The following three examples in (d), (e), and (f) are not in CNF

d) $\neg(X1 \vee \neg X2)[NOTproblem]$
e) $\neg X2 \vee (X1 \wedge X3)(ANDproblem)$
f) $X1 \wedge (X2 \vee (X1 \wedge X3))(ORproblem)$

These three formulas can be converted to equivalent CNF as in (g), (h), and (i).

g) $\neg X1 \wedge X2$
h) $(\neg X2 \vee X1) \wedge (\neg X2 \vee X3)$
i) $X1 \wedge (X2 \vee X1) \wedge (X2 \vee X3)$

CNF is used in SAT problems for detecting conflict and to remember partial assignments that do not work.

(In the context-free grammar chapter, we learnt about the Chomsky normal form. Do not mess this CNF with the conjunctive normal form.)

2 SAT: A SAT problem is called 2 SAT if the number of literals in each disjunction (clause) is exactly 2.
3 SAT: A SAT problem is called 3 SAT if the number of literals in each disjunction (clause) is exactly 3.

SAT is in NP because any assignment of Boolean values to Boolean variables that are claimed to satisfy the given logical expression can be verified in polynomial time by a deterministic Turing machine. (Can be proved)

Prove that 2 SAT problem is in P
Proof: Let ϕ be an instance of 2 SAT. Generate a graph $G(\phi)$ by the following rules.
1. Vertices of the graph G are the variables and their negations. If the number of variables is n, then the number of nodes is 2n.
2. If there is a clause $(\neg\alpha \vee \beta)$ or $(\beta \vee \neg\alpha)$ in ϕ, then an edge $\alpha \to \beta$ is added in G.
3. If there is an edge from $\alpha \to \beta$ in G, then an edge $\neg\beta \to \neg\alpha$ is added in G.
4. If there is a clause $(\alpha \vee \beta)$ in ϕ, then two edges $\neg\alpha \to \beta$ and $\neg\beta \to \alpha$ are added in G.

Let us take an example $\phi = (x_1 \vee x_2) \wedge (x_1 \vee \neg x_3) \wedge (\neg x_1 \vee x_2) \wedge (x_2 \vee x_3)$.
1. There are three variables $x_1, x_2, andx_3$ in ϕ. Thus, the graph contains 6 nodes namely $x_1, \neg x_1, x_2, \neg x_2, x_3, and \neg x_3$.
2. $(x_1 \vee \neg x_3)$ is in the form $(\beta \vee \neg\alpha)$. Thus, an edge $x_3 \to x_1$ is added to G. According to the rule (c),an edge $(\neg x_1 \to \neg x_3)$ is also added to G.
3. By the same rule, $x_1 \to x_2$ and $(\neg x_2 \to \neg x_1)$ are also added to G.
4. $(x_1 \vee x_2)$ is in the form $(\alpha \vee \beta)$. Thus, two edges $(\neg x_1 \to x_2)$ and $(\neg x_3 \to x_1)$ are added to the graph.
5. By the same rule, $(\neg x_2 \to x_3)$ and $(\neg x_3 \to x_2)$ are added to the graph for $(x_2 \vee x_3)$.

ϕ is unsatisfi able if and only if there is a variable x such that there are edges from x to $\neg x$ and $\neg x$ to x in G.

Suppose such x exists. x may have any value between 0 and 1. Consider x = 1 and an edge x to $\neg x$. (Consider x as α and $\neg x$ as β.) It means there is a clause $(\neg x \vee \neg x)$ or $(\neg x \vee \neg x)$ (both are the same)in ϕ. The clause $(\neg x \vee \neg x)$ returns 0 if x = 1. This makes the total ϕ as 0 [because the sub-clauses are attached with $\wedge$] which means unsatisfiable.