Accelerated Failure Time Model

Interpretation of regression coefficients

Accelerated Failure Time (AFT) model

- The AFT model assumes that
  $Y = \log(T) = \beta_0 + \beta_1 x_1 + \cdots + \beta_p x_p + \sigma\varepsilon.$
- Why is it called accelerated failure time model?
- Then in distribution
  $\mathrm{T}= \exp\left(\beta_1 x_1 + \cdots + \beta_p x_p\right) \exp(\beta_0 + \sigma\varepsilon).$
  or $\mathrm{T} = \eta_x T_0$
  - $T_0 = \exp(\beta_0 + \sigma\varepsilon)$, the baseline survival time for $x_1 = \cdots = x_p = 0.$
  - $\eta_x = \exp(\beta_1 x_1 + \cdots + \beta_p x_p)$

Accelerated Failure Time (AFT) model

- AFT model gives that
  $\mathrm{T} = \eta_x T_0$
  - One year for T_0 is equivalent to η_x years for T.
- The constant $1/\eta_x$: acceleration factor, the relative speed to move along the time axis compared with baseline.
  - If η_x=2 or $1/\eta_x$=1/2, the subject ages at half speed of the baseline (decelerating).
  - If η_x=1/2 or $1/\eta_x$=2, the subject ages at twice speed of the baseline (accelerating).

Accelerated Failure Time (AFT) model

- Recall that AFT model gives
$$T = \eta_x T_0.$$
- Let μ be the mean survival time. Then
$$\mu = \eta_x \mu_0$$
  - If η_x=2, the expected survival time of subject with covariate value $\boldsymbol{x}$ is twice of that of the baseline.
  - If η_x=1/2, the expected survival time of subject with covariate value $\boldsymbol{x}$ is half of that of the baseline.

Accelerated Failure Time (AFT) model

- Let $t_{(p)}$ be the p-th population percentile. Then
$$t_{(p)} = \eta_x t^0_{(p)}.$$

In particular, the median survival time
$$t_{(0.5)} = \eta_x t^0_{(0.5)}.$$
  - If η_x=2, the median survival time of subject with covariate value $\boldsymbol{x}$ is twice of that of the baseline.
  - If η_x=1/2, the median survival time of subject with covariate value $\boldsymbol{x}$ is half of that of the baseline.

Summary: properties of AFT model

- Let η_x= $\exp(\beta_1 x_1 + \cdots + \beta_p x_p)$.
- AFT model implies that
$$T = \eta_x T_0$$
$$\mu = \eta_x \mu_0$$
$$t_{(0.5)} = \eta_x t^0_{(0.5)}$$

Interpretation of coefficients

- Consider β_k .
- Suppose increase k-th covariate by one unit: x_k to $x_k + 1$ while holding other covariates fixed.
  - $T_{x_k} = \exp(\beta_1 x_1 + \cdots + \beta_k x_k + \cdots + \beta_p x_p) \exp(\beta_0 + \sigma\varepsilon)$.
  - $T_{x_k+1} = \exp(\beta_1 x_1 + \cdots + \beta_k (x_k + 1) + \cdots + \beta_p x_p) \exp(\beta_0 + \sigma\varepsilon)$.
- Then in distribution,

$$T_{x_k+1} = e^{\beta_k} T_{x_k}.$$

- The effect of one unit increase in k-th covariate is to multiply the survival time by e^{β_k} .
  - $\mu_{x_k+1} = e^{\beta_k} \mu_{x_k}$.
  - Or the percentage of change in mean survival time is

$$\frac{\mu_{x_k+1} - \mu_{x_k}}{\mu_{x_k}} = e^{\beta_k} - 1.$$

An example

- If x_k is a treatment indicator and β_k=0.6, then

$$\mu_{x_{k=1}} = e^{\beta_k} \mu_{x_{k=0}} = 1.82 \mu_{x_{k=0}}.$$

  - The expected survival time of patients who received the treatment is 1.82 times that of patients who did not receive the treatment.
- Or $\frac{\mu_{x_k=1} - \mu_{x_k=0}}{\mu_{x_k=0}}$= exp(0.6)-1=0.82
  - The expected survival time of patients who received the treatment is 82% longer than patients who did not receive the treatment.

An example

- If x_k is age and β_k=-0.6, then

$$\mu_{x_k+1} = e^{\beta_k} \mu_{x_k} = 0.55 \mu_{x_k}.$$

$$\frac{\mu_{x_k+1} - \mu_{x_k}}{\mu_{x_k}} \quad \exp(-0.6)\text{-}1\text{=-}0.45.$$

  - With one year increase in age, the expected survival time decreases by 45%.

AFT model

- The AFT model assumes that

$$Y = \log(T) = \beta_0 + \beta_1 x_1 + \cdots + \beta_p x_p + \sigma\varepsilon.$$

$$T = \eta_x T_0 \text{ with } \quad \eta_x = \exp(\beta_1 x_1 + \cdots + \beta_p x_p).$$

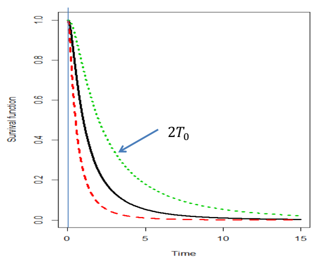
- We can show that, in AFT model,

$$S(t) = S_0\left(\frac{1}{\eta_x} t\right),$$

$$\mathrm{h}(t) = \frac{1}{\eta_x} h_0\left(\frac{1}{\eta_x} t\right).$$

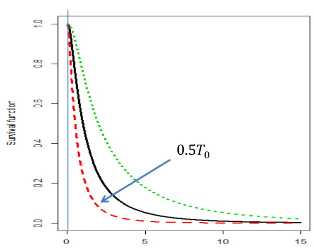
An example: lognormal

Survival functions for baseline T_0 , $2T_0$ and $0.5T_0$



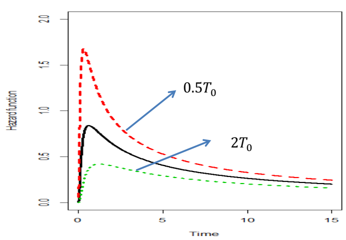
An example: lognormal

Survival functions for baseline T_0 , $2T_0$ and $0.5T_0$



An example: lognormal

Hazard functions for baseline T_0 , $2T_0$ and $0.5T_0$



Summary

- In AFT model, the effects of covariates are multiplicative on survival time.
- In AFT models, survival times all follows the same family of distribution, with different scale parameters.
- Interpretation of regression coefficients.
- The effects of covariates on survival and hazard functions.