In-Class Quiz 4

1 Problem 1

Buses arrive at a certain stop according to a Poisson process with rate λ. If you take the bus from that stop then it takes a time R, measured from the time at which you enter the bus, to arrive home. If you walk from the bus stop, it takes a time W to arrive home. Suppose that your policy when arriving at the bus stop is to wait up to a time s, and if a bus has not yet arrived by that time then you walk home.

1. Compute the expected time from when you arrive at the bus stop until you reach home.
2. Show that if $W < \frac{1}{\lambda} + R$ then the expected time of part (a) is minimized by letting $s = 0$; if $W > \frac{1}{\lambda} + R$ then it is minimizes by letting $s = \infty$ (that is, you continue to wait for the bus); and when $W = \frac{1}{\lambda} + R$ all values of s give the same expected time.

2 Problem 2

A competition is started at time 0 for a group of n competitors. Each competitor is allowed to work until completing the contest. It is known that each competitor's time to complete the contest is exponentially distributed with density $f_X(x) = \lambda e^{-\lambda x}; x \geq 0$. The times $X_1, ..., X_n$ are IID.

1. Let Z be the time at which the last competitor finishes. Show that Z has a distribution function $F_Z(z)$ given by $[1 - e^{-\lambda z}]^n$.
2. Let T_1 be the time at which the first competitor leaves. Show that the probability density of T_1 is given by $n\lambda e^{-n\lambda t}$. For each $i, 2 \leq i \leq n$, let T_i be the interval from the departure of the $i - 1^{th}$ competitor to that of the i^{th}. Show that the density of each T_i is exponential and find the parameter of that exponential density. Explain why T_i s are independent.