Statistical Machine Learning
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CE 40-956

TH-Quiz 6 (Point Processes)

Due May 12, 2020 (11:59 pm)

1. Suppose that $\{N_1(t), t \geq 0\}$ and $\{N_2(t), t \geq 0\}$ are independent Poisson processes with rates λ_1 and λ_2. Show that $\{N_1(t) + N_2(t), t \geq 0\}$ is a Poisson process with rate $\lambda_1 + \lambda_2$. Also, show that the probability that the first event of the combined process comes from $\{N_1(t), t \geq 0\}$ is $\lambda_1/(\lambda_1 + \lambda_2)$, independently of the time of the event.

2. Buses arrive at a certain stop according to a Poisson process with rateλ. If you take the bus from that stop then it takes a time R, measured from the time at which you enter the bus, to arrive home. If you walk from the bus stop the it takes a time W to arrive home. Suppose that your policy when arriving at the bus stop is to wait up to a time s, and if a bus has not yet arrived by that time then you walk home.

   (a) Compute the expected time from when you arrive at the bus stop until you reach home.

   (b) Show that if $W < 1/\lambda + R$ then the expected time of part (a) is minimized by letting $s = 0$; if $W > 1/\lambda + R$ then it is minimizes by letting $s = \infty$(that is, you continue to wait for the bus); and when $W = 1/\lambda + R$ all values of s give the same expected time.

   (c) Give an intuitive explanation of why we need only consider the cases s $= 0$ and $s = \infty$ when minimizing the expected time.