Modelling and Simulation of Marine Craft

KINEMATICS

Edin Omerdic
Senior Research Fellow
Mobile & Marine Robotics Research Centre
University of Limerick

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 691980.

Outline

- Kinematics
  - Reference Frames
    - ECEF Frame
    - NED Frame
    - BODY Frame
    - UNITY Frame
    - UTM
  - Transformations BODY – NED
    - Euler Angles
    - Quaternions
    - Quaternions from Euler Angles
    - Euler Angles from Quaternions
  - Transformations ECEF – NED
    - Lat-Lon from ECEF Coordinates
    - ECEF Coordinates from Lat-Lon

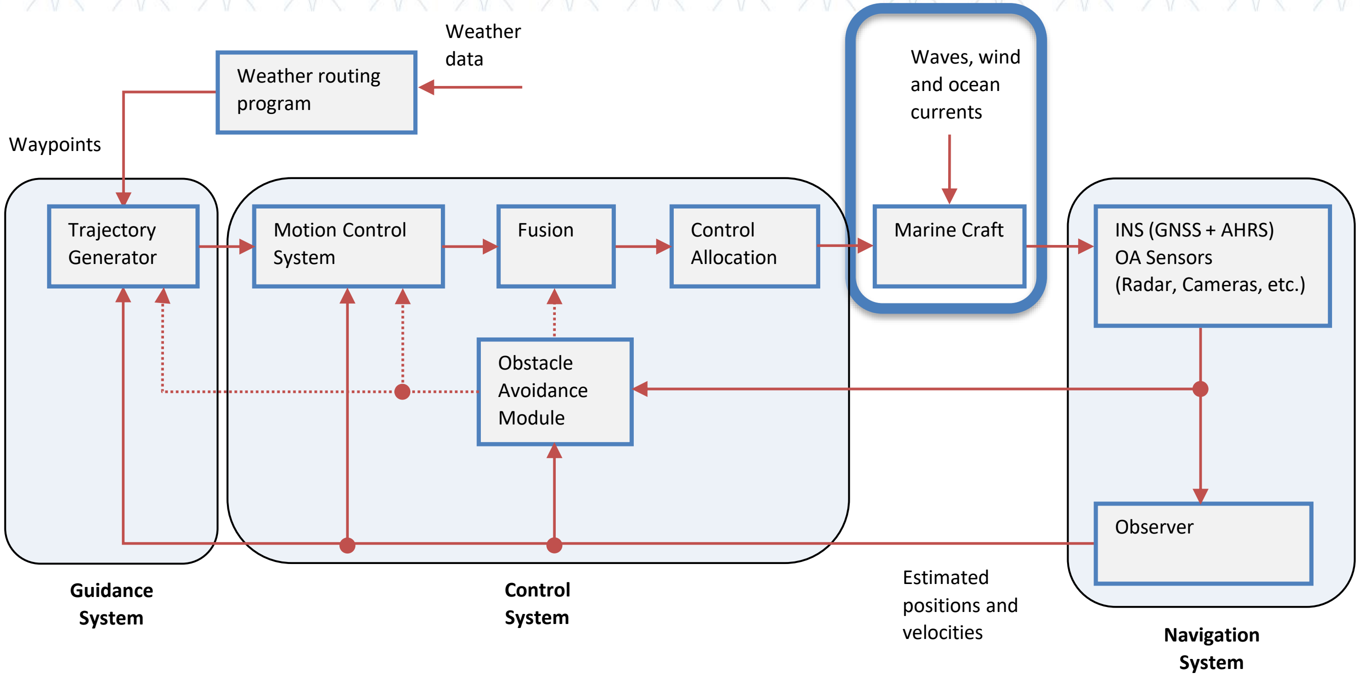
GNC Signal Flow

University *of* Limerick

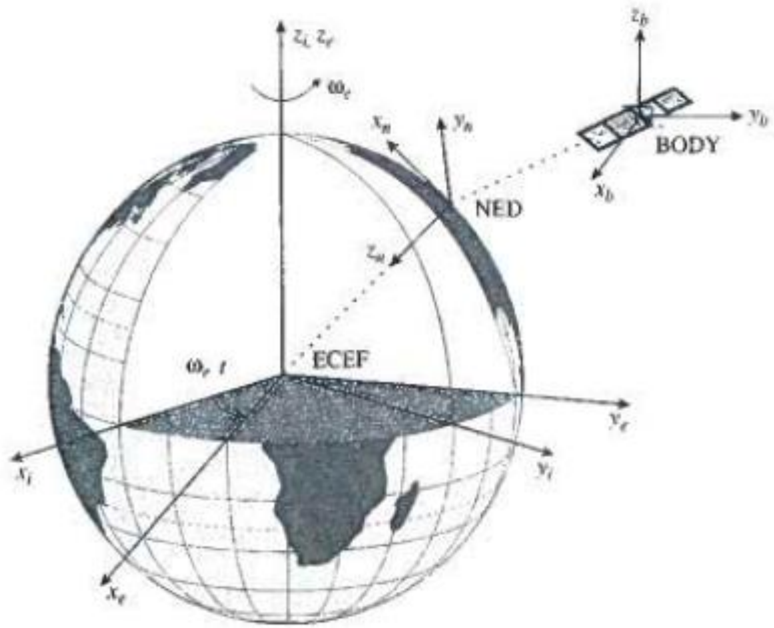
Mobile & Marine Robotics Research Centre

EXCELLABUST

GNC Signal Flow



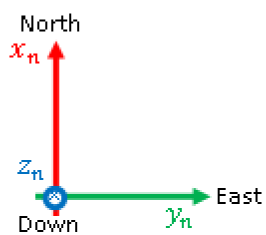
ECEF Frame

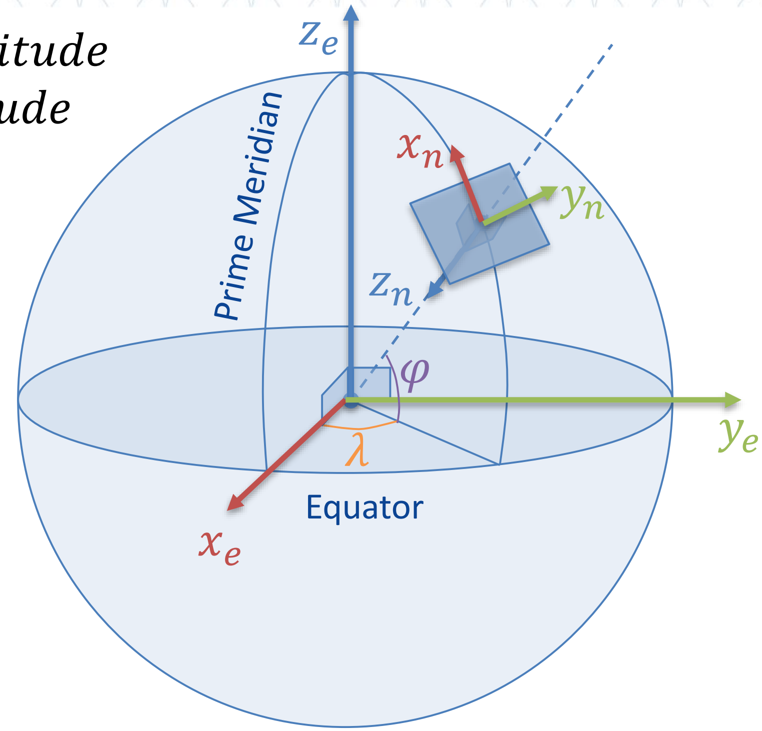


Symbol	$\{\mathrm{e}\} = (x_\mathrm{e}, y_\mathrm{e}, z_\mathrm{e})$
Definition	The Earth-centered Earth-fixed frame which rotate around North-South axis with angular speed $w_e = 7.2921 * 10^{-5}\ rad/s$
Type	Right-handed
Axes	x_e (from origin through intersection of Prime Meridian and Equator) y_e z_e (from origin through North pole)
Origin	Location of ECEF origin o_e is fixed to the Centre of the Earth.
Assumption	For marine craft moving at relatively low speed the Earth rotation can be neglected and $\{\mathrm{e}\}$ can be considered to be inertial, such that Newton's laws apply.
Application	Typically used for global guidance, navigation and control (for example, to describe motion and location of ships in transit between continents).

NED Frame

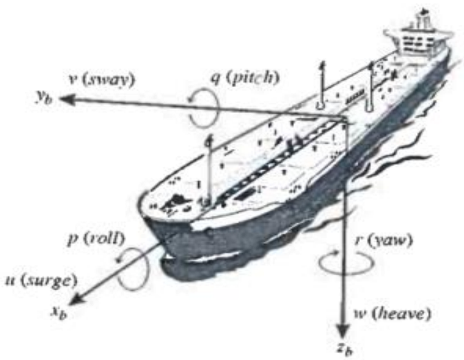
λ – *Longitude*
φ – *Latitude*





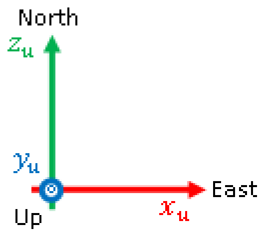
Symbol	$\{n\} = (x_n, y_n, z_n)$
Definition	Tangent plane on the surface of Earth reference ellipsoid WGS 1984.
Type	Right-handed
Axes	x_n= North y_n= East z_n= Downwards, normal to the surface.
Origin	Location of NED origin o_n relative to ECEF frame $\{e\} = (x_e, y_e, z_e)$ is determined using Lat and Lon.
Assumption	$\{n\}$ is inertial, such that Newton's laws apply.
Application	The position and orientation of the craft are described relative to $\{n\}$.

BODY Frame



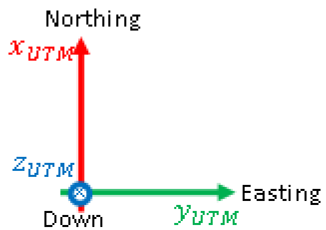
Symbol	$\{b\} = (x_b, y_b, z_b)$
Definition	Moving frame fixed to the craft.
Type	Right-handed
Axes	x_b= longitudinal axis (directed toward front); y_n= transversal axis (directed toward starboard); z_n= normal axis (directed toward bottom)
Origin	Location of BODY origin o_b is usually chosen to coincide with a point midships in the water line for marine crafts.
Application	The linear and angular velocities of the vessel are described relative to $\{b\}$.

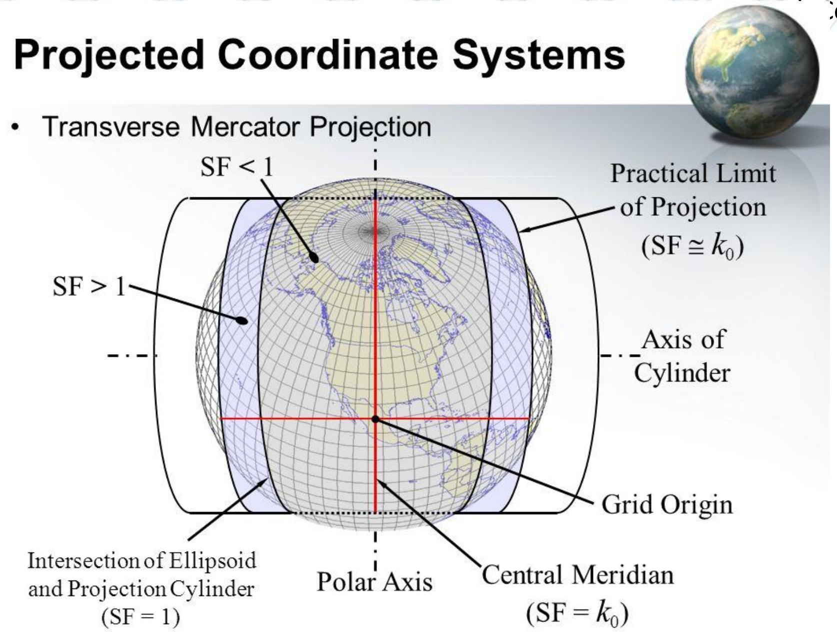
UNITY Frame



Symbol	$\{u\} = (x_u, y_u, z_u)$
Definition	Frame used for 3D visualisation in Unity 3D.
Type	Left-handed
Axes	x_u= East y_u= Up z_u= North.
Origin	Location of UNITY origin o_u relative to ECEF frame $\{e\} = (x_e, y_e, z_e)$ is determined using Lat and Lon.
Application	The position and orientation of all vessels are sent to Unity 3D for visualisation.

UTM

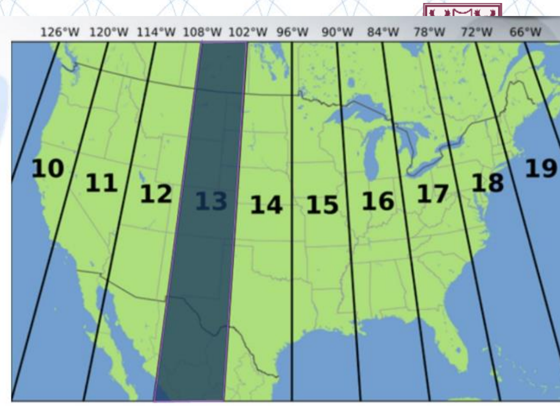


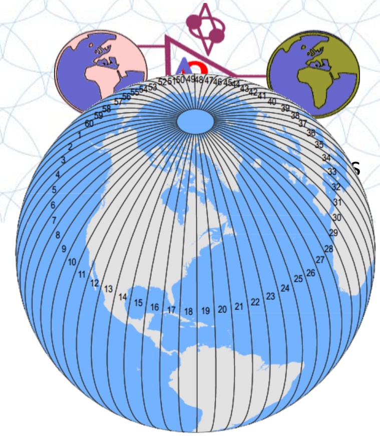


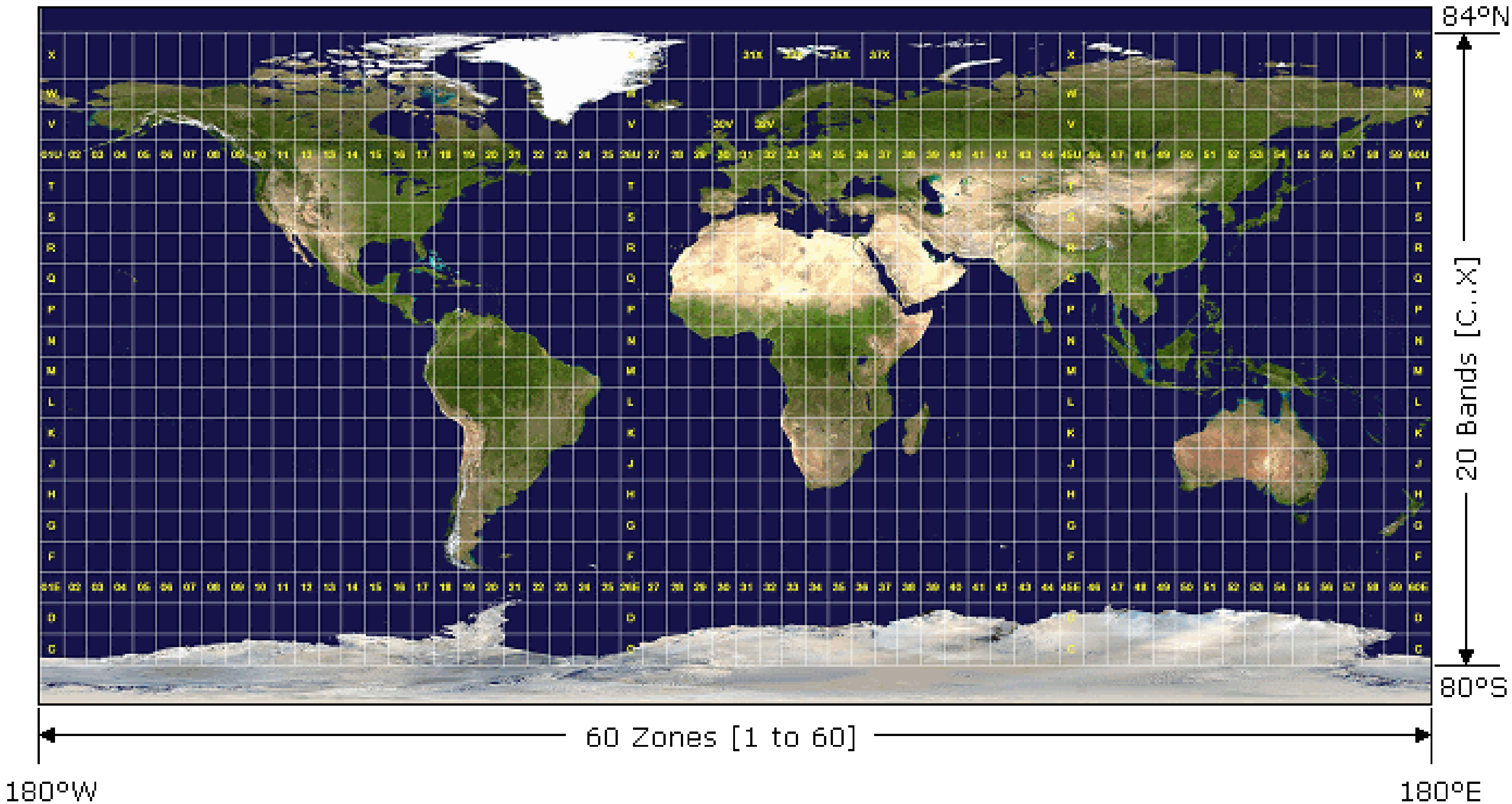
Symbol	$\{UTM\} = (x_{UTM}, y_{UTM}, z_{UTM})$
Definition	The UTM system divides the Earth into 60 zones and uses a secant transverse Mercator projection in each zone (conformal projection) to give location on the surface of the Earth. Each zone is segmented into 20 latitude bands.
Type	Right-handed
Axes	x_{UTM}= Northing y_{UTM}= Easting z_{UTM}= Downwards, normal to the surface.
Origin	Location of UTM origin o_{UTM} is shifted such that Northing and Easting coordinates are always positive. For this purpose artificial offset False Easting is added in Northern hemisphere, while both False Easting and False Northing are used in Southern hemisphere. In vertical plane the UTM origin is at sea level.

Kinematics
Reference Frames

UTM (cont.)







Notation

$$v^{to} = R^{to}_{from}\, v^{from}$$

ECEF position $\quad p^e_{b/e} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3$

Longitude and latitude $\quad \Theta_{en} = \begin{bmatrix} l \\ \mu \end{bmatrix} \in S^2$

NED position $\quad p^n_{b/n} = \begin{bmatrix} N \\ E \\ D \end{bmatrix} \in \mathbb{R}^3$

Attitude (Euler angles) $\quad \Theta_{nb} = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} \in S^3$

Body-fixed linear velocity $\quad v^b_{b/n} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \in \mathbb{R}^3$

Body-fixed angular velocity $\quad \omega^b_{b/n} = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \in \mathbb{R}^3$

Body-fixed force $\quad f^b_b = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \in \mathbb{R}^3$

Body-fixed moment $\quad m^b_b = \begin{bmatrix} K \\ M \\ N \end{bmatrix} \in \mathbb{R}^3$

$\boldsymbol{p}^{UTM}_{b/UTM} = \begin{bmatrix} x_{UTM} \\ y_{UTM} \\ z_{UTM} \end{bmatrix}$	Position of vessel $\{o_b\}$ in UTM	$\boldsymbol{p}^n_{b/n} = \begin{bmatrix} x_n \\ y_n \\ z_n \end{bmatrix}$	Position of vessel $\{o_b\}$ in NED
$\boldsymbol{v}^b_{b/n} = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$	Linear velocity of $\{o_b\}$ with respect to *{n}* expressed in *{b}*	$\boldsymbol{v}^n_{b/n}$	Linear velocity of $\{o_b\}$ with respect to *{n}* expressed in *{n}*
$\boldsymbol{\theta}_{nb} = \begin{bmatrix} R \\ P \\ Y \end{bmatrix}$	Attitude (Euler angles) between *{n}* and *{b}*	$\boldsymbol{w}^b_{b/n} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$	Angular velocity of $\{b\}$ with respect to *{n}* expressed in *{b}*
$\boldsymbol{p}^{UTM}_{n/UTM}$	Position of NED origin $\{o_n\}$ in UTM	$\dot{\boldsymbol{\theta}}_{nb} = \begin{bmatrix} \dot{R} \\ \dot{P} \\ \dot{Y} \end{bmatrix}$	Euler rate vector
$\boldsymbol{R}^n_b(\theta_{nb})$	Transformation matrix of linear velocity vector $\boldsymbol{v}^b_{b/n}$ from *{b}* to $\boldsymbol{v}^n_{b/n}$ in *{n}*	$\boldsymbol{T}_\theta(\boldsymbol{\theta_{nb}})$	Transformation matrix of angular velocity vector $\boldsymbol{w}^b_{b/n}$ from *{b}* to Euler rate vector $\dot{\boldsymbol{\theta}}_{nb}$ in *{n}*

$$\boldsymbol{\lambda} \times \boldsymbol{a} := \boldsymbol{S}(\boldsymbol{\lambda})\boldsymbol{a}$$

$$\boldsymbol{S}(\boldsymbol{\lambda}) = -\boldsymbol{S}^\top(\boldsymbol{\lambda}) = \begin{bmatrix} 0 & -\lambda_3 & \lambda_2 \\ \lambda_3 & 0 & -\lambda_1 \\ -\lambda_2 & \lambda_1 & 0 \end{bmatrix}, \quad \boldsymbol{\lambda} = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix}$$

Euler's Rotation Theorem:

Geometry perspective

Any displacement of a rigid-body in 3D, such that a point on the rigid body remains fixed, is equivalent to a single rotation about some axis that passes through the fixed point.

Linear Algebra perspective

Any two Cartesian coordinate systems in 3D with a common origin are related by a rotation about some fixed axis passing through the origin.

Theorem 2.1 (Euler's Theorem on Rotation)
Every change in the relative orientation of two rigid bodies or reference frames $\{\mathcal{A}\}$ and $\{\mathcal{B}\}$ can be produced by means of a simple rotation of $\{\mathcal{B}\}$ in $\{\mathcal{A}\}$.

$$v^n_{b/n} = R^n_b v^b_{b/n}, \quad R^n_b := R_{\lambda,\beta}$$

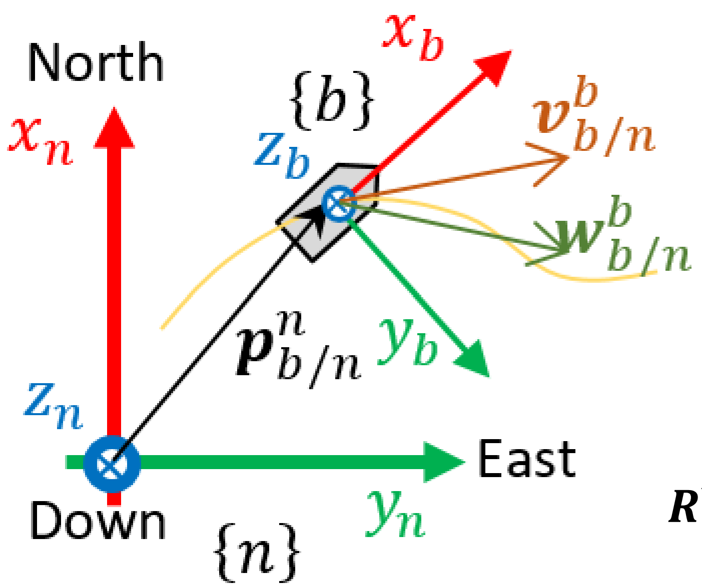
$$R_{\lambda,\beta} = I_{3\times3} + \sin(\beta) S(\lambda) + [1 - \cos(\beta)]\, S^2(\lambda)$$

$$S^2(\lambda) = \lambda\lambda^\top - I_{3\times3}$$

Linear Velocity Transformation

$$\dot{\boldsymbol{p}}^n_{b/n} = \boldsymbol{v}^n_{b/n} = \boldsymbol{R}^n_b(\boldsymbol{\theta}_{nb})\boldsymbol{v}^b_{b/n} = \boldsymbol{R}_{z,Y}\boldsymbol{R}_{y,P}\boldsymbol{R}_{x,R}\boldsymbol{v}^b_{b/n}$$

$$\boldsymbol{R}_{x,R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cR & -sR \\ 0 & sR & cR \end{bmatrix}, \boldsymbol{R}_{y,P} = \begin{bmatrix} cP & 0 & sP \\ 0 & 1 & 0 \\ -sP & 0 & cP \end{bmatrix}, \boldsymbol{R}_{z,Y} = \begin{bmatrix} cY & -sY & 0 \\ sY & cY & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



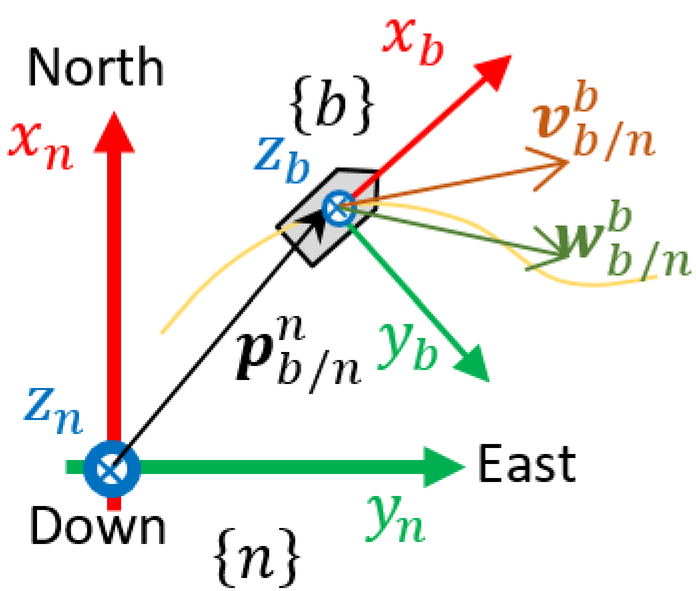
$$\boldsymbol{R}^n_b(\boldsymbol{\theta}_{nb})^{-1} = \boldsymbol{R}^n_b(\boldsymbol{\theta}_{nb})^T$$

$$\boldsymbol{R}^n_b(\boldsymbol{\theta}_{nb}) = \begin{bmatrix} cYcP & -sYcR + cYsPsR & sYsR + cYcRsP \\ sYcP & cYcR + sYsPsR & -cYsR + sPsYcR \\ -sP & cPsR & cPcR \end{bmatrix}$$

Angular Velocity Transformation

$$\dot{\boldsymbol{\theta}}_{nb} = \boldsymbol{T}_{\theta}(\boldsymbol{\theta}_{nb})\boldsymbol{w}^{b}_{b/n} = \begin{bmatrix} 1 & sRtP & cRtP \\ 0 & cR & -sR \\ 0 & sR/cP & cR/cP \end{bmatrix}\boldsymbol{w}^{b}_{b/n}$$

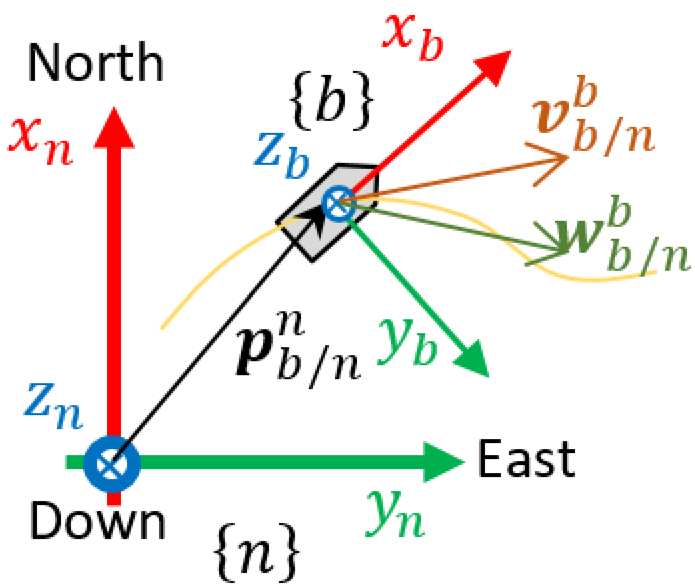
Singularity for P=±90˚



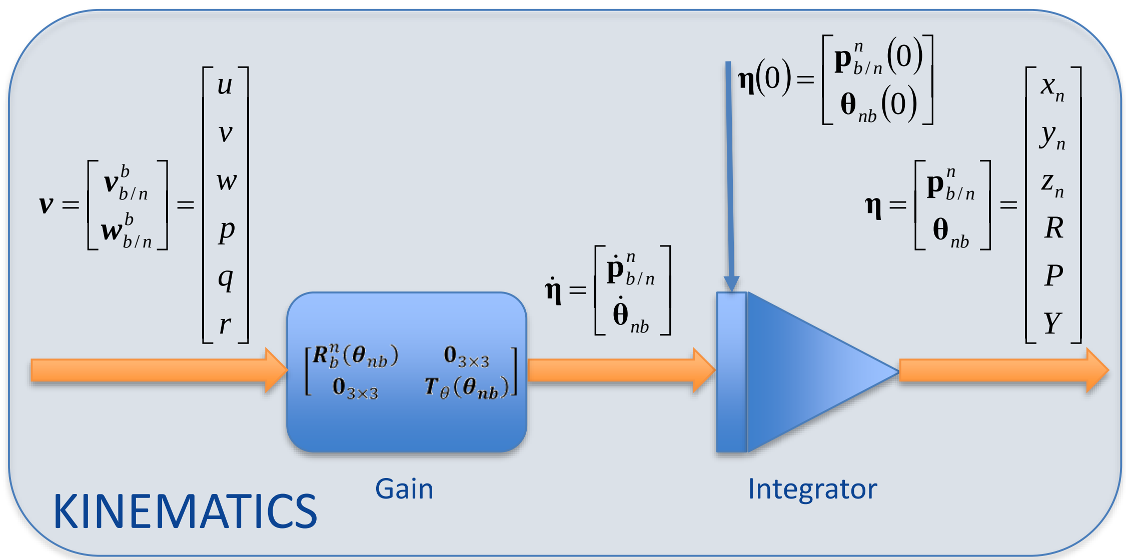
6 DOF Kinematic Equations

$$\begin{bmatrix} \dot{\boldsymbol{p}}^n_{b/n} \\ \dot{\boldsymbol{\theta}}_{nb} \end{bmatrix} = \begin{bmatrix} \boldsymbol{R}^n_b(\boldsymbol{\theta}_{nb}) & \boldsymbol{0}_{3\times3} \\ \boldsymbol{0}_{3\times3} & \boldsymbol{T}_\theta(\boldsymbol{\theta}_{nb}) \end{bmatrix} \begin{bmatrix} \boldsymbol{v}^b_{b/n} \\ \boldsymbol{w}^b_{b/n} \end{bmatrix}$$

$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}_{\Theta}(\boldsymbol{\eta})\boldsymbol{\nu}$$



Simulation Model
Attitude representation: **Euler Angles**



$\boldsymbol{H} \equiv \mathbb{R}^4$ Basis: ***1,i,j,k*** $i^2 = j^2 = k^2 = ijk = -1$

$ij = k, \quad ji = -k,$
$jk = i, \quad kj = -i,$
$ki = j, \quad ik = -j,$

$$q = \overbrace{a\mathbf{1}}^{\text{Scalar part}} + \underbrace{b\boldsymbol{i} + c\boldsymbol{j} + d\boldsymbol{k}}_{\text{Vector part}}$$

"Real" quaternions: $a\mathbf{1} + 0\boldsymbol{i} + 0\boldsymbol{j} + 0\boldsymbol{k}$

"Pure" quaternions: $0\mathbf{1} + b\boldsymbol{i} + c\boldsymbol{j} + d\boldsymbol{k}$

Noncommutativity of quaternion multiplication

×	**1**	***i***	***j***	***k***
1	1	*i*	*j*	*k*
i	*i*	−1	*k*	−*j*
j	*j*	−*k*	−1	*i*
k	*k*	*j*	−*i*	−1

Addition:

$$(a_1\mathbf{1} + b_1\boldsymbol{i} + c_1\boldsymbol{j} + d_1\boldsymbol{k}) + (a_2\mathbf{1} + b_2\boldsymbol{i} + c_2\boldsymbol{j} + d_2\boldsymbol{k})$$
$$= (a_1 + a_2)\mathbf{1} + (b_1 + b_2)\boldsymbol{i} + (c_1 + c_2)\boldsymbol{j} + (d_1 + d_2)\boldsymbol{k}$$

Scalar Multiplication:

$$\alpha(a\mathbf{1} + b\boldsymbol{i} + c\boldsymbol{j} + d\boldsymbol{k}) = \alpha a\mathbf{1} + \alpha b\boldsymbol{i} + \alpha c\boldsymbol{j} + \alpha d\boldsymbol{k}$$

Quaternion Multiplication:

$$(a_1\mathbf{1} + b_1\boldsymbol{i} + c_1\boldsymbol{j} + d_1\boldsymbol{k})(a_2\mathbf{1} + b_2\boldsymbol{i} + c_2\boldsymbol{j} + d_2\boldsymbol{k})$$
$$= (a_1a_2 - b_1b_2 - c_1c_2 - d_1d_2)\mathbf{1}$$
$$+ (a_1b_2 + b_1a_2 + c_1d_2 - d_1c_2)\boldsymbol{i}$$
$$+ (a_1c_2 - b_1d_2 + c_1a_2 + d_1b_2)\boldsymbol{j}$$
$$+ (a_1d_2 + b_1c_2 - c_1b_2 + d_1a_2)\boldsymbol{k}$$

$$q = (\overbrace{r}^{\text{Scalar part}}, \underbrace{\vec{v}}_{\text{Vector part}})$$

Addition: $(r_1, \vec{v}_1) + (r_2, \vec{v}_2) = (r_1 + r_2, \vec{v}_1 + \vec{v}_2)$

Scalar Multiplication: $\alpha(r, \vec{v}) = (\alpha r, \alpha\vec{v})$

Quaternion Multiplication: $(r_1, \vec{v}_1)(r_2, \vec{v}_2) = (r_1 r_2 - \vec{v}_1 \cdot \vec{v}_2, r_1\vec{v}_2 + r_2\vec{v}_1 + \vec{v}_1 \times \vec{v}_2)$

$$q = (\overbrace{r}^{\text{Scalar part}}, \underbrace{\vec{v}}_{\text{Vector part}}) = r\mathbf{1} + v_x\boldsymbol{i} + v_y\boldsymbol{j} + v_z\boldsymbol{k}$$

Conjugate: $q^* = (r, -\vec{v}) = r\mathbf{1} - v_x\boldsymbol{i} - v_y\boldsymbol{j} - v_z\boldsymbol{k}$

$$qq^* = (r, \vec{v})(r, -\vec{v}) = \left(r^2 + |\vec{v}|^2, \vec{0}\right) = \left(r^2 + v_x^2 + v_y^2 + v_z^2\right)\mathbf{1} + 0\boldsymbol{i} + 0\boldsymbol{j} + 0\boldsymbol{k}$$

$$q^*q = (r, -\vec{v})(r, \vec{v}) = \left(r^2 + |\vec{v}|^2, \vec{0}\right) = qq^*$$

$$(q_1q_2)^* = q_2^*q_1^*$$

Norm: $\|q\| = \sqrt{qq^*} = \sqrt{(r^2 + |\vec{v}|^2)} = \sqrt{\left(r^2 + v_x^2 + v_y^2 + v_z^2\right)}$

$$\|pq\|^2 = (pq)(pq)^* = pqq^*p^* = p\|q\|^2p^* = pp^*\|q\|^2 = \|p\|^2\|q\|^2$$

Inverse: $q^{-1} = \dfrac{q^*}{\|q\|^2}$

$$q^{-1}q = qq^{-1} = 1$$

Unit Quaternion: $\|q\| = 1 \Rightarrow r^2 + |\vec{v}|^2 = 1 \Rightarrow \exists \theta \in [0, \pi]: \cos\theta = r, \sin\theta = |\vec{v}|$

$$\|q\| = 1 \Rightarrow q = (r, \vec{v}) = \overbrace{\mathrm{r}}^{\cos\theta} + \overbrace{|\vec{v}|}^{\sin\theta}\underbrace{\frac{\vec{v}}{|\vec{v}|}}_{\hat{\vec{v}}} = \cos\theta + \sin\theta\hat{\vec{v}} = e^{\left(0,\theta\hat{\vec{v}}\right)}$$

Polar Decomposition:

$$q = \|q\|\overbrace{\frac{q}{\|q\|}}^{\text{Unit Quaternion}} = \|q\|\overbrace{\left(\cos\theta + \sin\theta\hat{\vec{v}}\right)}^{\text{Unit Quaternion}} = \|q\|\overbrace{e^{\left(0,\theta\hat{\vec{v}}\right)}}^{\text{Unit Quaternion}}$$

Power:

$$q^p = \|q\|^p e^{\left(0,(p\theta)\hat{\vec{v}}\right)} = \|q\|^p\left(\cos(p\theta) + \sin(p\theta)\hat{\vec{v}}\right)$$

Euler Formula (General Case):

$$\boxed{e^{(r,\vec{v})} = e^{(r,\vec{0})+\left(0,|\vec{v}|\hat{\vec{v}}\right)} = e^{(r,\vec{0})}e^{\left(0,\theta\,\hat{\vec{v}}\right)} = e^r e^{\left(0,\theta\,\hat{\vec{v}}\right)} = e^r\left(\cos\theta\,, \sin\theta\,\hat{\vec{v}}\right)}$$

Euler Formula ("Pure" Quaternion):

$$q = (0, \vec{v})$$

$$q^2 = (0, \vec{v})(0, \vec{v}) = \left(-|\vec{v}|^2, \vec{0}\right)$$

$$q^3 = \left(-|\vec{v}|^2, \vec{0}\right)(0, \vec{v}) = (0, -|\vec{v}|^2 \vec{v})$$

$$q^4 = (0, -|\vec{v}|^2 \vec{v})(0, \vec{v}) = \left(|\vec{v}|^4, \vec{0}\right)$$

$$q^5 = \left(|\vec{v}|^4, \vec{0}\right)(0, \vec{v}) = (0, |\vec{v}|^4 \vec{v})$$

$$q^6 = (0, |\vec{v}|^4 \vec{v})(0, \vec{v}) = \left(-|\vec{v}|^6, \vec{0}\right)$$

$$q^7 = \left(-|\vec{v}|^6, \vec{0}\right)(0, \vec{v}) = (0, -|\vec{v}|^6 \vec{v})$$

$$\vdots$$

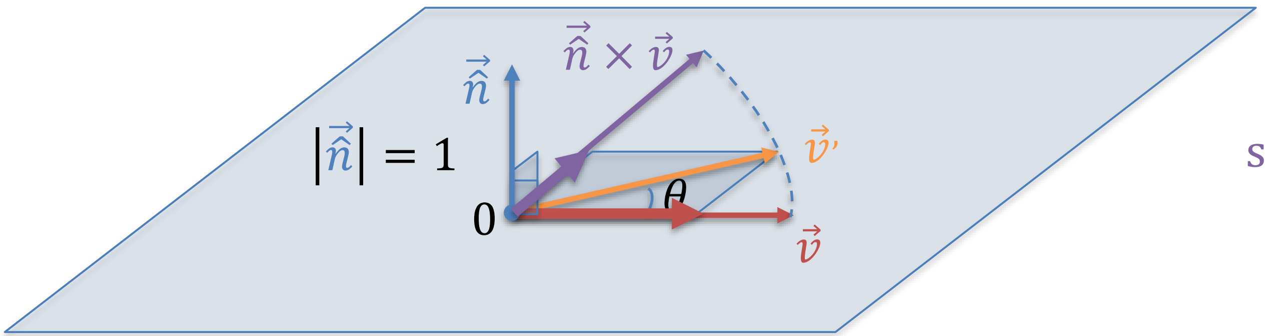
$$e^q = 1 + q + \frac{q^2}{2!} + \frac{q^3}{3!} + \frac{q^4}{4!} + \frac{q^5}{5!} + \frac{q^6}{6!} + \frac{q^7}{7!} \cdots$$

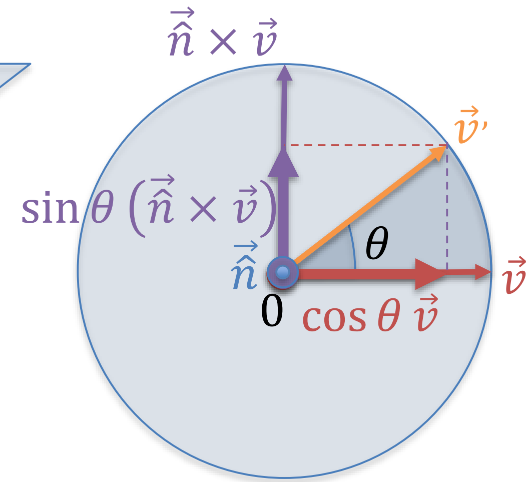
$$e^{(0,\vec{v})} = (1,0) + (0, \vec{v}) + \frac{(-|\vec{v}|^2, 0)}{2!} + \frac{(0, -|\vec{v}|^2 \vec{v})}{3!} + \frac{(|\vec{v}|^4, 0)}{4!} + \frac{(0, |\vec{v}|^4 \vec{v})}{5!} \cdots$$

$$\boxed{e^{(0,\vec{v})} = \left(\left(1 - \frac{|\vec{v}|^2}{2!} + \frac{|\vec{v}|^4}{4!} \cdots\right), \left(1 - \frac{|\vec{v}|^2}{3!} + \frac{|\vec{v}|^4}{5!} \cdots\right) \frac{\vec{v}}{|\vec{v}|}\right) = \left(\cos|\vec{v}|\,, \sin|\vec{v}|\; \hat{v}\right)}$$

Special case: Rotation About Perpendicular Axis

$$\boxed{\vec{v}' = \cos\theta\,\vec{v} + \sin\theta\,(\vec{\hat{n}} \times \vec{v})}$$





Solution using Quaternions:

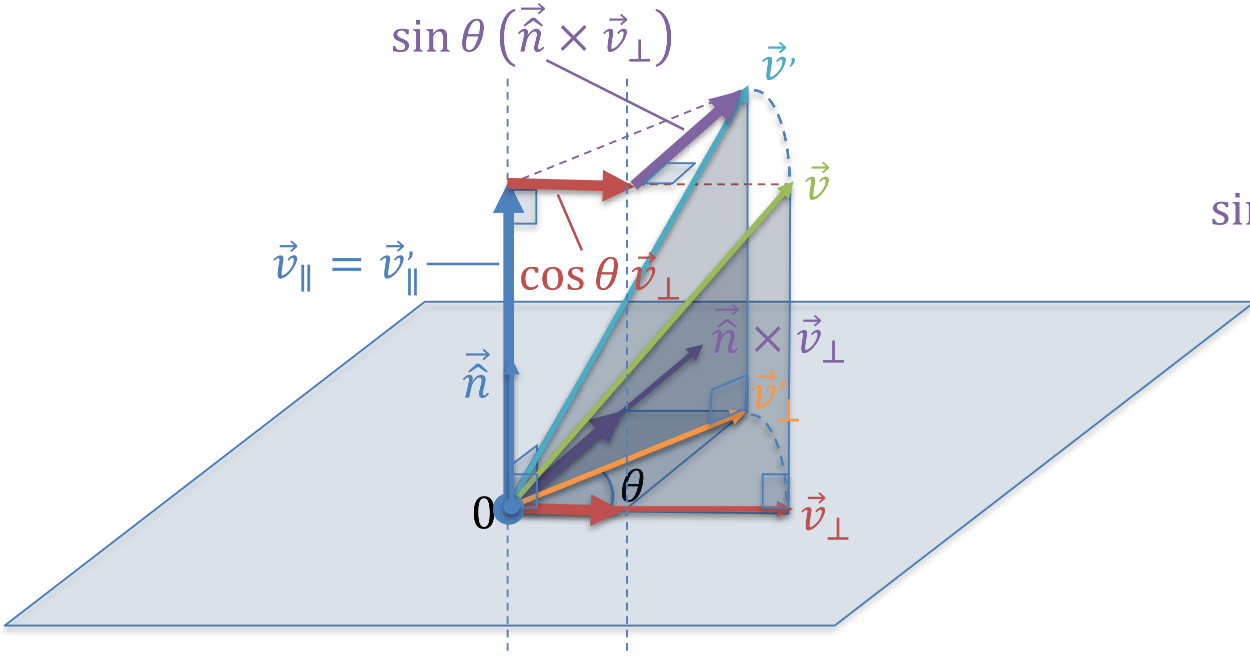
$$\left.\begin{aligned} v &= (0, \vec{v}) \\ n &= (0, \vec{\hat{n}}) \end{aligned}\right| \quad \begin{aligned} nv &= (0, \vec{\hat{n}})(0, \vec{v}) = (0, \vec{\hat{n}} \times \vec{v}) \\ n^2 &= (0, \vec{\hat{n}})(0, \vec{\hat{n}}) = (-1, \vec{0}) \end{aligned}$$

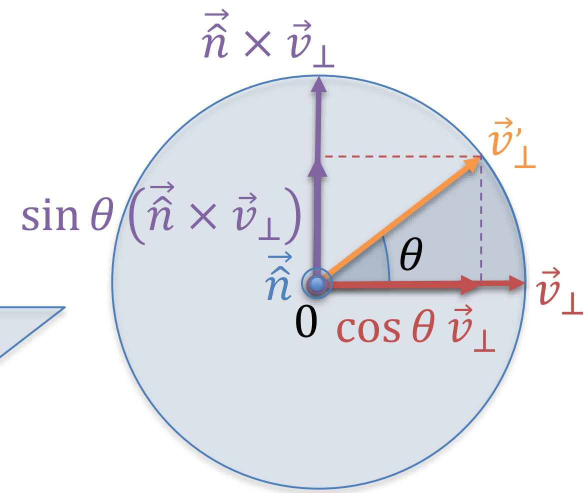
$$v' = (0, \vec{v}') = \left(0, \cos\theta\,\vec{v} + \sin\theta\,(\vec{\hat{n}} \times \vec{v})\right) = (\cos\theta\,, \sin\theta\,\vec{\hat{n}})(0, \vec{v}) = e^{\left(0, \theta\vec{\hat{n}}\right)}(0, \vec{v}) = e^{\left(0, \theta\vec{\hat{n}}\right)} v$$

$$\boxed{v' = e^{\left(0, \theta\vec{\hat{n}}\right)} v}$$

Rotation About General Axis

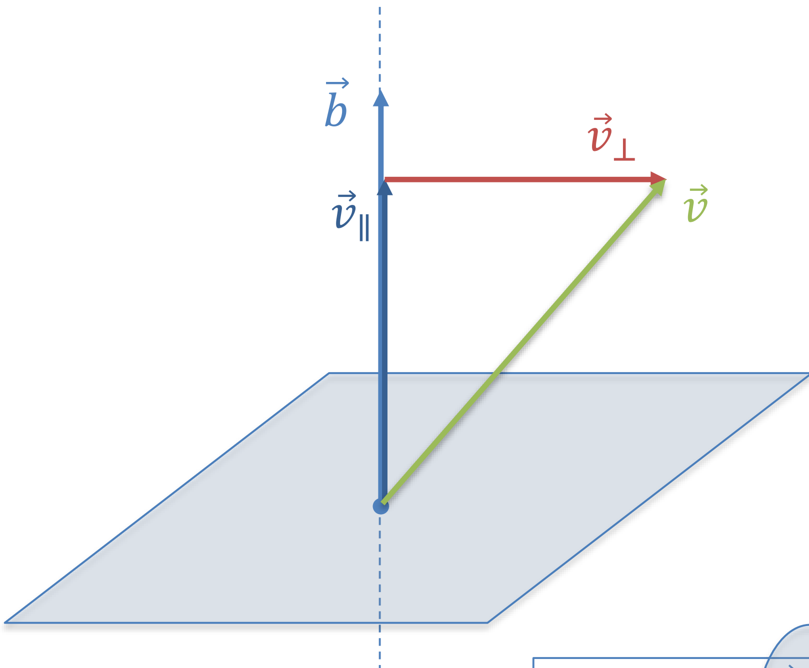
$$\vec{v}'_{\perp} = \cos\theta\,\vec{v}_{\perp} + \sin\theta\,(\vec{\hat{n}} \times \vec{v}_{\perp})$$





$$\vec{v}' = \vec{v}'_{\parallel} + \vec{v}'_{\perp} = \vec{v}_{\parallel} + \cos\theta\,\vec{v}_{\perp} + \sin\theta\,(\vec{\hat{n}} \times \vec{v}_{\perp})$$

Vector Projection onto Line



$$\vec{v}_{\parallel} = \alpha \vec{b}$$

$$\vec{v} = \vec{v}_{\parallel} + \vec{v}_{\perp} \Rightarrow \vec{v}_{\perp} = \vec{v} - \vec{v}_{\parallel} = \vec{v} - \alpha \vec{b}$$

$$\vec{b} \perp \vec{v}_{\perp} \Rightarrow \vec{b} \cdot \vec{v}_{\perp} = 0$$

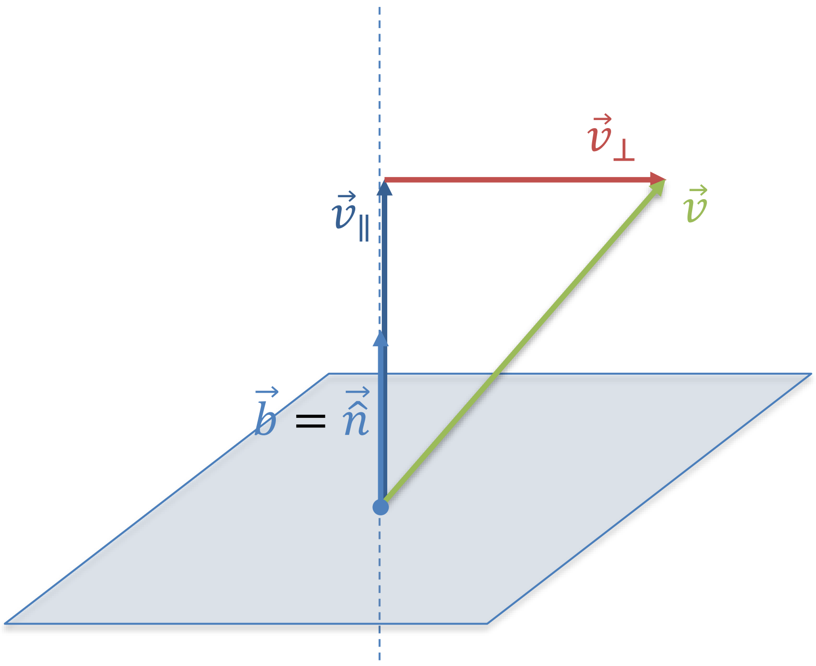
$$\vec{b} \cdot \left(\vec{v} - \alpha \vec{b}\right) = 0$$

$$\vec{b} \cdot \vec{v} - \alpha\left(\vec{b} \cdot \vec{b}\right) = 0$$

$$\boxed{\alpha = \frac{\vec{b} \cdot \vec{v}}{\vec{b} \cdot \vec{b}} = \frac{\vec{b}^T \vec{v}}{\vec{b}^T \vec{b}}}$$

$$\boxed{\vec{v}_{\parallel} = \alpha \vec{b} = \underbrace{\frac{\vec{b} \cdot \vec{v}}{\vec{b} \cdot \vec{b}}}_{\text{Scalar}} \vec{b} = \frac{\vec{b}^T \vec{v}}{\vec{b}^T \vec{b}} \vec{b} = \vec{b} \frac{\vec{b}^T \vec{v}}{\vec{b}^T \vec{b}} = \underbrace{\frac{\vec{b} \vec{b}^T}{\vec{b}^T \vec{b}}}_{\text{Projection Matrix}} \vec{v}}$$

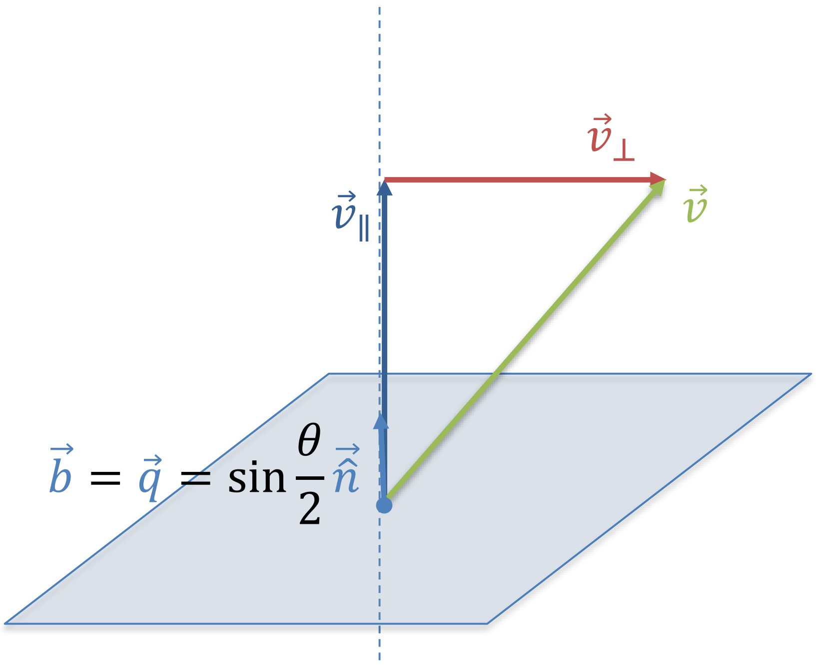
Special Case: $\vec{b} = \vec{\hat{n}} \qquad \left|\vec{\hat{n}}\right| = 1$



$$\vec{v}_\parallel = \frac{\vec{\hat{n}} \cdot \vec{v}}{\vec{\hat{n}} \cdot \vec{\hat{n}}}\vec{\hat{n}} = \frac{\vec{\hat{n}} \cdot \vec{v}}{\left|\vec{\hat{n}}\right|^2}\vec{\hat{n}} = \underbrace{\left(\vec{\hat{n}} \cdot \vec{v}\right)}_{\text{Scalar}}\vec{\hat{n}}$$

$$\vec{v}_\parallel = \frac{\vec{\hat{n}}\vec{\hat{n}}^T}{\vec{\hat{n}}^T\vec{\hat{n}}}\vec{v} = \frac{\vec{\hat{n}}\vec{\hat{n}}^T}{\left|\vec{\hat{n}}\right|^2}\vec{v} = \underbrace{\vec{\hat{n}}\vec{\hat{n}}^T}_{\text{Projection Matrix}}\vec{v}$$

Special Case: $\vec{b} = \vec{q} = \sin\frac{\theta}{2}\vec{\hat{n}}$ $\qquad |\vec{q}| = \sin\frac{\theta}{2}$

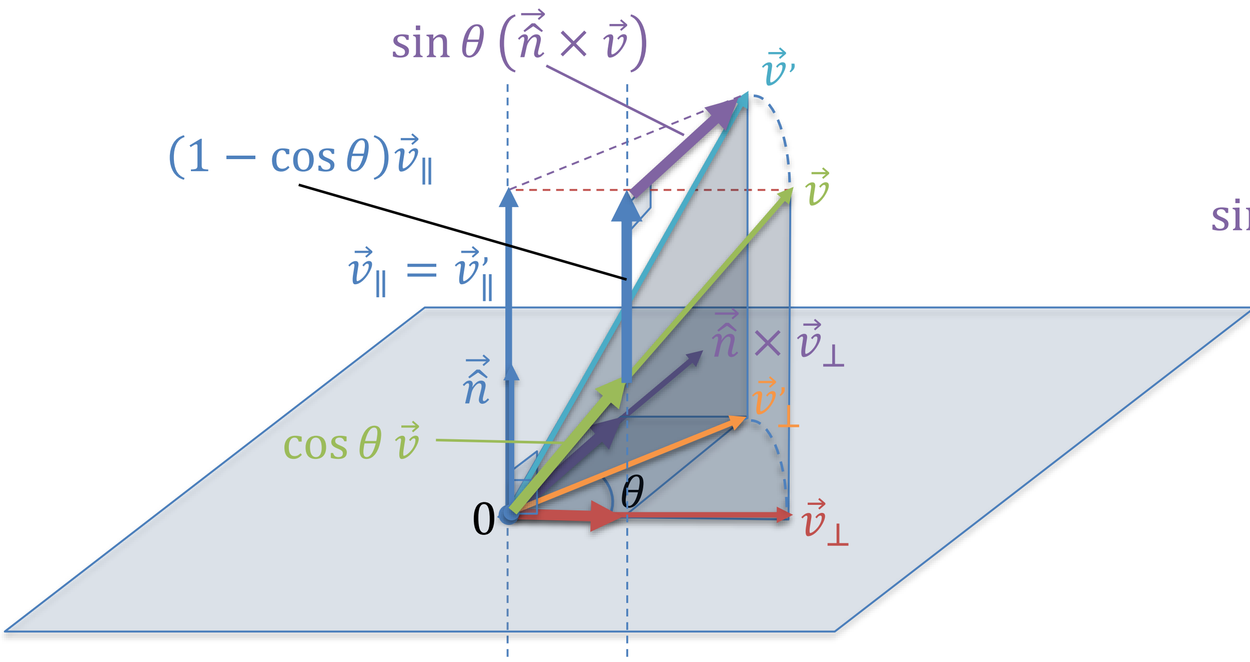


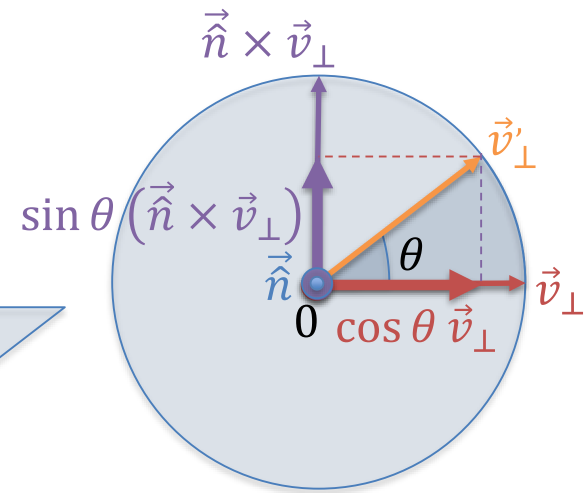
$$\vec{v}_\parallel = \frac{\vec{q}\cdot\vec{v}}{\vec{q}\cdot\vec{q}}\vec{q} = \frac{\vec{q}\cdot\vec{v}}{|\vec{q}|^2}\vec{q} = \underbrace{\frac{\vec{q}\cdot\vec{v}}{\sin^2\frac{\theta}{2}}}_{\text{Scalar}}\vec{q}$$

$$\vec{v}_\parallel = \frac{\vec{q}\vec{q}^T}{\vec{q}^T\vec{q}}\vec{v} = \frac{\vec{q}\vec{q}^T}{|\vec{q}|^2}\vec{v} = \underbrace{\frac{\vec{q}\vec{q}^T}{\sin^2\frac{\theta}{2}}}_{\text{Projection Matrix}}\vec{v}$$

Rotation About General Axis

$$\vec{v}'_{\perp} = \cos\theta\,\vec{v}_{\perp} + \sin\theta\,(\vec{\hat{n}}\times\vec{v}_{\perp})$$





$$\vec{v}' = \vec{v}'_{\parallel} + \vec{v}'_{\perp} = \vec{v}_{\parallel} + \cos\theta\,\vec{v}_{\perp} + \sin\theta\,(\vec{\hat{n}}\times\vec{v}_{\perp}) = \vec{v}_{\parallel} + \cos\theta\,(\vec{v}-\vec{v}_{\parallel}) + \sin\theta\,\left(\vec{\hat{n}}\times(\vec{v}-\vec{v}_{\parallel})\right)$$

$$= (1-\cos\theta)\vec{v}_{\parallel} + \cos\theta\,\vec{v} + \sin\theta\,(\vec{\hat{n}}\times\vec{v}) - \sin\theta\,\overbrace{\vec{\hat{n}}\times\vec{v}_{\parallel}}^{\vec{0}}$$

$$\boxed{\vec{v}_{\parallel} = (\vec{v}\cdot\vec{\hat{n}})\vec{\hat{n}}}$$

Rodrigues Rotation Formula: $\boxed{\vec{v}' = \cos\theta\,\vec{v} + (1-\cos\theta)\overbrace{(\vec{v}\cdot\vec{\hat{n}})\vec{\hat{n}}}^{\vec{v}_{\parallel}} + \sin\theta\,(\vec{\hat{n}}\times\vec{v})}$

Rotation About General Axis: Solution using Quaternions

$$v = (0, \vec{v})$$
$$n = (0, \vec{\hat{n}})$$
$$v_\parallel = (0, \vec{v}_\parallel)$$
$$v_\perp = (0, \vec{v}_\perp)$$
$$v' = (0, \vec{v}')$$

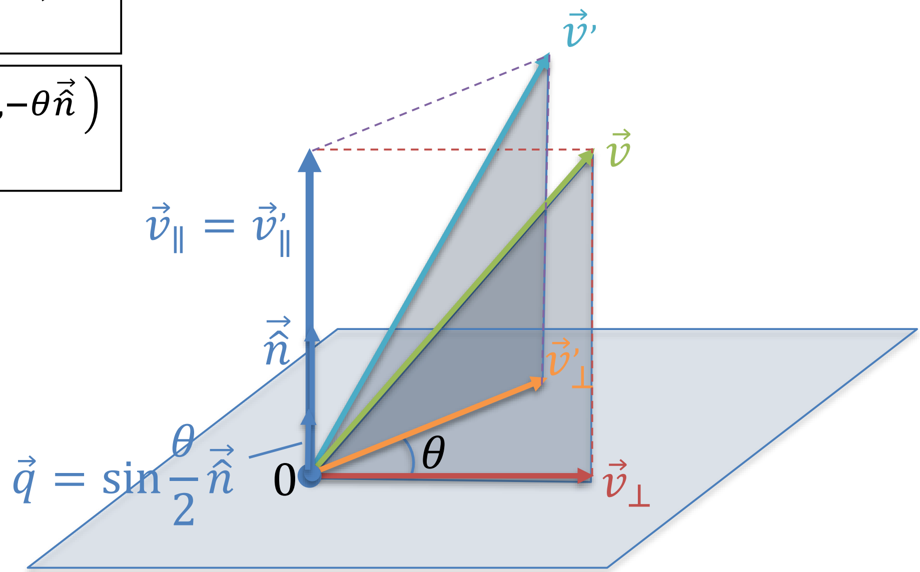
Lema 1: $e^{\left(0,\theta\vec{\hat{n}}\right)} v_\parallel = v_\parallel e^{\left(0,\theta\vec{\hat{n}}\right)}$

Lema 2: $e^{\left(0,\theta\vec{\hat{n}}\right)} v_\perp = v_\perp e^{\left(0,-\theta\vec{\hat{n}}\right)}$

$$\vec{v}' = \vec{v}'_\parallel + \vec{v}'_\perp = \vec{v}_\parallel + \vec{v}'_\perp$$

$$v' = (0, \vec{v}') = \left(0, \vec{v}'_\parallel + \vec{v}'_\perp\right) = \left(0, \vec{v}_\parallel + \vec{v}'_\perp\right)$$

$$v' = v_\parallel + v'_\perp = v_\parallel + e^{\left(0,\theta\vec{\hat{n}}\right)} v_\perp$$



$$v' = e^{\left(0,\frac{\theta}{2}\vec{\hat{n}}\right)} e^{\left(0,-\frac{\theta}{2}\vec{\hat{n}}\right)} v_\parallel + e^{\left(0,\frac{\theta}{2}\vec{\hat{n}}\right)} e^{\left(0,\frac{\theta}{2}\vec{\hat{n}}\right)} v_\perp = e^{\left(0,\frac{\theta}{2}\vec{\hat{n}}\right)} v_\parallel e^{\left(0,-\frac{\theta}{2}\vec{\hat{n}}\right)} + e^{\left(0,\frac{\theta}{2}\vec{\hat{n}}\right)} v_\perp e^{\left(0,-\frac{\theta}{2}\vec{\hat{n}}\right)}$$

Lema 1

Lema 2

$$\boxed{v' = e^{\left(0,\frac{\theta}{2}\vec{\hat{n}}\right)} (v_\parallel + v_\perp) e^{\left(0,-\frac{\theta}{2}\vec{\hat{n}}\right)} = e^{\left(0,\frac{\theta}{2}\vec{\hat{n}}\right)} v e^{\left(0,-\frac{\theta}{2}\vec{\hat{n}}\right)}}$$

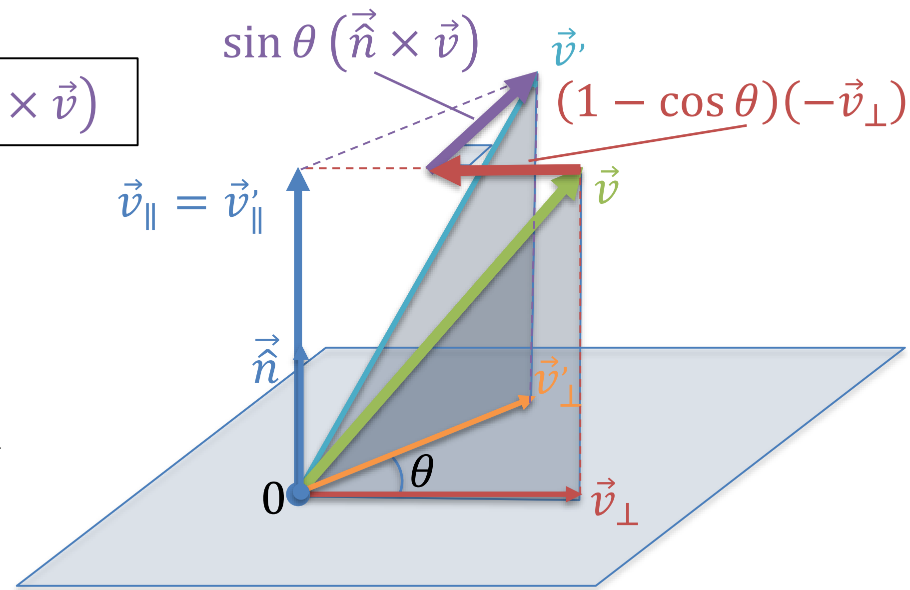
$$\boxed{\vec{v}_\parallel = \frac{\vec{q}\cdot\vec{v}}{\vec{q}\cdot\vec{q}}\vec{q}}$$

Rodrigues Rotation Formula: $\boxed{\vec{v}' = \cos\theta\,\vec{v} + (1-\cos\theta)(\vec{v}\cdot\vec{n})\vec{n} + \sin\theta\,(\vec{n}\times\vec{v})}$

$$\vec{v}' = (1-\cos\theta)\left((\vec{v}\cdot\vec{n})\vec{n} - \vec{v} + \vec{v}\right) + \cos\theta\,\vec{v} + \sin\theta\,(\vec{n}\times\vec{v})$$

$$= (1-\cos\theta)\left((\vec{v}\cdot\vec{n})\vec{n} - \vec{v}\right) + (1-\cos\theta+\cos\theta)\vec{v} + \sin\theta\,(\vec{n}\times\vec{v})$$

$$\boxed{\vec{v}' = \vec{v} + (1-\cos\theta)\overbrace{\left(\underbrace{(\vec{v}\cdot\vec{n})\vec{n}}_{\vec{v}_\parallel} - \vec{v}\right)}^{-\vec{v}_\perp} + \sin\theta\,(\vec{n}\times\vec{v})}$$



$$\vec{n}\times\vec{v} \coloneqq \boldsymbol{S}(\boldsymbol{n})\mathbf{v}$$

$$\vec{n}\times(\vec{n}\times\vec{v}) = (\vec{v}\cdot\vec{n})\vec{n} - \vec{v} = -\vec{v}_\perp$$

$$\boldsymbol{S}(\boldsymbol{n})(\boldsymbol{S}(\boldsymbol{n})\mathbf{v}) = \boldsymbol{S}^2(\boldsymbol{n})\mathbf{v} = \left(\boldsymbol{n}\boldsymbol{n}^T - \boldsymbol{I}_{3\times3}\right)\mathbf{v}$$

$$= \left(\boldsymbol{n}\boldsymbol{n}^T\right)\mathbf{v} - \boldsymbol{I}_{3\times3}\mathbf{v} = \mathbf{v}_\parallel - \mathbf{v} = -\mathbf{v}_\perp$$

$$\mathbf{v}' = \underbrace{\left(\boldsymbol{I}_{3\times3} + (1-\cos\theta)\boldsymbol{S}^2(\boldsymbol{n}) + \sin\theta\,\boldsymbol{S}(\boldsymbol{n})\right)}_{\boldsymbol{R}_{\boldsymbol{n},\theta}}\mathbf{v}$$

Rotation Formula using Quaternions:

$$\boxed{v' = e^{\left(0,\frac{\theta}{2}\vec{\hat{n}}\right)}(v_{\parallel} + v_{\perp})e^{\left(0,-\frac{\theta}{2}\vec{\hat{n}}\right)} = e^{\left(0,\frac{\theta}{2}\vec{\hat{n}}\right)}ve^{\left(0,-\frac{\theta}{2}\vec{\hat{n}}\right)} = qvq^*}$$

$$q = e^{\left(0,\frac{\theta}{2}\vec{\hat{n}}\right)} = \left(\cos\frac{\theta}{2}, \sin\frac{\theta}{2}\vec{\hat{n}}\right) = (q_0, \vec{q}), \quad q_0^2 + |\vec{q}|^2 = 1$$

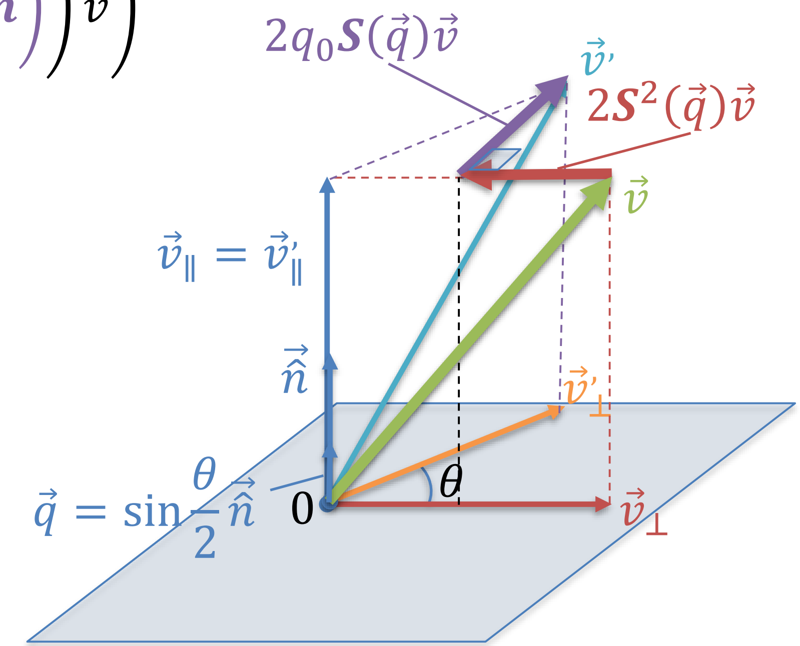
$$v' = (0, (\boldsymbol{I}_{3\times3} + (1-\cos\theta)\boldsymbol{S}^2(\boldsymbol{n}) + \sin\theta\boldsymbol{S}(\boldsymbol{n}))\vec{v}) =$$

$$= \left(0, \left(\boldsymbol{I}_{3\times3} + 2\sin^2\frac{\theta}{2}\boldsymbol{S}^2(\boldsymbol{n}) + 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\boldsymbol{S}(\boldsymbol{n})\right)\vec{v}\right)$$

$$v' = \left(0, \left(\boldsymbol{I}_{3\times3} + 2\boldsymbol{S}^2\left(\sin\frac{\theta}{2}\boldsymbol{n}\right) + 2\cos\frac{\theta}{2}\boldsymbol{S}\left(\sin\frac{\theta}{2}\boldsymbol{n}\right)\right)\vec{v}\right)$$

$$\boxed{v' = (0, (\underbrace{\boldsymbol{I}_{3\times3} + 2\boldsymbol{S}^2(\vec{q}) + 2q_0\boldsymbol{S}(\vec{q})}_{\boldsymbol{R}_b^n(\boldsymbol{q}) = \boldsymbol{R}_{(q_0,\vec{q})}})\vec{v})}$$

$$\boldsymbol{S}^2(\vec{q}) = \vec{q}\vec{q}^T - |\vec{q}|^2\boldsymbol{I}_{3\times3}$$



EXCELLABUST

Kinematics

Transformations BODY-NED

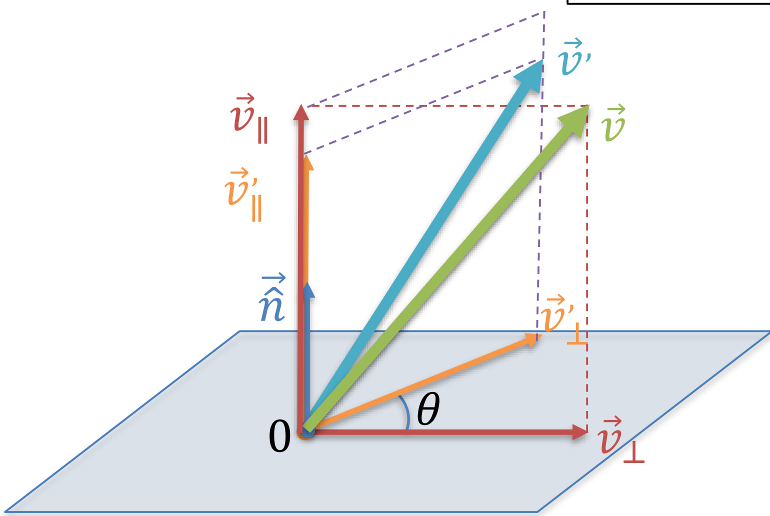
Quaternions

University of Limerick

Mobile & Marine Robotics Research Centre

Wrong Formula:

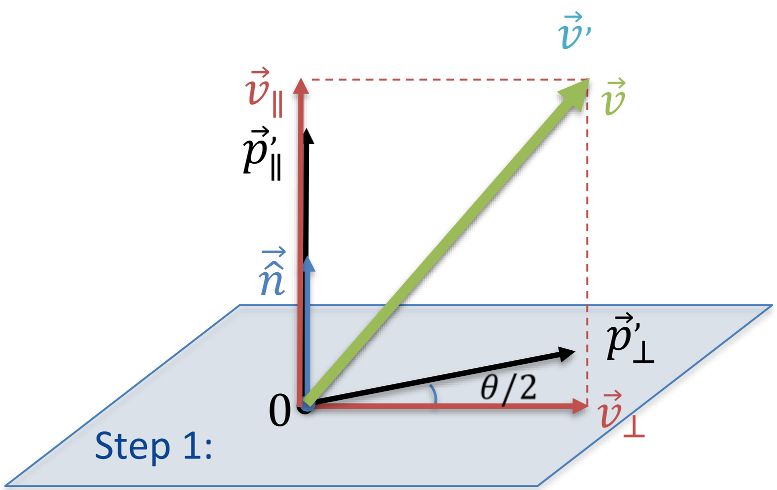
$$v' = e^{\left(0,\theta\vec{\hat{n}}\right)}(v_{\parallel} + v_{\perp}) = \underbrace{e^{\left(0,\theta\vec{\hat{n}}\right)}v_{\parallel}}_{\vec{v}'_{\parallel}\ \times} + \underbrace{e^{\left(0,\theta\vec{\hat{n}}\right)}v_{\perp}}_{\vec{v}'_{\perp}\ \checkmark}$$

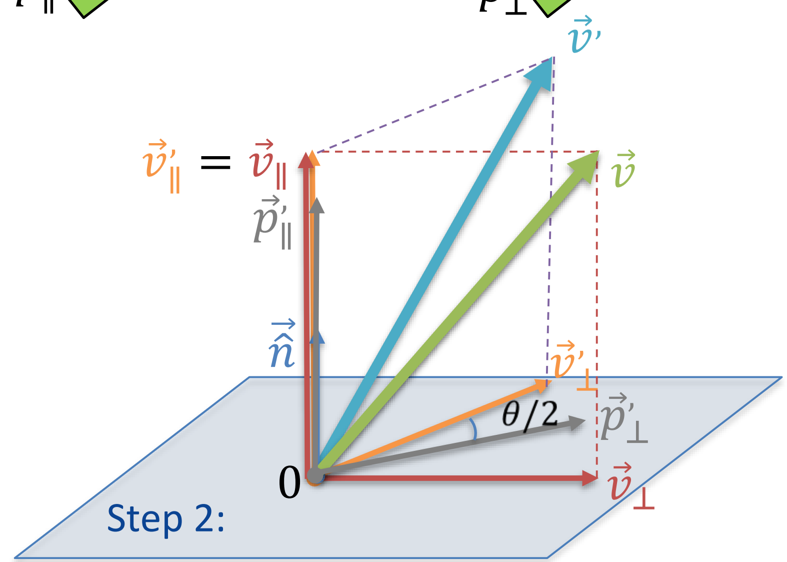


Correct Formula:

$$v' = e^{\left(0,\frac{\theta}{2}\vec{\hat{n}}\right)}(v_{\parallel} + v_{\perp})e^{\left(0,-\frac{\theta}{2}\vec{\hat{n}}\right)} =$$

$$= \underbrace{\left[e^{\left(0,\frac{\theta}{2}\vec{\hat{n}}\right)}v_{\parallel}\right]}_{\vec{p}'_{\parallel}\ \checkmark} e^{\left(0,-\frac{\theta}{2}\vec{\hat{n}}\right)} + \underbrace{\left[e^{\left(0,\frac{\theta}{2}\vec{\hat{n}}\right)}v_{\perp}\right]}_{\vec{p}'_{\perp}\ \checkmark} e^{\left(0,-\frac{\theta}{2}\vec{\hat{n}}\right)}$$





Rotation Formula using Quaternions:

$$\boxed{v' = e^{\left(0,\frac{\theta}{2}\vec{\hat{n}}\right)}(v_\parallel + v_\perp)e^{\left(0,-\frac{\theta}{2}\vec{\hat{n}}\right)} = e^{\left(0,\frac{\theta}{2}\vec{\hat{n}}\right)}v e^{\left(0,-\frac{\theta}{2}\vec{\hat{n}}\right)} = qvq^*}$$

$$q = e^{\left(0,\frac{\theta}{2}\vec{\hat{n}}\right)} = \left(\cos\frac{\theta}{2}, \sin\frac{\theta}{2}\vec{\hat{n}}\right) = (q_0, \vec{q}), \quad q_0^2 + |\vec{q}|^2 = 1$$

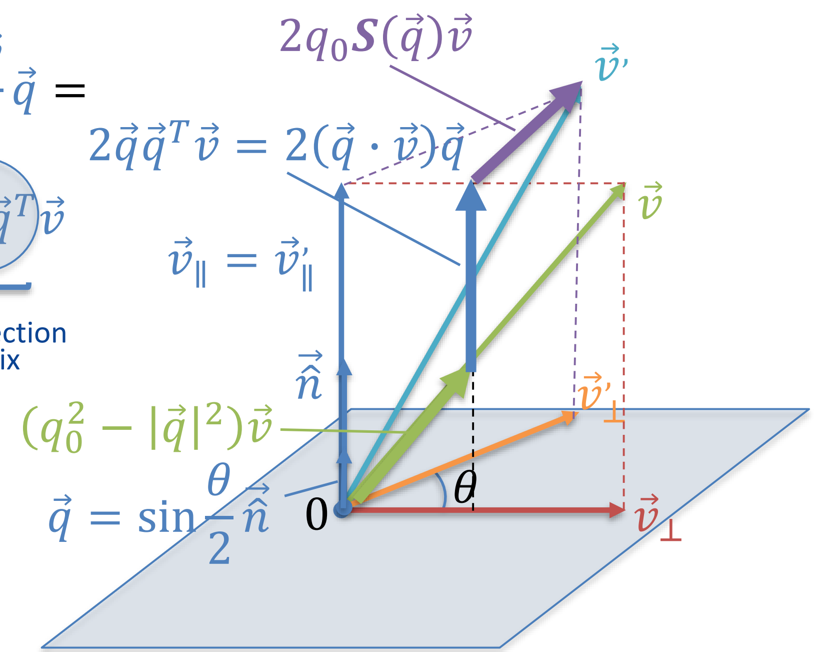
Quaternion Rotation Operator:

$$v' = L_q(v) \coloneqq qvq^* = (q_0, \vec{q})(0, \vec{v})(q_0, -\vec{q}) = \left(0, (q_0^2 - |\vec{q}|^2)\vec{v} + 2(\vec{q}\cdot\vec{v})\vec{q} + 2q_0(\vec{q}\times\vec{v})\right)$$

$$(q_0^2 - |\vec{q}|^2)\vec{v} = (q_0^2 + |\vec{q}|^2 - 2|\vec{q}|^2)\vec{v} = \left(1 - 2\sin^2\frac{\theta}{2}\right)\vec{v} = \cos\theta\,\vec{v}$$

$$(1-\cos\theta)\vec{v}_\parallel = \left(2\sin^2\frac{\theta}{2}\right)\frac{\vec{q}\cdot\vec{v}}{\vec{q}\cdot\vec{q}}\vec{q} = \left(2\sin^2\frac{\theta}{2}\right)\frac{\vec{q}\cdot\vec{v}}{|\vec{q}|^2}\vec{q} =$$

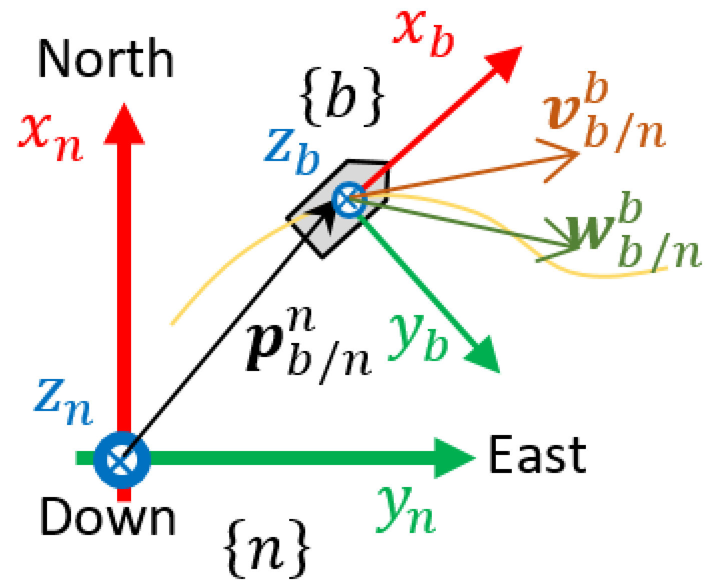
$$= \left(2\sin^2\frac{\theta}{2}\right)\frac{\vec{q}\cdot\vec{v}}{\sin^2\frac{\theta}{2}}\vec{q} = \underbrace{2(\vec{q}\cdot\vec{v})}_{\text{Scalar}}\vec{q} = 2\vec{q}(\vec{q}^T\vec{v}) = \underbrace{2\vec{q}\vec{q}^T}_{\text{Projection Matrix}}\vec{v}$$



$$\boxed{v' = \left(0, \left((q_0^2 - |\vec{q}|^2)\boldsymbol{I}_{3\times3} + 2\vec{q}\vec{q}^T + 2q_0\boldsymbol{S}(\vec{q})\right)\vec{v}\right)}$$

Linear Velocity Transformation

$$\dot{\boldsymbol{p}}^n_{b/n} = \boldsymbol{v}^n_{b/n} = \boldsymbol{R}^n_b(\boldsymbol{q})\boldsymbol{v}^b_{b/n}$$



$$\boldsymbol{R}^n_b(\boldsymbol{q}) = \boldsymbol{R}_{(q_0,\vec{q})} = \boldsymbol{I}_{3\times 3} + 2\boldsymbol{S}^2(\vec{q}) + 2q_0\boldsymbol{S}(\vec{q})$$

$$q = (q_0, \vec{q}) = q_0\mathbf{1} + q_1\boldsymbol{i} + q_2\boldsymbol{j} + q_3\boldsymbol{k}$$

$$q_0 = \cos\frac{\theta}{2}$$

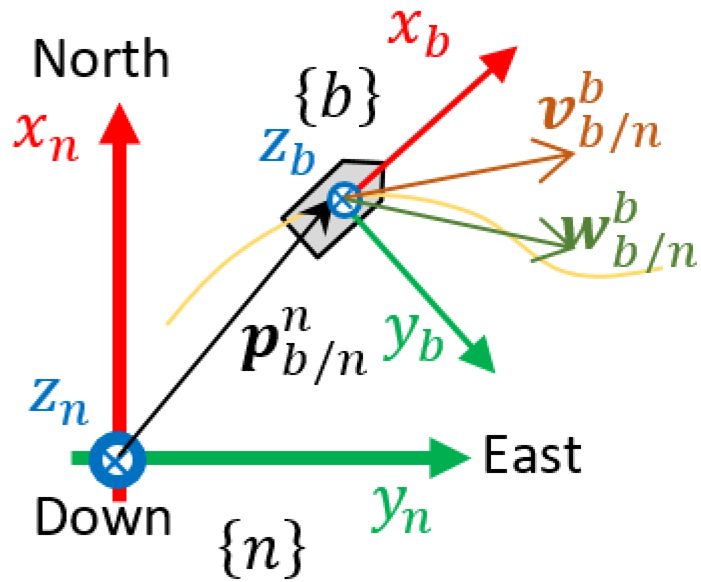
$$\vec{q} = \sin\frac{\theta}{2}\vec{\hat{n}}$$

$$\boldsymbol{R}^n_b(\boldsymbol{q})^{-1} = \boldsymbol{R}^n_b(\boldsymbol{q})^T$$

$$\boldsymbol{R}^n_b(\boldsymbol{q}) = \begin{bmatrix} 1-2(q_2^2+q_3^2) & 2(q_1q_2-q_3q_0) & 2(q_1q_3+q_2q_0) \\ 2(q_1q_2+q_3q_0) & 1-2(q_3^2+q_1^2) & 2(q_2q_3-q_1q_0) \\ 2(q_1q_3-q_2q_0) & 2(q_2q_3+q_1q_0) & 1-2(q_1^2+q_2^2) \end{bmatrix}$$

Angular Velocity Transformation

$$\dot{\boldsymbol{R}}_b^n(\boldsymbol{q}) = \boldsymbol{R}_b^n(\boldsymbol{q})\boldsymbol{S}\left(\boldsymbol{w}_{b/n}^b\right) \Rightarrow \boxed{\dot{q} = \boldsymbol{T}_q(\boldsymbol{q})\boldsymbol{w}_{b/n}^b}$$



$$q = (q_0, \vec{q}) = q_0\mathbf{1} + q_1\boldsymbol{i} + q_2\boldsymbol{j} + q_3\boldsymbol{k}$$

$$q_0 = \cos\frac{\theta}{2} \qquad \boldsymbol{T}_q^T(\boldsymbol{q})\boldsymbol{T}_q(\boldsymbol{q}) = \frac{1}{4}\boldsymbol{I}_{3\times3}$$

$$\vec{q} = \sin\frac{\theta}{2}\vec{\hat{n}}$$

$$\boldsymbol{T}_q(\boldsymbol{q}) = \frac{1}{2}\begin{bmatrix} -\vec{q}^T \\ q_0\boldsymbol{I}_{3\times3} + \boldsymbol{S}(\vec{q}) \end{bmatrix}$$

$$\boldsymbol{T}_q(\boldsymbol{q}) = \frac{1}{2}\begin{bmatrix} -q_1 & -q_2 & -q_3 \\ q_0 & -q_3 & q_2 \\ q_3 & q_0 & -q_1 \\ -q_2 & q_1 & q_0 \end{bmatrix}$$

Angular Velocity Transformation

Quaternions

$$\boldsymbol{T}_{\boldsymbol{q}}(\boldsymbol{q}) = \frac{1}{2}\begin{bmatrix} -q_1 & -q_2 & -q_3 \\ q_0 & -q_3 & q_2 \\ q_3 & q_0 & -q_1 \\ -q_2 & q_1 & q_0 \end{bmatrix}$$

Euler Angles

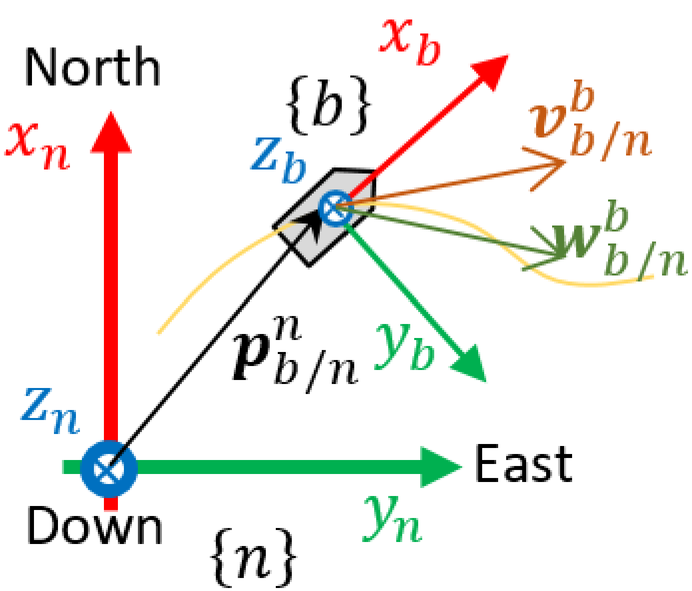
$$\boldsymbol{T}_{\theta}(\boldsymbol{\theta_{nb}}) = \begin{bmatrix} 1 & sRtP & cRtP \\ 0 & cR & -sR \\ 0 & sR/cP & cR/cP \end{bmatrix}$$

Singularity for P=±90°

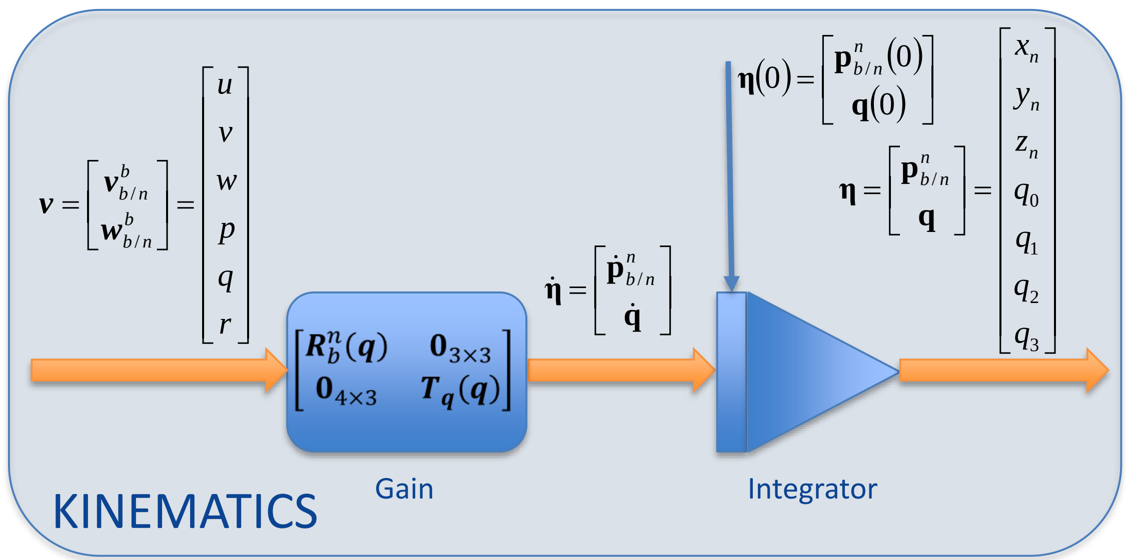
6 DOF Kinematic Equations

$$\begin{bmatrix} \dot{\boldsymbol{p}}^n_{b/n} \\ \dot{\boldsymbol{q}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{R}^n_b(\boldsymbol{q}) & \boldsymbol{0}_{3\times3} \\ \boldsymbol{0}_{4\times3} & \boldsymbol{T}_q(\boldsymbol{q}) \end{bmatrix} \begin{bmatrix} \boldsymbol{v}^b_{b/n} \\ \boldsymbol{w}^b_{b/n} \end{bmatrix}$$

$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}_q(\boldsymbol{\eta})\boldsymbol{v}$$



Simulation Model
Attitude representation: **Quaternions**



Euler Angles → Quaternions

$$\boldsymbol{q} = \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} c(R/2)c(P/2)c(Y/2) + s(R/2)s(P/2)s(Y/2) \\ s(R/2)c(P/2)c(Y/2) - c(R/2)s(P/2)s(Y/2) \\ c(R/2)s(P/2)c(Y/2) + s(R/2)c(P/2)s(Y/2) \\ c(R/2)c(P/2)s(Y/2) - s(R/2)s(P/2)c(Y/2) \end{bmatrix}$$

Quaternions → Euler Angles

$$\boldsymbol{\theta}_{\boldsymbol{nb}} = \begin{bmatrix} R \\ P \\ Y \end{bmatrix} = \begin{bmatrix} \operatorname{atan2}\left(2(q_2 q_3 + q_1 q_0), 1 - 2(q_1^2 + q_2^2)\right) \\ \operatorname{asin}\left(2(q_0 q_2 - q_3 q_1)\right) \\ \operatorname{atan2}\left(2(q_0 q_3 + q_1 q_2), 1 - 2(q_2^2 + q_3^2)\right) \end{bmatrix}$$

NED → ECEF

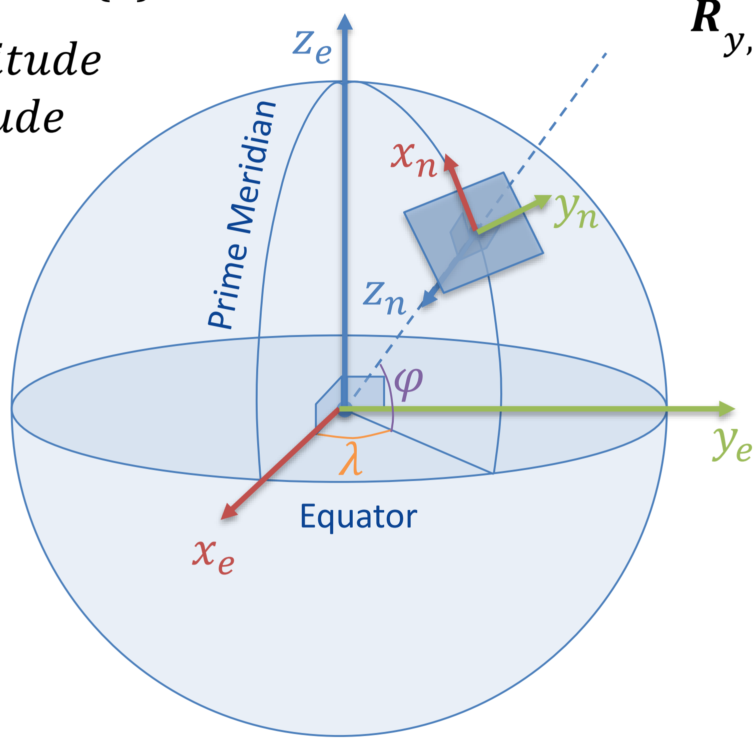
$$\boldsymbol{R}_{z,Y} = \begin{bmatrix} c\lambda & -s\lambda & 0 \\ s\lambda & c\lambda & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

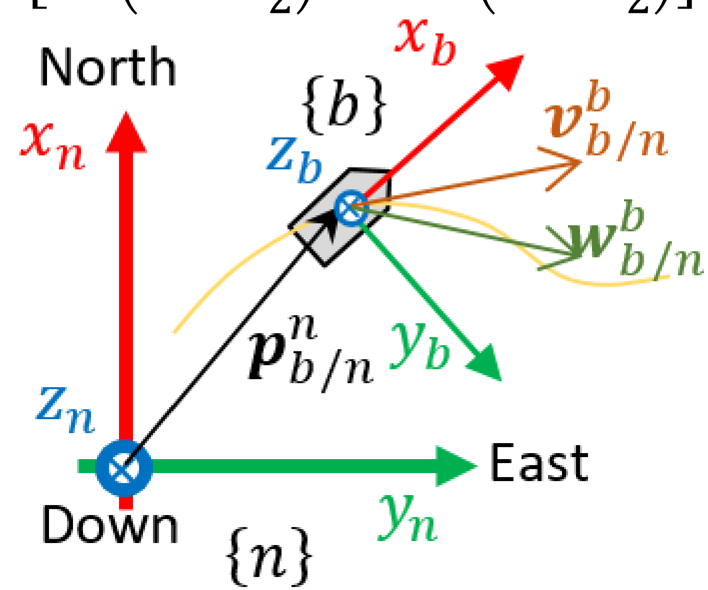
$$\dot{\boldsymbol{p}}^e_{b/e} = \underbrace{\boldsymbol{R}^e_n(\boldsymbol{\theta}_{en})\overbrace{\boldsymbol{R}^n_b(\boldsymbol{\theta}_{nb})\underbrace{\boldsymbol{v}^b_{b/n}}_{\{b\}}}^{\{n\}}}_{\{e\}} = \underbrace{\boldsymbol{R}_{z,\lambda}\boldsymbol{R}_{y,-\varphi-\frac{\pi}{2}}}_{\boldsymbol{R}^e_n(\boldsymbol{\theta}_{en})}\,\dot{\boldsymbol{p}}^n_{b/n}$$

$$\boldsymbol{R}_{y,-\varphi-\frac{\pi}{2}} = \begin{bmatrix} c\left(-\varphi-\frac{\pi}{2}\right) & 0 & s\left(-\varphi-\frac{\pi}{2}\right) \\ 0 & 1 & 0 \\ -s\left(-\varphi-\frac{\pi}{2}\right) & 0 & c\left(-\varphi-\frac{\pi}{2}\right) \end{bmatrix}$$

$\lambda - Longitude$

$\varphi - Latitude$





NED → ECEF

$$\underbrace{\dot{\boldsymbol{p}}^{e}_{b/e}}_{\text{ECEF}} = \boldsymbol{R}^{e}_{n}(\boldsymbol{\theta}_{en})\underbrace{\dot{\boldsymbol{p}}^{n}_{b/n}}_{\text{NED}} = \underbrace{\begin{bmatrix} -c\lambda s\varphi & -s\lambda & -c\lambda c\varphi \\ -s\lambda s\varphi & c\lambda & -s\lambda c\varphi \\ c\varphi & 0 & -s\varphi \end{bmatrix}}_{\boldsymbol{R}^{e}_{n}(\boldsymbol{\theta}_{en})} \underbrace{\begin{bmatrix} \dot{x}_n \\ \dot{y}_n \\ \dot{z}_n \end{bmatrix}}_{\dot{\boldsymbol{p}}^{n}_{b/n}} \begin{array}{l} \text{North Velocity} \\ \text{East Velocity} \\ \text{Down Velocity} \end{array}$$

ECEF → NED

$$\underbrace{\dot{\boldsymbol{p}}^{n}_{b/n}}_{\text{NED}} = \boldsymbol{R}^{n}_{e}(\boldsymbol{\theta}_{en})\underbrace{\dot{\boldsymbol{p}}^{e}_{b/e}}_{\text{ECEF}} = {\boldsymbol{R}^{e}_{n}}^{T}(\boldsymbol{\theta}_{en})\dot{\boldsymbol{p}}^{e}_{b/e} = \underbrace{\begin{bmatrix} -c\lambda s\varphi & -s\lambda s\varphi & c\varphi \\ -s\lambda & c\lambda & 0 \\ -c\lambda c\varphi & -s\lambda c\varphi & -s\varphi \end{bmatrix}}_{\boldsymbol{R}^{n}_{e}(\boldsymbol{\theta}_{en})} \underbrace{\begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{z}_e \end{bmatrix}}_{\dot{\boldsymbol{p}}^{e}_{b/e}}$$

Speed & Heading

$$\text{Speed} = \sqrt{\dot{x}_n^2 + \dot{y}_n^2}$$

$$\text{Heading} = \tan^{-1}\frac{y_n}{x_n}$$

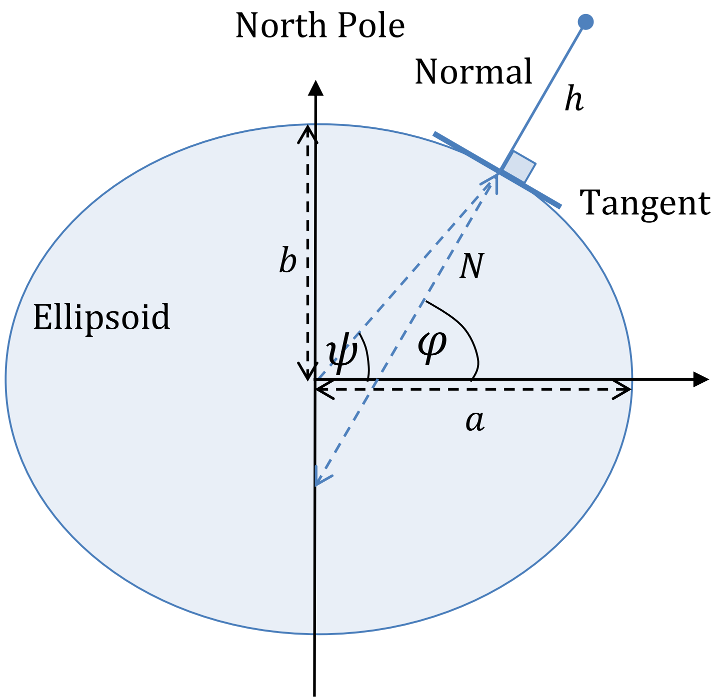
Geodetic Latitude vs Geocentric Latitude

Raw GNSS (GPS, GLONASS and Gallileo) output $\boldsymbol{p}^{\boldsymbol{e}}_{b/e}$ is given in Cartesian ECEF frame.

It is presented to users in terms of longitude λ, latitude φ and height h relative to WGS-84 ellipsoid.

$\varphi - Geodetic\ Latitude$
$\psi - Geocentric\ Latitude$

$$\psi(\varphi) = \tan^{-1}\left((1 - e^2)\tan\varphi\right)$$



WGS-84 parameters:

$a = 6378137.0m\ (semi - major\ axis)$

$b = 6356752.3142m\ (semi - minor\ axis)$

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}\ (eccentricity)$$

$$e' = \sqrt{\frac{a^2 - b^2}{b^2}}$$

$f = (a - b)/a$ ("flattening" $parameter$)

$e^2 = 2f - f^2$

1. Mean Sea Level (MSL) Datum (height relative to geoid)
2. GPS height (height above WGS-84 ellipsoid)

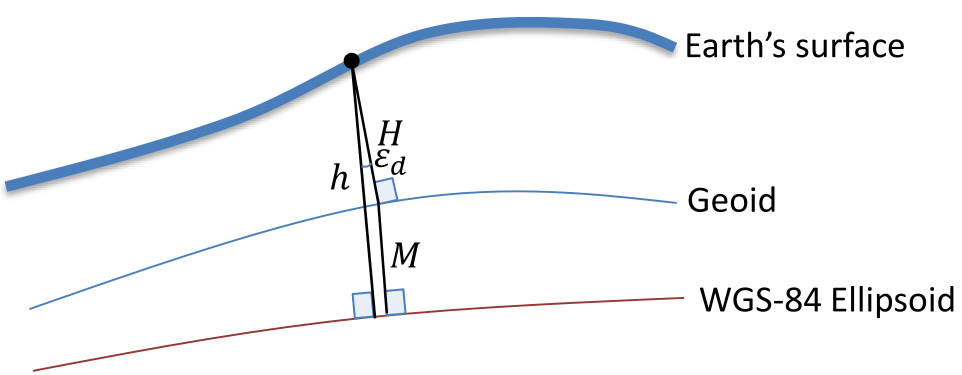
$h - ellipsoidal\ height\ (geodetic)$

$H - orthometric\ height\ (MSL)$

$M - geoid\ separation\ (undulation)\quad |M| \leq 100m$

$\varepsilon_d - deflection\ of\ the\ vertical \approx 0$

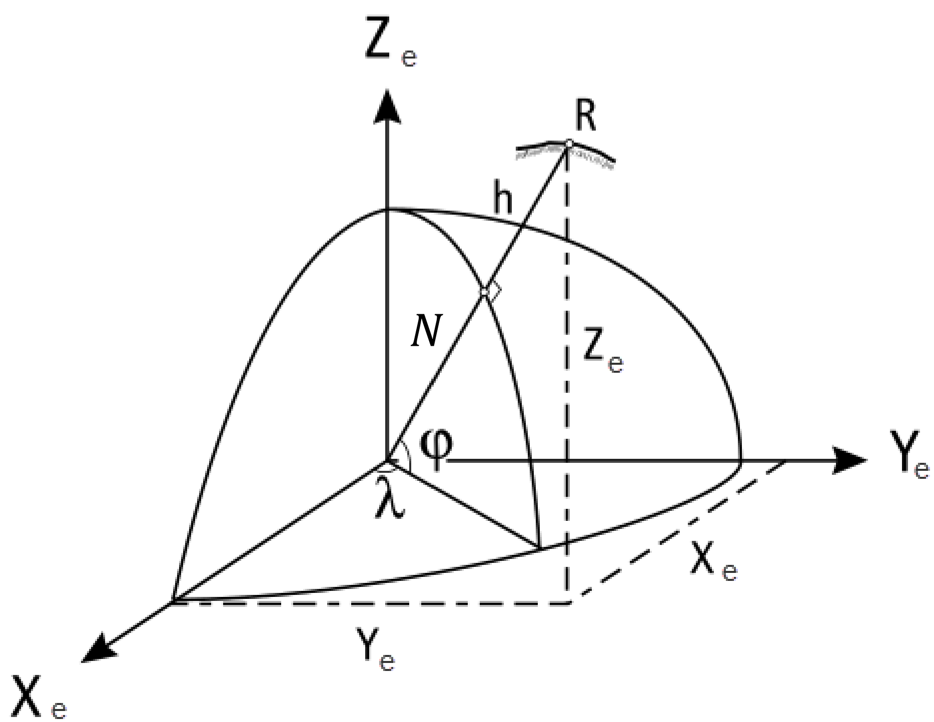
$$\boxed{h \approx H + M}$$



Lat-Lon → ECEF Coordinates

$$N = \frac{a^2}{\sqrt{(a^2\cos^2\varphi + b^2\sin^2\varphi)}} \qquad (Prime\ Vertical\ of\ Curvature\ (m))$$

$$\boldsymbol{p}_{b/e}^{\boldsymbol{e}} = \begin{bmatrix} x_e \\ y_e \\ z_e \end{bmatrix} = \begin{bmatrix} (N+h)\cos\varphi\cos\lambda \\ (N+h)\cos\varphi\sin\lambda \\ \left(\frac{b^2}{a^2}N + h\right)\sin\lambda \end{bmatrix}$$



ECEF Coordinates → Lat-Lon

$$p = \sqrt{x_e^2 + y_e^2}$$

$$\theta = \tan^{-1} \frac{z_e a}{p b}$$

$$\lambda = \tan^{-1} \frac{y_e}{x_e}$$

$$\varphi = \tan^{-1} \frac{z_e + e'^2 b \sin^3 \theta}{p - e^2 a \cos^3 \theta}$$

$$h = \frac{p}{\cos \varphi} - N$$

EXCELLABUST

EXCELLING LABUST IN MARINE ROBOTICS

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 691980.

FER
UNIVERSITY OF ZAGREB
FACULTY OF ELECTRICAL ENGINEERING AND COMPUTING

LABUST
LABORATORY FOR UNDERWATER SYSTEMS AND TECHNOLOGIES

Consiglio Nazionale delle Ricerche

UNIVERSITY of LIMERICK
OLLSCOIL LUIMNIGH

Universitat de Girona
Institut de Recerca en Visió per Computador i Robòtica

GIRONA UNDERWATER VISION AND ROBOTICS