# In-Class Quiz 1 (Gaussian Process)

## 1 Problem 1

### 1.1 a

Show that if a gaussian process is WSS then it is SSS.

### 1.2 b

consider the following gaussian process:

$$X_0 \sim \mathcal{N}(0, \sigma^2)$$
$$X_n = \frac{1}{2}X_{n-1} + Z_n, n \geq 1$$

where $Z_1, Z_2, ...$ are iid $\mathcal{N}(0,1)$ and independent of $X_0$. find $\sigma$ such that $X_n$ is SSS: [Hint: You can assume that there is some value of $\sigma$ that this process is SSS]

### 1.3 Solution a

A random process $X(t)$ is said to be SSS if all it's finite order distributions are time invariant. i.e., the joint cdf (pdf, pmf) of $X(t_1), X(t_2), ..., X(t_k)$ and $X(t_1+\tau), X(t_2+\tau), ..., X(t_k+\tau)$ are the same for all k, all $t_1, ..., t_k$ and all time shift $\tau$.

A random process$X(t)$ said to be wide-sense stationary WSS if it's mean and autocorrelation function are time invariant.

- $E[X(t)] = \mu$ independent of t.
- $R_x(t_1, t_2)$ is only a function of $t_1 - t_2$.

For Gaussian random processes, WSS $\Rightarrow$ SSS, since the process is completely specified by its mean and autocorrelation functions.

### 1.4 Solution B

This process is gaussian so it is SSS if it is WSS.

$$E[X_n] = 0$$

Assume $n \geq m$:

$$R_x[n,m] = E[X_n X_m] = E[(\frac{1}{2}X_{n-1} + Z_n)(\frac{1}{2}X_{m-1} + Z_m)]$$
$$= E[\frac{1}{4}X_{n-1}X_{m-1} + \frac{1}{2}X_{n-1}Z_m + \frac{1}{2}X_{m-1}Z_n + Z_m Z_n]$$

For the second term:

$$E[\frac{1}{2}X_{n-1}Z_m] = E[\frac{1}{2}(\frac{1}{2}X_{n-2} + Z_{n-1})Z_m]$$

At some point $n - i = m$ and we know that $E[Z_i^2] = 1$

$$E[\frac{1}{2}X_{n-1}Z_m] = 2^{(-(n-m))}$$

So we have:

$$R_x[n, m] = \frac{1}{4}R_x[n-1, m-1] + 2^{-(n-m)}$$

If this process wants to be wss then $R_x[m, n] = R_x[m-1, n-1]$:

$$R_x[n, m] = \frac{4}{3}2^{-(n-m)}$$

Let $m = n$:

$$R_x[n, n] = E[X_n^2] = Var(X_n) = Var(X_0) = \sigma^2 = \frac{4}{3}$$