Life-Table

Life-table

- The life-table is a summary of the survival data grouped into convenient intervals.
- Suppose the data are grouped into k intervals
$$[b_0, b_1), [b_1, b_2), \cdots, [b_{k-1}, b_k)$$
- The life-table presents the number of failures and censored times falling in each interval.
  - c_j: number of censored individuals
  - d_j: number of failure (event) times
  - n_j: number of individuals at risk at the start of the jth interval.

An example

Year since entry	n_j: # of alive at start of interval	d_j: # of death	c_j: # of censoring
[0, 1)	146	27	3
[1,2)	116	18	10
[2,3)	88	21	10
[3,4)	57	9	3
[4,5)	45	1	3
[5,6)	41	2	11
[6,7)	28	3	5
[7,8)	20	1	8
[8,9)	11	2	1
[9,10)	8	2	6

Life-table Method

- How to estimate $S(1)$?
- Note $\hat{S}(1) = 1 - \hat{m}_1$, $\hat{m}_1$ is the mortality rate during first year.
  - If all censoring occurred at the beginning of an interval, then
$$\hat{m}_1 = \frac{d_1}{n_1 - c_1}$$
  - If all censoring occurred at the end of an interval, then
$$\hat{m}_1 = \frac{d_1}{n_1}$$
- Often censoring occurred during the interval, so a compromise is
$$\hat{m}_1 = \frac{d_1}{n_1 - c_1/2}.$$
  - $e_1 = n_1 - c_1/2$ is called effective risk size.
- Therefore, $\hat{s}(1) = 1 - \frac{d_1}{n_1 - \frac{c_1}{2}} = 1 - \frac{27}{146 - 3/2}$=0.81

Life-table Method

- How to estimate $S(2)$?
- Similar to K-M estimator,
$$S(2) = P(T > 2) = P(T > 2 \mid T > 1)P(T > 1)$$
$$=(1\text{-}m_2)S(1)$$
  - $m_2 = P(1 < T \leq 2 \mid T > 1)$, the mortality rate during second year given survival to the beginning of year 2.
- We can estimate
$$\hat{m}_2 = \frac{d_2}{n_2 - c_2/2}$$
- Therefore
$$\hat{s}(2) =(1\text{-}\hat{m}_2)\,\hat{s}(1)=(1 - \frac{d_2}{n_2 - c_2/2})\hat{s}(1)$$
$$=(1 - \frac{18}{116 - 10/2})\times 0.81=0.68$$

Life-table method: summary

The life-table estimator of the survival function at the end of jth interval $\left[b_{j-1}, b_j\right)$ is given as

$$\hat{S}(b_j) = \hat{S}(b_{j-1})(1 - \frac{d_j}{n_j - c_j/2}).$$

with $\hat{S}(0) = 1$.

Life-table: example

Year since entry	n_j	d_j	c_j	= -	=
[0, 1)	146	27	3	-	
[1,2)	116	18	10	-	
[2,3)	88	21	10	-	
[3,4)	57	9	3	-	
[4,5)	45	1	3	-	
[5,6)	41	2	11	-	

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