2D Finite Differences

November 19, 2019

1 2D Finite Differences

```
In [1]: #import external modules
        import numpy as np
        import matplotlib.pyplot as plt
        from scipy import sparse
        from scipy.special import jv, j1, iv
```

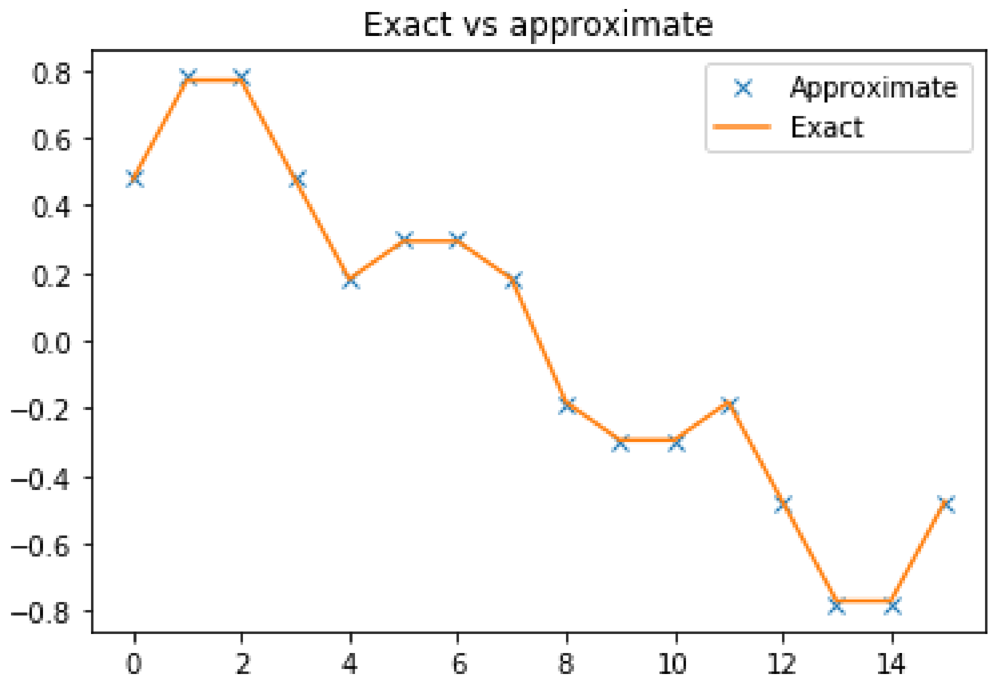
1.1 Code Deliverable

```
In [2]: def finiteDifference2D(f, ua, ub, uc, ud, N):

            h = 1/(N+1)

            #define 1D meshpoints
            x = np.linspace(0,1,N+2)
            y = np.linspace(0,1,N+2)

            x = x[1:-1]
            y = y[1:-1]

            #create 2d meshgrid from 1d x and y coords
            X, Y = np.meshgrid(x, y)

            #compute f(X,Y)
            func = f(X, Y)*(-h**2)
            func = func.flatten().T

            #bottom
            func[0:N] = func[0:N] - ua(x)
            #top
            func[(N**2 - N) : N**2] = func[(N**2 - N) : N**2] - ub(x)
            #right side
            func[np.arange(N - 1, N**2, N)] = func[np.arange(N - 1, N**2, N)] - uc
            #left side
            func[np.arange(0 , (N**2 - N + 1),  N)] = func[np.arange(0 , (N**2 - N + 1),  N)]
```

```
    #define vectors for main diagonal and off diagonals
    main_diag = 2*np.ones((N, 1)).ravel()
    off_diag = -1*np.ones((N, 1)).ravel()

    #create sparse matrix
    diagonals = [main_diag, off_diag, off_diag]
    S = sparse.diags(diagonals, [0,-1,1], shape=(N,N)).toarray()

    #create identity matrix
    I = np.identity(N)

    #create kron matrix
    L = np.kron(S, I) + np.kron(I, S)
    #solve function
    U = np.linalg.solve(L, func)

    #return approximations
    return U, X, Y
```

2 Exercise

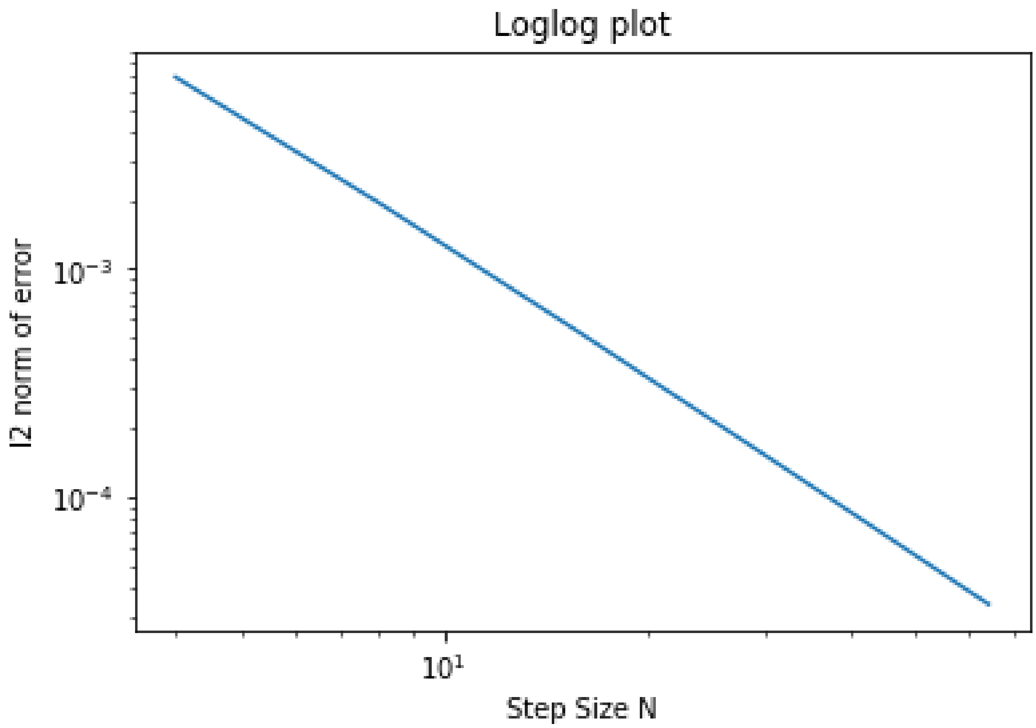
```
In [3]: #define exact and f function
        exact = lambda x, y: np.sin(np.pi*x)*np.cos(np.pi*y)
        f = lambda x, y: -(2*np.pi**2*np.sin(np.pi*x)*np.cos(np.pi*y))

        #boundary conditions
        uc = 0
        ub = lambda x: np.sin(np.pi*x)
        ua = lambda x: -(np.sin(np.pi*x))
        ud = 0

        #call 2d solver
        U, X, Y = finiteDifference2D(f, ua, ub, uc, ud, 4)

        plt.plot(U, 'x',label='Approximate')
        plt.plot(exact(X,Y).flatten().T, label='Exact')
        plt.title('Exact vs approximate')
        plt.legend()

Out[3]: <matplotlib.legend.Legend at 0x1eb22ef3470>
```




```
In [4]: def finiteDiffConvergence2D(f, ua, ub, uc, ud):

            """
            This function performs a convergence study
            """

            #array of different values of N
            N = [4, 8, 16, 32, 64]

            #initialize empty array to hold error
            x = []

            #loop through N values in Narray
            for h in N:
                #call finite different functions
                U, X, Y = finiteDifference2D(f, ua, ub, uc, ud, h)
                #find error from approximate value and exact
                E = U - exact(X,Y).flatten().T

                #calculate the l2 and linf norm
                x.append(np.linalg.norm(E, 2)/h)

            #return norm
            return x, N
```

In [5]:
```
import matplotlib.pyplot as plt
%matplotlib inline

def plotLoglog(X, NN):
    """
    This function plots a convergence study on the approximated solutions

    Inputs:
        X - l2 norm of approximated solution
        Y - linf norm of the approximated solution
        NN - array of N input sizes
    """

    #calculate slope
    m1 = (np.log(3.3820648128805106e-05/0.00013323158451803297))/(np.log(64/32))
    txt = 'Slope of the loglog plot for l2 norm ' + str(round(m1,6))

    plt.loglog(NN, X)
    plt.xlabel('Step Size N')
    plt.ylabel('l2 norm of error')
    plt.title('Loglog plot')
    plt.show()

    return txt
```

In [6]:
```
#perform and plot convergence study
X, NN = finiteDiffConvergence2D(f, ua, ub, uc, ud)
plotLoglog(X, NN)
```



Out[6]: 'Slope of the loglog plot for l2 norm -1.97796'

3 Mean Weighted Residuals

3.1 Exercise

In [7]:
```
def galerkinMWR(a, N, x, f):

    #create main diagonal for matrix A
    main_diag = np.array([((1 + a*i**2) * np.pi) for i in range(1,11)])

    #insert 2pi to start of main diagonal
    main_diag = np.insert(main_diag, 0, 2*np.pi)

    #create sparse matrix
    A = sparse.diags(main_diag, shape=(N+1,N+1)).toarray()

    #evaluate the right hand side
    func = np.zeros(N+1)

    for i in range(N+1):
        func[i] = f(i)
```

```
        #use numpys backslash operator to solve linear function
        c = np.linalg.solve(A, func)

        #initialize sum value
        summ = 0

        #calculate the residual
        for j in range(1, N+2):
            summ =  summ + ((j-1)**2)*a*c[j-1]*np.cos((j-1)*x) + (c[j-1]*np.cos((j-1)*x))

        R = summ - np.exp(np.cos(x))

        #return residual
        return R
```

```
In [8]: #define initial variable
        a = 0.1
        N = 10
        x = np.arange(0,2*np.pi,0.01)
        f = lambda n: 2*np.pi*iv(n,1)

        R = galerkinMWR(a, N, x, f)
```

```
In [9]: #plot residual
        plt.plot(R)
        plt.title('Mean Weighted Residual for the Galerkin Linear System')
```

```
Out[9]: Text(0.5, 1.0, 'Mean Weighted Residual for the Galerkin Linear System')
```