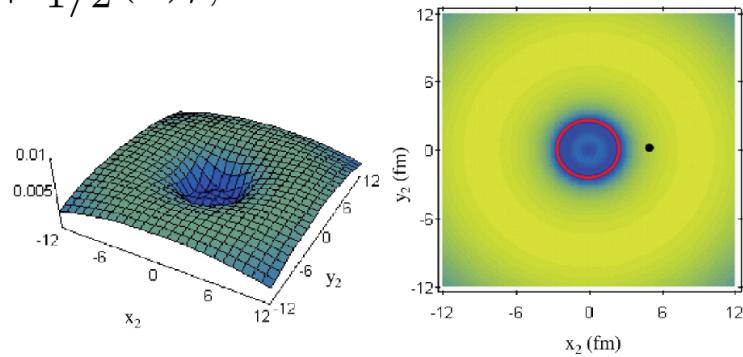
A. Idini (Darmstadt)

F. Barranco (Sevilla)

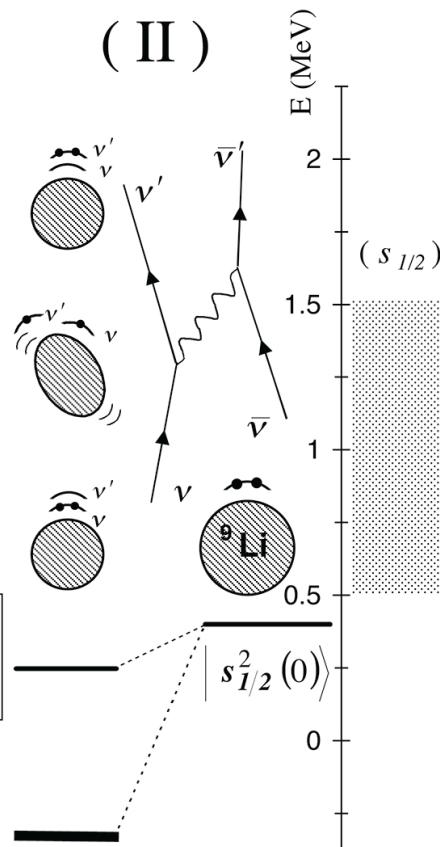
E. Vigezzi (Milano)

a)

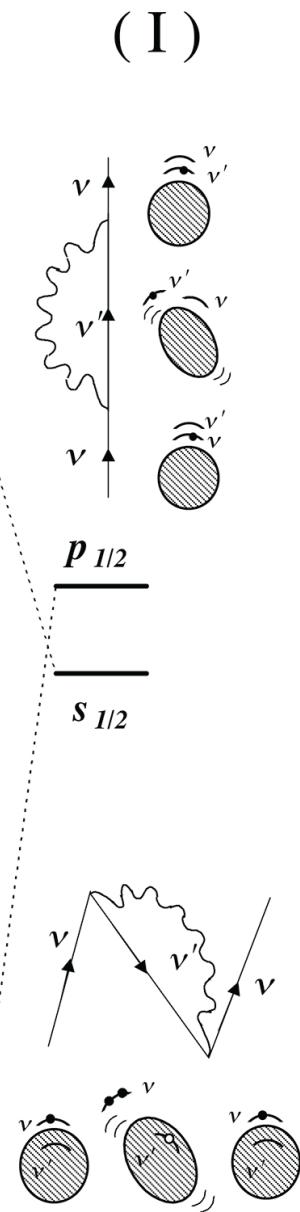
$$|s_{1/2}^2(0)\rangle, r_1 = 5 \text{ fm}$$



(II)



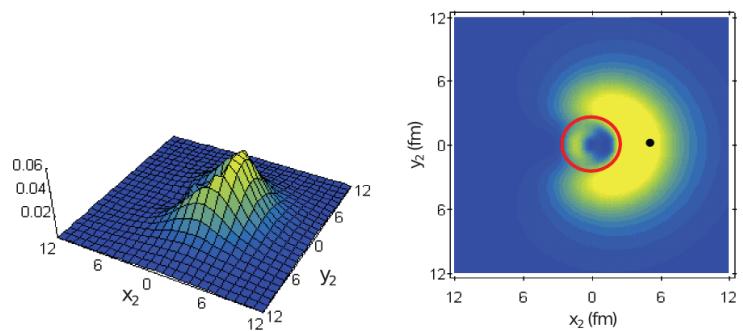
(I)



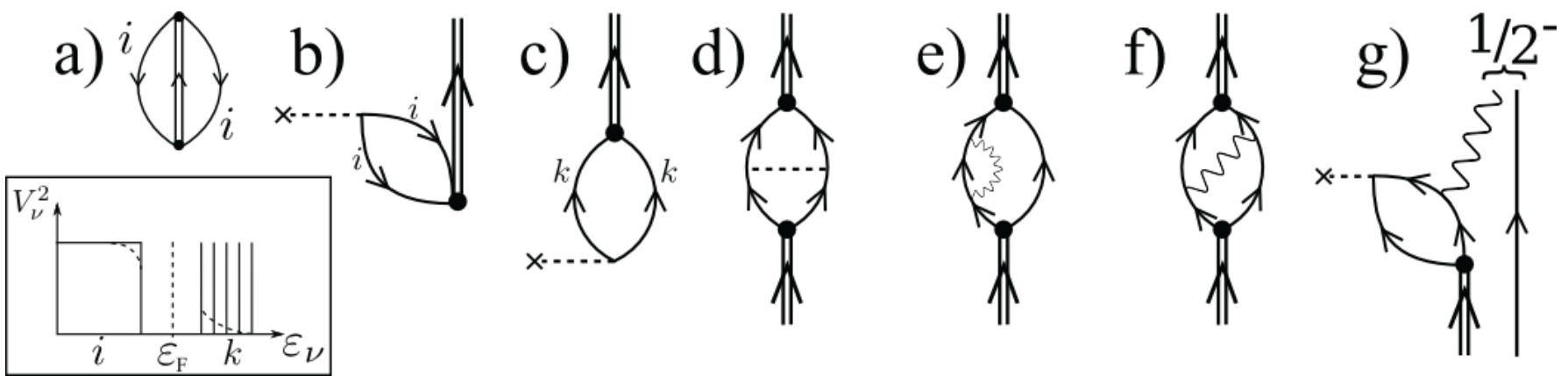
b)

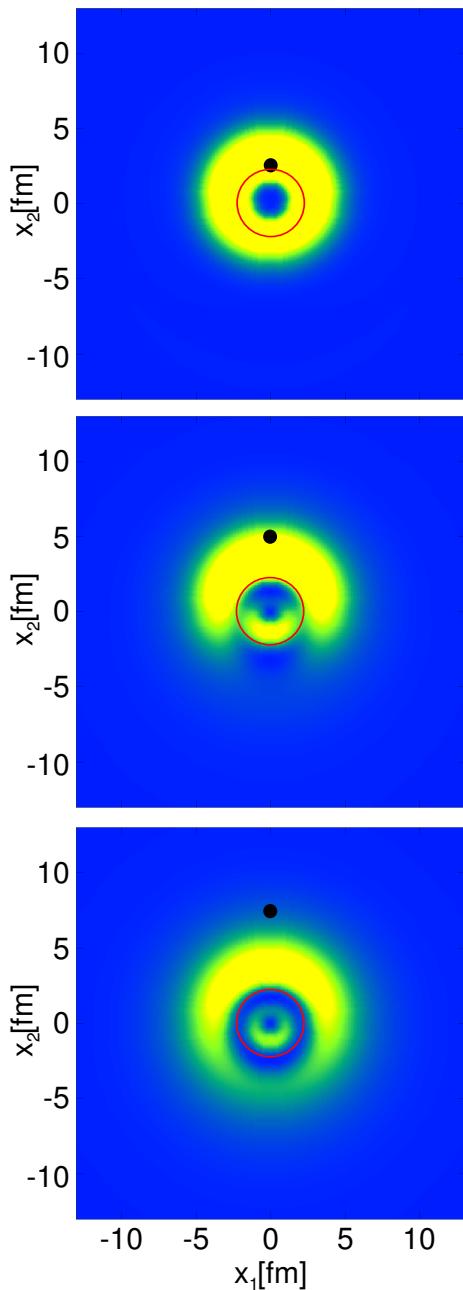
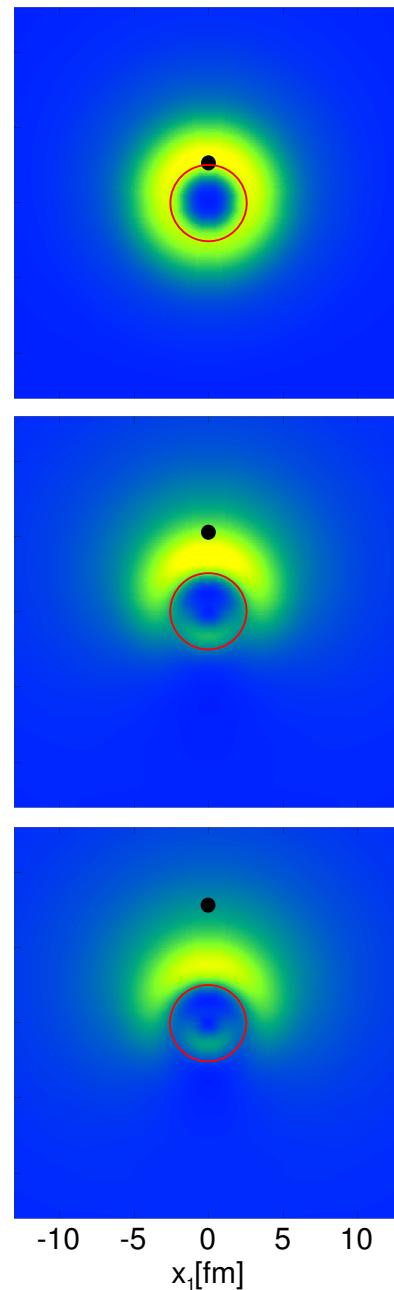
$$\begin{aligned} |\tilde{0}\rangle = & |0\rangle + 0.7 \left| (p_{1/2}, s_{1/2})_{1^-} \otimes 1^-; 0 \right\rangle \\ & + 0.1 \left| (s_{1/2}, d_{5/2})_{2^+} \otimes 2^+; 0 \right\rangle \end{aligned}$$

$$r_1 = 5 \text{ fm}$$



$$\begin{aligned} |0\rangle = & 0.45 |s_{1/2}^2(0)\rangle \\ & + 0.55 |p_{1/2}^2(0)\rangle \\ & + 0.04 |d_{5/2}^2(0)\rangle \end{aligned}$$



$^{12}\text{Be}(\text{gs})$  $^{11}\text{Li}(\text{gs})$  $^{12}\text{Be}(\text{exc})$ 