Accelerated Failure Time Model

Examples

Accelerated Failure Time (AFT) model

- AFT model assumes that

$Y = \log(T) = \beta_0 + \beta_1 x_1 + \cdots + \beta_p x_p + \sigma\varepsilon.$

- A different distribution of ε gives different regression model.

Distribution of ε	Distribution of T
Extreme values	Weibull ($\sigma = 1$ reduce to exponential)
Log-Gamma	Gamma
Logistic	Log-logistic
Normal	Log-normal

Exponential regression model

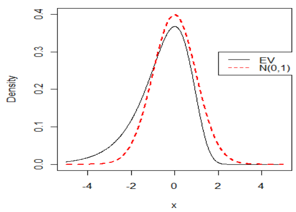
- The exponential model has the form

$$Y = \log(T) = \beta_0 + \beta_1 x_1 + \cdots + \beta_p x_p + \sigma\varepsilon,$$

  - $\sigma = 1$
  - ε ~ Standard extreme value distribution.
- What is the extreme value distribution?
  - If $T \sim Exp(1)$, then $\varepsilon = \log(T) \sim Standard\ EV$.
  - If $\varepsilon \sim Standard\ EV$, then $T = exp(\varepsilon) \sim Exp(1)$.
- Note that the extreme value random variable ε is not restricted to positive.

Standard extreme value distribution

Density of standard extreme value distribution
$f(t) = \exp(t - \exp(t))$.



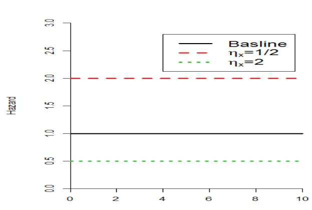
Exponential regression model

- The exponential model has the form
  $Y = \log(T) = \beta_0 + \beta_1 x_1 + \cdots + \beta_p x_p + \sigma\varepsilon,$
  - $\sigma = 1$
  - $\varepsilon \sim$ Standard extreme value distribution.
- The survival time
  $T \sim \mathrm{Exp}(\lambda)$ with constant hazard $\lambda = \exp\{-\beta_0 - \boldsymbol{x}^T\beta\}$.
- For the baseline survival time,
  $T_0 \sim \mathrm{Exp}(\lambda_0)$ with constant hazard $\lambda_0 = \exp\{-\beta_0\}$.
- Also a proportional hazard model
  $\frac{h(t|x)}{h_0(t)} = \frac{\exp\{-\beta_0 - \boldsymbol{x}^T\beta\}}{\exp\{-\beta_0\}} = e^{-\boldsymbol{x}^T\beta} = \frac{1}{\eta_x}$
  - $h_0(t)$ baseline hazard
  - $h(t \mid \boldsymbol{x})$ hazard function with covariate $\boldsymbol{x}$.
  - $\eta_x = e^{x^T\beta}$

Exponential regression model

Hazard functions

$h(t \mid x) = h_0(t)\frac{1}{\eta_x}$

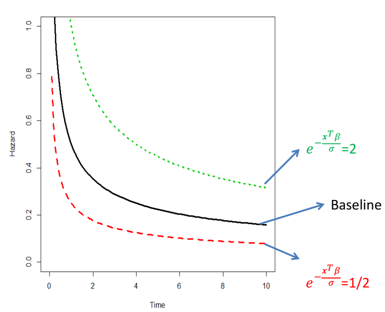


Weibull regression model

- The Weibull regression model assumes that
  $Y = \log(T) = \beta_0 + \beta_1 x_1 + \cdots + \beta_p x_p + \sigma\varepsilon$
  - σ is a unknown scale parameter
  - $\varepsilon \sim$ Extreme value distribution.
- Compared with Exponential regression, there is one additional unknown parameter σ.
- When σ=1, it reduces to Exponential regression.
- T$\sim$ $Weibull\ (\alpha, \lambda)$ with $\alpha = 1/\sigma, \lambda = \exp\{-\frac{\beta_0 + x^T\beta}{\sigma}\}$
- Again a proportional hazard model:

$$\frac{h(t|x)}{h_0(t)} = \frac{\alpha t^{\alpha-1}\exp\{-\frac{\beta_0 + x^T\beta}{\sigma}\}}{\alpha t^{\alpha-1}\exp\{-\frac{\beta_0}{\sigma}\}} = e^{-\frac{x^T\beta}{\sigma}}.$$

Weibull regression model



Weibull regression model

In summary, the Weibull regression model has two interesting properties:

- $h(t \mid \boldsymbol{x}) = h_0(t)\exp(-\boldsymbol{x}^T\beta/\sigma)$: a proportion hazard model and covariates act multiplicatively on the hazard.
- $T = T_0\exp(\boldsymbol{x}^T\beta)$: an accelerated failure time model and covariates act multiplicatively on survival time.
- The Weibull model is the only one which yields both a proportional hazard model and an accelerated failure time model.
- The coefficients change signs in two models. why?

Summary

- Exponential and Weibull regression models
  - A popular parametric regression model
  - Both AFT model PH model
- Other parametric regression model such as log-logistic, log-normal and Gamma regressions.