

Algebra: Chapter 0 Exercises

Chapter 1, Section 4

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Problem 4.1. Composition is defined for *two* morphisms. If more than two morphisms are given, e.g.:

$$A \xrightarrow{f} B \xrightarrow{g} C \xrightarrow{h} D \xrightarrow{i} E$$

then one may compose them in several ways, for example:

$$(ih)(gf), \quad (i(hg))f, \quad i((hg)f), \quad \text{etc.}$$

so that at every step one is only composing two morphisms. Prove that the result of any such nested composition is independent of the placement of the parentheses.

Solution. Let $Z_m \in \text{Obj}(C)$ and $f_m \in \text{Hom}(Z_m, Z_{m+1})$ for every $m \in \mathbb{N}$. Let n be the number of morphisms we're composing. We will use induction on n .

Base case: Suppose $n = 3$. Then, since C is a category, we have $f_1(f_2f_3) = (f_1f_2)f_3$.

Induction: Suppose that all parenthesizations of f_1, \dots, f_{n-1} under composition are equivalent. Then, let $\alpha =$ ■