Exercise 2.1

For each of the following topics:

prime numbers, fractions, complex numbers,

(i) write five short sentences; (ii) ask five questions. The sentences should give a definition or state a fact; the questions should have mathematical significance, and preferably possess a certain degree of generality. (complete without using any mathematical symbol.)

Bad: Is 41 a prime number? [Specific and insignificant.]
Good: Why is 1 not a prime number?
Good: Is a fraction the same as a rational number?
Bad: What is the real part of $2 + 3i$?
Good: What is the real part of i^i?

Exercise 2.2

Define five interesting finite sets.

Bad: The set of natural numbers less than 10.

Good: The set of the proper subsets of a finite set.

Exercise 2.3

The following expressions define sets. Turn words into symbols, using standard or Zermelo definitions. (Represent geometrical objects, e.g., planar curves, by their cartesian equations.)

1. The set of negative odd integers.
2. The set of natural numbers with three decimal digits.
3. The set of rational numbers which are the ratio of consecutive integers.
4. The set of rational points in the closed unit cube.
5. The complement of the open unit disc in the complex plane.
6. The set of vectors of unit length in three-dimensional euclidean space.
7. The set of circles in the plane, passing through the origin.
8. The set of hyperbolae in the plane, whose asymptotes are the coordinate axes.
9. The set of lines tangent to the unit circle.

Exercise 2.4

The following expressions define sets. Turn symbols into words.

1. $\{x \in \mathbb{Q} \ : \ 0 < x < 1\}$
2. $\{1/(2n+1) \ : \ n \in \mathbb{Z}\}$
3. $\{m2^{-k} \ : \ m \in 1 + 2\mathbb{Z}, \ k \in \mathbb{N}\}$
4. $\{x \in \mathbb{R} \backslash \mathbb{Z} \ : \ x^2 \in \mathbb{Z}\}$
5. $\{z \in \mathbb{C} \backslash \mathbb{R} \ : \ z^2 \in \mathbb{Z}\}$
6. $\{z \in \mathbb{R} \ : \ |Re(z)| + |Im(z)| \leq 1\}$
7. $\{(m, n) \in \mathbb{Z}^2 \ : \ m|n\}$
8. $\{(x, y, z) \in \mathbb{R}^3 \ : \ xyz = 0\}$
9. $\{(x_1, ..., x_n) \in \mathbb{R}^n \ : \ \sum_k x_k = 0\}$
10. $\{x \in \mathbb{R} \ : \ \sin(2\pi x) = 0\}$