# Exercise 2.1

#### • Prime Numbers

- 1. There are 25 prime numbers less than 100.
- 2. The only even prime number is 2.
- 3. No prime number greater than 5 ends with a 5.
- 4. The set of prime numbers is infinite.
- 5. The largest known prime number as of now is  $2^{82,589,933}$  with 24,862,048 digits.
- 6. How do you know if a number is prime?
- 7. Why is it important to find the largest prime number?
- 8. How do you prove a number is prime?
- 9. Is there a largest prime number?
- 10. Why is 1 not a prime number?

#### • Fractions

- 1.  $\frac{9/10}{den}$  is greater than  $\frac{8}{9}$ .
- 2. An improper fraction is always 1 or greater than 1.
- 3. A fraction is a numerical quantity that is not a whole number.
- 4. A fraction is proper if the numerator is less than the denominator.
- 5. Any integer number can be shown as a fraction.
- 6. What is the difference between a fraction and a rational number?
- 7. What are the equivalent fractions of  $\frac{2}{3}$ ?
- 8. How to solve fractions?
- 9. Are fractions always less than 1?
- 10. How can you tell which fraction is greater?

### • Complex Numbers

- 1. The 16th century Italian mathematician Gerolamo Cardano introduced complex numbers in his attempts to find solutions to cubic equations.
- 2. The complex numbers cannot be ordered since the square of the imaginary unit i is 1.
- 3. You can't compare two complex numbers in some cases.
- 4. The product of two imaginary numbers is always a negative real number.
- 5. The set of integers is closed under addition, multiplication, and exponentiation, but not division.
- 6. Is 2i a complex number?
- 7. Can complex numbers be ordered?
- 8. Are all numbers complex?
- 9. Can we compare two complex numbers?
- 10. What is the real part of  $i^{i+5}$ ?

# Exercise 2.2

- 1. The set of the trivial subsets of a finite set.
- 2. The set of solutions of a quadratic equation.
- 3. The set of moons around planet mars.
- 4. The set of drops of water in ocean.
- 5. The set of sand grains on land.

## Exercise 2.3

- 1.  $\{2z+1: z \in Z^-\}$
- $2. \ \{n \in N : 99 < n < 1000\}$
- 3.  $\{\frac{x+1}{x} : x \in Z^*\}$
- 4.  $\{(x, y, z) \in Q^3 : x^2 + y^2 + z^2 \le 1\}$
- 5.  $\{z \in C : |z| \ge 1\}$
- 6.  $\{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\}$
- 7.  $\{(x-a)^2 + (y-b)^2 = R^2 : a^2 + b^2 = R^2\}$
- 8.  $\{y = \frac{k}{x} : k, x, y \in R\}$
- 9.  $\{ax + by + 1 = 0 : a,b \text{ values result in tangency.}\}$

### Exercise 2.4

- 1. The set of rational points in the open unit interval.
- 2. The set of reciprocals of odd integers.
- 3. The set of rationals whose numerator is odd and its denominator is a power of 2.
- 4. The set of real roots of integer numbers excluding integers.
- 5. The imaginary axis in the complex plane, excluding the origin.
- 6. The set of complex numbers where sum of real and imaginary parts is less than 1.
- 7. The set of integer pairs whose first component divides the second.
- 8. The set of vectors in the three-dimensional space where at least one coordinate is zero.
- 9. The set of points in a euclidean space whose coordinates have zero sum.
- 10. The set of all integers divided by 2.