Determination of unit normal vectors of aspherical surfaces given unit directional vectors of incoming and outgoing rays

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Received October 4, 2011; revised November 9, 2011; accepted November 9, 2011; posted November 14, 2011 (Doc. ID 155984); published January 17, 2012

Snell's law describes the relationship between the incidence angle and reflection (or refraction) angle of a light ray impinging on the interface between two different isotropic media. In this paper, Snell's law is used to derive the unit normal vectors of an aspherical surface given a knowledge of the unit directional vectors of the incoming and outgoing rays. The proposed method has important applications in the design and fabrication of aspherical surfaces since the surface normal vectors determine not only the optical performance of the surface but also the cutting tool angles required to machine the surfaces.

OCIS codes: 080.1753, 080.2720, 080.2740, 080.3620, 080.4225, 080.4228.

1. INTRODUCTION

In the geometrical optics field, an aspherical surface is defined as a surface whose profile is not a portion of a sphere, plane, or cylinder. Its complex surface profiles yield fewer optical aberrations than elements with spherical and/or flat surfaces. Furthermore, a single aspherical element can often replace a much more complex multielement system, thereby reducing the size and weight of the device and potentially lowering the cost. As a result, reflective aspherical surfaces [1] and aspherical lenses [2] are commonly used to realize compact beam-shaping devices with uniform irradiance. Braat and Greve [3] designed a system with two aspherical surfaces using the Wasserman–Wolf method [4]. Zhao and Burge [5] developed pupil astigmatism criteria for correcting quadratic field-dependent aberrations and demonstrated their use in the design of two optical systems with three and four aspherical surfaces, respectively. In designing their aspherical surfaces, it was assumed that the slope of the current segment was aligned along the tangent of the preceding segment {line 13 after Eq. (7) of [5]}. In practice, however, this assumption does not always hold, and thus the actual optical performance of the system is therefore lower than the predicted performance. However, to the best of our knowledge, the literature lacks a method for determining the actual unit normal vectors of an aspherical surface such that a more accurate prediction of the optical performance can be obtained.

The fabrication of aspherical surfaces requires the use of high-precision manufacturing techniques in order to achieve the necessary surface accuracy and smoothness. Small glass or plastic aspherical lenses are generally fabricated using precision molding techniques [6]. By contrast, large aspherical lenses are typically produced using grinding and polishing techniques. Single-point diamond turning [7] provides an alternative means of fabricating large aspherical surfaces and typically results in a better metallurgical structure than that produced by polishing and lapping. However, in using such a method, precise tool angle settings must be determined in advance in order to obtain the desired surface profile [8,9]. The setting angles are determined by both the tool geometry and the normal vectors of the aspherical surface. Thus, a knowledge of the normal vectors of an aspherical surface is important not only in predicting the optical performance of the surface (as described above) but also in formulating the numerical code required to machine the surface during the fabrication process. Accordingly, the present study proposes a method for deriving explicit expressions for the unit normal vectors of an aspherical surface given a knowledge of the unit directional vectors of the incoming and outgoing rays. In accordance with the homogeneous coordinate notation adopted in this study, the ith incidence point and its unit directional vector are written as column matrices $\bar{\mathbf{P}}_i = [P_{ix} \quad P_{iy} \quad P_{iz} \quad 1]^T$ and $\bar{\ell}_i = [\ell_{ix} \quad \ell_{iy} \quad \ell_{iz} \quad 0]^T$, respectively. The validity of the proposed approach is demonstrated by analyzing the spot diagrams of an aspherical mirror and an aspherical lens, respectively.

2. UNIT NORMAL VECTOR AT INCIDENCE POINT ON REFRACTIVE SURFACE

The most notable benefit of aspherical refractive surfaces is their ability to correct spherical aberration [5]. As shown in Fig. 1, the design of a refractive aspherical surface requires a knowledge of the incidence angle θ_i and refraction angle $\underline{\theta}_i$ corresponding to the unit directional vectors of the incoming and outgoing rays, i.e., $\bar{\ell}_{i-1}$ and $\bar{\ell}_i$, respectively. In accordance with Snell's law, the refraction phenomenon at the interface between two different isotropic media can be modeled as follows:

$$\bar{\ell}_{i-1} \cdot \bar{\ell}_i = C(\theta_i - \underline{\theta}_i) = C\theta_i C\underline{\theta}_i + S\theta_i S\underline{\theta}_i, \qquad (1)$$

$$S\underline{\theta}_i = (\xi_{i-1}/\xi_i)S\theta_i = N_i S\theta_i, \tag{2}$$

where S and C denote sine and cosine, respectively. ξ_i is the refractive index of medium i, and $N_i = \xi_{i-1}/\xi_i \neq 1$ is the refractive index of medium $i-1$ relative to that of medium i. In geometrical optics, the incidence angle θ_i and refraction angle $\underline{\theta}_i$ are both confined to the range of 0°–90°. Therefore, one can have the following four equations from Eqs. (1) and (2) since the trigonometric functions of θ_i and $\underline{\theta}_i$ are positive:

$$S\theta_i = \frac{\sqrt{1-(\bar{\ell}_{i-1}\cdot\bar{\ell}_i)^2}}{\sqrt{N_i^2+1-2N_i(\bar{\ell}_{i-1}\cdot\bar{\ell}_i)}}, \tag{3a}$$

$$C\theta_i = \frac{|N_i-(\bar{\ell}_{i-1}\cdot\bar{\ell}_i)|}{\sqrt{N_i^2+1-2N_i(\bar{\ell}_{i-1}\cdot\bar{\ell}_i)}}, \tag{3b}$$

$$S\underline{\theta}_i = \frac{N_i\sqrt{1-(\bar{\ell}_{i-1}\cdot\bar{\ell}_i)^2}}{\sqrt{N_i^2+1-2N_i(\bar{\ell}_{i-1}\cdot\bar{\ell}_i)}}, \tag{3c}$$

$$C\underline{\theta}_i = \frac{|1-N_i(\bar{\ell}_{i-1}\cdot\bar{\ell}_i)|}{\sqrt{N_i^2+1-2N_i(\bar{\ell}_{i-1}\cdot\bar{\ell}_i)}}. \tag{3d}$$

Referring to Fig. 1, to determine the unit normal vector $\bar{\mathbf{n}}_i$ at any incidence point on a refractive surface, it is first necessary to compute the unit common normal $\bar{m}_i$ of the unit directional vectors $\bar{\ell}_{i-1}$ and $\bar{\ell}_i$ of the incoming and outgoing rays, respectively. Given the assumption $\theta_i \neq \underline{\theta}_i$, $\bar{m}_i$ can be obtained as

$$\bar{m}_i = [m_{ix} \quad m_{iy} \quad m_{iz} \quad 0]^T = \frac{\bar{\ell}_{i-1}\times\bar{\ell}_i}{S(\theta_i-\underline{\theta}_i)}. \tag{4}$$

To simplify the expression of the unit normal vector $\bar{n}_i$, it is useful to have Eq. (5) obtained by taking the post cross product of Eq. (4) by $S(\theta_i-\underline{\theta}_i)\bar{\ell}_{i-1}$, i.e.,

$$\begin{aligned} S(\theta_i-\underline{\theta}_i)(\bar{m}_i\times\bar{\ell}_{i-1}) &= (\bar{\ell}_{i-1}\times\bar{\ell}_i)\times\bar{\ell}_{i-1} \\ &= \bar{\ell}_i(\bar{\ell}_{i-1}\cdot\bar{\ell}_{i-1}) - \bar{\ell}_{i-1}(\bar{\ell}_i\cdot\bar{\ell}_{i-1}) \\ &= \bar{\ell}_i - \bar{\ell}_{i-1}C(\theta_i-\underline{\theta}_i). \end{aligned} \tag{5}$$

As shown in Fig. 1, the unit normal vector $\bar{\mathbf{n}}_i$ can be obtained by rotating $-\bar{\ell}_{i-1}$ about $\bar{m}_i$ by an angle θ_i {Eq. (1.70) of [10]}. This leads to

$$\bar{n}_i = \begin{bmatrix} n_{ix} \\ n_{iy} \\ n_{iz} \\ 0 \end{bmatrix} = \begin{bmatrix} m_{ix}^2(1-C\theta_i)+C\theta_i & m_{iy}m_{ix}(1-C\theta_i)-m_{iz}S\theta_i & m_{iz}m_{ix}(1-C\theta_i)+m_{iy}S\theta_i & 0 \\ m_{ix}m_{iy}(1-C\theta_i)+m_{iz}S\theta_i & m_{iy}^2(1-C\theta_i)+C\theta_i & m_{iz}m_{iy}(1-C\theta_i)-m_{ix}S\theta_i & 0 \\ m_{ix}m_{iz}(1-C\theta_i)-m_{iy}S\theta_i & m_{iy}m_{iz}(1-C\theta_i)+m_{ix}S\theta_i & m_{iz}^2(1-C\theta_i)+C\theta_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\ell_{i-1x} \\ -\ell_{i-1y} \\ -\ell_{i-1z} \\ 0 \end{bmatrix}. \tag{6}$$

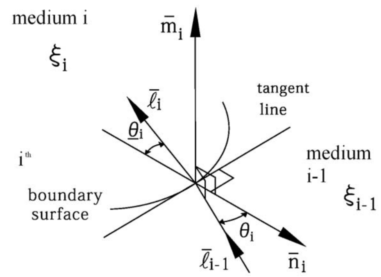

Fig. 1. Schematic illustration showing that unit normal vector $\bar{\mathbf{n}}_i$ of refractive surface at incidence point $\bar{\mathbf{P}}_i$ can be determined by rotating unit directional vector $-\bar{\ell}_{i-1}$ about $\bar{m}_i$ by an angle θ_i.

Utilizing Eq. (5), Eq. (6) can be simplified as follows:

$$\begin{aligned} \bar{n}_i &= \begin{bmatrix} n_{ix} \\ n_{iy} \\ n_{iz} \\ 0 \end{bmatrix} \\ &= \left[\frac{C(\theta_i-\underline{\theta}_i)S\theta_i}{S(\theta_i-\underline{\theta}_i)} - C\theta_i\right]\begin{bmatrix} \ell_{i-1x} \\ \ell_{i-1y} \\ \ell_{i-1z} \\ 0 \end{bmatrix} - \left[\frac{S\theta_i}{S(\theta_i-\underline{\theta}_i)}\right]\begin{bmatrix} \ell_{ix} \\ \ell_{iy} \\ \ell_{iz} \\ 0 \end{bmatrix} \\ &= \left[\frac{(C\theta_i C\underline{\theta}_i + S\theta_i S\underline{\theta}_i)S\theta_i}{S\theta_i C\underline{\theta}_i - C\theta_i S\underline{\theta}_i} - C\theta_i\right]\begin{bmatrix} \ell_{i-1x} \\ \ell_{i-1y} \\ \ell_{i-1z} \\ 0 \end{bmatrix} \\ &\quad - \left[\frac{S\theta_i}{S\theta_i C\underline{\theta}_i - C\theta_i S\underline{\theta}_i}\right]\begin{bmatrix} \ell_{ix} \\ \ell_{iy} \\ \ell_{iz} \\ 0 \end{bmatrix}, \end{aligned} \tag{7}$$

where $S\theta_i$, $C\theta_i$, $S\underline{\theta}_i$, and $C\underline{\theta}_i$ are given in Eqs. (3a), (3b), (3c), and (3d), respectively.

3. UNIT NORMAL VECTOR AT INCIDENCE POINT ON REFLECTIVE SURFACE

Referring to Fig. 2, to determine the unit normal vector $\bar{n}_i$ at any incidence point on a reflective surface, it is first necessary to calculate the common normal vector $\bar{m}_i$ of the unit directional vectors of the incoming and outgoing rays, i.e., $\bar{\ell}_{i-1}$ and $\bar{\ell}_i$, respectively. Given the assumption $\theta_i \neq 0$, $\bar{m}_i$ can be obtained as

$$\bar{m}_i = [m_{ix} \quad m_{iy} \quad m_{iz} \quad 0]^T = \frac{\bar{\ell}_i\times\bar{\ell}_{i-1}}{S(2\theta_i)}, \tag{8}$$

where the incidence angle θ_i is determined by

$$C(2\theta_i) = -\bar{\ell}_{i-1}\cdot\bar{\ell}_i. \tag{9}$$

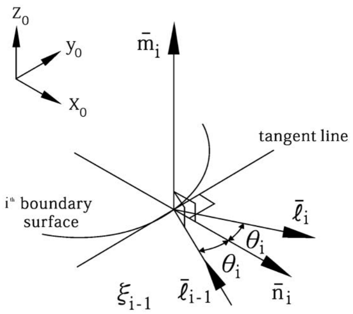

Fig. 2. Schematic illustration showing that unit normal vector $\bar{\mathbf{n}}_i$ of a reflective surface at incidence point $\bar{\mathbf{P}}_i$ can be determined by rotating unit directional vector $-\bar{\ell}_{i-1}$ about $\bar{m}_i$ by an angle θ_i.

Note that Eq. (9) is also applicable when the incidence angle is equal to zero.

To simplify the expression of the unit normal vector $\bar{n}_i$, it is useful to have the following equation obtained by taking the post cross product of Eq. (8) by $S(2\theta_i)\bar{\ell}_{i-1}$, i.e.,

$$\begin{aligned} S(2\theta_i)(\bar{m}_i \times \bar{\ell}_{i-1}) &= (\bar{\ell}_i \times \bar{\ell}_{i-1}) \times \bar{\ell}_{i-1} \\ &= \bar{\ell}_{i-1}(\bar{\ell}_i \cdot \bar{\ell}_{i-1}) - \bar{\ell}_i(\bar{\ell}_{i-1} \cdot \bar{\ell}_{i-1}) \\ &= -C(2\theta_i)\bar{\ell}_{i-1} - \bar{\ell}_i. \end{aligned} \tag{10}$$

As shown in Fig. 2, the unit normal vector $\bar{n}_i$ can be determined by rotating $-\bar{\ell}_{i-1}$ about $\bar{m}_i$ by an angle θ_i [see Eq. (6)]. Using Eq. (10) to simplify Eq. (6), the following expression is obtained for the unit normal vector $\bar{n}_i$ at any incidence point on the reflective surface when $\bar{\ell}_{i-1}$ and $\bar{\ell}_i$ are given:

$$\begin{aligned} \bar{n}_i = \begin{bmatrix} n_{ix} \\ n_{iy} \\ n_{iz} \\ 0 \end{bmatrix} &= \frac{1}{2C\theta_i}\left(\begin{bmatrix} \ell_{ix} \\ \ell_{iy} \\ \ell_{iz} \\ 0 \end{bmatrix} - \begin{bmatrix} \ell_{i-1x} \\ \ell_{i-1y} \\ \ell_{i-1z} \\ 0 \end{bmatrix} \right) \\ &= \frac{1}{\sqrt{2(1 - \bar{\ell}_{i-1} \cdot \bar{\ell}_i)}}\left(\begin{bmatrix} \ell_{ix} \\ \ell_{iy} \\ \ell_{iz} \\ 0 \end{bmatrix} - \begin{bmatrix} \ell_{i-1x} \\ \ell_{i-1y} \\ \ell_{i-1z} \\ 0 \end{bmatrix} \right). \end{aligned} \tag{11}$$

It should be noted that Eqs. (7) and (11) are applicable to all three-dimensional optical systems. One has to note that a knowledge of the unit tangential vector $\bar{\boldsymbol{\varepsilon}}_i$ is required in order to smooth the designed aspherical surface. When the surface is designed only in the $y_0 z_0$ plane, the first component of $\bar{\mathbf{n}}_i$ (i.e., $\mathbf{n}_{ix}$) vanishes. Thus, the unit tangential vector $\bar{\boldsymbol{\varepsilon}}_i$ of the designed generation curve is given by

$$\bar{\boldsymbol{\varepsilon}}_i = [0 \quad \varepsilon_{iy} \quad \varepsilon_{iz} \quad 0]^T = \pm[0 \quad n_{iz} \quad -n_{iy} \quad 0]^T. \tag{12}$$

4. ILLUSTRATIVE EXAMPLES

The purpose of this section is to design an aspherical mirror and an aspherical lens to validate the proposed computation scheme by evaluating their spot diagrams.

Example A. Consider the concave aspherical mirror shown in Fig. 3, designed to correct spherical aberration in accordance with Fermat's principle. Assume that the geometrical distances along the optical axis of the mirror are as follows: $\lambda_{1/\mathrm{axis}} = 400$ mm, $\lambda_{2/\mathrm{axis}} = 100$ mm, and $\lambda_{3/\mathrm{axis}} = 50$ mm. Assume further that a spherical wavefront originating from an axial source point $\bar{\mathbf{P}}_0 = [0 \quad P_{0y} \quad P_{0z} \quad 1]^T = [0 \quad -\lambda_{1/\mathrm{axis}} \quad 0 \quad 1]^T$ is incident on the reflective surface with a unit direction vector of

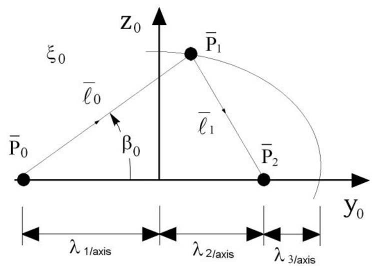

Fig. 3. Use of aspherical mirror to correct all spherical aberration of any order by means of Fermat's principle.

$$\bar{\ell}_0 = \begin{bmatrix} 0 \\ \ell_{0y} \\ \ell_{0z} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ C\beta_0 \\ S\beta_0 \\ 0 \end{bmatrix}, \tag{13}$$

where β_0 is the spherical angle measured from the y_0 axis. The unit directional vector $\bar{\ell}_1$ of the reflected ray is given by

$$\begin{aligned} \bar{\ell}_1 = \begin{bmatrix} 0 \\ \ell_{1y} \\ \ell_{1z} \\ 0 \end{bmatrix} &= \frac{1}{\sqrt{(P_{2y} - P_{1y})^2 + (P_{2z} - P_{1z})^2}} \begin{bmatrix} 0 \\ P_{2y} - P_{1y} \\ P_{2z} - P_{1z} \\ 0 \end{bmatrix} \\ &= \frac{1}{\sqrt{(\lambda_{2/\mathrm{axis}} + \lambda_{1/\mathrm{axis}} - C\beta_0\lambda_1)^2 + (S\beta_0\lambda_1)^2}} \\ &\quad \times \begin{bmatrix} 0 \\ \lambda_{2/\mathrm{axis}} + \lambda_{1/\mathrm{axis}} - C\beta_0\lambda_1 \\ -S\beta_0\lambda_1 \\ 0 \end{bmatrix}, \end{aligned} \tag{14}$$

where $\bar{P}_1 = [0 \quad P_{1y} \quad P_{1z} \quad 1]^T = \bar{P}_0 + \bar{\ell}_0\lambda_1 = [0 \quad -\lambda_{1/\mathrm{axis}} + C\beta_0\lambda_1 S\beta_0\lambda_1 1]^T$, and $\bar{P}_2 = [0 \quad P_{2y} \quad P_{2z} \quad 1]^T = [0 \quad \lambda_{2/\mathrm{axis}} \quad 0 \quad 1]^T$..

If the aspherical mirror shown in Fig. 3 is to correct spherical aberration of any order, Fermat's principle states that the optical path length must be constant for any ray originating from source point $\bar{P}_0$ and reflected to image point $\bar{P}_2$, i.e.,

$$\begin{aligned} \mathrm{OPL}_{\mathrm{system}} &= \xi_0\lambda_1 + \xi_0\lambda_2 \\ &= \xi_0\lambda_1 + \xi_0\sqrt{(\lambda_{2/\mathrm{axis}} + \lambda_{1/\mathrm{axis}} - C\beta_0\lambda_1)^2 + (S\beta_0\lambda_1)^2} \\ &= \xi_0(\lambda_{1/\mathrm{axis}} + \lambda_{2/\mathrm{axis}} + 2\lambda_{3/\mathrm{axis}}) = \text{constant}, \end{aligned} \tag{15}$$

where λ_1 and λ_2 are the geometrical distances from $\bar{P}_0$ to $\bar{P}_1$ and from $\bar{P}_1$ to $\bar{P}_2$, respectively. In Eq. (15), there exists one unknown (λ_1), which must be determined for each ray $[\bar{\mathbf{P}}_0 \quad \bar{\ell}_0]^T$. It is well known that the solution of this design is one part of the following elliptic curve:

$$\frac{(y_0 + 150)^2}{(300)^2} + \frac{z_0^2}{\left(50\sqrt{11}\right)^2} = 1. \tag{16}$$

The validity of the method proposed in this study for computing the unit normal vectors of the aspherical surface shown in Fig. 3 can be verified by comparing the spot position and spot

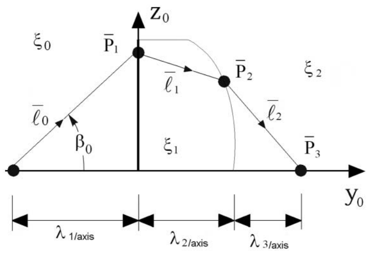

Fig. 4. Use of plano-aspherical lens to correct all spherical aberration of any order by means of Fermat's principle.

size characteristics of the resulting spot diagram with those obtained when designing the surface using the method proposed by Zhao and Burge [5]. The spot position is given by the centroid $[x_{\text{centroid}} \quad y_{\text{centroid}} \quad z_{\text{centroid}} \quad 1]^T$, while the spot size is measured by the root-mean-square (rms) radius of the rays in the spot from the centroid. Our numerical results show that any ray leaving one focus point $\bar{\mathbf{P}}_0 = [0 \quad -\lambda_{1/\text{axis}} \quad 0 \quad 1]^T = [0 \quad -400 \quad 0 \quad 1]^T$ of this elliptic mirror always passes through the second focus point, i.e., $\bar{\mathbf{P}}_2 = [0 \quad \lambda_{2/\text{axis}} \quad 0 \quad 1]^T = [0 \quad 100 \quad 0 \quad 1]^T$. Given the assumption in [5] that the slope of the current segment lies along the tangent of the previous segment, the centroid and spot size for the aspherical mirror shown in Fig. 3 are found via ray tracing to be $[0 \quad 100 \quad 0.000015 \quad 1]^T$ and $\text{rms}^2 = 0.00375$, respectively, when the rays are reflected by the right-half portion of the mirror and $\bar{\ell}_0$ is confined to the range of $-35.7° \le \beta_0 \le 35.7°$ with an increment of $\Delta\beta_0 = 0.5°$. It is noted that these results deviate from the proposed method, i.e., a spot centroid located at $\bar{\mathbf{P}}_2$ and a spot radius of $\text{rms}^2 = 0$. Furthermore, it is noted that the deviation of the results based on the assumption in [5] increases as the value of $\Delta\beta_0$ is increased.

Example B. Consider the plano-aspherical lens shown in Fig. 4; designed to correct spherical aberration of any order in accordance with Fermat's principle. Assume that the source ray $[\bar{\mathbf{P}}_0 \quad \bar{\ell}_0]^T = [0 \quad -\lambda_{1/\text{axis}} \quad 0 \quad 0 \quad C\beta_0 \quad S\beta_0]^T$ is incident on the flat boundary surface with an angle of $-5° \le \beta_0 \le 5°$. Assume also that the following geometrical and optical parameters apply: $\lambda_{1/\text{axis}} = 200$ mm, $\lambda_{2/\text{axis}} = 10$ mm, $\lambda_{3/\text{axis}} = 50$ mm, $\xi_0 = \xi_2 = 1$, and $\xi_1 = 1.65$. For a given source ray $[\bar{\mathbf{P}}_0 \quad \bar{\ell}_0]^T$, the refracted ray $[\bar{\mathbf{P}}_1 \quad \bar{\ell}_1]^T$ at the first boundary surface is determined by Snell's law. Furthermore, the incidence point $\bar{\mathbf{P}}_2$ of the refracted ray on the aspherical surface can be expressed as $\bar{\mathbf{P}}_2 = \bar{\mathbf{P}}_1 + \bar{\ell}_1\lambda_2 = [0 \quad P_{1y} + \ell_{1y}\lambda_2 \quad P_{1z} + \ell_{1z}\lambda_2 \quad 1]^T$, where λ_2 is the only unknown in the design. The unit directional vector $\bar{\ell}_2$ at the aspherical surface can be computed as

$$\bar{\ell}_2 = \begin{bmatrix} 0 \\ \ell_{2y} \\ \ell_{2z} \\ 0 \end{bmatrix} = \frac{1}{\sqrt{(P_{3y} - P_{2y})^2 + (P_{3z} - P_{2z})^2}} \begin{bmatrix} 0 \\ P_{3y} - P_{2y} \\ P_{3z} - P_{2z} \\ 0 \end{bmatrix}$$
$$= \frac{1}{\sqrt{[\lambda_{2/\text{axis}} + \lambda_{3/\text{axis}} - (P_{1y} + \ell_{1y}\lambda_2)]^2 + (P_{1z} + \ell_{1z}\lambda_2)^2}} \times \begin{bmatrix} 0 \\ \lambda_{2/\text{axis}} + \lambda_{3/\text{axis}} - (P_{1y} + \ell_{1y}\lambda_2) \\ -(P_{1z} + \ell_{1z}\lambda_2) \\ 0 \end{bmatrix}, \tag{17}$$

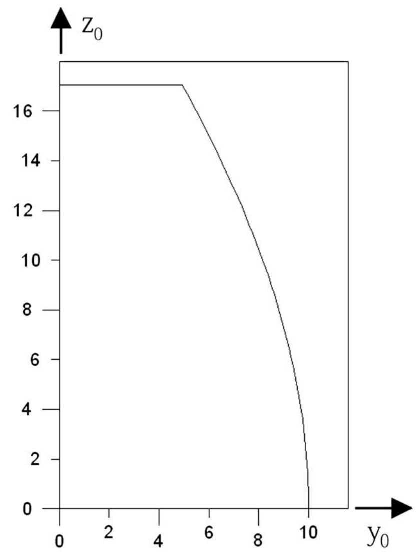

Fig. 5. Generation curve of single lens having one flat refractive surface and one aspherical refractive surface ($\lambda_{1/\text{axis}} = 200$ mm, $\lambda_{2/\text{axis}} = 10$ mm, $\lambda_{3/\text{axis}} = 50$ mm, $\xi_0 = \xi_2 = 1$, and $\xi_1 = 1.65$).

where $\bar{P}_2 = [0 \quad P_{2y} \quad P_{2z} \quad 1]^T = \bar{P}_1 + \bar{\ell}_1\lambda_2 = [0 \quad P_{1y} + \ell_{1y}\lambda_2 P_{1z} + \ell_{1z}\lambda_2 1]^T$ and $\bar{\mathbf{P}}_3 = [0 \quad P_{3y} \quad P_{3z} \quad 1]^T = [0 \quad \lambda_{2/\text{axis}} + \lambda_{3/\text{axis}} \quad 0 \quad 1]^T$.

According to Fermat's principle, the optical path length must be constant for any ray originating from the source point $\bar{\mathbf{P}}_0 = [0 \quad -\lambda_{1/\text{axis}} \quad 0 \quad 1]^T$ and arriving at image point $\bar{\mathbf{P}}_3$ if all the spherical aberration of any order are to be corrected, i.e.,

$$\begin{aligned} \text{OPL}_{\text{system}} &= \xi_0\lambda_1 + \xi_1\lambda_2 + \xi_2\lambda_3 \\ &= \xi_0\lambda_{1/\text{axis}} + \xi_1\lambda_{2/\text{axis}} + \xi_2\lambda_{3/\text{axis}}. \end{aligned} \tag{18}$$

The unknown, λ_2, can be obtained from Eq. (18) by using numerical methods. The resulting generation curve is shown in Fig. 5. Applying a ray tracing technique to evaluate the image quality of the rays originating from $\bar{\mathbf{P}}_0 = [0 \quad -200 \quad 0 \quad 1]^T$ subject to the assumption made in [5], the spot centroid and spot size are found to be $[0 \quad 60 \quad 0.00079 \quad 1]^T$ and $\text{rms}^2 = 0.000085$, respectively, for incident angles in the range of $-5° \le \beta_0 \le 5°$ with an increment of $\Delta\beta_0 = 0.5°$. In other words, the centroid and spot size deviate from the proposed method of $\bar{\mathbf{P}}_3 = [0 \quad 60 \quad 0 \quad 1]^T$ and $\text{rms}^2 = 0$, respectively.

5. CONCLUSIONS

This paper has presented a method for computing the unit normal vectors of a reflective or refractive aspherical optical surface given a knowledge of the unit directional vectors of the incoming and outgoing rays, respectively. The proposed method is useful not only in predicting the optical performance of aspherical elements but also in generating the numerical code required to machine aspherical surfaces using a single-point diamond turning technique.

ACKNOWLEDGMENTS

The authors gratefully acknowledge the financial support provided to this study by the National Science Council of Taiwan under grant NSC 99-2221-E-006-023-MY2.

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