

TH Quiz 1 (Prequisites + Nonparametric Bayesian)

Due March 3, 2020 (11:59 pm)

Linear Algebra:

1. A positive definite matrix A can be defined as one for which the quadratic form

$$X^*AX$$

is positive for any real value of the vector X . Show that a necessary and sufficient condition for A to be positive definite is that all of the eigenvalues λ_i of A are positive.

Stochastic Processes:

1. On a chessboard, determine the expected number of moves that it takes a knight, starting in one of four corners of the chessboard, to return to its initial position, assuming that each knight move is equally likely to be any of its legal moves at the time.
2. A transition matrix P is said to be *doubly stochastic* if each column sum is 1; that is, if

$$\sum_i P_{i,j} = 1, \text{ for all } j$$

If such a chain is irreducible and has states $1, \dots, m$ find its stationary probabilities.

Nonparametric Bayesian:

1. Given prior density $g(\mu)$ and observation $X \sim Poi(\mu)$, you compute $g(\mu|x)$, the posterior density of μ given x . Later you are told that x could only be observed if it were greater than 0. Does this change the posterior density of μ given x ?
2. What are the main differences between parameter and nonparametric models?