
Name: Std. Number:

Quiz 3 (Dirichlet Processes)

Questions

1. if $P \sim \mathrm{DP}(\alpha)$, then, for any measurable sets A and B show that:
   (a) $E[P(A)] = \bar{\alpha}(A)$
   (b) $\mathrm{var}(P(A)) = \frac{\bar{\alpha}(A)\bar{\alpha}(A^c)}{1+|\alpha|}$
   (c) $\mathrm{cov}(P(A), P(B)) = \frac{\bar{\alpha}(A \bigcap B) - \bar{\alpha}(A)\bar{\alpha}(B)}{1+|\alpha|}$

2. Assume a Dirichlet process prior, $DP(\alpha)$, for distribitions G on X.Show that for any measurable disjoint subsets A_1 and A_2 of X, $\mathrm{corr}(G(A_1), G(A_2))$ is negative. Is the negative correlation for random probabilities induced by the DP prior a restriction? Discuss.

3. Sequence of variables $X_1, X_2, X_3, \ldots, X_n$ is exchangeable if the joint distribution is invariant to permutation. An infinite sequence is infinitely exchangeable if any subsequence is exchangeable.
   (a) Show that CRP is infinitely exchangeable
   (b) Discuss the relationship of infinitely exchangeable to i.i.d