Algebra: Chapter 0 Exercises Chapter 2, Section 3

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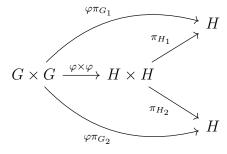
Problem 3.1. Let $\varphi: G \to H$ be a morphism in a category \mathbf{C} with products. Explain why there is a unique morphism

$$(\varphi \times \varphi) : G \times G \to H \times H$$

compatible in the evident way with the natural projections.

Solution. I'm going to assume the author means the following:

Since $H \times H$ is a product in \mathbf{C} , we have that $G \times G$ along with two "copies" of $\varphi : G \to H$ admits a unique morphism $(\varphi \times \varphi) : G \times G \to H \times H$ that makes the following diagram commute:



where π_{G_1} and π_{G_2} are the canonical projections of $G \times G$ onto G.

Problem 3.2. Let $\varphi: G \to H$ and $\psi: H \to K$ be morphisms in a category with products, and consider morphisms between the products $G \times G$, $H \times H$, and $K \times K$ as in Exercise 3.1. Prove that

$$(\psi\varphi)\times(\psi\varphi)=(\psi\times\psi)(\varphi\times\varphi)$$

Solution. To demonstrate this result, first stare at this diagram, mapping $G \times G$ to the product $H \times H$ and $H \times H$ to the product $K \times H$.

Compare it to the following diagram, mapping $G \times G$ straight to $K \times K$:

$$G \xrightarrow{\psi\varphi} K$$

$$\uparrow \qquad \uparrow$$

$$G \times G \xrightarrow{(\psi\varphi)\times(\psi\varphi)} K \times K$$

$$\downarrow \qquad \downarrow$$

$$G \xrightarrow{\psi\varphi} K$$

Notice that we have morphisms $(\psi\varphi) \times (\psi\sigma)$ and $(\psi \times \psi)(\varphi \times \varphi)$ from $G \times G$ to $K \times K$ determined by the universal property for products with regards to $K \times K$. These must be equal per Problem 3.1.

Problem 3.3. Show that if G and H are both *abelian* groups, then $G \times H$ satisfies the universal property for coproducts in \mathbf{Ab} .

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