

### TH Quiz 5 (Pint Processes)

**Due May 03, 2020 (11:59 pm)**

1. Let  $N(t)$  be a Poisson point process with intensity  $\lambda=2$ , and let  $X_1, X_2, \dots$  be the corresponding inter-arrival times.
  - a. Given that the third arrival occurred at time  $t=2$ , find the probability that the fourth arrival occurs after  $t=4$ .
  - b. Consider the process at time  $t=10$ . Let  $T$  be the first arrival after  $t=10$ . Find  $E(T)$  and  $\text{Var}(T)$ .
  - c. If  $N(t)$  has rate  $\lambda$ , what is the distribution of arrival times  $T_1, T_2, \dots$ . In particular, for  $n=1, 2, 3, \dots$ , find  $E[T_n]$  and  $\text{var}(T_n)$ .
  - d. How do you generate the samples of arrival times by using i.i.d exponentially distributed random variables.
2. Assume we have two independent temporal point processes, with histories  $H_1(t)$  and  $H_2(t)$ , using intensities  $\lambda_1(t)$  and  $\lambda_2(t)$ , respectively. Let characterize the joint history  $H(t) = H_1(t) \cup H_2(t)$ , then find the corresponding intensity function  $\lambda(t)$  by setting up the proper differential equation.