

# Linear Algebra Done Right Exercises

## Chapter 5, Section A

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**Problem 13.** Suppose  $V$  is finite-dimensional,  $T \in \mathcal{L}(V)$ , and  $\varphi \in \mathbb{F}$ . Prove that there exists  $\alpha \in \mathbb{F}$  such that  $|\varphi - \lambda| < \frac{1}{1000}$  and  $T - \alpha I$  is invertible.

*Solution.* Let  $X$  be the set of  $\alpha \in \mathbb{F}$  such that  $|\varphi - \lambda| < \frac{1}{1000}$ , and let  $E$  be the set of eigenvalues of  $T$ . We know that  $T - \alpha I$  being invertible is equivalent to  $\alpha$  being an eigenvalue of  $T$ , so we just have to prove that there is an  $\alpha \in X$  that is not in  $E$ ; that is,  $X \not\subseteq E$ . This is clear due to the fact that  $|E| < |X|$ , since  $E$  is finite as a result of  $V$  being finite-dimensional. ■