Lecture 3: Collective communication

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CME 194
Stanford University

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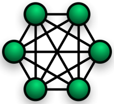
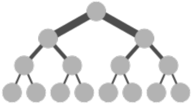
Outline

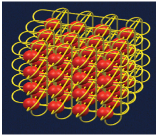
Basic communication cost model

Recall $\alpha + \beta n$ model:

- Can only send one message at a time
- Cost depends on *latency*, α, message length, *n*, and *bandwidth*, $1/\beta$

Will assume network is fully-connected, 

but, in practice, topology is usually *fat-tree* 

or *multi-dimensional torus* 

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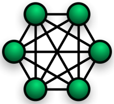

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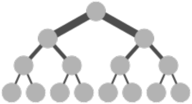

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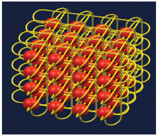

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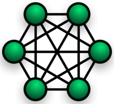

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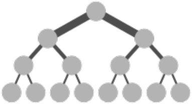

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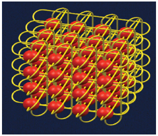

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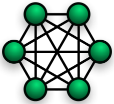
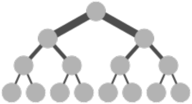
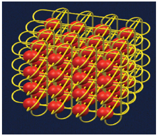
Will assume network is fully-connected, 

but, in practice, topology is usually *fat-tree* 

or *multi-dimensional torus* 

Binomial tree broadcast (short messages)

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There are $\log_2 p$ stages

Total cost is $(\alpha + \beta n) \log_2 p$

$\beta n \log_2 p$ should be avoided when n is large...

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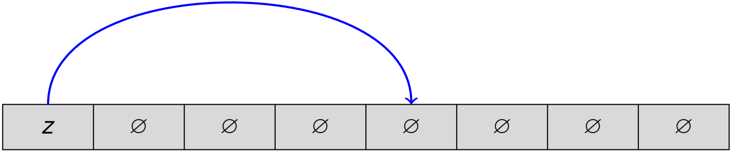

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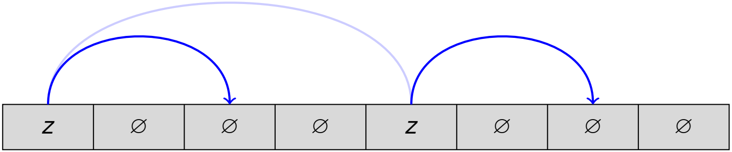

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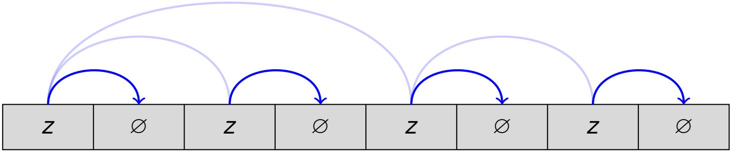

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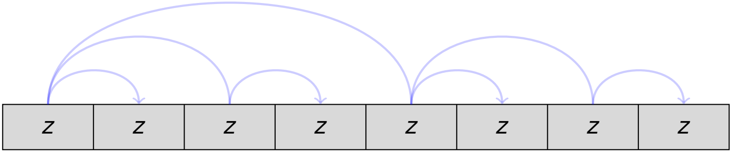

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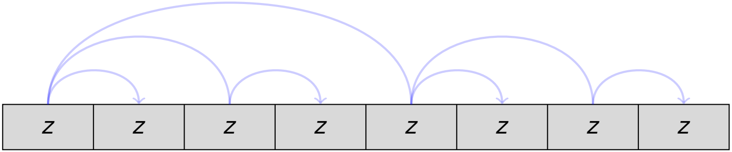

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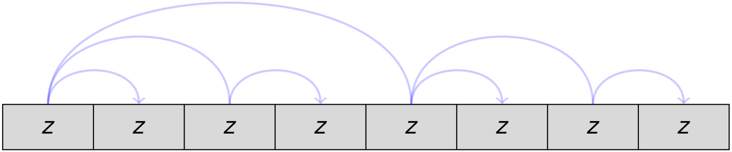

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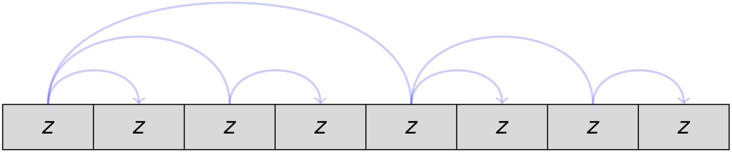

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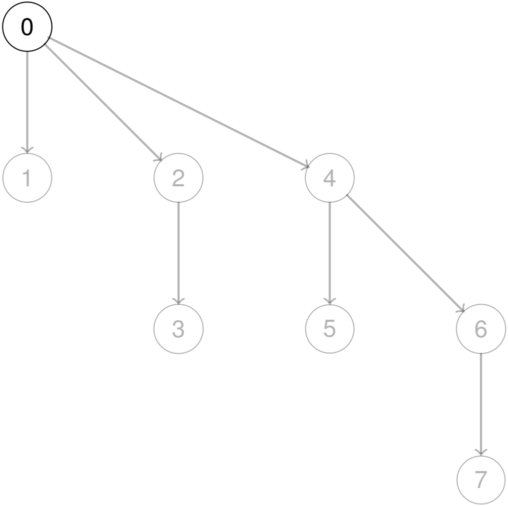

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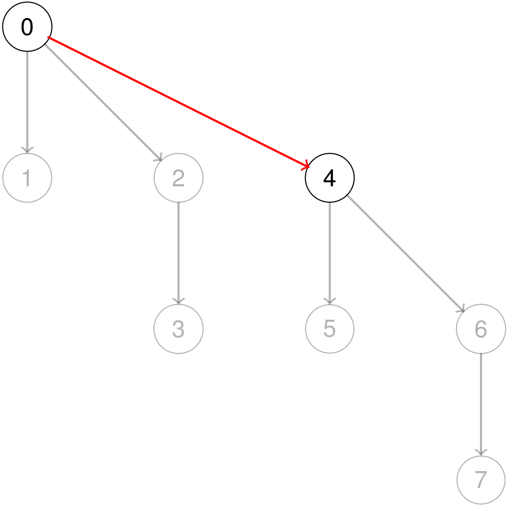

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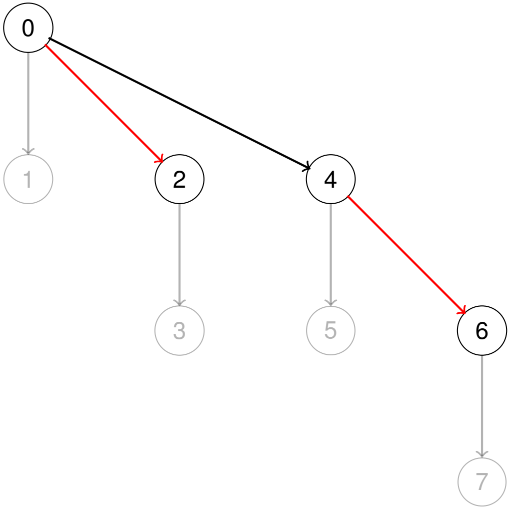

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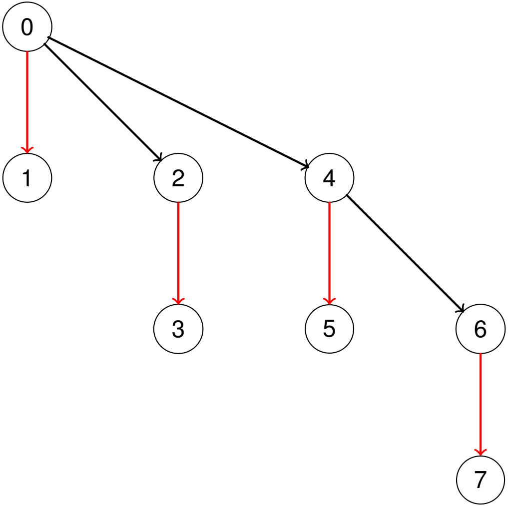

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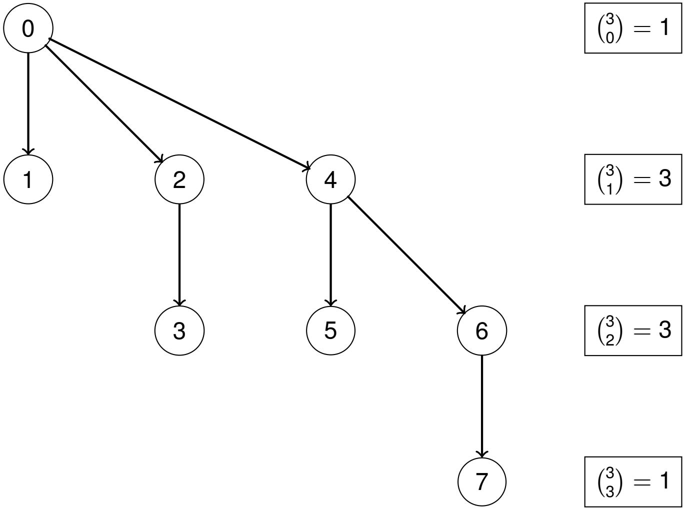

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Lower volume cost than $\alpha \log_2 p + \beta n \log_2 p$ binomial Broadcast

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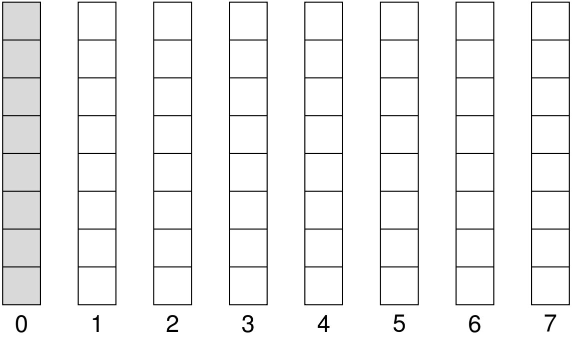

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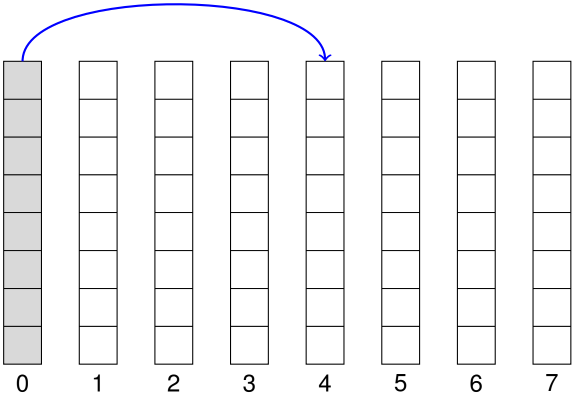

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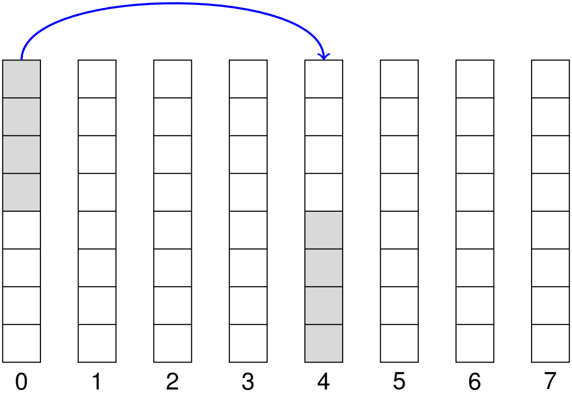

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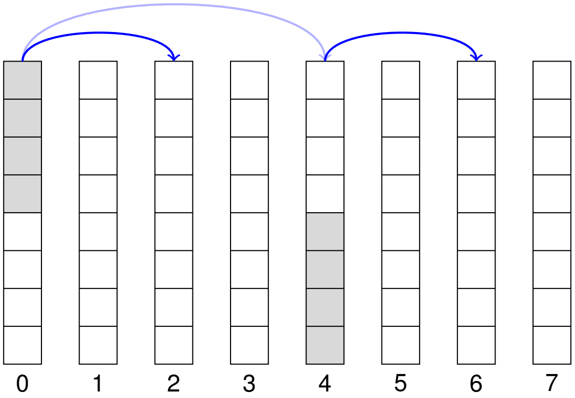

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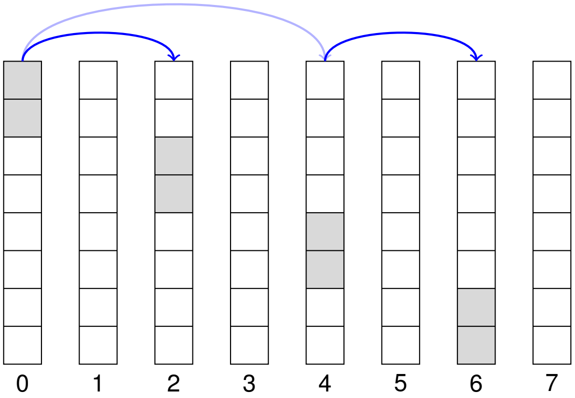

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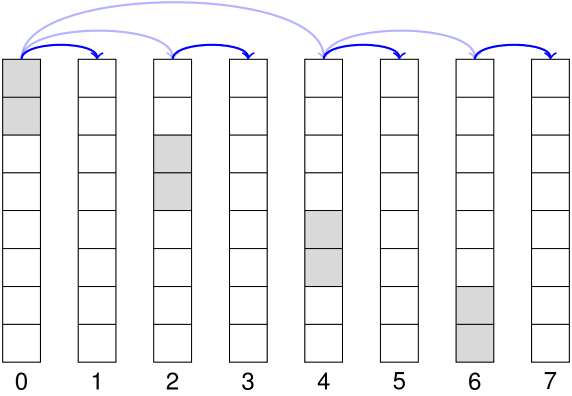

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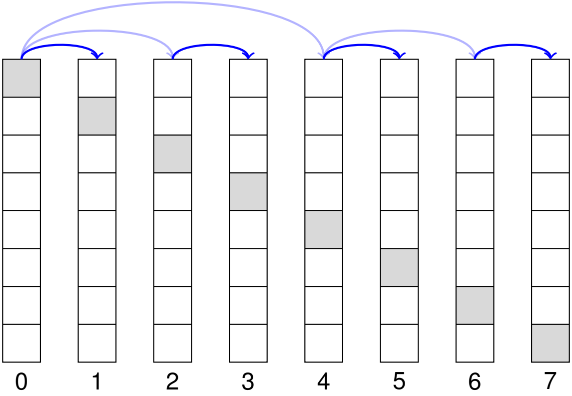

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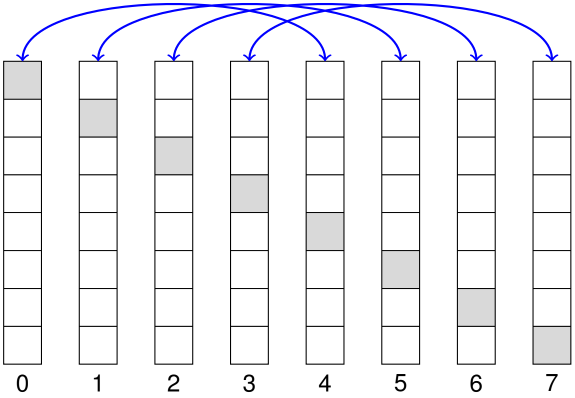

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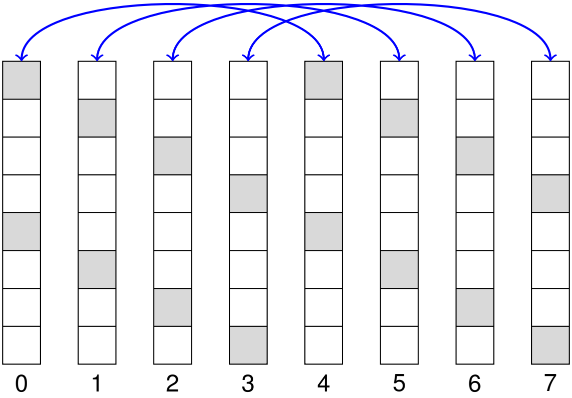

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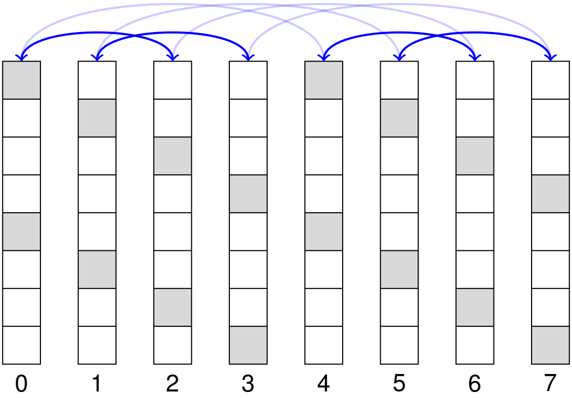

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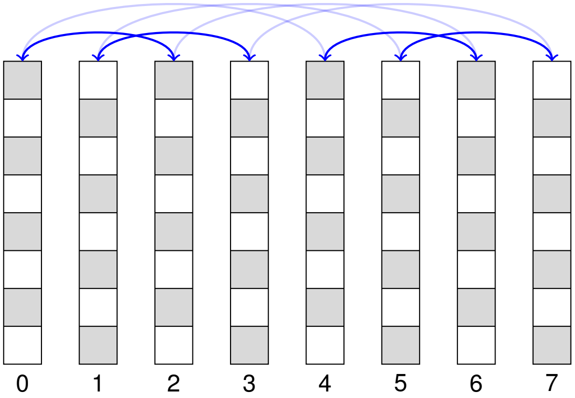

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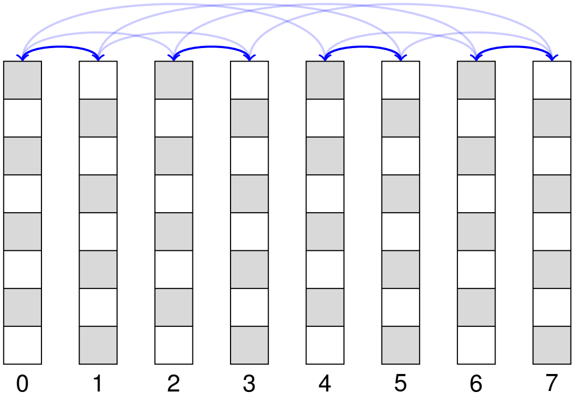

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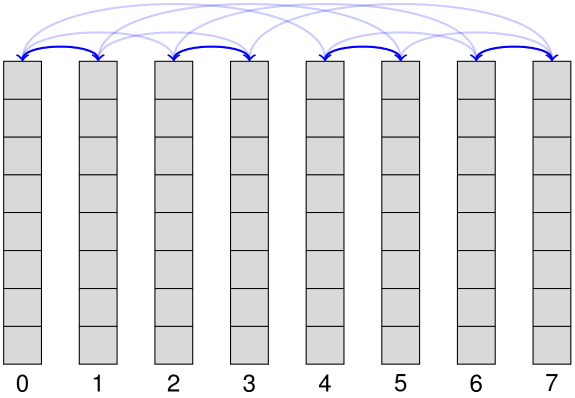

MPI_Bcast

`MPI_Bcast( void* buffer, int n, MPI_Datatype type, int root, MPI_Comm comm )`

Broadcasts *n* items of type `type` from process with rank `root` with respect to communicator `comm`.

Typically switches from binomial to Fox/vdG algorithm when *n* is sufficiently large.

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Reduce

Each process has vector of same length and we would like to give sum to single process:

$$x = \sum_{i=0}^{p-1} \hat{x}_i$$

Can run Broadcast algorithms in reverse (with summation).

Therefore, same communication costs!

Can of course replace summation with other binary operations, e.g., entrywise maximum or product

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MPI_Reduce( void* sendBuf, void* recvBuf, int
n, MPI_Datatype type, MPI_Op op, int root,
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Reduces each process in communicator `comm`'s *n* items from `sendBuf` into process `root`'s `recvBuf` using binary operation implied by `op`, e.g., `MPI_SUM` or `MPI_MAX`.

Related routines are `MPI_Allreduce` (give every process the result) and `MPI_Reduce_scatter` (give every process a subset of the result)

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Example: distributed inner-product

$$\alpha := y^H x$$

- Each process gets subset of x and y, e.g., x_i and y_i
- Then $\alpha = \sum_{i=0}^{p-1} y_i^H x_i = \sum_{i=0}^{p-1} \hat{\alpha}_i$
- Have each process form $\hat{\alpha}_i := y_i^H x_i$ then Reduce
- Work is $2\frac{n}{p} + (\alpha + \beta) \log_2 p$ with binomial Reduce

Setting $y = x$ yields parallel $\|x\|_2$.

Setting $y = \text{ones}(n, 1)$ yields parallel $\|x\|_1$.

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Example: distributed inner-product

$$\alpha := y^H x$$

- Each process gets subset of x and y, e.g., x_i and y_i
- Then $\alpha = \sum_{i=0}^{p-1} y_i^H x_i = \sum_{i=0}^{p-1} \hat{\alpha}_i$
- Have each process form $\hat{\alpha}_i := y_i^H x_i$ then Reduce
- Work is $2\frac{n}{p} + (\alpha + \beta)\log_2 p$ with binomial Reduce

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Example: distributed $\| \cdot \|_\infty$

$$\|x\|_\infty = \max_j \chi_j$$

If we give each process subset of x, say x_i, then

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- Generate random distributed vector x and compute its two-norm
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