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**Exercise 5.1** Write each symbolic sentence in two ways:

(i) without any symbol, apart from  $f$ .

(ii) with symbols only, using quantifiers.

1.    i The value of  $f$  at the origin is rational.  
      ii  $\exists r \in Q, f(0) = r$ .
2.    i The value of function  $f$  at each point is its corresponding point.  
      ii  $\forall x \in R, f(x) = x$ .
3.    i The function  $f$  is constant.  
      ii  $\exists y \in R, \forall x \in R, f(x) = y$ .
4.    i The value of function  $f$ , for all integers, is zero.  
      ii  $\forall x \in Z, f(x) = 0$ .
5.    i The zeros of  $f$  include all integers.  
      ii  $\forall x \in Z, f^{-1}(0) = x$ .
6.    i Both integers and natural numbers have the same image under  $f$ .  
      ii  $\forall x \in Z, \forall y \in N f(x) = f(y)$ .
7.    i The rational numbers have no rational value under  $f$ .  
      ii  $\nexists x \in Q, f(x) \in Q$

**Exercise 5.3** Write each symbolic sentence without symbols, apart from  $f$ .

1. The zeros of  $f$  include all even integers.
2. The function  $f$  is identically zero for negative values of the argument.
3. The composition of the function  $f$  with itself is the Identity function.
4. The function  $f$  is non-zero at the closed interval of zero to one.
5. The value of function  $f$  is not rational for any natural number.