

Algebra: Chapter 0 Exercises

Chapter 1, Section 4

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Problem 4.1. Composition is defined for *two* morphisms. If more than two morphisms are given, e.g.:

$$A \xrightarrow{f} B \xrightarrow{g} C \xrightarrow{h} D \xrightarrow{i} E$$

then one may compose them in several ways, for example:

$$(ih)(gf), \quad (i(hg))f, \quad i((hg)f), \quad \text{etc.}$$

so that at every step one is only composing two morphisms. Prove that the result of any such nested composition is independent of the placement of the parentheses.

Solution. We will use induction on the number of morphisms being composed, which we will denote as n .

Base case: Suppose $n = 3$. Then, since \mathbf{C} is a category, we have $f_1(f_2f_3) = (f_1f_2)f_3$.

Induction: Suppose that all groupings of f_1, \dots, f_{n-1} are equivalent. Then, let $\alpha =$ ■