Take-Home Quiz 1 (Gaussian Process)

1 Problem 1

Assume X, Y are joint Gaussian with zero mean and their variances are σ_X^2, σ_Y^2 respectively. let normalized covariance matrix be ρ.

1.1 A

let $V = Y^3$ find joint p.d.f. of $f_{X|V}(x|v)$.

1.2 B

let $U = Y^2$ find joint p.d.f. of $f_{X|U}(x|u)$.

2 Problem 2

2.1 A

Assume $X_1 \sim \mathcal{N}(0, \sigma_1^2)$, $X_2 \sim \mathcal{N}(0, \sigma_2^2)$ are independent. show $X_1 + X_2 \sim \mathcal{N}(0, \sigma_1^2 + \sigma_2^2)$.

2.2 B

Assume W_1, W_2 are normalized iid gaussian random variables. show $\alpha_1 W_1 + \alpha_2 W_2 \sim \mathcal{N}(0, \alpha_1^2 + \alpha_2^2)$

2.3 C

Show that any linear combination of normalized iid gaussian random variables is gaussian random variable.

3 Problem 3

Let X, Y be iid normalized gaussian random variables. let $Z = |Y|Sgn(x)$ and $Sgn(x)$ is sign function where is 1 for $x \geq 0$ and -1 otherwise. Show that X, Z are each gaussian but are not jointly gaussian.