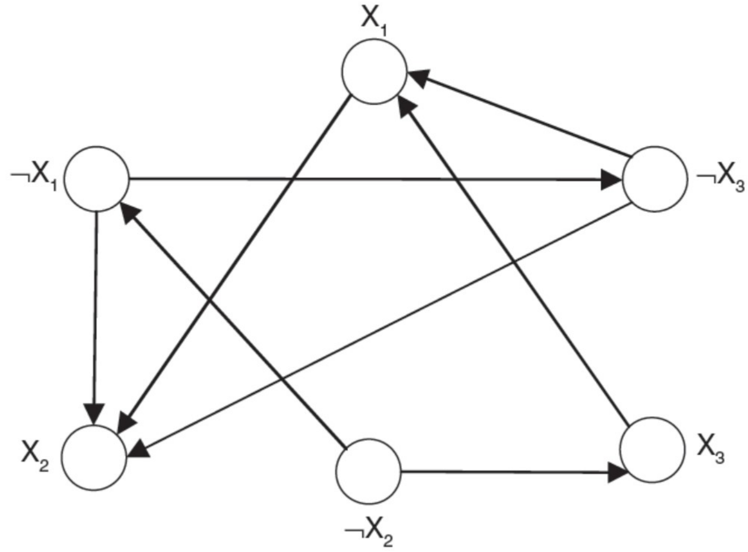

The same thing is true for an edge $\neg x$ to x in G.

For proving the 'only if' part, we have to take the help of contradiction. Consider ϕ is unsatisfi able.

1. If there is a path from α to $\neg\alpha$ for a node α, then α must be assigned to false.
2. If there is no path from α to $\neg\alpha$ for a node α, then those nodes which are reachable from α must be assigned to false. [According to the rule that if there is an edge $\alpha \to \beta$, then there are clauses $(\neg\alpha \vee \beta)$ or $(\beta \vee \neg\alpha) in \phi$.]
3. Repeat this for all the nodes.

Now consider the previous graph. There is no edge from x_3 to $\neg x_3$.

If x_3 is true, then x_1 must be false, because this was added for the sub-clause $(x_1 \vee \neg x_3)$ in ϕ. There is an edge $x_1 \to x_2$ for the clause $(x_1 \vee x_2)$ in ϕ. If x_1 is true, then x_2 is false, which means x_3 must be false and $\neg x_3$ must be true.

There is an edge $(\neg x_2 \to x_3)$ for the clause $(x_2 \vee x_3)$ in ϕ. If x_2 is false, then x_3 must be false.

Here, we are getting the contradiction for x_3 and $\neg x_3$.

It proves that step (2) cannot exist if ϕ is unsatisfiable, which justifies that ϕ is unsatisfiable if and only if there is a variable x such that there are edges from x to $\neg x$ and $\neg x$ to x in G.

For a 2 SAT problem, if there are n variables, then the existence of an edge from x to $\neg x$ and $\neg x$ to x can be found within 2n steps. This signifies that 2 SAT is in P.

Prove that 3 SAT problem is in NP

Proof : A SAT problem ϕ is called 3 SAT if the number of literals in each clause is exactly 3. Let F be a CNF SAT problem. We have to convert F to a 3 CNF SAT problem F' such that if F is satisfiable, then F' is also satisfiable. Let the problem F contain clauses $C_1, C_2 \ldots . C_k$. Here, three cases of C may occur.

1. Ci contains exactly 3 literals.
2. Ci contains less than 3 literals.
3. Ci contains more than 3 literals.

We have to concentrate on cases (2) and (3).

Solution for Case (2): If Ci contains less than 3 literals, then the number of literals may be 2 or 1. If it contains 1 literal, say L_1, then replace this by $(L_1 \vee L_1 \vee L_1)$.