Exercise 5.1 Write each symbolic sentence in two ways:
(i) without any symbol, apart from f.
(ii) with symbols only, using quantifiers. (You may assume that $f : \mathbb{R} \to \mathbb{R}$.)

1. $f(0) \in \mathbb{Q}$

(i) The value of f at the origin is rational.
(ii) $\exists r \in \mathbb{Q},\ f(0) = r$.

2. $f(\mathbb{R}) = \mathbb{R}$
3. $\#f(\mathbb{R}) = 1$
4. $f(\mathbb{Z}) = \{0\}$
5. $f^{-1}(\{0\}) = \mathbb{Z}$
6. $f(\mathbb{Z}) = f(\mathbb{N})$
7. $f(\mathbb{Q}) \cap \mathbb{Q} = \emptyset$.

Exercise 5.3 Write each symbolic sentence without symbols, apart from f .

1. $\forall x \in \mathbb{Z},\ f(2x) = 0$

The zeros of f include all even integers.

2. $\forall x \in \mathbb{R}^+,\ f(-x) = 0$

The function f is identically zero for negative values of the argument.

3. $\forall x \in \mathbb{R},\ f(f(x)) = x$
4. $\forall x \in [0, 1],\ f(x) \neq 0$
5. $\forall x \in \mathbb{N},\ f(x) \notin \mathbb{Q}$