Weighted Logrank Test

Logrank test

- We have introduced the logrank test for comparing two survival time distributions.
- The logrank statistic is given by

Logrank= $\frac{[\sum_{i=1}^{D}(d_{i1}-e_{i1})]^2}{\sum_{i=1}^{D} v_{i1}} \sim \chi_1^2$.

where d_{i1} is observed # of events, e_{i1} is expected# of events for group 1 at time t_i.

Weighted Logrank test

- However, it is not obvious that each time point should receive the same weight in the linear combination.
- It may be more appropriate to give less weight to a later time point with a smaller size of at-risk individuals.
- One natural extension is the family of Weighted Logrank Test of the form

$$\frac{[\sum_{i=1}^{D} w_i(d_{i1} - e_{i1})]^2}{\sum_{i=1}^{D} w_i^2 v_{i1}}$$

Here $\{w_i\}$ are weights chosen to inflate or deflate various time points.

Choices of weights

- **Wilcoxon test** (or Gehan test) uses
$$w_i = Y_i$$
  - weights equal to the total number at risk at t_i
  - greater weight to earlier failure times
- **Peto-Peto test** uses
$$w_i = \hat{S}_{KM}(t_i)$$
  - weights equal to the KM estimate of survival at t_i using the pooled sample
  - greater weight to earlier failure times.

Choices of weights

- Fleming and Harrington test uses
$$w_i = [\hat{S}_{KM}(t_i)]^{\rho}\ [1 - \hat{S}_{KM}(t_i)]^{\gamma}$$
  - ρ= γ= gives the usual Logrank statistics
  - ρ=1, γ=0 gives the Peto-Peto test
  - $\rho > 0$, γ =0 greater weight to earlier failure times
  - $\rho = 0, \gamma$>0 greater weight to later failure times

Choice of weights

Weight	Test
$w_i = 1$	Logrank test
$w_i = Y_i$	Wilcoxon test
$w_i = \hat{S}_{KM}(t_i)$	Peto-Peto test
$w_i = [\hat{S}_{KM}(t_i)]^{\rho}\ [1 - \hat{S}_{KM}(t_i)]^{\gamma}$	Fleming-Harrington generalization test
ρ=0,γ=0	Logrank test
ρ=1,γ=0	Peto-Peto test
$\rho > 0$, γ =0	More sensitive to early difference
$\rho = 0$, γ >0	More sensitive to late difference

Choice of weight

- Weighted logrank test enables us to inflate the early or late differences.
- It is not reasonable to look at the survival curves first, then choose weights (data snooping).
- How to choose weights:
  - Is proportional hazard reasonable assumption? If yes, then go with logrank test.
  - If not, consider what survival differences are most scientifically meaningful (early or late).
    - Lung cancer: early difference
    - Prostate cancer: late difference