

# DATA FILES FOR THE ALGEBRAIC NOVIKOV, ADAMS, AND ADAMS-NOVIKOV SPECTRAL SEQUENCES

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ABSTRACT. This document describes the structure of some comma-separated-value (CSV) files that contain detailed information about the algebraic Novikov, Adams, and Adams-Novikov spectral sequences, in both the classical and  $\mathbb{C}$ -motivic contexts.

This document describes the structure of some comma-separated-value (CSV) files that contain detailed information about the algebraic Novikov, Adams, and Adams-Novikov spectral sequences, in both the classical and  $\mathbb{C}$ -motivic contexts. These files are auxiliary to the projects described in [1], [2], [3], and [4].

See the cited documents for more mathematical details. The remainder of this document describes the structure of the CSV files.

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## 1. CLASSICAL ADAMS SPECTRAL SEQUENCE

**Adams-classical-E2.csv:** Each line of the file corresponds to an element in the classical Adams  $E_2$ -page. This data is used to produce the chart appearing in [3].

*name:* Human-readable name of an element. (Beware that naming conventions have changed over time.)

*stem:* The stem of an element. This is the horizontal coordinate in a standard Adams chart.

*Adams filtration:* The Adams filtration of an element. This is the vertical coordinate in a standard Adams chart.

*shift:* Used for display purposes in reference to the chart in [3], when more than one element occurs with the same bidegree. Lower values correspond to dots on the left.

*h0info:* Information about special behavior of an  $h_0$  extension.

loc means that an element is  $h_0$ -periodic.

p means that an  $h_0$  extension is not known to occur.

*h0target:* Value of an  $h_0$  extension. An empty cell indicates that there is no  $h_0$  extension.

*h1target:* Value of an  $h_1$  extension. An empty cell indicates that there is no  $h_1$  extension.

*h2target:* Value of an  $h_2$  extension. An empty cell indicates that there is no  $h_2$  extension.

*drinfo:* Information about an Adams differential. An integer value  $r$  indicates a  $d_r$  differential.

p means that a differential is not known to occur.

*drtarget:* Value of an Adams  $d_r$  differential.

**Adams-classical-Einfy.csv:** Each line of the file corresponds to an element in the classical Adams  $E_\infty$ -page. This data is used to produce the chart appearing in [3].

*name:* Human-readable name of an element. (Beware that naming conventions have changed over time.)

*stem:* The stem of an element. This is the horizontal coordinate in a standard Adams chart.

*Adams filtration:* The Adams filtration of an element. This is the vertical coordinate in a standard Adams chart.

*shift:* Used for display purposes in reference to the chart in [3], when more than one element occurs with the same bidegree. Lower values correspond to dots on the left.

*h0info:* Information about special behavior of an  $h_0$  extension.

loc means that an element is  $h_0$ -periodic.

h means that there is a hidden 2 extension.

h ? means that there is a possible hidden 2 extension.

p means that an  $h_0$  extension is not known to occur.

*h0target:* Value of an  $h_0$  extension. An empty cell indicates that there is no  $h_0$  extension.

*h1info:* Information about special behavior of an  $h_1$  extension.

h means that there is a hidden  $\eta$  extension.

h ? means that there is a possible hidden  $\eta$  extension.

p means that the  $h_1$  extension is not known to occur.

*h1target:* Value of an  $h_1$  extension. An empty cell indicates that there is no  $h_1$  extension.

*h2info:* Information about special behavior of an  $h_2$  extension.

h means that there is a hidden  $\eta$  extension.

p means that the  $h_2$  extension is not known to occur.

*h2target:* Value of an  $h_2$  extension. An empty cell indicates that there is no  $h_2$  extension.

*drinfo:* Information about an Adams differential. An integer value  $r$  indicates a  $d_r$  differential.

p means that the differential is not known to occur.

*drtarget:* Value of an Adams  $d_r$  differential.

**Adams-classical-Einfy-extn.csv:** Each line of the file corresponds to a hidden extension by 2,  $\eta$ , or  $\nu$  in the classical Adams  $E_\infty$ -page. This data is used to produce the chart appearing in [3].

*source:* Source of an extension. (Beware that naming conventions have changed over time.)

*type:* Type of extension.

**h0** means an extension by 2.

**h1** means an extension by  $\eta$ .

**h2** means an extension by  $\nu$ .

*stem:* The stem of the source of an extension. This is the horizontal coordinate in a standard Adams chart.

*Adams filtration:* The Adams filtration of the source of an extension. This is the vertical coordinate in a standard Adams chart.

*info:* Information about special behavior of an extension.

**?** means that an extension is not known to occur.

*target:* Target of an extension.

*source<sub>x</sub>, source<sub>y</sub>, target<sub>x</sub>, target<sub>y</sub>:* Used for display purposes in reference to the chart in [3], when a curved hidden extension is necessary. Gives the tangent vectors at the source and target.

2.  $\mathbb{C}$ -MOTIVIC ADAMS SPECTRAL SEQUENCE

**Adams-motivic-E2.csv:** Each line of the file corresponds to an element in the motivic Adams  $E_2$ -page. This data is used to produce the chart appearing in [3].

*name:* Human-readable name of an element. (Beware that naming conventions have changed over time.)

*stem:* The stem of an element. This is the horizontal coordinate in a standard Adams chart.

*Adams filtration:* The Adams filtration of an element. This is the vertical coordinate in a standard Adams chart.

*weight:* The motivic weight of an element.

*tautorsion:* Indicates the  $\tau$  module structure of an element.

0 means that an element is  $\tau$ -periodic.

Any other integer  $k$  means that an element is annihilated by  $\tau^k$ .

*shift:* Used for display purposes in reference to the chart in [3], when more than one element occurs with the same bidegree. Lower values correspond to dots on the left.

*h0info:* Information about special behavior of an  $h_0$  extension.

p means that an  $h_0$  extension is not known to occur. t means that an  $h_0$  extension equals  $\tau$  times an element.

t followed by an integer  $k$  means that an  $h_0$  extension equals  $\tau^k$  times an element.

*h0target:* Value of an  $h_0$  extension. An empty cell indicates that there is no  $h_0$  extension.

loc means that an element is  $h_0$ -periodic.

*h1info:* Information about special behavior of an  $h_1$  extension.

p means that an  $h_1$  extension is not known to occur. t means that an  $h_1$  extension equals  $\tau$  times an element.

t followed by an integer  $k$  means that an  $h_1$  extension equals  $\tau^k$  times an element.

*h1target:* Value of an  $h_1$  extension. An empty cell indicates that there is no  $h_1$  extension.

loc means that an element is  $h_1$ -periodic.

*h2info:* Information about special behavior of an  $h_2$  extension.

p means that an  $h_2$  extension is not known to occur. t means that an  $h_2$  extension equals  $\tau$  times an element.

t followed by an integer  $k$  means that an  $h_2$  extension equals  $\tau^k$  times an element.

*h2target*: Value of an  $h_2$  extension. An empty cell indicates that there is no  $h_2$  extension.

*drinfo*: Information about an Adams  $d_2$  differential.

**free** means that the target of the differential is not displayed on the chart, typically because it is  $h_1$ -periodic.

**p** means that a differential is not known to occur.

**t** means that a differential equals  $\tau$  times an element.

**t** followed by an integer  $k$  means that a differential equals  $\tau^k$  times an element.

*drtarget*: Value of an Adams  $d_2$  differential.

**Adams-motivic-E3.csv:** Each line of the file corresponds to an element in the motivic Adams  $E_3$ -page. This data is used to produce the chart appearing in [3]. This file takes the same format as **Adams-motivic-E2.csv**.

**Adams-motivic-E4.csv:** Each line of the file corresponds to an element in the motivic Adams  $E_4$ -page. This data is used to produce the chart appearing in [3]. This file takes the same format as **Adams-motivic-E2.csv**.

**Adams-motivic-E5.csv:** Each line of the file corresponds to an element in the motivic Adams  $E_5$ -page. This data is used to produce the chart appearing in [3]. This file takes the same format as **Adams-motivic-E2.csv**.

**Adams-motivic-E6.csv:** Each line of the file corresponds to an element in the motivic Adams  $E_6$ -page. This data is used to produce the chart appearing in [3]. This file takes the same format as **Adams-motivic-E2.csv**.

**Adams-motivic-Einfy.csv:** Each line of the file corresponds to an element in the motivic Adams  $E_\infty$ -page. This data is used to produce the chart appearing in [3]. This file takes the same format as **Adams-motivic-E2.csv**.

**Adams-motivic-Einfy-extn.csv:** Each line of the file corresponds to a hidden extension by  $\tau$  in the  $\mathbb{C}$ -motivic Adams  $E_\infty$ -page. This data is used to produce the chart appearing in [3].

*source:* Source of an extension. (Beware that naming conventions have changed over time.)

*stem:* The stem of the source of an extension. This is the horizontal coordinate in a standard Adams chart.

*Adams filtration:* The Adams filtration of the source of an extension. This is the vertical coordinate in a standard Adams chart.

*weight:* The motivic weight of an element.

*info:* Information about special behavior of an extension.

? means that an extension is not known to occur.

*target:* Target of an extension.

*source<sub>x</sub>, source<sub>y</sub>, target<sub>x</sub>, target<sub>y</sub>:* Used for display purposes in reference to the chart in [3], when a curved hidden extension is necessary. Gives the tangent vectors at the source and target.



## 3. ADAMS-NOVIKOV SPECTRAL SEQUENCE

**ANSS-v1periodic-E2.csv:** Each line of the file corresponds to a  $v_1$ -periodic element in the Adams-Novikov  $E_2$ -page. This data is used to produce the chart appearing in [4].

*name:* Human-readable name of an element. (Beware that naming conventions have changed over time.)

*stem:* The stem of an element. This is the horizontal coordinate in a standard Adams-Novikov chart.

*Adams-Novikov filtration:* The Adams-Novikov filtration of an element. This is the vertical coordinate in a standard Adams-Novikov chart.

*order:*  $\log_2$  of the order of an element.

*h1info:* Information about special behavior of an  $h_1$  extension.  
*loc* means that an element is  $h_1$ -periodic.

*h1target:* Value of an  $h_1$  extension. An empty cell indicates that there is no  $h_1$  extension.

*h2target:* Value of an  $h_2$  extension. An empty cell indicates that there is no  $h_2$  extension.

*drinfo:* Information about an Adams-Novikov differential.  
 An integer  $r$  means that there is a  $d_r$  differential.

*drtarget:* Value of an Adams-Novikov differential.

**ANSS-v1periodic-Einfy.csv:** Each line of the file corresponds to a  $v_1$ -periodic element in the Adams-Novikov  $E_\infty$ -page. This data is used to produce the chart appearing in [4].

*name:* Human-readable name of an element. (Beware that naming conventions have changed over time.)

*stem:* The stem of an element. This is the horizontal coordinate in a standard Adams-Novikov chart.

*Adams-Novikov filtration:* The Adams-Novikov filtration of an element. This is the vertical coordinate in a standard Adams-Novikov chart.

*order:*  $\log_2$  of the order of an element.

*h1target:* Value of an  $h_1$  extension. An empty cell indicates that there is no  $h_1$  extension.

*h2target:* Value of an  $h_2$  extension. An empty cell indicates that there is no  $h_2$  extension.

**ANSS-v1periodic-Einfy-extn.csv:** Each line of the file corresponds to a hidden extension between  $v_1$ -periodic elements in the Adams-Novikov  $E_\infty$ -page. This data is used to produce the chart appearing in [4].

*source:* Source of an extension. (Beware that naming conventions have changed over time.)

*type:* Type of extension.  
h0 means an extension by 2.

*stem:* The stem of the source of an extension. This is the horizontal coordinate in a standard Adams-Novikov chart.

*Adams-Novikov filtration:* The Adams-Novikov filtration of the source of an extension. This is the vertical coordinate in a standard Adams-Novikov chart.

*target:* Target of an extension.

**ANSS-E2.csv:** Each line of the file corresponds to an element in the Adams-Novikov  $E_2$ -page, excluding  $v_1$ -periodic elements. This data is used to produce the chart appearing in [4].

*name:* Human-readable name of an element. (Beware that naming conventions have changed over time.)

*stem:* The stem of an element. This is the horizontal coordinate in a standard Adams-Novikov chart.

*Adams-Novikov filtration:* The Adams-Novikov filtration of an element. This is the vertical coordinate in a standard Adams-Novikov chart.

*order:*  $\log_2$  of the order of an element.

*shift:* Used for display purposes in reference to the chart in [4], when more than one element occurs with the same bidegree. Lower values correspond to dots on the left.

*h1info:* Information about special behavior of an  $h_1$  extension.  
An integer  $k$  means that the  $h_1$  extension equals  $2^k$  times a generator.

*h1target:* Value of an  $h_1$  extension. An empty cell indicates that there is no  $h_1$  extension.

*h2info:* Information about special behavior of an  $h_2$  extension.  
An integer  $k$  means that the  $h_2$  extension equals  $2^k$  times a generator.

*h2target:* Value of an  $h_2$  extension. An empty cell indicates that there is no  $h_2$  extension.

*drinfo:* Information about an Adams-Novikov differential.  
An integer  $r$  means that there is a  $d_r$ -differential.  
? means that a differential is not known to occur.

*drtarget:* Value of an Adams-Novikov differential.

**ANSS-E4.csv:** Each line of the file corresponds to an element in the Adams-Novikov  $E_4$ -page, excluding  $v_1$ -periodic elements. This data is used to produce the chart appearing in [4]. This file takes the same format as **ANSS-E2.csv**.

**ANSS-E6.csv:** Each line of the file corresponds to an element in the Adams-Novikov  $E_6$ -page, excluding  $v_1$ -periodic elements. This data is used to produce the chart appearing in [4]. This file takes the same format as **ANSS-E2.csv**.

**ANSS-Einfy.csv:** Each line of the file corresponds to an element in the Adams-Novikov  $E_\infty$ -page, excluding  $v_1$ -periodic elements. This data is used to produce the chart appearing in [4]. This file takes the same format as **ANSS-E2.csv**.

**ANSS-Einfy-extn.csv:** Each line of the file corresponds to a hidden extension by 2,  $\eta$ , or  $\nu$  in the Adams-Novikov  $E_\infty$ -page. This data is used to produce the chart appearing in [4].

*source:* Source of an extension. (Beware that naming conventions have changed over time.)

*type:* Type of extension.

**h0** means an extension by 2.

**h1** means an extension by  $\eta$ .

**h2** means an extension by  $\nu$ .

*stem:* The stem of the source of an extension. This is the horizontal coordinate in a standard Adams-Novikov chart.

*Adams-Novikov filtration:* The Adams-Novikov filtration of the source of an extension. This is the vertical coordinate in a standard Adams-Novikov chart.

*info:* Information about special behavior of an extension.

**?** means that an extension is not known to occur.

*target:* Target of an extension.

*source<sub>x</sub>, source<sub>y</sub>, target<sub>x</sub>, target<sub>y</sub>:* Used for display purposes in reference to the chart in [4], when a curved hidden extension is necessary. Gives the tangent vectors at the source and target.

4.  $h_1$ -BOCKSTEIN SPECTRAL SEQUENCE FOR THE ALGEBRAIC NOVIKOV  $E_2$ -PAGE

**algNovikov-h1periodic-E0.csv** Each line of the file corresponds to an element in the  $E_0$ -page of the  $h_1$ -Bockstein spectral sequence that converges to part of the algebraic Novikov  $E_2$ -page. This data is used to produce the chart appearing in [2].

*name:* Human-readable name of an element. (Beware that naming conventions have changed over time.)

*stem:* The stem of an element. This is the horizontal coordinate in a standard Adams chart.

*Adams filtration:* The Adams filtration of an element. This is the vertical coordinate in a standard Adams chart.

*weight:* The motivic weight of an element.

*cell:* Indicates whether an element is detected by the top cell or the bottom cell of the cofiber of  $\tau$ .

0 means that an element is in the image in Ext of inclusion of the bottom cell.

1 means that an element maps non-trivially in Ext under projection to the top cell.

*shift:* Used for display purposes in reference to the chart in [3], when more than one element occurs with the same bidegree. Lower values correspond to dots on the left.

*h1info:* Information about special behavior of an  $h_1$  extension.

? means that an  $h_1$  extension is not known to occur.

h means that an  $h_1$  extension is hidden, in the sense that its source is detected by the top cell of the cofiber of  $\tau$ , while its target is detected by the bottom cell.

*h1target:* Value of an  $h_1$  extension. An empty cell indicates that there is no  $h_1$  extension.

loc means that an element is  $h_1$ -periodic.

*drinfo:* Information about a Bockstein differential.

An integer  $r$  means that there is a Bockstein  $d_r$  differential.

? means that a differential is not known to occur.

*drtarget:* Value of a Bockstein differential.

**algNovikov-h1periodic-Einfy.csv** Each line of the file corresponds to an element in the  $E_\infty$ -page of the  $h_1$ -Bockstein spectral sequence that converges to part of the algebraic Novikov  $E_2$ -page. This data is used to produce the chart appearing in [2]. This file takes the same format as **algNovikov-h1periodic-E0.csv**.



## 5. ALGEBRAIC NOVIKOV SPECTRAL SEQUENCE

**algNovikov-E2.csv:** Each line of the file corresponds to an element in the algebraic Novikov  $E_2$ -page. This data is used to produce the chart appearing in [2].

*name:* Human-readable name of an element. (Beware that naming conventions have changed over time.)

*stem:* The stem of an element. This is the horizontal coordinate in a standard Adams chart.

*Adams filtration:* The Adams filtration of an element. This is the vertical coordinate in a standard Adams chart.

*weight:* The motivic weight of an element.

*cell:* Indicates whether an element is detected by the top cell or the bottom cell of the cofiber of  $\tau$ .

0 means that an element is in the image in Ext of inclusion of the bottom cell.

1 means that an element maps non-trivially in Ext under projection to the top cell.

*h0info:* Information about special behavior of an  $h_0$  extension.

**h** means that an  $h_0$  extension is hidden, in the sense that its source is detected by the top cell of the cofiber of  $\tau$ , while its target is detected by the bottom cell.

*h0target:* Value of an  $h_0$  extension. An empty cell indicates that there is no  $h_0$  extension.

**loc** means that an element is  $h_0$ -periodic.

*h1info:* Information about special behavior of an  $h_1$  extension.

**?** means that an  $h_1$  extension is not known to occur.

**h** means that an  $h_1$  extension is hidden, in the sense that its source is detected by the top cell of the cofiber of  $\tau$ , while its target is detected by the bottom cell.

*h1target:* Value of an  $h_1$  extension. An empty cell indicates that there is no  $h_1$  extension.

**loc** means that an element is  $h_1$ -periodic.

*h2info:* Information about special behavior of an  $h_2$  extension.

**h** means that an  $h_2$  extension is hidden, in the sense that its source is detected by the top cell of the cofiber of  $\tau$ , while its target is detected by the bottom cell.

*h2target:* Value of an  $h_2$  extension. An empty cell indicates that there is no  $h_2$  extension.

*drinfo*: Information about an algebraic Novikov differential.

An integer  $r$  means that there is  $d_r$  differential.

? means that a differential is not known to occur.

*drtarget*: Value of an algebraic Novikov differential.

**algNovikov-E3.csv:** Each line of the file corresponds to an element in the algebraic Novikov  $E_3$ -page. This data is used to produce the chart appearing in [2]. This file takes the same format as **algNovikov-E2.csv**.

**algNovikov-E4.csv:** Each line of the file corresponds to an element in the algebraic Novikov  $E_4$ -page. This data is used to produce the chart appearing in [2]. This file takes the same format as **algNovikov-E2.csv**.

**algNovikov-E5.csv:** Each line of the file corresponds to an element in the algebraic Novikov  $E_5$ -page. This data is used to produce the chart appearing in [2]. This file takes the same format as **algNovikov-E2.csv**.

**algNovikov-Einfy.csv:** Each line of the file corresponds to an element in the algebraic Novikov  $E_\infty$ -page. This data is used to produce the chart appearing in [2].

*name:* Human-readable name of an element. (Beware that naming conventions have changed over time.)

*stem:* The stem of an element. This is the horizontal coordinate in a standard Adams chart.

*Adams filtration:* The Adams filtration of an element. This is the vertical coordinate in a standard Adams chart.

*weight:* The motivic weight of an element.

*cell:* Indicates whether an element is detected by the top cell or the bottom cell of the cofiber of  $\tau$ . An empty cell means that an element lies beyond the range that has been analyzed.

B means that an element is in the image in homotopy of inclusion of the bottom cell.

T means that an element maps non-trivially in homotopy under projection to the top cell.

? means that it is not known whether an element is detected by the bottom cell or the top cell.

! means that there is a hidden value of inclusion of the bottom cell or of projection to the top cell.

*h0info:* Information about special behavior of an  $h_0$  extension.

h means that there is a hidden 2 extension.

*h0target:* Value of an  $h_0$  extension. An empty cell indicates that there is no  $h_0$  extension.

loc means that an element is  $h_0$ -periodic.

*h1info:* Information about special behavior of an  $h_1$  extension.

h means that there is a hidden  $\eta$  extension.

*h1target:* Value of an  $h_1$  extension. An empty cell indicates that there is no  $h_1$  extension.

loc means that an element is  $h_1$ -periodic.

*h2info:* Information about special behavior of an  $h_2$  extension.

h means that there is a hidden  $\nu$  extension.

*h2target:* Value of an  $h_2$  extension. An empty cell indicates that there is no  $h_2$  extension.

**algNovikov-Einfy-extn.csv:** Each line of the file corresponds to a hidden extension by 2,  $\eta$ , or  $\nu$  in the Adams-Novikov  $E_\infty$ -page. This data is used to produce the chart appearing in [2]. Not all hidden extensions appear in this file; only the ones that require curved lines are listed.

*source:* Source of an extension. (Beware that naming conventions have changed over time.)

*type:* Type of extension.

**h0** means an extension by 2.

**h1** means an extension by  $\eta$ .

**h2** means an extension by  $\nu$ .

*stem:* The stem of the source of an extension. This is the horizontal coordinate in a standard Adams-Novikov chart.

*Adams filtration:* The Adams-Novikov filtration of the source of an extension. This is the vertical coordinate in a standard Adams-Novikov chart.

*weight:* The motivic weight of an element.

*target:* Target of an extension.

*source<sub>x</sub>, source<sub>y</sub>, target<sub>x</sub>, target<sub>y</sub>:* Used for display purposes in reference to the chart in [2], when a curved hidden extension is necessary. Gives the tangent vectors at the source and target.

## 6. MACHINE GENERATED DATA

**Adams-motivic-E2-machine.csv:** Each line of the file corresponds to an  $\mathbb{F}_2[\tau]$ -module generator of the  $\mathbb{C}$ -motivic Adams  $E_2$ -page.

*name:* An arbitrary name of the form  $\{\mathbf{a-b}\}$  assigned by machine to a generator. The value of  $\mathbf{a}$  is the Adams filtration of the generator, while the value of  $\mathbf{b}$  is an arbitrary number.

*stem:* The stem of an element. This is the horizontal coordinate in a standard Adams chart.

*Adams filtration:* The Adams filtration of an element. This is the vertical coordinate in a standard Adams chart.

*weight:* The motivic weight of an element.

*tautorsion:* Indicates the  $\tau$  module structure of a generator.

0 means that an element is  $\tau$ -periodic.

Any other integer  $k$  means that a generator is annihilated by  $\tau^k$ .

*h0info:* Information about special behavior of an  $h_0$  extension.

An integer  $k$  means that an  $h_0$  extension equals  $\tau^k$  times a generator.

*h0target:* Value of an  $h_0$  extension. An empty cell indicates that there is no  $h_0$  extension.

*h1info:* Information about special behavior of an  $h_1$  extension.

An integer  $k$  means that an  $h_1$  extension equals  $\tau^k$  times a generator.

*h1target:* Value of an  $h_1$  extension. An empty cell indicates that there is no  $h_1$  extension.

*h2info:* Information about special behavior of an  $h_2$  extension.

An integer  $k$  means that an  $h_2$  extension equals  $\tau^k$  times a generator.

*h2target:* Value of an  $h_2$  extension. An empty cell indicates that there is no  $h_2$  extension.

*h3info:* Information about special behavior of an  $h_3$  extension.

An integer  $k$  means that an  $h_3$  extension equals  $\tau^k$  times a generator.

*h3target:* Value of an  $h_3$  extension. An empty cell indicates that there is no  $h_3$  extension.

**algNovikov-machine.csv:** Each line of the file corresponds to an element in the algebraic Novikov  $E_2$ -page.

*name:* An arbitrary name assigned by machine to a generator.

*stem:* The stem of an element. This is the horizontal coordinate in a standard Adams chart.

*Adams filtration:* The Adams filtration of an element. This is the vertical coordinate in a standard Adams chart.

*weight:* The motivic weight of an element.

*h0target:* Value of an  $h_0$  extension in the Adams-Novikov  $E_2$ -page. An empty cell indicates that there is no  $h_0$  extension. Beware that these are not extensions in the algebraic Novikov  $E_2$ -page.

*h1target:* Value of an  $h_1$  extension in the Adams-Novikov  $E_2$ -page. An empty cell indicates that there is no  $h_1$  extension. Beware that these are not extensions in the algebraic Novikov  $E_2$ -page.

**loc** indicates that an element is  $h_1$ -periodic.

*h2target:* Value of an  $h_2$  extension in the Adams-Novikov  $E_2$ -page. An empty cell indicates that there is no  $h_2$  extension. Beware that these are not extensions in the algebraic Novikov  $E_2$ -page.

*h3target:* Value of an  $h_3$  extension in the Adams-Novikov  $E_2$ -page. An empty cell indicates that there is no  $h_3$  extension. Beware that these are not extensions in the algebraic Novikov  $E_2$ -page.

*drinfo:* Information about an algebraic Novikov differential.  
An integer value  $r$  indicates a  $d_r$  differential.

*drvalue:* Value of an algebraic Novikov  $d_r$  differential.

**ANSS-cofiber-2-machine.csv:** Each line of the file corresponds to an element in the Adams-Novikov  $E_2$ -page for the cofiber of 2.

*name:* An arbitrary name assigned by machine to a generator.

*cell:*

B indicates that an element lies in the image of the bottom cell.

T indicates that an element projects non-trivially to the top cell.

*image:* Indicates the pre-image of an element under inclusion of the bottom cell, or the value under projection to the top cell.

*stem:* The stem of an element. This is the horizontal coordinate in a standard Adams chart.

*Adams filtration:* The Adams filtration of an element. This is the vertical coordinate in a standard Adams chart.

*weight:* The motivic weight of an element.

*Adams-Novikov filtration:* The Adams-Novikov filtration of an element. This is the vertical coordinate in a standard Adams-Novikov chart.

*h1target:* Value of an extension by  $[1-0]$ , i.e., by  $h_1$ . An empty cell indicates that there is no extension.

*h2target:* Value of an extension by  $[1-1]$ , i.e., by  $h_2$ . An empty cell indicates that there is no extension.

*h3target:* Value of an extension by  $[1-2]$ , i.e., by  $h_3$ . An empty cell indicates that there is no extension.

*theta2:* Value of an extension by  $v_2^{-1}[1-0]$ , i.e., by the element that maps to  $h_2^2$  under projection to the top cell.

*theta3:* Value of an extension by  $[1-3]$ , i.e., by the element that maps to  $h_3^2$  under projection to the top cell.

*theta4:* Value of an extension by  $[1-4]$ , i.e., by the element that maps to  $h_4^2$  under projection to the top cell.

*theta5:* Value of an extension by  $[1-5]$ , i.e., by the element that maps to  $h_5^2$  under projection to the top cell.



**ANSS-conversion-machine.csv:** Converts between arbitrary names for elements in the Adams-Novikov  $E_2$ -page used in the two previous machine-generated files.

*ANSS-cofiber-2-machine:* Used in the *image* column of **ANSS-cofiber-2-machine.csv**.

*algNovikov-machine:* Used in the *name* column of **algNovikov-machine.csv**.

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- [1] Daniel C. Isaksen, Guozhen Wang, and Zhouli Xu, *More stable stems* (2019), preprint.
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