In The Name of God the Compassionate the Merciful

Engineering Mathematics Final Project

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Project Description

This project consists of two parts:

1. **Generate ECG signals with Fourier series**
2. **Solve the PDE equation**

The explanation of each is as follows on the next pages.

You can carry out the project in groups of one or two, and before starting the project, each group **must** announce its grouping to the teaching assistants.

Implement projects in either **Matlab** or **Python** programming languages. Projects should have a complete **report** containing any information about implementations and results, and it is also a **key** part of the project's grading.

After the deadline, each group should explain its project with the **oral** presentation that will be announced later.

The project's deadline is **99/11/10** and will not be extended!

Stay successful and safe!

Part 1 – Generate ECG signals with Fourier series

Electrocardiogram (ECG) is the depiction of the hearts recorded electrical activity, obtained by attaching electrodes that are specifically designed to place at particular points of human body.

An electrocardiogram is composed of several specific parts. Those parts are connected by an isoelectric line, a section where the heart is inactive for a short time, and this part of the ECG signal is given a zero voltage level.

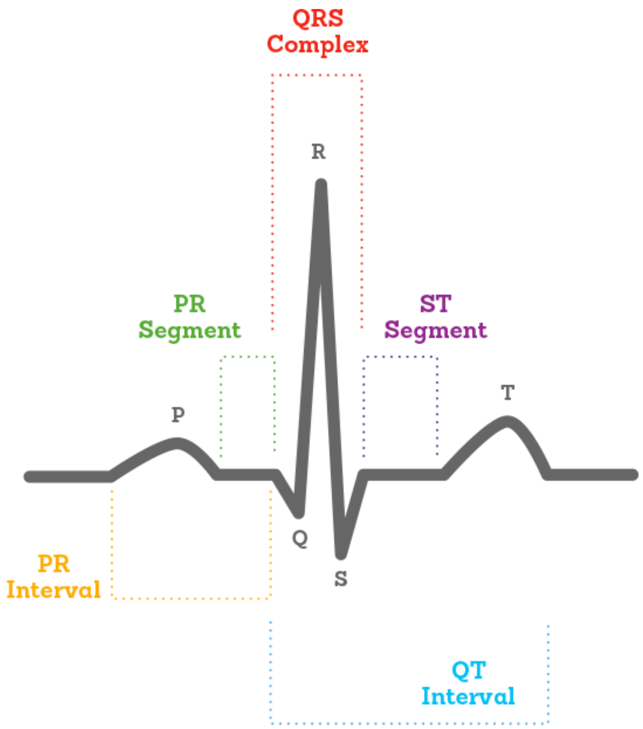

Figure 1 – ECG signal

P-wave

The p wave is the first element of the ECG signal. This positive wave is given by activities sinoatrial node and indicates the atria's electrical activation, i.e., their depolarization. The amplitude can take different values from 0, when this wave is not observable, to a value of 0.3 mV. The duration is about 100 ms.

P-R interval

This interval includes atrial activity, i.e., the P wave, the passage of impulses through the A V node, i.e., the PR segment, and ends at the beginning of the activity chambers, i.e., in the ventricular

QRS complex. We can say that this interval describes the atrioventricular conduction. The length of this interval is in the healthy population ranges between 120 and 200 ms.

QRS complex

The QRS complex is a graphical representation of depolarization of the ventricles. It consists of waves Q, R, and S. Q wave, which is the first negative part of the electrocardiogram, reaching a quarter of the amplitude of the new wave R and takes less than 30 ms and its amplitude is about 25 percentage of R Wave. R wave, positive displacement follow-up wave Q, can size up to 1.6 mV, but can also be completely missing, which almost always refers to the heart's pathological state. The last wave of the complex is also negative and is called S. Its amplitude value is 0.8 mV, and takes about 50 ms. Complex QRS is given by range 50 to 110 ms.

S-T segment

S - T segment is normally horizontal and isoelectric segment between the QRS complex and the T wave which its duration is about 50 ms.

T-wave

In this part of the heart, the muscle is electrically returned to the idle state. This process is also called repolarization. The amplitude of this wave takes to 0.5 - 0.8 mV and lasts until 0.42 ms.

What is the approach of project?

As we know, The Fourier series is used for the decomposition of periodic functions. ECG signal is not periodic because the heart is not a machine. No two electrocardiograph records are the same; there are artifacts and various changes, and the very heartbeat is not regular. We can describe an ECG signal by repetitive function.

In this project, this fact is overlooked, and the ECG signal will be seen as a periodic function, and it will be decomposed using the Fourier series. **Firstly** you should analyze figure 1 and observe that it composes a repeating sine wave (P waves, T, U) and triangular (QRS complex) form, which can further simplify and divide into nine parts. **You** should describe every part of that as a mathematical function.

Then by combining the Fourier series of every part, you can make the desired ECG signal.

Bonus: After generating your ECG signal, you can use the movie for Matlab implementation and the animation for python to better visualize the output and make it similar to real ECG signals.

Part 2 – Solve the PDE equation

In this part, you should solve **one** of the following PDE equations and plot the solution in the x-u plane for various t.

After that, investigate the behavior as $t \to \infty$, which is the steady-state behavior of the equation.

I) Heat equation with initial-boundary-value:

$$\begin{aligned} &u_t = 2u_{xx} \\ &u(x,0) = -\sin(3\pi x) + \frac{1}{4}\sin(6\pi x) \\ &u(0,t) = u(1,t) = 0. \end{aligned}$$

II) Heat equation with initial-boundary-value:

$$\begin{aligned} &u_t = u_{xx} \\ &u(x,0) = 3 + \cos(2\pi x) \\ &u_x(0,t) = u_x(3,t) = 0. \end{aligned}$$

II) Heat equation with initial-boundary-value:

$$\begin{aligned} &u_t = u_{xx} \\ &u(x,0) = 7\cos(\frac{5x}{2}) \\ &u_x(0,t) = u(\pi,t) = 0. \end{aligned}$$