Determination of unit normal vectors of aspherical surfaces given unit directional vectors of incoming and outgoing rays: comment

Antonín Mikš and Pavel Novák*

Czech Technical University in Prague, Faculty of Civil Engineering, Department of Physics, Thakurova 7,16629 Prague, Czech Republic
**Corresponding author: pavel.novak.3@fsv.cvut.cz*

Received February 24, 2012; revised April 13, 2012; accepted April 18, 2012;
posted May 21, 2012 (Doc. ID 163701); published June 21, 2012

In a recent paper by Lin and Tsai [J. Opt. Soc. Am. A **29**, 174 (2012)] there is presented a rather complicated method for derivation of the unit normal vectors of an aspherical surface given the knowledge of the unit directional vectors of the incoming and outgoing rays. In our comment we present a much simpler method that leads to compact equations suitable for practical implementation.

OCIS codes: 080.1753, 080.2720, 080.2740, 080.3620, 080.4225, 080.4228.

Reference [1] presents a relatively complicated method for determination of the unit normal vector to an aspherical surface at a given point in the case that we know the directional vectors of the incoming (incident) and outgoing (reflected or refracted) ray at that point. Let us show a considerably simpler and faster method for the calculation of the direction vector of the normal to the optical surface at the above-mentioned case. The investigated situation is illustrated schematically in Fig. 1 (the meaning of individual symbols is explained further). As is well known, Snell's law of refraction at the interface between two isotropic optical media is given by the equation [2]

$$n \sin \varepsilon = n' \sin \varepsilon', \tag{1}$$

where ε is the angle of incidence, ε' is the angle of refraction, n is the refractive index of the optical medium in front of the interface, and n' is the refractive index of the optical medium behind the interface. The preceding equation can be expressed in vector form. It holds that [2]

$$n'(\mathbf{s}' \times \mathbf{g}) = n(\mathbf{s} \times \mathbf{g}), \tag{2}$$

where $\mathbf{s}(s_x, s_y, s_z)$ is the unit directional vector of the incident ray, $\mathbf{s}'(s'_x, s'_y, s'_z)$ is the unit directional vector of the refracted ray, and $\mathbf{g}(g_x, g_y, g_z)$ is the unit normal vector to the interface between the two media pointing from medium 1 with refractive index n into medium 2 with refractive index n' (note that the unit normal vector as introduced in [1] has the opposite direction from vector $\mathbf{g}$), whereas the components of vectors (s_x, s_y, s_z), (s'_x, s'_y, s'_z), and (g_x, g_y, g_z) are the directional cosines of those vectors (see Fig. 1). By taking the cross product of both sides of Eq. (2) with vector $\mathbf{g}$, we obtain, after some modifications, the following expression of Snell's law of refraction:

$$\mathbf{s}' = \mu\mathbf{s} + \mathbf{g}\sqrt{1 - \mu^2[1 - (\mathbf{gs})^2]} - \mu\mathbf{g}(\mathbf{gs}), \tag{3}$$

where $\mu = n/n'$ and $(\mathbf{gs}) = g_x s_x + g_y s_y + g_z s_z$ denotes the dot (scalar) product of vectors $\mathbf{g}$ and $\mathbf{s}$. In the case of reflection it holds that [2]

$$\mathbf{s}'' = \mathbf{s} - 2\mathbf{g}(\mathbf{gs}), \tag{4}$$

where $\mathbf{s}''$ denotes the unit directional vector of the reflected ray (see Fig. 1). In order to determine the unit normal vector to the reflective surface (e.g., aspherical surface) at the incidence point assuming we know the unit directional vectors of the incident and reflected rays, we will use Eq. (4). By a simple modification we have

$$\mathbf{g} = \frac{\mathbf{s} - \mathbf{s}''}{2(\mathbf{gs})}. \tag{5}$$

If we calculate the dot products of vector $\mathbf{g}$ with both sides of Eq. (4), we can express the dot product $(\mathbf{gs})$ in the form

$$(\mathbf{gs}) = \sqrt{\frac{1 - (\mathbf{ss}'')}{2}}. \tag{6}$$

By substitution of the previous equation into Eq. (5) we obtain

$$\mathbf{g} = \frac{\mathbf{s} - \mathbf{s}''}{\sqrt{2[1 - (\mathbf{ss}'')]}}, \tag{7}$$

and the problem for the case of reflection is solved.

In order to determine the unit normal vector to the interface between the two isotropic optical media (e.g., aspherical surface) at the incidence point assuming we know the unit direction vectors of the incident ray $\mathbf{s}$ and refracted ray $\mathbf{s}'$ we will use Eq. (3). From Eq. (3) we obtain

$$\mathbf{g} = \frac{\mathbf{s}' - \mu\mathbf{s}}{\sqrt{1 - \mu^2[1 - (\mathbf{gs})^2]} - \mu(\mathbf{gs})}. \tag{8}$$

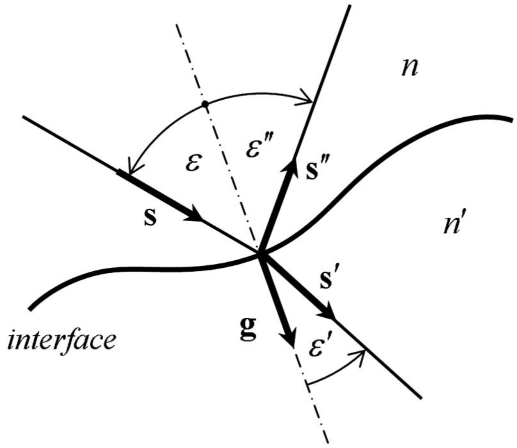

Fig. 1. Reflection and refraction at the interface between two optical media with different refractive indices.

If we calculate the dot products of vector **s** with both the left-hand side and the right-hand side of Eq. (3), we have

$$\mu(\mathbf{gs})^2 + \mathbf{ss}' - \mu = (\mathbf{gs})\sqrt{1 - \mu^2[1 - (\mathbf{gs})^2]}. \tag{9}$$

From the previous equation one can express the value of the dot product (**gs**). It holds that

$$(\mathbf{gs}) = \frac{|(\mathbf{ss}') - \mu|}{\sqrt{1 + \mu^2 - 2\mu(\mathbf{ss}')}}. \tag{10}$$

By inserting Eq. (10) into Eq. (8), we can calculate the desired unit normal vector **g** to the surface. Equations (8) and (10) represent the solution of the problem for the case of refraction.

As can be seen, our derivation of equations for calculation of the unit normal vector to the interface between the two isotropic optical media with different refractive indices is much shorter, and the derived equations are a lot simpler, than those presented in [1]. Because of their compact and simple form, Eqs. (8) and (10) for the case of refraction are more suitable for practical implementation since they have less computational costs than Eqs. (7) and (3) given in [1].

REFERENCES

1. P. D. Lin and Ch.-Y. Tsai, “Determination of unit normal vectors of aspherical surfaces given unit directional vectors of incoming and outgoing rays,” J. Opt. Soc. Am. A **29**, 174–178 (2012).
2. M. Herzberger, *Modern Geometrical Optics* (Interscience, 1958).