Mathematical Writing
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Exercise V
Pouya Aghahoseini

Exercise 1 Rewrite each symbolic sentence using the quantifier $\exists$.

1. $Y \not\supseteq X \Leftrightarrow \exists x \in X, x \notin Y$
2. $\#X > 1 \Leftrightarrow \exists x, y \in X, x \neq y$
3. $X \bigcap Y \neq \emptyset \Leftrightarrow \exists x \in X, y \in Y, x = y$
4. $z \in X \times Y \Leftrightarrow \exists z = (x, y), x \in X, y \in Y$
5. $X \neq Y \Leftrightarrow (\exists x \in X, x \notin Y) \wedge (\exists y \in Y, y \notin X)$
6. $\#(X \triangle Y) > 0 \Leftrightarrow (\exists x \in X, x \notin Y) \wedge (\exists y \in Y, y \notin X)$

Exercise 2 Write each sentence with symbols, using at least one quantifier.

1. The integer n is a cube. $\Leftrightarrow \exists k \in Z, n = k^3$
2. The equation $f(x) = 0$ has a rational solution. $\Leftrightarrow \exists x \in Q, f(x) = 0$
3. The fraction $\frac{a}{b}$ is not reduced. $\Leftrightarrow \nexists k \in Z, k|a, k|b.$
4. The unit circle has a rational point. $\Leftrightarrow \exists x, y \in Q|x^2 + y^2 = 1.$
5. The polynomial $P(x)$ has no integer roots. $\Leftrightarrow \nexists x \in Z|P(x) = 0.$

Exercise 3 The following expressions define sets; turn symbols into words.

1. $\{x \in N : \forall m, n \in N, (x|mn) \Rightarrow (x|m \vee x|n)\}$
   The set of prime numbers, together with the integer 1.
2. $\{x \in N : \forall m \in N, x|m^2 \Rightarrow x|m\}$
   The set of natural numbers, except squres.
3. $\{x \in N : \forall m \in N, x|m^3 \Rightarrow x|m^2\}$
   The set of natural numbers, except cubes.

Exercise 4 Decide if each sentence is true or false, hence write its negation with symbols. Then rewrite both with words.

1. $\forall n \in N, \frac{1}{n} \notin N$
   (False.) $\exists n \in N, \frac{1}{n} \in N$
   The reciprocal of a natural number is not a natural number.
   There is a natural number whose reciprocal is also a natural number.
2. $\forall n \in N, \sqrt{n} \in R \setminus Q$
   (False.) $\exists n \in N, \sqrt{n} \in R \setminus Q$
   All natural numbers square roots are irrational.
   There is a natural number whose square roots is irrational.

3. $\forall n \in Z, 2|n(n+1)$
   (True.) $\forall n \in Z, 2 \nmid n(n+1)$
   The product of each consecutive pair of integers is even.
   There exist a pair of consecutive integers which their product is odd.

4. $\forall n \in N, \exists r \in Q, n < r^2 < n+1$
   (True.) $\forall n \in N, \forall r \in Q, n < r^2 < n+1$
   There exists a square of a rational number between two consecutive natural numbers.
   The square of all rational numbers are between two consecutive natural numbers.