In-Class Quiz 2 (DP and IBP)

1) Describe the Dirichlet process from the Chinese Restaurant Process context (i.e What is the alternative view of DP related to CRP?).

Solution. Consider Chinese Restaurant Process, first costumer sits at the first table. mth customer sits a table drawn from the following distribution

$$\text{P(previously occupied table i}|\mathcal{F}_{m-1}) \propto n_i$$
$$\text{P(the next unoccupied table}|\mathcal{F}_{m-1}) \propto \alpha$$

So, Obviously the observations are not independent. However, It can be shown that the procedure is exchangeable. It means that Observations given a latent distribution are independent. This distribution $\mathcal{P}$ is drawn from Dirichlet Process.

In summary, this means that we get an equivalent procedure to the above algorithm:

1. Draw a distribution $\mathcal{P}$ from DP
2. Draw observations independently from $\mathcal{P}$

2) Show that Chinese Restaurant Process is an exchangeable distribution.

The marginal distribution of CPR is

$$P(X) \propto \frac{\prod_{i=1}^{k}(m_i - 1!)}{N!}$$

Where m_i is # of people on i-th table. As you you can see the distribution is invariant under the permutation of customers! Means that it doesn't depend on the order of customers, which is basically the definition of an exchangeable distribution. This proof is enough for the quiz, but if you want to try to compute the marginal distribution, you can see the process of obtaining the equation 5 from this paper.

3) Briefly explain what is the Indian Buffet Process in NPB context?

Solution. We may assume that data observations are influencing from a latent model in which each observation X_i has a latent feature vector f_i. Due to the bayesian approach we consider a prior over F, P(F) and define F as $F = \begin{bmatrix} f_1^T & f_2^T & \dots & f_N^T \end{bmatrix}^T$. We can factorize F as $F = Z \bigotimes V$ where Z is a spare binary matrix indicating which features are possessed by each object. The IBP is defining a prior over this sparse binary matrix.