lab02 Random Processes

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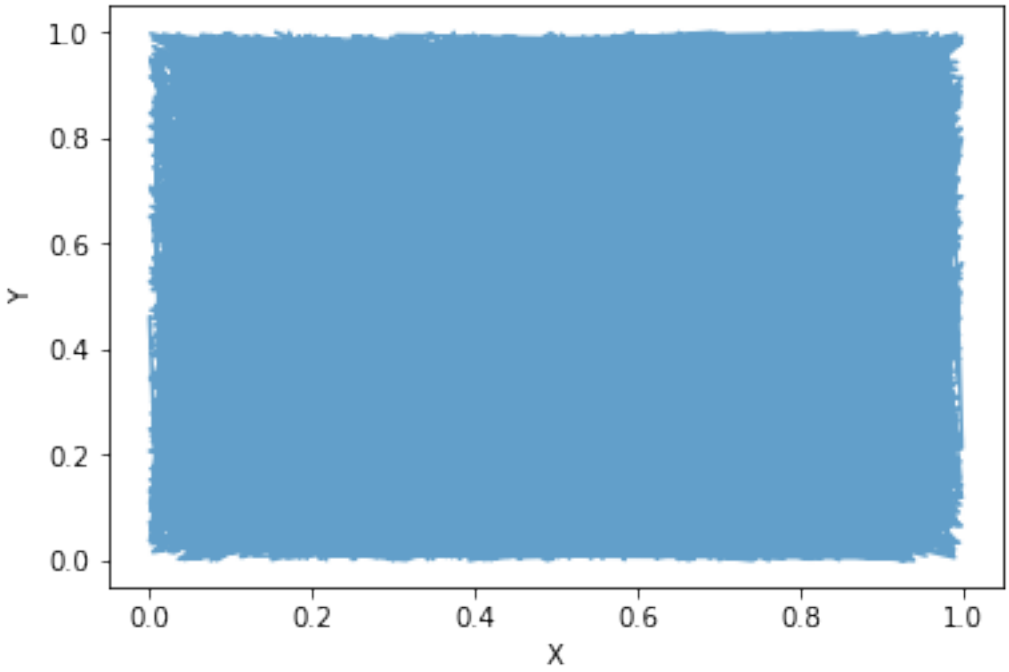
1 Random Number generator

1.1 Uniform Random Cordinates

Generate 10,000 uniform random cordinates, store then in an array, and plot them.

In [1]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
%matplotlib inline

#initialize uniform random variables for x and y
x = np.random.uniform(0, 1, 10000)
y = np.random.uniform(0, 1, 10000)

#plot random coordinates
plt.plot(x, y, alpha = 0.7); plt.ylabel('Y'); plt.xlabel('X')
```

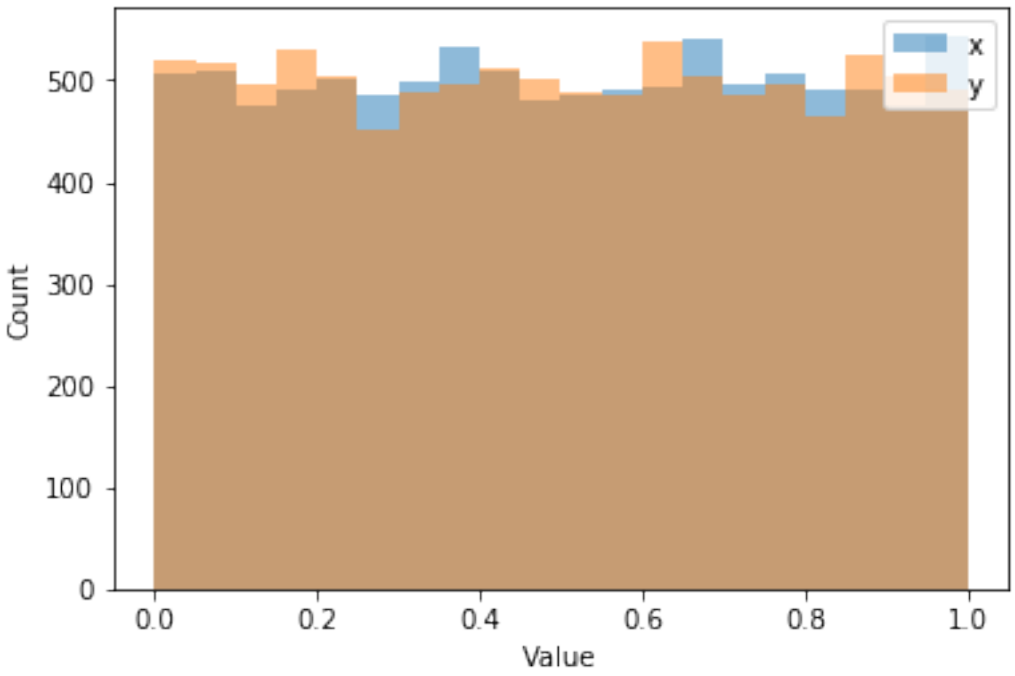
Out[1]:
```
Text(0.5, 0, 'X')
```



1.2 Histogram Visuals

Generate histograms of the random samples and show them in the same plot.

```
In [2]: #plot histogram for x and y uniform random numbers
        plt.hist(x, 20, alpha = 0.5, label = 'x')
        plt.hist(y, 20, alpha = 0.5, label = 'y')
        plt.xlabel('Value')
        plt.ylabel('Count')
        plt.legend(loc='upper right')

Out[2]: <matplotlib.legend.Legend at 0x1faffe9ab38>
```



1.3 Randomness of Samples

Determine the randomness of the samples. Calculate the mean of x and y, and compare the results to the expected value of the two random variables which are both 0.5.

```
In [3]: #Create expected value
        expected_value = 0.5

        #calculate the mean of x and y random uniform values
        x_mean = np.mean(x)
        y_mean = np.mean(y)

        #compare expected values with mean of x and y values
        comp_x = expected_value - x_mean
        comp_y = expected_value - y_mean

        #print results
        print('Mean of random sample x: {}\nMean of random sample y: {}'.format(x_mean,y_mean)

        print('Expected value minus mean of random sample x: {}\nExpected value minus mean of r
```

```
Mean of random sample x: 0.5012426220950413
Mean of random sample y: 0.497738238021686
Expected value minus mean of random sample x: -0.0012426220950413347
```

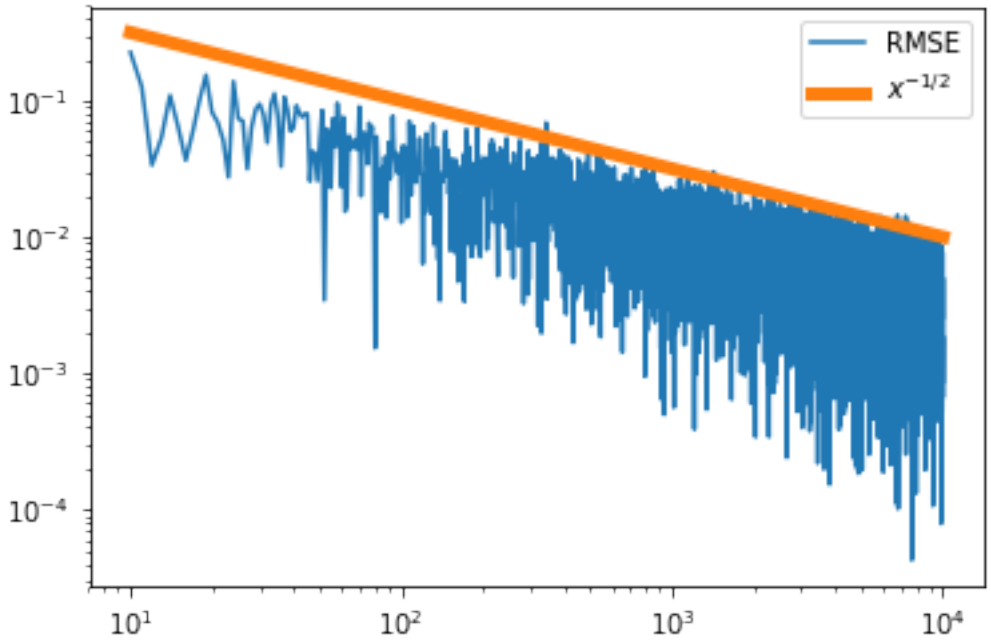
```
Expected value minus mean of random sample y: 0.002261761978314014
```

```
In [4]: def RMSE(arr):
            """
            Inputs:
                arr - array of random points with size Nx2
            Outputs:
                x_mean - mean of X column
                y_mean - mean of Y column
                calc - the root mean squared error
            """
            #split the array my columns of x and y
            x = arr[:, 0]
            y = arr[:, 1]

            #calculate the mean of x and y
            x_mean = np.mean(x)
            y_mean = np.mean(y)

            #calculate the mean squared error
            calc = np.sqrt((x_mean - 0.5)**2 + (y_mean - 0.5)**2)

            #return expected values
            return x_mean, y_mean, calc
```

```
In [5]: # output the mean of each columns of the array
        # and the root-mean-squared-error

        #initialize S Using 5 just to show mean of columns and rmse
        S = 5

        #uniform array of size Sx2
        array = np.random.uniform(size=(S, 2))

        #call RMSE function
        x_mean, y_mean, calc = RMSE(array)

        #output results
        print("The mean of the first column is {}.\n\n The mean of the second column is: {}.\n\
```

```
The mean of the first column is 0.555448219221686.

 The mean of the second column is: 0.280718828273788.

 The root-mean-squared-error is:
 0.22618297302952892
```

```
In [6]: #initial vraiables
        #row size for array of Nx2
        dat = []

        #uniform random array of size Nx2
        for N in range(10, 10000):

            #create uniform random array for each value of N
            arr = np.random.uniform(size=(N, 2))

            #call RMSE function and append values to python list
            x_mean, y_mean, rmse = RMSE(arr)
            dat.append(rmse)

In [7]: x = np.linspace(10, 10000, 9990)
        plt.loglog(x, dat, label='RMSE')
        plt.loglog(x, x**(-1/2), linewidth=5,label='$x^{-1/2}$')
        plt.legend()

Out[7]: <matplotlib.legend.Legend at 0x1fa80101e80>
```



2 Monty Hall Problem

2.1 List All Outcomes And Find Probabilities of Each Outcome

Assume the contestant picks door 1. Since there are 3 possible locations of the car and two choices (switch or keep), there are a total of 3*2 = 6 outcomes. List all six outcomes and determine the

result for each of them. Calculate the pobability of winning the car for each of the two strategies (switch or keep).

```
In [8]: data = {'Behind door 1':['Goat', 'Goat', 'Car'],
                'Behind door 2':['Goat', 'Car', 'Goat'],
                'Behind door 3':['Car', 'Goal', 'Goat'],
                'Result if staying':['Wins goat', 'Wins goat', 'Wins car'],
                'Result if switching':['Wins car', 'Wins car','Wins goat']}

In [9]: df = pd.DataFrame(data)
        df

Out[9]:   Behind door 1 Behind door 2 Behind door 3 Result if staying  \
        0          Goat          Goat           Car         Wins goat
        1          Goat           Car          Goal         Wins goat
        2           Car          Goat          Goat          Wins car

          Result if switching
        0            Wins car
        1            Wins car
        2           Wins goat
```

The probability of winning the car if the contestant does not switch his/her initial choice is 1/3. The probability of winning the car if the contestant does switch his/her initial choice is 2/3.

2.2 Monty Hall Function

```
In [10]: import random

         def montyHall(switch_door, n_doors):
             """
             This function simulates the monty hall game. Contestant chooses door at random
             in hopes that his selected door reveals a prize. The host then picks a different
             door to show the contestant that the door does not contain the prize. The
             contestant then has the choice to switch his door to the last remaining door
             or stick with his selection.
             Input:
                 switch_door - Boolean value to determine if the candidate switces doors
                 n_doors - number of doors to be open in a game
             Output:
                 result - boolean value that determines if the candidate won or lost. True
                 for won, False for lost
             """

             #randomly chosen int 1, 2, or 3 to represent door with car and
             #contestant's choice
             car_door = random.randint(1, n_doors)
             contestant_door = random.randint(1, n_doors)
```

```
            #This creates a list of doors [1, 2, 3]
            doors = list(range(1, n_doors+1))#must do n_doors+1 since range is exclusive at
                                             #the end point

            #continue to loop through the list of doors [1,2,3] until the host picks a door
            #that hasn't been selected yet
            while len(doors) > 2:

                #randomly chose a door to open
                #going to use choice function in the random package to "open" door
                host_door = random.choice(doors)#makes a random selection in doors [1,2,3]

                #the host can never open the door with the car or the door that the
                #contestant chose
                if ((host_door == car_door) or (host_door == contestant_door)):
                    #continue within the while loop until the host picks a door that isnt the
                    #car or contestant door
                    continue

                #host picks a door, remove the door from the list of remaining_doors
                doors.remove(host_door)#remove the host door from the list of doors

            #If the contestant switches their choice
            if switch_door:#if the boolean value == True

                #there are two doors left. The contestant will never choose the same door,
                #so I remove that door as a choice. Make remaining_doors into list so you
                #can remove contestant_door
                #remember here the door only has the contestant door and the car door
                available_doors = list(doors)
                available_doors.remove(contestant_door)#remove contestants initial choice

                #change the candidate_door to the only door available
                #pop returns the only value in the list as original datatype element
                contestant_door = available_doors.pop()

            #create boolean that labels if the candidate won or lost
            result = (contestant_door == car_door)

            #return boolean true or false
            return result

In [11]: def run_N_montyHall(N, switch_door, n_doors):
            """
            This function runs the monty hall problem for N number of times
            Input:
                N - Number of tests to run
                switch_door - Boolean value to determine if the candidate switces doors
```

```
        n_doors - number of doors to be open in a game
    Output:
        num_wins - Number of times the contestant won

    """

    #num_wins is the counter. Intialize to 0
    num_wins = 0

    #run tests N times
    for i in range(N):

        #if montyHall returns true increment win column
        if montyHall(switch_door, n_doors):
            #update number of wins
            num_wins += 1

    #return the number of wins
    return num_wins
```

In [12]:
```
#assign initial conditions
num_door = 3
num_test = 1000

#run tests without and with switch
num_wins_without_switch = run_N_montyHall(num_test, False, num_door)
num_wins_with_switch = run_N_montyHall(num_test, True, num_door)

#calculate probability
prob_without_switch = num_wins_without_switch/num_test
prob_with_switch = num_wins_with_switch/num_test

# #report outputs
data = {"Number of iterations":[num_test,num_test],
        "Number of doors":[num_door,num_door],
        "Number of wins":[num_wins_without_switch, num_wins_with_switch],
        "Percentage (%)":[prob_without_switch*100, prob_with_switch*100]}
df = pd.DataFrame(data, index=['Without Switching', 'With Switching'])
df
```

Out[12]:

	Number of iterations	Number of doors	Number of wins	Percentage (%)
Without Switching	1000	3	345	34.5
With Switching	1000	3	672	67.2

2.3 Generalize Monty Hall Function

In [13]:
```
#assign initial conditions
#For part three generalize code to simulate when there are 100 doors
num_door = 100
num_test = 10000

#run tests without and with switch
num_wins_without_switch = run_N_montyHall(num_test, False, num_door)
num_wins_with_switch = run_N_montyHall(num_test, True, num_door)

#calculate probability
prob_without_switch = num_wins_without_switch/num_test
prob_with_switch = num_wins_with_switch/num_test

# #report outputs
data = {"Number of iterations":[num_test,num_test],
        "Number of doors":[num_door,num_door],
        "Number of wins":[num_wins_without_switch, num_wins_with_switch],
        "Percentage (%)":[prob_without_switch*100, prob_with_switch*100]}
df = pd.DataFrame(data, index=['Without Switching', 'With Switching'])
df
```

Out[13]:

	Number of iterations	Number of doors	Number of wins	Percentage (%)
Without Switching	10000	100	100	1.0
With Switching	10000	100	9880	98.8