Indian Institute of Technology, Indore

Department of Astronomy, Astrophysics and Space Engineering (DAASE)
AA 608N - Astrostatistics

Assignment-2

Prepared by: Hemanth Potluri
Roll No: 2203121011
Date: March 13, 2023

Markov chain is the representation of a sequence of possible events in which the probability of each event depends upon the state attained in the previous event.

$$P_{ij} = P(X_{n+1} = j | X_n = i)$$

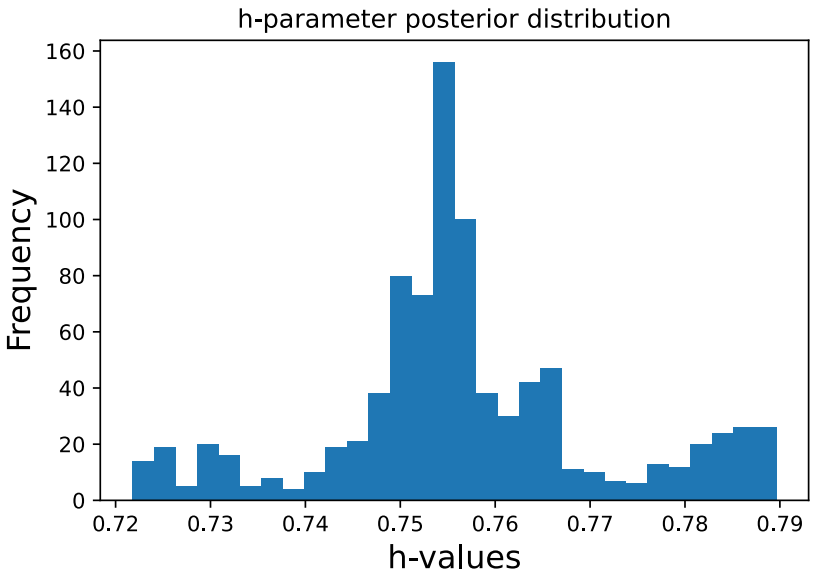
where
P_{ij}= Transition probability of the system from state i to j
X(random variable)=State of the system

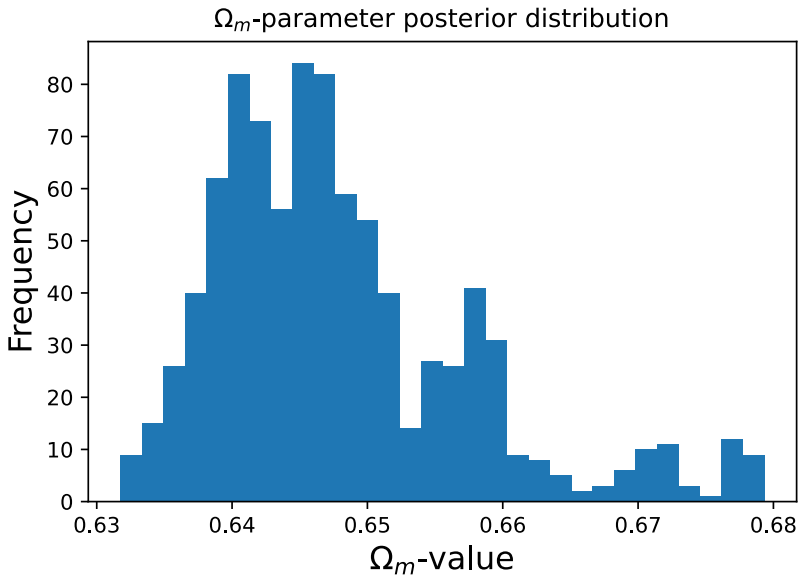
The conditionality on knowing our current state, having more information about the past state variable, does not change the transition probability to the next stage.

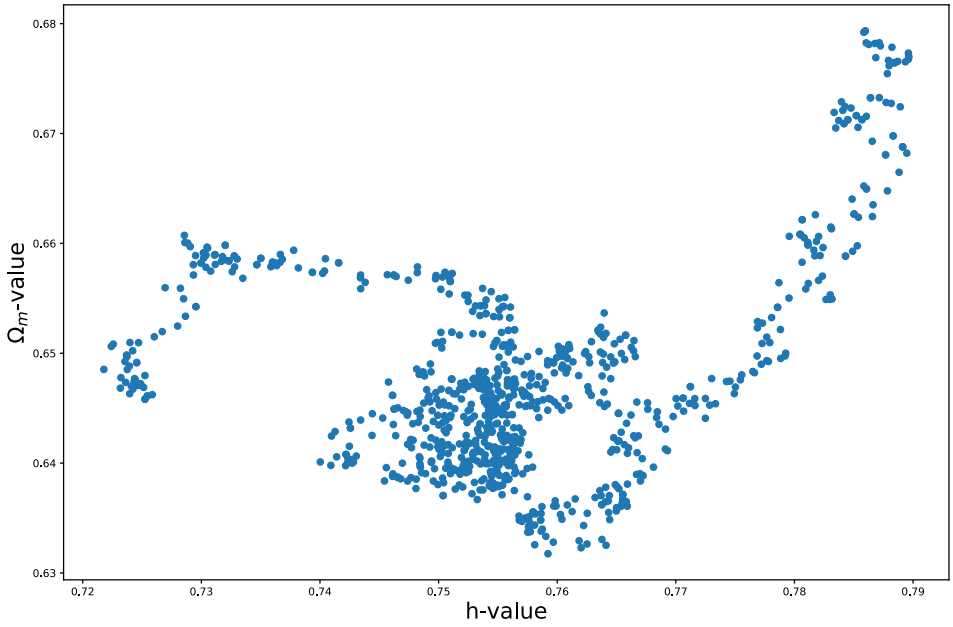
Once we assure that the information contained in the description of our state captures all the relevant information, we can make predictions about the future evolution.

From the given likelihood we have to estimate the values of h and Ω_m
We assume a Gaussian proposal distribution, where the new point is selected from a Gaussian distribution centered around the old point.
The starting of Markov chain for parameter h and Ω_m is taken as 0.8 and 0.7 respectively.

Markov chain with very small proposal distribution:

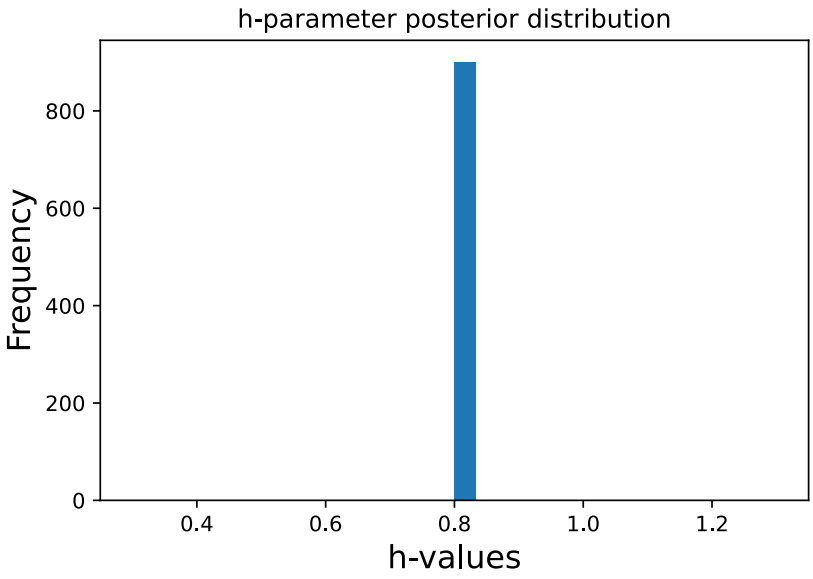


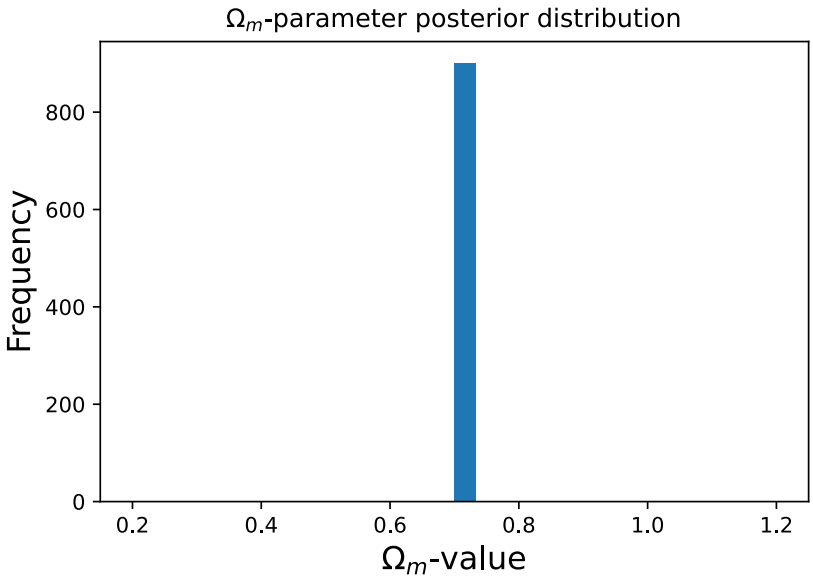

Ω_m-parameter posterior distribution
Frequency
0
10
20
30
40
50
60
70
80
0.63
0.64
0.65
0.66
0.67
0.68
Ω_m-value

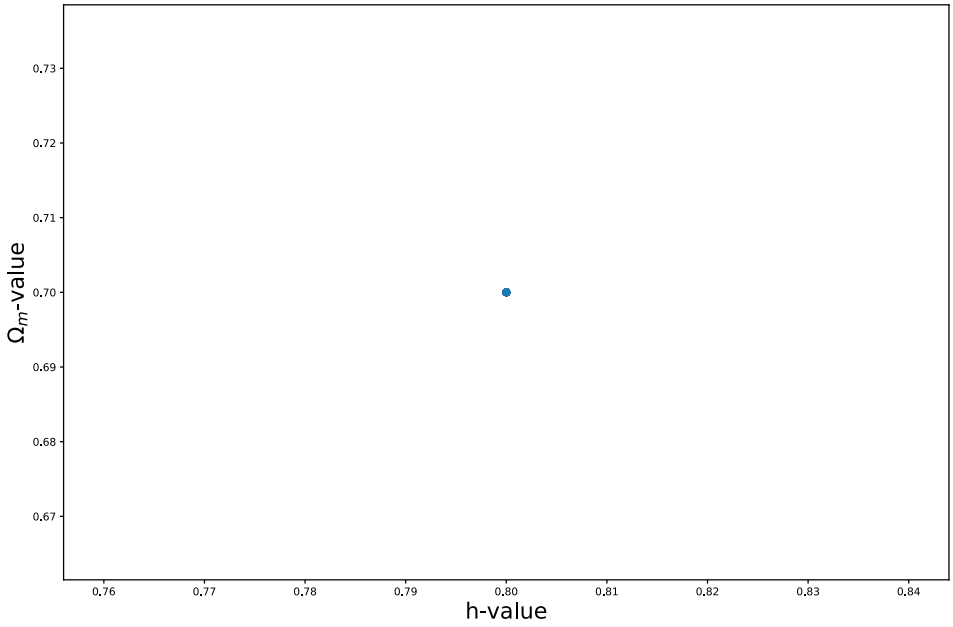


With a small proposal distribution almost all the trial points are accepted and confined into a limited space of target distribution. We need a large number of sample points to cover the entire target distribution.

Markov chain with very large proposal distribution:

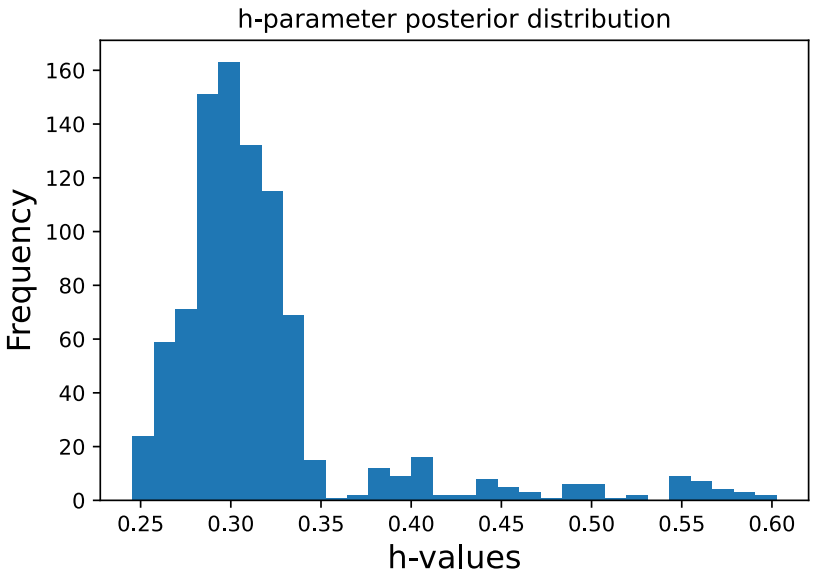


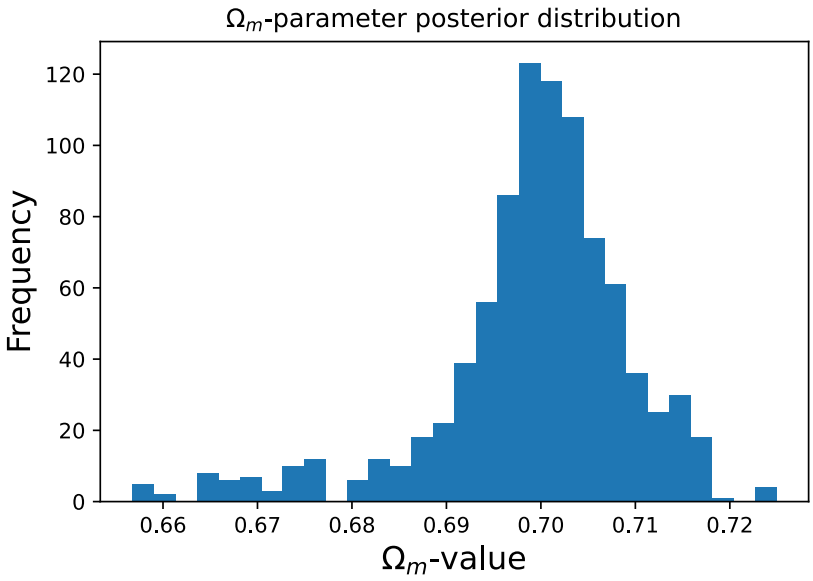

Ω_m-parameter posterior distribution
Frequency
0
200
400
600
800
0.2
0.4
0.6
0.8
1.0
1.2
Ω_m-value

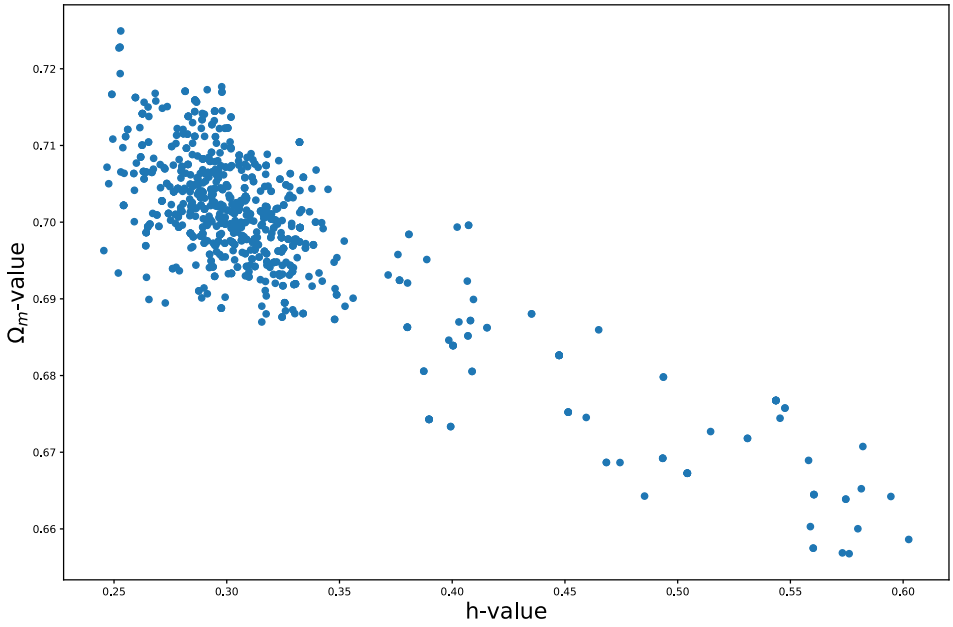


With a large proposal distribution the trail points fails to sample the target distribution.

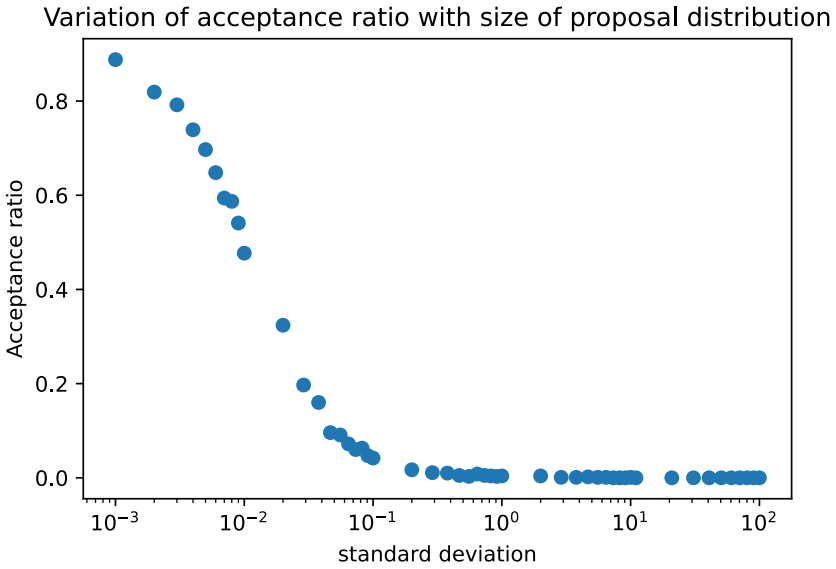
Markov chain with very reasonable proposal distribution:




Ω_m-parameter posterior distribution
Frequency
0
20
40
60
80
100
120
0.66
0.67
0.68
0.69
0.70
0.71
0.72
Ω_m-value


0.72
0.71
0.70
0.69
0.68
0.67
0.66
Ω_m-value
0.25
0.30
0.35
0.40
0.45
0.50
0.55
0.60
h-value

Change in acceptance probability with change in proposal distribution:



For a reasonable proposal distribution(standard deviation of gaussian is 0.01):
mean of the parameter 'h' under the posterior distribution: 0.3538617473236557
mean of the parameter 'Ω_m' under the posterior distribution: 0.6947788286635479
variance of the parameter 'h' under the posterior distribution: 0.015232767971123468
variance of the parameter 'Ω_m' under the posterior distribution: 0.00029234593152191625