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Simulation of the Mathematical Model of a Quad Rotor Control System using Matlab Simulink

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Abstract— Quad rotor vehicles are gaining prominence as Unmanned Aerial Vehicles (UAVs) owing to their simplicity in construction and ease of maintenance. They are being widely developed for applications relating to reconnaissance, security, mapping of terrains and buildings, etc. The control of the quad rotor is a complex problem. As a precursor to developing a model based design, the simulation of the mathematical model of the quad rotor is implemented. This will facilitate easier implementation of the model based design using Matlab Simulink©.

NOMENCLATURE

F_i	Force due to i^{th} rotor
I_{xx}	Moment of inertia about x axis
I_{yy}	Moment of inertia about y axis
I_{zz}	Moment of inertia about z axis
J_r	Rotor Moment of inertia
PWM_i	Pulse Width Modulation signal corresponding to i^{th} motor
U_1	Net rotor thrust in vertical direction
U_1^*	Effective force in vertical direction
U_2	Component of force responsible for pure roll
U_3	Component of force responsible for pure pitch
U_4	Component of force responsible for pure yaw
a_i	x intercept of force vs pwm graph corresponding to i^{th} motor
b_i	Slope of force vs pwm graph corresponding to i^{th} motor
b_{cw}	Thrust coefficient of rotor rotating in clock wise direction
b_{aw}	Thrust coefficient of rotor rotating in anti - clock wise direction
b	Thrust coefficient of rotor
d_{cw}	Drag coefficient of rotor rotating in clock wise direction
d_{aw}	Drag coefficient of rotor rotating in anti - clock wise direction
$e_{altitude}$	Altitude error
e_{roll}	Roll error
e_{pitch}	Pitch error
e_{yaw}	Yaw error
g	Acceleration due to gravity
l	Arm length
k_{Palt}	Proportional constant for altitude

$k_{p_{roll}}$	Proportional constant for roll
$k_{p_{pitch}}$	Proportional constant for pitch
$k_{p_{yaw}}$	Proportional constant for yaw
$k_{i_{alt}}$	Integral constant for altitude
$k_{i_{roll}}$	Integral constant for roll
$k_{i_{pitch}}$	Integral constant for pitch
$k_{i_{yaw}}$	Integral constant for yaw
$k_{d_{alt}}$	Derivative constant for altitude
$k_{d_{roll}}$	Derivative constant for roll
$k_{d_{pitch}}$	Derivative constant for pitch
$k_{d_{yaw}}$	Derivative constant for yaw
m	Mass
u_1	Actuation signal from altitude controller
u_2	Actuation signal from roll controller
u_3	Actuation signal from pitch controller
u_4	Actuation signal from yaw controller
$\ddot{x}$	Acceleration in the x direction
$\ddot{y}$	Acceleration in the y direction
$\ddot{z}$	Acceleration in the z direction
ϕ	Angular movement about the x axis, Roll
θ	Angular movement about the y axis, Pitch
ψ	Angular movement about the x axis, Yaw
τ_i	Torque due to the i^{th} motor
Ω_i	RPM of the i^{th} motor

Introduction

A quad rotor is an aerial vehicle that generates lift with four rotors. It is an inherently unstable system in the open loop [1]. The motion in x and y directions is coupled with pitch and roll. Thus, developing a closed loop control for maintaining the stability of the quad rotor is essential. The modelling of the system is a precursor to implementing the control strategy. The mathematical model of the system gives the state of the quad rotor in accordance with the physical parameters such as velocity, force, thrust, etc.

This paper begins with an introduction to quad rotor motion, model based design and the mathematical modelling. In the next section, the coefficients from the quad rotor structure are incorporated into the modelling of the quad rotor. The implementation of the modeling using Simulink is explained in the subsequent section. The results of the modeling and the implications of it on the control strategy are discussed next, followed by conclusions from the study.

Quad rotor modeling

The quad rotor motion is controlled by varying the rpm of the motors and not by using any mechanical actuators. The craft requires active control of six degrees of freedom to fly.

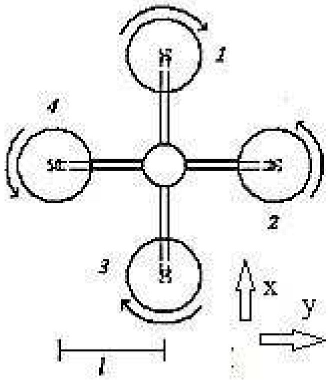

Figure 1. Quad Rotor Layout top view

The quad rotor layout is shown in Figure 1. There are two arms, each having motors at both their ends. The motors 1 and 3 rotate in the clockwise direction while the motors 2 and 4 rotate in the anti-clockwise arrangement [2]. The motors at opposite ends of the same arm rotate in the same direction to prevent torque imbalance during linear flight.

Model based design is used to develop the control system for the quad rotor.

The quad rotor model is simulated using Matlab Simulink, which represents the change in state of the quad rotor when any disturbance occurs or input is provided.

A. Mathematical Model

The mathematical model of the quad rotor can be represented with the following equations [2], [3], [4], [5], [6]:-

$$\ddot{x} = (\cos(\phi)\sin(\theta)\cos(\psi) + \sin(\phi)\sin(\psi))\frac{U_1}{m} \quad (2.1)$$

$$\ddot{y} = (\cos(\phi)\sin(\theta)\sin(\psi) - \sin(\phi)\cos(\psi))\frac{U_1}{m} \quad (2.2)$$

$$\ddot{z} = -g + (\cos(\phi)\cos(\theta))\frac{U_1}{m} \quad (2.3)$$

$$\ddot{\phi} = \dot{\theta}\dot{\psi}\left[\frac{I_{yy} - I_{zz}}{I_{xx}}\right] + \frac{J_r}{I_{xx}}\dot{\theta}\Omega_d + \frac{1}{I_{xx}}U_2 \quad (2.4)$$

$$\ddot{\theta} = \dot{\phi}\dot{\psi}\left[\frac{I_{zz} - I_{xx}}{I_{yy}}\right] - \frac{J_r}{I_{yy}}\dot{\phi}\Omega_d + \frac{1}{I_{yy}}U_3 \quad (2.5)$$

$$\ddot{\psi} = \dot{\theta}\dot{\phi}\left[\frac{I_{xx} - I_{yy}}{I_{zz}}\right] + \frac{1}{I_{yy}}U_4 \quad (2.6)$$

B. Assumptions and Decoupling

The quad rotor is assumed to have very low linear and angular velocities when in motion and assumed not to tilt beyond 15^0 in pitch and roll. The quad rotor is always flying at near hovering conditions and Coriolis and rotor moment of inertia terms can be neglected [7],[4].

Attitude is controlled by manipulating the four degrees of freedom involved – viz. altitude (z), roll (ϕ), pitch (θ) and yaw (ψ). The equations representing the four degrees of freedom are [4], [6]:-

$$\ddot{z} = -g + (\cos(\phi)\cos(\theta))\frac{U_1}{m} \quad (2.7)$$

$$\ddot{\phi} = \frac{1}{I_{xx}}U_2 \quad (2.8)$$

$$\ddot{\theta} = \frac{1}{I_{yy}}U_3 \quad (2.9)$$

$$\ddot{\psi} = \frac{1}{I_{yy}}U_4 \quad (2.10)$$

Implementation

The mathematical model of the quad rotor was implemented using Matlab Simulink. Matlab Function Blocks were used for different modules of the simulation.

A. Reference Model Creator Block (Manual input block):

This block is used to obtain the reference values for rotational parameters - roll, pitch and yaw – and the translational parameter– z, which are manual inputs in the simulation. The block can be modified to accept input from an outer control loop for position in future.

B. Controller Block

This block is used to compare the reference values of the rotational parameters - roll, pitch and yaw - and the translational parameter, z, with the actual values obtained as feedback from the quad rotor's mathematical model. The error obtained from the comparison is given to a PID control block which performs the control action and the necessary control signals are generated. Four PID controllers control the parameters of the system. The outputs from this block are [4]:-

u_1=actuation signal from altitude controller
u_2=actuation signal from roll controller
u_3=actuation signal from pitch controller
u_4=actuation signal from yaw controller

The PID controller equations obtained from Matlab Simulink are:-

$$\frac{u_1(s)}{e_{altitude}} = k_{p_{alt}} + k_{i_{alt}}\frac{1}{s} + k_{d_{alt}}\frac{s}{0.1s+1} \quad (3.1)$$

$$\frac{u_2(s)}{e_{roll}} = k_{p_{roll}} + k_{i_{roll}}\frac{1}{s} + k_{d_{roll}}\frac{s}{0.1s+1} \quad (3.2)$$

$$\frac{u_3(s)}{e_{pitch}} = k_{p_{pitch}} + k_{i_{pitch}}\frac{1}{s} + k_{d_{pitch}}\frac{s}{0.1s+1} \quad (3.3)$$

$$\frac{u_4(s)}{e_{yaw}} = k_{p_{yaw}} + k_{i_{yaw}}\frac{1}{s} + k_{d_{yaw}}\frac{s}{0.1s+1} \quad (3.4)$$

C. PWM Signal Generation Block

The signals from the controller block are given to the PWM Signal Generation block. This block generates the PWM signals to be given to the four motors of the quad rotor. The equations used for this calculation is available in literature and are as follows [4]:-

$$F_1 = \frac{1}{4}u_1 + \frac{1}{2}u_3 + \frac{b}{4d}u_4 \qquad (3.5)$$

$$F_2 = \frac{1}{4}u_1 - \frac{1}{2}u_3 + \frac{b}{4d}u_4 \qquad (3.6)$$

$$F_3 = \frac{1}{4}u_1 + \frac{1}{2}u_2 - \frac{b}{4d}u_4 \qquad (3.7)$$

$$F_4 = \frac{1}{4}u_1 - \frac{1}{2}u_2 - \frac{b}{4d}u_4 \qquad (3.8)$$

The thrust factor for the clockwise and anti-clockwise rotating propellers is different, and the equations were modified as:-

$$F_1 = \frac{1}{4}u_1 + \frac{1}{2}u_3 + \frac{b_{cw}}{4dcw}u_4 \qquad (3.9)$$

$$F_2 = \frac{1}{4}u_1 - \frac{1}{2}u_3 + \frac{b_{aw}}{4d_{aw}}u_4 \qquad (3.10)$$

$$F_3 = \frac{1}{4}u_1 + \frac{1}{2}u_2 - \frac{b_{cw}}{4d_{cw}}u_4 \qquad (3.11)$$

$$F_4 = \frac{1}{4}u_1 - \frac{1}{2}u_2 - \frac{b_{aw}}{4d_{aw}}u_4 \qquad (3.12)$$

The PWM signal to be generated based on the input to the controller [4]:-

$$PWM_1 = \frac{1}{b_1}(F_1 - a_1) \qquad (3.13)$$

$$PWM_2 = \frac{1}{b_2}(F_2 - a_2) \qquad (3.14)$$

$$PWM_3 = \frac{1}{b_3}(F_3 - a_3) \qquad (3.15)$$

$$PWM_4 = \frac{1}{b_4}(F_4 - a_4) \qquad (3.16)$$

D. Motor Dynamics Block

This block is used for estimating the thrust generated by the motor for a particular input PWM signal. The block uses a first order algebraic equation, and converts the input PWM signal to equivalent force.

The relation between the PWM signal and the thrust developed by the motor is [4]:-

$$F_i = b_i.PWM_i + a_i \qquad (3.17)$$

The output from the block represents the resultant force acting on the system resolved into components responsible for altitude, roll, pitch and yaw. To achieve this the equations are [2],[4], [5], [8],[6]:-

$$U_1 = b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \qquad (3.18)$$

$$U_2 = b(-\Omega_2^2 + \Omega_4^2) \qquad (3.19)$$

$$U_3 = b(-\Omega_1^2 + \Omega_3^2) \qquad (3.20)$$

$$U_4 = d(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2) \qquad (3.21)$$

The equations were modified for implementation as follows.

The force and torque produced by the motors can be related to their RPM, Ω as:-

Force due to the i^{th} motor $= F_i = b_{cw}\Omega_i^2$ (3.22)

Torque due to the i^{th} motor $= \tau_i = d_{cw}\Omega_i^2$ (3.23)

Where i = 1 or 3

Force due to the i^{th} motor $= F_i = b_{aw}\Omega_i^2$ (3.24)

Torque due to the i^{th} motor $= \tau_i = d_{aw}\Omega_i^2$ (3.25)

Where i = 2 or 4

The output from the block is:-

$$U_1 = b_{cw}(\Omega_1^2 + \Omega_3^2) + b_{aw}(\Omega_2^2 + \Omega_4^2) \quad (3.26)$$

$$U_1^* = U_1 - mg \quad (3.27)$$

$$U_2 = b_{aw}(-\Omega_2^2 + \Omega_4^2) \quad (3.28)$$

$$U_3 = b_{cw}(-\Omega_1^2 + \Omega_3^2) \quad (3.29)$$

$$U_4 = -b_{cw}(\Omega_1^2 + \Omega_3^2) + b_{aw}(\Omega_2^2 + \Omega_4^2) \quad (3.30)$$

E. Mathematical Model Block

The forces from the motor dynamics block are the inputs to the mathematical model block. This block gives the positional parameters of the system. Three of the decoupled transfer functions that represent roll, pitch and yaw are [4], [6]:-

$$\frac{\phi(s)}{U_2(s)} = \frac{1}{s^2 I_{xx}} \quad (3.31) \qquad \frac{\theta(s)}{U_3(s)} = \frac{1}{s^2 I_{yy}} \quad (3.32)$$

$$\frac{\psi(s)}{U_4(s)} = \frac{1}{s^2 I_{zz}} \quad (3.33)$$

A transfer function for the altitude was introduced. This is:-

$$\frac{z(s)}{U_1^*(s)} = \frac{\cos(\phi)\cos(\theta)}{s^2 m} \quad (3.34)$$

F. Inertial Measurement Unit Block

The acceleration and torque of the system is interpreted in terms of rotational and translational parameters using this block. It is used for providing feedback to the controller.

Results

The PID parameters were tuned and the values are shown in Table I. The plots illustrate the ability of the control system in stabilizing the quad rotor. The four single input single output loops for altitude, roll, pitch and yaw, work simultaneously to effectively control the quad rotor.

TABLE I. PID GAIN VALUES

	Proportional Gain (Kp)	Integral Gain (Ki)	Derivative Gain (Kd)
ROLL	0.0763	-0.0070	0.0987
PITCH	0.0763	-0.0070	0.0987
YAW	0.0763	-0.0070	0.0987
ALTITUDE	-0.0195	-0.0099	-0.0831

In each of the plots shown, the x axis represents time. The duration of the simulation is 35s.

The plot in Figure 2 represents the reference input given for the altitude and the graph in Figure 3 represents the actual altitude of the system. Here, the y axis represents the altitude of the quad rotor in metres.

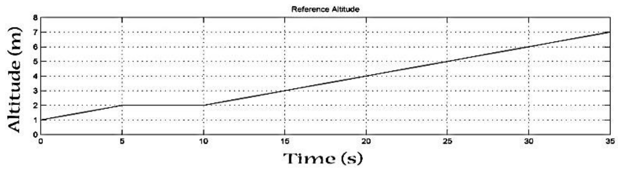

Figure 2. Reference Altitude

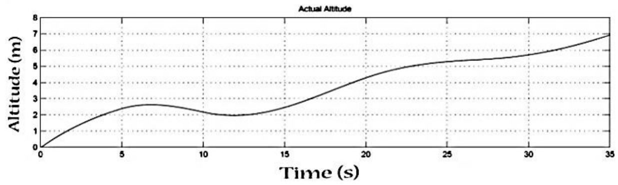

Figure 3. Actual Altitude

The plots in Figures 4 to 9 represent the reference values and the actual values of the rotational parameters – viz. roll, pitch and yaw. The y axis represents the angle of the respective parameter in radians.

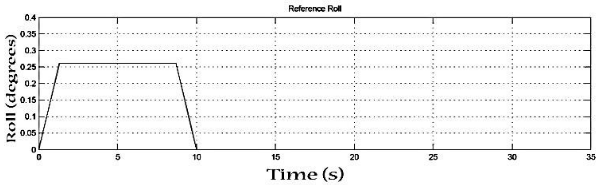

Figure 4. Reference Roll

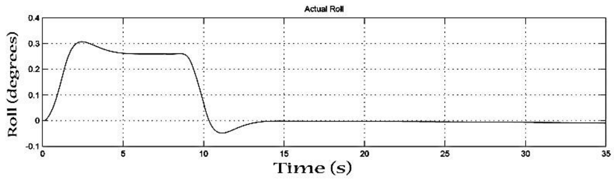

Figure 5. Actual Roll

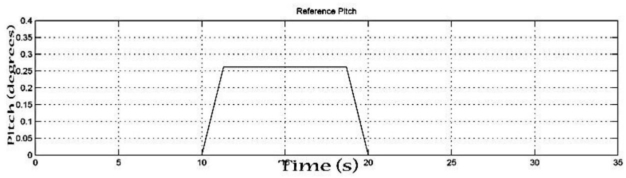

Figure 6. Reference Pitch

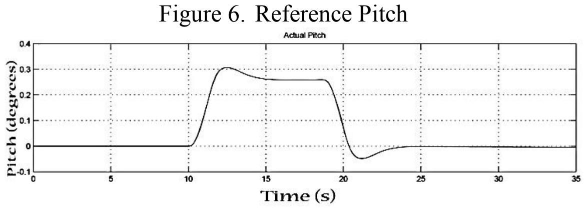

Figure 7. Actual Pitch

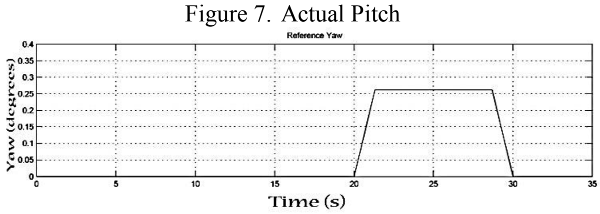

Figure 8. Reference Yaw

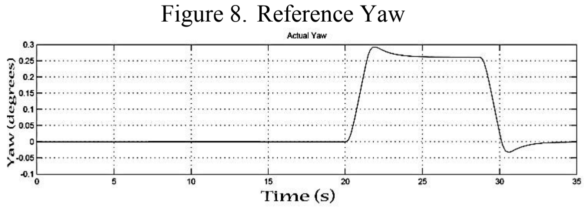

Figure 9. Actual Yaw

Conclusion

The paper describes the simulation developed as a precursor to a model based design of quad rotor control system. The equations available in the literature that are relevant to the simulation were examined and appropriate modifications have been made to represent the system behavior better. The plots in Figures 2 to 9 prove the ability of the control system to stabilize the quad rotor. Future work involves comparison of various control algorithms for quad rotor control and conversion of the simulation to a control system to be used on the actual flying quad rotor.

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