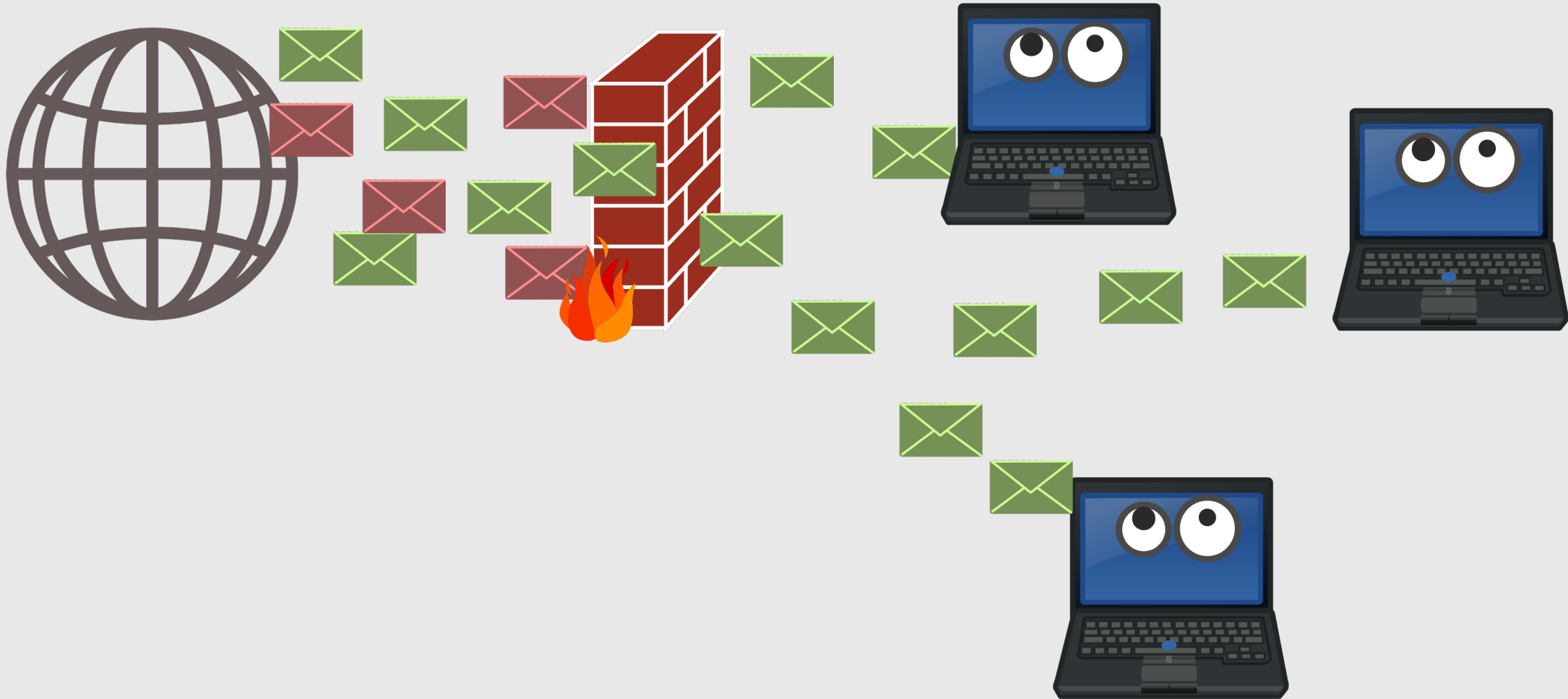


# Self-Adjusting Partially Ordered Lists

Vamsi Addanki, Macej Pacut, **Arash Pourdamghani**, Gábor Rétvári,  
Stefan Schmid and Juan Vanerio

## A Real World Need



# A Real World Need




N	Protocol	Src. IP	Dst. IP	Src. Port	Dst. Port	Action
1	TCP	10.12.12.0/24	20.0.0.1/32	ANY	80	DENY
2	TCP	0.0.0.0/0	20.0.0.1/32	ANY	80	ACCEPT
3	IP	0.0.0.0/0	20.0.0.1/32			DENY
4	UDP	0.0.0.0/0	0.0.0.0/0	1000-2000	1000-2000	ACCEPT
5	UDP	20.0.0.0/24	10.0.10.0/24	ANY	3306	ACCEPT
6	TCP	10.12.12.0/24	0.0.0.0/0	21	21	DENY
7	IP	10.0.0.0/16	20.0.0.0/20			ACCEPT
8	IP	0.0.0.0/0	0.0.0.0/0			DENY

# Packet Classification

N	Protocol	Src. IP	Dst. IP	Src. Port	Dst. Port	Action
1	TCP	10.12.12.0/24	20.0.0.1/32	ANY	80	DENY
2	TCP	0.0.0.0/0	20.0.0.1/32	ANY	80	ACCEPT
3	IP	0.0.0.0/0	20.0.0.1/32			DENY
4	UDP	0.0.0.0/0	0.0.0.0/0	1000-2000	1000-2000	ACCEPT
5	UDP	20.0.0.0/24	10.0.10.0/24	ANY	3306	ACCEPT
6	TCP	10.12.12.0/24	0.0.0.0/0	21	21	DENY
7	IP	10.0.0.0/16	20.0.0.0/20			ACCEPT
8	IP	0.0.0.0/0	0.0.0.0/0			DENY

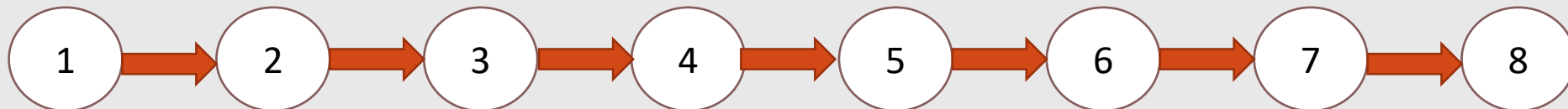
# Packet Classification & Orders



N	Protocol	Src. IP	Dst. IP	Src. Port	Dst. Port	Action
1	TCP	10.12.12.0/24	20.0.0.1/32	ANY	80	DENY
2	TCP	0.0.0.0/0	20.0.0.1/32	ANY	80	ACCEPT
3	IP	0.0.0.0/0	20.0.0.1/32			DENY
4	UDP	0.0.0.0/0	0.0.0.0/0	1000-2000	1000-2000	ACCEPT
5	UDP	20.0.0.0/24	10.0.10.0/24	ANY	3306	ACCEPT
6	TCP	10.12.12.0/24	0.0.0.0/0	21	21	DENY
7	IP	10.0.0.0/16	20.0.0.0/20			ACCEPT
8	IP	0.0.0.0/0	0.0.0.0/0			DENY

## Model: A List

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8	IP	0.0.0.0/0	0.0.0.0/0			DENY



# Model: A List

N	Protocol	Src. IP	Dst. IP	Src. Port	Dst. Port	Action
1	TCP	10.12.12.0/24	20.0.0.1/32	ANY	80	DENY
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5	UDP	20.0.0.0/24	10.0.10.0/24	ANY	3306	ACCEPT
6	TCP	10.12.12.0/24	0.0.0.0/0	21	21	DENY
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8	IP	0.0.0.0/0	0.0.0.0/0			DENY



# Temporal structure

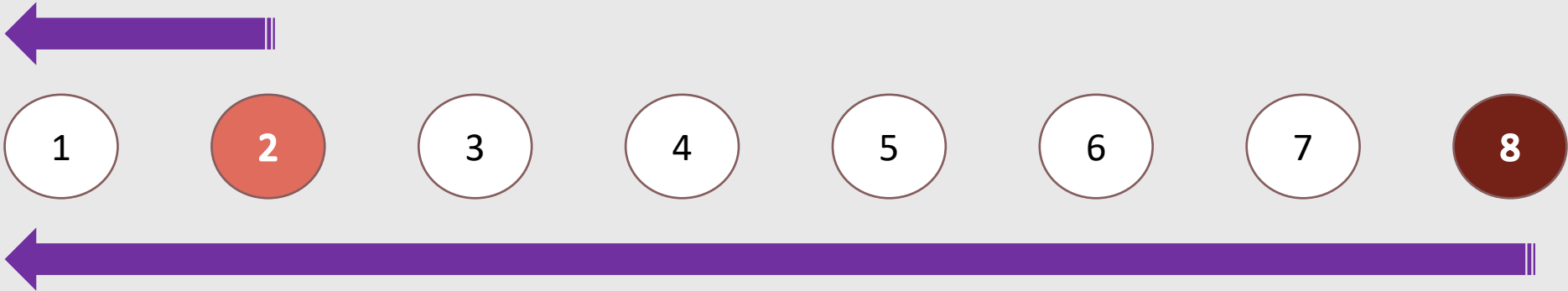
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6	TCP	10.12.12.0/24	0.0.0.0/0	21	21	DENY
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8	IP	0.0.0.0/0	0.0.0.0/0			DENY





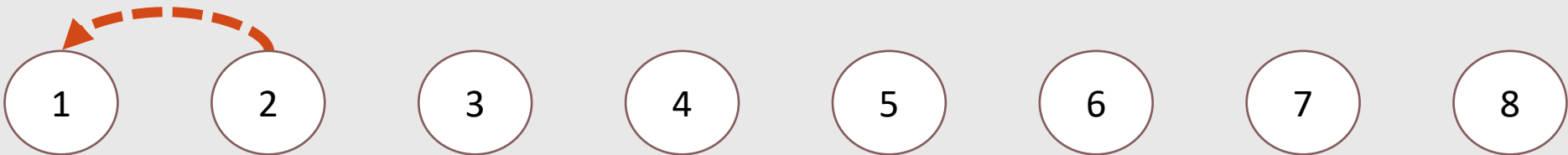
# Is Self-adjustment Possible?

N	Protocol	Src. IP	Dst. IP	Src. Port	Dst. Port	Action
1	TCP	10.12.12.0/24	20.0.0.1/32	ANY	80	DENY
2	TCP	0.0.0.0/0	20.0.0.1/32	ANY	80	ACCEPT
3	IP	0.0.0.0/0	20.0.0.1/32			DENY
4	UDP	0.0.0.0/0	0.0.0.0/0	1000-2000	1000-2000	ACCEPT
5	UDP	20.0.0.0/24	10.0.10.0/24	ANY	3306	ACCEPT
6	TCP	10.12.12.0/24	0.0.0.0/0	21	21	DENY
7	IP	10.0.0.0/16	20.0.0.0/20			ACCEPT
8	IP	0.0.0.0/0	0.0.0.0/0			DENY



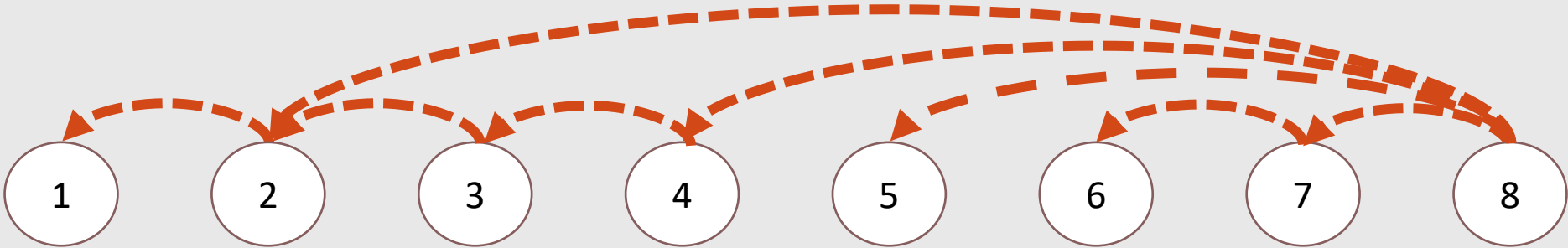
# Enforced Orders

N	Protocol	Src. IP	Dst. IP	Src. Port	Dst. Port	Action
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4	UDP	0.0.0.0/0	0.0.0.0/0	1000-2000	1000-2000	ACCEPT
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6	TCP	10.12.12.0/24	0.0.0.0/0	21	21	DENY
7	IP	10.0.0.0/16	20.0.0.0/20			ACCEPT
8	IP	0.0.0.0/0	0.0.0.0/0			DENY



# Enforced Orders

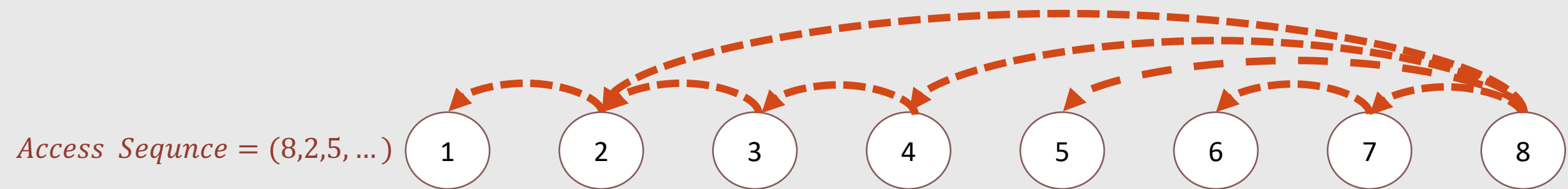
N	Protocol	Src. IP	Dst. IP	Src. Port	Dst. Port	Action
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2	TCP	0.0.0.0/0	20.0.0.1/32	ANY	80	ACCEPT
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6	TCP	10.12.12.0/24	0.0.0.0/0	21	21	DENY
7	IP	10.0.0.0/16	20.0.0.0/20			ACCEPT
8	IP	0.0.0.0/0	0.0.0.0/0			DENY



## Formal Question

### □ Inputs:

- A set of **items** in a link list
- A set of “enforced” **orderings** between items
- An **access sequence** revealed over time



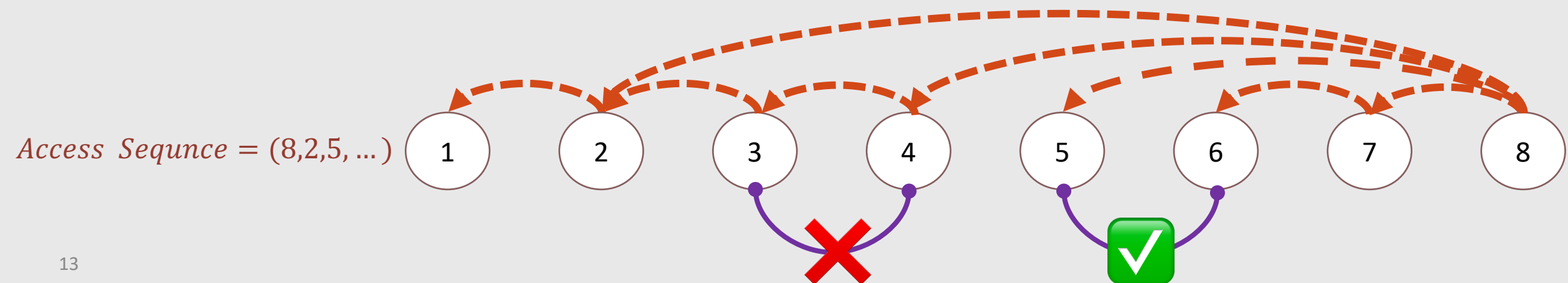
# Formal Question

## □ Inputs:

- A set of **items** in a link list
- A set of “enforced” **orderings** between items
- An **access sequence** revealed over time

## □ Operations:

- Swapping position of two items while respecting orders



# Formal Question

## ❑ Inputs:

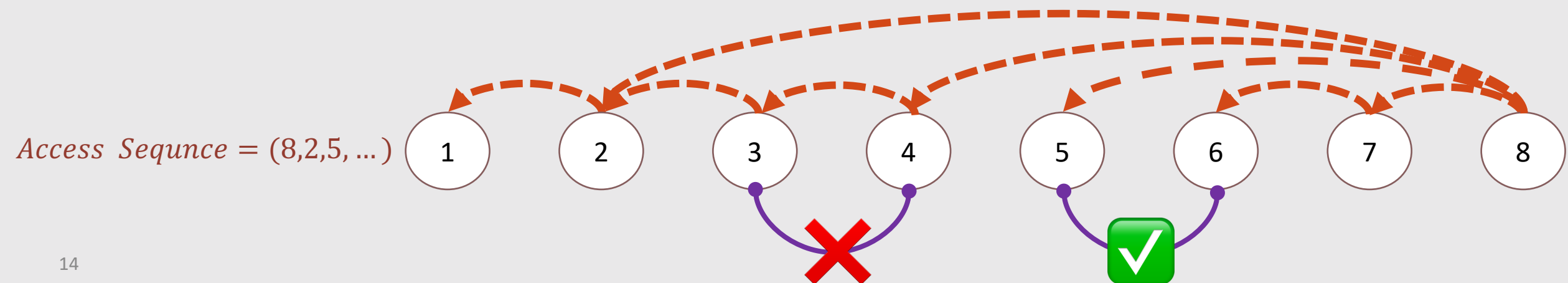
- A set of **items** in a link list
- A set of “enforced” **orderings** between items
- An **access sequence** revealed over time

## ❑ Operations:

- Swapping position of two items while respecting orders

## ❑ Objective:

- Minimizing the **total (access + reconfiguration) cost** while respecting orders



# Our Approach

- ❑ Going beyond worst-case analysis

## Our Approach

- ❑ Going beyond worst-case analysis
- ❑ **Constant competitiveness**, given a constant  $c$ :

Total cost of an algorithm  $\leq_{\forall \text{ inputs}} c \cdot$  Total cost of the optimal offline algorithm



## Our Approach

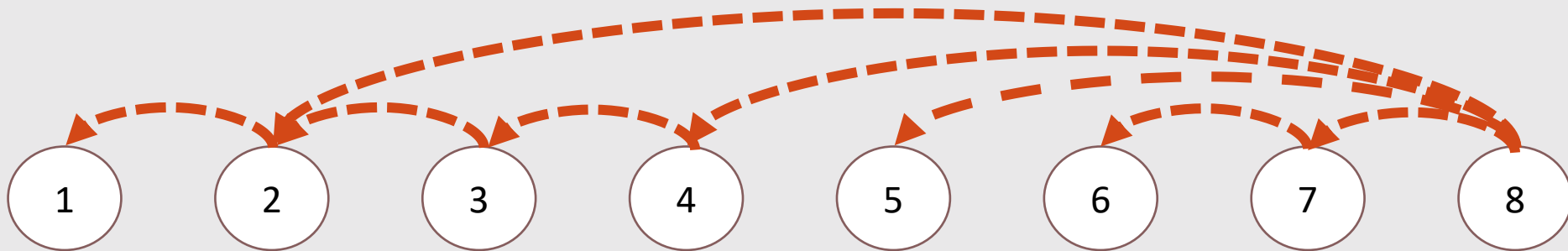
- ❑ Going beyond worst-case analysis
- ❑ **Constant competitiveness**, given a constant  $c$ :

Total cost of an algorithm  $\leq_{\forall \text{ inputs}} c \cdot$  Total cost of the optimal offline algorithm

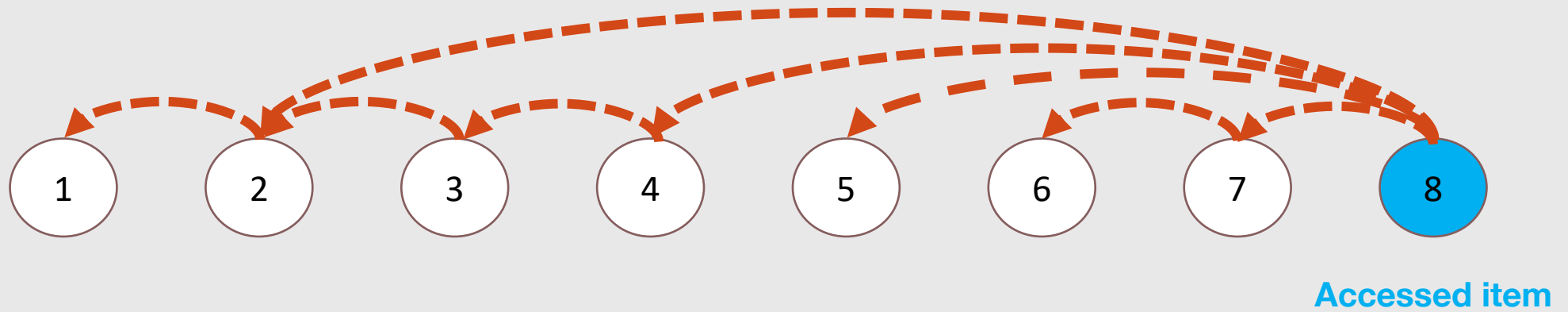
- ❑ Our observation:

**Local moves ensure constant competitiveness!**

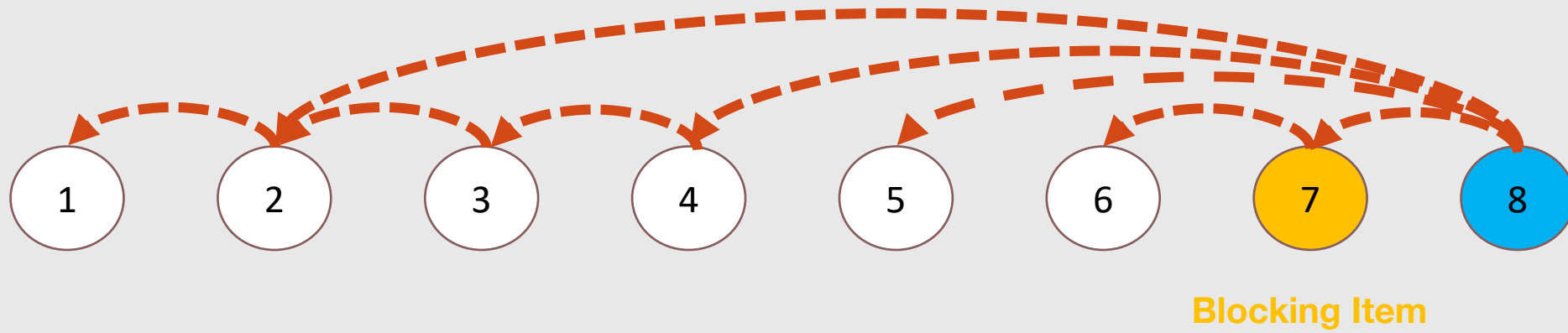
# Deterministic Algorithm: Move-Recursively-Forward (MRF)



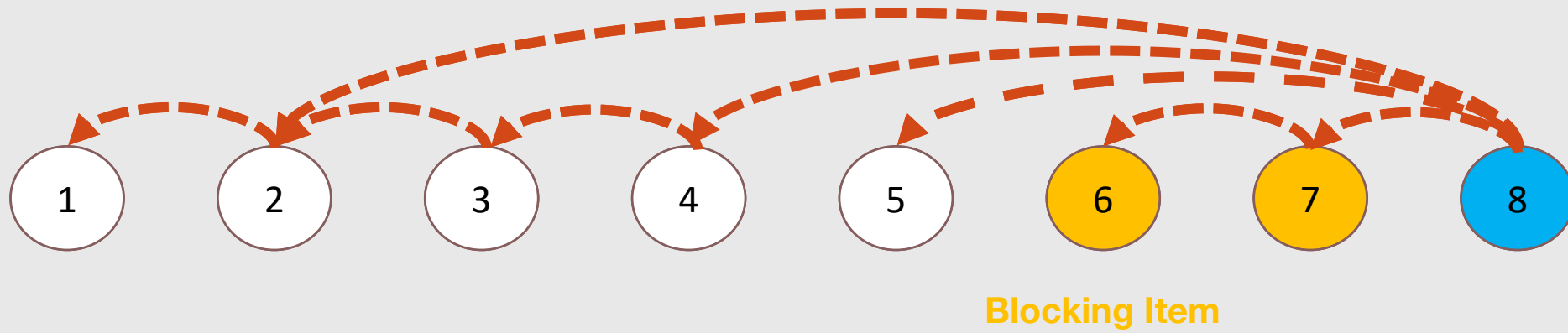
# Deterministic Algorithm: Move-Recursively-Forward (MRF)



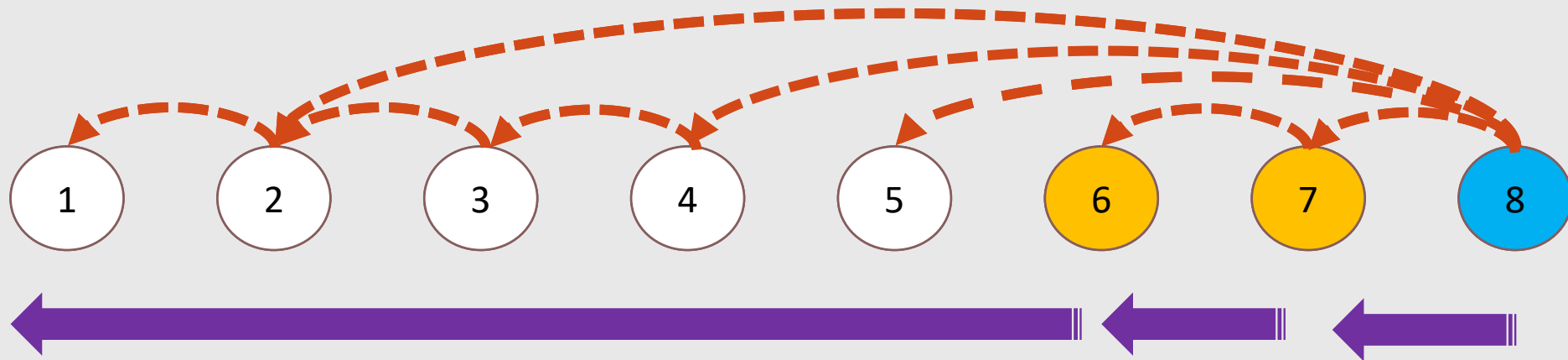
# Deterministic Algorithm: Move-Recursively-Forward (MRF)



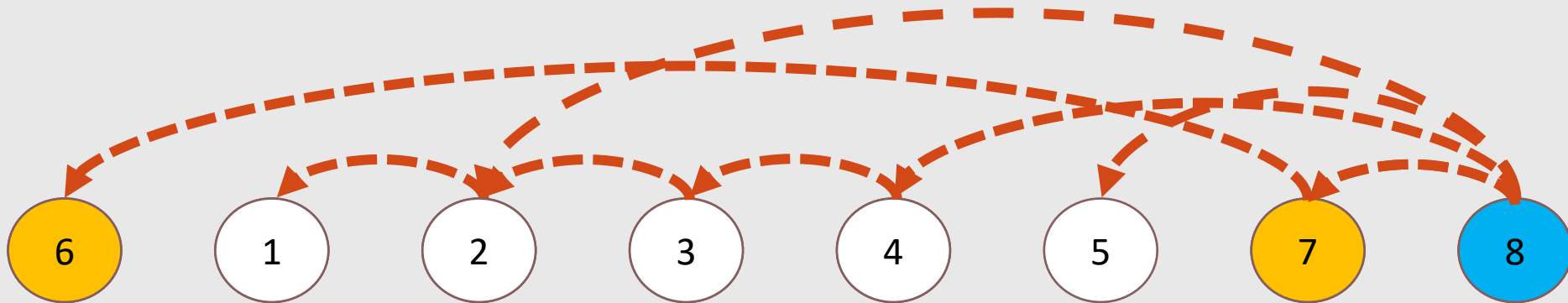
# Deterministic Algorithm: Move-Recursively-Forward (MRF)



## Deterministic Algorithm: Move-Recursively-Forward (MRF)



## Deterministic Algorithm: Move-Recursively-Forward (MRF)



## MRF Analysis

□ Upper bound:

MRF is strictly 4-competitive.



## MRF Analysis

- ❑ Upper bound:

MRF is strictly 4-competitive.

- ❑ General proof idea:

Potential function analysis based on inversions

# MRF Analysis

❑ Upper bound:

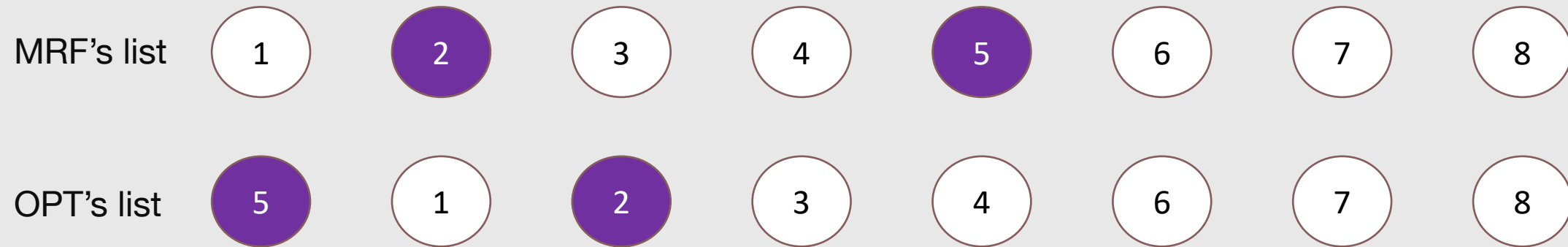
MRF is strictly 4-competitive.

❑ General proof idea:

Potential function analysis based on inversions

❑ An inversion:

Mismatch between MRF's list and OPT's list



## MRF Analysis

- ❑ Upper bound:

MRF is strictly 4-competitive.

- ❑ General proof idea:

Potential function analysis based on inversions

- ❑ An inversion:

Mismatch between MRF's list and OPT's list

- ❑ Challenge:

Identifying set of inversions after MRF moves

## Lower Bound Analysis

- ❑ Lower bound:
  - For deterministic case,
  - Given an  $\epsilon$  based on structure of partial orders

Any online algorithm is  $3 - \epsilon$  competitive.

## Lower Bound Analysis

### ❑ Lower bound:

- For deterministic case,
- Given an  $\epsilon$  based on structure of partial orders

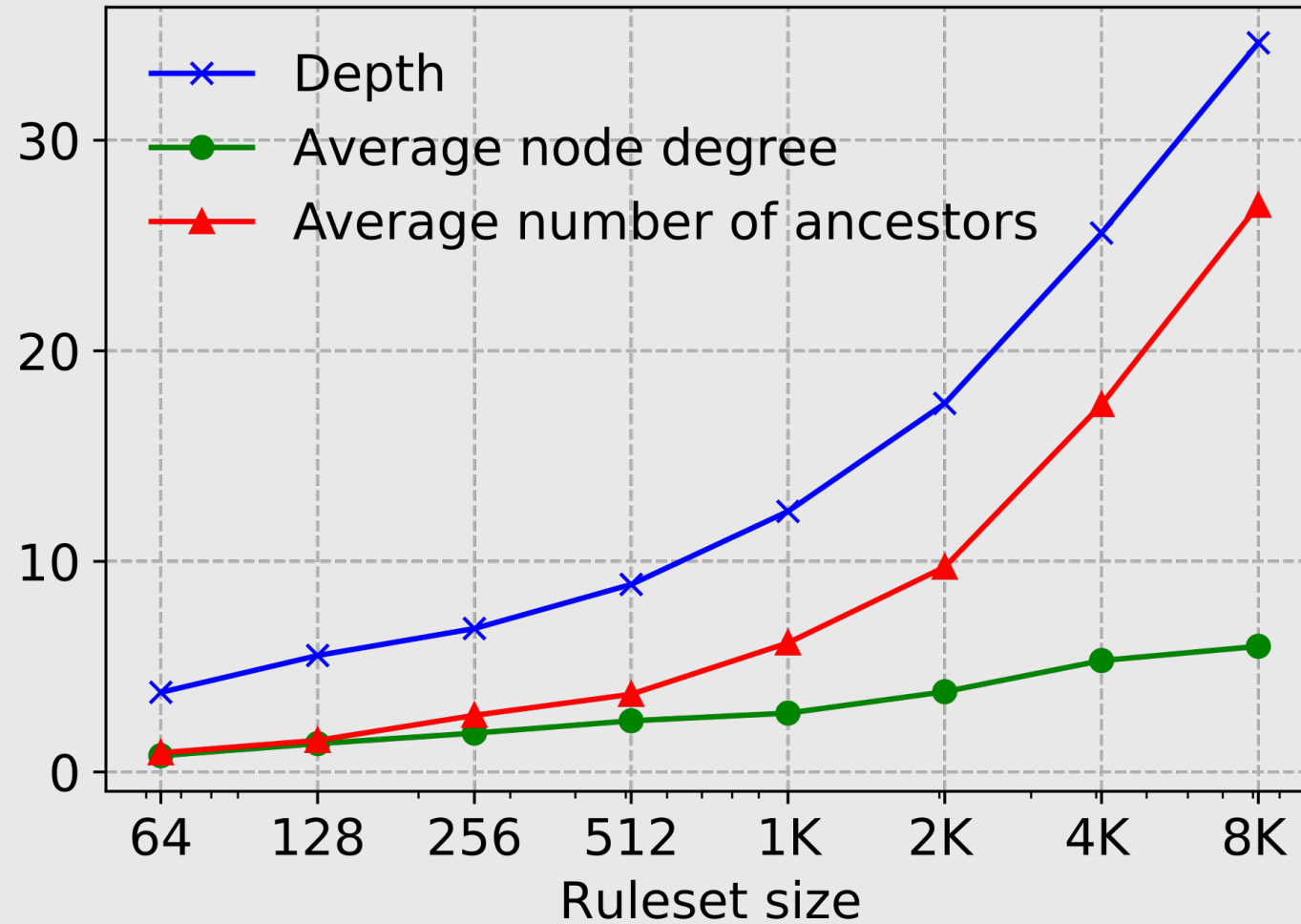
Any online algorithm is  $3 - \epsilon$  competitive.

### ❑ Proof idea:

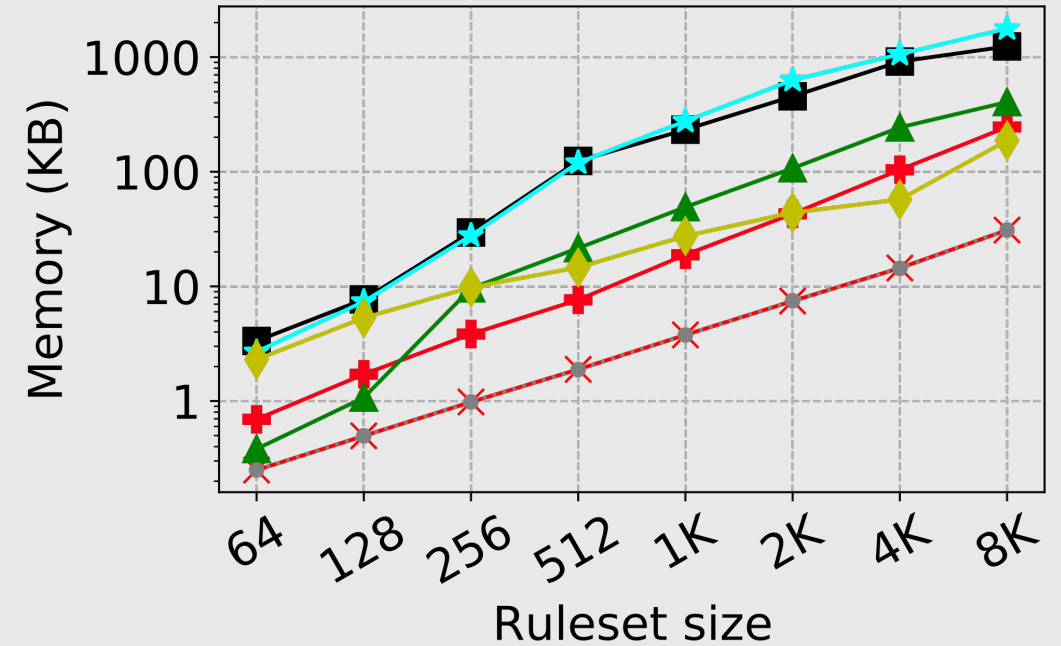
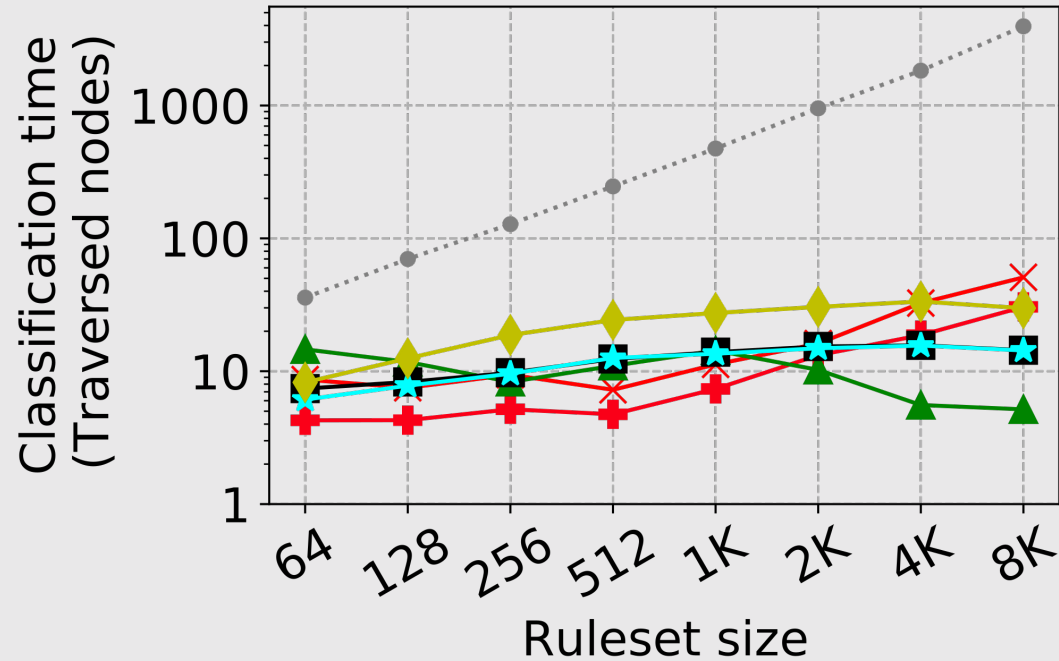
Constructing two special request sequences.

- One for algorithms with “static” strategy
- One for algorithms with “dynamic” strategy

## Real World Performance: Dataset



## Real World Performance: High Locality



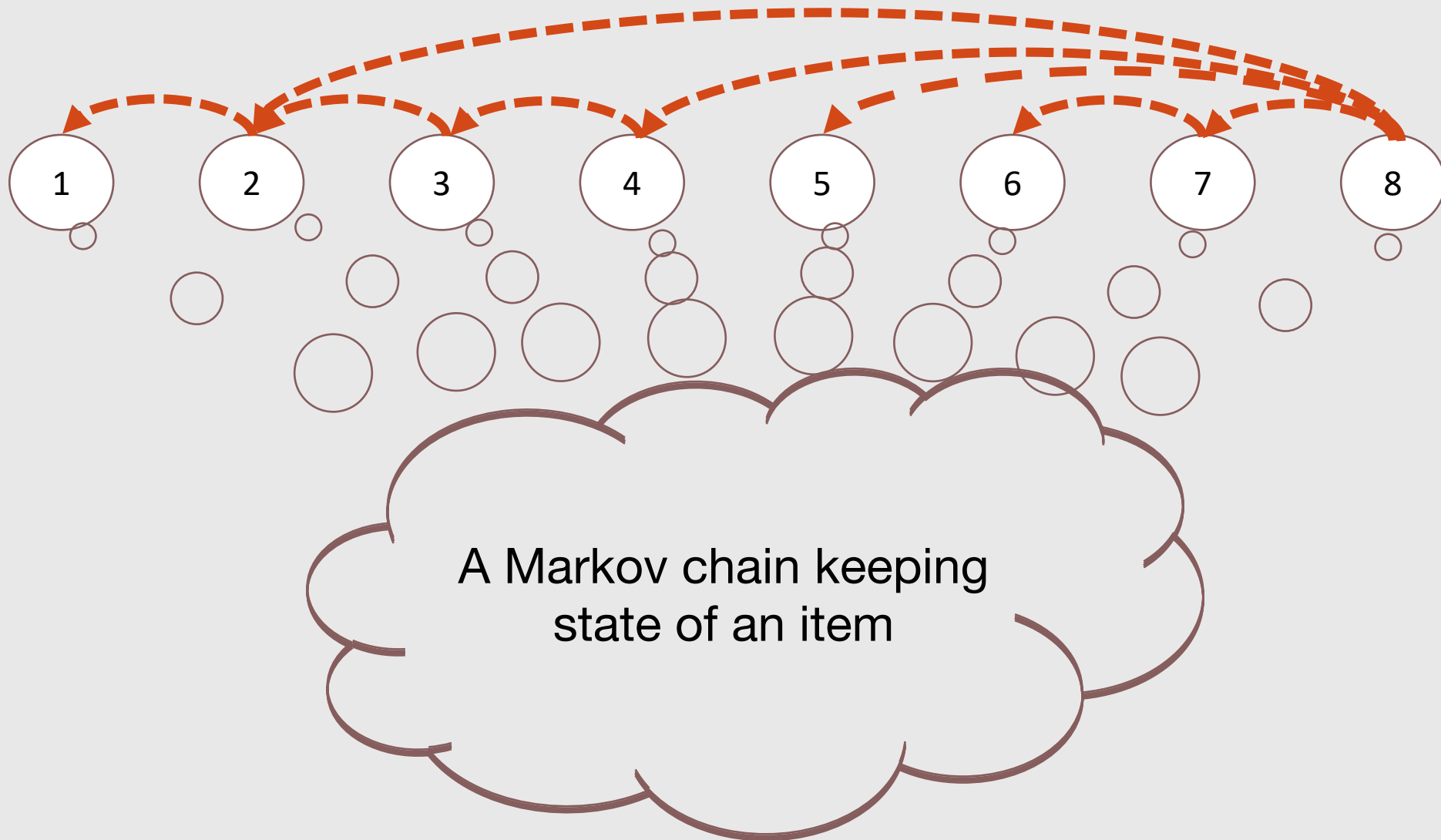
-----●----- Static-List      × MRF      —▲— Efficuts  
—■— Hypercuts      —★— Hicuts      —◆— Cutsplit

# Transforming Traditional List Access Approaches

Algorithm	Randomized	Competitive Factor
Move-to-Front[Sleator & Tarjan, Commun. ACM'85]	✗	4
BIT [Reingold et al. Algorithmica 94]	✓	3
COUNTER [Reingold et al. Algorithmica 94]	✓	2.75
RANDOM-RESET [Reingold et al. Algorithmica 94]	✓	$\sqrt{7}$



## Randomized Version



# Thank You

