Ljung-Box Q-Test

The sample autocorrelation function (ACF) and partial autocorrelation function (PACF) are useful qualitative tools to assess the presence of autocorrelation at individual lags. The Ljung-Box Q-test is a more quantitative way to test for autocorrelation at multiple lags *jointly* [1]. The null hypothesis for this test is that the first *m* autocorrelations are jointly zero,

$$H_0 : \rho_1 = \rho_2 = ... = \rho_m = 0.$$

The choice of *m* affects test performance. If *N* is the length of your observed time series, choosing $m \approx \ln(N)$ is recommended for power [2]. You can test at multiple values of *m*. If seasonal autocorrelation is possible, you might consider testing at larger values of *m*, such as 10 or 15.

The Ljung-Box test statistic is given by

$$Q(m) = N(N + 2)\sum_{h=1}^{m}\frac{\hat{\rho}_h^2}{N-h}.$$

This is a modification of the Box-Pierce Portmanteau "Q" statistic [3]. Under the null hypothesis, Q(*m*) follows a χ_m^2 distribution.

You can use the Ljung-Box Q-test to assess autocorrelation in any series with a constant mean. This includes residual series, which can be tested for autocorrelation during model diagnostic checks. If the residuals result from fitting a model with *g* parameters, you should compare the test statistic to a χ^2 distribution with *m* – *g* degrees of freedom. Optional input arguments to `lbqtest` let you modify the degrees of freedom of the null distribution.

You can also test for conditional heteroscedasticity by conducting a Ljung-Box Q-test on a squared residual series. An alternative test for conditional heteroscedasticity is Engle's ARCH test (`archtest`).

References

[1] Ljung, G. and G. E. P. Box. "On a Measure of Lack of Fit in Time Series Models." *Biometrika*. Vol. 66, 1978, pp. 67–72.

[2] Tsay, R. S. *Analysis of Financial Time Series*. 3rd ed. Hoboken, NJ: John Wiley & Sons, Inc., 2010.

[3] Box, G. E. P. and D. Pierce. "Distribution of Residual Autocorrelations in Autoregressive-Integrated Moving Average Time Series Models." *Journal of the American Statistical Association*. Vol. 65, 1970, pp. 1509–1526.

See Also

`archtest` | `lbqtest`

Related Examples

- Detect Autocorrelation
- Detect ARCH Effects

More About

- Autocorrelation and Partial Autocorrelation
- Engle's ARCH Test
- Residual Diagnostics
- Conditional Mean Models