Take-Home Quiz 2 (DP and IBP)

1. Consider an Indian buffet Process with parameter α.

   - **a**) The primary Definition of IBP is non-exchangeable. Explain the method that makes this process exchangeable.
   - **b**) Proof following properties.

     - Adopting the above method makes the process exchangeable.
     - Number of ones in each row follow Poisson(α)
     - Expected total number of ones is αN

2. Proof that the $DP(\alpha + \sum_i^n \delta_{X_i})$ - process is a version of posterior distribution given an i.i.d samples $X_1, X_2, ..., X_n$ from the $DP(\alpha)$ process.

3. In this problem, we want to investigate the number of distinct values in random samples $X_1, X_2, ..., X_n$ from a Dirichlet process. You can consider the DP as a CRP procedure with the parameter α in this problem.

   - Calculate the expected number of occupied tables. (hint: answer $\approx \alpha \log(\frac{\alpha+n}{\alpha})$)
   - Assume that a restaurant has 5 customer currently which the first, second and fifth customer seat on new tables. calculate the probability of this event given the parameter α.
   - Show that the probability of drawing k tables in n draws is given by:

$$P(k \text{ tables}|\alpha, n) = S(n, k)\frac{\alpha^k \Gamma(\alpha + n)}{\Gamma(\alpha + n)}$$

   where $S(n, k)$ is a Stirling number of the first kind.