Statistical Learning
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CE 40-956

Name: Std. Number:

Quiz 2 (Gaussian Processes)

Questions

1. Let $x(t)$ be a normal process with zero expected value. If $x(t)$ is passed through a nonlinear system and we have $y(t) = x(t)^2$:

   (a) Show that
   $$S_y(\omega) = 2\pi R_x^2(0)\delta(\omega) + 2S_x(\omega) * S_x(\omega)$$
   ($S(\omega)$ is the spectral density of a process)

   (b) If $S_x(\omega)$ is an ideal low-pass filter, what does $S_y(\omega)$ look like?

2. Let $X(t) = R\cos(2\pi ft + \theta)$ where R is a Rayleigh rv and the rv θ is independent of R and uniformly distributed over the interval 0 to 2π.

   (a) Show that $E[X(t)] = 0$

   (b) Show that $E[X(t)X(t+\tau)] = \frac{1}{2}E[R^2]\cos(2\pi f\tau)$.

   (c) Show that $X(t)$; $t \in \mathbb{R}$ is a Gaussian process.