In-Class Quiz 4 (Review)

1 Problem 1(GP)

In the figure below(1), there are 5 functions drawn from 5 different gaussian processes. For all Gaussian processes mean is zero. The covariance matrices are all listed below. Match each function with the covariance matrix it's most likely has been drawn.

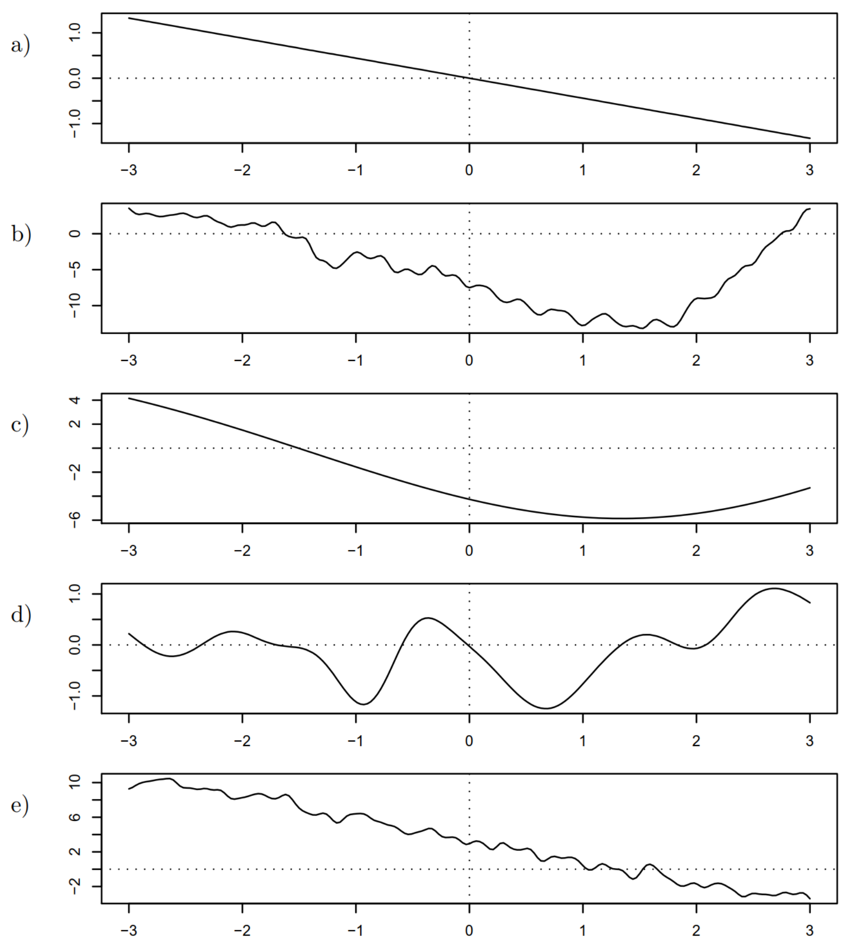

Figure 1: Functions of question 1.

(a) $Cov(y_{i1}, y_{i2}) = 0.5^2 exp(-((x_{i1} - x_{i2})/0.5)^2)$

(b) $Cov(y_{i1}, y_{i2}) = x_{i1} x_{i2}$

(c) $Cov(y_{i1}, y_{i2}) = 5^2 + 5^2 x_{i1} x_{i2} + 0.5^2 exp(-((x_{i1} - x_{i2})/0.1)^2)$

(d) $Cov(y_{i1}, y_{i2}) = 0.7^2 exp(-((x_{i1} - x_{i2})/0.1)^2) + 8^2 exp(-((x_{i1} - x_{i2})/2)^2)$

(e) $Cov(y_{i1}, y_{i2}) = 8^2 exp(-((x_{i1} - x_{i2})/5)^2)$

1.1 solution

(a) $Cov(y_{i1}, y_{i2}) = 0.5^2 exp(-((x_{i1} - x_{i2})/0.5)^2) \Rightarrow (d)$

(b) $Cov(y_{i1}, y_{i2}) = x_{i1} x_{i2} \Rightarrow (a)$

(c) $Cov(y_{i1}, y_{i2}) = 5^2 + 5^2 x_{i1} x_{i2} + 0.5^2 exp(-((x_{i1} - x_{i2})/0.1)^2) \Rightarrow (e)$

(d) $Cov(y_{i1}, y_{i2}) = 0.7^2 exp(-((x_{i1} - x_{i2})/0.1)^2) + 8^2 exp(-((x_{i1} - x_{i2})/2)^2) \Rightarrow (b)$

(e) $Cov(y_{i1}, y_{i2}) = 8^2 exp(-((x_{i1} - x_{i2})/5)^2) \Rightarrow (c)$

2 Problem 2(DP)

Consider a Chinese Restaurant Process with the parameter α and consider customer clusters as below:

$$c_1, c_1, c_2, c_3, c_2, c_2, c_3, c_3, c_1$$

a. Compute the probability of this occurrence.

b. Give an example of another permutation, which the assignments be equal to the above permutation. Compute the probability of this occurrence.

c. What do you conclude from the result of the above two parts?

2.1 solution

(a) $\frac{\alpha}{\alpha} \times \frac{1}{1+\alpha} \times \frac{\alpha}{2+\alpha} \times \frac{\alpha}{3+\alpha} \times \frac{1}{4+\alpha} \times \frac{2}{5+\alpha} \times \frac{1}{6+\alpha} \times \frac{2}{7+\alpha} \times \frac{2}{8+\alpha} = \frac{\alpha^3 \times 2^3}{\prod_{i=0}^{8}(\alpha+i)}$

(b) $c_1, c_1, c_1, c_2, c_2, c_2, c_3, c_3, c_3$ (Many other examples are acceptable)

$$P = \frac{\alpha}{\alpha} \times \frac{1}{1+\alpha} \times \frac{2}{2+\alpha} \times \frac{\alpha}{3+\alpha} \times \frac{1}{4+\alpha} \times \frac{2}{5+\alpha} \times \frac{\alpha}{6+\alpha} \times \frac{1}{7+\alpha} \times \frac{2}{8+\alpha} = \frac{\alpha^3 \times 2^3}{\prod_{i=0}^{8}(\alpha+i)}$$

(c) Exchangeability property!!