Name: Std. Number:

Quiz 4 (Dirichlet Process)

Questions

1. Stochastic process can be seen as an indexed collection of random variables. It can be considered as a collection of random variables $\{X_t\}_{t \in T}$ where T is the index set and for each t, X_t is a function from one measure space $(\Omega, \mathcal{F})$ to another measure space $(\Omega', \mathcal{F}')$. In this setting, how can we define Dirichlet Process? Define index set and domain and target measure spaces. (hint: see [**?**] and read about Kolmogorov extension theorem)
Solution: The index set of a DP is a field of an arbitrary measure space. For example consider $(\mathcal{X}, \mathcal{A}, \alpha)$ as a measure space. For each $A \in \mathcal{A}$ consider a random variable P_A from $(\Omega, \mathcal{F})$ to $(\mathbb{R}, \mathcal{C})$ where $\mathcal{C}$ is the Borel σ-field on $\mathbb{R}$. In this way P can be considered a stochastic process with index set $\mathcal{A}$. We will call it a Dirichlet Process if for any measurable partition of $\mathcal{X}$ like $(A_1, \ldots, A_k)$, the joint distribution of $(P_{A_1}, \ldots, P_{A_k})$ has dirichlet distribution with parameter $(\alpha(A_1), \ldots, \alpha(A_k))$. So P can be considered as a random probability measure on $(\mathcal{X}, \mathcal{A})$. The existence of $(\Omega, \mathcal{F})$ is proved by Kolmogorov existence theorem.

References

[1] Ferguson, Thomas S. "A Bayesian analysis of some nonparametric problems." The annals of statistics (1973): 209-230.