Lab 2 Watermarking

November 18, 2019

To run this lab simply run all cells below.

In this lab, I perform Image watermarking using vector quantization. I use the K-Means clustering algorithm to reconstruct a version of an image using a smaller number of elements. Using the K-Means procedure I watermark the image to perform two imporvements to extract the watermark.

0.0.1 Blocking Image

Start by dividing the 512x512 Truman image into 1024 blocks, each of which are 16x16 in size. These blocks are then converted into vectors of length 256.

```
In [1]: import numpy as np
        from PIL import Image
        import matplotlib.pyplot as plt
        %matplotlib inline

        def readImage(img):
            """
            This function divides the 512x512 image into 1024 blocks, each of which
            are 16x16 in size. Once the image is seperated into blocks, each block
            is converted into vectors of length 256.
            Inputs: img - numpy array of image pixels
            Outputs: X - numpy array of size 256x1024
            """

            #preallocate size for X
            X = np.zeros([256, 1024])

            #initialize k
            k = 0

            #step size
            step = 16

            #loop through pixels of image
            for i in range(0, 512, step):
                for j in range(0, 512, step):
```

```
                #split into 1024 blocks of 16x16
                block = img[i:i+step, j:j+step]
                #reshape each block into a vector of length 256
                X[:,k] = np.reshape(block, (1, 256))#total of 1024 vectors
                #sum up k
                k +=1

        #return 1024 vectors of length 256
        return X
```

0.0.2 K-Means Algorithm

The K-Means algorithm is modularized for repeated use. I randomly initialize k centers by picking out 20 block vectors from the set of block vectors constructed earlier. In the function closestCentroids I compute the Euclidean Distance and label each block vector with the number of the center it is close to. In the function updateGenerator I recompute the centers by taking the average of the block vectors assigned to each group. The kmeans function computes the sum of the change between the new centers and the old ones.

```
In [2]: #function to calculate the closest centroids
        def closestCentroids(img, centroids):

            #initialize empty list to hold minimum generators
            generator = np.zeros([256, 20])

            assignment = []

            #iterate through length of the original image
            for i in range(img.shape[1]):

                #variable iterates and stores each row of image matrix
                vals = img[:,i]

                #using list comprehension to calculate the euclidean distance from image and e
                euclid = [abs(vals[0] - centroids[0][j]) + abs(vals[1] - centroids[1][j]) for j

                #add min euclidian distance
                generator[:,np.argmin(euclid)] = vals

                assignment.append(np.argmin(euclid))

            #return min generators
            return generator, assignment

In [3]: #function to update generators for each iteration of k means
        def updateGenerators(image_blocks, nclusters, cluster_assignments):

            #empty array to hold new generators
```

```
        new_generators = np.zeros([256, 20])

        #loop through the length of number of clusters
        for i in range(nclusters):

            indexes = np.where(np.array(cluster_assignments) == i)[0]

            averages = np.average(image_blocks[:,indexes], axis=1)

            #append the average generator for each dimension
            #return a vector with 3 vals
            new_generators[:, i] = averages

        #return new generators
        return new_generators
```

```
In [4]: #import external modules
        import random

        #function to perform k means clustering
        def kmeans(img, nclusters):

            #create initial generators from original image matrix
            generator = img[:,0:20]

            #define iterations
            iterations = 10

            mval = 100000
            tol = 1e-6

            #run k means for 10 iterations
            while mval >= tol:

                #calculate closest generators
                generator, assignment = closestCentroids(img, generator)

                #update new generators
                new_generator = updateGenerators(img, nclusters, assignment)

                mval = np.linalg.norm(abs(new_generator - generator)/(new_generator + 1e-16))

                generator = new_generator

            return np.array(generator), assignment
```

```
In [5]: def reconstructImage(generator, assignment):
```

```
    #preallocate size for X
    approx_image = np.zeros([512, 512])

    #initialize k
    k = 0

    m,n = generator.shape

    #step size
    step = 16

    #loop through pixels of image
    for i in range(0, 512, step):
        for j in range(0, 512, step):
            #split into 1024 blocks of 16x16
            block = generator[:,assignment[k]]
            #reshape each block into a vector of length 256
            approx_image[i:i+step, j:j+step] = np.reshape(block, (16, 16))#total of 10
            #sum up k
            k +=1

    return approx_image
```

```
In [6]: def plotApproxImage(approximation, title):

            #generalized plots to show approximate image
            plt.imshow(approximation, cmap='gray')
            plt.title(title)
```

Run K-Means on the group of block vectors with K=20 and plot the image/

```
In [7]: def runKmeans(image):

            #block original image
            image = readImage(image)

            #kmeans with 20 clusters on the block image
            nclusters = 20
            generator, assignment = kmeans(image, nclusters)
            approximate_image = reconstructImage(generator, assignment)

            return approximate_image
```

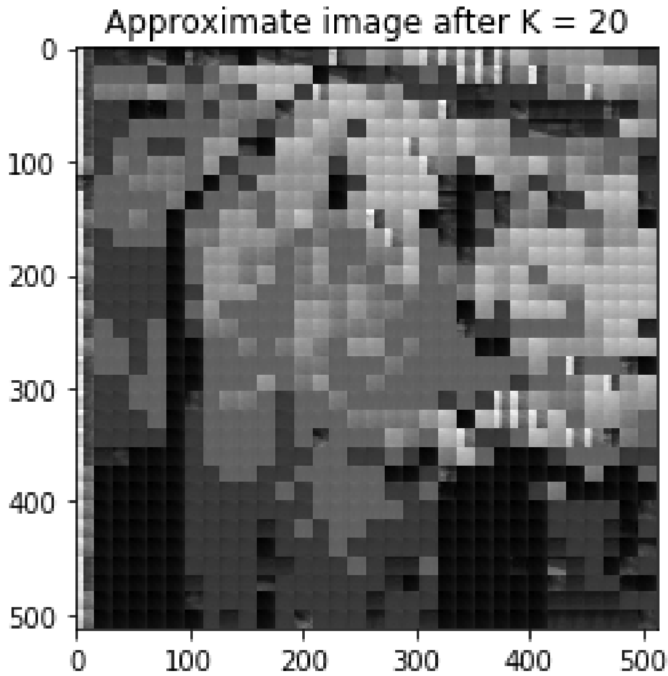
```
In [8]: #read in original image
        img = np.array(Image.open('TrumanEatsLunch.jpg'))

        #run kmeans
        approximate_image = runKmeans(img)
```

```
C:\Users\cspoe\Anaconda3\lib\site-packages\numpy\lib\function_base.py:356: RuntimeWarning: Mear
  avg = a.mean(axis)
C:\Users\cspoe\Anaconda3\lib\site-packages\numpy\core\_methods.py:78: RuntimeWarning: invalid v
  ret, rcount, out=ret, casting='unsafe', subok=False)
```

0.0.3 First K-Means Run on Original Image

In [9]:
```
#create figure showing the approximated image after applying
#k-means when k = 20 first run
plotApproxImage(approximate_image, 'Approximate image after K = 20')
```

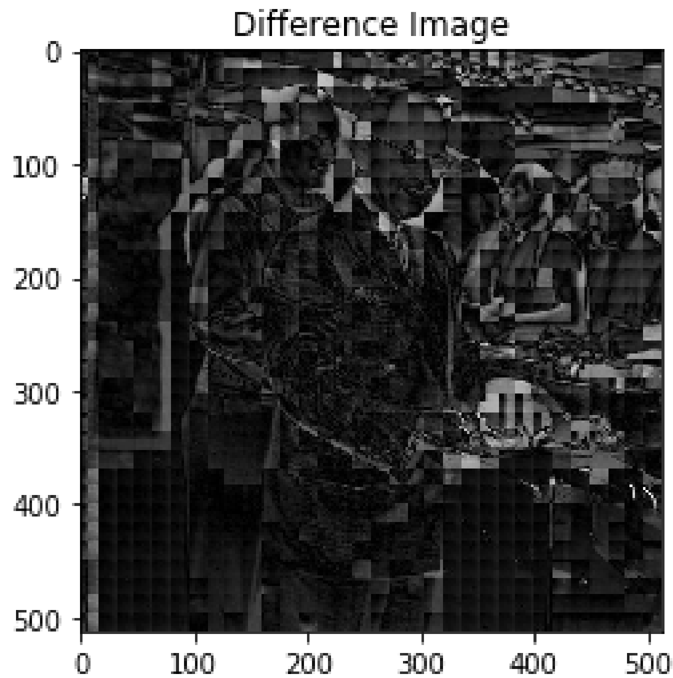


0.0.4 Difference Image

In [10]:
```
#figure out how different approximate image from the original image
#take pixel by pixel subtraction
def differencing(orignal_image, approximate_image):
    return abs(orignal_image - approximate_image)
```

In [11]:
```
#figure out how different reconstructed image is from original image
#call differencing function to take a pixel by pixel subtraction
difference = differencing(img, approximate_image)
```

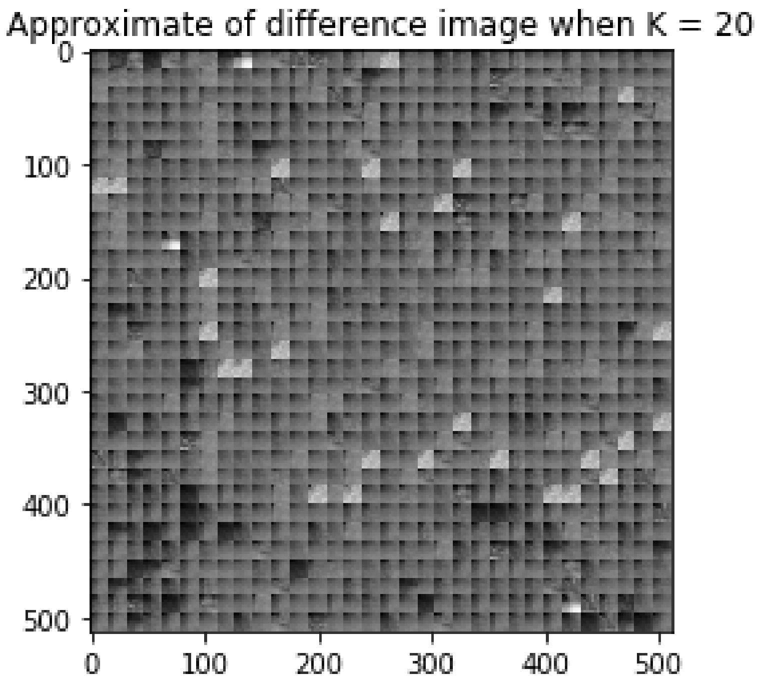
In [12]:
```
#create plot out of this difference
plotApproxImage(difference, 'Difference Image')
```



0.0.5 Second K-Means Run on Difference Image

```
In [13]: #run K-Means on the difference image
         reconstructed_image = runKmeans(difference)
```

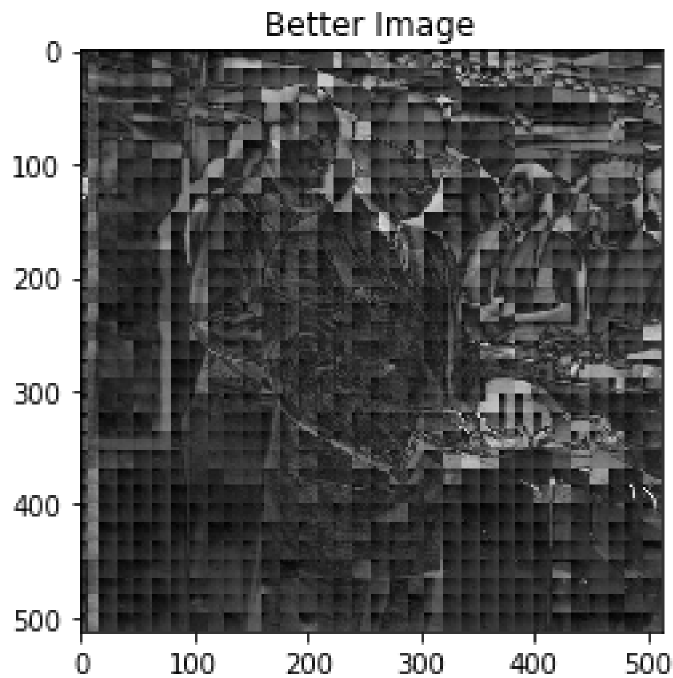
```
In [14]: #reconstruct an approximate difference image out of the
         #centers of the second k means run by replacing each block in the image
         #with the center it was assigned to
         plotApproxImage(reconstructed_image, 'Approximate of difference image when K = 20')
```



0.0.6 Final Reconstruction Improvements

```
In [15]: #add the reconstructed_image and the reconstructed difference image together
         better_image = difference + reconstructed_image
```

```
In [16]: #plot better image
         plotApproxImage(better_image, 'Better Image')
```



0.0.7 Watermarking Preparation

```
In [17]: #load in watermarking image
         smiley = np.array(Image.open('smiley.jpg'))

         # #binarize watermarking image
         W = (smiley > 128).astype(int)
```

```
In [18]: #read in original image
         truman = np.array(Image.open('TrumanEatsLunch.jpg'))
         #block image
         img = readImage(truman)
         #run k-Means a third time on the original image
         nclusters = 20
         generator, assignment = kmeans(img, nclusters)
```

0.0.8 The Index Matrix

```
In [19]: def indexesMatrix(assignment):

             #construct an "indexes matrix" representation of the image by
             #reshaping the assignments array into a matrix size of 32x32
             return np.array(assignment).reshape(32, 32)
```

```
In [20]: indexes = indexesMatrix(assignment)
```

0.0.9 The Variance Matrix

```
In [21]: def varianceMatrix(indexes):


             variance = np.zeros_like((indexes))

             #construct the variance matrix
             N = len(indexes)
             for i in range(2, N-1):
                 for j in range(2, N-1):

                     variance[i,j] = np.var((indexes[i,j], indexes[i-1,j], indexes[i+1,j], inde
                     variance[0,j] = np.var((indexes[0,j], indexes[0,j-1], indexes[1,j], indexe
                     variance[-1,j] = np.var((indexes[-1,j], indexes[-1,j-1], indexes[-1,j+1],
                     variance[i,0] = np.var((indexes[i,0], indexes[i+1,0], indexes[i-1,0], inde
                     variance[i,-1] = np.var((indexes[i,-1], indexes[i+1,-1], indexes[i-1,-1],i
                     variance[0,0] = np.var((indexes[0,0], indexes[0,1], indexes[1,0], indexes
                     variance[-1,-1] = np.var((indexes[-1,-2], indexes[-2,-2], indexes[-2,-1],
                     variance[-1, 0] = np.var((indexes[-1,0], indexes[-2,0], indexes[-1,1], inc

             #pick a threshold. This is set to the global median of the variance
             T = np.median(np.median(variance))

             return variance, T

In [22]: variance, T = varianceMatrix(indexes)
```

0.0.10 The Polarities Matrix

```
In [23]: def polarityMatrix(variance, T):
             #using threshold T construct polarities matrix of size 32x32
             #each element is either a 0 or 1
             polarities = np.zeros_like((variance))

             m,n = variance.shape

             for i in range(m):
                 for j in range(n):
                     polarities[i,j] = 0 if variance[i,j] <= T else 1

             return polarities

In [24]: polarities = polarityMatrix(variance, T)
```

0.0.11 Watermark Permutation

```
In [25]: #randomly permute the watermark image
         key1 = np.zeros_like(polarities)
```

```
Wp = np.zeros_like(polarities)

for i in range(polarities.shape[0]):
    key1[i,:] = np.random.permutation(polarities.shape[0])
    Wp[i,:]  = W[i,key1[i,:]]
```

0.0.12 Embedding the Watermark

In [26]:
```
#Embedding the watermark into the polarities matrix by the XOR operation
key2 = np.zeros_like(polarities)

m, n = polarities.shape

for i in range(m):
    for j in range(n):
        key2[i,j] = int(polarities[i,j] != Wp[i,j])
```

0.0.13 Image Approximation

In [27]:
```
#Image approximation
#run K-Means once again on the original image when k=20
#read in original image
img = np.array(Image.open('TrumanEatsLunch.jpg'))
image = readImage(img)

#kmeans with 20 clusters on the block image
nclusters = 20
generator, assignment = kmeans(image, nclusters)
approximate_image = reconstructImage(generator, assignment)
```

In [28]:
```
#Re-compute the variance and polarities matrices from this
indexes = indexesMatrix(assignment)
variance, T = varianceMatrix(indexes)
polarities_approx = polarityMatrix(variance, T)
```

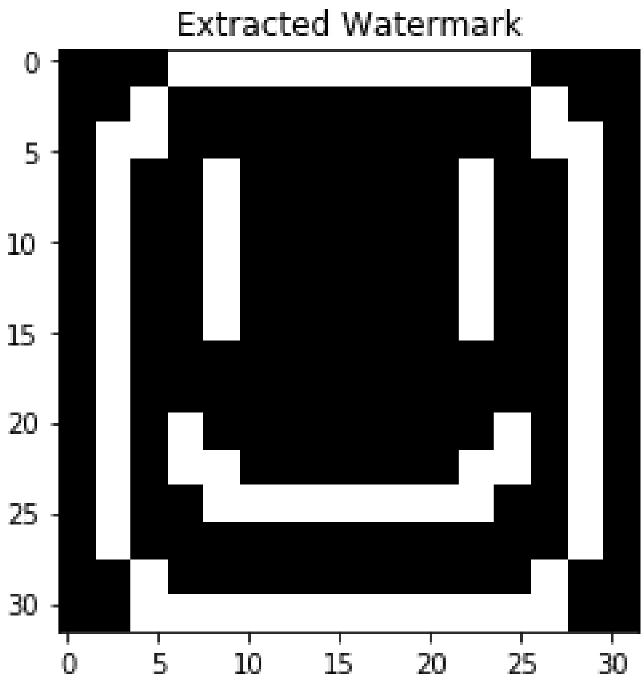
0.0.14 Extract the Watermark

In [29]:
```
#extract the watermark from the approximate polarities matrix using the inverse opera
for i in range(m):
    for j in range(n):
        Wp[i,j] = int(key2[i,j] != polarities_approx[i,j])

for k in range(m):
    W[k,key1[k,:]] = Wp[k,:]
```

Final extracted watermark

In [30]:
```
plotApproxImage(W, 'Extracted Watermark')
```


Extracted Watermark
0
5
10
15
20
25
30
0
5
10
15
20
25
30