Mathematical Writing **Exercise VIII**
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Exercise 7.1 You are given cryptic proofs of mathematical statements. Rewrite them in a good style, with plenty of explanations.

1. A point on the first line is represented by the position vector

$$l(\lambda) = (4, 5, 1)^T + \lambda(1, 1, 1)^T = (4 + \lambda, 5 + \lambda, 1 + \lambda)^T$$

for some real number λ. Similarly, a point on the second line has position vector

$$m(\mu) = (5, 4, 0)^T + \mu(2, 3, 1)^T + (5 + 2, 43,)^T$$

for some real μ. For the lines to intersect, there must exist values of λ and μ for which the two position vectors are the same, namely

$$l(\lambda) = m(\mu)$$

The above vector equation corresponds to three scalar equations:

$$4 + \lambda = 5 + 2\mu$$

$$5 + \lambda = -4 - 3\mu$$

$$1 + \lambda = \mu$$

Eliminating μ from equations, we obtain

$$4 + \lambda = 5 + 2(1 + \lambda)$$

which yields the solution $\lambda = 3$, $\mu = 2$. This solution satisfies all three equations. Substituting these values in first two equations, we obtain

$$l(3) = m(2) = (1, 2, 2)^T$$

which is the position vector of the common point of the two lines.

2. A point where the function assumes its minimum or maximum value is where its derivative equals zero. Also if the second derivative of a function at the corresponding point is positive or negative, the function is curved outward or invard respectively. The derivative function of f is

$$f(x) = 12x(x2 + x + 1)$$

and the only root of it is 0. the second derivative of f is always positive which implies that function f is curved outward and its minimum is at point 0. by evaluating the value of f at 0, we find out that is is positive. therefore we made sure that the function never assumes negative values and is always positive.

3. We have the line equation as

$$l(x) : y = ax - (\frac{a-1}{2})^2$$

and the parabola equation as

$$p(x) : y = x^2 + x$$

In order to assert whether the line is tangent to the parabola, we need to check two contraints:
at a given point x_0:

$$(i) l(x_0) = p(x_0)$$

$$(ii) l^{'}(x_0) = p^{'}(x_0)$$

by solving the first equation with some straight forward calculations we find that $x_0 = \frac{a-1}{2}$ and x_0 also satisfies the second equation which involves the second derivatives. Eq.(i) implies that line and parabola intersect at x_0 and Eq.(ii) implies that their slopes are equal, therefore they are tangent.

Exercise 7.2 The following text has several faults: (a) explain what they are; (b) write an appropriate revision.

(a) The statement of the theorem is imprecise in several respects.
The nature of the numbers x and y is not specified (the inequality would be meaningless for complex numbers). The case in which one of x or y is zero should be excluded, since in this case the left-hand side of the inequality is undefined. The statement is false unless the inequality is made non-strict; indeed the equality holds for infinitely many values of x and y.
There are several flaws in the proof
The basic deduction is carried out in the wrong direction, which proves nothing. (Proving that P True gives no information about P.) The assertion 'the last equation is trivially true' is, in fact, false for $x = y$. The writing is inadequate, without sufficient explanations, and also imprecise (the expression $(xy)2 > 0$ is an inequality, not an equation).

(b) Revised statement: Theorem: For all nonzero real numbers x and y, the following holds:

$$\frac{x^2 + y^2}{|xy|} \geq 2$$

Proof: Let x and y be real numbers, with $xy = 0$. We shall deduce our result from the inequality $(xy)^2 \geq 0$. We begin with the chain of implications:

$$(xy)^2 \geq 0x^2 2xy + y^2 \geq 0x^2 + y^2 \geq 2xy$$

, Now, since xy is non-zero, we divide both sides of the rightmost inequality by —xy—, to obtain

$$\frac{x^2 + y^2}{|xy|} \geq 2\frac{xy}{|xy|}$$

The expression $xy/|xy|$ is equal to 1 or 1, and by choosing an appropriate sign we can ensure that $xy/|xy| = 1$. Our proof is complete.

Exercise 7.4 Write the
first few sentences of the proof of each statement, introducing all relevant notation in an appropriate order, and identifying the RTP.

1. Let X be a compact set, and let f be a continuous function over X. RTP: f is uniformly continuous.
2. Let F be a field, and let V be a valuation of f with prime charachteristics. RTP: V is non-archimedean.
3. Let f(x) be a polynomial with integer coefficients, and assume that f is not a constant. RTP: There is an integer n such that f(n) is not prime.
4. Let X:[a,b] be a closed interval, and assume f(x) be a real function continuous in X. RTP: for all points x in X, f(x) is between maximum and minimum of f.
5. Let X be a subset of a metric space, and assume $X^{'}$ be its complement. RTP: (i) We prove that X is open if $X^{'}$ is closed.
(ii) We prove that $X^{'}$ is closed if X is open.